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Indecisiveness aversion and preference for commitment*

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Abstract

We present a system of behavioral axioms for preferences over menus that is motivated by three assumptions. First, the decision maker is uncertain *ex ante* (i.e. at the time of choosing a menu) about her *ex post* (i.e. at the time of choosing an option within her chosen menu) preferences over options, and she anticipates that this subjective uncertainty will only resolve after the *ex post* stage. Second, she is averse to *ex post* indecisiveness (i.e. to having to choose between options that she cannot rank with certainty). Third, when evaluating a menu she discards options that are dominated (i.e. inferior to another option whatever her *ex post* preferences may be) and restricts attention to the undominated ones. Under these assumptions, the decision maker has a preference for commitment in the sense of preferring menus with fewer undominated alternatives. We derive a representation in which the decision maker's uncertainty about her *ex post* preferences is captured by means of a subjective state space, which in turn determines which options are undominated in a given menu, and in which the decision maker fears, whenever indecisive, to choose an option that will turn out to be the worst (undominated) one according to the realization of her *ex post* preferences.

Keywords. Opportunity sets, subjective uncertainty, indecisiveness, dominance.
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1 Introduction

Consider a two-stage decision situation. In the first stage, the decision maker has to choose a menu (or opportunity set). In the second stage, she chooses an option from this menu. We refer to these two stages as the *ex ante* and *ex post* stage, respectively. We assume that the decision maker is uncertain *ex ante* about her *ex post* preferences over options. Standard models in the literature on opportunity sets use this assumption in order to motivate a desire for flexibility (Kreps, 1979; Nehring, 1999; Dekel, Lipman, and Rustichini, 2001; Dekel, Lipman, Rustichini, and Sarver, 2007a; Ozdenoren, 2002; Epstein, Marinacci, and Seo, 2007). According to these models, larger menus can never be worse than smaller ones when a decision maker expects to learn her *ex post* preferences before actually having to choose an option. In contrast to these approaches, we consider a decision maker who anticipates that her uncertainty about her *ex post* preferences will only resolve after she will have chosen an option. Such a decision maker will find herself at the *ex post* stage, at least for some menus, in a situation of indecisiveness, i.e. of having to choose an option without being certain which option she prefers. We assume that the decision maker is averse to such situations of indecisiveness and, therefore, prefers smaller menus to larger ones, to the extent that smaller menus enable her to avoid these situations.

As an illustrative example, consider Bethy, who is a manager of a small division in a large company. She is faced with the problem of assigning the execution of a project to one of the employees. Right now she can only choose among the employees in her division, whom she knows well and has previously observed in similar projects. She is rather certain that Alan would be the best person to entrust with the project. However, just before Bethy can make the decision, the CEO of the company contacts her and suggests that she now has the possibility to pick an employee not just from her own division, but from the entire company staff. Bethy has only limited knowledge of the staff outside of her division. In particular, she knows that Bob, Chris and Dave are well suited to execute the project, but she finds these three candidates hard to compare: e.g., Bob would be excellent on the financial side of the project, but Chris would do better than Bob when it comes to marketing, and Dave is not so good when it comes to marketing or finance but has outperformed the other two in terms of creativity in the past. Bethy knows that all of these dimensions might be relevant for the success of the project, but the current situation makes it difficult to forecast which one would be most important. She is faced with a hard choice: she has to make an important decision (for the company, for her career and that of the person who will be in charge of the project), and take full responsibility for this decision in front of the CEO, without being able to confidently go for either one of the possible options. In fact, she would have much preferred sticking to her division, which would have avoided her this situation of indecisiveness altogether. Thus she would willing to forego candidates that are potentially better than Alan (in fact, she may even be sure that, e.g., Bob is superior to Alan in all regards) in order to avoid the pain of having to choose in a situation of indecisiveness.[1].

Extending previous work from Guerdjikova and Zimper (2008), we propose and axiomatize a representation of preferences over opportunity sets that captures this aversion to indecisiveness. More specifically, the decision maker's *ex ante* uncertainty about her *ex post* preferences is captured by means of a subjective state space, each subjective state corresponding to a utility function over options. Since the decision maker anticipates that she will only learn the subjective state after she will have chosen an option, she is indecisive between two options whenever the first one is ranked above the second one in some subjective state whereas the second one is ranked above the first one in some other subjective state. When evaluating a menu, the decision maker first discards all options that are dominated by some other option in the menu (i.e. ranked below this other option in all subjective states), as these options are clearly irrelevant for her final choice. Restricting attention to undominated options, then, the representation captures indecisiveness aversion by evaluating the menu, in each subjective state, by the utility of the worst option in this state. To give an intuition, this is as if the decision maker viewed herself confronted with a malevolent nature that would first select the subjective state, then manage to have her choose the worst possible option in this subjective state, and then only reveal her the subjective state. Finally, an increasing aggregator transforms the vector of subjective-state contingent utilities into the *ex ante* utility of the menu.

Our representation exhibits a preference for commitment, once one restricts attention to undominated alternatives. In other words, the decision maker always prefers a menu with a smaller set of undominated alternatives. It is noteworthy that our representation does not identify which option will eventually be chosen by the decision maker (not even contingently on the subjective state, since in fact the decision maker does not know the subjective state at the time of choosing an option). Thus our notion of indecisiveness aversion arises from the fact of having to choose without knowing one's preferences rather than from the outcome of this choice. In this regard, our model differs from models of temptation (Gul and Pe-

[1]In this sense, the decision maker who conforms to our theory prefers to avoid taking responsibility for her decisions. This interpretation was suggested to us by Klaus Nehring.

sendorfer, 2001; Dekel, Lipman, and Rustichini, 2007), regret (Sarver, 2008), costly contemplation (Ergin and Sarver, 2008), or thinking aversion (Ortoleva, 2008).

The paper is organized as follows. In section 2 we present our setup and utility representation and compare it to Dekel, Lipman, and Rustichini (2001)'s ordinal utility representation. In section 3 we introduce our axiom system and derive our representation result. Section 4 concludes. All proofs are relegated to the appendix.

2 Setup and representation

Let B be a finite set of prizes and let $\Delta(B)$ be the set of all probability distributions (*lotteries*) over B which stand for the options of our approach. Given $\beta, \beta' \in \Delta(B)$ and $\lambda \in [0,1]$, we define the λ-mixture of β and β' as usual and denote it by $\lambda\beta + (1-\lambda)\beta'$. A non-empty subset x of $\Delta(B)$ is interpreted as an *opportunity set* or *menu*, i.e. as the commitment to choose some lottery $\beta \in x$ at a given later date. We refer to the choice of a menu as the *ex ante* stage and to the (implicit) choice of a lottery within the chosen menu as the *ex post* stage. We endow the set of lotteries with the Euclidean metric and the set of menus with the Hausdorff metric.

We restrict attention to menus that are *polytopes*, i.e. convex hulls of (non-empty) finite sets of lotteries.[2] Let X denote the set of all such menus. We consider a decision maker endowed with a weak preference relation $\succsim$ over X, capturing her *ex ante* ranking of menus. From $\succsim$ we define the strict preference relation $\succ$ and the indifference relation $\sim$ as usual. We look for an ordinal utility representation of $\succsim$ as follows:

Definition. *An* ***indecisiveness averse representation*** *of* $\succsim$ *consists in*

(i) *A non-empty, closed, convex set* $U \subseteq \mathbb{R}^B$ *of utility functions such that, for all* $\beta, \beta' \in \Delta(B)$,

$$\{\beta\} \sim conv(\{\beta, \beta'\}) \Leftrightarrow [\forall u \in U, u \cdot \beta \geq u \cdot \beta']. \tag{1}$$

(ii) *A functional* $c : X \to X$ such that[3]

$$c(x) = \{\beta \in x | \nexists \beta' \in X, U \cdot \beta' > U \cdot \beta\}, \tag{2}$$

[2]We can think of these menus as determined by a finite set of linear constraints or, equivalently, we can think of the decision maker as considering finite menus but being able to randomize between options. the proof of our representation theorem relies on this restriction to polytopes (see appendix).

[3]In the appendix it is shown that if x is a polytope then so is $c(x)$ (a fact which is not true for arbitrary compact menus).

where $U \cdot \beta' > U \cdot \beta$ *means* $u \cdot \beta' \geq u \cdot \beta$ *for all* $u \in U$ *and* $u \cdot \beta' > u \cdot \beta$ *for some* $u \in U$.

(iii) *An aggregator* $g : \mathbb{R}^U \rightarrow \mathbb{R}$, *continuous and weakly increasing on*

$$U(X) = \{(\min_{\beta \in c(x)} u \cdot \beta)_{u \in U} | x \in X\}, \tag{3}$$

such that for all $x, x' \in X$,

$$x \succsim x' \Leftrightarrow g\left(\left(\min_{\beta \in c(x)} u \cdot \beta\right)_{u \in U}\right) \geq g\left(\left(\min_{\beta \in c(x')} u \cdot \beta\right)_{u \in U}\right). \tag{4}$$

The interpretation of the representation is that the decision maker envisions a set of possible *ex post* preferences. Each of these is an expected-utility preference represented by a von Neumann-Morgenstern utility function $u \in U$, so U can be interpreted as a (subjective) state space. A lottery β dominates a lottery β' if and only if β has a higher expected utility than β' regardless of the *ex post* utility function. As we noticed in the introduction, dominated lotteries are never chosen and their addition does not influence the evaluation of a menu. Therefore, we can interpret condition (1) to say that if we add the dominated lottery β' to the singleton menu β, the resulting menu *conv* $(\{\beta; \beta'\})$ will be exactly as good as $\{\beta\}$. Simultaneously, we require this condition to *characterize* the dominance relation between lotteries.

For a given set of utility functions U, the functional c identifies the undominated alternatives of each set in X. Since dominated alternatives are never chosen, the decision maker should be indifferent between choosing an option out of $x \in X$, or out of $c(x)$. Hence, for the purposes of our representation, only the set of undominated alternatives is relevant.

For each *ex post* utility function, the decision maker evaluates a menu x by the lowest possible expected utility an undominated lottery in x can give her. This reflects her aversion towards indecisiveness. It is as if the decision maker pictures herself choosing the worst possible option w.r.t. any possible realization of her ex post utility function. Finally, the different possible *ex post* utility functions are aggregated through the increasing function g.

Remark 1. Note that the set U of *ex post* utility functions plays a double role in the representation. First, it determines the mapping $x \rightarrow c(x)$, i.e. the set of undominated options for each set x. The larger U, the larger $c(x)$ for a given x. Second, it determines the mapping $c(x) \rightarrow (\min_{\beta \in c(x)} u \cdot \beta)_{u \in U}$. The larger $c(x)$, the lower $\min_{\beta\in c(x)} u\cdot\beta$ for each $u \in U$ and, hence, the lower the *ex ante* utility of x since g is increasing. In the extreme case where the decision maker does not anticipate any possibility of being indecisive, U reduces to a singleton so that the second role then disappears and

$$x \succsim x' \Leftrightarrow \max_{\beta\in x} u\cdot\beta \geq \max_{\beta\in x'} u\cdot\beta.$$

That is, we are brought back to standard indirect utility for which opportunity sets are ranked according to their optimal alternatives.

Remark 2. Recall that Dekel, Lipman, and Rustichini (2001) derive the following ordinal utility representation for preferences over opportunity sets:

$$x \succsim x' \Leftrightarrow h\left(\left(\max_{\beta\in x} v\cdot\beta\right)_{v\in V}\right) \geq h\left(\left(\max_{\beta\in x'} v\cdot\beta\right)_{v\in V}\right)$$

where $V \subseteq \mathbb{R}^B$ is a subjective state space and $h : \mathbb{R}^V \to \mathbb{R}$ is an aggregator. Moreover, h is decreasing if and only if preferences satisfy *preference for commitment*, i.e.,

$$x \subseteq x' \text{ implies } x \succsim x'.$$

Now let $U = -V$ and define $g : \mathbb{R}^U \to R$ by, for all $z \in \mathbb{R}^V$ $g(z) = h(-z)$. Then, for all $x, x' \in X$,

$$x \succsim x' \Leftrightarrow g\left(\left(\min_{\beta\in x)} u\cdot\beta\right)_{u\in U}\right) \geq g\left(\left(\min_{\beta\in x')} u\cdot\beta\right)_{u\in U}\right),$$

and g is increasing if and only if h is decreasing. Consequently, whenever the set of undominated options coincides with the full opportunity set, i.e., $c(x) = x$, our representation formally coincides (by an appropriate change of variables) with Dekel, Lipman, and Rustichini (2001)'s ordinal utility representation. This formal equivalence, however, does no longer hold whenever $c(x) \neq x$.

3 Axioms and result

3.1 The Dominance Relation

In order to axiomatize our ideas, we first derive from $\succsim$ a *dominance* relation $\succsim^*$ over lotteries as follows: for all $\beta, \beta' \in \Delta(B)$,

$$\beta \succsim^* \beta' \Leftrightarrow \{\beta\} \sim conv(\{\beta, \beta'\}).$$

We want to interpret this relation as saying that the decision maker *decisively* prefers β over β'. In terms of our intended representation, this will correspond to the case in which the utility associated with β is higher than that associated with β', for all von Neumann-Morgenstern utility functions in the set U.

To understand the intended interpretation, note that if the decision maker knows that whatever her *ex post* preferences, she will weakly prefer β to β', then adding β' to the singleton menu $\{\beta\}$ should neither improve nor worsen this menu (the convex hull is just to have a menu in X). In fact, in this case we should also have $conv(\{\beta, \beta'\}) \succsim \{\beta'\}$.

Conversely, suppose that the decision maker does not *decisively* weakly prefer β to β'. This may be the case for two reasons. First, she may decisively strictly prefer β' to β ($\beta \succ^* \beta'$). In this case, adding β' to the singleton menu $\{\beta\}$ should improve this menu, so we should have $conv(\{\beta, \beta'\}) \succ \{\beta\}$ (as well as $\{\beta'\} \sim conv(\{\beta, \beta'\})$).

Second, she may have no decisive preference between β and β'. In this case, under our assumption that she dislikes indecisiveness, we should have $\{\beta\} \succ conv(\{\beta, \beta'\})$ and $\{\beta'\} \succ conv(\{\beta, \beta'\})$. In both cases, we do not have $\{\beta\} \sim conv(\{\beta, \beta'\})$, justifying the above definition (we will make this justification more precise below by deriving all these properties of $\succsim^*$ from our axioms). It is important to emphasize that the dominance relation $\succsim^*$ can (and in general will) be incomplete. We use $\bowtie^*$ to denote incomparability between two options.

Remark 3. Note that our dominance preference relation closely corresponds to Kreps (1979)'s "domination" relation, and also has a similar interpretation. The only difference is that in the absence of decisive preferences /dominance, the decision maker prefers larger menus in Kreps' model whereas she prefers smaller menus in our model. Of course, this just reflects the fact that Kreps assumes that the decision maker expects to learn her *ex post* preferences before choosing a lottery whereas we assume she does not.

Let us now define the set $\tilde{c}(x)$ of *undominated* lotteries in a menu $x \in X$ by

$$\tilde{c}(x) = \{\beta \in x | \nexists \beta' \in x, \beta' \succ^* \beta\}.$$

Under our assumption that the decision maker rules out all dominated lotteries, she should be indifferent between choosing a lottery in x or in $\tilde{c}(x)$. Clearly, if $\succsim$ admits an indecisiveness averse representation then $c(x) = \tilde{c}(x)$ for all $x \in X$.

3.2 Axioms

The intuitive discussion above motivates the following axioms on the preference relation $\succsim$ over X and $\succsim^*$ over $\Delta(B)$, respectively.[4]

Axiom 1 (Weak order). $\succsim$ *is complete and transitive.*

Axiom 2 (Dominance transitivity). $\succsim^*$ *is transitive.*

Axiom 3 (Dominance independence). *For all $\beta, \beta', \beta'' \in \Delta(B)$ and $\lambda \in (0,1)$, if $\beta \succsim^* \beta'$, then $\lambda\beta + (1-\lambda)\beta'' \succsim^* \lambda\beta' + (1-\lambda)\beta''$.*

Axiom 4 (Dominance continuity). *For all $\beta, \beta', \beta'', \beta''' \in \Delta(B)$, the set $\{\lambda \in [0,1] | \lambda\beta + (1-\lambda)\beta' \succsim^* \lambda\beta'' + (1-\lambda)\beta'''\}$ is closed.*

Axiom 5 (Indecisiveness aversion). *For all $x, x' \in X$, if for all $\beta \in \tilde{c}(x)$, there exists $\beta' \in \tilde{c}(x')$ such that $\beta \succsim^* \beta'$, then $x \succsim x'$.*

Axiom 6 (Undominated continuity). *For all $x, x', (x_n)_{n\geq 1}, (x'_n)_{n\geq 1} \in X$ such that $\tilde{c}(x_n) \to \tilde{c}(x)$ and $\tilde{c}(x'_n) \to \tilde{c}(x')$, if $x_n \succsim x'_n$ for all $n \geq 1$, then $x \succsim x'$.*

Axiom 1 is standard and without it a representation of preferences by a real-valued function is impossible. Axiom 2 requires that the dominance preference relation $\succsim^*$ defined above is transitive, i.e. if β dominates β', β' dominates β'', then β dominates β''. While this appears to be a desirable property of the dominance relation, it is important to note that it is not implied by the transitivity of $\succsim$: indeed, it may be that $\{\beta\} \sim conv(\{\beta; \beta'\})$, $\beta' \sim conv(\{\beta'; \beta''\})$ and yet $\beta \not\sim conv(\{\beta; \beta''\})$. Hence, the need for Axiom 2.

Axiom 3 requires the dominance relation to satisfy independence. The interpretation of this axiom is standard: when the two lotteries β and β' are mixed in equal proportions with a third β'', the decision maker faces the choice between β and

[4] The reader should keep in mind that $\succsim$ is the only primitive preference relation of our approach whereby we use $\succsim^*$, completely determined by $\succsim$, for notational and iterpretational reasons only.

β' with probability λ and the (trivial) choice between β'' and β'' with probability $(1-\lambda)$. Hence, if β dominates β', so should $\lambda\beta+(1-\lambda)\beta''$ dominate $\lambda\beta'+(1-\lambda)\beta''$. Axiom 4 imposes a continuity property on the dominance relation, by requiring that the better- and the worse-sets of this relation are closed.

Axiom 5 captures the main assumption of our approach, the fact that the decision maker dislikes situations of indecisiveness. Below we show that under Axiom 5, the decision maker prefers sets x, which have smaller (w.r.t. inclusion) sets of undominated options. Hence, the decision maker is worse-off if undominated, but incomparable options are added to her choice set. To understand the intuition behind this result, consider two lotteries, β and β' such that $\beta \bowtie^* \beta'$. Consider the two sets, $\{\beta\}$ and $conv\left(\{\beta;\beta'\}\right)$. Note that $\tilde{c}\left(\{\beta\}\right)=\{\beta\}$ and $\tilde{c}\left(conv\{\beta;\beta'\}\right)= conv\{\beta;\beta'\}$. Since $\beta \succsim^* \beta$, but $\beta' \not\succsim^* \beta$ the axiom implies that $\{\beta\} \succsim conv\left(\{\beta;\beta'\}\right)$. Furthermore, since $\beta \not\succsim^* \beta'$, we have $\{\beta\} \succ conv\left(\{\beta;\beta'\}\right)$. Similarly, we obtain $\{\beta'\} \succ conv\left(\{\beta;\beta'\}\right)$.

A further implication of Axiom 5 is that a set is evaluated only based on the undominated options contained in it. To understand this, compare the sets x and $\tilde{c}\left(x\right)$. Since $\tilde{c}\left(x\right)=\tilde{c}\left(\tilde{c}\left(x\right)\right)$, it follows that the condition of the Axiom is trivially satisfied and we obtain $x \sim \tilde{c}\left(x\right)$. Hence, consistent with our intuition, the decision maker who conforms to Axiom 5 acts as if he discards all dominated options in a given opportunity set.

Axiom 6 is a continuity condition imposed on preferences over sets consisting of undominated options. Since preferences over arbitrary sets can be reduced to preferences over their respective sets of undominated options, this is the right notion of continuity required for our representation.

3.3 Representation Theorem

We are now ready to state our representation theorem:

Theorem. *There exists an indecisiveness averse representation of* $\succsim$ if and only if $\succsim$ *satisfies axioms 1–6.*

The proof of the theorem is relegated to the appendix. Here we provide a brief sketch of the proof.

First, recall that $\succsim^*$ is an incomplete preference relation. Axioms 2, 3 and 4 correspond to the axioms used by Dubra, Maccheroni, and Ok (2004). This implies that there exists a non-empty, closed and convex set U such that:

$$\beta \succsim^* \beta' \text{ iff } U \cdot \beta \geq U \cdot \beta'.$$

The set U represents the set of subjective states of the decision maker and the dominance relation indeed indicates that the comparison between β and β' does not depend on the realized subjective state.

We can, therefore, conclude that the two definitions of sets of undominated options, $c(x)$ and $\tilde{c}(x)$ are, in fact, equivalent. Since $conv\,(c\,(x))$ belongs to X, and since Axiom 5 implies that the decision maker discards dominated options, we obtain $conv\,(c\,(x)) \sim x$ for all x. We can, therefore restrict attention to comparisons between sets of the form $conv\,(c\,(x))$ for some x. These preferences are complete and transitive (by Axiom 1) and satisfy continuity (by Axiom 6). Therefore, they can be represented by a continuous utility function, which by Axiom 5 is decreasing with respect to set inclusion. The remainder of the proof consists in showing that this function will take the form $g\left(\left(\min_{\beta\in c(x)} u\cdot\beta\right)_{u\in U}\right)$.

4 Conclusion

We have analyzed a representation of preferences over opportunity sets capturing the notion of indecisiveness aversion. In our representation, the decision maker's uncertainty about her *ex post* preferences is captured by means of a subjective state space. Since this uncertainty does not resolve before the choice of option, it gives rise to indecisiveness at the *ex post* stage. More specifically, the decision maker discards options that are clearly dominated, and evaluates the remaining set of undominated options pessimistically, as if he would get the worst possible option in all subjective states. This gives rise to a preference for commitment, in the sense of preferring menus with fewer undominated options.

Our representation is ordinal in the sense that our aggregator is only required to be monotone and continuous. It is natural to look for a more specific representation in which the aggregator has a linear form. That is to say, we could look for a positive measure μ on U such that, for all $x, x' \in X$,

$$x \succsim x' \Leftrightarrow \int_{u\in U}\left(\min_{\beta\in c(x)} u\cdot\beta\right) d\mu\,(u) \geq \int_{u\in U}\left(\min_{\beta\in c(x')} u\cdot\beta\right) d\mu\,(u)\,. \qquad (5)$$

One thing to note about this representation is that it is not truly linear. This is because it is not true that $c(\lambda x + (1-\lambda)x') = \lambda c(x) + (1-\lambda)c(x')$ in general. In fact, it is only true that $c(\lambda x + (1-\lambda)x') \subseteq \lambda c(x) + (1-\lambda)c(x')$ but the converse does not hold because, roughly speaking, by mixing between two menus one gets rid of some undominated options. Therefore, this representation does not imply the independence axiom, but only the following, weaker axiom: For all $x_1, x_2, \bar{x}, y_1, y_2 \in$

X and $\lambda \in (0,1)$ such that $\text{conv}(c(y_i)) = \text{conv}(\lambda c(x_i) + (1-\lambda)c(\bar{x}))$, $i = 1,2$, if $x_1 \succsim x_2$ then $y_1 \succsim y_2$. This makes it tempting to try to work on the class $\{\text{conv}(c(x)) | x \in X\}$ and parallel the proof of Dekel, Lipman, and Rustichini (2001); Dekel, Lipman, Rustichini, and Sarver (2007a)'s linear utility representation theorem. However, since this class is not convex, a similar argument to theirs (in particular for lemma S11 in Dekel, Lipman, Rustichini, and Sarver, 2007b) is not at hand in our model. We leave the problem of axiomatizing a linear representation of indecisivness averse preferences for future research.

Appendix: proofs

We start with two lemmas:

Lemma 1. *Assume that there exists $U \subseteq \mathbb{R}^B$ such that $\succsim^*$ satisfies (1). Then:*

1. *For all $x \in X$, $c(x) = \tilde{c}(x)$.*
2. *For all $x \in X$, $conv(c(x)) \in X$.*

Proof of lemma 1.

1. Follows immediately from (1) and the definitions of $c(x)$ and $\tilde{c}(x)$.

2. First, $c(x)$ in nonempty since x is compact (Eliaz and Ok, 2006, lemma 3). Since a polytope has only finitely many faces and each of these faces is closed, it is sufficient to show that $c(x)$ is a union of faces of x. Let $\beta \in c(x)$. We know that β belongs to the relative interior of some face f of x (Rockafellar, 1970, theorem 18.2). It is sufficient to show that $f \subseteq c(x)$. Suppose there exists $\beta' \in f$ such that $\beta' \notin c(x)$. Then, clearly, $\beta' \neq \beta$. Moreover, by part 1 of the lemma, there exists $\bar{\beta}' \in x$ such that $U \cdot (\bar{\beta}' - \beta') > 0$. Now, since β belongs to the relative interior of f, there exists $\beta'' \in f$ and $\lambda \in (0,1)$ such that $\beta = \lambda\beta' + (1-\lambda)\beta''$ (Rockafellar, 1970, theorem 6.4). Let $\bar{\beta} = \lambda\bar{\beta}' + (1-\lambda)\beta''$. Then $U \cdot (\bar{\beta} - \beta) = \lambda U \cdot (\bar{\beta}' - \beta') > 0$, so $\beta \notin c(x)$, a contradiction. Consequently, (4) is indeed well-defined since $c(x)$ is non-empty and closed for all $x \in X$.□

Lemma 2. *Assume $\succsim$ satisfies axioms 1–6. Then:*

1. *For all $\beta, \beta', \beta'' \in \Delta(B)$ and $\lambda \in (0,1)$, $\beta \succsim^* \beta'$ if and only if $\lambda\beta + (1-\lambda)\beta''^{*}\lambda\beta' + (1-\lambda)\beta''$.*
2. *For all $x, x' \in X$, if $c(x) \subseteq c(x')$, then $x \succsim x'$.*
3. *For all $x \in X$, $conv(c(x)) \in X$.*
4. *For all $x \in X$, $x \sim conv(c(x))$.*
5. *For all $\beta, \beta' \in \Delta(B)$,*

$$\beta \sim^* \beta' \Leftrightarrow \{\beta\} \sim conv(\{\beta, \beta'\}) \sim \{\beta'\},$$
$$\beta \succ^* \beta' \Leftrightarrow \{\beta\} \sim conv(\{\beta, \beta'\}) \succ \{\beta'\},$$
$$\beta \bowtie^* \beta' \Leftrightarrow [\{\beta\} \succ conv(\{\beta, \beta'\}) \text{ and } \{\beta'\} \succ conv(\{\beta, \beta'\})].$$

Proof of lemma 2.

1. Follows from axioms 1–4 (Dubra, Maccheroni, and Ok, 2004, lemma 1).

2. Follows immediately from axiom 5.

3. By axioms 1–4, there exists a non-empty, closed, convex set $U \subseteq \mathbb{R}^B$ such that $\succsim^*$ satisfies (1) (Dubra, Maccheroni, and Ok, 2004). Hence the result follows from lemma 1.2.

4. By parts 2 and 3 of the lemma, it is sufficient to prove that $\tilde{c}(conv(\tilde{c}(x))) = \tilde{c}(x)$. First, we show that for all $\beta \in x$, there exists $\beta' \in \tilde{c}(x)$ such that $\beta'^*\beta$. Let $y = \{\bar{\beta} \in x | \bar{\beta} \succsim^* \beta\}$. Since x is compact and $\succsim^*$ is continuous (Dubra, Maccheroni, and Ok, 2004, proposition 1), y is compact and, hence, there exists $\beta' \in y$ such that $\bar{\beta} \succ^* \beta'$ for no $\bar{\beta} \in y$ (Eliaz and Ok, 2006, lemma 3). Suppose $\bar{\beta} \succ^* \beta'$ for some $\bar{\beta} \in x \setminus y$. Since $\beta'^*\beta$ by definition of y, it follows that $\bar{\beta} \succ^* \beta$ by transitivity of $\succsim^*$, so $\bar{\beta} \in y$, a contradiction. Hence $\beta' \in \tilde{c}(x)$.

Now, by definition, $\tilde{c}(x) = \{\beta \in x | \nexists \beta' \in x, \beta'^*\beta\}$ and $\tilde{c}(conv(\tilde{c}(x))) = \{\beta \in conv(\tilde{c}(x)) | \nexists \beta' \in conv(\tilde{c}(x)), \beta'^*\beta\}$. Let $z = \{\beta \in conv(\tilde{c}(x)) | \nexists \beta' \in x, \beta'^*\beta\}$. Then $z = \tilde{c}(x) \cap conv(\tilde{c}(x)) = \tilde{c}(x)$. We show that $\tilde{c}(conv(\tilde{c}(x))) = z$. Clearly, $z \subseteq \tilde{c}(conv(\tilde{c}(x)))$ since $conv(\tilde{c}(x)) \subseteq x$. Conversely, let $\beta \in conv(\tilde{c}(x)) \setminus z$. Then there exists $\beta' \in x$ such that $\beta'^*\beta$. By the argument above, there then exists $\beta'' \in \tilde{c}(x)$ such that $\beta''^*\beta'$ and, hence, $\beta''^*\beta$, so $\beta \notin \tilde{c}(conv(\tilde{c}(x)))$. Hence $\tilde{c}(conv(\tilde{c}(x))) \subseteq z$.

5. The indifference property follows immediately from the definition of $\succsim^*$. Now, for all $\beta, \beta' \in \Delta(B)$, we obviously have $\tilde{c}(\{\beta\}) = \{\beta\}$ and $\tilde{c}(\{\beta'\}) = \{\beta'\}$. Moreover, by part 1 of the lemma,

$$\tilde{c}(conv(\{\beta, \beta'\})) = \begin{cases} \{\beta\} & \text{if } \beta \succ^* \beta', \\ \{\beta'\} & \text{if } \beta'^*\beta, \\ conv(\{\beta, \beta'\})) & \text{if} \beta \sim^* \beta' \text{ or } \beta \bowtie^* \beta'. \end{cases}$$

We now show that $\beta \succsim^* \beta'$ implies $conv(\{\beta, \beta'\}) \succsim \{\beta'\}$. Suppose $\beta \succsim^* \beta'$ and $\{\beta'\} \succ conv(\{\beta, \beta'\})$. Then $\{\beta\} \sim conv(\{\beta, \beta'\})$ by definition of $\succsim^*$ and, hence, $\{\beta'\} \succ \{\beta\}$ by transitivity of $\succsim$. But since $\tilde{c}(\{\beta\}) = \{\beta\}$, $\tilde{c}(\{\beta'\}) = \{\beta'\}$, and $\beta \succsim^* \beta'$, we have $\{\beta\} \succsim \{\beta'\}$ by axiom 5, a contradiction. This establishes the strict preference property as well as the $\Leftarrow$ part of the noncomparability property. For the $\Rightarrow$ part, assume $\beta \bowtie^* \beta'$. Then $\tilde{c}(\{\beta\}) \subseteq \tilde{c}(conv(\{\beta, \beta'\}))$ and $\tilde{c}(\{\beta'\}) \subseteq \tilde{c}(conv(\{\beta, \beta'\}))$, so we have $\{\beta\} \succsim conv(\{\beta, \beta'\})$ and $\{\beta'\} \succsim conv(\{\beta, \beta'\})$ by part 2 of the lemma. Suppose these two preferences are in fact indifferences. Then $\beta \sim^* \beta'$, a contradiction. Hence one of the two must be strict. Suppose the other one is an indifference. Then we face the same contradiction as above. Hence both preferences are strict.□

Proof of the Representation Theorem. Obviously, axiom 1 is necessary for a representation to exist. Given this axiom, we know that $\succsim^*$ is reflexive and, hence, axioms 2–4 are necessary and sufficient for the existence of a non-empty, closed, convex set $U \subseteq \mathbb{R}^B$ such that $\succsim^*$ satisfies (1) (Dubra, Maccheroni, and Ok, 2004). It remains to prove that axioms 5 and 6 are necessary and sufficient for the existence of a continuous and weakly increasing aggregator $g : U(X) \to \mathbb{R}$ such that $\succsim$ satisfies (4). It is easy to check that these axioms are necessary. The remainder of this section is devoted to the sufficiency proof.

Assume $\succsim$ satisfies axioms 1–6. Let $C = \{conv(c(x))|x \in X\}$. Clearly, for all $x \in X$ and $u \in U$, we have $\min_{\beta \in c(x)} u \cdot \beta = \min_{\beta \in conv(c(x))} u \cdot \beta$. Hence, by lemma 2.3–2.4, it is sufficient to find a continuous and weakly increasing aggregator g such that, for all $x, x' \in C$,

$$x \succsim x' \Leftrightarrow g\left(\left(\min_{\beta \in x} u \cdot \beta\right)_{u \in U}\right) \geq g\left(\left(\min_{\beta \in x'} u \cdot \beta\right)_{u \in U}\right).$$

Since C is a subset of a separable metric space (Klein and Thompson, 1984), axioms 1 and 6 imply the existence of a continuous utility function $v : C \to \mathbb{R}$ such that, for all $x, x' \in C$, $x \succsim x'$ if and only if $v(x) \geq v(x')$ (Debreu, 1954). We now claim that for all $x, x' \in C$, if $\min_{\beta \in x} u \cdot \beta \geq \min_{\beta \in x'} u \cdot \beta$ for all $u \in U$, then $x \succsim x'$. If the claim is correct, then we can define the aggregator $g : U(C) = U(X) \to R$ by, for all $(r_u)_{u \in U} \in U(C)$, $g((r_u)_{u \in U}) = v(x)$ for any $x \in C$ such that $(\min_{\beta \in x} u \cdot \beta)_{u \in U} = (r_u)_{u \in U}$. Moreover, it is clear that g is then weakly increasing, so the proof is complete.

To prove the claim, let $x \in C$ and define the sets

$$y = \{\gamma \in \mathbb{R}^B | \forall u \in U, u \cdot \gamma \geq \min_{\beta \in x} u \cdot \beta\},$$
$$z = \{\gamma \in \mathbb{R}^B | \exists \beta \in x, U \cdot \gamma \geq U \cdot \beta\}.$$

By (1) and axiom 5, it is sufficient to show that $z = y$. Define the set $k = \{\gamma \in \mathbb{R}^B | U \cdot \gamma \geq 0\}$. Then k is a closed convex cone and, more precisely, is the polar of the cone generated by $-U$. Clearly, $z = x + k$. Since x is a polytope and k is closed and convex, z is closed and convex (Rockafellar, 1970, theorem 20.3) and, hence, is equal to the intersection of all closed half-spaces containing it (Rockafellar, 1970, theorem 11.5). For all $u \in \mathbb{R}^B$, define the set $h_u = \{\gamma \in \mathbb{R}^B | u \cdot \gamma \geq \inf_{\gamma' \in z} u \cdot \gamma'\}$. Clearly, we have $z = \bigcap\{h_u | u \in \mathbb{R}^B\} = \bigcap\{h_u | u \in V\}$, where $V = \{u \in \mathbb{R}^B | \inf_{\gamma' \in z} u \cdot \gamma' > -\infty\}$. By definition, $-V$ is the barrier cone of z and, hence, is the polar of the recession cone of z (Rockafellar, 1970, corollary 14.2.1). But since x is a polytope, the recession cone of z is the recession cone of k and, since k is a cone, the recession cone of k is k. Thus $-V$ is the polar of k and, hence is the cone generated by $-U$. Since U is convex, this latter cone is $\bigcup\{\lambda U | \lambda \geq 0\}$ and, since $h_u = h_{\lambda u}$ for all $\lambda > 0$ by definition, we have $z = \bigcap\{h_u | u \in U\}$. Finally, since k is a cone and x is a polytope, $\inf_{\gamma' \in z} u \cdot \gamma' > -\infty$ implies $\inf_{\gamma' \in z} u \cdot \gamma' = \min_{\gamma' \in x} u \cdot \gamma'$, so the latter equality implies $z = y$.

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