# Bonn Econon Discussion Papers

Discussion Paper 21/2009

## An experimental methodology testing for prudence and third-order preferences

by

Sebastian Ebert, Daniel Wiesen

September 2009

Bonn Graduate School of Economics
Department of Economics
University of Bonn
Kaiserstrasse 1
D-53113 Bonn

Financial support by the
*Deutsche Forschungsgemeinschaft (DFG)*
through the
*Bonn Graduate School of Economics (BGSE)*
is gratefully acknowledged.

*Deutsche Post World Net* is a sponsor of the BGSE.

# An experimental methodology testing for prudence and third-order preferences

September 5, 2009

Sebastian Ebert
Bonn Graduate School of Economics, University of Bonn, sebastianebert@uni-bonn.de

Daniel Wiesen
Laboratory for Experimental Economics, University of Bonn, daniel.wiesen@uni-bonn.de

We propose an experimental method to test individuals for prudence (i.e. downside risk aversion) outside the expected utility framework. Our method relies on a novel representation of compound lotteries which allows for a systematic parameterization that captures the full generality of prudence. Therefore, we develop a general technique for lottery calibration in experiments. Since we investigate a very subtle third-order property we test our method in the laboratory employing a factorial design. We find that it yields robust results and that prudence is observed on the aggregate as well as on the individual level. Further we show that preferences based on statistical moments, in particular skewness seeking, can at most approximately explain individuals' behavior in the experiment.

*Key words*: Decision making under uncertainty, risk preferences, prudence, downside risk, statistical moments, laboratory experiment

*JEL classification*: D81, C91

## 1. Introduction

It is often misunderstood that risk aversion only partially captures an individual's risk attitudes. Prudence[1] is an entirely independent and complementary preference to risk aversion and has far reaching implications on economic behavior. Since Kimball (1990) coined the term prudence and showed that it is necessary and sufficient for a precautionary savings motive[2], the interest in the

[1] We refer to prudence as defined in Eeckhoudt and Schlesinger (2006) which within the expected utility framework corresponds to third-degree risk aversion as defined in Ekern (1980).

[2] That means, the awareness of uncertainty in future payoffs will raise an individual's optimal saving today.

concept has increased rapidly.

Meanwhile, prudence has proved to be important in numerous decision contexts, in particular in the area of insurance and finance. For example, a prudent (or downside risk averse) portfolio manager prefers high uncertainty in her return during good times (when returns are high) rather than in bad times (when returns are low, such as they are during a financial crisis). The stylized example illustrates that prudence, unlike risk aversion, is linked intuitively to the skewness of a probability distribution and other statistical measures of downside risk such as value-at-risk.[3]

Another important behavioral implication that is also influential for health economics has been found by Eeckhoudt and Gollier (2005). Prudence influences preventive action undertaken to reduce the probability of an adverse effect to occur. Likewise, prudence has been shown to be an important factor in preventive care decisions within a medical decision making context (see Courbage and Rey 2006).

Until now there is no experimental benchmark method to test individuals for prudence. Moreover, there are only a few empirical studies so far that can be related to prudence (e.g. Mao 1970 and Unser 2000). A first laboratory experiment was conducted by Tarazona-Gomez (2003). These studies test for prudence as defined within the expected utility theory (EUT) only if a Taylor expansion of the utility function is truncated (at third order) prior to taking expectations.[4] This preference framework, referred to as *truncated expected utility* in the following, is a special case of *moment preferences* where individuals' decisions between two prospects only depend on the first few statistical moments of these prospects. As in the study of prudence only prospects with equal mean and variance will be compared, such *third-order moment preferences* are equivalent to a preference for or against skewness. That is, in this setting prudence is equivalent to *skewness seeking*.

[3] More precisely, the example links to downside risk aversion as defined in Menezes et al. (1980) which is equivalent to prudence. A less stylized example would be life-cycle investment behavior (e.g. Gomes and Michaelides 2005). A recent article that illustrates the importance of prudence for insurance demand in the case of state-dependent background risk is Fei and Schlesinger (2008).

[4] This expectation of the Taylor expansion of a general utility function (evaluated at random wealth) was first presented in Arditti (1967); see also Kraus and Litzenberger (1976) and Scott and Horvath (1980).

This, however, is an assumption as rough as setting risk aversion equal to disliking variance. Thus, as Brockett and Kahane (1992) and Brockett and Garven (1998) show, these studies do not really test for prudence. Our experiment not only avoids this simplification but also evaluates whether it is empirically justified. Although moment preferences, in general, are incompatible with EUT they are widely assumed in economic modeling due to their simplicity and tractability.[5] This is also why this theory is still an active area of research, see for example Eichner (2008). To our knowledge, the only experiment that aims to test for prudence in a slightly more general manner is the one of Deck and Schlesinger (2008).[6]

In this paper we introduce an experimental method to test for prudence which can be applied in other experiments, together with a suitable test for risk aversion, to obtain a more complete picture of individuals' risk attitudes. For this, prudence is defined according to Eeckhoudt and Schlesinger (2006) as a preference for proper risk apportionment, i.e. prudent individuals prefer to 'disaggregate the harms' of a sure loss and a zero-mean risk. This understanding of prudence is 'model-free' in the sense that it applies inside and outside the EUT framework.

We test for prudence giving detailed consideration to the theoretical and experimental challenges. The lotteries employed cover a wide scope of systematically chosen parameter values and we propose a new representation for compound lotteries that allows for a sufficiently complex parameterization. In spite of this necessary complexity the representation is simple enough so that it is applicable in experiments. Indeed, by testing the method itself by use of a factorial design, we show that the decision task is robust towards different framing.

The experimental data show that prudence can be observed on the aggregate level, i.e. 65% of subjects' decisions are prudent. 47% of individuals (34 out of 72) made a significant number of prudent choices and thus are classified as prudent. Similarly, 8% of subjects are classified as imprudent. Highly influential is the (statistical) structure of the zero-mean risks in the definition

[5] For example, they underly a large number of classical and also modern portfolio choice models such as Kraus and Litzenberger (1976) or Briec et al. (2007).

[6] This experiment will be discussed more thoroughly later.

of Eeckhoudt and Schlesinger. This is not surprising as the statistical properties of the prudence lotteries are mainly driven by the zero-mean risk. In particular, its skewness determines whether the prudent or imprudent lottery choice has a higher kurtosis. This again makes clear that there is more to prudence than skewness seeking. Indeed, we find that prudence and skewness seeking do not coincide in practice. Thus lotteries used to test for prudence must feature a sufficient degree of complexity such that the broad scope of Eeckhoudt and Schlesinger's definition of prudence is respected.

The paper proceeds as follows. Section 2 analyzes the lotteries underlying the experiment, motivates the parameter choices and outlines the novel lottery calibration technique applied in this paper. In Section 3 the research questions are stated. Section 4 describes the experimental design and procedure. In Section 5 results from the experiment and the questionnaire are provided and Section 6 concludes.

## 2. Prudence Lotteries and Simple Lottery Approximation

In this section we define the lotteries underlying our experiment and relate them to prudence. The lotteries of Mao (1970) represent the class of lotteries that can only be interpreted as testing for skewness seeking and that are statistically similar to the lotteries used in Deck and Schlesinger (2008). The comparison to the 'stronger' lotteries of Eeckhoudt and Schlesinger (2006) gives insight on what there (theoretically) is more to prudence than skewness. As both types of lotteries are presented to subjects in our experiment we will see whether this difference remains in practice.

We first will define binary lotteries in general and the ones of Mao. Then we will prove some results on the latter. After that we will give the definition of prudence in Eeckhoudt and Schlesinger (2006) and compare their lotteries to the ones of Mao. We close the section by showing how to calibrate the lotteries of Eeckhoudt and Schlesinger and Mao to each other appropriately. For this, Theorem 1 will be used that shows the existence and uniqueness of a binary lottery with given first three moments. Thus, every random variable with finite first three moments can be approximated up to the third order by a binary lottery. The calibration technique introduced in this work might find application in any other lottery experiments. All proofs are given in Appendix B.

DEFINITION 1. A (simple) binary lottery denoted by $L = L(p, x_1, x_0)$ is defined as the random variable

$$L = X \cdot x_1 + (1 - X) \cdot x_0$$

where $x_1, x_0 \in \mathbb{R}$, $x_1 > x_0$ and $X$ is a Bernoulli-distributed random variable with parameter $p \in (0,1)$, called the (up-) probability of the lottery.

For uniqueness of representation we refer to $p$ as the up-probability and the payoff that occurs with probability $p$ is always higher then the one occurring with $1-p$. We also exclude the trivial cases $(x_1 = x_0)$ and $p \in \{0,1\}$ in our definition of a binary lottery. Expectation, variance, skewness and (non-excess) kurtosis of a random variable are denoted by $\mathbb{E}[\cdot]$, $\mathbb{V}[\cdot]$, $\nu[\cdot]$ and $\kappa[\cdot]$, respectively. In recognition of Mao (1970) we define the following.

DEFINITION 2. Two binary lotteries $L_X = L_X(p_X, x_1, x_0)$ and $L_Y = L_Y(p_Y, y_1, y_0)$ constitute a *Mao lottery pair* or a *Mao pair* if $p_X = 1 - p_Y$, $\mathbb{E}[L_X] = \mathbb{E}[L_Y]$ and $\mathbb{V}[L_X] = \mathbb{V}[L_Y]$.

Note that for the Mao lottery pair, if $L_X$ has its high payoff associated with the *high* probability, then $L_Y$ has its high outcome associated with the *small* probability, and vice versa.

The following proposition gives a statistical characterization of the Mao lotteries. The proof is instructive, i.e. it shows how to design Mao lotteries. We denote the (non-standardized) $n^{th}$ *central moment* or $n^{th}$ *moment about the mean* of a random variable $X$ by $\mu_n(X) := \mathbb{E}[(X - \mathbb{E}[X])^n]$. The *standardized* $n^{th}$ *central moment* is then given by $\mu_n^S(X) := \frac{\mu_n(X)}{(\mathbb{V}(X))^{\frac{n}{2}}}$. Note that in this notation $\mathbb{V}(X) = \mu_2(X)$, $\nu(X) = \mu_3^S(X)$ and $\kappa(X) = \mu_4^S(X)$.[7]

PROPOSITION 1 **(Statistical Characterization of Mao lottery pairs)**. *Consider a pair of Mao lotteries given by* $L_X = L_X(p, x_1, x_0)$ *and* $L_Y = L_Y(1-p, y_1, y_0)$*. Then for all* $n \in \mathbb{N}$ *it is*

$$\mu_n^S(L_X) = (-1)^n \mu_n^S(L_Y),$$

*in particular*

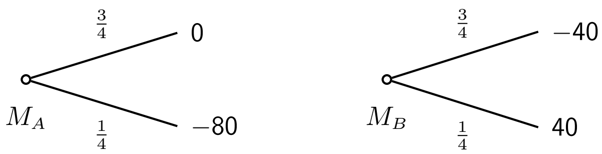
$$\nu(L_X) = -\nu(L_Y) \text{ and } \kappa(L_X) = \kappa(L_Y).$$

[7] The expectation equality of the Mao lotteries is not needed in the proof. Also note that it is not implied by this Proposition as $\mu_1^S(\cdot) \equiv 0$, i.e. no statement is made about the first moment.

Definition 3 (**Mao preference**). An individual that prefers the lottery of a Mao pair with the positive skewness (denoted by $M_B$) over the one with the negative skewness (denoted by $M_A$) is said to be Mao preferent or that it has the Mao preference.

Figure 1 provides an illustrative example of a Mao pair.

**Figure 1 Example of a Mao lottery pair** $(M_A, M_B)$




*Note.* The lotteries above correspond to the Mao pair in question MAO1 of the experiment. A Mao preferent individual prefers lottery $M_B$ (with a skewness of $+1.15$) over lottery $M_A$ (with a skewness of $-1.15$).

Intuition on how skewness manifests in binary lotteries is given in the claim and proof of Theorem 1. We now introduce the lotteries of Eeckhoudt and Schlesinger (2006) that are used for the definition of prudence.

Definition 4 (**Eeckhoudt-Schlesinger Prudence**). Let $X$ be a Bernoulli distributed random variable with parameter $p = \frac{1}{2}$ and let $k > 0$. Let $\epsilon$ be a non-degenerate random variable independent of $X$ with $\mathbb{E}[\epsilon] = 0$. Consider the random variables (lotteries)

$$A_3 = X \cdot (0) + (1 - X) \cdot (-k + \epsilon) \textit{ and } B_3 = X \cdot (-k) + (1 - X) \cdot \epsilon.$$

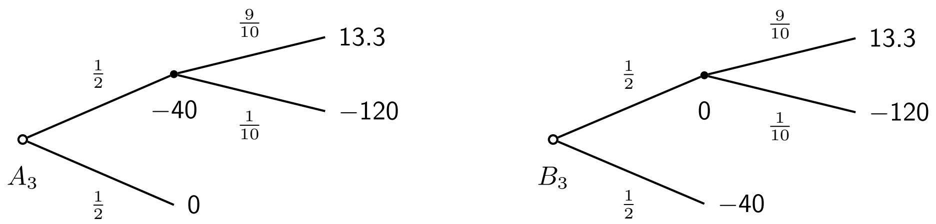
These two lotteries as a pair are called (Eeckhoudt-Schlesinger) prudence lottery pair or ES pair. An individual is called prudent if she or he prefers $B_3$ over $A_3$ for all values of $k$, for all random variables $\epsilon$ and for all wealth levels $x$.[8]

An example of an ES pair is given in Figure 2.

Eeckhoudt and Schlesinger (2006) interpret prudence as 'proper risk apportionment'. In Figure 2

[8] In fact, $k$ may also be chosen negative such that $-k$ corresponds to a gain. In this case the prudent choice would be the lottery $A_3$ and the imprudent choice would be the lottery $B_3$. To avoid confusion here we stay with positive values of $k$. In the experiment, however, we test for consistency regarding this variation. Of course, the decisions of a truly prudent individual should not be affected by it.

**Figure 2 Example of a prudence lottery pair** $(A_3, B_3)$




*Note.* The lotteries above correspond to the ES pair in question ES1 of the experiment. In the example $\epsilon$ is left-skewed implying that lottery $A_3$ has a larger kurtosis than lottery $B_3$. The lotteries in Figures 1 and 2 have equal mean and variance as well as $\nu(B_3) - \nu(A_3) = \nu(M_B) - \nu(M_A)$.

the prudent option ($B_3$) implies that the additional zero-mean risk $\epsilon$ (i.e. the second lottery) occurs in the good state of the 50/50 gamble (i.e. in the state where no fixed amount is lost), whereas in the imprudent option $A_3$ the zero-mean risk occurs in the bad state. Intuitively, a prudent choice implies a 'disaggregation of harms'. Eeckhoudt and Schlesinger (2006) show that this preference is equivalent to downside risk aversion as defined in Menezes et al. (1980) or to third-degree risk aversion as defined in Ekern (1980) within the expected utility framework. The latter also clarifies the relation to the prudence measures defined in Kimball (1990) and Crainich and Eeckhoudt (2008). We state the following fact without proof as it only requires statistical standard calculations.[9]

**Fact 1** *Consider an arbitrary Eeckhoudt-Schlesinger lottery pair in Definition 4. $A_3$ and $B_3$ have equal expectation and variance and thus $\mathbb{V}(A_3) = \mathbb{V}(B_3) =: \sigma^2$ is well-defined. Furthermore,*

$$\nu(B_3) - \nu(A_3) = \frac{3k\mathbb{E}[\epsilon^2]}{2\sigma^3} > 0 \tag{1}$$

*and*

$$\kappa(B_3) - \kappa(A_3) = \frac{2k\mathbb{E}[\epsilon^3]}{\sigma^4}. \tag{2}$$

*In particular, the prudent lottery choice has the higher skewness.*

From Proposition 1 and Fact 1 we see that both prudence and the Mao preference imply positive skewness to be beneficial to the individual. Unlike the Mao preference, prudence requires that

[9] The statements for the first three moments are also given in Crainich and Eeckhoudt (2008).

the lottery with the higher skewness is preferred no matter whether it has the smaller or higher kurtosis. If, however, third-order moment preferences or third-order truncated expected utility is assumed, prudence and the Mao preference are equivalent. As mentioned earlier the experiment of Tarazona-Gomez (2003) made use of this assumption.

The zero-mean risks (the $\epsilon$'s) of the lotteries that were employed in the experiment of Deck and Schlesinger (2008) throughout were binary lotteries with 50/50 probabilities. It is easy to see from Fact 1 that this constantly implies the same kurtosis of the two prudence lotteries, i.e. $\kappa(B_3) - \kappa(A_3) = 0$. Moreover, Roger (2009) shows the signs of all moments of prudence lotteries with symmetric $\epsilon$'s (in particular binary $\epsilon$'s with 50/50 probabilities) to coincide with those we derived in Proposition 1 for the Mao lotteries.[10] Thus, from a statistical point of view, prudence lotteries with symmetric zero-mean risks are essentially equivalent to those of Mao and are far off from being as general as the lotteries defined in Eeckhoudt and Schlesinger (2006).

It is the aim of this paper to employ the prudence lotteries in their full generality. Of course, in the experiment one must restrict to a simple subset of parameter choices. However, it should be sufficiently general to capture the requirements in Definition 4 reasonably well. In the experiment we will ask subjects also to decide between Mao lottery pairs that act as representatives for this type of skewness preference that is (theoretically) not sufficient to imply prudence.

To have the prudence and Mao lotteries in the same parameter range we now carry out how Mao lottery pairs must be calibrated so that they are close to the prudence pairs.[11] We start with the following theorem stating that a binary lottery with non-trivial variance and otherwise arbitrary first three moments always exists and the moments uniquely determine the lottery. It implies that every non-degenerate probability distribution with finite first three moments can be

[10] Further, the expression for kurtosis we gave in Fact 1 shows that Roger's result can not be generalized to arbitrary, i.e. asymmetric, zero-mean risks.

[11] This is a general issue in lottery choice experiments and has been shown to be important, for example, in the context of multiple price list formats to elicit risk preferences (see Harrison and Rutström 2008). We will show that this calibration has an effect on subjects' decisions in Subsection 5.4.

approximated up to the third moment by a binary lottery and this approximating lottery is unique. Simple lotteries are one of the main tools to examine decisions under uncertainty and for testing associated theories like expected utility or prospect theory. Therefore, the following theorem might find application in many experiments and, in particular, is useful for calibration issues. The given equations facilitate to construct exactly the lottery an experimenter is looking for, conveniently.

THEOREM 1. *For constants $E \in \mathbb{R}$, $V \in \mathbb{R}_{>0}$ and $S \in \mathbb{R}$ there exists exactly one binary lottery $L_X = L_X(p, x_1, x_0)$ such that $\mathbb{E}[L_X] = E$, $\mathbb{V}[L_X] = V$ and $\nu[L_X] = S$. Setting $\tilde{S} := 4 + S^2$ its parameters are given by*

$$p = \begin{cases} \frac{\tilde{S}+\sqrt{\tilde{S}^2-4\tilde{S}}}{2\tilde{S}} & \text{if } S < 0 \\ \frac{1}{2} & \text{if } S = 0 \\ \frac{\tilde{S}-\sqrt{\tilde{S}^2-4\tilde{S}}}{2\tilde{S}} & \text{if } S > 0 \end{cases}$$

$$x_1 = E + \sqrt{\frac{V \cdot (1-p)}{p}}$$

$$x_0 = E - \sqrt{\frac{V \cdot p}{1-p}}.$$

Now we use this Theorem to calibrate the Mao and ES pairs to each other and define the approximation of an ES pair with a Mao pair. The Mao pair in Figure 1 and the ES pair in Figure 2 exemplify such an approximation. That is, all four lotteries depicted have equal mean and variance and the differences in skewness between the ES pair and the Mao pair are also equal.[12]

PROPOSITION 2 **(Approximation of an ES pair with a Mao pair)**. *Consider a prudence lottery pair $(A, B)$ with finite first three moments. For every $S > 0$ there exists exactly one Mao lottery pair $(M_A, M_B)$ such that*

$$\mathbb{E}[M_A] = \mathbb{E}[A] \text{ and } \mathbb{E}[M_B] = \mathbb{E}[B],$$
$$\mathbb{V}[M_A] = \mathbb{V}[A] \text{ and } \mathbb{V}[M_B] = \mathbb{V}[B] \text{ as well as}$$
$$\nu[M_A] = -S \text{ and } \nu[M_B] = S.$$

[12] The numerical values of the statistical moments for ES pairs and Mao pairs are given in Table 1 and Table A.1 in Appendix A, respectively.

*For*

$$S = \frac{\nu[B] - \nu[A]}{2}$$

*the difference in skewness of the prudence pair equals the difference in skewness of the Mao pair and the quadratic error* $\Delta := (\nu[B] - \nu[M_B])^2 + (\nu[A] - \nu[M_A])^2$ *is minimal.*

DEFINITION 5. For a given ES pair $(A, B)$ the Mao pair $(M_A, M_B)$ that was shown to exist in Proposition 2 is called the *Mao approximation* or the *approximating Mao pair* of the ES pair $(A, B)$.

## 3. Research questions

The main purpose of our experimental analysis is to find a methodology to test for prudence. Moreover, our experimental methodology facilitates to answer the research questions presented in this section.

As explained in Section 2, the Mao lotteries are in structure similar to the ones employed in earlier studies (e.g. Tarazona-Gomez, 2003). In our experimental methodology they serve as a benchmark to the ES lotteries that helps to investigate the relationship of prudence and skewness seeking.

**Research question 1** *What is the relationship between prudence and the Mao preference?*

Answering research question 1 is of particular interest for several reasons. If prudence and the Mao preference are equivalent, the first three statistical moments (or expected truncated utility) seem to characterize prudence sufficiently well, giving credit to the studies mentioned earlier. Moreover, this would support the assumption of moment preferences up to order three in general.[13] If Mao preferent individuals do not exhibit prudence, this implies that prudence is a stronger property, in practice and in theory. Then it is not sufficient to ask lotteries based on the first three moments to test for prudence. In particular, as shown in Section 2, then no binary lottery can be sufficiently complex to test for prudence.

Note again that Eeckhoudt and Schlesinger's definition for prudence (Definition 4) is very broad

[13] Note, for example that most portfolio choice models are built upon the assumption of moment preferences.

in scope. Prudence is defined for any random variable $\epsilon$, any outcome $k$, any wealth level $x$ and, of course, is robust towards framing of the decision task.[14] Concerning the robustness towards framing, we test whether it makes a difference if the task is to add the zero-mean risk $\epsilon$ or the fixed amount $-k$ to a state of the 50/50 gamble, given that the other item ($-k$ or $\epsilon$, respectively) is already present in one state. This relates to the intuition of Eeckhoudt and Schlesinger's definition of prudence as 'proper risk apportionment'. Further, having $k$ positive and negative can also be interpreted as a framing issue.[15] We state the following research questions.

**Research question 2** *Are individuals' decisions independent of whether the fixed amount $k$ corresponds to a gain or a loss?*

**Research question 3** *Are individuals' decisions influenced by the wealth level $x$?*

**Research question 4** *Are individuals' decisions influenced by different framing related to the idea of proper risk apportionment?*

**Research question 5** *Does the choice of the zero-mean random variable $\epsilon$ matter for individuals' decisions?*

On the one hand, we estimate the fraction of *prudent choices*, i.e. we analyze subjects' decisions on the aggregate. The factorial design allows us to investigate what kind of *questions* will be more likely to be answered prudently. This is linked to the question of what prevents subjects from being prudent or what might cause rather prudent subjects to deviate from their preference. On the other hand, we analyze every individual on its own. That is, our experiment allows us to classify every individual to be imprudent, prudent, or indifferent. The experiment is followed by a survey which investigates how prudence is related to individuals' attitudes, e.g. towards risk aversion, loss aversion and demand on precautionary savings.

[14] This is the reason why a 'binary lottery preference' can be equivalent to signing the derivatives of the utility function or to stochastic dominance – looking more closely, the lottery preference is not that simple. In particular, the fact that the zero-mean risks are arbitrary adds a large amount of stochastic freedom to these lotteries.

[15] This will be explained at length in Section 4.

## 4. Experimental design and procedure

The computerized experiment consists of three main stages.[16] In total, each subject makes her individual choices over 34 lottery pairs. The lottery outcomes are disclosed in Taler, our experimental currency. One Taler is worth € 0.15 (about $0.20). Decisions are incentivized by a random-choice payment technique; this means that *one* out of 34 decisions is randomly drawn to determine solely a subject's payoff. The lottery chosen by the individual in the randomly determined decision is actually played out at the end of the experiment.

In stage ES, we test subjects for prudence. Subjects reveal their preferences over Mao pairs in stage MAO. In stage RIAV, we determine subjects' degree of risk aversion employing the well established method by Holt and Laury (2002). An extensive questionnaire comprising questions on loss aversion, precautionary savings and demographic attributes succeeds the experiment. We now describe the experimental stages in more detail.

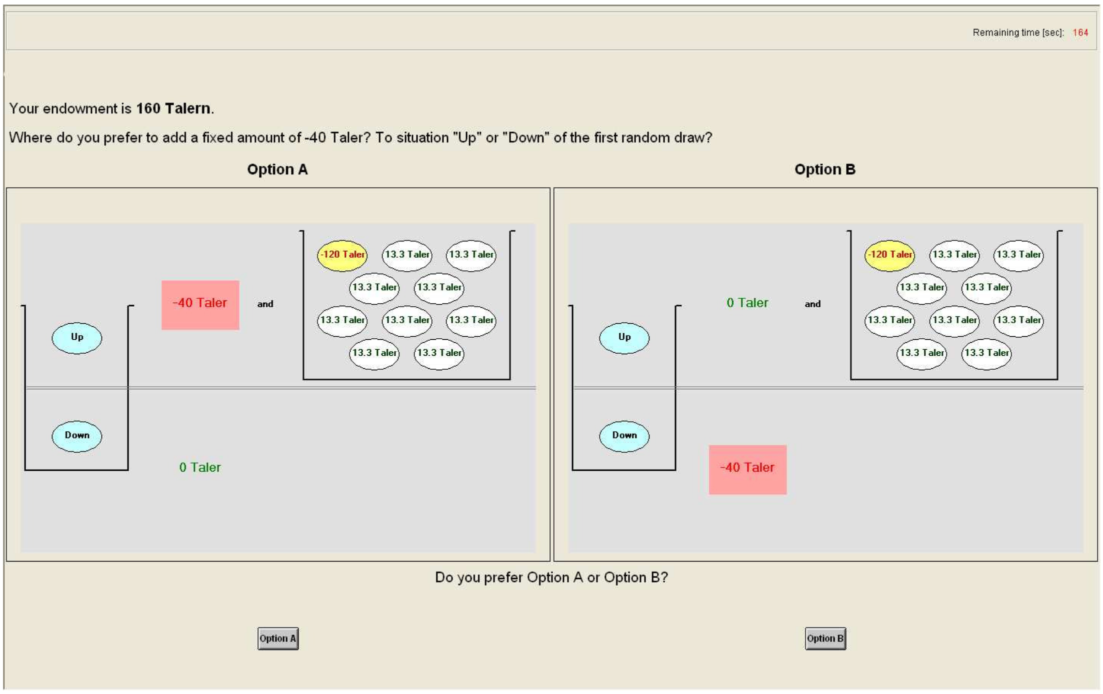
### 4.1. Prudence test embedded in a factorial design – Stage ES

In stage ES we test whether individuals are prudent according to Definition 4. To this end subjects are asked to make preference choices over the 16 ES pairs ES1, ES2, ..., ES16.

We introduce a new ballot box representation to display the compound lotteries of the ES pairs. Figure 3 shows, as an example, how question ES1 (that has already been illustrated more formally in Figure 2) appears on subjects' decision screens. It must be understood as follows: Option A and Option B are displayed in the left and right panel of Figure 3, respectively. For both options the 50/50 gamble is depicted as a ballot box that contains two blue balls named "Up" and "Down". The displays of both Option A and Option B themselves are spatially separated, each into an upper panel containing the "Up-ball", and into a lower panel containing the "Down-ball". Now consider Option A. If the draw from the first ballot box is "Up", then a loss of $-40$ Taler occurs to the subject and a second lottery (the zero-mean risk $\epsilon$) succeeds. $\epsilon$ is also displayed in a ballot

[16] For programming the experimental software zTree was used (Fischbacher 2007).

**Figure 3 Example of lottery display in stage ES (Question ES1)**




box format with 10 balls in total. Yellow balls imply a loss (here: −120 Taler colored in red) and white balls a gain (here: 13.3 Taler colored in green). In situation "Down" no second lottery follows and no loss occurs. For Option B, if the draw from the first ballot box is "Up" no loss occurs and a second lottery succeeds (the same $\epsilon$ depicted in Option A). If the draw is "Down", a loss of −40 Taler occurs.[17]

This ballot box representation interlinks decisions at the computer screen with the real-world lottery play at the end of the experiment (see Figure A.1 in Appendix A). Further, it is able to display asymmetric zero-mean risks and all probabilities are naturally visualized.

To facilitate testing for the broad scope of the prudence preference as defined by Eeckhoudt and Schlesinger, we employ a completely randomized $2^4$ factorial design; that means four factors, each at two levels.[18] The factors relate to Research questions 2 to 5. The factorial setup allows us to analyze the influence of each factor and the interactions of factors on subjects' responses. A response

[17] The order of subjects' decision screens occurred for each subject in a different randomized order and also the position of the prudent option being either left or right on the screen was randomized.

[18] For a detailed description of factorial designs see, e.g., Montgomery (2005).

can be either "prudent" or "imprudent". The factors are as follows: sign of $k$ (Factor A), wealth level $x$ (Factor B), framing of the decision task (Factor C) and composition of $\epsilon$ (Factor D).[19]

Along the illustration in Figure 3 we now explain how the factors of the factorial design translate into subjects' decision screens. The outcomes of the 50/50 gamble are 0 Taler and −40 Taler, displayed as a red bill, implying that Factor A is at level $k_1 = 40$.[20] Hence, in the example the imprudent choice is Option A, as the additional zero-mean risk occurs in the bad state. The alternative level of Factor A is $k_2 = -40$. Then the fixed amount is a gain which is displayed as a green bill. With Factor A we essentially test for an experimental framing effect (Research question 2) and whether individuals really exhibit the intuition of proper risk apportionment. Whenever $k$ equals 40 Taler, then zero is the bad outcome, whereas when $k$ is equal to −40 Taler, 0 Taler is the good outcome. Whenever a subject consistently prefers the option where $\epsilon$ is added to outcome 0 Taler we could conjecture that this is due to framing concluding that 0 is a so-called focal point.

Factor B tests for a wealth effect according to Research question 3 and has the levels $x_1 = 160$ or $x_2 = 80$ (in Taler). Wealth levels may be interpreted as an endowment subjects receive in order to accommodate possible negative lottery outcomes. The wealth level on subjects' screen is indicated in the upper left corner. In Figure 3 it is set to 160 Taler.

In the example, the decision between the imprudent Option A and the prudent Option B is whether in the up-state or in the down-state a fixed loss of 40 Taler is preferred *given* that the additional risk will be in the up-state. That is, the question on the decision screen is "*Where do you prefer to add a fixed amount of* −40 *Taler? To situation "Up" or "Down" of the first random draw?*" At the other level of Factor C subjects are asked to which situation – either 0 or $-k$ – of the 50/50 gamble to add a random draw ($\epsilon$).[21] Thus the two levels of Factor C are "add $k$" (a sure reduction or increase in wealth) or "add $\epsilon$" (a zero-mean random variable). Factor C directly relates to the intuition behind Eeckhoudt and Schlesinger's prudence definition of proper risk apportionment. Moreover, Factor C determines whether it is the fixed amounts or the zero-mean risks that

[19] Columns 6 to 10 in Table 1 show a complete design layout including the arrangement of factors.

[20] Note that by Definition 4 the fixed amount is $-k$, i.e. it corresponds to a loss if $k > 0$ and to a gain if $k < 0$.

[21] A complete description of subjects' decision task is given in subjects' instructions in Appendix C.

occur in state "Up" of the 50/50 gamble in both options. Note that Factor C is purely checking for a framing issue as the lotteries across levels of Factor C are statistically identical. The interaction of Factors A and C leads to even more different framing situations.[22]

With Factor D we test for a moment effect of the fourth order or, *equivalently*, if prudence is invariant under variation of the $\epsilon$'s (Research question 5). In the following we show how our particular choices of the zero-mean risks in the experiment are motivated by the statistical properties they induce on the prudence lottery pairs. Since the relationship between mean, variance and skewness of the ES pair is fixed for all parameter choices (compare Fact 1), and since we test for a third-order property, it is natural to consider how the ES pairs differ in their kurtosis. When speaking of kurtosis difference we consider $\kappa(B_3) - \kappa(A_3)$, i.e. if it is negative this means that the *prudent* choice has the smaller kurtosis. According to Fact 1 the kurtosis difference translates one-to-one into the skewness of the zero-mean risk. The skewness of a binary lottery is one-to-one to its up-probability.[23] In our example, $\epsilon$ is left-skewed, $\nu(\epsilon) = -2.97$, such that $\kappa(B_3) - \kappa(A_3) < 0$. If $\epsilon$ in the example had the signs of the outcomes switched it would be right-skewed and the prudent option had the higher kurtosis. As $\epsilon$ has mean zero, skewness has the following interpretation. A left-skewed $\epsilon$ yields a small gain with high probability and a large loss with a small probability. Further, as we display the $\epsilon$ as a ballot box containing 10 balls, skewness translates one-to-one to the amount of draws implying losses or gains, respectively. Indeed, in the example $\epsilon$ implies a loss of 120 Taler with 10% chance and a gain of 13.3 Taler with 90% chance. Thus, Factor D can take the levels "$\kappa(B_3) - \kappa(A_3) > 0$" and "$\kappa(B_3) - \kappa(A_3) < 0$". However, any of the mentioned equivalent interpretations (kurtosis difference, skewness of the zero-mean risk, composition of the ballot box) is captured by Factor D. The practical interpretations of kurtosis difference support our theoretical argument in Section 2 that restricting to one particular choice of a binary and symmetric $\epsilon$ is a

[22] This way we test for quite a variety of different framing within our ballot box representation of the ES lottery pairs what might seem confusing. This is just the point as we will show that our procedure to test for prudence is robust towards this confusion.

[23] For this and the following arguments see Theorem 1 and its proof in Appendix B.

rather severe limitation for a procedure that aims to test for prudence.

To sum up, by specifying the four factors as above, we systematically account for the whole

**Table 1 ES pairs with their underlying factors and their statistical properties**

| | $\epsilon$ | | | | | Factors | | | | Statistical properties | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ES pair | $p$ | $z_1$ | $1-p$ | $z_0$ | Treat. comb. | A | B | C | D | $\mathbb{E}[A_3] = \mathbb{E}[B_3]$ | $\mathbb{V}(A_3) = \mathbb{V}(B_3)$ | $\nu(B_3) - \nu(A_3)$ | $\kappa(B_3) - \kappa(A_3)$ |
| ES1 | 0.90 | 13.33 | 0.10 | −120.00 | abc | 40 | 160 | add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ | −20.00 | 1,200.00 | 2.30 | −9.48 |
| ES2 | 0.10 | 120.00 | 0.90 | −13.33 | abcd | 40 | 160 | add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ | −20.00 | 1,200.00 | 2.30 | 9.48 |
| ES3 | 0.80 | 12.00 | 0.20 | −48.00 | ab | 40 | 160 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ | −20.00 | 688.00 | 1.92 | −3.50 |
| ES4 | 0.20 | 48.00 | 0.80 | −12.00 | abd | 40 | 160 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ | −20.00 | 688.00 | 1.92 | 3.50 |
| ES5 | 0.70 | 12.00 | 0.30 | −28.00 | ac | 40 | 80 | add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ | −20.00 | 568.00 | 1.48 | −1.33 |
| ES6 | 0.30 | 28.00 | 0.70 | −12.00 | acd | 40 | 80 | add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ | −20.00 | 568.00 | 1.48 | 1.33 |
| ES7 | 0.60 | 8.00 | 0.40 | −12.00 | a | 40 | 80 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ | −20.00 | 448.00 | 0.60 | −0.15 |
| ES8 | 0.40 | 12.00 | 0.60 | −8.00 | ad | 40 | 80 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ | −20.00 | 448.00 | 0.60 | 0.15 |
| ES9 | 0.90 | 13.33 | 0.10 | −120.00 | bcd | −40 | 160 | add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ | 20.00 | 1,200.00 | 2.30 | 9.48 |
| ES10 | 0.10 | 120.00 | 0.90 | −13.33 | bc | −40 | 160 | add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ | 20.00 | 1,200.00 | 2.30 | −9.48 |
| ES11 | 0.80 | 12.00 | 0.20 | −48.00 | bd | −40 | 160 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ | 20.00 | 688.00 | 1.92 | 3.50 |
| ES12 | 0.20 | 48.00 | 0.80 | −12.00 | b | −40 | 160 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ | 20.00 | 688.00 | 1.92 | −3.50 |
| ES13 | 0.70 | 12.00 | 0.30 | −28.00 | cd | −40 | 80 | add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ | 20.00 | 568.00 | 1.48 | 1.33 |
| ES14 | 0.30 | 28.00 | 0.70 | −12.00 | c | −40 | 80 | add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ | 20.00 | 568.00 | 1.48 | −1.33 |
| ES15 | 0.60 | 8.00 | 0.40 | −12.00 | d | −40 | 80 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ | 20.00 | 448.00 | 0.60 | 0.15 |
| ES16 | 0.40 | 12.00 | 0.60 | −8.00 | (1) | −40 | 80 | add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ | 20.00 | 448.00 | 0.60 | −0.15 |

*Note.* This table describes the prudence lottery pairs ES1, ES2, … ES16 in stage ES. $\epsilon$ is the zero-mean risk with its up-state $z_1$, its down-state $z_0$ and the respective probabilities $p$ and $1-p$ shown in columns 2 to 5. Column 6 (treatment combinations) indicates which factor(s) is (are) at its high level; (1) denotes that all factors are at their low level. The explicit arrangement of factors A, B, C and D is given in columns 7 to 10. The remaining columns provide statistical properties of the ES pairs.

complexity of the definition of prudence as in Eeckhoudt and Schlesinger (2006). Further, we can test which conditions in the definition of prudence have a severe impact on individuals' decisions such that they must be accounted for when testing for prudence. A complete overview of the 16 ES pairs, their statistical properties and the arrangement of factors is provided in Table 1.

## 4.2. Stage MAO

In this stage we investigate whether subjects are Mao preferent in order to answer Research question 1. Applying Proposition 2, we obtain 8 different pairs of Mao lotteries between which subjects have to state their preference. There are only 8 pairs, as the change in the kurtosis (Factor D) does not affect these lotteries (see Proposition 1). Thus lottery pair MAO1 is the approximation of both lottery pairs ES1 and ES2, lottery pair MAO2 approximates ES3 and ES4 and so on. As the Mao lotteries also imply negative outcomes, subjects are endowed with a certain amount of money equal to the wealth level $x$ in the corresponding ES pairs. The Mao pairs are shown in Table A.1 in Appendix A.[24]

For the Mao lottery pairs we choose a graphical representation similar to the one proposed by Camerer (1989). Here, probabilities are depicted on the vertical and outcomes of the lotteries on the horizontal axis. The outcome-probability area nicely illustrates the weight of an outcome in a particular lottery and facilitates comparisons across lottery pairs. Possible gains are colored in green and losses in red. For an example see the screenshot in the instructions in Appendix C.

### 4.3. Stage RIAV

In stage RIAV we apply the widely prevalent method invented by Holt and Laury (2002) to test for risk aversion. Here, subjects are asked to make 10 choices – either Option A or Option B – between paired lotteries. The 10 lottery pairs are displayed in a table visible to the subject all at once. The underlying logic is as follows: When the probability of the high-payoff outcome increases enough by walking down the table, a person should cross over to Option B. Option A always comprises the less risky lottery but is associated with a lower expected value for the last six choices than Option B. This implies that a risk neutral agent should choose Option A in the first four choices and switch to Option B afterwards. A risk averse individual switches after the fourth choice, whereas a risk seeking individual switches to Option B before the fourth choice. See Table A.2 in Appendix A for the complete set of the lottery pairs in stage RIAV.

[24] Analogous to stage ES, the order of subjects' decision screens is randomly permutated in stage MAO and the position of the Mao preferent option is randomized.

### 4.4. Procedural details

The experiment was conducted at the *B*onn*E*con*L*ab. Overall 72 students of the University of Bonn from various fields participated in 9 experimental sessions. The stage order was varied across sessions (see Table 2 for a complete overview). Each session lasted for about 90 minutes. Subjects earned on average € 18.50 (about $ 24.70).

The procedure of the experiment was as follows: Firstly, experimenters extensively introduced

**Table 2 Session overview**

| Session | Date | Stage order | Number of subjects |
|---|---|---|---|
| 1 | 2008/12/10 | MAO-ES-RIAV | 6 |
| 2 | 2008/12/19 | MAO-ES-RIAV | 6 |
| 3 | 2009/01/16 | MAO-ES-RIAV | 8 |
| 4 | 2009/01/16 | MAO-ES-RIAV | 8 |
| 5 | 2009/01/20 | ES-MAO-RIAV | 10 |
| 6 | 2009/01/20 | ES-MAO-RIAV | 8 |
| 7 | 2009/02/11 | MAO-ES-RIAV | 10 |
| 8 | 2009/02/11 | ES-MAO-RIAV | 8 |
| 9 | 2009/02/11 | ES-MAO-RIAV | 8 |

the decision task and the entire procedure of the experiment to subjects. Secondly, before each experimental stage started, subjects were asked to answer control questions testing their understanding of the decision task. In particular, they were familiarized with the illustration of lotteries and their outcomes as well as probabilities. Only when subjects had answered these questions correctly they were allowed to proceed to the decision stages of the experiment. Then, thirdly, subjects made the decisions in the experimental stages. Fourthly, subjects answered questions of the questionnaire. For answering the questionnaire subjects received € 4.00 in addition to their earnings from the experiment (comparable to a show-up fee). Finally, each subject's payoff was determined by a random-choice payment technique.[25] As already mentioned, subjects made a series of 34 choices, each with substantial monetary consequences, and final earnings are based on just

[25] See Laury (2005) for an excellent survey on the random-choice payment method.

*one* of these choices selected at random after all have been completed.

The random choice was made by drawing one ball out of a set of balls numbered between 1 and 34 from a ballot box referring to a lottery pair from stage ES, MAO or RIAV. The subject's lottery choice in this randomly drawn lottery pair was actually played out. In stages MAO and ES, the outcome was allocated to the subjects' wealth level in that decision, i.e. subjects could charge the coupon they obtained in the beginning. The ES lotteries were played out using ballot boxes resembling the lotteries displayed on subjects' decision screens (see the photograph in Figure A.1). The binary lotteries in stages MAO and RIAV were played out using a ballot box with 100 balls numbered from 1 to 100. If, e.g., the up-state had a likelihood of 90%, a draw of the balls numbered $1, 2, \ldots, 90$ implied the corresponding up-payoff.

## 5. Experimental results

In this section we present the results of the experiment. We first describe subjects' behavior on the aggregate level in all three experimental stages. Moreover, we report results from the factor analysis, the test of moment preferences and analysis of behavior at the individual level. Finally, we provide an analysis of subjects' individual characteristics elicited by our questionnaire and their association to decisions in the experiment.

### 5.1. Preliminary analysis

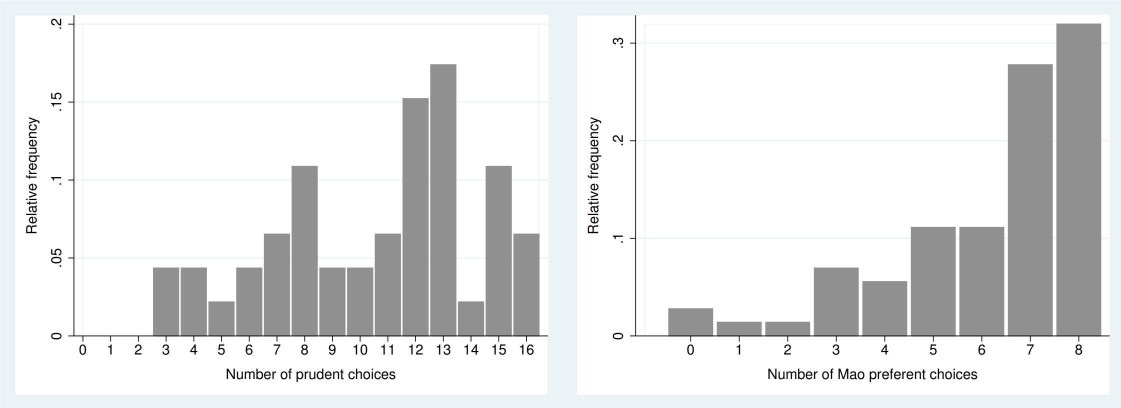
A brief overview on subjects' aggregate behavior is provided in this subsection.[26] In our experimental data we find substantial evidence for prudence. Overall, 65.10% of subjects' responses are prudent. The left panel in Figure 4 illustrates the relative frequencies of subjects' prudent choices.

[26] To rule out possible stage order effects we compare responses from sessions with stage order MAO-ES with responses from sessions with stage order ES-MAO. The null hypothesis that both samples are drawn from the same distribution cannot be rejected (for ES-responses: $p = 0.413$ and for MAO-responses: $p = 1.000$, two-sided two-sample Kolmogorov-Smirnov test). Note that, this analysis is mandatory in order to merge samples from sessions with different stage orders for our overall analysis.

On average, 10.42 of the choices are prudent with a standard deviation of 3.65. The median (mode) of prudent choices is 11 (13). Observed behavior in stage ES differs significantly from random behavior ($p = 0.0000$, two-sided one-sample Wilcoxon signed-rank test).[27]

In stage MAO, 77.08% of all questions have been answered in a Mao preferent way. The right

**Figure 4 Distribution of the number of prudent and Mao preferent choices by subjects**




*Note.* The left panel of the figure above shows relative frequencies of the number of subjects' prudent choices. In the right panel, relative frequencies of the number of subjects' Mao preferent choices are indicated ($N = 72$).

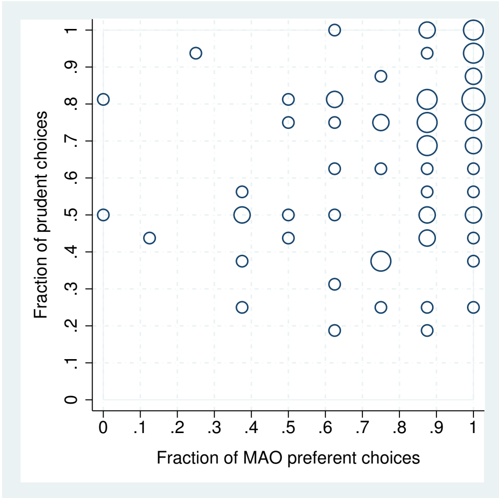
panel in Figure 4 illustrates the relative frequencies of subjects' Mao preferent choices. Each subject has been, on average, Mao preferent in 6.16 out of 8 questions with a standard deviation of 2.01. The median (mode) of Mao preferent choices is 7 (8). Also this behavior differs significantly from random behavior ($p = 0.0000$, one-sample Wilcoxon signed-rank test). We find a positive relation between fractions of Mao preferent choices and fractions of prudent choices per subject (see Figure 5) and a significant positive correlation of $\rho = 0.2844$ ($p = 0.0155$, Spearman rank correlation test).[28]

In stage RIAV we tested subjects for risk aversion by applying the procedure of Holt and Laury (2002). 63 subjects switch from choosing Option A to Option B after the fourth pairwise decision and, thus, are risk averse. 4 subjects are classified as risk seeking, whereas 5 subjects chose

[27] In the following all statistical tests are *two-sided* if not indicated differently.

[28] A more in-depth analysis of the relation between lotteries in stages MAO and ES is provided in Subsection 5.4.

**Figure 5 Relative frequency of Mao preferent and prudent choices by subject**




*Note.* This scatter plot contrasts the fractions of Mao preferent and prudent choices per subject. The circumference of circles varies with the number of observations ($N = 72$).

Option A four times and are, thus, classified as risk neutral.[29]

## 5.2. Within subject analysis

This subsection is concerned with Research question 1 and the relationship of prudence and skewness seeking. We have already shown that there is a positive correlation (which is a symmetric association measure). We now show that the actual relation is asymmetric. To this end we categorize subjects' responses in stages ES and MAO according to the frequency of prudent and Mao preferent choices, respectively. A subject who answered 12 or more (4 or less) out of 16 questions prudently is categorized as *prudent* (*imprudent*). Subjects who answered 5 to 11 questions prudently are classified as *indifferent*. Subjects are classified as *Mao preferent* (*not Mao preferent*) if they have answered 7 or 8 (0 or 1) out of 8 questions in favor of the lottery with the positive (negative) skewness. When answering 2 to 6 questions in a Mao preferent manner subjects are allotted to the category *indifferent*. This categorization relies on the fact that for $h \geq 12$ successes (prudent choices) in $N = 16$ cases the null hypothesis can be rejected that

[29] One subject was excluded due to multiple switching points.

there is no difference between the probability of making a prudent choice and the probability of making an imprudent choice ($p < 0.0768$, binomial test). The same holds true for $h \geq 7$ Mao preferent choices out of $N = 8$ questions ($p < 0.0703$, binomial test).[30] When we say a subject is prudent this essentially means that she answered significantly *more* questions prudently than imprudently. Note that prudence is a very strong property in terms of requirements and, thus, it is not surprising that individuals may behave prudent in certain situations and in others they do not.

**Table 3 Contingency table on categories**

| | Not Mao preferent | Indifferent | Mao preferent | Total |
|---|---|---|---|---|
| Imprudent | 0 | 3 | 3 | 6 |
| Indifferent | 2 | 13 | 17 | 32 |
| Prudent | 1 | 10 | 23 | 34 |
| Total | 3 | 26 | 43 | 72 |

To answer Research question 1 we set up a contingency table showing absolute frequencies about subjects' categories (see Table 3). Let us first analyze prudence and the Mao preference separately. 34 (47.22%) of all 72 subjects are prudent whereas only 6 (8.33%) are imprudent. Note again that this gives a very different picture as from looking at the aggregate responses only. The Mao preference is more widely observed than prudence as 43 (59.72%) of all subjects exhibit it whereas only 3 (4.17%) do not. This complies with our arguments made in Sections 2 and 3 as it shows that also empirically the Mao preference is a weaker property than prudence. The difference in prudent and Mao preferent observations immediately indicates that indeed Mao lotteries are not sufficient to test for prudence.

The conditional frequency $f(\text{Mao preferent}\,|\text{prudent})$ that a prudent individual exhibits the Mao

[30] In this sense Deck and Schlesinger (2008) were not able to classify an individual as prudent with statistical significance unless it answered 6 out of 6 questions in a prudent manner.

preference is 67.65% whereas $f$(not Mao preferent |prudent) is only 2.94%.[31] The chance for a prudent individual to be Mao preferent thus is about 23 times higher than being not Mao preferent. The analysis of the reverse statement does not provide such a clear-cut picture. The conditional frequency $f$(prudent |Mao preferent) is given by 53.49% whereas $f$(imprudent |Mao preferent) equals 6.98%. Thus the chance of being prudent given that an individual is Mao preferent is about 8 times higher for an individual that is not Mao preferent. This result, however, is not very reliable as there are only 3 subjects that were not Mao preferent. Still, we see that knowing about an individual's preference towards the Mao lotteries gives some information about whether the individual is prudent. The result also hints in the 'right' direction as being Mao preferent increases the probability of being prudent. In summary, we can state the following result.

**Result 1** *Most prudent individuals exhibit the Mao preference whereas Mao preferent individuals may not be prudent.*

We conclude that the Mao preference (which is equivalent to prudence under third-order preferences) is not sufficient to make conclusions whether an individual is prudent. Thus, there is more to prudence than to skewness seeking.

### 5.3. Influences on prudent behavior – Factor analysis

To investigate what types of ES questions are more likely to be answered prudently we fix a particular question and analyze the total of 72 individual responses to this question. Table A.3 shows relative frequencies of prudent choices per question as well as the underlying factor levels for each question. In general, we find that the particular choice of the prudence lottery pair has a strong impact on subjects' decisions so that, indeed, the broad scope of Eeckhoudt and Schlesinger's prudence definition must be taken into account.

In order to determine what particular elements in the definition of prudence cause these differences we investigate Factors A, B, C and D according to Research questions 2 to 5. The analysis

[31] If we exclude subjects that were indifferent at least at one stage these numbers become 95.6% and 4.2%, respectively.

is carried out at an aggregate level. Fixing one factor and analyzing aggregate responses given the two distinct factor levels reveals the proportions of prudent choices shown in the upper panel of Table 4.

As formulated in Research question 2 we are interested whether the fixed amount $k$ being a gain or a loss (Factor A) influences subjects' decisions. When $k$ is a loss, 66.32% of responses are prudent, whereas slightly less responses are prudent (63.89%) when $k$ is a gain. Test statistics in of a Wilcoxon signed-rank test and a Fisher-Pitman permutation test for paired replicates in Table 4 show that this difference is insignificant ($p = 0.5253$ and $p = 0.5008$, respectively).

**Result 2** *Subjects' decisions are robust towards different outcomes of the 50/50 gamble, i.e. whether the fixed amount $k$ is a gain or a loss. Implicitly, 0 as a focal point did not influence behavior.*

Considering Factor B, 64.76% of choices are prudent if the wealth level $x$ is high ($x_1 = 160$) and 65.45% of choices are prudent if $x$ is low ($x_2 = 80$) what indicates an insignificant difference (see the test results in Table 4).

**Result 3** *Subjects' decisions are robust towards different wealth levels.*

Research question 4 asks whether a framing of the decision task (Factor C) influences subjects' decisions. The level of Factor C influences prudent choices substantially, as 67.36% of the choices are prudent if the level is "add $\epsilon$" and 62.85% if the level is "add $-k$".[32] Test statistics show that differences are significant below a 10% level.

**Result 4** *Framing of the decision task weakly influences subjects' decisions. At weak significance more subjects answer questions prudently if the zero-mean risk ($\epsilon$) has to be added to the 50/50 gamble compared to the fixed amount ($k$).*

In essence, Result 4 shows that the decision task involving subjects' conscious consideration about another risky event (random draw) leads to more prudent choices, whereas when asked

[32] Keep in mind that Factor C also determines whether the fixed amounts or the zero-mean risk occur in the same or in the different state of Option A and B.

to add a fixed amount subjects make slightly more imprudent choices. Table 4 also shows weak significance for Interaction AC. Weakly significant more choices are prudent whenever i) the fixed amount is a loss ($k_1 = 40$) and subjects are asked to "add $\epsilon$" and ii) the fixed amount is a gain ($k_2 = -40$) and they are asked to "add $-k$".

Factor D considered in Research question 5 is most significant (see Table 4). At its low level

**Table 4 Analysis of prudent choices for different levels of factors and interactions of factors**

| | | Factor level | Relative frequency of prudent choices | $p$-value (Wilcoxon signed- rank test) | $p$-value (Fisher-Pitman permutation test) |
|---|---|---|---|---|---|
| Factors | A | $k_1 = 40$ | 0.6632 | 0.5253 | 0.5008 |
| | | $k_2 = -40$ | 0.6389 | | |
| | B | $x_1 = 160$ | 0.6476 | 0.9863 | 0.8362 |
| | | $x_2 = 80$ | 0.6545 | | |
| | C | add $-k$ | 0.6285 | 0.0697 | 0.0677 |
| | | add $\epsilon$ | 0.6736 | | |
| | D | $\kappa(B_3) - \kappa(A_3) < 0$ | 0.6163 | 0.0154 | 0.0121 |
| | | $\kappa(B_3) - \kappa(A_3) > 0$ | 0.6858 | | |
| Interactions | AB | $k_1, x_1$ and $k_2, x_2$ | 0.6406 | 0.5911 | 0.4409 |
| | | $k_1, x_2$ and $k_2, x_1$ | 0.6615 | | |
| | AC | $k_1$, add $-k$ and $k_2$, add $\epsilon$ | 0.6285 | 0.0697 | 0.0690 |
| | | $k_1$, add $\epsilon$ and $k_2$, add $-k$ | 0.6736 | | |
| | AD | $k_1, \kappa(B_3) - \kappa(A_3) < 0$ and $k_2, \kappa(B_3) - \kappa(A_3) > 0$ | 0.6962 | 0.0098 | 0.0165 |
| | | $k_1, \kappa(B_3) - \kappa(A_3) > 0$ and $k_2, \kappa(B_3) - \kappa(A_3) < 0$ | 0.6059 | | |
| | BC | $x_1$, add $-k$ and $x_2$, add $\epsilon$ | 0.6580 | 0.7375 | 0.6140 |
| | | $x_1$, add $\epsilon$ and $x_2$, add $-k$ | 0.6441 | | |
| | BD | $x_1, \kappa(B_3) - \kappa(A_3) < 0$ and $x_2, \kappa(B_3) - \kappa(A_3) > 0$ | 0.6458 | 0.6487 | 0.7098 |
| | | $x_1, \kappa(B_3) - \kappa(A_3) > 0$ and $x_2, \kappa(B_3) - \kappa(A_3) < 0$ | 0.6563 | | |
| | CD | add $-k$, $\kappa(B_3) - \kappa(A_3) < 0$ and add $\epsilon$, $\kappa(B_3) - \kappa(A_3) > 0$ | 0.6337 | 0.1443 | 0.1288 |
| | | add $-k$, $\kappa(B_3) - \kappa(A_3) > 0$ and add $\epsilon$, $\kappa(B_3) - \kappa(A_3) < 0$ | 0.6684 | | |

(negative kurtosis difference), 68.58% of subjects' choices are prudent. If Factor D is at its high level (positive kurtosis difference), 61.63% of choices are prudent. For questions ES9 (largest positive kurtosis difference in the experiment) and ES10 (largest negative kurtosis difference, other factors like in ES9) 50.00% and 75.00% of answers are prudent, respectively (see Table A.3). Note again that a negative kurtosis difference is equivalent to $\epsilon$ being left-skewed, i.e. the ballot box displayed on subjects' screens contains more white balls (implying a small gain) than yellow balls (implying a high loss).

**Result 5** *The particular choice of the zero-mean risk $\epsilon$ strongly influences subjects' decisions. Significantly more subjects decide prudently if the prudent choice has the smaller kurtosis.*

Intuitively, Result 5 shows more choices to be prudent whenever the imprudent option exhibits more probability mass attributed to extreme outcomes compared to the prudent option. Equivalently, more choices are prudent if the zero-mean risk is left-skewed. Another intuition is, if the zero-mean risk is considered as particularly "bad" when it is left-skewed, there is a higher necessity to be prudent. Thus, the properties of the zero-mean risk do influence subjects' behavior such that Research question 5 has to be answered in the affirmative. Result 5 is a major finding of our experiment. It emphasizes the importance to use "sufficiently" complex lotteries when testing for prudence. We constructed such lotteries by controlling for the first four moments in Eeckhoudt and Schlesinger's definition of the prudence lotteries. Factor D shows again that there is more to prudence than to the 'pure' third-order concept of skewness seeking.

Further, significantly more prudent choices (69.62% compared to 60.59%) occur when i) the fixed amount is a loss and the prudent choice has the smaller kurtosis or ii) the fixed amount is a gain and the imprudent choice has the smaller kurtosis. This is proven by the significant Interaction AD (see Table 4).

### 5.4. Testing for moment preferences

We now perform a test on moment preferences. Remember that Mao pairs are approximations of ES pairs in the sense of Proposition 2. That is, the first three moments of the lotteries in question MAO1 correspond to the moments of the lotteries in questions ES1 and ES2, and in this sense lottery pairs MAO2 correspond to lottery pairs ES3 and ES4, and so on. We investigate whether there is a stronger association between subjects' decisions over such corresponding lottery pairs than to those over the other ones. For each ES question – paired with any Mao question – we set up 8 contingency tables. That equals 128 $2 \times 2$-contingency tables, in total, among which are 16 tables for corresponding Mao and ES pairs.

As a measure of association we use the *phi coefficient* ($r_\phi$).[33] Each contingency table comprises the four categories i) prudent, Mao preferent, ii) prudent, not Mao preferent, iii) imprudent, Mao preferent and iv) imprudent, not Mao preferent.

The results shown in Table A.4 are that for 7 out of 16 comparisons the degree of association between the Mao and the corresponding ES pair is stronger compared to the remaining ones. For 4 out of these 7 associations the difference is significant (at a 6% level) as indicated by test results of a one-sided Fisher-Pitman permutation test (see last column of Table A.4). The probability that the degree of association of a corresponding lottery pair is largest by coincidence is one out of eight. For 7 successes out of 16 observations, the null hypothesis that the probability of a success on a single trial is 1/8 has to be rejected ($p = 0.0019$ two-sided binomial-test).

**Result 6** *For a significant number of ES pairs the number of prudent choices is 'closest' to the number of Mao preferent choices to the corresponding Mao pair. We find that the first three statistical moments have some prediction value but at the same time they are far from completely explaining subjects' decision behavior.*

Recall from Subsection 5.1 that we have already observed a general positive correlation between prudence and the Mao preference which under third-order moment preferences are equivalent. Result 6 again could be interpreted in the way that moments do have some 'approximate' explanation value. Note also that the weak association between moments and preferences observed here supports the necessity of appropriate lottery calibration. This way it can be ruled out that measured effects between experimental stages are only due to different parameter ranges (such as different expected wealth levels) among lotteries. Still, the results are in line with the theoretical findings of Brockett and Garven (1998) that subjects' decisions in the experiment can not be explained completely by the first three moments only. In particular, prudence is not well-captured by skewness seeking.

[33] In general, $r_\phi$ is used as a measure of association or relation between two sets of attributes measured on a nominal scale, each of which may only take two values. Note that $r_\phi = 1$ implies a perfect positive correlation, $r_\phi = 0$ no correlation and $r_\phi = -1$ a perfect negative correlation.

### 5.5. Individual characteristics and questionnaire data

In this subsection we report results from the questionnaire and study the association between prudence and risk aversion. The age of the 72 participants is, on average, 24.25 years; the youngest individual being 19 the oldest 42 years. Among the participants are 41 female and 31 male subjects. To determine who is prudent we use our classification mentioned above. Here, 34 subjects are prudent and 38 subjects are *non-prudent* being either imprudent (6) or indifferent (32). The prudent subjects are, on average, slightly younger (23.73 years) than the non-prudent subjects (24.71 years); see Table 5.

In order to analyze whether prudent individuals tend to make more precautionary savings as suggested by Kimball (1990) we ask two questions on precautionary savings in our questionnaire. Question PREC1 is about retirement pensions. With equal chance the hypothetical situations i) a retirement pension will be paid out and ii) no retirement pension will be paid out occur. It is assumed that a subject earns € 1,000 a month. Subjects are asked how much (€ 0, € 10, …, € 100) of their current earnings they are willing to save in a pension plan each month to account for possible poverty after retirement. The second question on precautionary savings (PREC2) is about a situation where an employee can account for a shortfall in wage payments. In a hypothetical situation the employer promises to the employee to double the wage within six months. The chance that the wage is not paid is 50%. Subjects are asked how much to save monthly (€ 0, € 100, € 200, …, € 600)

**Table 5 Descriptive statistics on questionnaire data**

| | Age | | PREC1 | | | PREC2 | | | LA $(0,1)$ |
|---|---|---|---|---|---|---|---|---|---|
| Category | Mean | Std. Dev. | Mean | Median | Std. Dev. | Mean | Median | Std. Dev. | Mean |
| Prudent (N=34) | 23.73 | 2.5739 | 62.35 | 60.00 | 27.53 | 305.88 | 300.00 | 163.19 | 0.4706 |
| Non-prudent (N=38) | 24.71 | 3.8267 | 60.26 | 60.00 | 28.43 | 242.11 | 300.00 | 153.57 | 0.5263 |

*Notes.* This table shows descriptive statistics of questionnaire items for prudent and non-prudent individuals. PREC 1 and PREC 2 are questions on precautionary savings differing in presented scenarios. LA categorizes subjects as either loss averse (1) or non loss averse (0) according to their answers to questions LA1 and LA2 (see footnote 34).

out of a wage of € 1,000 in order to accommodate the possible shortfall. Descriptive statistics in Table 5 indicate that in both situations prudent individuals tend to save more on average. In question PREC1 prudent individuals save insignificantly more than non-prudent individuals, namely € 62.35 compared to € 60.26, on average ($p = 0.6981$, Mann-Whitney U-test). In question PREC2, prudent (non-prudent) individuals save on average € 305.88 (€ 242.11). The difference for question PREC2 is more substantial, although not significant ($p = 0.1063$).

We tested subjects for loss aversion using two questions from Kahneman and Tversky (1979).[34] Out of all individuals 63 (87.50%) made the risk averse choice in the gain domain which is similar to Kahneman and Tversky's findings. In the question set in a loss domain 58.83% (41.17%) of the subjects chose the safer option (risky option), i.e. the effect we observe is less pronounced. Distinguishing prudent from non-prudent individuals we observe that the former tend to be less loss averse (see the last column in Table 5). This finding is in line with the theoretical results of Ågren (2006) and Maier and Rüger (2009).[35]

Theoretically, prudent individuals can be risk averse, risk neutral or risk loving. This is confirmed by our experimental data where a substantial proportion of subjects is risk averse (87.50%). Among the prudent (non-prudent) subjects are 1 (3) risk neutral, 29 (34) risk averse and 3 (1) risk loving. In particular, the degree of risk aversion of prudent and non-prudent individuals does not differ substantially ($p = 0.8106$, Mann-Whitney U-test).

## 6. Conclusion

In this paper we thoroughly implement the lotteries of Eeckhoudt and Schlesinger (2006) to test individuals for prudence and third-order moment preferences in a laboratory setting. The challenge

[34] Subjects were asked to answer the following questions: LA1) Option A: with 90% chance one wins € 3,000 vs. Option B: with 45% chance one wins € 6000; LA2) Option A: with 90% chance one looses € 3,000 vs. Option B: with 45% chance one looses € 6000. A loss averse individual will be risk averse in the gain domain and risk seeking in the loss domain.

[35] In general, the literature on the relationship between prudence and reference-dependent preferences is scarce. A major contribution was recently made by Maier and Rüger (2009). However, while their definition of reference-dependent preferences is rather general, their analysis of prudence is restricted to symmetric zero-mean risks.

lies in respecting the generality of the prudence lottery preference while the representation of the lotteries must be kept simple enough such that the choice task is easily accessible for experimental subjects.

Concerning the first aspect, we construct a set of 16 prudence lotteries that are controlled up to the fourth statistical moment. We show that this necessarily requires the zero-mean risks of the prudence lotteries to be asymmetric. This is realized in our method by choosing binary lotteries with *unequal* probabilities. Using binary lotteries with asymmetric probabilities and by systematically varying all parameters of the prudence lotteries, we really test for prudence and not for skewness seeking compared to earlier studies related to this issue. We also carry out several tests whether, empirically, prudence is sufficiently well captured by skewness seeking and find that this is not the case. First, one obtains quite different results from different lotteries that are very close up to the third order. Second, up to the fourth order subjects respond to differences in the prudence lotteries. However, in our experiment we also find a weak or 'approximative' association between statistical moments of prospects and preferences.

Concerning the second aspect, we propose a new ballot box representation of the compound prudence lotteries that reflects the full generality of prudence. At the same time it is very easy to understand and translates naturally from subjects' decision screens to the real world draw of the lotteries. As lottery experiments in general are highly sensitive with respect to framing we conduct robustness tests for our method. By means of a factorial design we test whether different types of framing affect subjects' decision behavior concerning the intuition of proper risk apportionment. For an interaction factor we observe weak significance. In particular, however, a gain-loss framing factor that induces a focal point is insignificant.

Implementation of our method in further studies requires asking individuals for 16 decisions, or less. As wealth and the fixed amount being a gain or a loss do not influence subjects' decisions significantly one could dispense with these variations. Hence, the method could be useful in any experiment (complementary to a test for risk aversion) to obtain a more complete picture of individuals' risk attitudes.

From the experimental data we observe prudence on the aggregate as well as on the individual level. 65% of responses are prudent while 47% of individuals answer significantly more questions prudent than imprudent whereas the converse holds only for 8% of the individuals. The aggregate fractions substantially vary under different parameter constellations in the definition of prudence. Questionnaire data reveals that prudent individuals tend to exhibit a stronger precautionary savings motive.

Moreover, this paper contains a calibration theorem for lotteries that allows the researcher to construct binary lotteries with desired first three moments. It is illustrated how this theorem can be used to construct different types of lotteries in the same parameter range and it is particularly suited to conduct tests of moment preference. We also present a statistical characterization of the lotteries of Mao (1970).

As the presented method to test for prudence is robust, it seems natural to also test for temperance. Our representation of the prudence lotteries can be adapted naturally to represent the temperance lotteries.

# Appendix A: Tables and figures

**Figure A.1 Sample of ballot boxes**


*Note.* This photograph shows an example of the ballot boxes used to determine subjects' payoffs at the end of the experiment from a decision made in stage ES, e.g., ES1 (compare to screenshot in Figure 3).

**Table A.1 Mao pairs and their statistical properties**

| | Option A | | | | Option B | | | | Statistical properties | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Mao pair | $p$ | $x_1$ | $1-p$ | $x_0$ | $p$ | $y_1$ | $1-p$ | $y_0$ | $\mathbb{E}[M_A] = \mathbb{E}[M_B]$ | $\mathbb{V}(M_A) = \mathbb{V}(M_B)$ | $\nu(M_B) = -\nu(M_A)$ |
| MAO1 | 0.75 | 0.00 | 0.25 | −80.00 | 0.75 | −40.00 | 0.25 | 40.00 | −20.00 | 1200.00 | 1.15 |
| MAO2 | 0.72 | −3.48 | 0.28 | −61.64 | 0.72 | −36.52 | 0.28 | 21.64 | −20.00 | 688.00 | 0.96 |
| MAO3 | 0.67 | −3.44 | 0.33 | −54.30 | 0.67 | −36.56 | 0.33 | 14.30 | −20.00 | 568.00 | 0.74 |
| MAO4 | 0.58 | −1.81 | 0.42 | −44.62 | 0.58 | −38.19 | 0.42 | 4.62 | −20.00 | 448.00 | 0.30 |
| MAO5 | 0.75 | 40.00 | 0.25 | −40.00 | 0.75 | 0.00 | 0.25 | 80.00 | 20.00 | 1200.00 | 1.15 |
| MAO6 | 0.72 | 36.52 | 0.28 | −21.64 | 0.72 | 3.48 | 0.28 | 61.64 | 20.00 | 688.00 | 0.96 |
| MAO7 | 0.67 | 36.56 | 0.33 | −14.30 | 0.67 | 3.44 | 0.33 | 54.30 | 20.00 | 568.00 | 0.74 |
| MAO8 | 0.58 | 38.19 | 0.42 | −4.62 | 0.58 | 1.81 | 0.42 | 44.62 | 20.00 | 448.00 | 0.30 |

**Table A.2 Lottery pairs according to Holt and Laury (2002) in stage RIAV**

| | Option A | | | | Option B | | | | Statistical properties | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $p$ | $x_1$ | $1-p$ | $x_0$ | $p$ | $y_1$ | $1-p$ | $y_0$ | $\mathbb{E}[A]$ | $\mathbb{E}[B]$ | $\mathbb{E}[A]-\mathbb{E}[B]$ | $\mathbb{V}[A]$ | $\mathbb{V}[B]$ |
| RIAV1 | 0.1 | 20 | 0.9 | 16 | 0.1 | 38.5 | 0.9 | 1 | 16.40 | 4.75 | 11.65 | 1.44 | 126.56 |
| RIAV2 | 0.2 | 20 | 0.8 | 16 | 0.2 | 38.5 | 0.8 | 1 | 16.80 | 8.50 | 8.30 | 2.56 | 225.00 |
| RIAV3 | 0.3 | 20 | 0.7 | 16 | 0.3 | 38.5 | 0.7 | 1 | 17.20 | 12.25 | 4.95 | 3.36 | 295.31 |
| RIAV4 | 0.4 | 20 | 0.6 | 16 | 0.4 | 38.5 | 0.6 | 1 | 17.60 | 16.00 | 1.60 | 3.84 | 337.50 |
| RIAV5 | 0.5 | 20 | 0.5 | 16 | 0.5 | 38.5 | 0.5 | 1 | 18.00 | 19.75 | $-1.75$ | 4.00 | 351.56 |
| RIAV6 | 0.6 | 20 | 0.4 | 16 | 0.6 | 38.5 | 0.4 | 1 | 18.40 | 23.50 | $-5.10$ | 3.84 | 337.50 |
| RIAV7 | 0.7 | 20 | 0.3 | 16 | 0.7 | 38.5 | 0.3 | 1 | 18.80 | 27.25 | $-8.45$ | 3.36 | 295.31 |
| RIAV8 | 0.8 | 20 | 0.2 | 16 | 0.8 | 38.5 | 0.2 | 1 | 19.20 | 31.00 | $-11.80$ | 2.56 | 225.00 |
| RIAV9 | 0.9 | 20 | 0.1 | 16 | 0.9 | 38.5 | 0.1 | 1 | 19.60 | 34.75 | $-15.15$ | 1.44 | 126.56 |
| RIAV10 | 1.0 | 20 | 0.0 | 16 | 1.0 | 38.5 | 0.0 | 1 | 20.00 | 38.50 | $-18.50$ | 0.00 | 0.00 |

**Table A.3 Frequencies of prudent choices by question in stage ES**

| | | | Factors | | | |
|---|---|---|---|---|---|---|
| ES pair | Abs. freq. | Rel. freq. | A | B | C | D |
| ES1 | 49 | 0.6806 | -40 | 160 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ |
| ES2 | 43 | 0.5972 | -40 | 160 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES3 | 44 | 0.6111 | -40 | 160 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ |
| ES4 | 51 | 0.7083 | -40 | 160 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES5 | 43 | 0.5972 | -40 | 80 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ |
| ES6 | 50 | 0.6944 | -40 | 80 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES7 | 52 | 0.7222 | -40 | 80 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ |
| ES8 | 50 | 0.6944 | -40 | 80 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES9 | 36 | 0.5000 | 40 | 160 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES10 | 54 | 0.7500 | 40 | 160 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ |
| ES11 | 45 | 0.6250 | 40 | 160 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES12 | 51 | 0.7083 | 40 | 160 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ |
| ES13 | 37 | 0.5139 | 40 | 80 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES14 | 50 | 0.6944 | 40 | 80 | Add $-k$ | $\kappa(B_3)-\kappa(A_3)<0$ |
| ES15 | 43 | 0.5972 | 40 | 80 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)>0$ |
| ES16 | 52 | 0.7222 | 40 | 80 | Add $\epsilon$ | $\kappa(B_3)-\kappa(A_3)<0$ |

**Table A.4 Correlation ($r_\phi$) between Mao and ES pairs**

| | MAO1 | MAO2 | MAO3 | MAO4 | MAO5 | MAO6 | MAO7 | MAO8 | $p$-value |
|---|---|---|---|---|---|---|---|---|---|
| ES1 | **0.1491** | 0.0087 | 0.0016 | 0.0888 | 0.0262 | 0.0940 | 0.1027 | 0.0170 | 0.0078 |
| ES2 | **0.0028** | 0.0194 | 0.0007 | 0.0032 | 0.0045 | 0.0036 | 0.0032 | 0.0005 | 0.7031 |
| ES3 | 0.2130 | **-0.0510** | 0.0400 | -0.0041 | 0.0117 | 0.2050 | 0.2273 | -0.1544 | 0.9688 |
| ES4 | 0.1050 | **0.0410** | 0.1480 | 0.0960 | -0.1035 | 0.1478 | -0.1409 | 0.1011 | 0.5859 |
| ES5 | 0.1315 | 0.0127 | **0.0974** | -0.0174 | -0.0029 | 0.1229 | 0.0769 | 0.0223 | 0.0547 |
| ES6 | 0.2800 | 0.1888 | **0.1312** | 0.0806 | -0.1176 | -0.0075 | 0.1282 | 0.0133 | 0.1953 |
| ES7 | 0.1272 | 0.0555 | 0.1654 | **0.1120** | -0.0127 | 0.1692 | 0.1663 | 0.0508 | 0.3750 |
| ES8 | 0.0836 | 0.1888 | 0.1312 | **0.0022** | -0.1176 | -0.0748 | 0.1282 | 0.1501 | 0.9297 |
| ES9 | 0.1206 | 0.1491 | -0.0702 | 0.0361 | **0.0342** | 0.0000 | 0.0327 | -0.0945 | 0.3672 |
| ES10 | 0.0348 | 0.0861 | 0.2026 | 0.0625 | **0.0987** | 0.0000 | 0.1322 | 0.0910 | 0.3125 |
| ES11 | 0.1713 | 0.0385 | 0.0544 | -0.0653 | -0.2561 | **0.0320** | -0.0253 | -0.1383 | 0.1406 |
| ES12 | -0.0940 | 0.0410 | 0.1480 | 0.0166 | 0.1223 | **-0.0569** | 0.0030 | 0.0318 | 0.9922 |
| ES13 | 0.2597 | 0.3107 | 0.2243 | 0.2659 | 0.2537 | 0.1413 | **0.3099** | 0.1112 | 0.0156 |
| ES14 | -0.1127 | 0.0270 | -0.0212 | 0.0022 | 0.1050 | -0.2094 | **0.1992** | -0.0551 | 0.0078 |
| ES15 | 0.1315 | 0.0887 | 0.1689 | 0.0562 | 0.0668 | -0.0035 | 0.1435 | **-0.2989** | 1.0000 |
| ES16 | -0.0075 | 0.1387 | 0.1654 | 0.1120 | 0.1400 | 0.1000 | 0.0933 | **0.0508** | 0.9766 |

*Note:* $p$-values are shown for a one-sided Fisher-Pitman permutation test for paired replicates.

## Appendix B: Proofs

**Proof of Proposition 1.** Let $n \in \mathbb{N}$ be arbitrary. It is well known that the $\mu_n(\cdot)$-operator is homogeneous of degree $n$ and translation invariant. The assumption $p_X = 1 - p_Y$ is equivalent to $X = 1 - Y$ such that $\mu_n(X) = (-1)^n \mu_n(Y)$ which for $n = 2$ just implies $\mathbb{V}(X) = \mathbb{V}(Y)$. Note that we can write $L_X = X \cdot x_1 + (1 - X) \cdot x_0 = (x_1 - x_0)X + x_0$ and thus $\mathbb{V}(L_X) = (x_1 - x_0)^2 \mathbb{V}(X)$. Analogously, we have $\mathbb{V}(L_Y) = (y_1 - y_0)^2 \mathbb{V}(Y)$. Since the Mao lotteries have equal variance we obtain $(x_1 - x_0)^2 = (y_1 - y_0)^2$ and because of the unique representation of binary lotteries (see Definition 1) this is equivalent to

$$x_1 - x_0 = y_1 - y_0. \tag{3}$$

Using once more homogeneity and translation invariance of the $\mu_n(\cdot)$-operator and plugging in yields

$$\mu_n(L_X) = (x_1 - x_0)^n \mu_n(X) = (y_1 - y_0)^n (-1)^n \mu_n(Y) = (-1)^n \mu_n(L_Y).$$

Because of the assumed variance equality the claim for $\mu_n^S(\cdot)$ follows immediately. □

**Proof of Theorem 1.** After calculating expectation, variance and skewness of a binary lottery as in Definition 1 we find that $L_X = L_X(p, x_1, x_0)$ has to suffice the following system of equations

$$E = px_1 + (1 - p)x_0 \tag{4}$$

$$V = (x_1 - x_0)^2 p(1 - p) \tag{5}$$

$$S = \frac{1 - 2p}{\sqrt{p(1 - p)}} \tag{6}$$

with $x_1 > x_0$ and $0 < p < 1$. It is natural to start with solving equation (6) for $p$. After squaring and some rearrangement one obtains

$$p^2(-S^2 - 4) + p(4 + S^2) - 1 = 0.$$

Setting $\tilde{S} := 4 + S^2$ the solutions to this quadratic equation are given by

$$p_{1/2} = \frac{\tilde{S} \overset{+}{-} \sqrt{\tilde{S}^2 - 4\tilde{S}}}{2\tilde{S}}, \tag{7}$$

where $p_1$ is the solution associated with the addition. It is easy to see that the expression under the square root is always positive. If $S = 0$ there is one solution, namely $p = \frac{1}{2}$. Otherwise there are two solutions. Both these solutions are strictly positive since $\sqrt{\tilde{S}^2 - 4\tilde{S} + 4 - 4} = \sqrt{(\tilde{S} - 2)^2 - 4)} \leq \tilde{S} - 2$ and thus

$$p_{1/2} \geq p_2 \geq \frac{\tilde{S} - (\tilde{S} - 2)}{2\tilde{S}} = \frac{1}{\tilde{S}} > 0.$$

All solutions are smaller than 1 since

$$p_1 < 1 \Longleftrightarrow \sqrt{\tilde{S}^2 - 4\tilde{S}} < \tilde{S}$$

what can be shown to be true for all $\tilde{S}$ (and thus for all $S$) by doing the quadratic expansion as in the previous calculation. Note that equation (6) is a square root equation and thus we have to verify the solutions. Obviously, if $S = 0$ then $p = 0.5$ is the unique solution. Otherwise, from equation (7) it follows that $p_1 > p_2$ and $p_1 + p_2 = 1$, i.e. $p_1 > 0.5$ and $p_2 < 0.5$. If $S < 0$ then $p_2$ does not solve equation (6) because $1 - 2p_2 > 0$, but $p_1$ does. Similarly, if $S > 0$ only $p_2$ solves equation (6). Thus in any case equation (6) has a unique solution in $(0, 1)$ (such that it is a probability) that will be denoted by $p$ in the following.[36]

The remainder of the proof is straightforward. For any $p$ obtained from equation (6) the system of equations (4) and (5) can be solved for a unique solution to obtain the expressions stated in the claim from which finally also $x_1 > x_0$ is evident. □

**Proof of Proposition 2.** By Theorem 1 there exists exactly one binary lottery $L_X \equiv M_A$ with $\mathbb{E}[M_A] = \mathbb{E}[A]$, $\mathbb{V}[M_A] = \mathbb{V}[A]$ and $\nu[M_A] = -S$. By Theorem 1 there also exists exactly one lottery $L_Y \equiv M_B$ whose expectation and variance equal that of $M_A$ and further $\nu[M_B] = +S$. From equation (6) one immediately obtains $p_X = 1 - p_Y$ such that by Definition 2 $(M_A, M_B)$ constitutes a Mao lottery pair that fulfills the requested moment conditions.

For the second part, note that by taking derivatives

$$\Delta = (\nu[B] - S)^2 + (\nu[A] - (-S))^2$$

indeed achieves its maximum at $S = \frac{\nu[B] - \nu[A]}{2}$. The difference in skewness of the Mao pair is $2S$ and as can be seen from the previous equation this indeed equals the skewness difference $\nu[B] - \nu[A]$ of the prudence pair. □

[36] We can see now how skewness is reflected in a binary lottery. It has zero skewness if and only if both states have equal probability. Otherwise, it is positively (negatively) skewed if the high payoff is associated with the low (high) probability.

## Appendix C: Instructions

[*translated from German for session order MAO-ES-RIAV*]

**Thank you very much for participating in this decision experiment!**

### General Information

In the following experiment, you will make a couple of decisions. Following the instructions and depending on your decisions, you can earn money. It is therefore very important that you read the instructions carefully.

You will make your decisions anonymously on your computer screen in your cubicle. During the experiment you are not allowed to talk to the other participants. Whenever you have a question, please raise your hand. The experimenter will answer your question in private in your cubicle. If you disregard these rules you can be excluded from the experiment. Then you receive no payment.

During the experiment all amounts are stated in Taler, the experimental currency. At the end of the experiment, your achieved earnings will be converted into Euro at an exchange rate of 1 Taler = € 0.15 and paid to you in cash.

### Structure of the Experiment and Your Decisions

In total, you will make 34 decisions throughout the experiment. In each decision you will decide upon which of **two different risky events – either Option A or Option B** – you prefer.

*An example of Option A could be as follows: With 50% chance you will lose 10 Taler or with 50% chance you will receive 20 Taler. Option B could be: With 20% chance you will receive 0 Taler and with 80% chance you will receive 10 Taler.*

The experiment consists of three stages that will be explained in detail in the following. To determine your payoff in the experiment, one of your decisions will be randomly chosen. This takes place after you have completed all your decisions. To this end, the experimenter picks one of 34 balls, marked with numbers from 1 to 34, out of a ballot box. Each number occurs only once in the ballot box, whereby the draw of a particular number is equally likely. The outcome of the risky event, that you have opted for, at the randomly chosen decision will afterwards be determined by another random draw. This procedure will be explained extensively when the stages of the experiment are described.

**Keep in mind that only one of your 34 decisions determines your payoff in the experiment. Therefore each of your single decisions can determine your entire payoff in the experiment.**

You make your decisions at the computer screen in the computer lab. For each decision you have a maximum of 3 minutes. After the experiment, the decision relevant for every participants's payoff and the outcome of the risky event will be determined by random draws for each participant in the seminar room. For this the experimenter will call upon participants one by one.

Note that some risky events comprise negative outcomes. For these questions you therefore receive coupons indicating an endowment (in Taler). You can charge the coupons when the outcome of the risky event is determined.

### Stage I

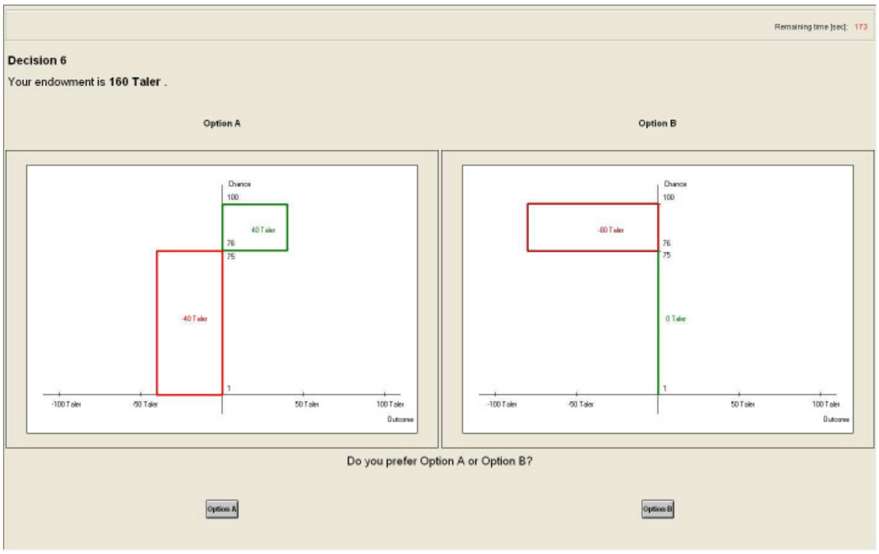
In the first stage of the experiment you are asked to make eight decisions. On each of the 8 sequent decision screens you decide which of the two risky events – Option A or Option B – you prefer.

For your decisions you receive an endowment in Taler, because the outcomes of the risky events in this stage can comprise losses. Accordingly, your payoff in this stage consists of two components:

**Endowment** and **Outcome of the chosen risky event**

How is the outcome of the (chosen) risky event determined in Stage I? To this end, there is another ballot box. This ballot box contains 100 balls marked with numbers from 1 to 100. Each number occurs only once, thus the draw of a particular number is equally likely.

An example of a decision screen is provided in the following figure:




*In Option A you will lose 40 Taler with 75% chance (balls 1 to 75) or with 25% chance you will receive 40 Taler (balls 76 to 100). In Option B you receive 0 Taler with 75% chance (balls 1 to 75) or you will lose 80 Taler with 25% chance (balls 76 to 100). Your endowment is 160 Taler in this example.*

*Now suppose that this decision was randomly drawn to determine your payoff.*

- *Suppose you have chosen **Option A** and assume that a ball is drawn from the ballot box with a number between 1 and 75. That means, you lose 40 Taler. Your resulting payoff, after allocating the endowment of 160 Taler for this decision to the lottery outcome, is 120 Taler. If a ball with a number between 76 and 100 is drawn, you receive 40 Taler. Under consideration of your endowment your payoff is 200 Taler.*
- *Suppose you have chosen **Option B** and assume that a ball is drawn from the ballot box with a number between 1 and 75. That means, you receive 0 Taler. Your resulting payoff after allocating the endowment of 160 Taler for this decision to the lottery outcome is 160 Taler. If a ball with a number between 76 and 100 is drawn, you lose 80 Taler. Under consideration of your endowment your payoff is 80 Taler.*

### Stage II

In the second stage of the experiment you make 16 decisions. Again, on each of the 16 sequent decision screens you decide which of the two risky events – either Option A or Option B – you prefer. In this stage risky events (may) comprise two random draws.

For each decision one random draw is given. This draw is as follows: With 50% chance the situation "Up" occurs or with 50% chance the situation "Down" occurs.

For your decisions you receive an endowment in Taler, because the outcomes of the risky events in this stage can also comprise losses. Accordingly, your payoff in this stage consists of two components:

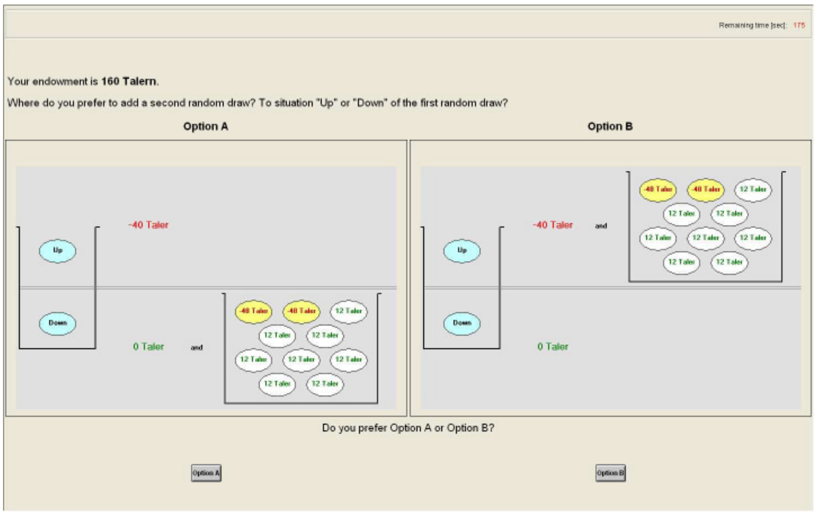
**Endowment** and **Outcome of the chosen risky event**

How is the outcome of the (chosen) risky event determined in Stage II? For the first random draw there are two balls in a ballot box – one marked with "Up" and the other with "Down". The draw of a particular ball is equally likely.

To determine your payoff in this stage **two** random draws may be necessary. At the second random draw one ball is drawn from another ballot box with 10 balls. The balls are either yellow or white. Note that the composition of yellow and white balls may change for different decisions in this stage. But within one decision, i.e. for Option A and Option B, the composition of yellow and white balls remains the same.

### Decision type 1

For 8 out of 16 decisions you are asked the following: Given what situation of the first random draw – either "Up" or "Down" – do you prefer a **second random draw**? An example is provided by the following screen:




*In Option A you lose 40 Taler, if situation "Up" occurs in the first random draw. If situation "Down" occurs, you receive 0 Taler and a second random draw succeeds. This second random draw is as follows: With 20% chance you lose 48 Taler and with 80% chance you receive 12 Taler. In Option B, you lose 40 Taler if in the first random draw the situation "Up" occurs and a second random draw succeeds (The second random draw is the same as in Option A). When situation "Down" occurs, you receive 0 Taler. For this decision you are endowed with 160 Taler.*

*Now suppose the decision from the example above is randomly drawn to determine your payoff.*
*Suppose you have chosen **Option A**.*

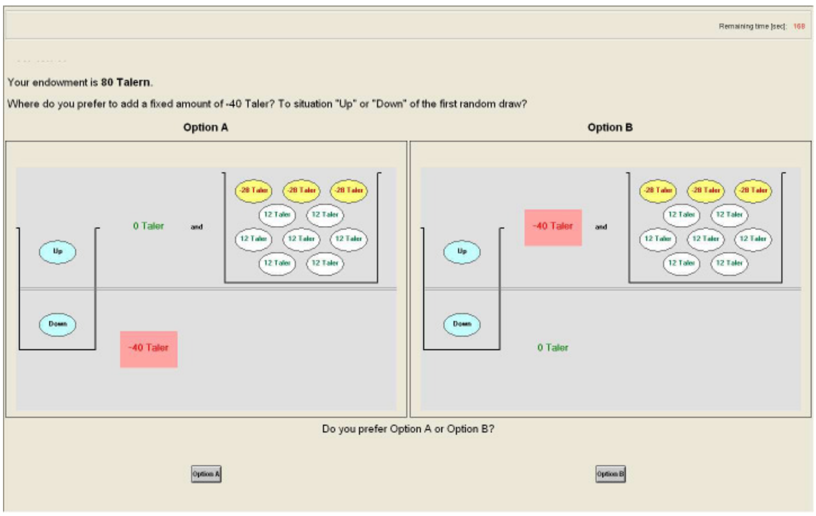
- *If in the first random draw the ball "Up" is drawn, you lose 40 Taler. After allocating your endowment of 160 Taler for this decision to the lottery outcome, your payoff is 120 Taler.*
- *If in the first random draw the ball "Down" is drawn, you receive 0 Taler and a second random draw succeeds.*
  - *If in the second random draw a <u>yellow</u> ball is drawn, you lose 48 Taler and your payoff after allocating your endowment is 112 Taler.*
  - *If in the second random draw a <u>white</u> ball is drawn, you receive 12 Taler and your payoff after allocating your endowment is 172 Taler.*

*Suppose you have chosen **Option B**.*

- *If in the first random draw ball "Up" is drawn, you lose 40 Taler and a second random draw succeeds.*
  - *If in the second random draw a <u>yellow</u> ball is drawn, you lose 48 Taler and your payoff after allocating your endowment is 72 Taler.*
  - *If in the second random draw a <u>white</u> ball is drawn, you receive 12 Taler and your payoff after allocating your endowment is 132 Taler.*
- *If in the first random draw "Down" is drawn, you receive 0 Taler. After allocating your endowment of 160 Taler for this decision to the lottery outcome your payoff is 160 Taler.*

### Decision type 2

For the remaining 8 out of 16 decisions in Stage II you are asked the following: To what situation do you prefer to add a **(fixed) amount** – either to situation "Up" where a second random draw succeeds or to situation "Down" where no second random draw succeeds. Note that the fixed amount can either be positive or negative. An example is provided by the following screen:



*In Option A, when situation "Up" occurs in the first random draw you receive 0 Taler and a second random draw succeeds. The second random draw is as follows: With 30% chance you lose 28 Taler and with 70% chance you receive 12 Taler. When situation "Down" occurs in the first random draw, you lose 40 Taler and no second random draw succeeds. In Option B if situation "Up" occurs in the first random draw you lose 40 Taler and a second random draw succeeds (The second random draw is the same as in Option A). When situation "Down" occurs, you receive 0 Taler and no second random draw succeeds. For this decision you are endowed with 80 Taler.*

*Now suppose the decision from the example above is randomly drawn to determine your payoff.*
*Suppose you have chosen **Option A**.*

- *If in the first random draw the ball "Up" is drawn, you receive 0 Taler and a second random draw succeeds.*

  — *If in the second random draw a yellow ball is drawn, you lose 28 Taler and your payoff after allocating your endowment is 52 Taler.*

  — *If in the second random draw a white ball is drawn, you receive 12 Taler and your payoff after allocating your endowment is 92 Taler.*

- *If in the first random draw the ball "Down" is drawn, you lose 40 Taler. After allocating your endowment of 80 Taler for this decision to the lottery outcome your payoff is 40 Taler.*

*Suppose you have chosen **Option B**.*

- *If in the first random draw the ball "Up" is drawn, you lose 40 Taler and a second random draw succeeds.*

  — *If in the second random draw a yellow ball is drawn, you lose 28 Taler and your payoff after allocating your endowment is 12 Taler.*

  — *If in this second a white ball is drawn, you receive 12 Taler and your payoff after allocating your endowment is 52 Taler.*

- *If in the first random draw the ball "Down" is drawn, you receive 0 Taler. After allocating your endowment of 80 Taler for this decision to the lottery outcome your payoff is 80 Taler.*

### Stage III

In Stage III you are asked to make 10 decisions on a single decision screen. The risky events between which you have to decide in this stage are displayed in a table format. In each row of the table you make one decision. For an illustration see the following figure:

Remaining time [sec]: 1799

| | Option A | | | | Option B | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Decision** | Chance of Outcome 1 | Outcome 1 (in Taler) | Chance of Outcome 2 | Outcome 2 (in Taler) | Chance of Outcome 1 | Outcome 1 (in Taler) | Chance of Outcome 2 | Outcome 2 (in Taler) | **Your decision** |
| 25 | 10% | 20.00 | 90% | 16.00 | 10% | 38.50 | 90% | 1.00 | Option A ○ ○ Option B |
| 26 | 20% | 20.00 | 80% | 16.00 | 20% | 38.50 | 80% | 1.00 | Option A ○ ○ Option B |
| 27 | 30% | 20.00 | 70% | 16.00 | 30% | 38.50 | 70% | 1.00 | Option A ○ ○ Option B |
| 28 | 40% | 20.00 | 60% | 16.00 | 40% | 38.50 | 60% | 1.00 | Option A ○ ○ Option B |
| 29 | 50% | 20.00 | 50% | 16.00 | 50% | 38.50 | 50% | 1.00 | Option A ○ ○ Option B |
| 30 | 60% | 20.00 | 40% | 16.00 | 60% | 38.50 | 40% | 1.00 | Option A ○ ○ Option B |
| 31 | 70% | 20.00 | 30% | 16.00 | 70% | 38.50 | 30% | 1.00 | Option A ○ ○ Option B |
| 32 | 80% | 20.00 | 20% | 16.00 | 80% | 38.50 | 20% | 1.00 | Option A ○ ○ Option B |
| 33 | 90% | 20.00 | 10% | 16.00 | 90% | 38.50 | 10% | 1.00 | Option A ○ ○ Option B |
| 34 | 100% | 20.00 | 0% | 16.00 | 100% | 38.50 | 0% | 1.00 | Option A ○ ○ Option B |

OK

Each risky event comprises two possible outcomes and two corresponding probabilities. You make your decision at the end of each row by indicating the risky event you prefer (either Option A or Option B).

When making your decisions you do not have to follow a particular order and you can change your decisions as often as desired within the time permitted.

The outcomes of the risky events in this stage do not comprise losses. Thus, for the decisions in this stage you do not receive an endowment. Accordingly, your payoff is as follows:

$$\boxed{\textbf{Outcome of the risky event}}$$

How is the outcome of the chosen risky event determined in Stage III? To determine the outcome there is a ballot box with 100 balls marked with numbers from 1 to 100 (analogously to Stage I). Each number occurs exactly once in the ballot box, i.e. the draw of a particular number is equally likely.

**Before the experiment will start now, please note:** You are asked comprehension questions before each stage starts. These questions should familiarize you with the decision task in each stage.

After the experiment, you are asked to answer a questionnaire. For answering the questionnaire you receive independently from your earnings during the experiment € 4.

## Acknowledgments

We thank Johannes Abeler, Aurélien Baillon, Armin Falk, Johannes Maier, Reinhard Selten, Klaus Utikal, Gari Walkowitz and participants of the topics course in experimental economics at the University of Bonn for helpful comments and suggestions. In particular we thank Louis Eeckhoudt for his numerous valuable suggestions. We are grateful to Timo Heinrich for his programming assistance and Rainer Rilke for assisting us during the experimental sessions. We gratefully acknowledge the financial support from the Bonn Graduate School of Economics. Financial support for Daniel Wiesen by the Konrad-Adenauer-Stiftung e.V. is gratefully acknowledged.

## References

Ågren, M. 2006. Prospect theory and higher moments. Working Paper, University of Uppsala.

Arditti, F. D. 1967. Risk and the required return on equity. *J. Finance* **22** 19–36.

Briec, W., K. Kerstens, O. Jokung. 2007. Mean-variance-skewness portfolio performance gauging: A general shortage function and dual approach. *Management Sci.* **53** 135–149.

Brockett, P. L., J. R. Garven. 1998. A Reaxamination of the relationship between preferences and moment orderings by rational risk-averse investors. *Geneva Papers Risk Insurance Theory* **23** 127–137.

Brockett, P. L., Y. Kahane. 1992. Risk, return, skewness and preference. *Management Sci.* **38** 851–866.

Camerer, C. F. 1989. An experimental test of several generalized utility theories. *J. Risk Uncertainty* **2** 61–104.

Courbage, C., B. Rey. 2006. Prudence and optimal prevention for health risks. *Health Econom.* **15** 1323–1327.

Crainich, D., L. Eeckhoudt. 2008. On the intensity of downside risk aversion. *J. Risk Uncertainty* **36** 267–276.

Deck, C., H. Schlesinger. 2008. Exploring higher order risk effects in the laboratory. Working Paper, CESifo Working Paper No. 2487.

Eeckhoudt, L., C. Gollier. 2005. The impact of prudence on optimal prevention. *Econom. Theory* **26** 989–994.

Eeckhoudt, L., H. Schlesinger. 2006. Putting risk in its proper place. *Amer. Econom. Rev.* **96** 280–289.

Eichner, T. 2008. Mean variance vulnerability. *Management Sci.* **54** 586–593.

Ekern, S. 1980. Increasing nth degree risk. *Econom. Lett.* **6** 329–333.

Fei, W., H. Schlesinger. 2008. Precautionary insurance demand with state-dependent background risk. *J. Risk Insurance* **75** 1–16.

Fischbacher, U. 2007. Z-tree: Zurich toolbox for readymade economic experiments. *Exper. Econom.* **10** 171–178.

Gomes, F., A. Michaelides. 2005. Optimal life-cycle asset allocation: Understanding the empirical evidence. *J. Finance* **60** 869–904.

Harrison, G. W., E. Rutström. 2008. Risk aversion in the laboratory. G. W. Harrison, J. C. Cox, eds., *Research in Experimental Economics 12: Risk Aversion in Experiments*. Emerald, Bingley, U.K., 41–196.

Holt, C. A., S. K. Laury. 2002. Risk aversion and incentive effects. *Amer. Econom. Rev.* **92** 1644–1655.

Kahneman, D., A. Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* **47** 263–292.

Kimball, M. S. 1990. Precautionary savings in the small and in the large. *Econometrica* **58** 53–73.

Kraus, A., R. H. Litzenberger. 1976. Skewness preference and the valuation of risky assets. *J. Finance* **31** 1085–1100.

Laury, S. K. 2005. Pay one or pay all: Random selection of one choice for payment. Working Paper, Andrew Young School of Policy Studies Research Paper Series 06-13.

Maier, J., M. Rüger. 2009. References-dependent risk preferences of higher orders. Working Paper, University of Munich.

Mao, J. C. T. 1970. Survey of capital budgeting: Theory and practice. *J. Finance* **25** 349–369.

Menezes, C., C. Geiss, J. Tressler. 1980. Increasing downside risk. *Amer. Econom. Rev.* **70** 921–932.

Montgomery, D. C. 2005. *Design and Analysis of Experiments*. John Wiley & Sons, New York.

Roger, P. 2009. Mixed risk aversion and preference for risk disaggregation: A story of moments. Working Paper, Strasbourg University.

Scott, R. C., P. A. Horvath. 1980. On the direction of preferences for moments of higher order than the variance. *J. Finance* **35** 915–919.

Tarazona-Gomez, M. 2003. Are individuals prudent? An experimental approach using lottery choices. Working Paper, Copenhagen Business School.

Unser, M. 2000. Lower partial moments as measures of perceived risk: An experimental study. *J. Econom. Psych.* **21** 253–280.