Bonn Econ Discussion Papers

Discussion Paper 13/2000

Group size and free riding when private and public goods are gross substitutes

by

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May 2000

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Group size and free riding when private and public goods are gross substitutes

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May 2000
(First version: December 1999)

Abstract

Using the traditional model of voluntary public good provision, it is shown that an expansion of group size exacerbates free riding tendencies as long as private consumption and the public good are strictly normal and weak gross substitutes. This result generalizes a previous Cobb-Douglas example with respect to preferences and asymmetric equilibria.

Keywords: private provision of public goods, group size

JEL-Classification: H 41

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I would like to thank Gábor Gyárfás and Christoph Lülfesmann for helpful comments. Financial support by the Deutsche Forschungsgemeinschaft, SFB 303 at the University of Bonn is gratefully acknowledged. The usual disclaimer applies.

1 Introduction

Private donations to public goods are subject to a free-rider problem. Following Olson (1965), it is often argued that this problem becomes worse as group size increases. Within the traditional model of voluntary public good provision, this popular claim has been illustrated by means of an example where all households are identical and preferences are Cobb-Douglas.[1] General results, however, are not available. Although most authors suggest the existence of counterexamples, no such example has been presented so far.[2] It is thus not clear which conditions are necessary and/or sufficient for the general presumption to hold.

This note is meant as a first step towards clarifying this question. It shows that the conventional claim is correct if private consumption and the public good are strictly normal and weak gross substitutes for all households. This finding is not restricted to identical individuals and does therefore generalize the previous Cobb-Douglas example not only with respect to preferences, but also with respect to asymmetric equilibria.

2 The model

The analysis is based on the orthodox paradigm for modelling the private supply of public goods. Following the approach of Fries et al. (1991), changes in the economy's size are represented by means of sequential replications of an initial economy which consists of a finite number of individuals (households) $i = 1, ..., I$. They may differ with respect to preferences and endowments. The distribution of these characteristics, however, does not change if $(n-1)$ replicas of each type enter the economy, such that total number of households grows to nI.

The individuals have initial endowments ω_i which can be used for either consuming x_i units of a composite private commodity or providing g_i units to the public good G. The marginal rate of transformation between the two goods is assumed to be constant and is normalized to unity. Fixing the commodity prices to $p_G = p_x = 1$, an individual of type i thus faces the budget constraint $\omega_i - x_i - g_i = 0$. The utility functions $U_i = U_i(x_i, G)$ are assumed to be strictly monotone, twice continuously differentiable, and strictly quasiconcave. When maximizing these functions subject

[1] See Laffont (1988, p. 39), Mueller (1989, p. 21), and Sandler (1992, p. 52). For another example with quasilinear preferences, see section 3 below.

[2] The claim that counterexamples exist is based on a graphical analysis presented by Cornes and Sandler (1996, p. 161). This analysis refers to an individual's response function and does not specify which type of preferences would generate the result.

to the budget constraint, the individuals take the total quantity G_{-i} provided by all other households as given[3]. Following the full-income analysis of Bergstrom et al. (1986, p. 32), the households' responses can then be written in the form

$$g_i = \max \{\gamma_i (\omega_i + G_{-i}) - G_{-i}, 0\}, \tag{1}$$

where $\gamma_i(\cdot)$ denote the individuals' demand functions for the public good. It is assumed that the two commodities x_i and G are strictly normal for all households. This implies $0 < \gamma_i' < 1$ and ensures existence and uniqueness of the Nash equilibrium $(g_1^N(n), ..., g_I^N(n))$.[4] Let $C(n)$ denote the set of contributing types, i.e. of those individuals who choose $g_i^N(n) > 0$ in the economy of size nI. Then the aggregate private supply can be written as follows:

$$G^N(n) := n \left(\sum_{i \in C(n)} g_i^N(n) \right).$$

This note is concerned with the comparison between $G^N(n)$ and a Pareto efficient provision level $G^*(n)$. The quantity $G^*(n)$, however, is not uniquely defined because it may depend on the distribution of the first-best utility levels. Since the results of this paper are based on a comparative statics analysis with respect to n, one of the first-best allocations has to be chosen. This choice is described here by fixing a vector of lump-sum taxes (transfers) τ_i, where $n(\sum_{i=1}^I \tau_i) = 0$. The first-best allocation is then defined by means of the Lindahl equilibrium which corresponds to the endowments $\tilde{\omega}_i := \omega_i - \tau_i$. While any positive vector $(\tilde{\omega}_1, ..., \tilde{\omega}_I)$ can be selected, the taxes τ_i are assumed to remain constant if the economy's size n changes.[5]

In Lindahl's model, the households pay personalized prices $p_i(n)$ for the public good and maximize utility subject to the constraint $x_i + p_i(n)G_i = \tilde{\omega}_i$, where p_x is still normalized to unity. This leads to the demand functions $G_i(p_i(n), \tilde{\omega}_i)$ and $x_i(p_i(n), \tilde{\omega}_i)$. In equilibrium, prices then have to satisfy the restrictions $n(\sum_{i=1}^I p_i(n)) = p_G = 1$ and $G_1(p_1(n), \tilde{\omega}_1) = ... = G_I(p_I(n), \tilde{\omega}_I)$. The quantity

$$G^*(n) := G_1(p_1(n), \tilde{\omega}_1) = ... = G_I(p_I(n), \tilde{\omega}_I)$$

is Pareto efficient and can thus be used for investigating the relationship between the economy's size and the households' tendencies for free riding.

[3]Note that G_{-i} does not only encompass the donations g_j of the types $j \neq i$, but also the donations g_i of the other $(n-1)$ individuals of type i.

[4]See Bergstrom et al. (1986, 1992) and, for a more general treatment, Cornes et al. (1999). Note that uniqueness of the Nash equilibrium implies symmetry among identical individuals. Hence, the equilibrium quantities $g_i^N(n)$ in an economy of size n are the same for all households of type i.

[5]Since all Pareto efficient allocations can be decentralized as a Lindahl equilibrium, the analysis allows us to choose any first-best allocation of an economy with size n. However, if this choice is made, the first-best allocation of the subsequent economy with size $n+1$ is determined as well.

3 Group size and free riding

Free riding leads to under-provision of the public good. This means that $G^*(n) - G^N(n) > 0$ or, equivalently, that $G^*(n)/G^N(n) > 1$. Both the theoretical and experimental literature usually take the relation $G^*(n)/G^N(n)$ as a measure of under-provision.[6] Hence, the claim that an expansion of group size exacerbates free riding tendencies is commonly translated into the hypothesis that the function $G^*(n)/G^N(n)$ is monotonically increasing in n.[7]

So far this hypothesis has been illustrated by means of two examples of a representative consumer economy. The first example can be found in Cornes and Sandler (1996). They assume quasilinear preferences $U(x_i, G) = x_i + f(G)$, where $f(G)$ is strictly concave. Hence G is neutral and $G^N(n)$ is independent of n (see McGuire, 1974). However, if G is neutral, Nash equilibria are not unique.[8] For this reason, the present analysis rules out the extreme case of zero income effects and assumes that both commodities are strictly normal. This assumption is satisfied by another example which can be found in Laffont (1988), Mueller (1989), and Sandler (1992). They use Cobb-Douglas preferences $U(x_i, G) = x_i^{\alpha} G^{1-\alpha}$ and show that $G^N(n)$ increases in n, but at a lower rate than the efficient quantity $G^*(n)$. However, the question remains whether this example illustrates just a possible outcome, or a general property of the model. In particular, it is not clear whether the assumption of symmetric equilibria is important for the result (see Sandler (1992), p. 194). The subsequent analysis is meant as a first step towards clarifying these questions. It shows that weak gross-substitutability of the two commodities (i.e. the assumption $\partial x_i(\cdot)/\partial p_i \geq 0$, $i = 1, ..., I$) is sufficient for a positive relationship between group size and the degree of free riding. Since Cobb-Douglas preferences imply $\partial x_i(\cdot)/\partial p_i = 0$, this finding generalizes the earlier example both with respect to preferences and asymmetric equilibria.

Proposition: *If the commodities x_i and G are strictly normal and weak gross sub-*

[6] This is because the effect of a variation in n can more easily be interpreted in terms of the per-capita quantities $G^*(n)/(nI)$ and $G^N(n)/(nI)$ than in terms of the aggregate quantities $G^*(n)$ and $G^N(n)$. Moreover, the relation $G^*(n+1)/G^N(n+1) > G^*(n)/G^N(n)$ implies $G^*(n+1) - G^N(n+1) > G^*(n) - G^N(n)$, but not vice versa.

[7] For a discussion of this issue, see Cornes and Sandler (1984, 1996), Sandler (1992), Olson (1992), and Pecorino (1999). With respect to the experimental (empirical) literature, see Isaac and Walker (1988), Isaac et al. (1994), and Lipford (1995). Since contributors do not 'free ride' in the literal sense, Cornes and Sandler (1984, 1996) call $G^*(n)/G^N(n)$ the 'index of easy riding'.

[8] Neutrality of G implies that the slope of an individual's response function is equal to -1. Hence, at least in the case of identical individuals, a continuum of Nash equilibria exists.

stitutes for all households, then

$$\frac{G^*(n+1)}{G^N(n+1)} > \frac{G^*(n)}{G^N(n)}, \quad \forall\, n \geq 1.$$

The intuition behind this result is straightforward: if n increases, prices do not change in the Nash-equilibrium. Hence, n affects $G^N(n)$ only through income effects which occur because some of the entrants donate to the public good. In first best, however, a change of n leads to a variation of the implicit (Lindahl-) prices imposed on the households. The relation between n and $G^*(n)$ is thus determined via price effects on the households' demand for the public good. As long as the two commodities are normal and gross substitutes, these price effects dominate the income effects. Therefore, $G^*(n)/G^N(n)$ has to increase monotonically in n.

The proof of the proposition is immediate from the following two lemmas which confirm the intuition given above. The first lemma shows that the normality assumption restricts the income effect underlying the relationship between n and $G^N(n)$ in such a way that $G^N(n)$ grows at a lower rate than n.

Lemma 1: *If the commodities x_i and G are strictly normal for all households, then*

$$\frac{G^N(n+1)}{G^N(n)} < \frac{n+1}{n}, \quad \forall\, n \geq 1.$$

Proof: The proof proceeds in two steps. First, it is shown that an increase of the aggregate supply G^N corresponds to a decrease of the individual donations g_i^N. This preliminary finding is then used to prove the result. Consider a contributing household of type $i \in C(n)$. Because of $g_i = \gamma_i\left(\omega_i + G_{-i}\right) - G_{-i}$ and $G_{-i} = G - g_i$, we have $G = \gamma_i\left(\omega_i + G - g_i\right)$. Total differentiation of this relationship gives

$$\frac{dg_i}{dG} = \frac{\gamma_i' - 1}{\gamma_i'} < 0, \tag{2}$$

where the inequality follows the assumption that x_i and G are strictly normal commodities. (For a similar finding with respect to weak normality of x_i, see Lemma 2 of Fries et al. (1991).) Now assume that the claim is not correct, which means

$$G^N(n+1) \geq \frac{n+1}{n} G^N(n). \tag{3}$$

This implies $G^N(n+1) > G^N(n)$. Because of equation (2), we thus have $g_i^N(n+1) < g_i^N(n)$ for all $i \in C(n)$ and $g_i^N(n+1) = g_i^N(n) = 0$ for all $i \notin C(n)$. Hence, $C(n+1) \subseteq C(n)$ and

$$G^N(n+1) = (n+1)\left(\sum_{i \in C(n+1)} g_i^N(n+1)\right) < (n+1)\left(\sum_{i \in C(n)} g_i^N(n)\right).$$

Since the expression on the right hand side of the inequality is equal to $(n+1)(1/n)G^N(n)$, this contradicts (3). ■

While Lemma 1 specifies an upper-bound with respect to the relative change of $G^N(n)$, the following result shows that $G^*(n)$ grows at least at the same rate as n as long as x_i and G are weak gross substitutes. Taken together, these two findings prove the proposition stated above.

Lemma 2: *If the commodities x_i and G are weak gross substitutes for all households, then*

$$\frac{G^*(n+1)}{G^*(n)} \geq \frac{n+1}{n}, \quad \forall\, n \geq 1.$$

Proof: Consider an arbitrary first-best allocation of an economy with size n. Since this allocation can be implemented by means of personalized (Lindahl-) prices $p_i(n)$ with $n\sum_{i=1}^{I} p_i(n) = 1$, we have $G^*(n) = G_i(p_i(n), \tilde{\omega}_i)$ and $x_i^*(n) = x_i(p_i(n), \tilde{\omega}_i)$ for all i. If the economy's size grows to $(n+1)$ the sum of personalized prices stays constant, which means $(n+1)\sum_{i=1}^{I} p_i(n+1) = n\sum_{i=1}^{I} p_i(n)$. Hence, there must be at least one type $j \in \{1, ..., I\}$ of households, such that

$$\frac{p_j(n+1)}{p_j(n)} \leq \frac{n}{n+1}. \tag{4}$$

This implies $p_j(n+1) < p_j(n)$. Since x_j and G are weak gross substitutes, we thus have $x_j^*(n+1) \leq x_j^*(n)$. (Note that this holds irrespective of whether the type j chooses an interior solution $x_j^*(n) > 0$ or a boundary solution $x_j^*(n) = 0$.) Using the household's budget constraint, this implies $p_j(n+1)G^*(n+1) \geq p_j(n)G^*(n)$. Hence,

$$\frac{G^*(n+1)}{G^*(n)} \geq \frac{p_j(n)}{p_j(n+1)} \geq \frac{n+1}{n}$$

where the second inequality follows from (4). ■

4 Conclusion

This note shows that an expansion of group size exacerbates free riding tendencies of the groups' members if private consumption and the public good are normal and gross substitutes. This result generalizes a previous Cobb-Douglas example and points to the direction one has to look for potential counterexamples. However, the question whether such a counterexample can indeed be constructed remains unresolved.

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