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by

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Party Platforms in Electoral Competition with Many Constituencies

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Abstract

This paper shows how political parties differentiate to reduce electoral competition. Two parties choose platforms in a unidimensional policy space, and then in a continuum of constituencies with different median voters candidates from the two parties compete in Hotelling-Downs competition. Departing from party platform is costly enough that candidates do not take the median voter's preferred position in each constituency. In equilibrium, parties acting in their candidates' best interests differentiate—when one party locates right of center, the other prefers to locate strictly left of center to carve out a “home turf,” constituencies that can be won with little to no deviation from party platform. Hence, Downsian competition that pulls candidates together pushes parties apart. Decreasing “campaign costs” increases party differentiation as the leftist party must move further from the rightist party to carve out its home turf. For a range of costs, parties take more extreme positions than their most extreme candidates. For small costs, parties are too extreme to maximise voter welfare, whereas for large costs they are not extreme enough.

Keywords Parties, median voter, Downsian competition.

JEL Classifications C72

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1 Introduction

Competition among political parties is surprisingly lacklustre. Approximately one-sixth of seats in the American House of Representatives go uncontested by one of the two major parties—Democrats or Republicans—each electoral cycle. American political parties systematically lose elections in politically unfavorable constituencies: Republicans win in the now-famous “red” states of the South, Mid-West, and South-West, while Democrats win in the remaining “blue” states, primarily on the two coasts. Contrary to the claims of Naderites, the two major parties differ in significant ways: virtually all Democrats in the Senate are less conservative than virtually all Republicans.¹ Why is political competition so uncompetitive?

This paper shows that rather than draw them together, political competition drives parties apart. Perhaps the most famous result in formal political theory is that two candidates competing in a first-past-the-post election for political office should jointly adopt the median voter’s preferred position (Hotelling 1929, Downs 1957, Black 1958). We use the Hotelling-Downs model of electoral competition as our point of departure to explore party positioning. What types of platforms do political parties acting in their candidates’ best interests espouse? We show that parties separate precisely because symmetric candidates compete so vigorously. They differentiate to reduce political competition.

We model a political party as a collection of candidates, each of whom campaigns in a different constituency. In each constituency, candidates from the two parties compete in a first-past-the-post election. Voters in every constituency have single-peaked preferences in the same unidimensional policy space; different constituencies have median voters with different ideal points. Knowing the distribution of median voters’ ideal points, the two political parties choose where to place their platforms in policy space. A party’s platform serves as its candidates’ default policy position. However, in any constituency, either candidate may depart from her party’s position at a cost: the further she moves from her party, the higher the candidate’s cost. Such costs could come about for any number of reasons, from campaign advertising—informing voters of a position different from your party’s requires costly advertising—to career concerns—departing from party platform diminishes prospects within the party, while toeing the party line becomes more costly the further away it lies. (For

¹Excluding the Democratic Senator Miller of Georgia, and the Republican Senators Chafee of Rhode Island and Snowe of Maine, all Democratic Senators had lower ADA scores in 2003 than all Republican Senators (<www.adaction.org>). ADA scores are commonly used in the political-science literature to measure of political orientation.

expositional simplicity, we refer to these costs of departing from party platform as campaign costs throughout.) Candidates trade campaign costs off against the private benefit of winning elections.

In this setting, we ask where political parties seeking to maximise their candidates' payoffs position their platforms. Our main result is that if campaign costs are high enough that candidates do not adopt the median voter's preferred position in each constituency, then parties do not jointly adopt the median voter's median preferred position; they non-cooperatively differentiate from each other. This happens because the closer the two parties' positions, the more intensely their candidates compete to win election in any given constituency. Consequently, each party has incentive to move away from the other—giving up heavily contested elections—in order to carve out politically sympathetic constituencies where it wins elections without much costly repositioning. Thus, while competition may drive candidates together, it drives parties apart.

In addition to providing their candidates with funds and organisational infrastructure, political parties also signal their candidates' policy positions. While candidates deviate from party policy to cater to their constituents—Republicans in Maryland espouse more liberal positions than their colleagues in Virginia—they clearly do not go so far as to adopt the median voter's preferred position. Parties systematically lose elections in politically unfavourable constituencies. Simple evidence for this comes from the correlation in American Senators' political affiliation. If voters elected candidates based upon their policy positions (or anything uncorrelated with party), and candidates took the median voter's position in every state, then given that roughly half of Senators come from each major party, the correlation in party affiliation between the junior and senior Senators across the fifty states would be zero: knowing a state's senior Senator's party affiliation would provide no information about the junior Senator's. The fact that the current correlation is 0.50 provides strong evidence that certain states favor candidates from one political party over the other.² This partisanship may derive from voters' having inherent preferences for candidates from one party over those from the other (begging the question of why parties enjoy such advantages in the first place) or because candidates depart from the median voter's position in systematic ways. The

²We exclude Vermont from the correlation as Senator Jeffords belongs to neither party. Of the remaining 98 Senators, 47 are Democrats and 51 Republican. The null hypothesis that their parties are uncorrelated can be formally tested through a chi-squared test that Senators' party affiliations are independent draws. The chi-squared statistic with one degree of freedom is 598.3, rejecting the null hypothesis at any conventional significance level.

closer a candidate's party lies to the median voter's preferred policy, the closer the candidate positions to the median voter; consequently, the candidate whose party platform lies closer to the median voter wins the election with higher probability. We favor the latter interpretation, which we capture in a simple way: candidates pay campaign costs to deviate from their party platform.

In our model, the candidate whose party platform lies closer to the median voter has an advantage in the election and is more likely to win. In some constituencies (parameterised by their different median voters) both candidates toe the party line, and the advantaged candidate wins the election with probability one. In others, candidates mix over policy positions; no candidate would undertake a costly departure from party platform only to lose the election with probability one. In these constituencies, the advantaged candidate sometimes loses but always wins with higher likelihood than the disadvantaged candidate. More importantly, by increasing her advantage, a candidate can decrease the campaign costs needed to win the election with any given probability. As the Republican party moves to the right, Republican candidates in states more conservative than their party need to depart less from party platform to fend off Democratic challengers.

When moving away from centre (the median voter's median position) parties acting in their candidates' best interests trade the expected number of elections their candidates win off against the costs their candidates pay to win those elections. Moving to the right costs the Republican party elections in the centre but also carves out a home turf on the right where elections can be won at little or no cost. Because elections in the centre are heavily contested by the Democrats, they can only be won at considerable cost.³ Each party's incentive to win any particular constituency depends not upon the value of election alone but rather upon its value net of campaign costs.

When campaign costs are high—candidates only reluctantly deviate from party platform—parties position themselves near the centre of the policy spectrum but sufficiently far apart that candidates from the leftist party do not contest elections in constituencies where the median voter lies right of the rightist party, and vice versa. Candidates only ever depart from their parties' positions to move to the centre. Consequently, the leftist party's position lies (weakly) to the left of all leftist candidates' positions, and likewise for the rightist party.

³Indeed, Ansolabehere, Snyder, and Stewart (2001) find the gap in candidates' policy positions is smallest in those Congressional districts whose voters split fifty-fifty in the vote for President: in contested constituencies, candidates must depart from their party platforms.

With high campaign costs, each party adopts a platform more extreme than its most extreme candidate.

As the costs of campaigning decrease, parties initially move apart from each other. Knowing that candidates from the other party will compete more intensely for any given realisation of the median voter, each party must move away from the other in order to carve out its home turf. But once costs fall below some threshold, parties' platforms jump back to the median voter's median position. This happens when campaign costs become sufficiently low that candidates adopt the median voter's position in every constituency, in which case parties minimise relocation costs by moving to the median voter's median position.

Because candidates do not adopt the median voter's preferred position in every constituency, electoral competition does not maximise voter welfare.⁴ A natural question is whether parties are too homogenous or too heterogenous for that purpose. Moving party platforms apart has two effects. When candidates retain their party platforms, separating platforms increases candidate diversity, which tends to raise welfare. But it also softens competition—candidates become less likely to take the median voter's preferred position—which tends to lower welfare. When campaigning is very costly, parties locate very near the centre of the policy spectrum and only in constituencies with median voters in the centre do candidates adopt the median voter's preferred position. In this case—candidates nearly always retain their party platforms—moving platforms apart increases voter welfare. On the other hand, when campaigning costs little and candidates differentiate to a high degree, then moving platforms together increases welfare by encouraging competition.

Candidates' costs from deviating from party policy play a crucial role in our analysis. We regard these costs as a reduced form of the many reasons why candidates may wish to mimic their parties. Most literally, advertising or publicising a new policy may be costly (buying television spots, etc.), doubly so as departing from party position may alienate donors. A candidate holding a position different from her party's may jeopardise leadership prospects. Or, these costs also could represent unpalatable payments or promises to special interests necessary to finance publicising a change from party policy. Political action committees—pressure groups—provide forty percent of funding for American Congressional elections (Herrnson 1997). The formal model does not depend on whether parties or candidates pay these costs.

⁴Locating at the median voter's position maximizes voter welfare when voters' utility decreases in the elected candidate's absolute distance from their bliss point. We restrict attention to these preferences in our welfare section but believe our qualitative results extend to other single-peaked preferences.

For the results, what matters most is that the overall game not be zero sum: fixing the number of elections they win, parties prefer that their candidates adhere to the party platform.⁵

Strictly speaking, in our model candidates prefer not to belong to either party so as to be able to adopt any position without cost. But for any number of reasons outside our formal model candidates benefit from party membership. Parties may reduce the costs of elections by sharing fixed costs across candidates. They may also enjoy legal privileges benefiting their candidates: party candidates automatically appear on the ballot in many elections, whereas unattached candidates often must submit petitions signed by enough voters. If elected, candidates who belong to a party enjoy more power through appointment to committees, etc. Likewise, we ignore candidates' party assignment. In our model, both candidates prefer to belong to the party closer to their constituency's median voter. Moreover, the candidate belonging to the party further away receives zero expected utility. Yet if candidates enjoy other benefits from campaigning, and there are enough potential candidates, then neither party will have trouble fielding a candidate. We believe that we sacrifice little in realism or applicability by assuming that for exogenous reasons the election in each constituency consists of one candidate from each of two parties.⁶

Several authors have explored the role of parties in electoral competition. Austen-Smith (1984) presents a model where in each of n constituencies, each of two parties fields a candidate in a first-past-the-post election. Voters recognise that government policy depends upon which party wins and where their party lies. Candidates seek to maximise vote shares. Austen-Smith shows that in a coalition-proof Nash equilibrium of the game candidates position in such a way as to position their party's policy at the median voter's bliss point. In general, candidates' positions differ across constituencies and parties. A crucial difference between his paper and ours is that his voters care about party location—turning party competition into Downsian competition—whereas our voters care about the position of the candidate elected in their own constituency. In that sense, his model may better resemble the British system of parliamentary democracy where MPs have only loose ties to their constituencies,

⁵At first pass, it may appear that parties would not care about candidates' leadership prospects; after all, someone will lead the party. However, the party would care if more talented leaders sometimes needed to deviate further from party platform to win election, thereby compromising their leadership prospects.

⁶Nevertheless, it is not essential to the model that parties always field candidates. In elections where both candidates maintain party position, nothing would change if the disadvantaged party decided not to field a candidate. In this case, parties would only run candidates in constituencies where their equilibrium probability of winning is positive.

and ours the American congressional system where congress members have much stronger ties to their constituencies. Certainly our model better reflects American gubernatorial elections, where voters care only about their own governors. Levy (2004) models parties in the citizen-candidate model of Besley and Coate (1997) and Osborne and Slivinski (1996) as being able to credibly commit to positions to which candidates could not individually. In her model, parties can only be effective in a multidimensional policy space as they work by allowing groups to exploit gains from trade from different preferences across issues.

Snyder and Ting (2002) provide a model where parties function as brand names. Voters have no information about candidates' positions other than their party membership (or lack thereof) and have preferences that depend upon both the mean and the variance of their beliefs about candidates' locations—they like candidates whom they expect to have a position near their own but dislike variance in their beliefs. Snyder and Ting show that when parties can reduce the variance of their members' positions by choosing extreme locations, then in equilibrium parties may prefer locating at the extremes than to locating at the centre.⁷

A number of authors have modified the Hotelling-Downs model of electoral competition in ways that produce differentiated candidates. Wittman (1983) and Calvert (1985) show that candidates may not converge when they care both about winning the election and about the position of the winner. Palfrey (1984) demonstrates that when two established candidates first take positions before a third candidate decides whether and where to enter the race, the established candidates differentiate to eliminate profitable entry opportunities. Chan (2001), Heidhues and Lagerlöf (2002), and Bernhardt, Duggan, and Squintani (2004) construct models where candidates separate due to asymmetric information about voters' preferences. Bernhardt and Ingberman (1985) model an incumbent with a reputation facing a challenger who cannot reveal his position with certainty. When voters dislike risk, the incumbent need not move to the median voter to defeat the challenger; hence, candidates differentiate. With the exception of Palfrey (1984), all of these models bear more resemblance to our model of candidate competition than our model of party competition. In our model, *candidates* differentiate because they start from different party platforms, which produces an effect similar to the asymmetric information or heterogenous preferences in these other papers. (Our candidates differentiate by using different strategies, which in some cases are mixed. In the context

⁷Of course, if moving away from center decreased variance—if extremist parties were more heterogenous rather than more homogenous as in Snyder and Ting (2002)—then parties in equilibrium would locate at the center.

of price competition, Bester, de Palma, Leininger, Thomas, and von Thadden (1996) have pointed out that common mixed strategies produce *ex post* differentiation.) By contrast, in our model *parties* differentiate despite symmetric starting points.

The intuition underlying our main result more closely resembles a literature in industrial organisation on price competition between duopolists. Hotelling (1929) analyses a model of two firms' choosing spatial locations knowing that consumers face transport costs. When prices are fixed, he shows that firms locate at the same position. But d'Aspremont, Gabszewicz, and Thisse (1979) show that when spatial location precedes price competition, firms exhibit maximal differentiation. While firms may wish to move together for given prices, they wish to separate to gain market power in order to put up prices; this latter effect dominates. Costly relocation in our model plays a role similar to price competition in d'Aspremont, Gabszewicz and Thisse's model. Konrad (1999) makes a related point in a model where firms first choose locations before competing in an all-pay auction for the right to sell a good to a customer whose location is initially unknown; the winning firm pays the cost of transportation to the consumer. Once firms have located and the consumer's position been revealed, competition between firms takes the form of an all-pay auction with heterogeneous, public valuations, a form of auction analysed by Baye, Kovenok, and de Vries (1993). Konrad (1999) shows that firms differentiate so as to minimise industry transport costs. One key difference from our paper is that in political competition, no candidate moves further than the median voter. This resembles a bid cap in an all-pay auction, which generates equilibria in the second stage of our game qualitatively different from those in Baye et al.

Section 2 introduces the formal model. Section 3 analyses candidates' final positions taking their initial positions as given. Section 4 analyses candidates' choice of initial conditions. Section 5 discusses voter welfare, and Section 6 concludes.

2 A Model

Two parties A and B compete in elections across a large number of constituencies. In each constituency, the parties field candidates, A and B , respectively, who compete in a first-past-the-post election with a continuum of voters. (For reasons described below, whether A denotes party or candidate will be clear from context.) The election game comprises two periods, 1 and 2. In period 1, each party chooses a platform in the policy space $[0, 1]$. In period 2, in

each of the continuum of elections, candidates from the two parties compete. The winner of each election receives a private benefit of $2V > 0$ and the loser nothing. A candidate's default position is her party's position. However, she may take a position different from her party's at a cost: if Candidate A 's party has platform a_1 , then choosing a_2 in the second period costs $|a_2 - a_1|$. The further a candidate moves from party platform, the higher the costs the candidate pays; for simplicity the marginal cost is constant and normalised to one. (As long as costs are linear, normalising marginal cost to one comes without loss of generality as candidates' behavior depends only on the ratio of V to the marginal cost of repositioning; hence, an increase in V can also be interpreted as a decrease in that marginal cost.) After observing candidates' period 2 positions, voters elect one of the two candidates. Thus, if Candidate A takes position a_2 with party platform a_1 and wins the election she receives a payoff of $2V - |a_2 - a_1|$; if she loses the election she receives $-|a_2 - a_1|$.

Voters are indexed in $[0, 1]$, where a voter at position i incurs a utility of $u_i(x) = -|x - i|$ when a candidate taking position x wins the election.⁸ Constituencies differ in the location of their median voters. The distribution of median voters across constituencies is uniform on $[0, 1]$. Parties wish to maximise their candidates' average payoffs across all constituencies.⁹

3 Electoral Competition

This section analyses candidates' location choices taking as given their party platforms. Let Candidate A and Candidate B 's party platforms be denoted a_1 and b_1 , respectively. Both candidates know the median voter's position, m . Because voter's preferences are single-peaked, whichever candidate locates closer to the median voter wins the election (Black, 1947). When candidates are equidistant from the median voter, we assume that each wins the election with probability one-half. When $|m - a_1| < |m - b_1|$ ($|m - a_1| < |m - b_1|$), then Party A is closer (further) from the median voter, and we refer to Candidate A as being advantaged (disadvantaged). In this section we assume that B is advantaged and further that $a_1 \leq b_1 \leq m$. This comes without loss in generality, since parties are symmetric and the

⁸Our characterisation of equilibrium holds for any preferences that satisfy a single-crossing property and any initial distribution of the median voter with a single peak. However, our welfare results in Section 5 depend upon voters' preferences (as well as upon more than the location of the median voter, as discussed in Section 5).

⁹The model can equally well be interpreted as one with a single constituency, where parties share the common prior that the median voter is uniformly distributed on $[0, 1]$.

winner of the election solely determined by the distances between the two candidates and the median voter and not their distance from each other. Therefore, whenever a median voter lies to the left of at least one of the candidates, we can think of the candidate closer to the median voter as Candidate A (and the other as Candidate B). Candidates' positions can be thought of as being left of the median voter with the same distance from the median voter as before.¹⁰

If Candidate A must incur costs larger than $2V$ to reach Party B 's platform ($b_1 - a_1 \geq 2V$), then Candidate A cannot challenge the election, and so both candidates optimally retain their initial positions. Let a_2 denote Candidate A 's position and b_2 Candidate B 's. Definition 1 organises elections into those where candidates move towards the median voter and those where they do not, allowing for mixed strategies.

Definition 1 *An election m is contested if $\Pr[a_2 = a_1] < 1$ or $\Pr[b_2 = b_1] < 1$. An election is uncontested if it is not contested.*

An election is uncontested if some candidate adheres to party platform with probability one. When $b_1 - a_1 \geq 2V$, elections are uncontested. On the other hand, if A can draw closer to the median voter than is B 's party at a cost of less than $2V$ ($b_1 - a_1 < 2V$), then both candidates' adhering to their party platforms can no longer be an equilibrium: elections are contested. In the case where $m - a_1 \leq V$, both candidates receive a positive payoff by moving to the median voter and winning with probability one-half. This constitutes an equilibrium, for each wishes to move to the median voter given that the other does the same. If $m - a_1 > V$, then A is unwilling to move to the median voter to win the election with probability one-half, and therefore it is not an equilibrium for both candidates to move to the median voter. When it is neither an equilibrium for both candidates to remain at their party platform nor an equilibrium for both to move to the median voter, then the equilibrium must be in mixed strategies.

If the distance between A and B is much smaller than that between A and the median voter (i.e. B has a large advantage) then both candidates mix over positions to the right of B : A tries to steal B 's election without moving all the way to the median voter, forcing B to move towards the median voter to fend off A . On the other hand, if the distance between

¹⁰One can interpret the period 2 game as a complete-information, common-value, all-pay auction with a bid cap (the median voter's position) and handicap (Candidate B 's advantage). We know of no paper addressing such auctions.

A and B is not much smaller than that between A and the median voter (i.e. B has a small advantage), then A finds it more profitable to directly adopt the median voter's position than to attempt to outmaneuver B ; in this case, each candidate retains her initial position with positive probability and jumps directly to the median voter with complementary probability. In both cases, candidates remain at their initial positions with positive probability. Because for some parameter configurations candidates' mixed-strategies are cumbersome to describe, we relegate precise characterisation of the equilibrium to Proposition 4 in the appendix.

Proposition 1 summarises equilibrium.

Proposition 1 *If $V < m - a_1$ candidates adhere to their party platforms with positive probability. Apart from these initial positions, their strategies have common support. The strategy of the advantaged Candidate B first-order stochastically dominates that of the disadvantaged Candidate A .*¹¹

Because the advantaged candidate's strategy first-order stochastically dominates the disadvantaged candidate's strategy, and clearly no candidate moves to the right of m , Proposition 1 implies that the advantaged candidate wins the election at least as often as the disadvantaged candidate. Without the normalisation that both candidates' initial positions lie to the left of m , Proposition 1 means that the distance between the disadvantaged candidate and the median voter first-order stochastically dominates the distribution between the advantaged candidate and the median voter.

Characterising first-period behavior requires only the candidates' continuation payoffs for each profile of first-period positions; these are (almost always) unique.

Proposition 2 *Assume $a_1 \leq b_1 \leq m$. The disadvantaged Candidate A 's expected payoff in the continuation equilibrium is as follows:*

$$U_A(a_1, b_1, m) = \begin{cases} V - m + a_1 & \text{if } m - a_1 < V \\ 0 & \text{otherwise.} \end{cases}$$

The advantaged Candidate B 's expected payoff in the continuation equilibrium is as follows:

$$U_B(a_1, b_1, m) = \begin{cases} \min\{2V, b_1 - a_1\} & \text{if } V \leq m - \frac{a_1 + b_1}{2} \\ 2(V - m + b_1) & \text{if } m - \frac{a_1 + b_1}{2} < V < m - a_1 \\ V - m + b_1 & \text{if } m - a_1 < V. \end{cases}$$

¹¹The distribution F first-order stochastically dominates the distribution G if for each x , $F(x) \leq G(x)$.

When $m - a_1 < V$, both candidates move to the median voter's position, in which case each gets one-half the value of winning the election minus the costs of relocation. When $m - a_1 > V$, because in equilibrium the disadvantaged candidate adheres to party platform with positive probability—where she receives a payoff of zero—she must get zero expected payoff.

The advantaged candidate's payoff depends upon her distance to the disadvantaged candidate as well as each candidate's distance to the median voter. To organise elections, we divide them into two classes that depend upon where the median voter is realised relative to party platforms.

Definition 2 *An election is extremal if $m \geq V + \frac{1}{2}(a_1 + b_1)$. An election is central if it is not extremal.*¹²

An election is extremal if the median voter lies sufficiently far from party platforms. If $b_1 - a_1 > 2V$, then $m \geq b_1 > a_1$ implies that $m > V + \frac{1}{2}(a_1 + b_1)$: uncontested elections are extremal. The only other extremal elections are those where A mixes to the right of B when A 's party platform lies far from the median voter as described above. Proposition 2 states that in extremal elections the advantaged candidate's expected payoff does not depend upon his distance from the median voter: either he wins for sure without moving and gets $2V$, or A challenges, and B 's expected payoff equals his initial distance from A .

In central elections, B 's payoff increases the closer his initial position to the median voter. However, the rate at which his payoff changes as a function of the distance between his initial position and the median voter is not constant. When $m - a_1 = V$, B moves to the median voter for sure, and A , who is indifferent between moving to the median voter and remaining at party platform, moves to the median voter with sufficiently high probability.¹³ As $m - a_1$ increases—holding everything else constant— A prefers to remain at her party platform. To keep A indifferent over moving to the median voter and remaining at her party platform, B cannot move to the median voter with probability one. To make B indifferent over moving to the median voter and remaining at his party platform, A must adhere to her party platform with sufficiently high probability. Thus, the probability that A moves to the

¹²Without the convention in this section that $a_1 \leq b_1 \leq m$, an election is extremal if $m \notin [\frac{a_1+b_1}{2} - V, \frac{a_1+b_1}{2} + V]$ or $|b_1 - a_1| > 2V$ (and central if not extremal).

¹³The probability that A stays at her initial position affects B 's equilibrium payoff, which explains why Proposition 2 excludes the case $m - a_1 = V$. For any a_1 and b_1 , the event that $m - a_1 = V$ occurs with zero probability and therefore does not affect parties' expected continuation payoffs.

median voter jumps down as $m - a_1$ moves through V , and so B 's expected payoff jumps up. For $m - a_1 < V$ —competition is tough— B benefits much less from being close to the median voter than when $m - a_1 > V$ —competition is weak.

To summarise, unless both candidates move to the median voter with probability one, the disadvantaged candidate receives a payoff of zero. In extremal elections, the advantaged candidate's payoff depends upon her distance from the disadvantaged candidate, while in central elections it depends upon her distance from the median voter. Among central elections, the advantaged candidate has more incentive to be close to the median voter when equilibrium is in mixed strategies than when it is in pure strategies: in the former case, being closer to the median voter reduces competition with the disadvantaged candidate, whereas in the latter it does not.

While mixed strategies in electioneering may strike some readers as unrealistic, we believe that they lend themselves to a natural interpretation. Suppose that over the course of a campaign each candidate makes a large number of speeches. In different speeches, the candidate may advocate different positions.¹⁴ On election morning, a newspaper samples from all these speeches and identifies the candidate's position; at this point, candidates can no longer campaign. On election day, voters read the newspaper, learn the candidates' positions, and vote. From Candidate B 's perspective, Candidate A plays a mixed strategy; B cannot best respond to A 's actual position but only the distribution from which it is drawn. Knowing that the newspaper samples randomly from her speeches, a candidate indifferent over her many policy positions is indifferent over all probability distributions over these positions; hence, her equilibrium policy distribution is indeed a best response to her opponent's behavior. In short, by appearing to endorse different positions, a candidate does something tantamount to mixing.¹⁵ Of course, a mixed equilibrium in our model can also be interpreted as the limit of pure-strategy equilibria of nearby incomplete-information games. In any event, the fact that equilibrium is in mixed strategies does not drive our main results below, as discussed in the conclusion.

¹⁴In equilibrium, the order of speeches plays no role.

¹⁵Aragones and Postlewaite (2002) show that candidates in a Downsian model may choose to be ambiguous when both voters and rival candidates cannot observe true positions. By contrast, under our interpretation, all the uncertainty is resolved before voters vote.

Changing the model to allow one candidate to move first, e.g. the advantaged candidate is a Stackelberg leader, would not affect equilibrium if the leader can still play a mixed strategy, as befits the interpretation here.

4 Platform Location

This section analyses parties' platform choice. Parties maximise their candidates' utility taking into account how their platform choice affects subsequent campaigning. When the private benefit of winning election, V , is not too large, parties adopt distinct platforms, one to the right of $\frac{1}{2}$ and the other to the left. In central elections, the advantaged candidate's utility decreases in her distance from the median voter; this gives parties incentive to minimise the expected distance between their platform and the median voter. In extremal elections, the advantaged candidate's utility increases the further her party's platform lies from the opponent's; this provides parties incentive to differentiate from each other. However, the fact that not all central elections affect candidates' payoff in the same way—candidates have more incentive to be near the median voter when equilibria are in mixed strategies—creates another motive for party differentiation. When choosing platforms, parties trade off these effects.

We begin by demonstrating why it cannot be an equilibrium for both parties to locate at one-half for all values of V . Suppose that Party B positions its platform at one-half, and consider A 's choosing between the platforms $\frac{1}{2}$ and $\frac{1}{2} - 2V$, for which we require $V < \frac{1}{4}$. Locating at one-half maximises the probability of winning; parties solely interested in winning would choose one-half as in the original Hotelling-Downs model.¹⁶ Yet, as argued in the previous section, electoral competition in the second period is toughest precisely when candidates begin from the same position. When their distance from the median voter exceeds V , each candidate receives an expected payoff of zero: competition eliminates all the private benefits of winning the election. When their distance from the median voters is less than V , each candidate adopts the median voter's position and wins with probability one-half. On average in these elections, the median voter's distance from one-half is $\frac{V}{2}$, so each party receives a payoff of $V - \frac{V}{2} = \frac{V}{2}$. The probability that the median voter lies within V of one-half is $2V$, so each party's expected payoff is V^2 . Now consider instead A 's choosing the platform $\frac{1}{2} - 2V$. Whenever the median voter lies to the left of A 's platform, B does not challenge the election, and A wins $2V$ without campaigning; such elections occur with probability $\frac{1}{2} - 2V$. Because with any other median voter A 's payoff is certainly non-negative,

¹⁶To see this, suppose that one party positions at $\frac{1}{2} - k_1$ and the other at $\frac{1}{2} + k_2$, where $k_1 > k_2 > 0$. The model is symmetric on $[0, 1 - (k_1 - k_2)]$, and therefore each wins half of these elections. Since the party at $\frac{1}{2} + k_2$ is advantaged in the remaining elections, which she wins with greater probability than her opponent by Proposition 1, she wins a larger share of elections. More generally, whichever candidate locates closer to one-half wins more elections, and hence equilibrium has both locating at one-half.

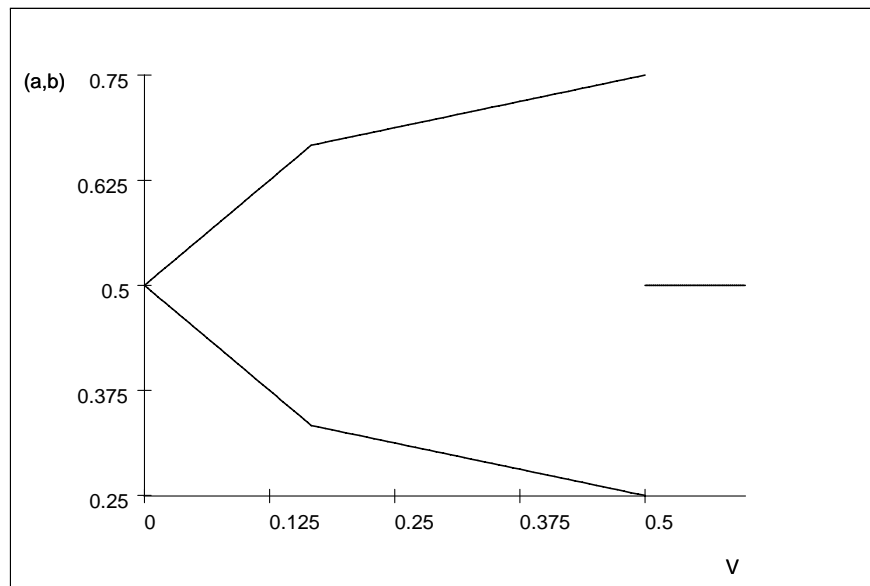
A's expected payoff from platform $\frac{1}{2} - 2V$ can be no smaller than $(\frac{1}{2} - 2V) 2V = V - 4V^2$. Since $V - 4V^2 > V^2$ whenever $V < \frac{1}{5}$, both parties do not choose platforms in the centre for small V .

Theorem 1 describes the unique payoff-symmetric, pure-strategy equilibrium in party platforms, denoted by a_1 and b_1 , illustrated in the figure below.¹⁷

Theorem 1 *The unique, pure-strategy, payoff-symmetric equilibrium in the first period is given by*

$$(a_1, b_1) = \begin{cases} (\frac{1}{2}, \frac{1}{2}) & \text{if } V \geq \frac{1}{2}, \\ (\frac{3}{8} - \frac{V}{4}, \frac{V}{4} + \frac{5}{8}) & \text{if } \frac{1}{2} \geq V \geq \frac{1}{6}, \\ (\frac{1}{2} - V, \frac{1}{2} + V) & \text{if } \frac{1}{6} \geq V. \end{cases}^{18}$$

When $V \geq \frac{1}{2}$, parties do not differentiate and locate at one-half. For high V winning the election with probability one-half is sufficiently valuable that candidates always adopt the median voter's position; as a result, neither party can soften competition by separating from the other.



Party Platforms as a Function of V

When $V < 1/6$, parties maintain an equilibrium distance of $2V$, the private value of winning the election. Central elections are contested, but extremal elections are not. From

¹⁷ For some values of V (but not others) payoff-asymmetric equilibria also exist. For instance, for sufficiently small V , $(a_1, b_1) = (\frac{1}{2}, \frac{1}{2} + 2V)$ is also an equilibrium. In addition, since the first-period game is symmetric with expected payoffs that are continuous in party platform, the game has a symmetric equilibrium, which is in mixed strategies. Naturally all these equilibria also involve candidate differentiation.

Definition 2, using the fact that in equilibrium $a_1 + b_1 = 1$, parties position in such a way that elections are central if and only if the median voter lies between the two parties' platforms. No candidate contests an election whose median voter lies on the other side of the opposing party platform. Parties have no incentive to take more extreme positions, which would not affect their payoff in extremal elections ($2V$) but would lower their payoff in central elections. Nor do they have incentive to take more central positions. To see this, suppose that Party B moves to the centre by $\varepsilon > 0$. First, this lessens B 's average distance from the median voter in central elections, increasing his expected payoff by ε times the likelihood of a central election in which B is advantaged. Second, it reduces the distance between candidates below $2V$, making all elections contested. In this case, B 's payoff in extremal elections falls from $2V$ to the distance between parties, $2V - \varepsilon$. This decreases B 's payoff by ε times the likelihood of an extremal election in which B is advantaged. For small V , the probability that the median voter is realised between the two candidates is small—most elections are extremal—and consequently candidates have more to lose than to gain by moving to the centre .

As V increases, each party wishes to move away from centre to maintain a distance of $2V$ from the other in order to secure its “home turf”; by moving too close to each other, parties would eliminate uncontested elections where their payoffs are highest. On the other hand, as V increases, this set of extremal elections shrinks. Consequently, parties' incentive to move to the centre to decrease their costs of winning contested, central elections grows larger by comparison. For $V < \frac{1}{6}$, the first effect dominates, and parties keep a distance of $2V$: parties differentiate as V increases.

For $V > \frac{1}{6}$, parties separate by less than $2V$ such that all elections are contested. To get an intuition for how V affects party platforms, suppose that parties start in equilibrium, and consider an increase in V . This expands the set of central elections, where candidates' payoffs depend upon their distance from the median voter. Such elections take two forms: either both candidates move to the median voter with probability one, or they maintain their party platforms with positive probability and move to the median voter with complementary probability. Among all central elections, the median, median voter is located at one-half regardless of V . Hence, if candidates had the same incentive to move to the median voter in all central elections, increasing V would not affect party platforms. But they do not. Section 3 describes how an advantaged candidate's payoff doubles as the disadvantaged candidate goes from adopting the median voter's position with probability one to mixing between the

median voter's position and party platform. Consequently, candidates have twice the incentive to move to the median voter in those central elections where equilibrium is in mixed strategies. As V increases, this region remains constant in size and moves to the exterior, away from one-half, at the rate V . Since parties care more about being near the median median voter in this region than in other central elections, as V increases they move away from one-half. But because the size of central elections with equilibrium in pure strategies grows as V increases, and the median, median voter in these elections remains at the midpoint between party platforms, parties move away from one-half at a rate slower than V .

To summarise, when the private benefit of election is low (or, alternatively, when campaign costs are high), the two parties maintain enough distance between them that the only contested elections are those with median voters lying between the parties. In other words, parties are always (weakly) more extreme than their members. As depicted in Figure 1, as V increases, each party moves away from one-half at the rate V . When the private benefit of election is high (or campaign costs are low), all elections are contested: parties separate by less than $2V$. Nevertheless, as V increases, electoral competition increases, which shifts the set of elections that a given party wins with probability greater than one-half away from one-half. Since it is in these elections that parties have the most incentive to locate close to the median voter, they move away from one-half as V increases.

Like in the standard Downsian model, parties here care about winning. But the fact that winning elections sometimes requires campaign expenditures means that parties care not about the private benefit of winning but rather the private benefit of winning net of campaign costs. In equilibrium, parties do not have incentive to move to the centre to provide their candidates with incentive to win more elections, for if winning such elections gave strictly positive payoffs, then candidates would choose to win them anyway; in other words, moving to the centre to capture these elections cannot have a first-order effect on expected payoffs. Rather, they care about being close to the median voter in elections where they are advantaged. A simple economic intuition underlies why their incentive to do so is higher in elections where equilibrium is in mixed strategies. Consider the effect on an advantaged party's payoff of being ε closer to the median voter. If both candidates travel to the median voter with probability one, then this increases the advantaged candidate's payoff by ε . Now consider a mixed equilibrium, fixing the strategy of the disadvantaged candidate. If the advantaged candidate always goes to the median voter, then being ε closer increases

her payoff by ε . But this leads the disadvantaged candidate never to go to the median voter, further increasing the advantaged candidate's payoff. Candidates have more incentive to be near the median voter when this diminishes electoral competition.

5 Voter Welfare

A natural question is how voter welfare depends upon party platform. Given candidates' incentive to move to the median voter in the second period, do parties differentiate too little or too much to maximise voter welfare?

Since voter i 's utility from policy x is $u_i(x) = -|x - i|$, the welfare-maximising policy (that which maximises the sum of voters' utilities) is located at the median voter's ideal point (independent of the distribution of voters).¹⁹ When $V > \frac{1}{2}$, candidates always move to the median voter with certainty, and so parties' platforms maximise voter welfare. But for $V < \frac{1}{2}$ candidates do not always move to the median voter with certainty, in which case party platforms may not maximise voter welfare. When candidates do not always adopt the median voter's preferred policy, voter welfare depends on more than the distance between the elected candidate and the median voter's ideal point. In this case, aggregating voters' utilities requires information about the distribution of voters' ideal points.

No two pairs of distinct party platforms can be ranked according to voter welfare without knowing more about the distribution of voters' bliss-points than its median. To illustrate, consider initial positions a_1 and b_1 , and assume that for each realisation of the median voter, m , slightly more than half the voters have bliss points at m , while the rest have bliss points at either a_1 or b_1 , whichever is closer to m . For all realisations of m , the winning candidate's policy is almost optimal. This follows from the fact that staying at the initial platform (or moving to any position between this and the median voter) leads to almost the same voter welfare as moving to the median voter. Thus, for some distribution of voters' ideal points (where the median ideal point is uniform on $[0, 1]$), (a_1, b_1) is almost optimal.

When $V = 1/2$, there exist two equilibria: one where both parties locate at one-half and each candidate moves to the median voter with probability one, and one where parties choose platforms at $\frac{1}{4}$ and $\frac{3}{4}$ and candidates only sometimes go to the median voter. Common platforms at one-half are better for voters than distinct platforms by the argument above. If

¹⁹An ε deviation away from the median voter would decrease utility for more than half of the voters by ε and increase utility for less than half the voters, also by ε .

we can rule out distributions of bliss points such as the one described above, a similar result holds for V slightly less than one-half: platforms at one-half are strictly better for voters than those chosen by the parties in equilibrium. With platforms at one-half, candidates would always move to any median voter other than those very close to the boundaries of the political spectrum. To these median voters candidates will move with high probability that can be made arbitrarily close to one, for V sufficiently close to one-half. On the other hand, given their parties' equilibrium platforms, candidates will never move to median voters close to zero or one with probability higher than $\frac{1}{2}$. As long as having the winning voter adopt the median voter's position is strictly better for voters than any other position, the costs of not moving to m cannot become arbitrarily small, and consequently voters are better off with common platforms at one-half.

For small V the situation is different. In equilibrium, platforms are located centrally at $\frac{1}{2} - V$ and $\frac{1}{2} + V$, and candidates maintain their party platforms in extremal elections, where strictly more than half of voters' bliss points are either to the left of $\frac{1}{2} - V$ or to the right of $\frac{1}{2} + V$. Moving platforms outward increases welfare in the likely event that m is realised outside $[\frac{1}{2} - V, \frac{1}{2} + V]$. When m is realised between party platforms, moving platforms outward decreases voters' welfare, since it increases the distance to most voters and reduces the probability (and distance) that candidates move towards them. Nevertheless, for small V these costs are outweighed by the gains from $m \notin [\frac{1}{2} - V, \frac{1}{2} + V]$. In this sense, we find that for small V parties platforms are too close together.

To formally state and prove these results we must exclude extreme distributions of voters' bliss points such as the one described above, which nevertheless leaves a very large class of distributions. A sufficient restriction is that the distributions have no gaps. Let $(F_m)_{m \in [0,1]}$ be a collection of distributions of voters' bliss points parameterised by its median m .

Definition 3 $(F_m)_{m \in [0,1]}$ is regular if there exists some constant $k > 0$ such that for each $\delta > 0$ and each $x, m \in (0, 1)$, $F_m(x + \frac{\delta}{2}) - F_m(x - \frac{\delta}{2}) > \delta k$.

Voters' bliss points are regular if they put mass around any policy in $(0, 1)$ that cannot become arbitrary small as m changes. In particular, whatever the median bliss point, the distribution of bliss points puts mass around that median.

Proposition 3 Assume that the collection of voters' distributions of bliss points is regular. If V is close to one-half then in equilibrium parties' platform choices are too extreme to maximise

welfare, *i.e.* voters prefer platforms at one-half. If V is close to zero then parties' equilibrium platform choices are too central to maximise welfare, *i.e.* voters prefer platforms further away from each other.

Proposition 3 provides limit results in the sense that for V close enough to one-half (zero) the parties' platform choices are too extreme (central). When campaign costs are very high—parties are tightly whipped—parties are too centrist. When campaign costs are very low—parties are very loosely whipped—parties are too extremist.

A different question is whether voters benefit from an increase in V , or equivalently from subsidised campaigning. Clearly, voter welfare is maximised if $V > \frac{1}{2}$ since candidates always take the median voter's position. For $V > \frac{1}{2}$, the effect of a marginal increase in V depends on the distribution of voters' bliss points in every constituency and is therefore ambiguous. If, however, voter welfare in each constituency is a linear function of the distance between the median voter and elected candidate, then increasing V raises voter welfare.

6 Conclusion

This paper provides a rationale for differentiation between ex ante identical political parties in a Hotelling-Downs-style model of electoral competition. In our model, parties are benevolent to their many purely-opportunistic candidates running in distinct constituencies with different voter preferences. Parties choose policy platforms that serve as their candidates' default campaign positions. Candidates may deviate from party platform to increase the probability of winning their constituency at a cost: the more they deviate, the higher these “campaign costs.” We find that parties do not adopt the platform that maximises the number of elections their candidates win. Instead, they take less central platforms and separate from each other to avoid costly campaigns in most constituencies. Each party carves out a “home turf,” constituencies where their candidates can win with little or no campaigning. Decreasing campaign costs causes parties to move further apart: because candidates campaign more vigorously, each party must move further from the other to carve out its home turf.

Our intention in this in this paper has been to re-explore the effects of political competition on electoral positioning. Many commentators have observed that political candidates seldom espouse the same policies. This paper provides an explanation for this finding in terms of political parties: candidates differ because their parties differ; parties differ to reduce political

competition and thereby avoid campaign costs (e.g., unpalatable promises to special interests). Other authors have offered other compelling reasons for political differentiation, and we do not suggest that party competition constitutes the sole reason for policy differentiation. However, we think it an important exercise to understand how electoral competition pulls candidates together and how it drives them apart.²⁰

A crucial assumption that drives party differentiation is that candidates incur costs by deviating from party platform. In this paper we assume that candidates pay these campaign costs whether or not they win election. But some reasons why candidates may find deviating from party policy costly (e.g. diminished leadership prospects) may loom larger when they win than when they lose. If campaign costs were paid only by winning candidates, our qualitative results would be unaffected. In this case too, the closer the parties' platforms, the more intense would be electoral competition (the higher candidates' campaign costs in equilibrium). Here, however, the disadvantaged candidate could move $2V$ from her platform before incurring a negative payoff. When $V > \frac{1}{4}$, both parties would choose platforms at $\frac{1}{2}$, and in every constituency both candidates would adopt the median voter's position with certainty. When $V \leq \frac{1}{4}$, parties would choose platforms at $\frac{1}{2} - V$ and $\frac{1}{2} + V$, and constituencies with extreme median voters would go uncontested. Here too the degree to which candidates differentiate increases in V (or, equivalently, decreases in campaign costs). The fact that in this model all continuation equilibria are in pure strategies underscores the fact that differentiation in our model is not an artifact of candidates' playing mixed strategies. The absence of mixing in this model also allows it the following, alternative interpretation: candidates have policy preferences; parties have none; and the winning party must pay a cost to attract a candidate whose preferred policy differs from its platform.

The structure of the main result (in Theorem 1) is also robust to the assumption that the median voter's bliss point is uniformly distributed. It holds for any continuous, single-peaked distribution of the median voter's location. As V increases from zero, the parties' equilibrium platforms separate more and more until V reaches a critical size, at which point parties locate at one-half. The intuition that underlies equilibrium when $V \leq \frac{1}{6}$ does not rely on uniformity. For V sufficiently small, parties separate their platforms by $2V$.²¹ Moving further apart only

²⁰An open, empirical question is how much candidates in elections without parties differ from each other relative to those in elections with parties.

²¹The maximum value of V for which parties maintain a distance of $2V$ does depend on the distribution and can be no larger than $\frac{1}{6}$, for the uniform distribution has more variance than any other single-peaked distribution.

diminishes payoffs for central median voters, whereas moving further together costs more in the many extremal elections than it benefits in the few central elections. However, unlike in our model, when $V > \frac{1}{6}$ comparative statics may no longer be monotone, as the intuition behind our arguments depended upon a comparison of the measure of constituencies where candidates always adopt the median voter’s preferred position to those where candidates mix between that and their party platforms.

While departing from party platform may affect candidates in any number of ways, it is essential to our model that deviating from party platform do more than cost votes in the election at hand. Suppose that there were no explicit campaign costs, but instead that each candidate’s probability of winning the election had the following properties: it decreased in her distance from the median voter; decreased in her distance from party platform; increased in her opponent’s distance from the median voter; and increased in the difference between her opponent’s position and party platform. Then parties would not differentiate and hence would locate at one-half. A party that deviated from one-half would find its candidates disadvantaged in most constituencies, and hence would win less than one half of constituencies. Nevertheless, we believe that in most elections deviating from party platform does not have this kind of zero-sum structure and therefore that our model captures an important aspect of electoral competition.

Parliamentary parties care about winning a majority of constituencies. Parties that traded payoff in our model off against the probability of winning a majority in a smooth way would adopt platforms of one-half in any payoff-symmetric equilibrium; if not, then one party could go from winning a majority with probability one-half to winning a majority with probability one by only an infinitesimal change in position. However, introducing noise into the model—for example, having voters vote based on idiosyncratic taste parameters—would restore party differentiation, as parties could no longer discretely change their probability of winning a majority with a small change in position.

In our model, party platforms are chosen to maximise their candidates’ average payoffs. If candidates chose their party platform in a majoritarian election, the result would be very different. Each candidate wishes her party would locate its platform at her constituency’s median voter regardless of the other party’s location. It is easy to verify that candidates’ preferences over their party’s location satisfy Gans and Smart’s (1996) single-crossing condition, implying that their median bliss point—the median, median voter—is a Condorcet winner.

This equilibrium differs so dramatically from ours because a majoritarian party would not trade gains in one constituency off against losses in another. Here candidates could make mutually advantageous trades, for instance by having more extremist candidates pay centrists' campaign costs in return for a move away from centre.

Finally, although we have interpreted our formal model in terms of parties and candidates, it can also be applied to a single election. Consider two candidates campaigning over the course of a long election process, where candidates learn information about voters' preferences as the campaign progresses. A stylised version of this strategic setting coincides with our two-period model: candidates take initial policy positions; they then learn the median voter's preferences; and last they take final positions, where departing from initial position is costly. Many of the reasons that motivate campaign costs in our original model apply equally well in this setting. Candidates will split the political spectrum at the outset of the campaign, each candidate betting on a median voter near her position that would allow her to win the election at minimal cost. Alternatively, the formal model also fits two long-term parties competing across a number of elections over time, where median voters vary but party positions remain constant.

One domain where our model can help shed light is to redistricting. If two parties had strictly conflicting interests, then a form of "no-trade theorem" should exist for redistricting: parties should never agree to redraw constituencies, for the one's advantage is the other's disadvantage. In our model, of course, parties do not have conflicting interests. Suppose that the parties, having already fixed their platforms, could agree to transform two constituencies with median voters located one-half into one with a median voter at zero and another with a median voter located at one. They would, for doing so increases each party's expected payoff: one uncontested constituency is more valuable than two heavily-contested ones.

7 Appendix

The following Lemma helps to characterise the continuation equilibria in the second period. As in Section 3 we assume without loss of generality that $a_1 \leq b_1 \leq m$. A strategy of candidate i is given by a cumulative distribution function F_i , $i \in \{A, B\}$, on $[0, 1]$, where $F_i(x)$ denotes the probability that candidate i chooses a policy platform smaller or equal to x . We say that the probability distribution F_i has an atom at x if it puts positive probability on x , i.e. x is chosen with probability $F_i(x) - \lim_{y \downarrow x} F_i(y) > 0$. For positions $x < y$ we say that F_i has a

gap between x and y if it puts no mass on the interval (x, y) , i.e. $F_i(y) = F_i(x)$.

Lemma 1 *Each continuation equilibrium $(F_A(x), F_B(x))$ must have the following properties:*

1. *If a candidate's strategy has a gap between $x \geq b_1$ and $y < m$ then it must have a gap between x and m , and also the other candidate's strategy must have a gap between x and m .*
2. *A candidate's strategy can only have an atom at her party platform or at m .*

Proof:

Note first that both candidates' strategies cannot have an atom at the same policy $x < m$. Suppose that both candidates move to $x < m$ with positive probability. Then a position slightly larger than x yields a higher payoff for candidate A (and likewise for candidate B), since the cost of moving there is only marginally larger whereas the probability of winning is significantly increased.

1. Suppose that the strategy of candidate X has a gap between x and y but none between x and m . Denote the position where the gap ends by z , i.e. $z := \arg \sup_w \{w | F_i(w) > F_i(x)\}$. We have $x \leq z < m$. Moving into a gap in the other candidate's strategy can never be optimal, for a small decrease in position would decrease costs without affecting the probability of winning. Therefore it must be that both candidates' strategies have a gap between x and z but not between x and $z + \varepsilon$ for all $\varepsilon > 0$. But since both cannot have atoms at $z \neq m$, at least one candidate could improve by moving into (x, z) (which would reduce costs without changing the winning probability).
2. Clearly, no candidate moves into (a_1, b_1) . If one candidate's strategy has an atom at $x \in (b_1, m)$, then there exists some $\varepsilon > 0$ such that the other candidate will not put mass in $(x - \varepsilon, x)$ (moving slightly above x would increase payoffs). But with a gap between $x - \varepsilon$ and x , the candidate with the atom at x could improve by moving into $(x - \varepsilon, x)$.

Q.E.D.

Proof of Proposition 1:

The statement that both strategies have common support between b_1 and m follows from Lemma 1. Furthermore, it is not possible for only one candidate to have an atom at m , for if so, then the other candidate must have a gap between some x and m (since otherwise moving to m is strictly better for that candidate than moving close to m), and hence it would be profitable to move from m into (x, m) .

To prove the first statement, first note that because $V < m - a_1$, A is unwilling to move to m . Either both candidates stay at their party platform for certain or, because strategies elsewhere have common support, they mix on (b_1, r) for some $r \leq m$ (with strict inequality when one candidate has an atom at m). B 's strategy must have an atom at b_1 , since otherwise A would not move to b_1 (where she would incur costs but never win). Assume now that A 's strategy does not have an atom at a_1 , which implies that candidate B has zero payoff in equilibrium. We distinguish two cases: both candidates have an atom at m , or neither has an atom at m (it is impossible that only one has an atom at m). In the latter case, both candidates win for sure by moving to r . If B moves to r in equilibrium, then her payoff there is zero and hence A 's payoff must be negative, which cannot happen in equilibrium. So suppose B moves to m with probability $P_B(m)$. Both candidates must mix on (b_1, r) with the same density; changing positions within this interval changes costs in the same way for both candidates and hence must also change winning probabilities in the same way. If we denote common mass that strategies put on (b_1, r) by M , i.e. $M := F_A(r) - F_A(b_1) = F_B(r) - F_B(b_1)$, then B 's payoff from moving to m is $2VM + V(1 - M) - m + b_1$, which must equal zero. A 's payoff from moving to m is $2V(1 - P_B(m)) + VP_B(m) - m + a_1$, which must equal her payoff at b_1 , $2V(1 - M - P_b(m)) - b_1 + a_1$. Hence,

$$\begin{aligned} 2V(1 - P_B(m)) + VP_B(m) - m + a_1 &= 2V(1 - M - P_b(m)) - b_1 + a_1 \\ \Rightarrow 2VM + VP_b(m) - m + b_1 &= 0 \\ \Rightarrow P_b(m) &= 1 - M. \end{aligned}$$

But this implies that B does not stay at b_1 with positive probability, a contradiction.

The last statement follows immediately from Proposition 4 below.

Q.E.D.

Proposition 4 *Let $a_1 \leq b_1 \leq m$. The following constitutes the unique continuation equilibrium:*

- If $b_1 - a_1 \geq 2V$, then both candidates retain party platform.
- If $b_1 - a_1 \leq 2V \leq m - a_1$, then both candidates retain their party platforms with probability $\frac{b_1 - a_1}{2V}$ and randomise continuously on $(b_1, 2V + a_1)$ with density $\frac{1}{2V}$.
- If $2m - a_1 - b_1 \geq 2V \geq m - a_1$, then both candidates retain their party platforms with probability $\frac{b_1 - a_1}{2V}$, take the median voter's position with probability $\frac{2V - m + a_1}{V}$, and randomise continuously on $(b_1, 2m - a_1 - 2V)$ with density $\frac{1}{2V}$.
- If $2m - 2a_1 \geq 2V \geq 2m - a_1 - b_1$, then candidate A retains her party platform with probability $\frac{V - m + b_1}{V}$ and takes the median voter's position with probability $\frac{m - b_1}{V}$. Candidate B retains his party platform with probability $\frac{m - V - a_1}{V}$ and takes the median voter's position with probability $\frac{2V + a_1 - m}{V}$.
- If $V \geq m - a_1$, then both candidates take the median voter's position.²²

Proof: To win the election A must move at least as far as b_1 . If $b_1 - a_1 \geq 2V$, the cost of doing so exceeds the benefit of winning the election, so A remains at a_1 . Henceforth, assume that $b_1 - a_1 < 2V$. From Lemma 1 we know that the strategies' support can only contain party platforms, an interval $[b_1, r]$, for some $b_1 < r < m$, and m . In particular, strategies cannot have atoms in $(b_1, r]$. Furthermore, since each position in $[b_1, r]$ yields the same expected payoff, whenever $b_1 < x < y < r$, for each $i \in \{A, B\}$, $2V(F_i(y) - F_i(x)) = y - x$: the value of the additional probability of winning at y relative to x must equal the increase in the cost of moving to y from x . If $V < m - a_1$ Proposition 1 implies that A 's payoff is zero, and therefore $F_B(b_1) = \frac{b_1 - a_1}{2V}$. Using these properties, we can find equilibrium in the different cases.

If $m - a_1 < V$ always moving to m results in a positive payoff for both parties and is therefore the only equilibrium.

If $m - a_1 > 2V$ it is never profitable for A to move to m . Hence B 's equilibrium strategy must be:

$$F_B(x) = \begin{cases} 0 & \text{if } x < b_1 \\ \min\left\{\frac{x - a_1}{2V}, 1\right\} & \text{if } b_1 \leq x, \end{cases}$$

²²Equilibrium is unique up to redefining cases by changing strict inequalities to weak ones. Doing so would not affect parties' expected continuation payoffs as the boundaries of these different regions occur with probability zero.

which determines A 's strategy.

If $V < m - a_1 < 2V$ both candidates move to m with a positive probability (if one did not, then the other could improve her payoff by moving to m). Again denote the probability that B moves to m as $\Pr_B(m)$. The fact that A makes zero profit implies $2V(1 - \Pr_B(m)) + V\Pr_B(m) = m - a_1$, which gives $\Pr_B(m) = \frac{2V - m + a_1}{V}$.

A might mix between a_1 and m or mix among $\{a_1\} \cup (b_1, r] \cup \{m\}$ with $b_1 < r \leq m$. If $1 - \Pr_B(m) = \frac{m - a_1 - V}{V} \leq \frac{b_1 - a_1}{2V}$, then A must confine her mixing to a_1 and m , since moving slightly beyond b_1 results in a negative payoff. This implies that B also only mixes between b_1 and m , and so

$$F_B(x) = \begin{cases} 0 & \text{if } x < b_1 \\ \frac{m - V - a_1}{V} & \text{if } b_1 \leq x < m \\ 1 & \text{if } x = m. \end{cases}$$

B 's payoff follows from the indifference condition $m - b_1 = V\Pr_A(m)$, where $\Pr_A(m)$ denotes the mass A puts on m , which gives $F_A(x)$.

If $\frac{m - a_1 - V}{V} > \frac{b_1 - a_1}{2V}$ we must have that $\Pr_B(b_1) = \frac{b_1 - a_1}{2V}$ and A mixes on $\{a_1\} \cup (b_1, r] \cup \{m\}$. A 's indifference condition gives that $\Pr_B(m) = \frac{2V - m + a_1}{V}$. Since $1 - \Pr_B(b_1) - \Pr_B(m) = \frac{2m - a_1 - b_1 - 2V}{2V}$, $r = 2m - a_1 - 2V$, which gives $F_B(x)$. A 's strategy follows from B 's indifference condition and the fact that both put the same mass on $(b_1, r]$, i.e.

$$0 = 2V \left(1 - \left(1 - \Pr_B(b_1) - \Pr_B(m) \right) \right) + V\Pr_A(m) - m + b_1.$$

Hence, $\Pr_A(m) = \frac{2V + a_1 - m}{V}$.

Q.E.D.

Proof of Proposition 2: Candidates' payoffs can be directly computed from the unique equilibrium given in Proposition 4. Q.E.D.

Proof of Theorem 1: Assume A 's party takes platform a_1 and B 's b_1 . Without loss of generality, $0 \leq a_1 \leq b_1 \leq 1$. Candidates' payoff for any realisation of m can be derived from Corollary 2 by a relabeling of variables: A 's payoff is $U_A(x, y, m)$, where

$$(x, y) = \begin{cases} (2m - b_1, 2m - a_1) & \text{if } m < a_1 \\ (2m - b_1, a_1) & \text{if } a_1 \leq m \leq \frac{a_1 + b_1}{2} \\ (a_1, 2m - b_1) & \text{if } \frac{a_1 + b_1}{2} < m \leq b_1 \\ (a_1, b_1) & \text{if } b_1 < m. \end{cases}$$

Using this, we can calculate A 's expected payoff $\bar{U}_A(a_1, b_1)$ from platforms (a_1, b_1) from Corollary 2. When $b_1 - a_1 \geq 2V$, candidates never depart from party platform for median

voters below $\frac{a_1+b_1}{2} - V$ (or those above $\frac{a_1+b_1}{2} + V$). For no realisations of m do both candidates either move to m or mix between party platform and m . Hence,

$$\bar{U}_A(a_1, b_1) = 2V \left(\frac{a_1 + b_1}{2} - V \right) + \int_{\frac{a_1+b_1}{2}-V}^{\frac{a_1+b_1}{2}} (b_1 - 2m + a_1) dm \quad (1)$$

and $\frac{\partial}{\partial a_1} \bar{U}_A(a_1, b_1) = V > 0$. Hence, $b_1 - a_1 \leq 2V$ in equilibrium.

When $b_1 - a_1 < 2V$, equilibrium can take any of the mixed forms described in Proposition 4. A 's payoff is nonzero if and only if $m < V + a_1$. If $b_1 - V < m < V + a_1$, both candidates move to m for sure. If $\frac{a_1+b_1}{2} - V < m < b_1 - V$, both parties mix on party platform with probability $\frac{V-m+b_1}{V}$ and the median voter with probability $\frac{m-b_1}{V}$ (note that since $b_1 - a_1 < 2V$ this can only happen left of a_1 and right of b_1). If $m < \frac{a_1+b_1}{2} - V$ candidate A receives a utility of $b_1 - a_1$. Using these observations, for $b_1 - a_1 < 2V$,

$$\begin{aligned} \bar{U}_A(a_1, b_1) &= (b_1 - a_1) \max \left\{ \frac{a_1 + b_1}{2} - V, 0 \right\} \\ &+ 2 \int_{\max \left\{ \frac{a_1+b_1}{2} - V, 0 \right\}}^{\max \{b_1-V, a_1\}, 0} (V + m - a_1) dm + \int_{\max \{b_1-V, a_1\}, 0}^{a_1} (V + m - a_1) dm \\ &+ 2 \int_{a_1}^{\max \{b_1-V, a_1\}} (V - m + a_1) dm + \int_{\max \{b_1-V, a_1\}}^{V+a_1} (V - m + a_1) dm. \end{aligned} \quad (2)$$

Since no symmetric, pure-strategy equilibrium exists in which $b_1 - a_1 > 2V$, we focus on $b_1 - a_1 \leq 2V$, which we divide into the following four cases: (i) $V \leq \frac{1}{2}$ and $b_1 - a_1 = 2V$; (ii) $V \leq \frac{1}{2}$ and $b_1 - a_1 \in (V, 2V)$; (iii) $V \leq \frac{1}{2}$ and $b_1 - a_1 \leq V$; and (iv) $V > \frac{1}{2}$.

(i) The only candidate for a (position-symmetric pure strategy) equilibrium in the first case is $a_1 = \frac{1}{2} - V$, $b_1 = \frac{1}{2} + V$. Given $b_1 = \frac{1}{2} + V$ according to (1) it cannot be profitable for A to decrease her position. An increase to $\hat{a}_1 \leq \frac{1}{2}$ leads to the following utility (since $\hat{a}_1 > \frac{1}{2} - V \Rightarrow \frac{\hat{a}_1+b_1}{2} - V > \frac{1}{2} - V > 0$):

$$\begin{aligned} \bar{U}_A(\hat{a}_1, b_1) &= (b_1 - \hat{a}_1) \left(\frac{\hat{a}_1 + b_1}{2} - V \right) + 2 \int_{\frac{\hat{a}_1+b_1}{2}-V}^{\hat{a}_1} (V + m - \hat{a}_1) dm \\ &+ 2 \int_{\hat{a}_1}^{b_1-V} (V - m + \hat{a}_1) dm + \int_{b_1-V}^{V+\hat{a}_1} (V - m + \hat{a}_1) dm, \end{aligned}$$

which increases up to $\hat{a}_1 = \frac{3}{10} + \frac{1}{5}V$ and decreases thereafter. Hence, increasing \hat{a}_1 is not profitable if $V \leq \frac{1}{6}$. An increase beyond $\hat{a}_1 = \frac{1}{2}$ is not profitable either; in this case,

$$\begin{aligned} \bar{U}_A(\hat{a}_1, b_1) &= (b_1 - \hat{a}_1) \left(\frac{\hat{a}_1 + b_1}{2} - V \right) + 2 \int_{\frac{\hat{a}_1+b_1}{2}-V}^{b_1-V} (V + m - \hat{a}_1) dm \\ &+ \int_{b_1-V}^{\hat{a}_1} (V + m - \hat{a}_1) dm + \int_{\hat{a}_1}^{V+\hat{a}_1} (V - m + \hat{a}_1) dm, \end{aligned}$$

which decreases in \hat{a}_1 for the considered region. Hence, if $V \leq \frac{1}{6}$, $a_1 = \frac{1}{2} - V$ and $b_1 = \frac{1}{2} + V$ constitute an equilibrium.

We treat the remaining cases—similarly examined by calculating the extrema of (2)—in somewhat less detail.

(ii) If $V \leq \frac{1}{2}$ and $2V > b_1 - a_1 > V$ then (2) is locally maximised with respect to a_1 at $a_1 = \frac{3}{5}b_1 - \frac{2}{5}V$. Combining this with symmetry, $b_1 = 1 - a_1$, we have $a_1 = \frac{3}{8} - \frac{1}{4}V$ and $b_1 = \frac{1}{4}V + \frac{5}{8}$ as a candidate equilibrium for $V \geq \frac{1}{6}$. Furthermore, a change in position is not profitable for A (while fixing $b_1 = \frac{1}{4}V + \frac{5}{8}$).

(iii) If $V < \frac{1}{2}$ and $b_1 - a_1 \leq V$, then (2) is maximised at $a_1 = 2V - b_1$, which cannot hold if $b_1 = 1 - a_1$.

(iv) If $V > \frac{1}{2}$, then $b_1 - a_1 < 2V$. Assume first that $b_1 - V \leq 0$, in which case

$$\bar{U}_A(a_1, b_1) = \int_0^{a_1} (V + m - a_1) dm + \int_{a_1}^{V+a_1} (V - m + a_1) dm$$

is maximised for $a_1 = V$. Hence, $a_1 = b_1 = \frac{1}{2}$ constitutes an equilibrium. To rule out the possibility of other permutation-symmetric equilibria when $V > \frac{1}{2}$, suppose $0 \leq b_1 - V \leq a_1$.

In equilibrium, a_1 maximises

$$\begin{aligned} \bar{U}_A(a_1, b_1) &= 2 \int_0^{b_1-V} (V + m - a_1) dm \\ &\quad + \int_{b_1-V}^{a_1} (V + m - a_1) dm + \int_{a_1}^{V+a_1} (V - m + a_1) dm, \end{aligned}$$

which implies $a_1 = 2V - b_1$. But equilibrium cannot have $0 \leq b_1 - V \leq a_1$. Nor can it have $a_1 \leq b_1 - V$, since a_1 must maximise

$$\begin{aligned} \bar{U}_A(a_1, b_1) &= 2 \int_0^{a_1} (V + m - a_1) dm + 2 \int_{a_1}^{b_1-V} (V - m + a_1) dm \\ &\quad + \int_{b_1-V}^{V+a_1} (V - m + a_1) dm, \end{aligned}$$

which occurs at $a_1 = \frac{1}{3}b_1$; the only candidate equilibrium is $a_1 = \frac{1}{4}$ and $b_1 = \frac{3}{4}$, which falls outside the considered range.

Q.E.D.

Proof of Proposition 3:

We provide a formal proof for the first statement. Since a complete proof of the second statement involves tedious case distinctions but is straightforward and similar, we only sketch its proof.

If parties choose platforms $a_1 = b_1 = \frac{1}{2}$ candidates will move to $m \in [\frac{1}{2} - V, \frac{1}{2} + V]$ with certainty; hence, no other platforms raise welfare for these median voters. Therefore it suffices to show that for $m \notin [\frac{1}{2} - V, \frac{1}{2} + V]$ welfare is higher when $a_1 = b_1 = \frac{1}{2}$ than in equilibrium. Assume $V = \frac{1}{2} - \varepsilon$, $\varepsilon > 0$, and consider $m > \frac{1}{2} + V = 1 - \varepsilon$ (by symmetry, the following arguments hold for $m \in [0, \varepsilon]$). By Proposition 4, for ε sufficiently small, candidates stay at $\frac{1}{2}$ with zero probability, mix on $[\frac{1}{2}, 2m - \frac{3}{2} - 2\varepsilon]$ with density $\frac{1}{1-2\varepsilon}$ and move to m with probability $\frac{3-2m-4\varepsilon}{1-2\varepsilon}$. In particular, we can get arbitrarily close to the optimum (i.e. to a situation where the winning candidate takes the median voter's position) by choosing ε small enough.

At parties' equilibrium positions, $a_1 = \frac{1}{4} + \frac{1}{4}\varepsilon$ and $b_1 = \frac{3}{4} - \frac{1}{4}\varepsilon$, when $m > \frac{1}{2} + V$ candidates remain at their initial positions with probability $\frac{\frac{1}{2}(1-\varepsilon)}{1-2\varepsilon}$, mix on $[\frac{3}{4} - \frac{1}{4}\varepsilon, 2m - \frac{5}{4} + \frac{7}{4}\varepsilon]$ with density $\frac{1}{1-2\varepsilon}$, and move to m with probability $\frac{\frac{5}{2} - \frac{7}{2}\varepsilon - 2m}{1-2\varepsilon}$.

To compare welfare under these two sets of platforms, we can replace any distributions of bliss points $(F_m)_{m \in [1-\varepsilon, 1]}$ with $(\tilde{F}_m)_{m \in [1-\varepsilon, 1]}$, which puts half its mass on m and distributes the remainder on $(-\infty, m)$ according to F_m . Because with either set of platforms candidates move no further than m , voter welfare does not change by replacing F_m with \tilde{F}_m . By symmetry, to analyse welfare we need only consider $m \geq \frac{1}{2}$. When $m < 1 - \varepsilon$ and both platforms are located at $\frac{1}{2}$, both candidates adopt the median voter's position and welfare is zero, its highest possible value. Hence, to show that voter welfare is higher with common platforms at one-half, we need only show that this holds conditional on the event that $m > 1 - \varepsilon$. Let $U_e(\varepsilon)$ be voter welfare in equilibrium and $U_c(\varepsilon)$ be welfare when parties locate at the centre, both conditional on the event that $m > 1 - \varepsilon$. First consider both platforms at one-half. When $m > 1 - \varepsilon$, the winning candidate locates at m with probability $1 - \left(1 - \frac{3-2m-4\varepsilon}{1-2\varepsilon}\right)^2 = 1 - \frac{(2m-2+2\varepsilon)^2}{(1-2\varepsilon)^2}$, and otherwise is distributed on $[\frac{1}{2}, 2m - \frac{3}{2} - 2\varepsilon]$ with density $\frac{x - \frac{1}{2}}{2(\frac{1}{2} - \varepsilon)^2}$. Thus

$$U_c(\varepsilon) = \frac{-1}{\varepsilon} \int_{1-\varepsilon}^1 \left[\begin{aligned} & \frac{1}{2} \left(\int_{\frac{1}{2}}^{2m - \frac{3}{2} - 2\varepsilon} (m-x) \frac{2x-1}{(1-2\varepsilon)^2} dx \right) \\ & + \int_{-\infty}^m \left(\int_{\frac{1}{2}}^{2m - \frac{3}{2} - 2\varepsilon} |i-x| \frac{2x-1}{(1-2\varepsilon)^2} dx + \left(1 - \frac{(2m-2+2\varepsilon)^2}{(1-2\varepsilon)^2}\right) (m-i) \right) d\tilde{F}_m(i) \end{aligned} \right] dm.$$

In equilibrium,

$$U_e(\varepsilon) = \frac{-1}{\varepsilon} \int_{1-\varepsilon}^1 \left[\begin{aligned} & \frac{1}{2} \left(\left(\frac{\frac{1}{2}(1-\varepsilon)}{1-2\varepsilon} \right)^2 (m - (\frac{3}{4} - \frac{1}{4}\varepsilon)) + \int_{\frac{3}{4}-\frac{1}{4}\varepsilon}^{2m-\frac{5}{4}+\frac{7}{4}\varepsilon} (m-x) \frac{x-\frac{1}{4}-\frac{1}{4}\varepsilon}{2(\frac{1}{2}-\varepsilon)^2} dx \right) \\ & + \int_{-\infty}^m \left(\left(\frac{\frac{1}{2}(1-\varepsilon)}{1-2\varepsilon} \right)^2 \left| \frac{3}{4} - \frac{1}{4}\varepsilon - i \right| + \int_{\frac{3}{4}-\frac{1}{4}\varepsilon}^{2m-\frac{5}{4}+\frac{7}{4}\varepsilon} |i-x| \frac{x-\frac{1}{4}-\frac{1}{4}\varepsilon}{2(\frac{1}{2}-\varepsilon)^2} dx \right. \\ & \quad \left. + \left(1 - \frac{(2m-\frac{3}{2}+\frac{3}{2}\varepsilon)^2}{(1-2\varepsilon)^2} \right) (m-i) \right) d\tilde{F}_m(i) \end{aligned} \right] dm.$$

It is readily verified that

$$\lim_{\varepsilon \rightarrow 0} U_c(\varepsilon) = \lim_{\varepsilon \rightarrow 0} \frac{-1}{\varepsilon} \int_{1-\varepsilon}^1 \int_{-\infty}^m \left(\begin{aligned} & \int_{\frac{1}{2}}^{2m-\frac{3}{2}-2\varepsilon} |i-x| \frac{2x-1}{(1-2\varepsilon)^2} dx \\ & + \left(1 - \frac{(2m-2+2\varepsilon)^2}{(1-2\varepsilon)^2} \right) (m-i) \end{aligned} \right) d\tilde{F}_m(i) dm$$

and

$$\lim_{\varepsilon \rightarrow 0} U_e(\varepsilon) = \frac{-1}{32} - \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \int_{1-\varepsilon}^1 \int_{-\infty}^m \left(\begin{aligned} & \left(\frac{\frac{1}{2}(1-\varepsilon)}{1-2\varepsilon} \right)^2 \left| \frac{3}{4} - \frac{1}{4}\varepsilon - i \right| \\ & + \int_{\frac{3}{4}-\frac{1}{4}\varepsilon}^{2m-\frac{5}{4}+\frac{7}{4}\varepsilon} |i-x| \frac{x-\frac{1}{4}-\frac{1}{4}\varepsilon}{2(\frac{1}{2}-\varepsilon)^2} dx \\ & + \left(1 - \frac{(2m-\frac{3}{2}+\frac{3}{2}\varepsilon)^2}{(1-2\varepsilon)^2} \right) (m-i) \end{aligned} \right) d\tilde{F}_m(i) dm.$$

If $\lim_{\varepsilon \rightarrow 0} (U_c(\varepsilon) - U_e(\varepsilon)) > 0$, then for ε sufficiently small $U_c(\varepsilon) > U_e(\varepsilon)$, meaning voter welfare would be higher with common platforms at the centre. As $\varepsilon \downarrow 0$, $U_c(\varepsilon) - U_e(\varepsilon)$ is smallest when \tilde{F}_m puts as much mass on $\frac{3}{4}$ as possible, i.e. if for some positive constant k (given by the regularity assumption)

$$\tilde{F}_m(i) = \begin{cases} 0 & \text{if } i < 0 \\ \frac{i}{k} & \text{if } 0 \leq i < \frac{3}{4} \\ \frac{1}{2} - \frac{i}{k} & \text{if } \frac{3}{4} \leq i < m \\ 1 & \text{if } i \geq m. \end{cases}$$

For such \tilde{F}_m we obtain

$$\lim_{\varepsilon \rightarrow 0} U_e(\varepsilon) < \frac{-1}{32} - \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \int_{1-\varepsilon}^1 \int_{-\infty}^m \left(1 - \frac{(2m-\frac{3}{2}+\frac{3}{2}\varepsilon)^2}{(1-2\varepsilon)^2} \right) (m-i) d\tilde{F}_m(i) dm < \frac{-1}{32} - \frac{3}{32} = -\frac{1}{8}$$

and

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} U_c(\varepsilon) &= \underbrace{\lim_{\varepsilon \rightarrow 0} \frac{-1}{\varepsilon} \int_{1-\varepsilon}^1 \int_{-\infty}^m \int_{\frac{1}{2}}^{2m-\frac{3}{2}-2\varepsilon} |i-x| \frac{x-\frac{1}{2}}{2(\frac{1}{2}-\varepsilon)^2} dx dF_m(i) dm}_{=0} \\ &\quad - \underbrace{\lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \int_{1-\varepsilon}^1 \int_{-\infty}^m \left(1 - \frac{(2m-2+2\varepsilon)^2}{(1-2\varepsilon)^2} \right) (m-i) dF_m(i) dm.}_{>-\frac{1}{8}} \end{aligned}$$

Therefore $\lim_{\varepsilon \rightarrow 0} (U_c(\varepsilon) - U_e(\varepsilon)) > 0$.

We now sketch the proof for small V . If $V < \frac{1}{6}$, parties choose platforms at $(\frac{1}{2} - V, \frac{1}{2} + V)$, and candidates stay at these positions whenever $m \in [0, \frac{1}{2} - V] \cup [\frac{1}{2} + V, 1]$. For $m \in [\frac{1}{2} - V, \frac{1}{2} + V]$, candidates move to m with probability $\frac{V+m-\frac{1}{2}}{V}$ and retain their platforms with probability $\frac{\frac{1}{2}-m}{V}$. We examine how welfare would change by moving platforms from $(\frac{1}{2} - V, \frac{1}{2} + V)$ to $(\frac{1}{2} - V - \varepsilon, \frac{1}{2} + V + \varepsilon)$, for $\varepsilon > 0$. In both cases, whenever $m \in [0, \frac{1}{2} - V - \varepsilon] \cup [\frac{1}{2} + V + \varepsilon, 1]$ candidates retain their platforms and are closer to the median voter for more dispersed platforms. For regular distributions of voters' bliss points this implies strictly higher voter welfare for these constituencies. If V is small this gain from the inframarginal elections $m \in [0, \frac{1}{2} - V - \varepsilon] \cup [\frac{1}{2} + V + \varepsilon, 1]$ is always larger than losses with respect to median voters in $[\frac{1}{2} - V - \varepsilon, \frac{1}{2} + V + \varepsilon]$.²³ This is because these constituencies are “very few”, and, hence, the latter effect is only of second order for $V = 0$ and $\varepsilon \rightarrow 0$.

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²³One can show that only for $m \in [\frac{1}{2} - V - \frac{\varepsilon}{2}, \frac{1}{2} + V + \frac{\varepsilon}{2}]$ is voter welfare lower when candidates are less centralised.

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