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Katrin Wesche *

Aggregation Bias in Estimating European Money Demand Functions

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* University of Bonn
Institut für Internationale Wirtschaftspolitik
Lennéstr. 37
D-53113 Bonn
Phone: +49 228 73-9223, Fax: +49 228 73-7953
E-Mail: wesche@iiw.uni-bonn.de

Abstract: *Recently, money demand functions for a group of European countries have been estimated and generally have been found to perform better than most national money demand functions. While parameter equality is a sufficient condition for valid aggregation of linear equations, in money demand estimation often log-linear specifications are used, so that aggregation is in effect nonlinear. This makes the relation between the aggregate and the individual equations more complicated. To investigate if the conditions for unbiased linear aggregation are also valid for logarithmic aggregation, a simulation study is performed.*

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Estimating macro relations always involves aggregation over individual decision making units. In most cases, data on the individual level cannot be obtained and it cannot be tested empirically if the necessary conditions for aggregation hold. Sometimes, however, data are available for different levels of aggregation. So it is interesting to ask which relation exists between the equations on the lower aggregation level and the aggregate equation.

This issue is of particular relevance in the case of European money demand. Recently, money demand functions for a group of European countries have been estimated and generally have been found to perform better than most national money demand functions.[1] The choice between an aggregate and a multi-country approach to money demand depends on the relative importance of the aggregation error versus the specification error. The aggregate approach will be preferable if the specification errors cancel out by aggregating. On the other hand, an aggregation bias is introduced if substantial differences exist between the individual equations. Therefore, the validity of aggregation is generally evaluated by testing for equality of the parameter estimates among the national money demand functions.[2]

Nevertheless, while parameter equality is a sufficient condition for valid aggregation in the case of linear aggregation, in money demand estimation often log-linear specifications are used, so that aggregation is in effect nonlinear. Here the situation becomes more complicated as the aggregate equation bears no mathematical relationship to the individual equations.

To look into the aggregation of money demand functions a simulation study is performed.

Aggregation Bias

In general, aggregation of micro relations leads to an aggregation bias because information is lost by imposing the same parameter values on all micro equations. In the case of linear aggregation, Theil (1954) has shown that the macro relation will at best perform as good as the estimated micro relations if the micro relations are correctly specified. So, even if the aim is to predict the behavior of the macro dependent variable, this can be better done by using a set of micro equations than by an aggregate equation. An aggregation bias is absent only in two special cases: First, if all micro relations have identical parameter values aggrega-

[1] See, e.g., Kremers and Lane (1990), Artis, Bladen-Howell, and Zhang (1993), and Monticelli and Strauss-Kahn (1991).
[2] See Lane and Poloz (1992), Angeloni, Cottarelli, and Levy (1994), Cassard, Masson, and Lane (1994).

tion will lead to no information loss; or second, if the independent variables remain in the same proportion over the whole sample period no aggregation bias results.

In aggregation theory it is generally assumed that the Klein-Nataf consistency requirement is met.[3] This means that the aggregate variables are defined as the sum of the micro variables or that the variables are aggregated with fixed, time-independent weights. While this requirement is intuitively plausible, it is often violated in empirical applications. If the micro relations are defined as log-linear equations, one would not define the aggregate variable as the sum of the variables in logarithms as this makes economically no sense. The aggregate variables rather would be defined as the logarithm of the sum of the micro variables and then a log-linear macro equation would be estimated.[4] This implies that aggregation is in effect nonlinear.

Unfortunately, in the case of nonlinear aggregation it is much more difficult, if not impossible, to derive mathematical relationships between aggregate and micro relations. This makes formal testing for certain parameter values unfeasible because the relation between macro and micro parameters is not known. Only simulations can give information on the aggregation bias incurred in special cases.

The model

To investigate the effect of logarithmic aggregation, a simple bivariate model is simulated, involving money and output. Since the model is intended to investigate the aggregation of money demand functions, the parameter values are chosen to reflect the stylized facts on money demand. On the disaggregate level, two countries are considered. Output for country *i* follows a random walk with drift.

$$y_t^i = \alpha + y_{t-1}^i + \eta_t^i,\ \eta_t^i \sim N(0,\sigma)$$

Money m^i_t and output for country *i* are cointegrated.

$$m_t^i - \beta y_t^i = \varepsilon_t^i,\ \varepsilon_t^i \sim N(0,\sigma)$$

The error terms η^i and ε^i are independently normally distributed with mean zero and standard deviation σ. For each country, 1000 observations are simulated. The first 200 observations are put aside to ensure that the estimation does not depend on the starting values. From the other 800 observations, a money demand equation for each country is estimated. Then the variables are aggregated and the macro relation is estimated. The fitted values from the micro

[3] See, e.g., Lovell (1973); Pesaran, Pierse, and Kumar (1989) or Lee, Pesaran, and Pierse (1990).

relations are aggregated and a composite residual is defined as the difference between the macro dependent variable and the aggregate micro predictions. The bias is computed as the difference between the sum of squared residuals from the aggregate relation and the sum of squared composite residuals. This procedure is repeated 1000 times to obtain the mean of the bias and its standard deviation.

To check the model, first linear aggregation is considered and the aggregate variables are defined as the sum of the micro variables. In this case, Theil's results are reproduced by the model. If the β coefficient is the same for both countries, the aggregation bias is not significantly different from zero. The same is true if output in one country is defined to be a fixed shared of output in the other country so that the distribution of the independent variables remains constant through time. As the parameter α does not appear in the cointegrating regression, its value is irrelevant for the aggregation bias.

Logarithmic aggregation

Next, the case of logarithmic aggregation is investigated. For the construction of the aggregate variables, the antilogs of m^i and y^i are taken, the variables are added up and then transformed into logarithms again before the macro equation is estimated. This procedure is also applied for the computation of the fitted values from the micro equations.

Interestingly, the Theil conditions do not translate to the case of logarithmic aggregation. Neither the equality of parameters nor the invariance of the distribution of the independent variables over time is a sufficient condition for the absence of an aggregation error. Instead, the aggregation bias depends on all parameter values of the model. Contrary to the linear case, where the aggregate equation can at least be as good as the micro models taken together, with logarithmic aggregation the aggregate equation may perform even better than the micro equations in certain parameter regions.

In the following, the simulation results are presented in detail. First, it is assumed that both countries are identical. The parameter α is fixed at 0.1 and different choices for β are investigated.[5] The mean aggregation bias and its standard deviation are shown in Fig. 1. While logarithmic aggregation in general leads to an aggregation bias even for identical micro equations, for an income elasticity between 1.0 and 1.5 it results in a small, but significant gain.

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[4] In the following, this procedure is referred to as logarithmic aggregation.
[5] The parameter values are in the range generally found in money demand estimations.

In contrast to the case of linear aggregation, the bias depends also on the variance of the random shocks (see Fig. 2a to 2d). With lower variance the aggregation bias approaches zero. This is intuitively plausible as, with identical countries, the aggregation bias results from the error terms and thus decreases with a lower error variance.

The aggregation bias depends on the value of α, the drift parameter in income, as well (see Fig. 3), though the results do not change much within the range of empirically plausible values for α. The more equations are aggregated, the more pronounced is the aggregation gain. This result can also be found in practice, see e.g., Arnold (1994), who aggregates money demand equations over OECD countries and finds a better performance of the aggregate equation the more countries are included.

Next, the distribution of the independent variables is kept fixed and the effect of different parameter values for β is investigated, so that the countries only differ in their income elasticity.[6] For the first country β is fixed at 1, for the second country β takes values between 0.5 and 2.0. Results are shown in Fig. 4. While in the linear case no aggregation bias results with a fixed distribution of the independent variables, this is not true for logarithmic aggregation. The aggregation bias increases with the values of β becoming more diverse.

The last simulation considers countries with different parameter values and without a fixed income distribution over time. In this case, linear aggregation would lead to a bias, too. Again for country one β is fixed at 1, for the second country β takes values between 0.95 and 1.05. The aggregation bias in the logarithmic case increases much faster with growing diversity of the parameters than in the linear case (see Fig. 5). But in the linear as in the logarithmic case the aggregation bias is much larger with different parameter values than the bias in the logarithmic case with identical parameter values.

Application

Since the simulations show that equality of the parameter estimates is not sufficient for the absence of an aggregation error, the model is applied to the estimation of a European money demand function for M1. Aggregation is performed over three countries, which are the most likely candidates for a core union, i.e., Germany, France and the Netherlands. The sample period is 1974:1 to 1994:4, data are quarterly. Real money, deflated by the consumer price index, is assumed to depend on an income variable and the opportunity cost of holding

[6] The parameter values for α and σ are kept at 0.1 and 1 for both countries.

money, represented by real GDP and the money market rate, respectively.[7] The variables are converted into Deutsche Mark with purchasing power parities for 1990, added up, and a European function is estimated. As the German currency union with the former GDR was found to have an influence on European money demand (see Falk and Funke, 1995), a dummy is included which takes the value of one from 1990:3 on. Since unit root tests showed that the variables are non-stationary the Engle-Granger method is applied for the estimation of the long run relation. Data are not seasonally adjusted, so four seasonal dummies are included in the estimation of the long run equation.

$$m_t - p_t = -1.9 + 1.01 y_t - 0.011 r_t + 0.0153 dummy + e_t$$

adj. $R^2 = 0.97$, DW= 0.62, SEE 0.0275

Parameter values are plausible and correspond to the results generally found in the literature.[8] To investigate stationarity of the residuals a Dickey-Fuller Test is performed, giving a test statistic of -4.627. Since this value exceeds the critical value of -3.844 for the 5% of significance (MacKinnon, 1991), the hypothesis of no cointegration is rejected.[9]

A simulation is run to check if aggregation leads to a bias even if parameter values across the national functions were equal. In the model described above a third equation for the interest rate is inserted, modeling the interest rate as a stationary AR(1) process with an autoregressive parameter near to, but below, unity.[10] Parameter values are chosen to reflect the empirical behavior of the time series used in the estimation and are assumed to be identical for all countries. Aggregation is performed over three countries. The simulation gives a mean bias of $3.35 \cdot 10^{-8}$ with a t-statistic of 61.74. While this bias is significantly different from zero, it is – compared to the bias caused by different β values in the linear as well as in the logarithmic case – negligibly small.

Conclusion

For logarithmic aggregation no simple relation for the absence of an aggregation bias can be defined. Specifically, the conditions for linear aggregation to be valid do not apply.

[7] Purchasing power parities are from OECD (1990). All other data are from the IFS CD-ROM.
[8] T-statistics are not shown because their distribution is non-standard.
[9] Results are almost the same if in the long run regression is run without seasonal dummies and an ADF-Test with four lags is performed on the residuals.
[10] Though in empirical applications the interest rate is often found to be non-stationary this is presumably due to the limited number of observations in empirical samples and the low power of unit root test to discriminate against the alternative of a parameter slightly lower than one. From a theoretical point of view, however, interest rates should be stationary since they cannot rise to infinity nor turn negative. Therefore it was chosen to model the interest rate as a stationary process.

Neither the requirement of equal parameter values nor the condition of a time invariant distribution of the independent variables is a sufficient condition for the absence of an aggregation bias. Nevertheless, for the parameter values generally encountered in money demand estimations the bias (which even may be an aggregation gain) is fairly small so that the Theil conditions can be taken as a good approximation in testing for aggregation bias. As the bias is increasing much faster with logarithmic aggregation than with linear aggregation, tests for parameter equality should perhaps use a more conservative significance levels because even small differences in parameter values lead to a fairly high aggregation bias. Nevertheless, results are sensitive to parameter values and in particular applications the existence of an aggregation error can only be checked for by simulations.

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Fig. 1

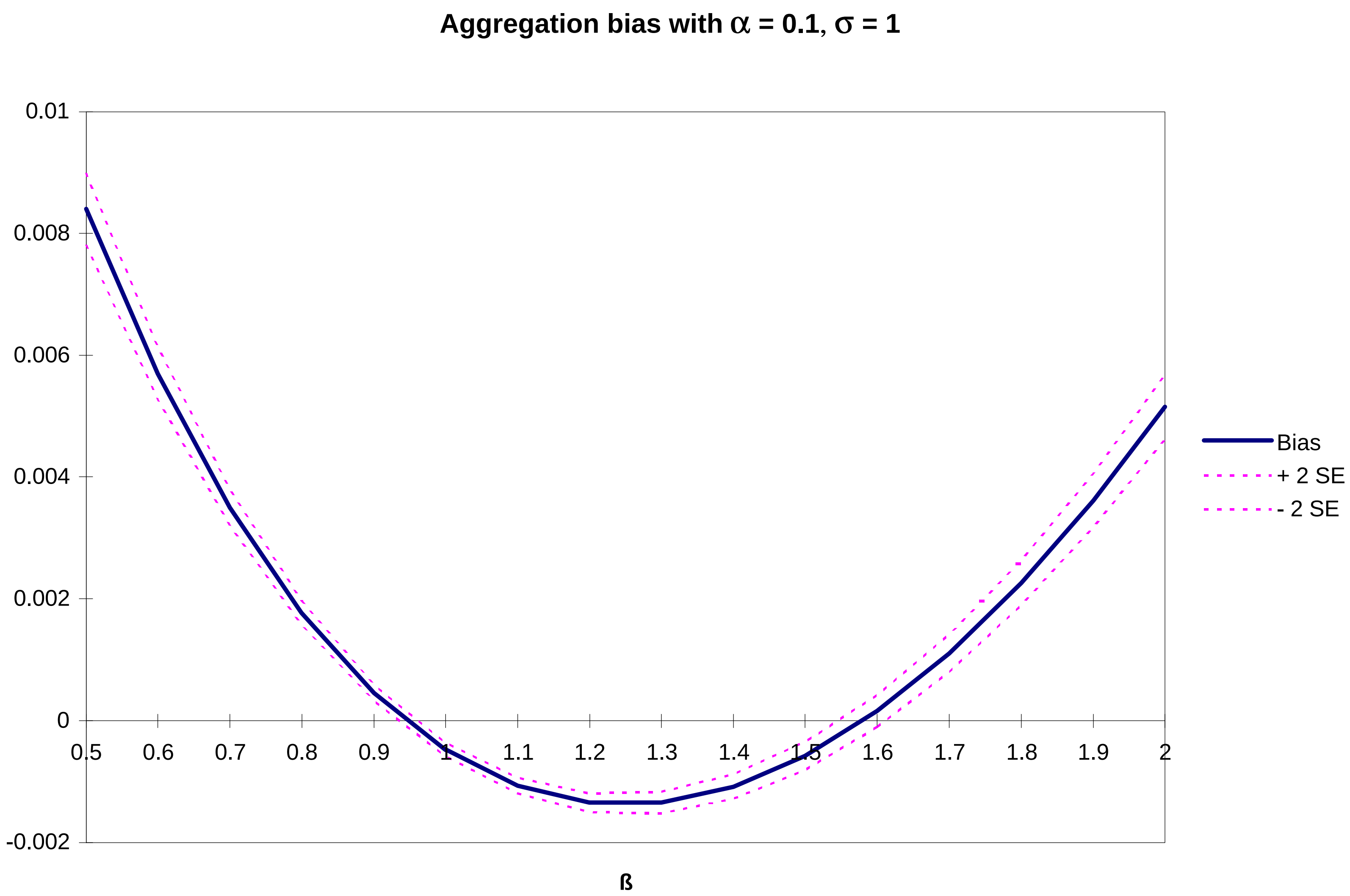
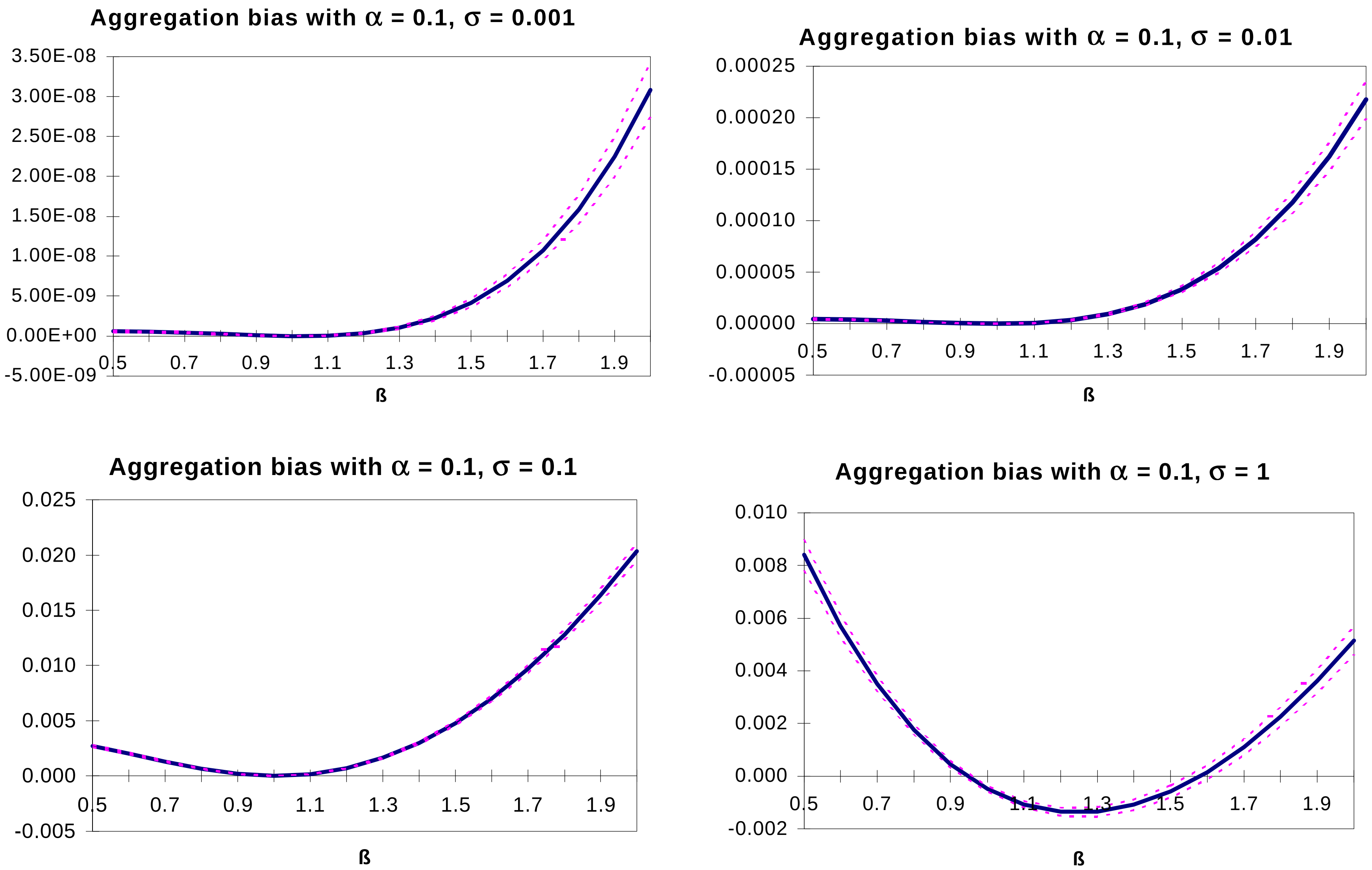
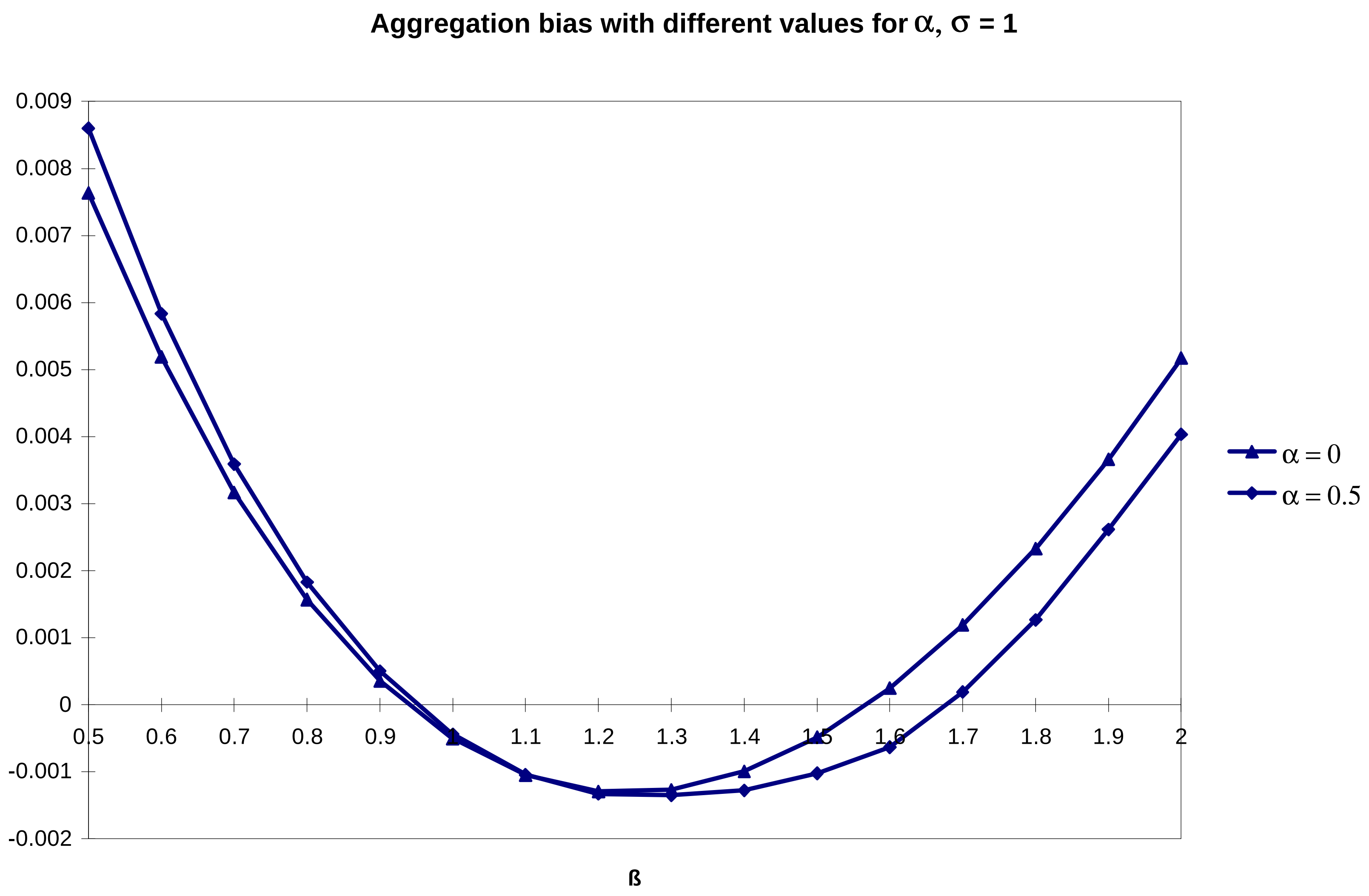

Aggregation bias with α = 0.1, σ = 1
0.01
0.008
0.006
0.004
0.002
0
-0.002
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
ß
Bias
+ 2 SE
- 2 SE

Fig. 2



Aggregation bias with α = 0.1, σ = 0.001
3.50E-08
3.00E-08
2.50E-08
2.00E-08
1.50E-08
1.00E-08
5.00E-09
0.00E+00
-5.00E-09
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
ß
Aggregation bias with α = 0.1, σ = 0.01
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
-0.00005
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
ß
Aggregation bias with α = 0.1, σ = 0.1
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
ß
Aggregation bias with α = 0.1, σ = 1
0.010
0.008
0.006
0.004
0.002
0.000
-0.002
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
ß

Fig. 3



Aggregation bias with different values for α, σ = 1
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
-0.001
-0.002
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
ß
α = 0
α = 0.5

Fig. 4

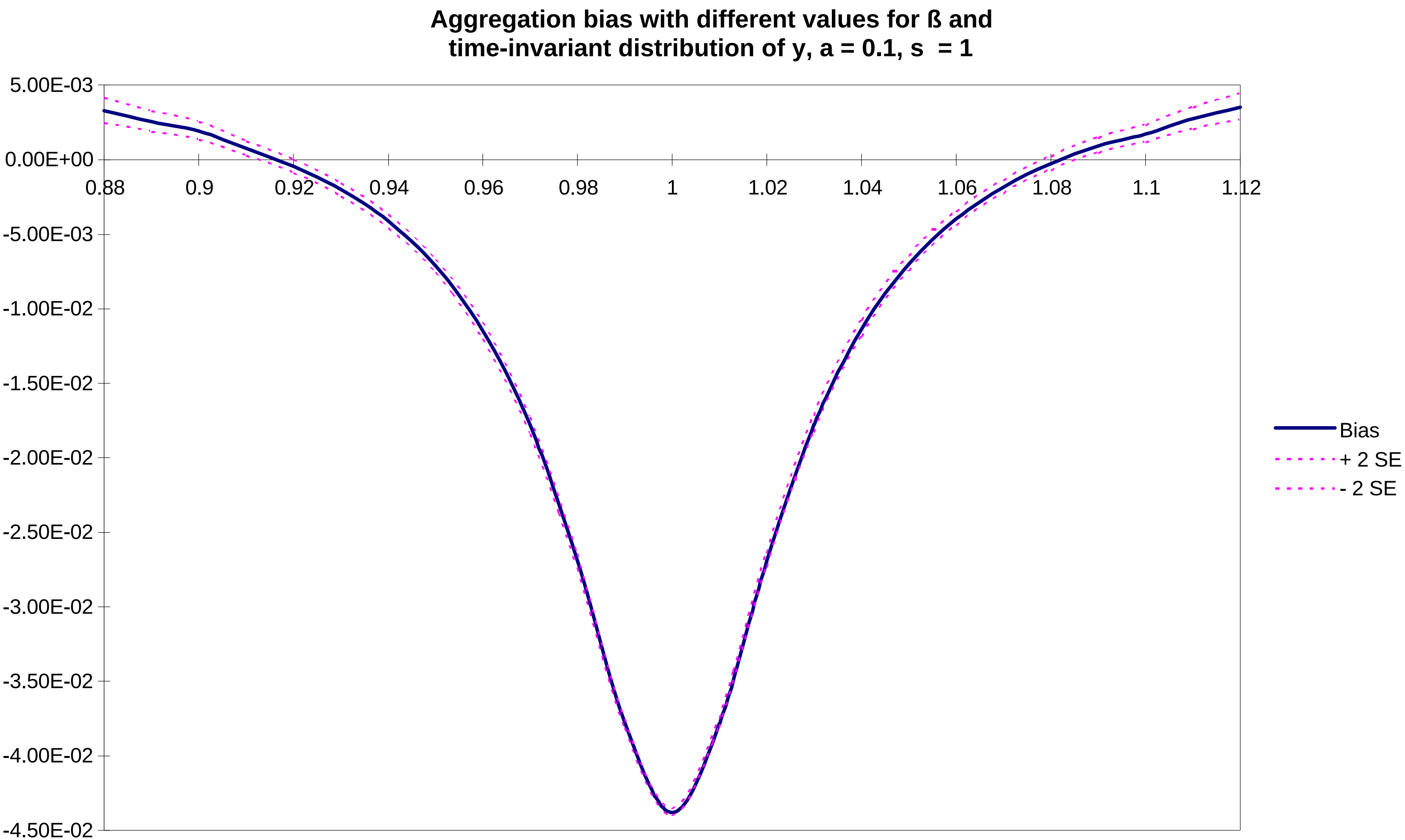
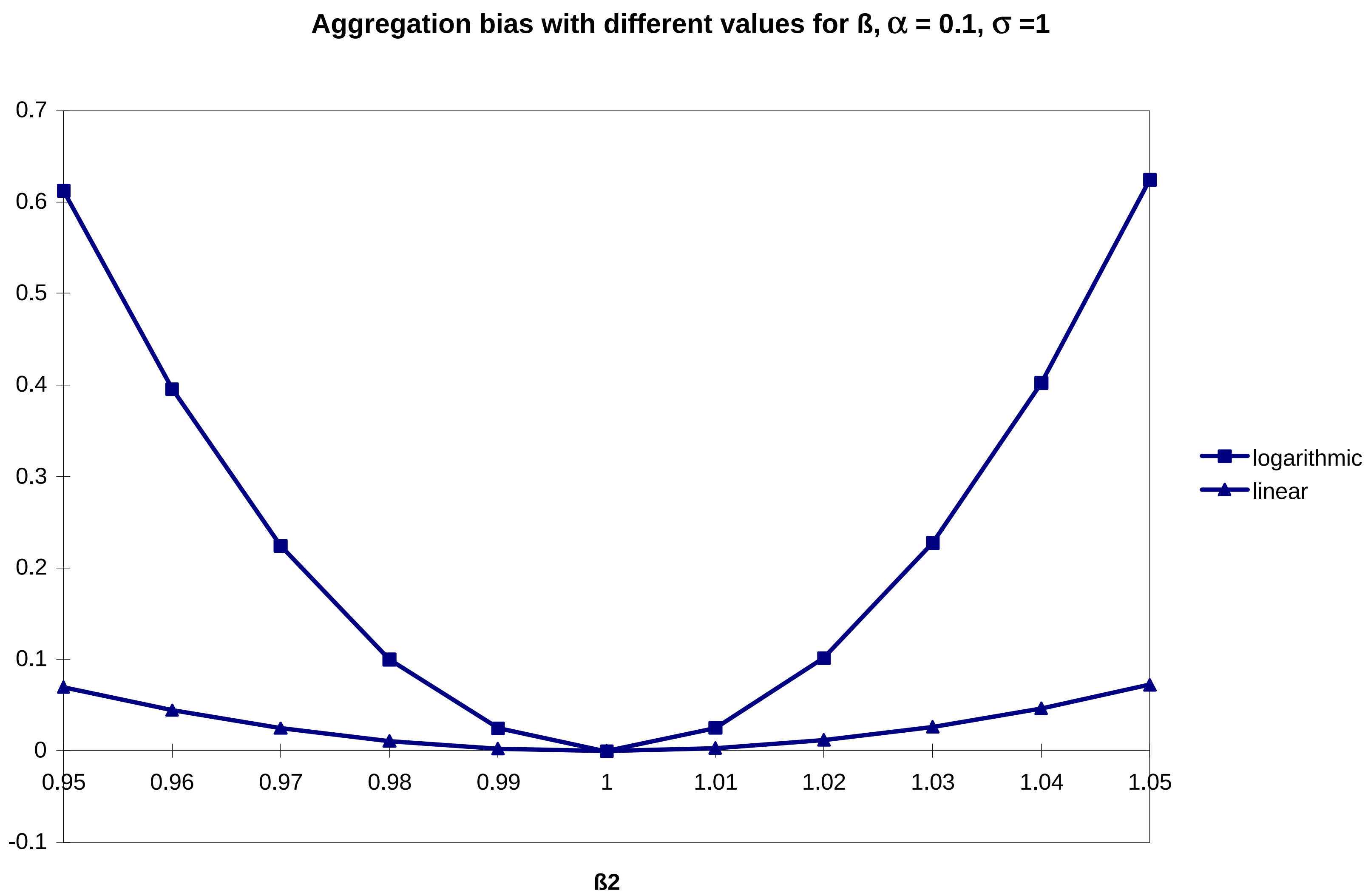

Aggregation bias with different values for ß and
time-invariant distribution of y, a = 0.1, s = 1
5.00E-03
0.00E+00
-5.00E-03
-1.00E-02
-1.50E-02
-2.00E-02
-2.50E-02
-3.00E-02
-3.50E-02
-4.00E-02
-4.50E-02
0.88
0.9
0.92
0.94
0.96
0.98
1
1.02
1.04
1.06
1.08
1.1
1.12
Bias
+ 2 SE
- 2 SE

Fig. 5


Aggregation bias with different values for ß, α = 0.1, σ =1
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.95
0.96
0.97
0.98
0.99
1
1.01
1.02
1.03
1.04
1.05
ß2
logarithmic
linear