
Horizontal and Vertical Tax Externalities in a Multicountry World

by

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Horizontal and Vertical Tax Externalities in a Multicountry World*  

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Abstract  

Recent contributions on tax competition recognize the interaction between both horizontal and vertical tax externalities in a single federation. In this paper, we extend the theoretical analysis to a framework with multiple federations (a Union). We show that the relative size of a federation in the Union determines not only the extent but also the direction of the tax inefficiency. The equilibrium state tax is lower in relative small countries but surprisingly, vertical externalities are more likely to dominate there, i.e. for a relative small federation, the non-cooperative local tax rate is lower than for a relative large federation but still higher than the one observed in absence of tax competition. This result seems to contradict recent theoretical findings where a lower equilibrium state tax is followed by a dominant horizontal externality.

JEL Codes: H21, H7, H3  

Key Words: Tax Competition, Horizontal Externalities, Vertical Externalities, Fragmentation, Fiscal Federalism

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1 Introduction

This paper identifies and focuses on a specific problem arising from single country models of tax competition and its implications for empirical work. The novelty of our approach is to show how horizontal and vertical tax externalities interact in the context of a multi-country world (a Union). The paper’s take is that the relative size of a federation in the Union determines not only the extent but also the direction of the tax inefficiency at state level. The equilibrium state tax is optimally set at a lower rate in countries that are small relative to the size of the Union but, surprisingly, the vertical externality is more likely to dominate in these countries, i.e. the non-cooperative state tax rate is still higher than the one observed in the absence of tax competition within the federation.

In the debate on tax competition and inefficiency in tax setting, two competing effects are likely to distort tax levels in opposite directions. In a context of a dominant horizontal tax externality, taxes and public expenditure are expected to be set inefficiently low in equilibrium. In contrast, a dominant vertical tax externality points towards excessively high tax levels. A relevant question is then which externality dominates the other and under what circumstances. Keen and Kotsogianis (2002) address this issue within a unified model featuring a single federation. According to this model, it is possible to associate the extent and direction of the tax inefficiency with the degree of fragmentation of the federation, i.e. the number of state governments (jurisdictions). State tax rates decrease with fragmentation if horizontal externalities dominate but increase with fragmentation if vertical exter-
nalities dominate\(^1\). The authors conclude therefore, that the direction of the tax inefficiency is a question of empirical nature. Based on this result, recent empirical literature on tax externalities is trying to verify whether vertical or horizontal tax externalities dominate in a given federation. Different tax definitions and econometric approaches have been explored to answer this question in a number of countries considered in isolation. Brülhart and Jametti (2006) use the relative size of municipalities (fragmentation) to test for the dominance of horizontal or vertical tax externalities in Switzerland. They find that municipalities that account for a smaller share of cantonal population have higher tax rates and conclude, therefore, that this result supports the hypothesis of dominant vertical externalities.

Besides the fact that ignoring the outside world may be a too unrealistic assumption\(^2\) when trying to test empirically models of tax competition, we argue in this paper that these empirical findings based on a single federation may simply lead to wrong conclusions also with regard to the direction of the tax inefficiency at the state level. In Section 2, we extend the model of Keen and Kotsogiannis (2002) to investigate the relative importance of tax externalities in a multicountry setting. The Union consists of a sub-set of \(l\) countries (federations) in the world, each of them consisting of \(N_l\) identical jurisdictions (state governments). The number of jurisdictions, however, can differ across federations. Therefore, a given federation \(L\) is small

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\(^1\)Whether equilibrium tax rates are too high or too low depend essentially on the relative elasticity of the tax base both, of the federal and state governments and on the relative government sizes. See Keen and Kotsogiannis (2002; 2004), and Brülhart and Jametti (2006)

\(^2\)Boadway (2001) finds that Ontario’s tax rate is not significantly influenced by any other provincial tax rate and argues that this may be due to the fact that Ontario’s major competitor for capital could be the United States. Karkalakos and Kotsogiannis (2007) find some evidence of U.S neighboring corporate income tax policy negatively affecting the tax setting of British Columbia, Ontario and Quebec.
or big relative to the size of the Union, depending on $R \equiv N_L / \sum N_t$, i.e. the number of jurisdictions in federation $L$ over the number of jurisdictions in the Union as a whole. There are two sources of horizontal externalities: between jurisdictions within a country and between jurisdictions across federations in the economic area. In our model, capital is not only perfectly mobile across jurisdictions in a single federation but across jurisdictions in the Union. We assume, further, that vertical externalities arise between state and federal government only within each federation. To explore the inefficiency in the tax setting of state governments under tax competition, we adopt as a benchmark the symmetric tax equilibrium in a given federation, that is, the equilibrium in which all states within a federation set the same tax rate. This benchmark, which coincides with the social optimum tax rate in single-country models of tax competition, let us explore the welfare gains of tax coordination in a given country when competition for mobile capital with other federations in the Union becomes relevant. To this extent, this paper is the first one that explores the welfare implications of enhancing tax competition between jurisdictions in a federation when both horizontal and vertical tax externalities are present and interaction with other jurisdictions outside the country is also possible, i.e. competition for capital among jurisdictions of different federations in the Union is allowed.

In Section 3, we show that whether vertical or horizontal tax externalities dominate can be determined on the basis of the relative importance of a given federation

\footnote{Wildasin (1989), Fuente and Gardner (1990) and Razin and Sadka (1991) have analyzed the potential gains from capital tax coordination, but only within highly simplified models of the Zodrow–Mieszkowski (1986) type, where only horizontal tax externalities between federal governments are considered. Sørensen (2004) presents a quantitative analysis of the gains from global and regional tax coordination also between federal governments in a model that includes income redistribution.}
in the corresponding economic area of competition (the Union). In particular, we find that the likelihood of a dominant vertical externality decreases with the relative size of the federation in the Union. This result, however, is independent of the noncooperative tax setting in equilibrium. In fact, our result is in line with other literature in the sense that for a relatively small federation, the non-cooperative state tax rate in equilibrium is set at a lower rate than that for a relative large federation\(^4\). More specifically, the equilibrium state tax is set at a lower rate if the country is small relative to the Union but the vertical externality is more likely to dominate, i.e. for a relatively small federation, the non-cooperative state tax rate will be lower than that for a relative large federation but the tax rate is still higher than the one observed in the absence of tax competition within the country. Section 4 summarizes and concludes.

Related literature: Several lines of research have emphasized the importance of horizontal and vertical tax externalities for the theory of fiscal federalism. Originally developed by Oates (1972) and formally modelled by Gordon (1983), Wilson (1986) and Zodrow and Mieszkowski (1986), the basic idea behind horizontal tax externalities is that, when a region increases its capital tax rate, some amount of capital is reallocated to other regions\(^5\). This capital movement represents a positive externality, implying a tendency for taxes and public expenditures to be set inefficiently low in equilibrium. Vertical tax externalities on the other hand, have

\(^4\)See for example Keen and Kanbur (1993)

\(^5\)See also Wildasin (1989), and Wilson (1999) for an extensive review of the literature on tax competition. Wildasin and Wilson (2004) describe some approaches to modelling the potential benefits of tax competition.
been examined more recently. They arise when different levels of government tax the same base. Each level of government neglects the adverse effect it has on the other by raising its tax rate, thereby causing the common tax base to shrink. This tax externality points towards excessively high state taxes. Recent literature includes Johnson (1988), Keen (1998), Wrede (2000), Hoyt (2001) and Dahlby and Wilson (2003). Both issues have been also addressed for the case in which policy makers are revenue-maximizing Leviathans in Wrede (1996) and Keen and Kotsogiannis (2003) and in the context of imperfectly competitive markets in Janeba (1998).

Our paper is technically closest to Keen and Kotsogiannis (2002) who analyze the inefficiency in the tax setting of state governments when horizontal and vertical tax externalities arise. Based on a single-country model composed by identical jurisdictions, they show that state taxes are too high or too low in equilibrium, depending on the relative elasticity of the supply of savings and the demand for capital, and on the extent to which state governments tax rents. While we also consider the interplay of horizontal and vertical externalities as first order when analysing the inefficiency in the tax setting of state governments, we also acknowledge that a broader view including the possibility of tax interaction with jurisdictions in a multicountry set-up is warranted. In this respect, only Grazzini and Petretto (2007) extend the single-country model to account for differences in the institutional structure. In their model, however, only competition among federal governments arises and interaction among jurisdictions across federations is not allowed. They find namely that

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a unitary country sets a higher federal capital tax rate in equilibrium than a federal country.

Several lines of research have emphasized the importance of country size in models of tax competition. Bucovetsky (1991), Wilson (1991), Keen and Kanbur (1993) and Eggert and Haufler (1998) explore the consequences of differences in country size for the distribution of the gains from tax coordination at federal level. Keen and Kanbur (1993) for example, develop a two-country model to address a range of consequent policy concerns, focusing particularly on the role of country size. In line with our result, the model presents a unique noncooperative equilibrium in which the smaller country charges a lower tax that the large country. Most relevant to our paper, Janeba and Wilson (2005) analyze the effects of decentralization in a model where horizontal and vertical tax externalities are present. They show that decentralization, by enhancing vertical tax competition may have a counterdistortionary role to offset the inefficiencies due to horizontal tax competition in terms of public goods underprovision.

Last, a number of empirical studies have motivated this paper. Besley and Rosen (1998), Hayashi and Boadway (2001), Esteller-More and Solé-Ollé (2001; 2002), Goodspeed (2000; 2002), Andersson et al. (2004) and Devereux et al. (2004) have applied different tax definitions and econometric approaches to test for vertical tax externalities in a single federation. Most prominently Brülhart and Jametti (2006) use a definition of relative jurisdictional size to test for the dominance of horizontal

\footnote{Although this result might not be directly comparable to ours since we analyze the effect of country size on the tax setting of state governments.}

\footnote{See Brueckner (2003) for an extensive review of the previous empirical literature on horizontal tax externalities.}
vs. vertical tax externalities in Switzerland. They find that municipalities that account for a smaller share of cantonal population set higher tax rates and conclude, surprisingly in terms of our contribution, that this result supports the hypothesis of a dominant vertical externality. Most relevant to our paper, Boadway (2001) suggests that US neighboring state tax policy may be relevant when testing for tax externalities at provincial level in Canada. Karkalakos and Kotsogiannis (2007) find some evidence in this direction. Relative to the empirical work, the paper’s contribution is two-fold. First, a formal model builds on previous literature on tax competition, most notably by allowing tax competition among jurisdictions in a multicountry set-up, as suggested by recent empirical results. Second, by looking specifically at relative country size, the formal framework validate the insights of recent theoretical results (lower state taxes in relative small countries) but also identify the limits of recent empirical and theoretical literatures (state taxes in relative small countries might be still higher than the ones observed in absence of tax competition).

2 The model

Assume a Union consisting of a sub-set of \( l > 1 \) countries (or federations) in the world, where each one consists of \( N_l \) identical jurisdictions (state governments) \( j \). The number of jurisdictions, however, can differ across federations. For example, given two possible federations Germany (\( DE \)) and Switzerland (\( CH \)), the number of local governments in each federation may differ \( N_{DE} \gtrless N_{CH}^{9} \). In each jurisdiction

\footnote{Our model builds on Keen and Kotsogiannis (2002) one-federation’s framework. We use their notation where possible, in order to facilitate comparability.}
a single firm produces a private good according to a strictly concave production function $F(k_{lj})$, where capital $k_{lj}$ is the only input. Capital is costlessly mobile not only across jurisdictions in a given federation but in the whole Union. Due to this feature, capital in the Union earns a unique post-tax return $\rho$ in each jurisdiction. In a given federation $L$, the central and state governments tax capital at a consolidated rate $\tau_{Lj} = T_L + t_{Lj}$ where $T_L$ denotes the federal tax rate on each unit of capital allocated in federation $L$ and $t_{Lj}$ denotes state’s tax rate on each unit of capital allocated in jurisdiction $j$. Normalizing the price of the private good to one, the arbitrage condition $F'(k_{lj}) = \rho + \tau_{lj}$, defines the demand for capital in each jurisdiction $j$ as $K_{lj} = K(\rho + \tau_{lj})$, with $K'(\rho + \tau_{lj}) = 1/F''(K_{lj}) < 0$. Further, rents arising in a given jurisdiction $\pi(\rho + \tau_{lj})$ are defined as the difference between the value of production and the cost of capital: $\pi(\rho + \tau_{lj}) = F(K_{lj}) - F'(K_{lj})K_{lj}$\textsuperscript{10}.

Taxes collected at the central and local levels are spent exclusively on the provision of two distinct publicly provided goods, which are produced with constant returns. The budget constraint faced by each local government is defined by

$$g_{lj} = t_{lj}K_{lj}(\rho + \tau_{lj}) \quad (1)$$

where $g_{lj}$ is a local publicly provided good. The central government in each of the countries in the union faces the following budget constraint:

$$G_l = \frac{1}{N_l} \sum_{j=1}^{N_l} T K_{lj}(\rho + \tau_{lj}) \quad (2)$$

\textsuperscript{10}For simplicity and without loss of generality, we assume that rents are untaxed.
where $G_i$ is the amount of federal publicly provided good spent in each jurisdiction within the federation. We assume here that the federal government allocates its total tax receipts equally across states and recall jurisdictions are identical within countries. We assume, further, that there are no intergovernmental transfers neither central-local government transfers nor between local governments.

As in Keen and Kotsogiannis (2002), there is a single consumer in each state $j$. He maximizes the intertemporal quasi-linear utility function $U_{ij}(C_1, C_2, g_{ij}, G_i) = u(C_1) + C_2 + \Gamma(g_{ij}, G_i)$, where $C_1$ and $C_2$ are private consumption in the first and second period respectively and $\Gamma(.)$ represents the utility she derives from the provision of local as well as federal public goods. Both $u(.)$ and $\Gamma(.)$ are strictly increasing concave functions. Each consumer is endowed with an identical amount of income $e$ at the beginning of the first period and in the second receives principal and interest on his savings, which are defined as $S(\rho)$, with $S' \geq 0$.

The indirect utility function can be written as:

$$U_{ij}(\rho, \tau_{ij}, g_{ij}, G_i) \equiv u_{ij}(e - S(\rho)) + (1 + \rho)S(\rho) + \pi(\rho + \tau_{ij}) + \Gamma(g_{ij}, G_i) \quad (3)$$

The after-tax rate of return $\rho$ in the Union is determined by the market-clearing condition

$$\sum_l N_l S(\rho) = \sum_l \sum_{j=1}^{N_l} K_{lj}(\rho + \tau_{ij}) \quad (4)$$

Recall that capital is costlessly mobile across countries within the Union, therefore, savings provide the stock of capital for the productive sector within the Union$^{11}$.

$^{11}$In Keen and Kotsogiannis' (2002) one-federation set-up, savings provide the stock of capital
A change in state $l_j$’s tax rate on $\rho$ is defined by

$$\frac{\partial \rho}{\partial \tau_{lj}} = \frac{K'_{lj}(\rho + \tau_{lj})}{\sum_i N_i S'_i(\rho) - \sum_i \sum_{j=1}^{N_i} K'_{lj}(\rho + \tau_{lj})}$$

(5)

If we impose symmetry of state tax rates within the federation, all states in the federation set the same tax but these tax rates could differ from the ones set by the state governments in other federations within the Union ($\tau_{lj} = \tau_l$, $\forall j$). In turn,

$$p'(\tau_l) = \frac{\partial \rho}{\partial \tau_l} = \frac{N_l K'_{l}(\rho + \tau_l)}{\sum_i N_i [S'_i(\rho) - K'_l(\rho + \tau_l)]} = N_l \frac{\partial \rho}{\partial \tau_{lj}} \in [-1, 0]$$

(6)

We investigate thereby the effect of uncoordinated vs coordinated state tax policy for a given federation in a multi-country context, disregarding at this point the potential welfare effects of international tax coordination\textsuperscript{12}. State and federal governments are assumed to be perfectly benevolent. They maximize the welfare of their own inhabitants and do not take into account the effect of their actions on residents of other states or countries outside the federation. The strategic policy variable of each policy maker is the tax rate at their disposal.

\textsuperscript{12}Within a multicountry set-up, we look at the welfare implications of state tax coordination in a given federation. Fuest and Huber (1999) and Cremer and Galvani (2000) even question the feasibility of tax coordination between federal governments. We disregard, therefore, the possibility of tax coordination at state level for all jurisdictions in the Union.
3 Vertical vs Horizontal Tax Externalities

3.1 Inefficiency in the setting of state taxes

Given the two potential sources of tax externalities, state taxes could be set under tax competition too low (if the horizontal externality dominates) or too high (if the vertical externality dominates) compared to the outcome in absence of tax competition within the federation. Write the welfare of the typical citizen in state $l_j$ using the indirect utility function (3) and the local and central government budget constraints (1) and (2),

$$W_{lj} = u_{lj}(e-S(\rho)) + (1+\rho)S(\rho) + \pi(\rho+\tau_{lj}) + \Gamma[t_{lj}K_{lj}(\rho+\tau_{lj}), \frac{1}{N_l}\sum_{j=1}^{N_l}TK_{lj}(\rho+\tau_{lj})]$$  \hspace{1cm} (7)

The first order condition of the government in state $l_j$, evaluated at the symmetric equilibrium, is

$$\frac{\partial W_{lj}}{\partial \tau_{lj}} = -K_{lj} + \Gamma_g[K_{lj} + t^*K'_{lj}(1 + \frac{1}{N_l}p')] + \frac{1}{N_l}\Gamma_gT^*K'_{lj}(1 + p') = 0$$  \hspace{1cm} (8)

Condition (8) defines the equilibrium state tax rate. Denoting welfare in a symmetric equilibrium by $W_t$ where $(\tau_{lj} = \tau_t, \forall j)$, the effect of a coordinated increase in all state taxes within a single federation is

$$W_{tl} = -K_{lj} + \Gamma_g[K_{lj} + t^*K'_{lj}(1 + p')] + \frac{1}{N_l}\Gamma_gT^*K'_{lj}(1 + p')$$  \hspace{1cm} (9)

The non-cooperative Nash equilibrium for the state tax rate is defined by equation
and setting equation (9) to zero implicitly defines the state tax rate that prevails when state governments coordinate their tax policy in a given federation\(^{13}\). The sign of \(W_{tt}\) evaluated at the non-cooperative state tax equilibrium indicates which externality dominates. For a dominant horizontal externality, \(W_{tt} > 0\) meaning that state taxes are too low. For a dominant vertical externality, \(W_{tt} < 0\) meaning that state taxes are too high under state tax competition within the federation. To investigate which externality dominates subtract (8) from (9),

\[
W_{tt} = [\Gamma g t^* p' + \Gamma G T^*(1 + p')] (1 - \frac{1}{N_t}) K'_{ij} \tag{10}
\]

Since \((1 - \frac{1}{N_t}) K'_{ij}\) is unambiguously negative and \(p' \in [-1, 0]\), the direction of inefficiency in the equilibrium state tax turns on the balance between the effects in the first bracketed term in (10). Further, in symmetric equilibrium \(G_l = T^* K (\rho + \tau_l)\) and \(g_l = t_l K (\rho + \tau_l)\) and therefore the vertical externality dominates if and only if

\[
|p'| < \frac{\Gamma G G}{\Gamma G G + \Gamma g g} \tag{11}
\]

Define \(R \equiv N_\mathcal{L}/\sum_l N_l \leq 1\) as the relative importance of a given federation \(\mathcal{L}\) in the Union, measured as the number of identical state governments in federation \(\mathcal{L}\), that is \(N_\mathcal{L}\) over the number of identical state governments in the Union as a whole, that is \(\sum_l N_l\). In the one-federation set-up \(\sum_l N_l = N_\mathcal{L}\), what is equivalent to \(R = 1\).

\(^{13}\)In a single-federation model, this benchmark coincides with the social optimum state tax rate. In a multicountry set-up, however, this is not necessarily true since state tax coordination between two countries (or a sub-set of them) in the Union could be welfare improving. We are grateful to Jürgen von Hagen for suggesting this interpretation. See Sørensen (2004) for a distinction between global tax coordination versus coordination among a subgroup of countries.
Proposition 1 For a given federation $L$ with $N_L$ identical state governments, the non-cooperative equilibrium state tax rate (implicitly defined in equation (8)) increases with the relative size of the federation $L$ in the Union (with $R$).

Proof. [Sketch: see appendix] First note that in a symmetric equilibrium $\tau_l = \tau, \forall l$ and therefore equation (6) can be rewritten as $p'(\tau) = R \frac{K'(\rho+\tau)}{|S'\rho - K'(\rho+\tau)|}$. The non-cooperative equilibrium state tax rate defined implicitly in (8) can be rewritten as $\Gamma_g = \frac{K - \frac{1}{N_L} \Gamma_g T^* K'(1+\rho')}{[K+\tau^* K'(1+\tau)]} = C$. See first that the right hand side of the last expression is increasing in $p'(\tau)$, that is $\frac{\partial C}{\partial p'(\tau)} < 0$ for a given $T^*$. From equation (6) in symmetric equilibrium we know that $\frac{\partial p'(\tau)}{\partial R} > 0$, and recall $\Gamma_{gg} < 0$. Therefore an increase in $R$ is associated with a higher amount of locally provided public goods and thereby a higher equilibrium state tax rate, that is $\frac{\partial \tau^*}{\partial R} > 0$. ■

The intuition goes as follows: As the number of federations (and their jurisdictions) in the Union increases, the effect of both horizontal and vertical tax competition on state taxes in federation $L$ go in the same direction. Horizontal tax competition is now increased because capital is costlessly mobile across a larger number of state governments in the Union and therefore, a lower equilibrium state tax rate is expected as $\sum L_i$ increases. Vertical tax competition also leads to lower equilibrium state tax rate as the number of federations in the Union increases. In principle, the effect of the vertical tax externality on the equilibrium state tax is independent of changes in the number of jurisdictions in the Union$^{14}$. An increase in the state tax rate in jurisdiction $jL$ would generate, however, a cost for all jurisdictions in terms

\footnote{An increase in the state tax rate in $jL$ would only reduce federal expenditure in this jurisdiction (as in all others in the country) by $1/N_L$. This, the vertical tax externality effect, remains unaffected since in our analysis we assume that $N_L$ remains fixed.}
of federal expenditure because of the capital outflow outside federation $L$ (the cost of the reduction in the federal tax base). This effect clearly increases with $\sum_i N_i$ because competition for capital is increased.

We are interested now to see how the direction in the inefficiency in the setting of state taxes (vertical vs horizontal tax externalities) is affected by the relative importance of a federation $L$ in the Union:

**Proposition 2** For a given federation $L$ with $N_L$ identical state governments, there exist a critical value $\bar{R}$ such that: (a) If $R \equiv N_L/\sum_i N_i < \bar{R}$ the vertical tax externality dominates and (b) if $R > \bar{R}$ the horizontal tax externality dominates.

**Proof.** First look at the expression $[\Gamma_g t^* p' + \Gamma_G T^*(1 + p')]$ in equation (10) and recall in symmetric equilibrium $p'(\tau) = R \frac{K'(\rho + \tau)}{S'(\rho) - K'(\rho + \tau)} \in [-1, 0]$, with $\frac{\partial p'(\tau)}{\partial R} > 0$. Since from Proposition 1, $\frac{\partial t^*}{\partial R} > 0$, it follows that the expression between brackets is decreasing in $|p'(\tau)|$ for given values of $T^*$, $\Gamma_G, \Gamma_g$. Therefore if there exists $\bar{R} < 1$ such that $[\Gamma_g t^*(\bar{R}) p'(\bar{R}) + \Gamma_G T^*(1 + p'(\bar{R}))] = 0$, for all $R < \bar{R}$ it follows that $W_{tL} < 0$ and the vertical tax externality dominates and for all $R > \bar{R}$, it follows that $W_{tL} > 0$ and the horizontal tax externality dominates. Further, if for the given values of $T^*, \Gamma_G, \Gamma_g$, and $R = 1$, it follows that $[\Gamma_g t^* p' + \Gamma_G T^*(1 + p')] \geq 0$, then the vertical tax externality dominates for all possible $R$. $\blacksquare$

The intuition goes as follows: Consider the case where only the horizontal tax externality is present as in Zodrow and Mietszkowski (1986). It is clear from (8) and (9) that as $R \to 0$, the state tax rate that prevails when state governments coordinate
their tax policies in a given federation converges to the non-cooperative equilibrium state tax rate, that is $t^{CO} \to t^*$. The state tax rate under cooperation within a federation converges to the non-cooperative one as the number of state governments outside the federation (but within the Union) increases. Consider now the case where only the vertical externality is present like for example in Dahlby et al (2000). Note that, in this case, the relation between the non-cooperative equilibrium and the one under cooperation keeps constant. For a given $N_L$, no matter what the value of $R$, the source of the vertical externality is to be found within the federation between the federal government and the $N_L$ state governments in the federation.

In the one federation set-up with $R = 1$, we know from condition (11) that the vertical externality dominates if and only if $|p'| < \frac{G + G}{G + G + G + g}$. In the context of the Union, however, we know that $\frac{\partial |p'(\tau)|}{\partial R} > 0$ and, therefore, the likelihood of a dominant vertical externality decreases with $R$, that is, if the number of jurisdictions in the federation $N_L$ represents a small enough part of the Union $\sum L N_l$, the vertical externality dominates. Note that if the vertical externality dominates in the one federation set-up with $R = 1$, it will dominate in the context of the Union with $R < 1$. In contrast, if the horizontal externality dominates in the one-federation set-up, it would be possible that actually vertical externalities turn to dominate as the number of jurisdictions in the Union increase ($R$ becomes small enough) or that the inefficiency associated with this dominant horizontal externality is less important and thereby the state tax rate is set closer to the one under state tax coordination within the federation.

**Corollary 1** For a given federation $L$ with $N_L$ identical state governments, the non-
cooperative equilibrium state tax rate and the likelihood of a dominant horizontal externality increases with $R$.

**Proof.** It follows immediately from Proposition 1 and Proposition 2. ■

4 Concluding remarks

A main issue in the recent trend towards economic integration has been the dramatic rise in international capital flows and the associated implications of fiscal competition. It has been argued, that a lack of tax coordination will lead to a "race to the bottom", as investors reallocate some of their capital to regions that undercut capital income taxes. More recently, the importance of vertical externalities in federal tax structures has been recognized. According to the last, sub-national governments underestimate the reduction in federal tax revenue they cause by raising their own tax rates, because the cost in terms of less federal expenditure is borne by all the citizens in the country, not only the residents of the jurisdiction that raises the tax. With both effects distorting the levels of taxation in opposite directions, the question whether state taxes are too low or too high in equilibrium becomes the relevant issue when asking for more tax coordination. The effects of this interrelationship between horizontal and vertical tax externalities have been extensively analyzed, however, only in single-country models. In this paper, we extend the analysis to a multicountry framework and focus on relative country size to study the effects of tax externalities. Three broad conclusions emerge:

(1) The relative size of a federation within the Union determines the extent to
which state governments set lower or higher taxes in equilibrium. We find that state
taxes are lower in countries that are small relative to the size of the Union. Recent
literature has come to similar conclusions when studying the effects of jurisdictional
size in models that account only for horizontal tax externalities. Country size and the
equilibrium federal tax have been found to be negatively correlated. This result has
been further extended to jurisdiction size and state taxes in single-country models.
The interaction of both levels of government (federal and states) in the federation
and mainly the multicountry setting, allows us to derive conclusions about the extent
of the equilibrium state tax when considering the relative size of the federation in
the context of the Union.

(2) The direction of the state tax inefficiency and the relative country size are
negatively correlated. The vertical externality, leading to excessively high equilib-
rium state tax when compared with the one that prevails when jurisdictions within
the federation coordinate their tax policy, is more likely to dominate in countries
that are small relative to the size of the Union. Together with the result stated in
(1), we show the state tax in equilibrium is lower in relatively small countries but
most likely still higher than the state tax rate that will prevails in absence of tax
competition within the federation.

(3) No strong case can be made on a priori grounds, however, that countries will
prefer state tax harmonization or competition at sub-national level. First, it is not
clear whether state taxes will be too high, too low or equally high in one or the
other situation, this outcome depending on the relative size of the federation that
takes part in the Union. Second, we are unable to compare outcomes of incomplete
state tax harmonization. Once we agree that competition for mobile factors may take place between sub-national governments in a multicountry set-up, a benchmark where state governments in a single federation coordinate their tax policy is far from a socially optimum. State tax coordination among two (or more) countries in a Union, if feasible, does not guarantees a welfare improving outcome. A more comprehensive and systematic analysis of the outcomes under the different possible features of tax coordination in the context of a Union would be very useful and clearly valuable for future research.

Last, a number of empirical contributions to the literature of tax externalities mentioned in the introduction have motivated this paper. Our results summarized in (1) and (2) are of crucial importance for the validity of the results based on empirical work that disregards international tax competition. If, as suggested by recent empirical contributions, there is room for international tax interactions, empirical work based on single-federation models could suggest the wrong conclusions about the leading tax externality. More extensive empirical work accounting for tax externalities in a multicountry set-up, clearly deserves a thorough treatment of its own.

5 Appendix

Proof of Proposition 1. We have to prove that \( \frac{\partial n}{\partial R} = \frac{\partial n}{\partial p'} \frac{\partial p'}{\partial R} \) where \( p' \in [1,0) \) as defined by expression (6) and \( R \equiv N_L/\sum_i N_i \). First take the derivative of Condition
(8) with respect to $p'$ to obtain $\frac{\partial t}{\partial p'}$:

$$
\frac{\partial t}{\partial p'} = -\frac{\frac{1}{N_L} K'_{L_J}(\Gamma_g t^* + \Gamma_G T^*)}{D} \tag{A.1}
$$

$$
D = -K_{L_J}'(1 + \frac{1}{N_L} p')(1 - 2\Gamma_g) + K_{L_J}''(1 + \frac{1}{N_L} p')[t^* \Gamma_g(1 + \frac{1}{N_L} p') + \frac{1}{N_L} \Gamma_G T^*(1 + p')] + \Gamma_g g(K_{L_J} + t^* K_{L_J}'(1 + \frac{1}{N_L} p'))^2 + \frac{1}{N_L} \Gamma_G G T^* K_{L_J}^2(1 + p')^2
$$

Recall $\Gamma(.)$ is a strictly increasing concave function with $\Gamma_{GG} < 0$, $\Gamma_{gg} < 0$, and such that $\Gamma_g > 1$, otherwise there is no $t^* > 0$ that solves Condition (8); then a sufficient condition for $\frac{\partial t}{\partial p'} < 0$ is that

$$
-K_{L_J}'(1 + \frac{1}{N_L} p')(1 - 2\Gamma_g) + K_{L_J}''(1 + \frac{1}{N_L} p')[t^* \Gamma_g(1 + \frac{1}{N_L} p') + \frac{1}{N_L} \Gamma_G T^*(1 + p')] < 0 \tag{A.2}
$$

Recalling Condition (8), Condition A.2 can be rewritten as

$$
E^K < \frac{1 - 2\Gamma_g}{1 - \Gamma_g} \tag{A.3}
$$

where $E^K \equiv \frac{KK''}{(K')^2}$ is (minus) the elasticity of the slope of the marginal product of capital schedule\textsuperscript{15}. It is straightforward to show that Condition A.3 holds for any strictly increasing concave function $F(.)$.

Take then the derivative of Expression (6) with respect to $R$ to obtain $\frac{\partial p'}{\partial R}$:

$$
\frac{\partial p'}{\partial R} = \frac{K_{L_J}'(S' - K_{L_J}')}{(S' - K_{L_J}')^2 - \frac{\partial t}{\partial p'} R[K_{L_J}'' S'(1 + \frac{1}{N_L} p') - \frac{1}{N_L} K_{L_J}' S'' p']}. \tag{A.4}
$$

\textsuperscript{15}See Keen and Kotsogiannis (2003)
with $S(\rho)$ strictly increasing concave function, $\frac{\partial \rho}{\partial R} < 0$ since $\frac{\partial n}{\partial \rho} < 0$. With $\frac{\partial n}{\partial \rho} < 0$ and $\frac{\partial \rho}{\partial R} < 0$, it follows immediately that $\frac{\partial n}{\partial R} > 0$. \hfill \blacksquare

References


