Bonn Econ Discussion Papers

Discussion Paper 37/2005

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by

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December 2005

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Financing of Competing Projects with Venture Capital*

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December 2005

*For valuable suggestions we are especially grateful to Avner Shaked and Frank Riedel. Further we would like to thank Stefan Amber, Ronald W. Anderson, Jan Bena, Dirk Engelmann, Andreas Ortmann, Jean Tirole, Viatcheslav Vinogradov, and the participants at the BGSE Workshop, 20th EEA Annual Congress, and 32nd EARIE Annual Conference for helpful comments. The financial support of the Grant Agency of the Czech Academy of Science is gratefully acknowledged. All errors are ours.

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Abstract

We analyze innovation race in a moral hazard setting. We develop a model in which two competing entrepreneurs work independently on the same project. The entrepreneurs do not possess any wealth of their own and their research is financed by a venture capitalist. The project, if successful, generates a prize, which is to be shared between the winning entrepreneur and the venture capitalist. The venture capitalist cannot observe the allocation of funds he provides, which creates a moral hazard problem. We compare a competitive setting with a benchmark case where the venture capitalist finances only one entrepreneur. We show that the venture capitalist can increase the efficiency of research (hence, his own expected profit from investments) and alleviate the moral hazard problem if he finances both entrepreneurs. This conclusion is unambiguous, when the entrepreneurs are at the same (the last) stage of R&D. It holds for a reasonably large range of parameters, when the entrepreneurs are at different stages of R&D.

Keywords: venture capital, moral hazard, optimal contract, innovation race
JEL Classification: G32, G34, O31

1 Introduction

One of the most important problems in venture capital financing is the separation of financing decisions, made by a venture capitalist, and allocation decisions, made by recipient firms or entrepreneurs. Venture capital funds are usually directed to projects of uncertain quality, where neither time nor financial recourses needed for successful completion of the project are known ex ante. As a rule, venture capitalists are actively involved in monitoring firms in their portfolio. Nevertheless, they can rarely control perfectly whether resources are allocated efficiently, since such control requires an expertise which often only an entrepreneur himself possesses. This creates a moral hazard problem: entrepreneurs tend to misallocate the funds provided by the venture capitalist. In particular, they either may divert part of the funds for their own uses, or may allocate them into activities that have high personal return but create little market value.

The venture capital literature has extensively discussed contractual arrangements which can be used by a venture capitalist in order to alleviate the moral hazard problem. These are, for example, convertible securities (Kaplan and Stromberg 2003), monitoring mechanisms (Gompers 1995), and stage financing (Bergemann and Hege 2000). Those mechanisms are efficient in mitigating the agency conflict. However, they are costly and complicated, which creates obstacles for efficient funding of research and development. In this paper, as opposed to the existing literature on venture capital, we investigate how a non-contractual mechanism, namely competition between firms in the portfolio of the venture capitalist, can be used to mitigate the agency conflict. The main question which we address is whether a venture capitalist can increase the efficiency of R&D and his own profit by creating competition between his portfolio entrepreneurs. Although we formulate the problem in terms of venture capital financing, it is of equal importance for grant agencies and for R&D process within the firms.

Casual empirical evidence suggests that venture capital funds and similar institutions sometimes indeed finance R&D race between competitive firms. It is also not unusual for grant agencies to finance competing research projects. For example, from 38,000 projects, the National Institute of Health proposed to grant and support in its 2006 budget, more than a quarter should be developed by competing teams.[1] Similarly, Vulcan Inc., a multi-division corporation owned by Microsoft co-founder Paul Allen, has contracted three competing agencies for the project Halo, aimed at the development of the problem-solving software.[2]

In this paper we investigate conditions, which make the financing of competing projects desirable. An obvious inefficiency created by competing projects is du-

[1]See the report of the National Institute of Health (2005).

[2]See `www.projecthalo.com` for details.

plication of efforts. However, competition also allows to increase the probability of success, since two entrepreneurs succeed more often, than one ("scale" effect of competition). Moreover, we argue that in a moral hazard setting competition is efficient in mitigating the agency problem, since it disciplines entrepreneurs and limits the rent which they can extract from the venture capitalist ("disciplining" effect of competition). Hence, competition is a beneficial arrangement from the venture capitalist's point of view, if its positive effects outweighs the duplication of R&D costs.

Analyzing the innovation race between competing entrepreneurs we consider a research process which consists of two sequential stages, which are observable and verifiable outcomes of R&D, such as patent or results of a test. This structure allows us to investigate the effect of competition between entrepreneurs which are at the same or at different stages of research. In the former case we focus on for situation where both entrepreneurs are on the last stage of R&D. We conclude, that in this situation the competition is unambiguously beneficial for the venture capitalist: he will always prefer to employ two entrepreneurs. In the latter case, one of the entrepreneurs is a leader (i.e., he is at the final stage of research) and the other is a follower (i.e., he is at the initial stage of research). In this case we also conclude that competition is an effective cure against moral hazard: it disciplines agents, by making diversion of funds less lucrative, than in the absence of competition, and increases the research horizon. We show that without moral hazard the venture capitalist would almost always finance only the most advanced entrepreneur. With moral hazard in place, however, the range of parameters where competition is beneficial increases significantly.

This paper is related to two strands of literature: the literature on innovation races and the literature on venture capital. It contributes to the former by considering innovation races in the moral hazard setting and to the latter by addressing the financing of competing projects.

The model which we develop in this paper shares several features with the literature on innovation races, such as costly experimentation framework and "learning-while-investing." However, majority of models of innovation race consider the investment decisions of rival firms that are modelled as independent entities, which carry all costs of R&D and expropriate the whole profit from the investment (Lee and Wilde 1980, Reinganum 1981, Choi 1991). We contribute to the existing literature by addressing innovation race in the moral-hazard setting. Namely, we consider a situation in which the financing decisions (made by the venture capitalist) and the allocation decisions (made by the firm or entrepreneur) are decentralized. Decentralization creates a moral hazard problem, since the entrepreneurs face a temptation to divert funds for private consumption, rather than to invest them in research and development.

In the literature on venture capital financing, we are aware of few papers that investigate the effect of competition between the portfolio firms of the venture capitalist. Inderst and Munnich (2003) consider the decisions of a wealth-constrained venture capitalist in a static framework. Portfolio firms in their model do not compete on the product market, but they do compete for the funds. The authors show that this setting improves the ability of the venture capitalist to deal with the agency problem. As opposed to Inderst and Munnich (2003), we consider the entrepreneurs, who compete for the introduction of a new product. Therefore, the setup of our model is close to the setup of an innovation race. We study the decision of the venture capitalist and the entrepreneurs in a dynamic framework, where in each period of time the entrepreneurs have to experiment (that is, to invest in research and development) in order to achieve success. The dynamic setting allows us to investigate how reward of the entrepreneurs and decisions of venture capitalist depend on time and the position, which each entrepreneur has in the race.

Levitt (1995) analyzes a problem of a principal whose payoff depends on the best of agents' outputs. Unlike in our model, the author investigates a static situation, which does not allow to consider a competition between the leader and the follower or to investigate how the terms of contract change with the number of completed stages. Likewise, the framework of the model does not allow the author to make any concrete conclusions in case when the production technologies of agents are independent. We address these issues in our model.

Our approach is closely related to that of Bergemann and Hege (1998, 2000, 2002). They investigate decision of a venture capitalist who finances a single entrepreneur under uncertainty about the quality of the project and investments needed for its successful realization. Bergemann and Hege (1998) analyze a model in which the quality of a project is not known and has to be resolved through a costly experiment. Their main result is that agency costs lead to inefficiently early stopping of the project. In their second model Bergemann and Hege (2000) extend these results and show the difference between relationship financing and arm-length financing. Finally, in the third model Bergemann and Hege (2002) investigate the value of staged financing. The authors show that use of financing rounds (stages) allows the increase of the funding horizon and makes it closer to the socially optimal horizon.

We use the framework of the Bergemann and Hege models and introduce competition between entrepreneurs. We show that competition has a disciplining effect and mitigates the agency problem. We argue that in the presence of competition the venture capitalist is willing to finance the research and development longer, than without competition. Finally, we develop the optimal contract for the case of competing entrepreneurs and investigate how the terms of the contract depend on the costs of R&D and on the probability of success. Our results hold in finite time and to justify this assumption we discuss the benefits of commitment to a finite research horizon and the commitment mechanism, which is relevant for venture capital-like institutions.

The structure of this paper is the following. We describe the set-up of the model in Section 2 and derive the sequentially optimal contract in Sections 3 and 4. We introduce strategic interaction among entrepreneurs in Section 5 and discuss the advantage of commitment to finite horizon in Section 6. Section 7 concludes. Proofs and results of numerical simulations can be found in Appendix A.

2 Description of the model

2.1 The model

There are two entrepreneurs with no wealth of their own. Both have an idea (a project) how to solve a particular problem. For example, they try to find a cure against a disease. Following Bergemann and Hege (2002), we assume that for a successful completion of the project, each entrepreneur has to complete N sequential stages. These stages are observable and verifiable outcomes, such as a patent, first version of a product, results of markets tests, etc. The stages are sequential in the sense that in order to enter in the k-th stage each entrepreneur has to finish successfully $(k-1)$ previous stages. Financing of the projects is done by venture capitalist who provides necessary funds. If all stages are completed, the project generates a prize R and the prize is to be divided between the venture capitalist and the winning entrepreneur. We assume that the winner has a monopoly over the project (he patents his invention), hence the second entrepreneur does not generate any value. Entrepreneurs and the venture capitalist are risk neutral individuals with common discount rate r.

In order to successfully finish a stage, the entrepreneur has to allocate an amount c (provided by the venture capitalist) into the project. In that case the stage is completed in the current period with probability p. With probability $1-p$ the entrepreneur does not succeed and has to invest further (conditional on the fact that his rival has not yet won the race).[3] Following Lee and Wilde (1980) and Reinganum (1981) we assume that event of achieving a success in each period is independent across the entrepreneurs and across time. Finance are provided by the venture capitalist, but allocation decisions are made by entrepreneurs. They can either invest funds or divert them for private uses. The venture capitalist is not able to observe the allocation decision. All he can observe is a success (completion of the current stage) or an absence of success (which can either mean

[3] We assume that the entrepreneurs are identical, which implies that the probability of success p is the same for both entrepreneurs.

that an entrepreneur has invested money but failed, or that he has diverted it).[4]

We will analyze a model where $N = 2$, i.e., in order to win the prize R an entrepreneur has to complete successfully two stages. Bergemann and Hege (2002) analyze a model with one entrepreneur and N stages. However, for more than one entrepreneur the analysis of the multistage game becomes extremely complicated. In spite of this limitation, our model enables us to illustrate the importance of competition between the entrepreneurs in venture capital financing. Finally, we will assume that $Rp > 2c$. This condition is necessary and sufficient for financing at least one entrepreneur.

2.2 Definitions and notations

We will call *regime* (i/j) a situation, where one entrepreneur has i successes (he has successfully completed i stages) and the other entrepreneur has j successes, where $i, j \in \{0, 1\}$. In this paper we consider a situation in which the entrepreneurs are initially in the regime (1/0), i.e., there is a *leader* (an entrepreneur, who already had one success) and a *follower* (an entrepreneur, who has zero successes). Therefore, we analyze financing in regimes (1/0) and (1/1).

We will use the following notation:

- C_k^{ij} denotes a contract is applied in regime (i/j), where contracts are indexed by $k = 1, 2, \ldots$. As will be clarified below, a contract between the venture capitalist and an entrepreneur specifies a reward of the entrepreneur in case of success, the maximal financing horizon, and a stopping rule.

- T_k^{ij} is the optimal financing horizon for regime (i/j) under the terms of contract C_k^{ij}. In the model the optimal financing horizon will be endogenously determined to maximize a profit of the venture capitalist.

- $V_{t,k}^{ij}$ denotes the value of the project at time t in regime (i/j) under the terms of contract C_k^{ij}.

- $E_{t,k}^{L}$ and $E_{t,k}^{F}$ are the value functions of the leader and the follower at time t in regime (1/0), for a contract C_k^{ij}. In regime (1/1) the value function is denoted $E_{t,k}^{11}$.

[4] Innovation process in this game can be interpreted as following. Each entrepreneur owns a coin (representing a project). He tosses the coin and counts a number of "heads" (successes) and "tails" (failures). The first entrepreneur who counts N heads wins the prize R. In order to make one toss, each entrepreneur has to pay a prescribed amount of money c. The venture capitalist provides money for both entrepreneurs in exchange for a share of the prize R.

- $s^L_{t,k}$ and $s^F_{t,k}$ are the rewards, which the leader, respectively the follower, earn upon successful completion of the current stage at time t in regime (1/0) according to the terms of a contract C^{10}_k. In regime (1/1) the reward is denoted as $s^{11}_{t,k}$.

Further, we will call *regime* (i) a situation, in which the venture capitalist finances only one entrepreneur, who is on i-th stage of R&D. The corresponding value of the project, value function of an entrepreneur and his reward and maximal financing horizon are denoted as V^i_t, E^i_t, s^i_t and T^i respectively.

Following Bergemann and Hege (2002) and Neher (1999), we assume that the venture capitalist can commit to financing the entrepreneurs for a maximum of T periods.[5] If this horizon was reached but no success had been achieved, the project would be irrevocably abandoned. We discuss this assumption in more detail in Section 6.

2.3 Moral hazard and contracting

We assume that there is a competitive market for innovative projects and a limited supply of venture capital. The venture capitalist can choose any entrepreneur from the pool of identical entrepreneurs. Therefore, in this world the venture capitalist has all the bargaining power, which also means that after paying an entrepreneur the incentive compatible compensation, he retains the residual payoff from the project.

The allocation of funds in this model is subject to moral hazard: in each period the entrepreneurs face a choice between allocating the funds into R&D and consuming them. The venture capitalist, however, is willing to finance R&D only if he can ensure that funds are allocated truthfully in each period of time. That is, the venture capitalist has to suggest such reward to both entrepreneurs, that they prefer to allocate the funds to R&D, rather than to divert them. Moreover, as the only verifiable outcome in each stage of the game is the completion of this stage, the incentive scheme should reward the entrepreneurs only if a stage was successfully completed.

There are several counteracting forces which determine size of the incentives payment. On one hand, by consuming funds the entrepreneurs receive the immediate utility c in each period. They also ensure themselves further financing, i.e., potential rent of c in the next period. Therefore, in each period of time the venture capitalist should promise the entrepreneurs a reward which is at least as large as the present value of all investments c which the entrepreneurs can consume. On the other hand, by consuming the funds rather than investing them,

[5] Note, that we use T to denote a *choice variable*, which is called *maximal financing horizon*, while T^{ij}_k is the optimal value of T for regime (i/j) and contract k.

each entrepreneur faces a risk that his rival wins the prize. This limits the option of each entrepreneur to deviate and to consume the funds. Therefore, competition might make it cheaper for the venture capitalist to meet the incentive compatibility constraints of the entrepreneurs.

In this paper we consider a model in which the venture capitalist initially faces the leader (an entrepreneur who has finished his first stage) and the follower (an entrepreneur who is at the initial stage of R&D). Therefore, we are interested in developing an optimal contract (or contracts) for the regimes (1/0) and (1/1). In each of the regimes (1/0) and (1/1), the venture capitalist has to suggest a new contract to the entrepreneurs. He also has to be able to decide whether to finance both of them, one of them or none and for how long. Each time the regime switches, new terms of a contract come in force. The terms of a contract stay valid until one of three events happens: (a) the regime changes, (b) at least one of the entrepreneurs wins, or (c) maximum time allowed for the current regime elapses. The terms of the contract should define the reward of each entrepreneur in case he achieves a success in the current regime. Further, the contract should define the maximal allowed time during which the entrepreneurs can invest in R&D in the current regime and the consequences to each entrepreneur in case the maximal time has elapsed but the regime has not changed.

A rule which determines when financing of one or both entrepreneurs should be abandoned is called a *stopping rule*. We consider a class of deterministic stoping rules, which use the only observable parameter of the model, i.e., the number of successes, to decide which entrepreneur should be financed and which should be stopped. For any regime (i/j), where $i \geq j$ the venture capitalist can choose the optimal contract (i.e the reward of the entrepreneurs and maximal financing horizon) out of the following classes of stopping rules:

- Rule 1: Finance both entrepreneurs until one of them wins or until the maximum allowed time T^{ij} expires. If neither entrepreneur has success, abandon financing of both.

- Rule 2: Finance both entrepreneurs until one of them wins or until the maximum allowed time T^{ij} expires. If neither entrepreneur has success, choose the leader in regime (1/0) and a random entrepreneur in regime (1/1) and finance him until time T^1 expires.

- Rule 3: Finance only one entrepreneur (the leader in regime (1/0) and in regime (1/1) a randomly chosen entrepreneur) until time T^1 expires.

In regime (1/1) the class of these stopping rules is limited to the three rules, described above. In regime (1/0), however, there are other stopping rules which the venture capitalist could use. Note that the above rules favor the leader in a sense that the leader always be financed at least as long as the follower. Potentially, the venture capitalist could use some stopping rule, which to the contrary favors the follower. Intuitively such rules should be less attractive for the venture capitalists than those where the leader is favored. In Appendix A we show, that any stopping rule which favors the follower indeed generates a smaller expected profit for the venture capitalist, that some stopping rule, which favors a leader. Hence, a rational venture capitalist will use no other stopping rules except those above.

3 Competition in the last stage of innovation race

To analyze the model we use the subgame perfect equilibrium concept. Hence, we develop a sequentially optimal dynamic contract, which maximizes the profit of the venture capitalist at each period of time and in each regime of the game. In this section we consider a situation in which innovation race is taking place between two entrepreneurs that are at the last stage of research and development and have the same probability of winning the final prize.

Solving the game at this stage means finding a contract, which

1. determines the share of each entrepreneur in case of success;
2. determines the maximum time for the research;
3. determines a stopping rule, which will be used if the maximum time elapses but no discovery is made. The stopping rule therefore dictates whether the financing of both entrepreneurs will be finished in that case or whether one of them will continue his experiment further.

In order to find this universally optimal contract we first develop an optimal contract for each of the three stopping rules and then compare the contracts across the stopping rules.

3.1 Value of the venture in the last stage

The venture capitalist's decision whether to finance one or two entrepreneurs and the choice of the maximal horizon of R&D depends on the expected profit which he receives in each case. This profit is the difference between the expected value of the project and the expected compensation of the entrepreneurs.

As our model is formulated in finite time, we can recover the value of the project recursively. Consider the first stopping rule, according to which two entrepreneurs are financed until one of them wins, but at most for T periods. This rule gives rise to a contract which we will denote C_1^{11}. In period t the expected value of the project can be written as

$$V_{t,1}^{11} = Rp(2-p) + \frac{(1-p)^2}{1+r} V_{t+1,1}^{11} - 2c.$$

This value is consists of three terms. The last term $2c$ represents the necessary investments by the entrepreneurs. With probability $1-(1-p)^2 = p(2-p)$ at least one of them makes a discovery (yielding the prize R) at period t. With probability $(1-p)^2$ neither of the entrepreneurs makes a discovery, so that the value of the project in period t is the discounted value of the project in period $(t+1)$, that is $\frac{1}{1+r} V_{t+1,1}^{11}$.

Considering transition to continuous time and solving the resulting differential equation, we obtain the following expression for value function in period t:[6]

$$V_{t,1}^{11} = \frac{2(Rp-c)}{r+2p}\left(1 - e^{-(r+2p)(T-t)}\right).$$

The expression for the value function consists of two factors. The first factor represents the expected payoff from the investment, discounted with a composite discount rate which combines time discount r and the uncertain arrival of success. The second factor shows how the value of the project decreases with time of discovery. The value functions the second and the third stopping rule can be derived analogically and are presented in Table 5 in Appendix B. They have a similar interpretation as described above.[7]

3.2 Incentives of the entrepreneurs in the last stage

In each period of time the entrepreneurs face a choice between diverting the funds provided by the venture capitalist for private needs and investing them into the project. In order to motivate entrepreneurs to allocate the funds truthfully into research and development, the venture capitalist has to promise them a reward which is at least as large as the stream of rent that an entrepreneur can receive from diverting the funds.

With our simple form if the R&D process, each entrepreneur has two available strategies: he can either "work" (that is, allocate funds into the project) or "shirk" (that is, divert all funds for private uses). He chooses among them based on the reward scheme. For the time being, we make the assumption that the entrepreneurs do not behave strategically, i.e., each of them believes that the other entrepreneur always "works", or allocates the funds into the project in each period.[8]

[6] For details on the derivation of the value function see Appendix A.

[7] The derivation of value function resembles Bergemann and Hege (2000).

[8] We discuss the strategic interaction in Section 5 and we show that it does not change the results, which we receive with the assumption of no-strategic interaction.

The venture capitalist should offer to each entrepreneur such share s_t^{11} that both of them find it incentive compatible to invest in each period, rather than consume funds. Consider the first stopping rule. According to this rule, financing of both entrepreneurs is terminated if no success occurred before time T_1^{11} elapses. In the terminal period the incentive compatibility constraint is

$$E_T^{11} = p(1-p)s_T^{11} + \frac{1}{2}p^2 s_T^{11} \geq c. \tag{1}$$

The left-hand side of (1) is the expected utility of an entrepreneur in case he invests money into the project. If he wins while his rival loses (which occurs with probability $p(1-p)$) the entrepreneur earns his share s_T^{11}. If there is a tie (with probability p^2), he earns his share with probability $\frac{1}{2}$. The right-hand side of (1) represents the utility which the entrepreneur receives if he consumes the funds. The incentive compatibility requires that the left-hand side (expected payoff from investing in the project) is at least as large as the right-hand side (payoff from diverting the funds).

Moving backward in time we obtain the following intertemporal incentive compatibility constraint for period t:

$$E_{t,1}^{11} = p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)}E_{t+1,1}^{11} \geq c + \frac{1-p}{1+r}E_{t+1,1}^{11}\,. \tag{2}$$

The left-hand side of (2) the inequality represents the expected utility of an entrepreneur, if he allocates the funds into the project at period t. The right-hand side represents the expected payoff of the entrepreneur from diverting funds at period t. The incentive to divert funds arises from two sources. First, an entrepreneur enjoys the utility c from consuming the funds rather than investing them. Second, by consuming the funds he ensures that financing of the project will continue in the next period with probability $(1-p)$. If he invests funds truthfully, however, the project will receive further financing with probability $(1-p)^2 < (1-p)$ and therefore by investing truthfully the entrepreneur cuts himself off from the future stream of rent. If there is only one entrepreneur, as in Bergemann and Hege (2002), then by diverting funds in period t he guarantees himself that the funding will continue in period $t+1$ with probability 1, unless it is the terminal period. In case of two entrepreneurs, however, funding of each is stochastic and depends on the fact that another entrepreneur has not yet reached success. Therefore, competition softens the incentive compatibility constraint and makes it easier for the venture capitalist to satisfy it.

The venture capitalist wants to pay each entrepreneur the minimal share which will force the entrepreneur to invest the funds rather than consume them. To determine the sequence of shares in each time $t = 1, 2, \ldots, T-1$, the venture capitalist has to solve the following minimization problem:

$$\begin{aligned} E_{t,1}^{11} = \min_{\{s_t^{11}\}} \quad & p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)}E_{t+1,1}^{11} \\ \text{s.t.} \qquad & p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)}E_{t+1,1}^{11} \geq c + \frac{1-p}{1+r}E_{t+1,1}^{11} \end{aligned}$$

Hence in optimum, the constraint is binding. Again, considering the transition to continuous time we derive expressions for the share, which the entrepreneur receives in case of success, and the value function which describes the expected utility of an entrepreneur in each time t, given that he allocates the funds into the project.[9] We obtain

$$s_t^{11} = \frac{c}{p} + E_{t,1}^{11}, \tag{3}$$

$$E_{t,1}^{11} = \frac{c}{r+p}\left(1 - e^{(r+p)(t-T)}\right). \tag{4}$$

As the entrepreneurs are identical, in the sense that they are at the same stage of R&D and have the same probability to complete the project, the value functions (and the shares) are identical for both entrepreneurs.

The compensation scheme, described by the value function $E_{t,1}^{11}$, guarantees that each entrepreneur uses the funds truthfully in each period in regime (1/1). The above expression is very intuitive. The first factor of $E_{t,1}^{11}$ represents the value of perpetuity which an entrepreneur would receive if he diverted the funds. The second factor represents a "punishment" for late discovery: the share of an entrepreneur decreases over time. The details about the derivation of value function $E_{t,1}^{11}$ can be found in Appendix A. Analogously it is possible to derive the share and compensation of the entrepreneurs for other stopping rules. The results are presented in Table 5 in Appendix B.

3.3 Optimal stopping time

For each stopping rule the venture capitalist maximizes his profit from the project, given that the incentive compatibility constraints of both entrepreneurs are satisfied. The choice variables of the venture capitalist are the shares of entrepreneurs and the time horizon.

Consider the first stopping rule, which requires that both entrepreneurs are financed until one of them wins or until the financing horizon elapses. In the previous section, we derived the value functions $V_{t,1}^{11}$ and $E_{t,1}^{11}$. The optimal time

[9] See Appendix A for derivation of value function and share of an entrepreneur.

horizon is derived from the following program:

$$\max_{T\in(0,\infty)} V_{0,1}^{11} - 2E_{0,1}^{11}\ .$$

The first order condition yields a unique solution to the maximization problem. We will denote the *optimal* financing horizon as T_1^{11}, where

$$T_1^{11} = -\frac{1}{p}\ln\frac{c}{Rp-c}\ .$$

We denote C_1^{11} the corresponding contract: Finance both entrepreneurs for T_1^{11} periods and abandon both is no success is made (see Table 5 in Appendix B for other details).

For the benchmark case with one entrepreneur, which corresponds to the third stopping rule, the optimal financing horizon is $T^1 = -\frac{1}{p}\ln\frac{c}{Rp-c}$ (see also Bergemann and Hege 2002). The resulting contract is denoted C_4. Since the optimal financing horizon depends on costs of R&D and on expected payoff, it is not surprising, that $T^1 = T_1^{11}$. Indeed, two entrepreneurs spend twice as much on R&D, but they also have twice as large probability of success,[10] so that the ratio of R&D costs to the expected payoff remains constant.

Note that for T^1 to be positive it is necessary that $Rp > 2c$. The intuition behind this restriction becomes clear when we re-write inequality as $R > \frac{2c}{p}$. Since the R&D in our model follows a Poisson process with parameter p, the expected time of discovery when a single entrepreneur is employed, is $\frac{1}{p}$. Hence, the requirement $R > \frac{2c}{p}$ means that the venture capitalist will finance the project only if the value of the prize is larger than the expected cost of R&D. Otherwise, it is not profitable for the venture capitalist to finance the project at all. From now on we will assume, that $Rp > 2c$.

Let us now consider second stopping rule. According to this rule both entrepreneurs will be financed until one of them wins, or until the maximal allowed time elapses. If no success was made, then one entrepreneur will be randomly chosen and financed for additional period of time. For simplicity we denote the expected profit, which the venture capitalist retains as $F(T) = V_{0,2}^{11} - 2E_{0,2}^{11}$, the functions $V_{0,2}^{11}$ and $E_{0,2}^{11}$ can be found in Table 5 in Appendix B. Maximizing the profit of the venture capitalist, we obtain the following first-order condition:

$$\begin{gathered} F'(T) = -(r+2p)\cdot B^{11}\cdot e^{-(r+2p)T} + (r+p)\cdot A^{11}\cdot e^{-(r+p)T} = 0, \\ \text{where} \qquad A^{11} = E_0^1 - \frac{2c}{r+p}, \qquad B^{11} = V_0^1 - \frac{2(Rp-c)}{r+2p}. \end{gathered} \tag{5}$$

[10]Intuition for this result is explained in Section 3.4.

Depending on the relation of A^{11} and B^{11} the optimal time can be finite or infinite. First note that B^{11} is always negative. Indeed the inequality $B^{11} < 0$ is equivalent to

$$\frac{Rp-c}{r+p}\left(1-e^{-T^1(r+p)}\right) < \frac{2(Rp-c)}{r+2p}.$$

which obviously holds for all values of parameters $p, r \in (0,1)$ satisfying the feasibility condition $Rp > 2c$.

If $(r+p)A^{11} \leq (r+2p)B^{11} < 0$, then the expected profit $F(T)$ is decreasing in T and the optimal research horizon is zero.[11] This situation if from the viewpoint of the venture capitalist equivalent to contract C_4.

If $(r+2p)B^{11} < (r+p)A^{11} < 0$, then the optimal research horizon is

$$T_2^{11} = -\frac{1}{p}\ln\frac{r+p}{r+2p}\,\frac{E_0^1 - \frac{2c}{r+p}}{V_0^1 - \frac{2(Rp-c)}{r+2p}}.$$

The corresponding contract is denoted C_2^{11}. According this contract, the venture capitalist commits to finance both entrepreneurs at most for T_2^{11} periods; if this time elapses without a success, then only one entrepreneur (randomly chosen) will be financed further. The terms of the contract are described in Table 5 in Appendix B.

On the other hand, if $A^{11} \geq 0$, the expected profit $F(T)$ is increasing in T and the optimal research horizon is infinite i.e., the venture capitalist is willing to finance the innovation race infinitely long. The corresponding contract is denoted C_3^{11}. This case corresponds to the favorable combinations of low costs of R&D and high probability of success. The condition $A^{11} \geq 0$ directly implies that (in expected terms) the venture capitalist would have to pay higher compensation to one entrepreneur than to two entrepreneurs, i.e., $E_0^1 > \frac{2c}{r+p}$. If this is the case, the venture capitalist always prefers a competitive arrangement to a single entrepreneur.

Remark 1. Note that for all contracts, the value functions and the cost functions at the optimal time are homogeneous of degree 1 in (c, R) and homogeneous of degree 0 in (c, p, r). Therefore, if we denote $W(c, p, r, R)$ the maximal value of the venture capitalist's objective function,[12] then

$$W(c,p,r,R) = R\cdot W\Big(\frac{c}{R}, p, r, 1\Big) = R\cdot W\Big(\frac{\bar{r}c}{Rr}, \frac{\bar{r}p}{r}, \bar{r}, 1\Big), \tag{6}$$

where $\bar{r}$ is some particular value of the discount rate. Hence any comparison of contracts for general values of parameters c, p, r, and R is equivalent to comparison

[11] Note that $F'' < 0$ and $F(0) > 0$.

[12] This is, for example, $V_{0,1}^{11} - 2E_{0,1}^{11}$ at time $T = T_1^{11}$ for contract C_1^{11}.

for parameters c and p with an arbitrary value of r and with $R = 1$.[13] Later, we use, without loss of generality, the value $\bar{r} = 0.05$ in numerical simulations.

3.4 The effect of competition

For each of the three stopping rules we can now specify a contract in terms of maximum time allowed for research and share of the prize, which each entrepreneur receives in case of success. As we showed in the previous section, some stopping rules generate more than one optimal contract. In any case, the terms of the contracts depend on the probability of success and the normalized costs (that is on the ratio $\frac{c}{R}$; see Remark 1). For each combination of parameters, the venture capitalist will choose among three contracts, corresponding to three stopping rules. The optimal contract then is the one which maximizes the residual payoff of the venture capitalist.

Proposition 1. *Let $Rp > 2c$. Then, in regime (1/1) the optimal contract is to finance both identical entrepreneurs for at most $T_1^{11} = -\frac{1}{p} \ln \frac{c}{Rp-c}$ periods and abandon financing of both if no success was made (such contract is denoted C_1^{11}).*

The proof of the proposition can be found in Appendix A. In order to understand the above result, recall that the R&D is modelled as a Poisson process. In each time the probability that at least one entrepreneur succeeds is $1-(1-p^2) = 2p - p^2$. According to the definition of the Poisson process, in continuous time the probability that two events (two successes) will occur in time interval $[t, t+\Delta]$ interval converges to zero, as $\Delta \to 0$. Therefore, the probability that at least one entrepreneur encounters a success in $[t, t+\Delta]$ can be approximated by $2\Delta p$.[14] Thus, at each time two entrepreneurs create twice as much value as one entrepreneur. At the same time, the expected reward to be paid to each of the competing entrepreneurs is less than the expected reward of a single entrepreneur:

$$E_1^{11} = \frac{c}{r+p}(1 - e^{-(r+p)(T-t)}) \quad < \quad \frac{c}{r}(1 - e^{-r(T-t)}) = E^1.$$

Let us summarize the effects which competition has for the R&D process and profits of the venture capitalist. When identical entrepreneurs are at the last stage of R&D, competition is unambiguously a beneficial arrangement from the point of view of the venture capitalist. It reduces the rent which each entrepreneur can extract from the venture capitalist: it "disciplines" the entrepreneurs, making

[13] In particular, given c, p, r, and R, we choose an arbitrary $\bar{r} > 0$ and consider new variables $\bar{c} = \frac{\bar{r}c}{Rr}$ and $\bar{p} = \frac{\bar{r}p}{r}$, which gives $W(c,p,r,R) = R \cdot W(\bar{c}, \bar{p}, \bar{r}, 1)$. We rename the variables to c and p by dropping the bar.

[14] This means, $2\Delta p$ is the first-order approximation of the probability. More precisely, the probability can be written as $2\Delta p + o(\Delta)$.

them work hard for less. The surplus of the venture capitalist is always larger under the arrangement with two competing entrepreneurs. The model predicts therefore, that the venture capitalist will always choose to finance competing entrepreneurs, if they are at the same (the last) stage of innovation race. This strong conclusion is partially a result of the assumption that the entrepreneurs are considered to be identical, which is a simplification of reality. In the next section show that this conclusion may not hold for entrepreneurs on the different stages of R&D, which is most likely the case that venture capitalists and similar institutions face.

4 Competition between leader and follower

When an innovation race starts in a regime with a leader and a follower, the entrepreneurs are not identical from the venture capitalist's point of view. Indeed, the leader has a higher probability of winning a prize. In this section we investigate whether and when the venture capitalist is willing to finance two competing entrepreneurs which are at different stages of R&D. Intuitively, competition can be beneficial, if the presence of the follower considerably limits the rent which the leader can extract from the venture capitalist. The follower has to be a credible threat in the sense that the probability that he makes a breakthrough and wins the race should be sufficiently high. On the other hand, the costs of R&D should be low, compared to the expected prize, so that the duplication of research efforts is justified.

In a game where two stages need to be completed, the leader is an entrepreneur who has finished the first stage of R&D (has already encountered one success), while the follower is his rival, who has still needs two successes, i.e., has zero successes. In our notations, this situation corresponds to regime (1/0). Analogically to regime (1/1) we investigate contracts, corresponding to three stopping rules. The contracts relevant for regime (1/0) are described in Table 6. As in regime (1/1), all value functions and cost functions are homogeneous of degree 1 in (c, R) and of degree 0 in (c, p, r).

An important observation is that the reward of the leader has to be higher than the reward of the follower. By diverting funds at some period of time, the leader can guarantee himself a rent $(c+\frac{pE_0^{11}}{1+r})$, where $\frac{pE_0^{11}}{1+r}$ is his expected payoff in the case when the follower makes the first success.[15] Therefore, the venture capitalist has to offer the leader an incentive compatible share, such that the leader's expected reward will be at least as large as the stream of rents $(c + \frac{pE_0^{11}}{1+r})$. On the other hand, if the follower consumes funds in period t, he can only guarantee himself a

[15] In regime (1/1) the optimal contract is C_1^{11}. Henceforth, for ease of notation when referring to the terms of this contract we will relax the index of a contract. That is $E_t^{11} := E_{t,1}^{11}$, $V_t^{11} := V_{t,1}^{11}$, $T^{11} := T_1^{11}$.

rent of c in this period. Therefore, his incentive compatible share should be lower than that of the leader.

For each stopping rule the value functions and optimal financing horizons are derived recursively from the terminal condition. In particular, the second stopping rule gives a raise to more than one contract, which differ with respect to optimal financing horizon. In this case the value of the venture and expected reward of the entrepreneurs are described by the following value functions:

$$\begin{aligned}
V_0^{10} &= \left(V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}\right)\cdot e^{-(r+2p)T} + \frac{p(R+V_0^{11})-2c}{r+2p}, \\
E_0^L &= E_0^1 \cdot e^{-(r+p)T} + \frac{c+pE_0^{11}}{r+p}(1-e^{-(r+p)T}), \\
E_0^F &= \frac{c}{r+p}(1-e^{-(r+p)T}).
\end{aligned}$$

Maximizing the expected surplus of the venture capitalist $G(T) = V_0^{10} - (E_0^L + E_0^F)$ with respect to stopping time T we obtain the following first order condition:

$$G'(T) = -(r+2p)B^{10}\cdot e^{-(r+2p)T} + (r+p)A^{10}\cdot e^{-(r+p)T},$$

where

$$A^{10} = E_0^1 - \frac{pE_0^{11}+2c}{r+p}, \qquad B^{10} = V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}. \tag{7}$$

Depending on the relation of A^{10} and B^{10}, the optimal financing horizon can be either zero, positive finite, or infinite. The following lemma summarizes the results; see Appendix A for its proof.

Lemma 1. *Let $Rp > 2c$. Then in regime $(1/0)$ the following statements hold:*

1. *If $A^{10} > 0$, then $B^{10} < 0$. In that case $G(T)$ in monotonically increasing and the optimal stoping time is infinite (contract C_3^{10}).*

2. *If $(r+2p)B^{10} < (r+p)A^{10} < 0$, then function $G(T)$ reaches maximum at time*
$$T_2^{10} = -\frac{1}{p}\ln\frac{r+p}{r+2p}\,\frac{E_0^1 - \frac{2c+pE_0^{11}}{r+p}}{V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}},$$
where $T_2^{10} > 0$. The corresponding contract is denoted C_2^{10}.

3. *If $A^{10} < 0$ and $(r+p)A^{10} < (r+2p)B^{10}$, then function $G(T)$ is monotonically increasing and the optimal stoping time is zero.*

Note that in case 3. the venture capitalist finances a single entrepreneur, i.e., $V_0^{10} = V_0^1$ and $E_0^L + E_0^F = E_0^1$. In other words, if the profit of the venture capitalist in the case when he finances both the leader and the follower is a monotonically decreasing function of time, he will prefer to finance the leader alone (contract C_4). In case 2. the venture capitalist finances both entrepreneurs until time T_2^{10} is reached and then abandon the follower and continue financing the leader for additional $T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$ periods. See Table 2 in Appendix B for further details on the contracts.

If the first stopping rule is applied, then the optimal stopping time is finite and the corresponding share of the entrepreneurs and value functions are described with contract C_1^{10}. According to this contract both entrepreneurs will be financed until maximal research horizon $T_1^{10} = -\frac{1}{p} \ln \frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}$ is reached or until the regime changes. Note that $2c + pE_0^{11} < p(R + V_0^{11}) - 2c$ is necessary for T_1^{10} to be positive. If the reverse inequality holds, then the optimal stopping time is zero and contract C_1^{10} degenerates to contract C_4.

There are conditions on parameters $(p, \frac{c}{R})$ that determine, whether a particular contract can be applied. For the contracts with finite stopping time (i.e., contracts C_1^{10}, C_2^{10}, C_4), these conditions require, that the optimal financing horizon is positive. For contract C_3^{10}, these conditions require, that parameters are such, that the optimal financing horizon is infinite. From now on we will call these condition the *feasibility conditions*. We will call a contract *feasible* in the range of parameters, where the corresponding feasibility conditions are satisfied. The range of parameters, where feasibility conditions for each contract are satisfied, is shown in Figure 3 in Appendix B.

Out of the pool of feasible contracts we then choose the one, which maximizes the profit of venture capitalist, i.e., we look for an optimal contract with respect to stopping rules. Investigation of feasibility conditions and optimality of contracts leads to Proposition 2. The proof of the proposition (partly numerical) can be found in Appendix A.

Proposition 2. *Let $Rp > 2c$. Then in regime $(1/0)$ the following statements hold:*

1. *If $A^{10} > 0$, then the feasible contracts are C_1^{10}, C_3^{10} and C_4. The optimal contract out of these contracts is C_1^{10}.*

2. *If $0 > A^{10}(r+p) > B^{10}(r+2p)$, then the feasible contracts are C_1^{10}, C_2^{10} and C_4. The optimal contract is C_1^{10}, if parameters are such that $T_1^{10} < T_2^{10} - \frac{1}{r+p} \ln \frac{2c+pE_0^{11}-E_0^1(r+p)}{2c+pE_0^{11}}$. Otherwise, contract C_2^{10} is optimal.*

3. *If $A^{10}(r+p) < B^{10}(r+2p)$ and $(2c + E_0^{11}) < p(R + V_0^{11}) - 2c$, then the feasible contracts are C_1^{10} and C_4. The optimal contract is C_4.*

4. If $A^{10}(r+p) < B^{10}(r+2p)$ *and* $(2c+E_0^{11}) > p(R+V_0^{11}) - 2c$*, then the only feasible (hence, the optimal) contract is* C_4*.*

Let us denote R_i the domain of parameters (p, c) where contract C_i^{10} is optimal, where $i = 1, 2, 3, 4$.[16] Proposition 2 shows that the domain R_3 is empty and hence the whole parameter space can be divided into three domains R_1, R_2, and R_4, as shown in Figure 1.[17]

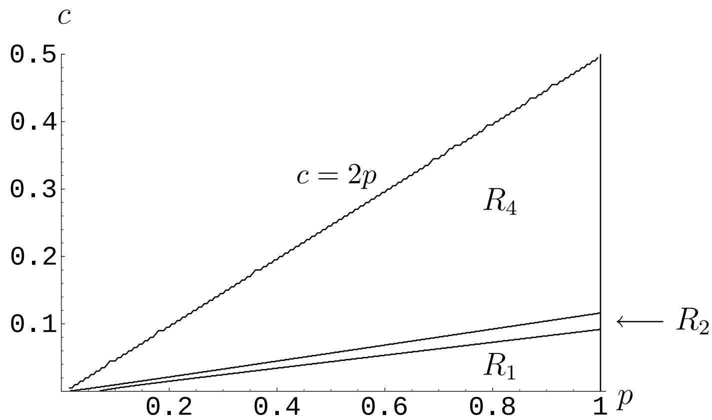

Figure 1: Regime (1/0): Division of the parameter space into three domains according to optimal contracts; for $r = 0.05$

As Figure 1 shows, the region R_1 corresponds to the most favorable combination of costs of R&D and the probability of success. In region R_4, on the contrary, for each success probability the costs of research and development are the highest. Finally, in region R_2 the combination of costs and success probability is moderately favorable. It is therefore intuitive that competition is a beneficial arrangement for the venture capitalist, if values of the parameters lie in the domain R_1 or R_2, while in domain R_4, competition is not beneficial, since the costs are too high to justify the duplication of research efforts.

[16] Region R_4 corresponds to contract C_4.

[17] To draw the domains R_1, R_2, and R_4 we considered fixed values of discount rate $r = 0.05$ and prize $R = 1$ and used numerical simulations. On a grid 0.001×0.001 and for values of parameters, such that $p \in [0, 1]$ and $c \in [0, p/2]$ we plotted the points where the constraints for each domain are satisfied. The results do not differ qualitatively for other values of discount rates. Moreover, again we can fix values of R and r and all statements remain true. See Proof of Proposition 2 for details.

4.1 The optimal contract in the absence of moral hazard

In order to investigate the effect that competition has on the decision to employ competing entrepreneurs, we compare the moral hazard setting with the benchmark case without moral hazard. In the latter case the venture capitalist can perfectly observe the allocation of funds and therefore the reservation utility of both entrepreneurs is zero (this is due to the assumption that the venture capitalist has all bargaining power). Hence, the expected payoff of the venture capitalist equals the expected value of the project. For all stopping rules the value of the project is maximized, if the time of financing is infinity.

In the regime (1/1), values of the project when competing entrepreneurs are financed (V^{11}) and a single entrepreneur is financed (V^1) are given by the following formulas:

$$V^{11} = \frac{2(Rp - c)}{r + 2p}, \qquad V^1 = \frac{Rp - c}{r + p}.$$

Obviously, $V^{11} > V^1$, so that the venture capitalist will always choose to finance competing entrepreneurs, if both are on the last stage of R&D. Note, that the necessary condition for a project to be financed in no moral hazard setting is $Rp > c$, as opposed to $Rp > 2c$ in the no moral hazard setting.

In the regime (1/0) the value of the project if both the leader and the follower are financed is

$$V^{10} = \frac{p(R + V^{11}) - 2c}{r + 2p}.$$

The venture capitalist will finance only the leader, if $V^1 \geq V^{10}$, which is equivalent to a condition:

$$\frac{c}{Rp - c} > \frac{p \cdot r}{(r + 2p)(r + p)}. \tag{8}$$

Otherwise, the venture capitalist will finance both entrepreneurs.

The division of the parameter space into two domains is shown in Figure 2. The border curve between single entrepreneur (SE) and competing entrepreneurs (CE) area satisfies condition $\frac{c}{Rp-c} = \frac{p \cdot r}{(r+2p)(r+p)}$.[18] The region above the line represents combinations of costs and success probability, where (8) holds, i.e., where the venture capitalist finances only the leader. If the combination of costs and probability is below the line, then the venture capitalist will prefer to finance both entrepreneurs.

[18]The areas were plotted for r is fixed at $r = 0.05$. The results does not change qualitatively for other reasonable discount rates.

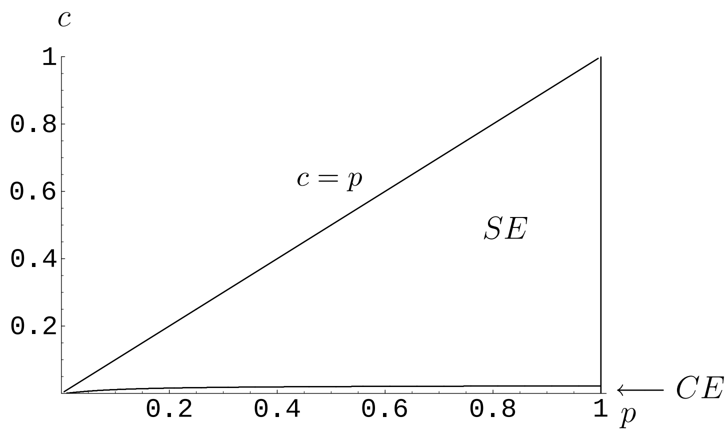

Figure 2: Regime (1/0), no-moral hazard case: Competing entrepreneurs (CE) vs a single entrepreneur (SE); $r = 0.05$

4.2 The effect of competition

In regime (1/1) the venture capitalist prefers to employ competing entrepreneurs, regardless whether the moral hazard is present or not. Without moral hazard this decision is motivated by the "scale" effect: with two entrepreneurs the probability of success is twice as large as with one entrepreneur. With moral hazard there is additional effect of competition, which we call the "disciplining" effect. This effect decreases the rent of each entrepreneur comparing to situation of no competition, so that in case of success the venture capitalist retains larger share of the prize.

Analysis of regime with the leader and the follower allows to understand the relative importance of the "scale" and "disciplining" effect in the presence of moral hazard. A comparison of Figures 1 and 2 shows that without moral hazard the range of parameters where the competition is beneficial is significantly smaller than in a moral hazard setting. In the absence of moral hazard the increased probability of success due to competition ("scale" effect) is most of the time not sufficient to justify financing of both the leader and the follower. However, the disciplining effect of competition in case of moral hazard is so important, that the venture capitalist will hire both the leader and the follower, although he does not gain much in terms of success probability. The venture capitalist nevertheless gains from the reduction of rent which he has to pay to both entrepreneurs. For certain combinations of costs and probability, the decrease in compensation of the leader due to competition is large enough to justify the financing of both entrepreneurs (domains R_1 and R_2). Naturally, the competition can be justified only if the follower is not too expensive to finance (c should be relatively small) and the reduction in the rent of the leader due to competition is significant (p

should be relatively large).

Another result of competition between entrepreneurs is increase in the total financing horizon of the project. For range of costs and probabilities where the venture capitalist chooses to finance both the leader and the follower (i.e., domains R_1 and R_2), the maximal financing horizon is longer with a competitive arrangement than with a single entrepreneur. Indeed, a single entrepreneur (the leader) would be financed for at most of T^1 periods according to contract C_4. If both entrepreneurs are employed, then the maximum financing horizon is $T_k^{10} + T^1 > T^1$, where $k = 1, 2$. Therefore, competition helps to alleviate one of the main problems created by moral hazard — the limitation of the research horizon. We have shown that the first best solution obtained in the absence of moral hazard is to finance the project infinitely long. The same result was obtained for the case of one entrepreneur by Bergemann and Hege (2002). Since the value of the project increases in the research horizon, the presence of moral hazard reduces this value. Competition, however, limits the amount of rent which the entrepreneurs can extract from the venture capitalist and hence allows the venture capitalist to set a longer financing horizon.

We conclude our analysis of regime (1/0) by noticing that the area R_4 is larger than the area $R_2 \cup R_1$.[19] Therefore, the model predicts that when the venture capitalist faces two entrepreneurs at different stages of R&D, he will more often employ only one of them (the leader), rather than both.[20] The venture capitalist will only finance the leader and the follower if the latter can be a credible threat to the former and if the costs of research and development are small enough compared to the expected payoff.

This prediction suggests an explanation why do venture capitalists not finance R&D race frequently. The venture capitalist-like institutions usually face projects (the entrepreneurs) which are at different stages of R&D and therefore tend to choose the more advanced of them. A different position in innovation race is an example of asymmetries between entrepreneurs. Our results indicate that the venture capitalist is less willing to finance a race between asymmetric entrepreneurs than between identical ones. Indeed, we have shown earlier that when entrepreneurs are identical, i.e., they are at the same stage of R&D, the venture capitalist will finance both.

In practice, the venture capitalist can face projects which are identical in terms of number of completed stages, if these are projects at the initial stage of R&D. Examples would be a revolutionary drug for which no prototypes exist, or software solutions based on an entirely new concept. In terms of our model, this situation would correspond to regime (0/0). However, investigation of this regime is beyond

[19]For $r = 0.05$ it is visible from Figure 1. The results are similar also for other values of r.

[20]This is true when all combinations of parameters are equally probable.

the scope of the present paper.

5 Strategic interaction

Up to this point we assumed that the entrepreneurs do not think strategically, i.e., that each entrepreneur believes that his rival always invests all funds into R&D. In other words, each entrepreneur believes that by diverting the funds in each period, he faces a probability p that his rival wins the prize in the meantime. With this assumption in hand, we have shown that competition softens the incentive compatibility constraints of the entrepreneurs and makes it cheaper for the venture capitalist to provide an incentive compatible reward scheme. As we have discussed, the incentive compatible reward of each entrepreneur is lower in the case of competition, than in the case of no competition.

However, if the entrepreneurs are well-trained game theorists and think strategically, they will take into account all possible strategies of the rival. Those can be either "work" (denote it w) or "shirk" (denote it s). Hence, in each period we can model the behavior of the entrepreneurs by a 2×2 game. The venture capitalist, naturally, wants to ensure the (w, w) equilibrium. Otherwise his investments are wasted. In this section we will show that in order to ensure the unique equilibrium (w, w) and rule out the equilibrium (s, s), it is sufficient to provide the entrepreneurs with a reward, which is incentive compatible under the (w, w) scheme, as considered before.

5.1 Strategic interaction in regime (1/1)

We start with the last stage of the game, i.e., regime (1/1). Consider the terminal period $T^{11} =: T$. Let s_T be the reward of an entrepreneur if he achieves a success. The payoff matrix of the game between two competing entrepreneurs is given in Table 1.

In order to ensure that (w, w) is a unique Nash equilibrium (in pure strategies), the reward s_T should be such that: $w \in BR(w)$ and $w \in BR(s)$, where BR stands for best response. Examining the payoffs, we can show that:

$$\begin{aligned} w \in BR(w), &\quad \text{iff} \quad s_T \geq \tfrac{c}{p - \frac{1}{2}p^2} =: s_T^w, \\ w \in BR(s), &\quad \text{iff} \quad s_T \geq \tfrac{c}{p} =: s_T^s. \end{aligned}$$

Since $s_T^s < s_T^w$ for all $p \in (0, 1)$, in the terminal period of the game the venture capitalist can ensure the unique equilibrium (w, w) by promising the entrepreneurs reward $s_T = s_T^w$.

Consider now some period of time $t \leq T - 1$ and assume that both entrepreneurs invest truthfully in each period $\tau = t + 1, \ldots, T$. We will determine such

	w	s
w	$(p - \frac{1}{2}p^2)s_T$ $(p - \frac{1}{2}p^2)s_T$	ps_T c
s	c ps_T	c c

Table 1: Normal form of the game: regime (1/1), $t = T$

s_t that in period t both entrepreneurs find it incentive compatible to invest funds rather than divert them. Table 2 shows a payoff matrix for period t.

	w	s
w	$(p - \frac{1}{2}p^2)s_t + \frac{(1-p)^2}{1+r}E^{11}_{t+1}$ $(p - \frac{1}{2}p^2)s_t + \frac{(1-p)^2}{1+r}E^{11}_{t+1}$	$ps_t + \frac{1-p}{1+r}E^{11}_{t+1}$ $c + \frac{1-p}{1+r}E^{11}_{t+1}$
s	$c + \frac{1-p}{1+r}E^{11}_{t+1}$ $ps_t + \frac{1-p}{1+r}E^{11}_{t+1}$	$c + \frac{E^{11}_{t+1}}{1+r}$ $c + \frac{E^{11}_{t+1}}{1+r}$

Table 2: Payoff matrix for competition with strategic interaction in regime (1/1)

Here $E^{11}_t = \frac{c}{r+p}(1 - e^{-(T-t)(r+p)})$ is the value of the expected reward of an entrepreneur, given that both entrepreneurs invest from time t on. As before, we need to determine s_t such that for each entrepreneur w is a best response to any strategy of a rival:

$$\begin{array}{lll} w \in BR(w), & \text{iff} & s_t \geq \frac{2c}{p(2-p)} + \frac{2p(1-p)}{p(2-p)(1+r)}E^{11}_{t+1} =: s^w_t, \\ w \in BR(s), & \text{iff} & s_t \geq \frac{c}{p} + \frac{E^{11}_{t+1}}{1+r} =: s^s_t. \end{array}$$

In order to ensure the unique equilibrium (w, w), the venture capitalist has to promise the entrepreneurs a share $s_t \geq \max\{s^s_t, s^w_t\}$.[21] It can be easily shown that

$$s^w_t > s^s_t, \quad \text{iff} \quad \frac{pE^{11}_{t+1}}{1+r} < c.$$

which always holds, since $pE^{11}_{t+1} = \frac{pc}{r+p}\left(1 - e^{-(r+p)[T^{11}-(t+1)]}\right) < \frac{pc}{r+p} < c$. Therefore, by promising the entrepreneurs a reward $s_t = s^w_t$ the venture capitalist ensures the equilibrium (w, w). Note that this reward is exactly the reward which we have calculated before, without accounting for strategic interaction.[22]

The result which we have established holds for any $t \leq T - 1$, therefore it holds in particular for $t = T - 1$. We have proved that in the terminal period the entrepreneurs will invest, if rewarded according to the (w, w) scheme. Therefore, they will also invest in period $(T - 1)$ if rewarded according to the (w, w) scheme. Recursively, we can prove that the result holds for any period t of regime $(1/1)$.

It is interesting to observe that if the entrepreneurs are compensated according to the (w, w) scheme, then in regime $(1/1)$ at each period of time the game resembles the Prisoners Dilemma game. The entrepreneurs can be better off if they divert the funds simultaneously in all periods. Indeed, in this case the expected payoff of each entrepreneur is $\frac{c}{r}(1 - e^{-rT})$, i.e., a properly discounted stream of rent c. If both entrepreneurs invest, then the expected reward of each is $\frac{c}{r+p}(1 - e^{-(r+p)T}) < \frac{c}{r}(1 - e^{-rT})$. But under the incentive scheme (w, w), "work" is always the best response to "shirk", therefore, a potentially attractive (for entrepreneurs) situation (s, s) is not a subgame perfect Nash equilibrium.

5.2 Strategic interaction in regime $(1/0)$

We continue our analysis by considering the regime $(1/0)$. If parameters belong to domain R_4, then the venture capitalist will finance only the most advanced entrepreneur (the leader). Therefore, we need to analyze the strategic interaction only in domains R_1 and R_2.

To prove that incentive scheme $(s^L_{t,k}, s^F_{t,k})$ induces a unique equilibrium (w, w) it is enough to show that for the follower "work" is always a best response, if he is compensated according to (w, w) scheme. We make our point clear below, analyzing first the domain R_1 and then domain R_2.

Table 3 represents the payoff matrix for the terminal period of the game in the regime $(1/0)$ of domain R_1. The first payoff is the payoff of the leader (row player) and the second payoff is the payoff of the follower (column player). For simplicity

[21] We will assume that when the entrepreneurs are indifferent between strategies "work" and "shirk", they choose to work.

[22] After transition to continuous time we receive: $s^w_t = \frac{2c}{p(2-p)} + \frac{2p(1-p)}{p(2-p)(1+r)}E^{11}_{t+1}$, which converges to $s^{11}_t = \frac{c}{p} + E^{11}_t$.

of notation, we relax the index of a contract k and denote $T := T_1^{10}$ the optimal financing horizon.

	w	s
w	$ps_T^L + p(1-p)\frac{E_0^{11}}{1+r}$ $p(1-p)(s_T^F + \frac{E_0^{11}}{1+r})$	ps_T^L c
s	$c + \frac{pE_0^{11}}{1+r}$ $p(s_T^F + \frac{E_0^{11}}{1+r})$	c c

Table 3: Normal form of the game in regime (1/0): domain R_1, $t = T$.

Investigating the payoff of the follower, we derive the following conditions:

$$\begin{aligned} &w \in BR(w), \quad \text{iff} \quad s_T^F \geq \tfrac{c}{p(1-p)} - \tfrac{E_0^{11}}{1+r} =: s_T^{F,w}, \\ &w \in BR(s), \quad \text{iff} \quad s_T^F \geq \tfrac{c}{p} - \tfrac{E_0^{11}}{1+r} =: s_T^{F,s}. \end{aligned}$$

It is obvious, that $s_T^{F,w}$ is always larger than $s_T^{F,s}$. Therefore, if the venture capitalist promises a reward $s_T^F = s_T^{F,w}$ to the follower, he ensures that the latter invests truthfully irrespective of the leader's strategy. This implies, that the leader will prefer to work, rather than shirk, if the following constraint is satisfied:

$$E_T^L = ps_T^L + p(1-p)\frac{E_0^{11}}{1+r} > c + \frac{pE_0^{11}}{1+r}.$$

The incentive compatibility constraint above is satisfied, if $s_T^L = \frac{c}{p} + \frac{p}{1+r}E_0^{11}$. Therefore, if in the last period of time both entrepreneurs are compensated according to (w, w) scheme, there is a unique equilibrium (w, w).

Going backwards in time, we can prove that "work" is the follower's dominant strategy if he is compensated according to (w, w) scheme. The payoff matrix in Table 4 shows payoffs of the follower (column player) in four strategic situations.

In the payoff matrix, $E_{t+1,1}^F$ is the expected payoff of the follower in period $t+1$, giving that both the leader and the follower invest in each period starting from the period $t+1$. Investigating the payoffs, we establish the condition for "work" to be a best response for the follower.

$$\begin{aligned} &w \in BR(w), \quad \text{iff} \quad s_t^F \geq \tfrac{c}{p(1-p)} + \tfrac{E_{t+1,1}^F - E_0^{11}}{1+r} =: s_t^{F,w} \\ &w \in BR(s), \quad \text{iff} \quad s_t^F \geq \tfrac{c}{p} + \tfrac{E_{t+1,1}^F - E_0^{11}}{1+r} =: s_t^{F,s}. \end{aligned}$$

	w	s
w	$p(1-p)(s_t^F + \frac{E_0^{11}}{1+r}) + \frac{(1-p)^2}{1+r} E_{t+1,1}^F$	$c + \frac{(1-p)E_{t+1,1}^F}{1+r}$
s	$p(s_t^F + \frac{E_0^{11}}{1+r}) + \frac{1-p}{1+r} E_{t+1,1}^F$	$c + \frac{E_{t+1,1}^F}{1+r}$

Table 4: Payoffs of the follower in regime (1/0), domain R_1.

Note that $s_t^{F,w}$ is greater than $s_t^{F,s}$. Thus, to ensure that the follower invests in period t, the venture capitalist has to promise him a reward of $s_t^F = s_t^{F,w}$, which is exactly the reward which he would pay under terms of contract C_1^{10}. Giving, that the follower works (i.e., invests) in each period of time, the venture capitalist has to promise a reward to the leader, such that the latter finds it incentive compatible to invest as well. This reward is exactly the reward of the leader, established by the terms of contract C_1^{10}. Hence, the presence of strategic interaction does not change results of the analysis of regime (1/0).

Finally, the analysis for domain R_2 closely resembles the analysis for domain R_1. The expected reward of the follower in any period of time is the same, as in domain R_1 up to the length of financing horizon. The length of the financing horizon however, does not influence the analysis of strategic interaction. Therefore, in domain R_2 the follower's dominant strategy is "work", if and only if he is promised the reward $s_t^F = \frac{c}{p(1-p)} + \frac{E_{t+1}^F - E_0^{11}}{1+r}$. Given that the follower invests in period t, the leader will invest if he is rewarded under (w, w) scheme. The incentive compatible shares and value function of the leader and the follower are described in contract C_2^{10}.

6 Finite horizon and commitment to stop

So far, we have assumed that the venture capitalist can choose the financing horizon for each regime and can commit to it. This means that if the maximum time allowed for experimentation in regime (i/j) elapses without success, then depending on terms of the contract either the project will be irrevocably abandoned, or the venture capitalist will abandon financing of follower. In this section we provide a rationale for that assumption.

If we assume that the venture capitalist cannot commit to stop the project after the maximal allowed time has elapsed, then he will finance the entrepreneurs infinitely long. Suppose that in regime (i/j) the contract between the venture capitalist and entrepreneurs determines some (optimal) time T^{ij}. If this time elapses but no success was made by any entrepreneur, the venture capitalist is willing to start the game from the beginning, as if the world is in the first period of regime (i/j). Indeed, all costs that the venture capitalists has incurred up to time T^{ij} are sunk, and the game has not changed since the venture capitalist made his optimal decision at $t = 0$ of regime (i/j). Because of this feature of our model (sunk costs and independent probability of success in each period), the venture capitalist is willing to finance the entrepreneurs infinitely long, if he enters the game once.

If the venture capitalist cannot commit to stopping the project, he also is not able to condition further financing on successful completion of predetermined stages or benchmarks. In a world, where commitment is not credible, the venture capitalists (if he decides to enter the game) will finance entrepreneurs until one of them wins the prize.

However, empirical literature on venture capital documents, that stage financing, which is conditional on successful completion of prescribed milestones, is one of the most important and commonly used control mechanisms in venture capital financing.[23] Therefore, the commitment assumption is not only realistic, but is essential for the ability of the venture capitalist to include the provision about the milestones into the contract.

Obviously, in our model the venture capitalist prefers committing to finite financing horizon. Commitment to stop financing of the project is an important punishment mechanisms, which allows to decrease compensation of the entrepreneurs and therefore to increase profits of the venture capitalist, comparing to situation with no commitment. In the model, however, there is no endogenous mechanism, which would make the ex-ante commitment credible ex-post. Hence, to justify the commitment power of the venture capitalist in our model, we make an assumption, that the venture capitalist is wealth-constrained.

This realistic assumption is well supported by the evidence about practice of the venture-capital funds. According to Inderst and Munnich (2003), the venture capital funds are normally close-ended, which means that funds are raised once from the investors and are directed afterwards into the portfolio of projects. The partnership agreements, which govern the venture capital funds, often contain a covenant that limits a possibility of the venture capitalist to raise further investments. Likewise, the partnership agreements restrict ability of the venture capitalist to transfer investments across projects and across different funds, run by the same partners. The wealth-constrained venture capitalist can credibly commit to limit resources directed to each of his portfolio projects and hence can commit to the finite financing horizon.

[23]See, for example, Kaplan and Stromberg (2003) and Sahlman (1990).

In the world described in our model, this commitment can be understood as the following. Ex ante, the venture capitalist is able to calculate the optimal period of time, during which he is willing to finance the project. He then commits a corresponding amount of money for this project and commits all other resources to his other portfolio projects. The partnership agreements restrict the ability of the venture capitalist to raise additional funds; therefore the commitment to stop the project is credible.

7 Conclusion

In this paper we study innovation race in the moral hazard setting. We explore a model where two entrepreneurs simultaneously develop a project which, if successful, generates a prize R. The project is developed in stages and the first entrepreneur who completes the second stage wins the prize. Research and development is financed by the venture capitalist, but the funds are allocated by the entrepreneurs. This creates a moral hazard problem: the entrepreneurs can divert the funds to their own uses. We investigate two possible states of the game: a state where one of the entrepreneurs is a leader and another is a follower, and a state, where both entrepreneurs are at the last stage of R&D.

We identify two effects which make the financing of competing entrepreneurs beneficial for the venture capitalist. First effect is the higher probability of success (scale effect) and the second is less obvious effect which competition has on incentives (disciplining effect). In order to highlight the importance of competition in the moral hazard setting we compare it with the benchmark setting of no moral hazard. The analysis reveals, that in the regime where both entrepreneurs are on the last stage of research both effects are important. Due to the scale effect financing of competing entrepreneurs is attractive in the absence of moral hazard. In the moral hazard setting, the presence of a competitor limits the rent which each entrepreneurs can extract from the venture capitalist. This disciplining effect reinforces scale effect, making the financing of competing entrepreneurs attractive also in the moral hazard setting.

However, in the regime with the leader and the follower, the scale effect appears to be of little importance, so that without moral hazard the follower will almost never be employed. Nevertheless, with moral hazard in place the presence of a competitor allows to reduce significantly the rent of the leader, which makes competition a beneficial arrangement for large range of parameters.

The key result of the paper, that competition positively affects incentives of the agents (i.e., relaxes their incentive constraint), is not surprising. This logic has been applied in various settings, including yardstick competition (Tirole 1997, pp. 41–42) or design of team incentives (Holmstrom 1982). We contribute to the literature by investigating the effect of competition on incentives in the dynamic framework, where only the winner's output matters to a principal. In this framework, competition has a positive incentive effect even when the research technologies are independent (which is not the case in yardstick competition). Since every period each entrepreneur faces a threat that his rival wins the current stage of research, the existence of a rival "disciplines" entrepreneurs and decreases the amount of rent which they can extract from the venture capitalist.

Analyzing the model, we derive a solution to the dynamic optimal contract problem and characterize the expected reward of the entrepreneurs, the expected value of the project and the expected profit of the venture capitalist in the closed form. Investigation of optimal contracts yields the following conclusions:

1. Competition disciplines entrepreneurs and makes diversion of funds for private use less lucrative then in a set up with one entrepreneur. Hence, it often allows the venture capitalist to retain larger surplus from the project, comparing with non-competitive situation.

2. If there is a leader and a follower in the innovation race, then optimal contract requires, that the leader is financed at least long as the follower and has higher expected reward.

3. Competition increases the maximal research horizon during which the venture capitalist is willing to finance entrepreneurs.

Our key finding therefore is that competition can be used by the venture capitalist as an effective cure against the moral hazard problem, in situation when the allocation of funds by the entrepreneurs is not observable. Hence, competition is a "natural" mechanism which allows to improve the efficiency of research and development. The existence of such mechanism is particulary important in those cases, where the use of complicated security schemes, which are developed in the venture capital literature, is difficult or not possible at all.

Our results can also be related to a setup where the entrepreneurs have different probability of success, in the sense that the same stage of R&D corresponds to identical probability of success, and a different stage of R&D corresponds to a asymmetric probability of success. Then our results suggest that as asymmetry between entrepreneurs increases, the positive effect of competition becomes less pronounced. Moreover, in setting with asymmetric entrepreneurs, the competition is beneficial if the value of output relative to costs is high and there is high chance that the asymmetry will be eliminated (namely probability of success is relatively high).

The logic of the model can be extended to the N-staged research process. It is intuitive, that the higher is the asymmetry between entrepreneurs in terms of completed stages, the less beneficial is competition between the leader and the follower. However, calculations becomes incredibly cumbersome, as number of stages increases. The direction for future research is to modify the model in order to investigate the competition for N-staged research.

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A Appendix: Proofs

A.1 Derivation of the value of the project in Section 3.1

Consider first the case where a venture capitalist provides financing to both entrepreneurs and abandons both if no success was made at time $t \leq T$ (first stopping rule).

At the terminal period, the value of the project is

$$V_T^{11} = R(1 - (1-p)^2) - 2c = 2(Rp - c) - Rp^2. \tag{9}$$

Going back in time we construct recursively the value of the project at time $t = T-1, \ldots, 2, 1$ as

$$V_t^{11} = 2(Rp - c) - Rp^2 + \frac{(1-p)^2}{1+r} V_{t+1}^{11}.$$

Following Bergemann and Hege (2000) we consider transition to continuous time:

$$V_t^{11} = 2(R\Delta p - \Delta c) - R(\Delta p)^2 + \frac{1 - 2\Delta p + (\Delta p)^2}{1 + \Delta r} V_{t+\Delta}^{11},$$

where Δ is a time which elapses between t and $t + \Delta$. Dividing the expression by Δ and taking $\Delta \to 0$ we receive the following differential equation:

$$V_t^{11}(r + 2p) = 2(Rp - c) + \dot{V}_t^{11}.$$

Solving the differential equation with initial (terminal) condition (9) we receive the expression for V_t^{11}:

$$V_t^{11} = \frac{2(Rp - c)}{r + 2p}(1 - e^{-(r+2p)(T-t)}).$$

The derivation of the value function for the case, where the second stopping rule applies is identical except for the boundary condition, which is

$$V_T^{11} = 2(Rp - c) - Rp^2 + \frac{1-p}{1+r} V_0^1.$$

Here $2(Rp - c) - Rp^2$ is the value of the project if success was achieved at period $t = T$ and $\frac{1-p}{1+r} V_0^1$ is the value of the project if the success was not achieved at the terminal period and one entrepreneur was randomly chosen to continue R&D.

A.2 Derivation of the value function and the share of an entrepreneur in Section 3.2

We will derive the value function for regime (1/1), first stopping rule. All other value functions which describe the reward of an entrepreneur can be derived analogically.

The minimization program, which allows us to determine the optimal share s_t and expected reward $E_{t,1}^{11}$ of the entrepreneur is given in Section 3:

$$\begin{aligned} E_{t,1}^{11} = \min_{\{s_t^{11}\}} &\quad p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)} E_{t+1,1}^{11} \\ \text{s.t.} &\quad E_{t,1}^{11} = p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)} E_{t+1,1}^{11} \geq c + \frac{1-p}{1+r} E_{t+1,1}^{11}. \end{aligned}$$

Since the venture capitalist wants to minimize the payment to each entrepreneur, the incentive compatibility constraint is binding. From this constraint we can express the share s_t:

$$ps_t - \frac{1}{2}p^2 s_t = c + \frac{p(1-p)}{1+r} E_{t+1,1}^{11}. \tag{10}$$

Considering transition of equality (10) to continuous time, we receive:

$$\Delta p s_t - \frac{1}{2}(\Delta p)^2 s_t = \Delta c + \frac{\Delta p(1-\Delta p)}{1+\Delta r} E_{t+\Delta,1}^{11}.$$

Finally, letting $\Delta \to 0$ we receive the continuous time expression for the share of an entrepreneur in period t:

$$s_t = \frac{c}{p} + E_{t,1}^{11}$$

If the incentive compatibility constraint is binding, then the solution to the minimization problem can be derived from the equality

$$E_{t,1}^{11} = c + \frac{1-p}{1+r} E_{t+1,1}^{11}.$$

Again, considering transition to continuous time we receive the differential equation:

$$(r+p)E_{t,1}^{11} = c + \dot{E}_{t,1}^{11}. \tag{11}$$

The terminal condition for this differential equation is given by the incentive compatibility constraint in the terminal period of regime (1/1):

$$E_T^{11} = p(1-p)s_T^{11} + \frac{1}{2}p^2 s_T^{11} = c.$$

Solving the differential equation (11) and using the above initial (terminal) condition, we receive the expression for the entrepreneurial value function:

$$E_{t,1}^{11} = \frac{c}{r+p}\left(1 - e^{(r+p)(t-T)}\right).$$

A.3 Stopping Rules

Consider the following stopping rules:

- Rule 4: In regime (1/0) employ both entrepreneurs and finance them at most for T periods. If the follower succeeds, then continue financing in regime (1/1). If the follower fails, then abandon the leader and continue financing the follower at most for $T^0 + T^1$ periods.

- Rule 5: In regime (1/0) freeze financing of the leader and continue financing the follower for at most T periods. If the follower succeeds, then continue financing in regime (1/1). If he fails, then stop both.

- Rule 6: In regime (1/0) freeze financing of the leader and continue financing the follower for at most T periods. If the follower succeeds, then continue financing in regime (1/1). If he fails, then stop the follower and continue financing the leader for at most T^1 periods.

- Rule 7: In regime (1/0) freeze financing of the leader and continue financing the follower for at most T periods. If the follower succeeds, then continue financing in regime (1/1). If he fails, then continue financing both in regime (1/0).

We will show that each of the above stopping rule delivers the venture capitalist lower profit, than one of the stopping rules 1 to 3. For the purpose of further discussion we simplify the notation and will denote optimal stopping time, corresponding to particular stopping rule by T_i, where $i = 1, \ldots, 7$.

Consider Rule 4. Its counterpart is Rule 2, according to which if no success was made the venture capitalist will abandon the follower and will finance only the leader. The latter case obviously generates larger profit to the venture capitalist, because after the follower is abandoned, there is only one stage to complete. However, if the leader is abandoned, two stages must be completed in order to reach a success. Formally, the profit of the venture capitalist if the second stopping rule is applied is (see Table 6)

$$V_{0,2}^{10} - E_{0,2}^{L} - E_{0,2}^{F} = \left(\frac{p(R+V_0^{11}) - 2c}{r+2p} - \frac{2c + pE_0^{11}}{r+p}\right)(1 - e^{-(r+p)T}) + (V_0^1 - E_0^1)e^{-(r+p)T}.$$

If the forth stopping rule is applied, than the difference with the case above is that the follower needs stronger incentives than in the former case and the leader needs the softer incentives. On the other hand, the value of the project will be lower. The profit of the venture capitalist under Rule 4 is

$$V_{0,4} - E^L_{0,4} - E^F_{0,4} =$$
$$= \left(\frac{p(R + V_0^{11}) - 2c}{r + 2p} - \frac{2(c + pE_0^{11})}{r + p} \right) (1 - e^{-(r+p)T}) + (V_0^0 - E_0^0) e^{-(r+p)T}.$$

Since $V_0^0 - E_0^0 < V_0^1 - E_0^1$ (the venture capitalist is always better off by employing the leader than the follower), it is clear that $V_{0,4} - E^L_{0,4} - E^F_{0,4} < V^{10}_{0,2} - E^L_{0,2} - E^F_{0,2}$ for any financing horizon T.

Rule 5 is clearly worse than Rule 6, which requires that when no change of the regime occurred, the follower will be abandoned and the leader will be financed further. Indeed, the constraint $Rp > 2c$ ensures that in regime (1/0) is always optimal to employ at least the leader. Moreover, the incentives of the follower under Rule 5 are the same as under Rule 6, since he will be financed beyond regime (1/0) only if he completes his first stage. Hence, Rule 6 does not change the incentive of the follower and allows to reach higher value of the project than the Rule 5.

On the other hand, Rule 6 is intuitively worse then the Rule 3. Indeed, according to the former rule, the venture capitalist finances a follower in a hope that he completes one (the first) stage. The venture capitalist, however, could as well finance the leader so that he completes his last stage and wins a prize. The latter case is obviously of a greater value to the venture capitalist. To prove this argument formally, we compare the surplus of the venture capitalist, which corresponds to the Rule 6 and the Rule 3. In the former case the surplus of the venture capitalist is

$$V_{0,6} - E^F_{0,6} - E^L_{0,6} = \left(V_0^1 - \frac{pV_0^{11} - c}{r + p} \right) \cdot e^{-(r+p)T} + \frac{pV_0^{11} - c}{r + p} -$$
$$\frac{c}{r}(1 - e^{-rT}) - \left(E_0^1 - \frac{pE_0^{11}}{r + p} \right) e^{-(r+p)T} - \frac{pE_0^{11}}{r + p}.$$

Taking the first derivative of the profit function, we receive a function $F(T)$:

$$F(T) = -(r+p) \left(V_0^1 - \frac{pV_0^{11} - c}{r + p} \right) \cdot e^{-(r+p)T} - ce^{-rT} + (r+p) \left(E_0^1 - \frac{pE_0^{11}}{r + p} \right) e^{-(r+p)T}.$$

If the function decreases in T, then the result is immediate: the venture capitalist will optimally set $T = 0$ and will prefer to employ the leader alone. If, on the contrary, the function has maximum, then the optimal stopping time T is:

$$T_6 = -\frac{1}{p}\ln\frac{c}{\left(E_0^1 - \frac{pE_0^{11}}{r+p}\right) - \left(V_0^1 - \frac{V_0^{11}-c}{r+p}\right)}$$

This contract is feasible, if $0 < c < \left(E_0^1 - \frac{pE_0^{11}}{r+p}\right) - \left(V_0^1 - \frac{V_0^{11}-c}{r+p}\right)$. Comparing the surplus of the venture capitalist under Rule 6 with the surplus which he retains if only the leader is employed (see contract C_4), we obtain the following equivalence:

$$\begin{aligned}
&V_{0,6} - E_{0,6}^F - E_{0,6}^L < V_0^1 - E_0^1 \Leftrightarrow \\
&\left((E_0^1 - \frac{pE_0^{11}}{r+p}) - (V_0^1 - \frac{pV_0^{11}-c}{r+p})\right)(1 - e^{-(r+p)T_6}) < \frac{c}{r}(1 - e^{-rT_6}).
\end{aligned}$$

Taking the feasibility constraints into account, we observe, that the left-hand side of the last inequality is smaller than $c\,(1 - e^{-(r+p)T_6})$. Hence,

$$\begin{aligned}
&V_{0,6} - E_{0,6}^F - E_{0,6}^L < V_0^1 - E_0^1 \Leftrightarrow \\
&c\,(1 - e^{-(r+p)T_6}) < \frac{c}{r}(1 - e^{-rT_6}).
\end{aligned}$$

The last inequality always holds, since for $r, p \in (0, 1)$ it is true, that:

$$c\,(1 - e^{-(r+p)T_6}) < \frac{c}{r+p}(1 - e^{-(r+p)T_6}) < \frac{c}{r}(1 - e^{-rT_6}).$$

Finally, the Rule 7 is an obvious counterpart of the Rule 1 or Rule 2, with the modification that it has an additional stage where only follower alone is financed, while the financing of the leader is frozen. After that, if the the follower did not succeed, both entrepreneurs will be financed either according to contract C_1^{10} or according to contract C_2^{10} (depending on parameters range, see Table 6 for feasibility constrains). Intuitively, the Rule 7 generates lower profit for the venture capitalist, than any of this contracts, because by freezing the financing of the leader he postpones the event of final success. The formal proof is analogical to the case of Rule 6 and is therefore not presented here.

A.4 Proof of Proposition 1

The proof is divided into two parts depending on the sign of A^{11}. If the parameters are such that $A^{11} > 0$, then the feasible contracts are C_1^{11}, C_3^{11} and C_4. On the other hand, if $A_{11} \leq 0$, then the available contracts are C_1^{11}, C_2^{11} and C_4. We will show that in both cases contract C_1^{11} is optimal.

First we show that contract C_1^{11} is always (regardless of A^{11}) preferred to contract C_4. Translated into profits, this is equivalent to the inequality $V_{0,1}^{11} -$

$2E^{11}_{0,1} > V^1_0 - E^1_0$, with $V^{11}_{0,1}$, $E^{11}_{0,1}$, V^1_0, and E^1_0 given in Table 5. After substitution, this can be rewritten as

$$\frac{2(Rp-c)}{r+2p} - \frac{Rp+c}{r+p} + \frac{c}{r} - \left(\frac{2(Rp-c)}{r+2p}e^{-2pT} - \frac{Rp+c}{r+p}e^{-pT} + \frac{c}{r}\right)e^{-rT} > 0.$$

Note that the optimal time is the same for both contracts and is equal to $T = T^1 = T^{11}_1 = -\frac{1}{p}\ln\frac{c}{Rp-c}$. Therefore, $e^{-pT} = \frac{c}{Rp-c}$. Using a substitution

$$x = \frac{c}{Rp-c}, \tag{12}$$

or equivalently $c = Rp\frac{x}{1+x}$, we the rewrite the above inequality as

$$\frac{Rp}{1+x}\left[\frac{2}{r+2p} - \frac{1+2x}{r+p} + \frac{x}{r} - \left(\frac{2x^2}{r+2p} - \frac{x(1+2x)}{r+p} + \frac{x}{r}\right)x^{r/p}\right] > 0.$$

Note that $e^{-pT} = x$ and the assumption $Rp > 2c > 0$ implies that $x \in (0,1)$. Multiplying the last inequality by $(r+2p)(r+p)r(1+x)/(Rp)$ yields

$$r^2 + (p-r)(2p+r)x + p(2rx - 2p - r)x^{1+r/p} > 0.$$

Denote the left-hand side of this inequality as $f(x)$.[24] Then

$$\begin{aligned} f'(x) &= (r+2p)[2rx^{1+r/p} - (p+r)x^{r/p} + (p-r)], \\ f''(x) &= (r+2p)\left[2r\left(1+\frac{r}{p}\right)x^{r/p} - (p+r)\frac{r}{p}x^{-1+r/p}\right]. \end{aligned}$$

First observe that $f(0) = r^2 > 0$, $f(1) = 0$, $f'(1) = 0$, $f''(1) = r(r+p)(r+2p)/p > 0$. Moreover, for $p \le r$, the function f is decreasing on interval $(0,1)$, since $f'(x) < (r+2p)[2rx^{r/p} - (p+r)x^{r/p} + (p-r)] = (r+2p)(p-r)(1-x^{r/p}) < 0$. Hence, $f(x) > f(1) = 0$, for $p \le r$.

On the other hand, for $p > r$ we have $f'(0) = (r+2p)(p-r) > 0$. Therefore, $f(x) > f(0)$ in some neighborhood of 0. Now, assume by contradiction that $f(x_0) = 0$ for some $x_0 \in (0,1)$. Then (according to *Rolle's theorem*) there exists some $x_1 \in (0, x_0)$ such that $f(x_1) = f(0)$, which implies that there exist some $x_2 \in (0, x_1)$ and $x_3 \in (x_0, 1)$ such that $f'(x_2) = f'(x_3) = 0 = f'(1)$. Therefore, the equation $f''(x) = 0$ has at least two solutions in interval $(0,1)$, which is a contradiction, since $f''(x) = 0$ only if $x = \frac{1}{2}$. This proves that contract C^{11}_1 is preferred to contract C_4.

Now, we will show that for $A^{11} > 0$, contract C^{11}_1 is preferred to C^{11}_3. Obviously the latter contract is a limiting case of the former, when the research horizon is

[24]Note that f is $\mathcal{C}^2$ on $(0,1]$.

infinity. However, for contract C_1^{11} the optimal time $T_1^{11} = -\frac{1}{p}\ln\frac{c}{Rp-c}$ is finite. Hence, contract C_1^{11} with research horizon T_1^{11} is more profitable for the venture capitalist than contract C_1^{11} with any other research horizon, including infinite research horizon.[25] Therefore, contract C_1^{11} is better than contract C_3^{11}.

It remains to prove that contract C_1^{11} is preferred to contract C_2^{11}, i.e., that $V_{0,1}^{11} - 2E_{0,1}^{11} > V_{0,2}^{11} - 2E_{0,2}^{11} > 0$, with $V_{0,1}^{11}$, $E_{0,1}^{11}$, $V_{0,2}^{11}$, and $E_{0,2}^{11}$ given in Table 5. This can be rewritten as follows:

$$\begin{aligned} & -\frac{2(Rp-c)}{r+2p}e^{-(r+2p)T_1^{11}} + \frac{2c}{r+p}e^{-(r+p)T_1^{11}} - \\ -&\left(V_0^1 - \frac{2(Rp-c)}{r+2p}\right)e^{-(r+2p)T_1^{11}} + \left(E_0^1 - \frac{2c}{r+p}\right)e^{-(r+p)T_1^{11}} > 0. \end{aligned}$$

Using again the substitution (12), we obtain $e^{-pT_1^{11}} = x$ and $e^{-pT_2^{11}} = x(p+r)/r \cdot [r-p+(r+p)x^{r/p}]/[r+(r+2p)x^{1+r/p}]$. Then, the above inequality can be, after multiplying by $r(p+r)(2p+r)(1+x)/(Rpx^{2+r/p})$, rewritten as follows:[26]

$$2r - [r-p+(r+p)x^{r/p}]\left[\frac{p+r}{r}\cdot\frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}\right]^{1+r/p} > 0.$$

Similarly as in the first part of this proof, denote the left-hand side of this inequality as $g(x)$. Observe that $g(1) = 0$ and that

$$\begin{aligned} g'(x) = \ & \frac{(r+p)(r+2p)}{p^2}\left[\frac{p+r}{r}\cdot\frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}\right]^{1+r/p} x^{-1+r/p} \times \\ & \times\frac{r^2(x-1)+p^2x(x^{r/p}-1)}{r+(r+2p)x^{1+r/p}}, \end{aligned}$$

which is negative, since $0 < x < 1$. Therefore, $g(x) > g(1) = 0$ for all $x \in [0,1)$, which completes the proof.

A.5 Proof of Lemma 1

1. Using the expressions for V_0^{11}, E_0^{11}, V_0^1, and E_0^1 from Table 5 in Appendix B, we obtain

$$\begin{aligned} A^{10} &= \frac{c}{r(p+r)^2}\big(p^2 - pr - r^2 - (r+p)^2e^{-rT} + pre^{-(r+p)T}\big), \\ B^{10} &= \frac{1}{(p+r)(r+2p)^2}\big[c(r+p)(r+2p) - pr(Rp-c) + \\ & \quad +(Rp-c)\big(-(r+2p)^2e^{-(r+p)T} + 2p(r+p)e^{-(r+2p)T}\big)\big], \end{aligned}$$

[25] One can easily see that $\frac{\mathrm{d}}{\mathrm{d}T}(V_{0,1}^{11} - 2E_{0,1}^{11}) < 0$ for $T > T_1^{11}$.

[26] Note that $r-p+(r+p)x^{r/p} > 0$, since $A^{11} = -Rpx/[r(r+p)(1+x)] \cdot [r-p+(r+p)x^{r/p}]$.

with T being the optimal stopping time for contracts C_1^{11} and C_4^{11} from regime (1/1), which is the same, i.e., $T = T^1 = T_1^{11} = -\frac{1}{p}\log\frac{c}{Rp-c}$.

Similarly as in the Proof of Proposition 1 we use the substitution (12), or $c = Rp\frac{x}{1+x}$. In addition, to simplify the expressions, we use another substitution

$$z = \frac{r}{p},$$

or $r = zp$. Given the conditions on parameters, we have $x \in (0,1)$ and $z > 0$. With this substitution, e^{-rT} simplifies to a nice form x^z and the above expressions can be rewritten as follows:

$$\begin{aligned} A^{10} &= \frac{x[1-z-z^2-(1+z)^2x^z+zx^{1+z}]}{(1+x)z(1+z)^2}, \\ B^{10} &= \frac{-z+(1+z)(2+z)x-(2+z)^2x^{1+z}+2(1+z)x^{2+z}}{(1+x)(1+z)(2+z)^2}. \end{aligned}$$

For simplicity denote $a(x)$ and $b(x)$ the numerators of A^{10} and B^{10}, respectively. Note that the signs of A^{10} and B^{10} are the same as the signs of $a(x)$ and $b(x)$, respectively.

We discuss two cases. First, when $1-z-z^2 \leq 0$, then $a(x) < 0$, since $-(1+z)^2x^z + zx^{1+z} = [-(1+z+z^2) - z(1-x)]x^z < 0$. Second, when the inequality $1-z-z^2 > 0$ holds, we will prove a stronger statement that this inequality already implies $b(x) < 0$, regardless of the sign of $a(x)$. Note that for $z > 0$, the condition $1-z-z^2 > 0$ is equivalent to $0 < z < \frac{1}{2}(\sqrt{5}-1) \approx 0.6180$. Obviously $b(0) = -z$ and $b(1) = 0$. Taking the derivatives of $b(x)$ we obtain

$$\begin{aligned} b'(x) &= (1+z)(2+z)\big[1-\big(z+2(1-x)\big)x^z\big], \\ b''(x) &= (1+z)(2+z)x^{-1+z}[2(1+z)x - z(2+z)]. \end{aligned}$$

Then $b'(0) = (1+z)(2+z) > 0$ and $b'(1) = (1+z)(2+z)(1-z) > 0$. The second derivative implies that b is concave in the interval $(0, x_1)$ and convex on $(x_1, 1)$, where $x_1 = \frac{z(2+z)}{2(1+z)} < \frac{1}{2}$, according to the assumption $1-z-z^2 > 0$. Therefore, b has a local maximum (denote it x_2) on interval $(0, x_1)$ and a local minimum on $(x_1, 1)$. Its possible shape is illustrated on Figure 4 in Appendix B Hence, in order to prove that $b(x) < 0$ on $(0,1)$ it remains to show that $b(x_2) < 0$. Although it is not possible to find a closed formula for x_2, we know that

$$x_2^z = \frac{1}{z+2(1-x_2)}.$$

Using this, we obtain

$$
\begin{aligned}
&2[z+2(1-x_2)]b(x_2) = \\
&\quad = \quad 2[-z+x_2(1+z)(2+z)]\,[z+2(1-x_2)] - \\
&\quad\quad\quad -2(2+z)^2x_2 + 4(1+z)x_2^2 = \\
&\quad = \quad -4(1+z)^2x_2^2 + z(z^2+4z+6)x_2 - z(2+z) = \\
&\quad = \quad -[2(1+z)x_2 - z(2+z)]^2 + z[z+2(1-x_2)](z^2+2z-2) < \\
&\quad < \quad 2z[z+2(1-x_2)](z^2+z-1) < 0.
\end{aligned}
$$

As a consequence, $A^{10} > 0$ implies that $G'(T) > 0$ for all $T \geq 0$. Hence the optimal stoping time is infinite.

2. The optimality condition $G(T) = 0$ can be rewritten as $e^{-pT} = \frac{(r+p)A^{10}}{(r+2p)B^{10}}$. The condition $(r+2p)B^{10} < (r+p)A^{10} < 0$ implies that $e^{-pT_2^{10}} \in (0,1)$, i.e., T_2^{10} is positive and finite. Moreover, we have $G''(T) = (r+2p)^2B^{10}e^{-(r+2p)T} - (r+p)^2A^{10}e^{-(r+p)T}$, which yields $G''(T_2^{10}) = (r+p)pA^{10}e^{-(r+p)T} < 0$.

3. We consider two cases. If $B^{10} \geq 0$, then obviously $G'(T) < 0$. If $B^{10} < 0$, then $G'(T) < [-(r+2p)B^{10} + (r+p)A^{10}]e^{-(r+p)T} < 0$ for all $T \geq 0$. Hence, $G(T)$ is monotonically decreasing and the optimal stoping time is zero.

A.6 Proof and numerical simulations for Proposition 2

1. Contract C_4 is feasible whenever $Rp > 2c$. If the second stopping rule is applied, the optimal stopping time is infinity (see discussion in Section 4) and contract C_3^{10} is feasible.

   The conditions $A^{10} > 0$ and $B^{10} < 0$ imply that $pE_0^{11} + 2c < (r+p)E_0^1$ and $(r+2p)V_0^1 < p(R+V_0^{11}) - 2c$ respectively. Moreover, from $V_0^1 > E_0^1 > 0$ we get $(r+p)E_0^1 < (r+2p)V_0^1$. Combining the inequalities, we obtain that

$$
\frac{pE_0^{11}+2c}{r+p} < \frac{p(R+V_0^{11})-2c}{r+2p} \Rightarrow T_1^{10} = -\frac{1}{p}\ln\frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c} > 0,
$$

   which means that contract C_1^{10} is feasible. We have proved that if the feasibility condition $A^{10} > 0$ is satisfied, then the pool of available contracts is C_4, C_1^{10}, C_3^{10}. Further we will compare the surplus which the venture capitalist retains with each contract, in order to choose the optimal one.

   Consider contracts C_4 and C_3^{10}. From the Proof of Lemma 1 we know that contract for $A^{10} > 0$, the contract C_3^{10} is optimal among all contracts with stoping rule 2. As contract C_4 is a degenerate case of this stoping rule (when the research horizon is zero), condition $A^{10} > 0$ then implies that $C_3^{10} \succ C_4$.[27]

Further, let us compare contract C_1^{10} and contract C_3^{10}. In case of contract C_1^{10} the surplus of the venture capitalist is maximized at finite stopping time, $T_1^{10} = -\frac{1}{p} \ln \frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}$. However, if the financing horizon is infinite, then C_1^{10} is identical to contract C_3^{10}. Hence, the former contract is always preferred to the latter.

In summary we get $C_1^{10} \succ C_3^{10} \succ C_4$. Hence, the optimal contract is C_1^{10}. Note that condition $A^{10} > 0$ implies that $E_0^1 > E_{0,1}^F + E_{0,1}^L$. In other words, competing entrepreneurs together require less compensation, than would a single entrepreneur.

2. Assume that $0 > A^{10}(r+p) > B^{10}(r+2p)$. According to Lemma 1, contract C_2^{10} is feasible. Recall, that

$$A^{10} = E_0^1 - \frac{pE_0^{11} + 2c}{r+p}, \qquad B^{10} = V_0^1 - \frac{p(R+V^{11}) - 2c}{r+2p}.$$

Hence, the inequality $A^{10}(r+p) > B^{10}(r+2p)$ implies that

$$0 < (pE_0^{11} + 2c) - E_0^1(r+p) < [p(R+V_0^{11}) - 2c] - V_0^1(r+2p).$$

Since $E_0^1(r+p) < V_0^1(r+2p)$, it necessarily must be that $pE_0^{11} + 2c < p(R+V_0^{11}) - 2c$. Hence, $T_1^{10} > 0$ and contract C_1^{10} is feasible as well. Therefore, the pool of contracts consists of C_1^{10}, C_2^{10} and C_4.

Let us first compare contracts C_1^{10} and C_2^{10}. The former contract is preferred to the latter, if and only if

$$V_{0,1}^{10} - V_{0,2}^{10} > \left(E_{0,1}^{(10),L} + E_{0,1}^{(10),F}\right) - \left(E_{0,2}^{(10),L} + E_{0,2}^{(10),F}\right), \tag{13}$$

where all value functions are given in Table 6 in Appendix B. After straightforward but tedious calculations we conclude that inequality (13) is equivalent to

$$T_1^{10} < T_2^{10} - \frac{1}{r+p} \ln \frac{2c + pE_0^{11} - E_0^1(r+p)}{2c + pE_0^{11}}.$$

In that case, contract C_1^{10} is optimal. Otherwise, the optimal contract is C_2^{10}.

[27] The relation "$\succ$" is used to denote preferences between contracts from the viewpoint of the venture capitalists, i.e., that one contract generates a larger profit for the venture capitalist than another one.

Note, that now it is sufficient to prove, that C_2^{10} is preferred to contract C_4, always when the feasibility condition $0 > A^{10}(r+p) > B^{10}(r+2p)$ holds. If this is the case, then C_1^{10} will be optimal, when $C_1^{10} \succ C_2^{10} \succ C_4$ and C_2^{10} will be optimal, when $C_2^{10} \succ C_1^{10}$ and $C_2^{10} \succ C_4$.

Contract C_2^{10} is better, than contract C_4, if and only if the following inequality holds:

$$V_0^1 e^{-(r+2p)T} + \frac{p(1+V_0^{11}) - 2c}{r+2p}\left(1 - e^{-(r+2p)T}\right) -$$
$$-E_0^1 e^{-(r+p)T} - \frac{2c + pE_0^{11}}{r+p}\left(1 - e^{-(r+p)T}\right) > V_0^1 - E_0^1,$$

This can be re-written in the form

$$A^{10}\left(1 - e^{-(r+p)T}\right) > B^{10}\left(1 - e^{-(r+2p)T}\right), \tag{14}$$

where A^{10} and B^{10} are defined above. Consider now two cases:

(a) If $0 > A^{10} > B^{10}$, then inequality (14) obviously holds, since $0 < \left(1 - e^{-(r+p)T}\right) < \left(1 - e^{-(r+2p)T}\right)$.

(b) If $A^{10} \leq B^{10} < 0$ we show numerically that (14) holds. In the numerical simulations we considered without loss of generality (see Remark 1) values $r = 0.05$ and $R = 1$. Using a grid 0.001×0.001 on the set of all positive (p, c), such that $p > 2c$ and $\frac{r+2p}{r+p} B^{10} < A^{10} \leq B^{10} < 0$, we plotted points where profit of the venture capitalist under contract C_2^{10} exceeds his profit under contract C_4. The simulations show that this is the case everywhere in the defined domain. Figure 5 illustrates the case for $r = 0.05, p = 0.5$, where $\Delta := A^{10}\left(1 - e^{-(r+p)T}\right) - B^{10}\left(1 - e^{-(r+2p)T}\right)$.

3. According to Lemma 1, condition $A^{10}(r+p) < B^{10}(r+2p)$ implies that contract C_2^{10} is not feasible. Moreover, condition $(2c + E_0^{11}) < p(R + V_0^{11}) - 2c$ implies that C_1^{10} is feasible. Therefore, we choose the optimal contract between C_1^{10} and C^4. Using numerical simulations, we have verified that in domain R_4, given that the feasibility conditions are satisfied for contract C_1^{10}, the venture capitalist prefers to finance the leader alone (contract C_4 is better than contract C_1^{10}). Again, the numerical simulations were performed for $r = 0.05$ and $R = 1$, using a grid of 0.001×0.001 for parameters (p, c).

4. If $(2c + E_0^{11}) > p(R + V_0^{11}) - 2c$, the only feasible (hence, optimal) contract is C_4.

B Appendix: Tables and figures

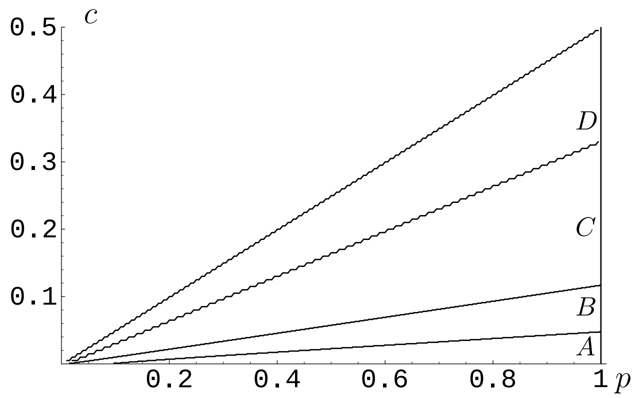

Figure 3: Feasibility of contracts in regime (1/0)

Notes to Figure 3:

1. Contract C_1^{10} is feasible in domains A, B and C;
2. Contract C_2^{10} is feasible in domain B;
3. Contract C_3^{10} is feasible in domain A;
4. Contract C_4 is feasible in domains A,B,C and D.

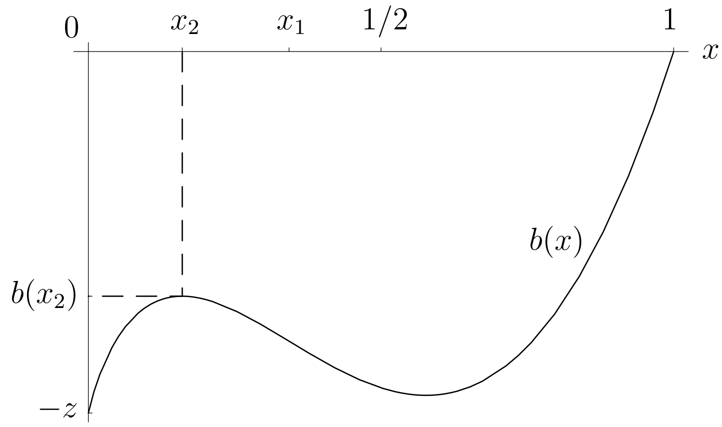

Figure 4: Shape of function $b(x)$

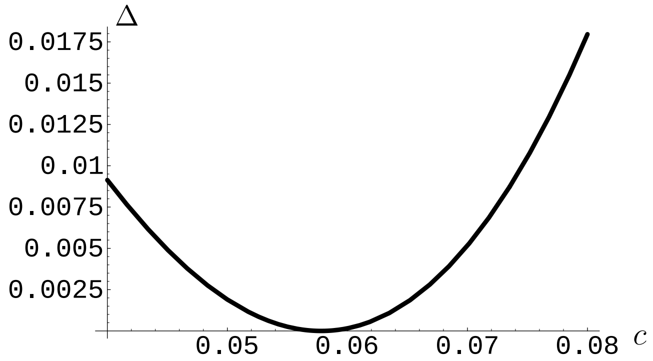

Figure 5: Regime (1/0): Illustration for Case 2; $p = 0.5$, $r = 0.05$.

Notes to Tables 5 and 6:

1. Stopping rules are the following:

   (a) Rule 1: both entrepreneurs are financed at most for T_k^{ij} periods.

   (b) Rule 2: both entrepreneurs are financed for at most T_k^{ij} periods. If no change of regime occurred, then in regime (1/0) the Follower is abandoned and the Leader is financed at most for additional T^1 periods. In regime (1/1), one of the entrepreneurs is randomly chosen and is financed for at most T^1 periods.

   (c) Rule 3: a single entrepreneur is financed for T^1 periods at most.

2. The feasibility conditions ensure that the optimal financing horizon is positive or infinite, depending on the contract. Recall that we always assume that $Rp > 2c$.

	C_1^{11}	C_2^{11}	C_3^{11}	C_4
Stoping rule	Rule 1	Rule 2	Rule 2	Rule 3
Share of entrep.	$s_{t,1}^{11} = \frac{c}{p} + E_t^{11}$	$s_{t,2}^{11} = \frac{c}{p} + E_t^{11}$	$s_{t,3}^{11} = \frac{c}{p} + E_t^{11}$	$s_t^1 = \frac{c}{p} + E_t^1$
Value fnct. of entrep.	$E_{t,1}^{11} = \frac{c}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$E_{t,2}^{11} = \left(\frac{1}{2}E_0^1 - \frac{c}{r+p}\right) \cdot$ $\cdot e^{-(r+p)(T-t)} + \frac{c}{r+p}$	$E_{t,3}^{11} = \frac{c}{r+p}$	$E_t^1 = \frac{c}{r}\left(1 - e^{-r(T-t)}\right)$
Value of the venture	$V_{t,1}^{11} = \frac{2(Rp-c)}{r+2p} \cdot$ $\cdot \left(1 - e^{-(r+2p)(T-t)}\right)$	$V_{t,2}^{11} = \left(V_0^1 - \frac{2(Rp-c)}{r+2p}\right) \cdot$ $\cdot e^{-(r+2p)(T-t)} + \frac{2(Rp-c)}{r+2p}$	$V_{t,3}^{11} = \frac{2(Rp-c)}{r+2p}$	$V_t^1 = \frac{Rp-c}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$
Optimal time	$T_1^{11} = -\frac{1}{p} \ln \frac{c}{Rp-c}$	$T_2^{11} = -\frac{1}{p} \ln \frac{r+p}{r+2p} \frac{E_0^1 - \frac{2c}{r+p}}{V_0^1 - \frac{2(Rp-c)}{r+2p}}$	$T_3^{11} \to \infty$	$T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$
Feasibility condit.		$0 > A^{11}(r+p) > B^{11}(r+2p)$	$A^{11} > 0$	

Table 5: Contracts and expected values: regime (1/1)

	C_1^{10}	C_2^{10}	C_3^{10}	C_4
Stoping rule	Rule 1	Rule 2	Rule 2	Rule 3
Share of the L.	$s_{t,1}^{11} = \frac{c}{p} + E_{t,1}^F - E_0^{11}$	$s_{t,2}^L = \frac{c}{p} + E_{t,2}^L$	$s_{t,3}^L = \frac{c}{p} + E_{0,3}^L$	$s_t^L = s_t^1 = \frac{c}{p} + E_t^1$
Value fnct. of the L.	$E_{t,1}^L = \frac{c+pE_0^{11}}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$E_{t,2}^L = \left(E_0^1 - \frac{c+pE_0^{11}}{r+p}\right) \cdot$ $\cdot e^{-(r+p)(T-t)} + \frac{c+pE_0^{11}}{r+p}$	$E_{t,3}^L = \frac{c+pE_0^{11}}{r+p}$	$E_t^1 = \frac{c}{r}\left(1 - e^{-r(T-t)}\right)$
Share of the F.	$s_{t,1}^F = \frac{c}{p} + E_{t,1}^F - E_0^{11}$	$s_{t,2}^F = \frac{c}{p} + E_{t,2}^F - E_0^{11}$	$s_{t,3}^F = \frac{c}{p} + E_{t,3}^F - E_0^{11}$	
Value fnct. of the F.	$E_{t,1}^F = \frac{c}{r+p} \cdot$ $\cdot (1 - e^{-(r+p)(T-t)})$	$E_{t,2}^F = \frac{c}{r+p}\left(1 - e^{-(r+p)(T-t)}\right)$	$E_{t,3}^F = \frac{c}{r+p}$	
Value of the venture	$V_{t,1}^{10} = \frac{p(R+V_0^{11})-2c}{r+2p} \cdot$ $\cdot \left(1 - e^{-(r+2p)(T-t)}\right)$	$V_{t,2}^{10} = \left(V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}\right) \cdot$ $\cdot e^{-(r+2p)(T-t)} + \frac{p(R+V_0^{11})-2c}{r+2p}$	$V_{t,3}^{10} = \frac{p(R+V_0^{11})-2c}{r+2p}$	$V_t^1 = \frac{Rp-c}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$
Optimal time	$T_1^{10} = -\frac{1}{p} \ln \frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}$	$T_2^{10} = -\frac{1}{p} \ln \frac{r+p}{r+2p} \frac{E_0^1 - \frac{2c+pE_0^{11}}{r+p}}{V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}}$	$T_3^{10} \to \infty$	$T_1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$
Feasibility cond.	$2c + E_0^{11} < p(R + V_0^{11}) - 2c$	$0 > A^{10}(r+p) > B^{10}(r+2p)$	$A^{10} > 0$	

Table 6: Contracts and expected values: regime (1/0)