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# Monetary Policy Transmission under Zero Interest Rates: An Extended Time-Varying Parameter Vector Autoregression Approach

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**Abstract**

This paper attempts to explore monetary policy transmission under zero interest rates by explicitly incorporating the zero lower bound (ZLB) of nominal interest rates into the time-varying parameter structural vector autoregression model with stochastic volatility (TVP-VAR-ZLB). Nominal interest rates are modeled as a censored variable with Tobit-type non-linearity and incorporated into the TVP-VAR framework. For estimation, an efficient Markov chain Monte Carlo (MCMC) method is constructed in the context of Bayesian inference. The model is applied to the Japanese macroeconomic data including the periods of the zero interest rates policy and the quantitative easing policy. The empirical results show that a dynamic relationship between monetary policy and macroeconomic variables is well detected through changes in medium-term interest rates, and not policy interest rates under the ZLB, although other macroeconomic dynamics are reasonably traced without considering the ZLB in an explicit manner.

**Keywords:** Monetary policy; Zero lower bound of nominal interest rates; Markov chain Monte Carlo; Time-varying parameter vector autoregression with stochastic volatility

**JEL classification:** C11, C15, E44, E52, E58

* Institute for Monetary and Economic Studies, Bank of Japan (Currently in the Personnel and Corporate Affairs Department <studying at Duke University>, E-mail: jouchi.nakajima@stat.duke.edu)

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# 1 Introduction

Central banks in major economies have reduced their policy interest rates to virtually zero levels in response to the recent financial and economic crisis. Under such circumstances, central banks have very little room for lowering short-term interest rates for further monetary easing. At the same time, it is concerned that the transmission mechanism of monetary policy is likely to be affected by the zero lower bound (ZLB) of nominal interest rates. To address that issue, this paper attempts to extend the time-varying parameter vector autoregression (TVP-VAR) model with stochastic volatility by explicitly incorporating the ZLB of nominal interest rates, thereby examining the possible changes in the dynamic relationship between monetary policy and macroeconomic variables.

A vector autoregression (VAR) is a standard econometric tool to be applied to a wide range of empirical analyses. Among those, a TVP-VAR model with stochastic volatility, proposed by Primiceri (2005), has become widely used in macroeconomic analyses. The TVP-VAR model with stochastic volatility enables us to take a very flexible specification of parameters, where the sources of time variation are both the coefficients and the variance covariance matrix of the innovations, to capture a possible time-varying behavior of the underlying structure in the concerned multivariate data. It can assess time variation in the VAR structure over time, which induces rich implications in the empirical investigation of economic data (see e.g., Nakajima (2011)). As shown by Primiceri (2005) and other related studies (e.g., Benati and Mumtaz (2005), Baumeister et al. (2008), and Nakajima et al. (2009)), the TVP-VAR model with stochastic volatility can incorporate a regime shift in the structure of the economy to some extent, because the parameters are assumed to follow a random walk process. In addition, it should be noted that stochastic volatility in disturbances plays an important role in improving the estimation precision for the sample period including the periods of extremely low interest rates.

Under zero-interest-rate circumstances, the transmission mechanism of monetary policy is unlikely to work through the interest rate channel in the same manner as normal times. Most VAR analyses including the TVP-VAR model with stochastic volatility, however, assume that nominal interest rates can take both positive and negative values for simplicity. While the TVP-VAR model allows us to quantify the transmission mechanism of monetary policy through the economy even if the data period includes some regime shifts, the model would require an explicit assumption in the case that the data include periods in which short-term nominal interest rates are close to zero, as pointed out by Nakajima et al. (2009). The asymmetry of fluctuations in nominal interest rates, taking only positive values in the real world, has the possibility to distort the estimation results under extremely low interest rates. Indeed, it is reasonable to consider that structural shocks to the interest rate equation vanish under such conditions, although the TVP-VAR models (or other reduced-form models) assume positive variances of structural shocks for all sample periods.

In the context of VAR analysis under zero interest rates in Japan, several studies investigate the effects of monetary policy during these periods. Kimura et al. (2003) estimate a VAR model with time-varying coefficients for the sample period of 1971 to 2002, and Fujiwara (2006) estimates a Markov switching VAR model for the period of 1985 to 2004, to examine the expansionary effect of the increase in the monetary base on the economy. Ugai (2007) discusses in his comprehensive survey on empirical studies, including the above mentioned articles, about the effects of the quantitative easing policy that the effects of expanding the monetary base, if any, are generally smaller than those stemming from the policy commitment in the period of zero interest rates.

As a more explicit treatment for the ZLB of policy interest rates, Kamada and Sugo (2006) estimate a private bank sector's financial intermediary function and use it as a monetary policy proxy that is not directly influenced by the ZLB in their VAR model instead of nominal short-term interest rate. Iwata and Wu (2006) (IW) incorporate the nominal interest rate lower bound into a structural VAR model and show insightful empirical results for Japanese macroeconomic data. IW treat an actually observed nominal interest rate as a censored variable with a certain lower bound, at which the nominal interest rate is regarded as essentially zero. This Tobit-type non-linear variable is incorporated into a constant parameter VAR system, which is estimated by the maximum likelihood method in their paper.[1]

This paper proposes to extend a TVP-VAR model with stochastic volatility by explicitly incorporating the ZLB of nominal interest rates (TVP-VAR-ZLB). That direction of the extension can be regarded as incorporating IW's non-negativity constraint on nominal interest rates into the TVP-VAR framework.[2] To estimate the TVP-VAR-ZLB model with stochastic volatility, an efficient Markov chain Monte Carlo (MCMC) method is constructed in the context of a Bayesian inference. Based on the algorithm of Primiceri (2005), several additional steps are added. Specifically, filtering and smoothing steps are extended to utilize the time-varying

[1] As a related study, Kitamura (2010) estimates an empirical dynamic stochastic general equilibrium (DSGE) model with IW's Tobit-type ZLB constraint using a particle filter approach.

[2] From another perspective, Franta (2011) examines the TVP-VAR model with sign restrictions to identify the monetary policy shocks in Japan for an explicit account of the ZLB constraint.

parameters in the presence of the ZLB of nominal interest rates.

As an empirical study, the TVP-VAR-ZLB model with stochastic volatility is applied to the Japanese macroeconomic data from 1977/Q1 to 2010/Q2, including the three episodes of monetary policy under the ZLB: the zero interest rates policy from 1999 to 2000, the quantitative easing policy from 2001 to 2006, and the policy responses to the recent global financial crisis from 2008 to the end of the sample period. The estimation results are also compared to the ones for the original TVP-VAR model with stochastic volatility that has no constraints on the nominal interest rates. The main conclusions from the empirical results are as follows: (i) the proposed model produces reasonable and remarkable time-varying impulse responses directly related to interest rates, and (ii) the assumption of the ZLB has a negligible effect on the results of the rest of the economy, as compared to the original TVP-VAR model with stochastic volatility. For the latter finding, in other words, the original TVP-VAR model with stochastic volatility works well enough to assess the time-varying relationship between macroeconomic variables, except for interest rates, even for the periods of the zero interest rates.

This paper is organized as follows. Section 2 proposes the TVP-VAR-ZLB model with stochastic volatility and the estimation procedure using the Bayesian inference and MCMC method. Section 3 provides the empirical results of the TVP-VAR-ZLB model with stochastic volatility for the Japanese macroeconomic variables and compares them to the results for the original TVP-VAR model with stochastic volatility.[3] Finally, Section 4 concludes.

# 2 Model and estimation methodology

## 2.1 TVP-VAR-ZLB model

Following IW's specification, a short-term nominal interest rate, denoted by $r_t$, is assumed to have a certain lower bound. It can be modeled as a censored variable with a latent variable $r_t^*$, namely,

$$r_t = \begin{cases} r_t^* & (\text{if } r_t^* > c) \\ c & (\text{if } r_t^* \leq c), \end{cases} \tag{1}$$

[3] As mentioned earlier, the incorporation of stochastic volatility into the TVP-VAR framework plays a crucial role in improving the accuracy and robustness of estimation results. To emphasize that point, we add the term of "with stochastic volatility" after the "TVP-VAR model" and "TVP-VAR-ZLB model." In the sections below, we use the "TVP-VAR model" and "TVP-VAR-ZLB model" to indicate those models with stochastic volatility for simplicity.

where $c \geq 0$ is a lower bound, and is assumed to be a small positive number at which nominal interest rates are regarded as essentially zero. That formation of a censored variable is a Tobit-type non-linearity, and IW referes to $r_t^*$ as the "implied interest rate." In the model, we observe the actual interest rate, which equals the implied interest rate when it stays above the lower bound.

We consider the VAR system that consists of three groups of variables: (i) macroeconomic variables such as inflation and output, (ii) nominal interest rates or implied interest rates, and (iii) some broad financial market variables such as medium- or long-term interest rates, stock market indices, and monetary base. As suggested by IW, it is crucially important to include those three groups of variables in identifying the VAR system. Define $y_t = (z_t', r_t, w_t')'$ and $y_t^* = (z_t', r_t^*, w_t')'$, where $z_t$ denotes a $k_z \times 1$ vector of macroeconomic variables, and $w_t$ denotes a $k_w \times 1$ vector of financial market variables ($k = k_z + k_w + 1$). The TVP-VAR-ZLB model is formulated as

$$y_t^* \;=\; B_{1t} y_{t-1} + \ldots + B_{st} y_{t-s} + A_t^{-1} \Sigma_t \, \varepsilon_t. \tag{2}$$

Note that the left-hand side of equation (2) contains the latent variable of the interest rate ($r_t^*$) which has no lower bound, and that the $y_t$'s in the right-hand side contain the actual level of the interest rate ($r_t$).

Stacking the elements in the rows of $B_i$s' to form $\beta$ (a $k(k+1)s \times 1$ vector), and defining $X_t = I_k \otimes (y_{t-1}', \ldots, y_{t-s}')$, where $\otimes$ denotes the Kronecker product, the model can be written as

$$y_t^* \;=\; X_t \beta_t + A_t^{-1} \Sigma_t \, \varepsilon_t, \tag{3}$$

where the coefficients $\beta_t$, and the parameters $A_t$, and $\Sigma_t$ are all time-varying.[4] There would be many ways to model the process for these time-varying parameters. Following Primiceri (2005), let $a_t = (a_{21}, a_{31}, a_{32}, a_{41}, \ldots, a_{k,k-1})'$ be a stacked vector of the lower-triangular elements in $A_t$ and $h_t = (h_{1t}, \ldots, h_{kt})'$ with $h_{jt} = \log \sigma_{jt}^2$, for $j = 1, \ldots, k$, and $t = s+1, \ldots, n$. We assume

[4]In a more general case, we can incorporate time-varying intercepts in equation (2) as in some literature on the TVP-VAR models. This case requires only the modifiction of defining $X_t := I_s \otimes (1, \mathbf{y}_{t-1}', \ldots, \mathbf{y}_{t-s}')$.

that the parameters in equation (3) follow a random walk process as follows:

$$
\begin{aligned}
\beta_{t+1} &= \beta_t + u_{\beta t}, \\
a_{t+1} &= a_t + u_{at}, \\
h_{t+1} &= h_t + u_{ht},
\end{aligned}
\qquad
\begin{pmatrix} \varepsilon_t \\ u_{\beta t} \\ u_{at} \\ u_{ht} \end{pmatrix} \sim N \left( 0, \begin{pmatrix} I & O & O & O \\ O & \Sigma_\beta & O & O \\ O & O & \Sigma_a & O \\ O & O & O & \Sigma_h \end{pmatrix} \right),
$$

for $t = s+1, \dots, n$, where $\beta_{s+1} \sim N(\mu_{\beta_0}, \Sigma_{\beta_0})$, $a_{s+1} \sim N(\mu_{a_0}, \Sigma_{a_0})$, and $h_{s+1} \sim N(\mu_{h_0}, \Sigma_{h_0})$.

## 2.2 Identification

The identification is assumed as the matrix $A^{-1}$ of a block lower triangular given by

$$
A_t^{-1} = \begin{pmatrix} A_{zz,t} & \mathbf{0} & O \\ \mathbf{a}'_{zr,t} & 1 & \mathbf{0}' \\ A_{zw,t} & \mathbf{a}_{rw,t} & A_{ww,t} \end{pmatrix},
$$

where $A_{zz,t}$ $(k_z \times k_z)$ and $A_{ww,t}$ $(k_w \times k_w)$ are the lower triangular matrices with diagonal elements equal to one, $A_{zw,t}$ is a $k_w \times k_z$ matrix, and $\mathbf{a}_{zr,t}$ and $\mathbf{a}_{rw,t}$ are $k_z \times 1$ and $k_w \times 1$ vectors, respectively.

All parameters are assumed to be time-varying except for the following assumption. If $r^*_{t-1} \le c$, *i.e.*, the nominal interest rate hits the lower bound, we assume that the simultaneous relation with the interest rate shock diminishes; namely,

$$
\mathbf{a}_{rw,t} = \mathbf{0}.
$$

Along this line of thought, for the periods when the nominal interest rate hits the lower bound, the structural shock to the interest rate should vanish. Thus, the $(k_z + 1)$-th element of $h_t$, which corresponds to the log-variance of the interest rate shock, is set equal to a very small value or zero. In addition, we assume that the innovations to all elements in the $(k_z + 1)$-th row of $B_{it}$, for $i = 1, \dots, s$ are all equal to zero for those periods.[5]

In that regard, several remarks are required for the specification of the TVP-VAR-ZLB

[5]In equation (2), $r^*_t$ does not directly depend on the lags of $r^*_t$ itself. However, because $r_t = r^*_t$ when $r^*_t > c$, we assume an interest rate smoothing induced by the lags of VAR system. When $r^*_t \le c$, all the coefficients related to $r^*_t$ is assumed to be equal to zero, therefore we do not follow the implicit movements of $r^*_t$ for the periods when the nominal interest rate hits the ZLB. See Ichiue and Ueno (2006, 2007) and Kitamura (2010) for the estimated trajectories of implied interest rates under the ZLB discussed using other models.

model. First, the assumption of the lower-triangular matrix for $A_t$ is the recursive identification for the VAR system. This specification is simple and widely used, although an estimation of structural models often needs more complicated identification to draw the structural implication of the economy as pointed out by Christiano et al. (1999) and other studies. In the TVP-VAR-ZLB framework, a standard estimation procedure is applicable for the model with non-recursive identification by a slight modification of the variables through the MCMC algorithm.

Second, the parameters are not assumed to follow a stationary process but the random walk process. As discussed by Nakajima (2011), the random walk assumption allows both temporary and permanent shifts in the parameters. The drifting parameter is meant to capture a possible non-linearity, such as a gradual change or a structural break. As a more realistic point, because the TVP-VAR model has considerably many parameters to estimate, we had better decrease the number of parameters by assuming the random walk process for the innovation of parameters. Most studies that use the TVP-VAR model assume the random walk process for time-varying parameters.

Third, the shocks to the innovations of the time-varying parameters are assumed to be uncorrelated among the parameters $\beta_t$, $a_t$, and $h_t$. That makes the estimation procedure easier and simpler. Here, we further assume that $\Sigma_\beta$, $\Sigma_a$, and $\Sigma_h$ are all diagonal matrices. The specification of dynamics here is adequate enough to permit the parameters to vary even if the shocks in the processes driving the time-varying parameters are uncorrelated.[6]

### 2.3 Estimation methodology

The MCMC methods have become popular in the empirical economic literature. Many works on empirical macroeconomics in recent years have been developed using the MCMC methods. The MCMC methods are considered in the context of Bayesian inference, and the goal is to assess the joint posterior distribution of the parameters of interest under certain prior probability densities that the researchers set in advance. Given data, we repeatedly sample a Markov chain whose invariant (stationary) distribution is the posterior distribution (see e.g., Chib and Greenberg (1996), and Chib (2001)).

As discussed by Primiceri (2005) and Nakajima (2011), the MCMC method plays an important role in estimating the TVP-VAR model because it has many parameters and state

[6] It shoud be noted that the estimation results do not differ significantly, regardless of asussming the correlation of $\beta_t$ over time. That is because the TVP-VAR model with stochastic volatility is flexible enough to capture gradual changes in the underlying structuter of the economy, as pointed out by Nakajima (2011).

variables in both linear and non-linear manner. Regarding the coefficients as state variables, the TVP-VAR model forms the state space model given the other parameters. The state space model has been extensively studied in many fields (see e.g., Harvey (1993), and Durbin and Koopman (2002b) for econometric issues). To estimate the state space model, several methods have been developed; for instance, the coefficients are easily estimated using a simple Kalman filter for a linear Gaussian state space model. However, because the TVP-VAR model has stochastic volatility in disturbances and the model forms a non-linear state space model, the maximum likelihood estimation requires heavy computation to repeat the filtering many times in evaluating the likelihood function for each set of parameters until we reach the maximum. Thus, alternatively, we take a Bayesian approach using the MCMC method for a precise and efficient estimation of the TVP-VAR model. Moreover, in this paper, the TVP-VAR-ZLB model includes the additional constraint of nominal interest rate lower bound. The MCMC estimation scheme plays a larger role in estimating the complicated structure of the TVP-VAR-ZLB model.

Let $y = \{y_t\}_{t=1}^n$, $r^* = \{r_t^*\}_{t=s+1}^n$ and $\omega = (\Sigma_\beta, \Sigma_a, \Sigma_h)$. We set the prior probability density as $\pi(\omega)$ for $\omega$. Given the data $y$, we draw a sample from the posterior distribution $\pi(\beta, a, h, r^*, \omega|y)$. The MCMC algorithm is proposed as follows:

1. Initialize $\beta, a, h, r^*$ and $\omega$.
2. Sample $\beta \,|\, a, h, r^*, \Sigma_\beta, y$.
3. Sample $\Sigma_\beta \,|\, \beta$.
4. Sample $a \,|\, \beta, h, r^*, \Sigma_a, y$.
5. Sample $\Sigma_a \,|\, a$.
6. Sample $h \,|\, \beta, a, r^*, \Sigma_h, y$.
7. Sample $\Sigma_h \,|\, h$.
8. Sample $r^* \,|\, \beta, a, h, y$.
9. Go to 2.

Given $r_t^*$, steps 2–7 are mainly the algorithms similar to the ones for the original TVP-VAR model developed by Primiceri (2005). However, the non-negative constraint of the ZLB requires several extensions to these algorithms and an additional step of sampling the implied interest rate $r_t^*$ from its conditional posterior distribution (Step 8). The details of the procedure are illustrated in the Appendix.

# 3 Empirical results for the Japanese economy

## 3.1 Data and settings

This section provides the empirical results of the TVP-VAR-ZLB model for the Japanese macroeconomic variables.[7] A four-variable TVP-VAR-ZLB model is estimated for the quarterly data from 1977/Q1 to 2010/Q2. The variable set $(p_t, x_t, r_t, b_t)$ is examined; $p_t$ is the inflation rate, $x_t$ is the output, $r_t$ denotes the short-term interest rates, and $b_t$ denotes the medium-term interest rates.[8] In the above notation, $z_t = (p_t, x_t)'$ and $w_t = b_t$.

Regarding the ZLB of nominal interest rates, we set $c$ in equation (1), *i.e.*, the lower bound of nominal interest rates, at 50 basis points, following IW. The time series of $r_t$ and the lower bound are plotted in Figure 1. The short-term interest rates in our dataset include the two periods when short-term nominal interest rates stay below 50 basis points (ZLB periods): the periods from 1995/Q4 to 2007/Q1 and from 2008/Q2 to the end of sample period (2010/Q2).[9] The short-term nominal interest rate went below 50 basis points in 1995/Q4 and below 10 basis points in 1999/Q2. The Bank of Japan introduced the zero interest rates policy in February 1999 and terminated it in August 2000. The quantitative easing policy was then implemented from March 2001 to March 2006. Under the recent financial crisis, the short-term nominal interest rate in this quarterly data went below 50 basis points again in 2008/Q2. The time

[7] For the applications of the VAR models using the Japanese macroeconomic data, see e.g., Miyao (2000, 2002), Kimura et al. (2003), Fujiwara (2006), Inoue and Okimoto (2008), and Nakajima et al. (2009).

[8] The inflation rate is taken from the consumer price index (CPI; general excluding fresh food, log-difference, the effects of the increase in the consumption tax removed, and seasonally adjusted). The output gap is a series of the differential between actual GDP and potential GDP, calculated by the Bank of Japan. The short-term interest rates are the overnight call rate (weighted average rates for each days trading). Except for the output gap, the monthly data is arranged to a quarterly base. The medium-term interest rates are a yield of the 5-year Japanese government bond. Up to 1988/Q1, the 5-year interest-bearing bank debenture, and from 1988/Q2, a series of the generic index of Bloomberg is used. For the medium-term interest rates, the (log-scale) difference of the original series from the trend of HP filter, that is, an interest rate gap from the trend, is computed for the variable of the estimation.

[9] From September 1995 to September 1998, the Bank of Japan's money market operations were directed to encourage the uncollateralized overnight call rate to move on average slightly below the official discount rate, which was set at 50 basis points at that time. After termininating the quantitative easing policy, the policy target rate was raised to 50 basis points in February 2007, and market-transacted rates, *i.e.*, weighted average rates for the overnight call transactions, remained at the levels higher than 50 basis points in the four quarters; from 2007/Q2 to 2008/Q1.

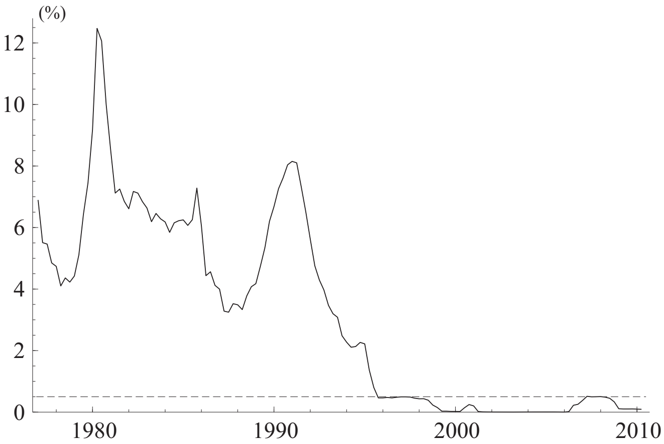



Figure 1: The short-term nominal interest rate (call rate, $r_t$, solid line) from 1977/1Q to 2010/2Q and the lower bound ($c$) at 50 basis points (dashed line).

series of the other three variables are plotted in Figure 2.

The autoregressive lag length is set as four,[10] and the following priors are assumed for the $i$-th diagonals of the covariance matrices:

$$(\Sigma_\beta)_i^{-2} \sim \text{Gamma}(40, 0.02),\ (\Sigma_a)_i^{-2} \sim \text{Gamma}(4, 0.02),\ (\Sigma_h)_i^{-2} \sim \text{Gamma}(4, 0.02).$$

For the initial state of the time-varying parameter, rather flat priors are set: $\mu_{\beta_0} = \mu_{a_0} = \mu_{h_0} = 0$ and $\Sigma_{\beta_0} = \Sigma_{a_0} = \Sigma_{h_0} = 10 \times I$. In comparison, the original TVP-VAR model is estimated with the same settings.

To compute the posterior estimates, we generate $M = 100{,}000$ draws after the initial 50,000 draws are discarded. To check the convergence of the MCMC algorithm, the convergence diagnostics (CD) of Geweke (1992) are computed. In the estimated results, the null hypothesis of the convergence to the posterior distribution is not rejected for the parameters at the 5% significance level based on the CD statistics, which assures that the iteration size is sufficient for the TVP-VAR-ZLB model in our settings.

As in IW, the general impulse response for the nonlinear VAR model (see e.g., Koop et al. (1996)) are computed in the case of the TVP-VAR-ZLB model due to the non-linearity of

[10] In a Bayesian inference, the marginal likelihood is used as a measure of the model fit. In our model, the marginal likelihood is estimated for up to six autoregressive lag lengths and the number of lag lenghts is determined based on the highest marginal likelihood. The computation of the marginal likelihood for the TVP-VAR model is explained by Nakajima et al. (2009). The computational results are generated using Ox version 4.02 (Doornik (2006)).

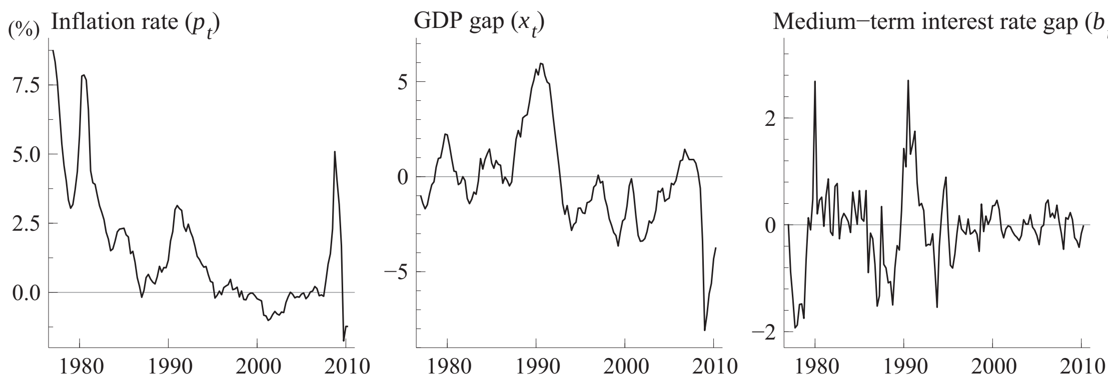



Figure 2: Inflation rate ($p_t$), output gap ($x_t$) and medium-term interest rate gap ($b_t$) from 1977/Q1 to 2010/Q2.

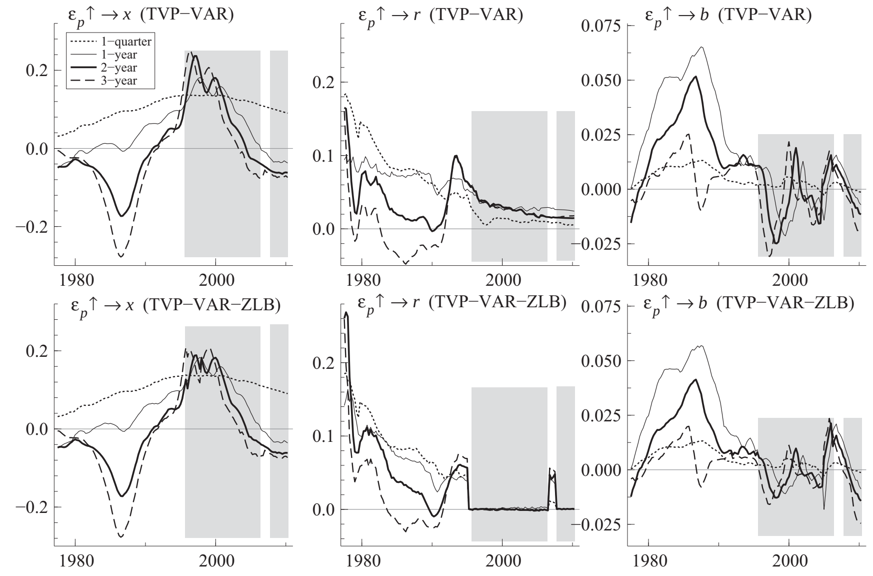
the observation equation. The response is obtained from the difference between the simulated forecasts with a current shock of a unit size and those without a shock. As for the initial condition, we divide the data into two subsets: (i) $r_t > c$ and (ii) $r_t = c$; the initial condition is randomly chosen from the same group as the current period on which the impulse response is drawn. The simulation is repeated 500 times for each time period and the impulse response is obtained as the average of the simulated sample.

## 3.2 Dynamic relationship between macroeconomic variables

Figure 3 shows the impulse responses to the (a) inflation $p_t$, (b) output $x_t$, (c) short-term interest rate $r_t$, and (d) medium-term interest rate $b_t$ shocks for the TVP-VAR and TVP-VAR-ZLB models. Based on the estimated TVP-VAR and TVP-VAR-ZLB models, the figure shows the impulse responses in the one-, two-, and three-year horizons as time series, thereby enabling us to examine the structural changes in the relationship between macroeconomic variables. Overall, the impulse responses of macroeconomic variables show significant variation over time, suggesting the effectiveness of applying the TVP-VAR model. In addition, the impulse responses to a positive short-term interest rates shock, estimated by the TVP-VAR-ZLB model, stay close to zero during the ZLB period, which corresponds to the shaded period in the figure.[11] Such estimates, however, are not obtained by the TVP-VAR model. That suggests the importance of explicitly incorporating the ZLB of nominal interest rates under

[11] Between two ZLB periods, we observe spikes of the impulse response. Because the short-term nominal interest rates are above the lower bound only in four quarters in this period, the estimated time-varying parameters related to the short-term nominal interest rates as well as the corresponding impulse responses are considered to have much uncertainty.

(a) Response to $p_t$ shock




(b) Response to $x_t$ shock

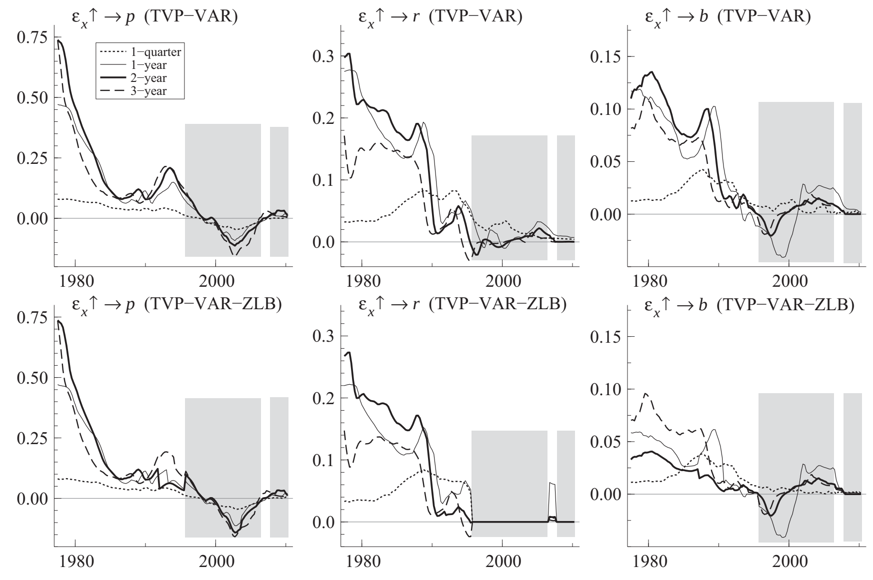
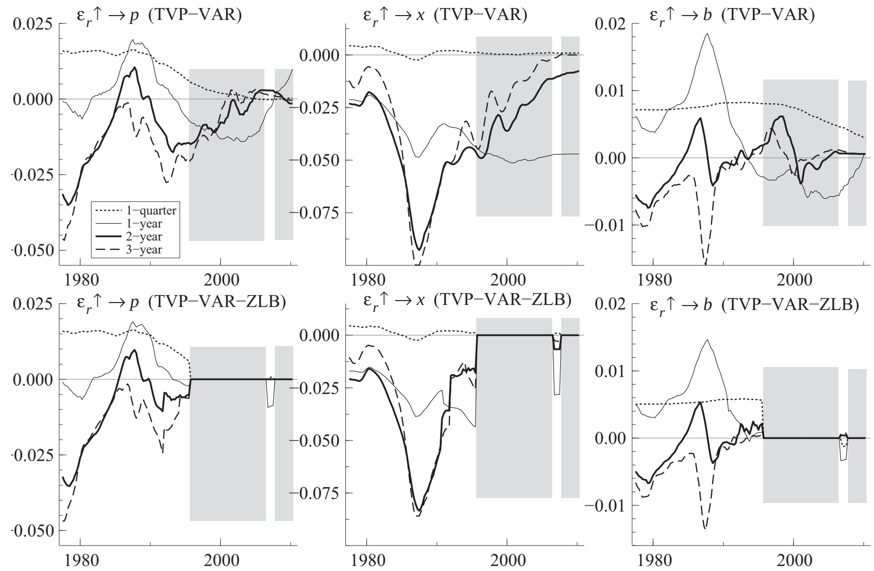



Figure 3: Time-varying impulse responses to the (a) inflation $p_t$ and (b) output $x_t$ shocks for the TVP-VAR and TVP-VAR-ZLB models. The one-year (solid), two-year (dashed), and three-year (dotted) horizons are plotted. The shaded periods refer to the period when nominal interest rates hit the ZLB in the model.

(c) Response to $r_t$ shock




(d) Response to $b_t$ shock

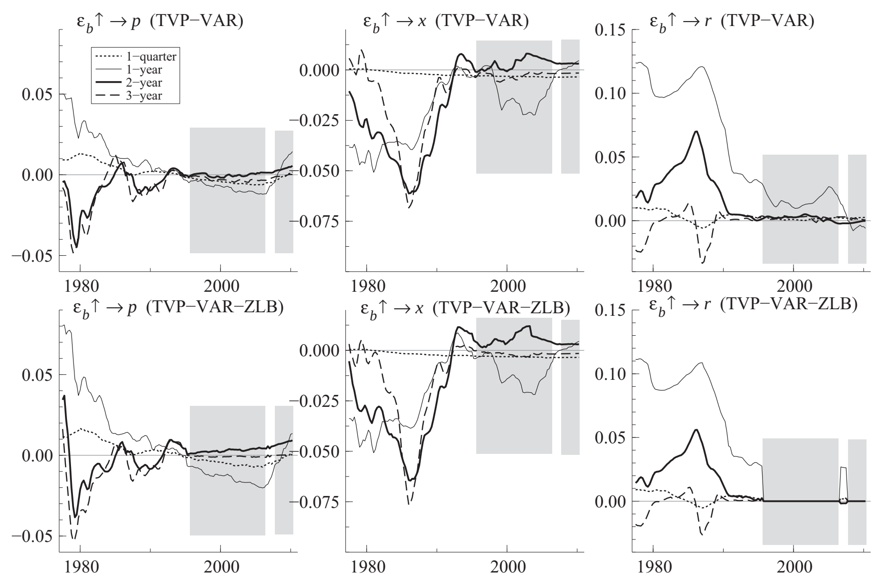



Figure 3: (continued) Time-varying impulse responses to the (c) nominal interest rate $r_t$ and (d) medium-term bond interest rate $b_t$ shocks for the TVP-VAR and TVP-VAR-ZLB models. The one-year (solid), two-year (dashed), and three-year (dotted) horizons are plotted. The shaded periods refer to the period when the nominal interest rates hit the ZLB in the model.

conditions of extremely low interest rates, although the impulse responses that are not directly related to short-term interest rates exhibit little difference between the two models. We see each panel in detail below.

First, looking at the responses to a positive inflation ($p_t$) shock in panel (a) of the figure, the impulse responses of output ($\varepsilon_p \to x$) are estimated as negative in the two- and three-year horizons in the 1980s by both models, and as positive in the 1990s. During the ZLB period, however, the impulse responses decline toward zero and remain around that level thereafter. Regarding the impulse responses of short-term interest rates to a positive inflation shock ($\varepsilon_p \to r$), the estimates from the TVP-VAR model stay positive for the ZLB period, although those from the TVP-VAR-ZLB model remain zero. As discussed above, it is reasonable to consider that the impulse responses of short-term interest rates diminish under the ZLB constraint of nominal interest rates, and therefore the TVP-VAR-ZLB model yields quite reasonable results. With regard to the impulse responses of medium-term interest rates ($\varepsilon_p \to b$), they are estimated as positive in one- and two-year horizons in the 1980s, but they decline in the 1990s and fluctuate around zero during the ZLB period. The fluctuations in the size and sign of the impulse responses are larger in the TVP-VAR-ZLB model than in the TVP-VAR model.

Second, panel (b) shows the impulse responses to a positive output ($x_t$) shock. The positive impulse responses of inflation ($\varepsilon_x \to p$) continue to decline in the 1980s and turn negative in the 2000s on both models. It is remarkable that the impulse responses in all horizons remain almost the same over time, suggesting that the initial impacts on inflation up to one year persist for at least three years. The impulse responses of short-term interest rates ($\varepsilon_x \to r$) are estimated as positive in the 1980s, but they decline rapidly in the late-1980s. With regard to the response of medium-term interest rates ($\varepsilon_x \to b$), they are estimated as positive, but exhibit a downward trend from the early-1980s to the 1990s. The estimated impulse responses from the TVP-VAR-ZLB model seem smaller than those from the TVP-VAR model, especially in the 1980s.

Third, the impulse responses to a positive short-term interest rates shock ($r_t$) are reported in panel (c). Looking at the estimates from the TVP-VAR-ZLB model, the impulse responses stay at zero during the ZLB period. Note, however, that there remain slight differences between the impulse responses of short-term interest rates and those of others, *i.e.*, output, inflation, and medium-term interest rates. In fact, although the impulse responses of short-term interest rates are estimated as statistically zero for the ZLB periods, quite a few of the simulated impulse responses show upward deviations from the ZLB. In contrast, the impulse responses of the other variables are exactly zero for those periods from the assumption in the model.[12]

Fourth, panel (d) shows the impulse responses to a positive medium-term interest rates shock ($b_t$). Looking at the impulse responses of output ($\varepsilon_b \to x$) and inflation ($\varepsilon_b \to p$), both responses significantly differ during the ZLB period from those at normal times. During the ZLB period, a positive medium-term interest rates shock produces negative impacts on both inflation and output in the one-year horizon. In other words, a kind of price puzzle in terms of inflation responses to the changes in medium-term interest rates disappears during the ZLB period. At the same time, output responses become more volatile during the ZLB, as can be seen as the fact that negative responses in the one-year horizon turn positive in the two-year horizon, and then return to approximately zero in the three-year horizon. The estimates from the TVP-VAR-ZLB model are clearer than those from the TVP-VAR model.

### 3.3 Discussion on monetary policy transmission under the ZLB

The above estimation results of the dynamic relationships between macroeconomic variables show both a time-varying nature and some differences between the estimates from the TVP-VAR model and the TVP-VAR-ZLB model. Based on those observations, we explore some implications on monetary policy transmission under the ZLB of nominal interest rates.

The estimation results suggest that the dynamic relation between inflation and output has considerable time variation through the sample period. The estimates of this variation are almost the same between the TVP-VAR and TVP-VAR-ZLB models, which implies that the estimated dynamics are mostly robust. For the response of the output to a inflation shock, the turnovers from negative to positive responses around the 1990s and the early 2000s indicate that a positive inflation shock does not offset the output growth. Because the responses diminish towards zero during the latter half of the ZLB period, the effect of inflation on output tends to be uncertain. For the responses of inflation to a output shock, the responses are negative after the late-1990s and stay around zero even in the mid-2000s when the output gap becomes positive, which is primarily consistent with the flattening of the Phillips curve. Such evidence implies that the weakening of the relations between economic activity and prices during the ZLB period.

By contrast, the estimated dynamic relations including short-term and medium-term interest rates are different between the TVP-VAR and TVP-VAR-ZLB models. The responses

[12] Regarding the possibility of asymmetric behavior between the positive and negative shocks of the economic variable on the system of the TVP-VAR-ZLB model, a robustness check to compare the positive and negative shocks reveals no relevant effect of the asymmetry.

related to short-term nominal interest rates are different by construction. The impulse responses related to medium-term interest rates are basically similar, although the sizes of the responses are mostly larger in the TVP-VAR-ZLB model. This evidence suggests a possibility that the effect of the policy commitment during the zero interest rate period can be regarded as the transmission mechanism through the medium-term interest rates when the ZLB is explicitly considered in the model. In other words, when we estimate the model without the ZLB constraint, the commitment effect may be incidentally estimated as a part of the transmission mechanism through the short-term interest rates.

## 4 Concluding remarks

This paper proposed the TVP-VAR-ZLB model, the TVP-VAR model with the ZLB of nominal interest rates. The non-negativity constraint of interest rates was modeled in Tobit-type non-linearity and successfully incorporated into the TVP-VAR framework. The efficient MCMC method was constructed for the TVP-VAR-ZLB model by extending the algorithm of Primiceri (2005). In application, an empirical investigation of the TVP-VAR-ZLB model using the Japanese macroeconomic data was provided. Based on the estimation results, the main conclusions included: (i) the proposed model produces reasonable and remarkable time-varying impulse responses directly related to short-term nominal interest rates, and (ii) the assumption of the ZLB has a negligible effect on the results of the rest of the economy, as compared to the original TVP-VAR model. From the latter finding, we concluded that the original TVP-VAR model worked well enough to assess the time-varying relations between macroeconomic variables, which are not directly related to short-term interest rates, even during the ZLB periods.

Beyond the TVP-VAR-ZLB framework developed in this paper, several empirical and methodological issues remain as future works. Nominal short-term interest rates themselves are modeled as the explicit indicator for the ZLB period based on the explicitly specified lower bound in the current study, although we can consider another indicator for the ZLB periods, such as other interest rates or other macroeconomic variables. Some variables may indicate that the beginning of the ZLB period would be some periods before the nominal short-term interest rates fall below the lower bound. Moreover, the lower bound of the indicators can be assumed to be unknown and estimated. Even in these challenging extensions, the TVP-VAR-ZLB model developed in this paper is expected to provide basic foundation of model assessment in macroeconomic analyses including the ZLB constraint.

## Appendix. MCMC algorithm for the TVP-VAR-ZLB model

This appendix illustrates the sequences of the conditional posterior distribution and the resulting MCMC algorithm for the TVP-VAR-ZLB model.

**Sample $\beta$**

To sample $\beta$ from the conditional posterior distribution $\pi(\beta|a, h, r^*, \Sigma_\beta, y)$, we write the model in the state space form as

$$\begin{aligned} y_t^* &= X_t\beta_t + A_t^{-1}\Sigma_t\varepsilon_t, \qquad t = s+1, \ldots, n, \\ \beta_{t+1} &= \beta_t + u_{\beta t}, \qquad t = s, \ldots, n-1, \end{aligned}$$

where $\beta_s = \mu_{\beta_0}$ and $u_{\beta s} \sim N(0, \Sigma_{\beta_0})$. Following Primiceri (2005), we sample $\beta$ from the joint posterior distribution $\pi(\beta_{s+1}, \ldots, \beta_n|a, h, r^*, \Sigma_\beta, y)$, using the simulation smoother (de Jong and Shephard (1995), Durbin and Koopman (2002a)).

To illustrate the simulation smoother, consider the state space model

$$\begin{aligned} Y_t &= Z_t\alpha_t + G_tu_t, \quad t = s+1, \ldots, n, \\ \alpha_{t+1} &= T_t\alpha_t + H_tu_t, \quad t = s, \ldots, n-1, \end{aligned}$$

where $u_t \sim N(0, I)$, $G_tH_t' = O$, $Y_t$ is an observation and $\alpha_t$ is a state variable. The simulation smoother draws $\eta = (\eta_s, \ldots, \eta_{n-1}) \sim \pi(\eta|y, \theta)$ where $\eta_t = H_tu_t$ for $t = s, \ldots, n-1$, and $\theta$ denotes the rest of the parameters in the model. We run the Kalman filter:

$$\begin{aligned} e_t &= Y_t - Z_ta_t, \quad D_t = Z_tP_tZ_t' + G_tG_t', \quad K_t = T_tP_tZ_t'D_t^{-1}, \\ L_t &= T_t - K_tZ_t, \quad a_{t+1} = T_ta_t + K_te_t, \quad P_{t+1} = T_tP_tL_t' + H_tH_t', \end{aligned}$$

for $t = s+1, \ldots, n$, where $a_{s+1} = T_s\alpha_s$ and $P_{s+1} = H_sH_s'$. Then, letting $\Lambda_t = H_tH_t'$, we run the simulation smoother:

$$\begin{aligned} C_t &= \Lambda_t - \Lambda_tU_t\Lambda_t, \quad \eta_t = \Lambda_tr_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, C_t), \quad V_t = \Lambda_tU_tL_t, \\ r_{t-1} &= Z_t'D_t^{-1}e_t + L_t'r_t - V_t'C_t^{-1}\varepsilon_t, \quad U_{t-1} = Z_t'D_t^{-1}Z_t + L_t'U_tL_t + V_t'C_t^{-1}V_t, \end{aligned}$$

for $t = n, n-1, \ldots, s+1$, with $r_n = U_n = 0$. Finally, we can draw $\eta_s = \Lambda_sr_s + \varepsilon_s$ and

$\varepsilon_s \sim N(0, C_s)$ with $C_s = \Lambda_s - \Lambda_s U_s \Lambda_s$. We construct the sample of $\{\alpha_t\}_{t=s+1}^{n}$ via the state equation using $\{\eta_t\}_{t=s}^{n-1}$ drawn through the simulation smoother.

Now, for sampling $\beta$ in the original TVP-VAR model, we coordinate the parameters as $Y_t = y_t^*$, $Z_t = X_t$, $T_t = I$, $G_t = (A_t^{-1}\Sigma_t, O)$, $H_t = (O, \Sigma_\beta^{1/2})$, for $t = s+1, \dots, n$, $T_s\alpha_s = \mu_{\beta_0}$, and $H_s = (O, \Sigma_{\beta_0}^{1/2})$.

Moreover, for the TVP-VAR-ZLB model, we arrange the matrix $H_t$ in order to satisfy the identification condition stated in Section 2.2 for period $t$ when the nominal interest rate hits the lower bound. Let $t^*$ denote the period such that $r_t = c$ and $k_\beta^*$ denotes the set of indexes in $\beta_t$ that correspond to all elements in the $(k_z+1)$-th row of $B_{it}$ for $i = 1, \dots, s$. Because the innovations to the rows $k_\beta^*$ of $\beta_{t^*}$ are restricted as equal to zero, we compute the $k_\beta^*$-th diagonal elements of $H_{t^*-1}$ as equal to zero.[13]

**Sample $a$**

To sample $a$ from the conditional posterior distribution, we consider the state space formulation below:

$$
\begin{aligned}
\hat{y}_t &= \hat{X}_t a_t + \Sigma_t \varepsilon_t, \qquad t = s+1, \dots, n, \\
a_{t+1} &= a_t + u_{at}, \qquad t = s, \dots, n-1,
\end{aligned}
$$

where $a_s = \mu_{a_0}$, $u_{as} \sim N(0, \Sigma_{a0})$, $\hat{y}_t = y_t^* - X_t\beta_t$, and

$$
\hat{X}_t = \begin{pmatrix}
0 & \cdots & & & & & & 0 \\
-\hat{y}_{1t} & 0 & 0 & \cdots & & & & \vdots \\
0 & -\hat{y}_{1t} & -\hat{y}_{2t} & 0 & \cdots & & & \\
0 & 0 & 0 & -\hat{y}_{1t} & \cdots & & & \\
\vdots & & & & \ddots & 0 & \cdots & 0 \\
0 & \cdots & & & 0 & -\hat{y}_{1t} & \cdots & -\hat{y}_{k-1,t}
\end{pmatrix},
$$

for $t = s+1, \dots, n$. We run the simulation smoother for sampling $a$, substituting the correspondences to the variables in the simulation smoother: $Y_t = \hat{y}_t$, $Z_t = \hat{X}_t$, $G_t = (\Sigma_t, O)$,

[13]Intuitively, each elements of $\beta_t$ runs in a random walk process for the periods in which the interest rates are above the lower bound, although when the interest rates hit the lower bound, those rows of $\beta_t$ stick to the most recent value. Moreover, when the interest rates rise above the lower bound, those rows of $\beta_t$ jump to the new level of series. For this jump, it is appropriate to set a rather diffused prior for those diagonal elements of $H_t$, because it is reasonable to allow the elements to move more than other periods, by enough room for the jump.

$T_t = I_{k_a}$, $H_t = (O,\, \Sigma_a^{1/2})$, and $H_0 = (O,\, \Sigma_{a_0}^{1/2})$, where $k_a$ is the number of rows of $a_t$. In addition, the identification constraints require to compute the relevant elements of $T_t$ and $H_t$ as equal to zero for the period $t^*$, as when sampling $\beta$.

**Sample $h$**

As for the stochastic volatility $h$, we make the inference for $\{h_{jt}\}_{t=s+1}^{n}$ separately for $j$ $(= 1, \ldots, k)$, because we assume that $\Sigma_h$ and $\Sigma_{h_0}$ are diagonal matrices. Let $y_{it}^*$ denote the $i$-th element of $A_t \hat{y}_t$. Then, we can write

$$\begin{aligned} y_{it}^* &= \exp(h_{it}/2)\varepsilon_{it}, \quad t = s+1, \ldots, n, \\ h_{i,t+1} &= h_{it} + \eta_{it}, \quad t = s, \ldots, n-1, \\ \begin{pmatrix} \varepsilon_{it} \\ \eta_{it} \end{pmatrix} &\sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & v_i^2 \end{pmatrix}\right), \end{aligned}$$

where $\eta_{is} \sim N(0,\, v_{i0}^2)$, $v_i^2$ and $v_{i0}^2$ are the $i$-th diagonal elements of $\Sigma_h$ and $\Sigma_{h_0}$, respectively, and $\eta_{it}$ is the $i$-th element of $u_{ht}$. We sample $(h_{i,s+1}, \ldots, h_{in})$ using the multi-move sampler (see Nakajima (2011)). The identification conditions require that the volatility $h_{it}$ stays zero for the zero interest rate periods.

**Sample $\omega$**

Sampling the diagonal elements of $\Sigma_\beta$, $\Sigma_a$, and $\Sigma_h$ is quite simple. When the priors are set as the inverse gamma distribution, the conditional posterior distribution of the diagonal elements of these matrices also forms the inverse gamma distribution.

**Sample $r^*$**

For the period $t$ such that $r_t = c$, we sample $r_t^*$ from its posterior distribution given by

$$\pi(r_t^* | \beta, a, h, y) \,\propto\, \exp\left(-\frac{1}{2}\hat{y}_t' \Omega_t^{-1} \hat{y}_t\right) I(r_t^* < c),$$

where $\Omega_t = A_t^{-1}\Sigma_t\Sigma_t' A_t'^{-1}$. Write the elements as $\hat{y}_t = (\hat{z}_t', \hat{r}_t^*, \hat{w}_t')'$ and

$$\Omega_t^{-1} = \begin{pmatrix} Q_{zz,t} & \mathbf{q}_{zr,t} & Q_{zw,t} \\ \mathbf{q}_{zr,t}' & q_{rr,t} & \mathbf{q}_{rw,t}' \\ Q_{zw,t}' & \mathbf{q}_{rw,t} & Q_{ww,t} \end{pmatrix},$$

Then, the posterior density can be rewritten as

$$\pi(r_t^*|\beta, a, h, y) \propto \exp\left\{ -\frac{q_{rr,t}}{2}\left(\hat{r}_t^* + \frac{\mathbf{q}_{zr,t}'\hat{z}_t + \mathbf{q}_{rw,t}\hat{w}_t}{q_{rr,t}}\right)^2 \right\} I(r_t^* < c).$$

The posterior draw is obtained by $r_t^* \,|\, \beta, a, h, \mathbf{y} \sim TN(\hat{\mu}_t,\, \hat{\sigma}_t^2)_{[\infty,\, c]}$, where

$$\hat{\mu}_t = \tilde{r}_t - \frac{\mathbf{q}_{zr,t}'\hat{z}_t + \mathbf{q}_{rw,t}\hat{w}_t}{q_{rr,t}}, \qquad \hat{\sigma}_t^2 = q_{rr,t}^{-1}.$$

and $\tilde{r}_t$ denotes the $(k_z+1)$-th element of $X_t\beta_t$.

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