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Joint Customer Data Acquisition and Sharing Among Rivals*

Nicola Jentzsch† Geza Sapi‡ Irina Suleymanova§

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Abstract

It is increasingly observable that in different industries competitors jointly acquire and share customer data. We propose a modified Hotelling model with two-dimensional consumer heterogeneity to analyze the incentives for such agreements and their welfare implications. In our model the incentives of firms for data acquisition and sharing depend on the willingness of consumers to switch brands. Firms jointly collect data on transportation cost parameters when consumers are relatively immobile between brands. However, the firms are unlikely to cooperatively acquire such data, when consumers are relatively mobile. Incentives to share information depend on the portfolio of data firms hold and consumer mobility. Data sharing arises with relatively mobile and immobile consumers - it is neutral for consumers in the former case, but reduces consumer surplus in the latter. Competition authorities ought to scrutinize such cooperation agreements on a case-by-case basis and devote special attention to consumer switching behavior.

JEL-Classification: D43; L13; L15; O30

Keywords: Information Sharing, Data Acquisition, Price Discrimination

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1 Introduction

Recent advances in information technologies allow firms to collect, analyze and share detailed information about customers and to use this information for targeted offers. The use of customer databases for price discrimination attracted the attention of regulators and privacy advocates alike. Two types of cooperation based on customer data are particularly wide-spread: i) cooperative data collection, and ii) information sharing.

There are several industries, where rivals cooperate in obtaining customer data. For example, national medical associations often provide uniform software solutions to members in order to manage patient medical records, which essentially standardizes customer data doctors acquire. Another example of cooperative data acquisition is the case of U.S. colleges, where education institutions cooperate in the College Board to jointly collect information on students for awarding institutional aid funds.

Beyond cooperation on data acquisition, the possibility to share customer data between competitors is also widely discussed in many industries. Airlines exchange detailed data on personal characteristics and travel details of passengers and target promotions to customers. Other examples include the retail industry, where firms join database cooperatives to share customer information for marketing purposes. Participants of information exchange include magazines and newspapers, which trade personal information about subscribers.

Joint customer data acquisition and information sharing initiated a heated debate between consumer privacy advocates, business groups, competition authorities and other regulators. At the same time, theoretical work on the topic is still evolving. We analyze the incentives of rival firms to cooperate on the acquisition and sharing of customer data, when firms use data to make targeted price offers. We also evaluate welfare effects of these practices in the context of a modified Hotelling model with competitive first- and third-degree price discrimination. We extend the standard model by introducing heterogeneity in consumer transportation costs. In addition, we allow firms to hold two different datasets on consumers reflecting i) brand preferences and ii) transportation cost parameters. Moreover, firms may only hold data on all consumers. We do not consider the case where firms hold data on a subset of consumers. Our approach applies well to markets, where a leading firm with a new technology is enabled to collect detailed customer profiles and to provide tailored services based upon these profiles, while competitors do not have the same ability. It also applies to newly liberalized markets, where the incumbent holds detailed purchase histories of all consumers. Depending on the data a firm holds, it offers uniform prices to all consumers, targets specific consumer groups (third-degree price discrimination) or sets individual prices (first-degree price discrimination). Firms may obtain data in addition to existing datasets and exchange data with the rival.

We are interested in three main questions: First, what type of data is acquired by both firms when firms agree to cooperatively collect data? Second, under what conditions is a firm holding a particular dataset willing to provide the competitor with access to it? Third, how does data acquisition cooperation and information sharing affect competition and welfare? To focus on the competitive effects of joint information acquisition and sharing, we assume that firms use data solely for price discrimination purposes. The important questions of collusion incentives and consumer privacy are beyond the scope of the present article.

We make the following contributions: By introducing heterogeneity in consumer transportation cost parameters into the standard Hotelling model, we show how incentives to acquire and share customer data depend on the *type* of information. Further, we allow firms to hold asymmetric data on consumers and derive incentives for partial information sharing.

Our results highlight the important role of the willingness of consumers to switch brands on the incentives of firms to jointly acquire data or to engage in information sharing. If a small price decrease can motivate a relatively large share of consumers to switch brands, cooperation between firms (holding similar types of customer data) for acquiring additional data does not take place. However, there is potential for information sharing, which is neutral for consumers and enhances social welfare. On the other hand, if consumers are generally loyal to their firms and price changes induce little switching, cooperation on data acquisition and sharing can be profitable. If such cooperation takes place, it is harmful to consumer surplus.

The main intuition of our results is as follows: If consumers are relatively mobile, a cooperation aimed at increasing the ability of firms to target individuals or specific groups is more likely to induce competition. This in turn provides little scope for using data for extracting rents, which makes cooperation unattractive for firms. Information sharing may still be profitable for firms, if it increases allocative efficiency, arising from the even allocation of consumers between firms. Equilibrium pricing strategies change with the mobility of consumers. When consumers are relatively immobile, price changes induce little switching. Firms can use customer data to extract rents from consumers, whereas the competition-intensifying effect of additional data is weak. Under these circumstances, consumers are likely to be harmed when firms cooperate by joint customer data acquisition or information sharing.

We conclude that competition authorities ought to scrutinize cooperation agreements between rival firms with respect to customer data acquisition and sharing on a case-by-case basis. Apart from the possibility that intensified information flows between rivals may facilitate collusion, a critical aspect concerning the competitive effects of a cooperation based upon customer data is whether consumers *are mobile enough* to render positive effects.

The rest of the article is organized as follows. Section 2 reviews the related literature. The model is presented in Section 3. In Section 4, we investigate the incentives of firms to cooperate in acquiring information on consumers. Section 5 turns to the analysis of information sharing, Section 6 concludes. Proofs are provided in the Appendix.

2 Related Literature

Despite the increasing importance of the acquisition and sharing of customer information among rivals, few theoretical articles directly addressed this issue.[1] Most relevant to our work are Liu and Serfes (2006) and Chen et al. (2001), who focus on the sharing of data on customer brand preferences between rivals. Liu and Serfes employ a two-period duopoly model with horizontally and vertically differentiated firms. In the first period, firms set uniform prices and collect information about customers. In the second period, firms use the information to make personalized offers. The authors show that information sharing takes place if firms are sufficiently asymmetric in customer bases. With sufficient asymmetry, the smaller firm has an incentive to share its customer information with the larger one. We take a different approach to model information exchange: By allowing firms to distinguish between consumer brand preferences and transportation cost parameters, we are able to address the question of partial information sharing, i.e., the exchange of only one type of information. In contrast to the results of Liu and Serfes is Chen et al. (2001), who show that firms engage in the sharing of customer data only when market shares are not too asymmetric and the level of customer targetability is low. Liu and Serfes (2006) as well as Chen et al. (2001) argue that it is the market shares of firms that drives information sharing. In our setup it is the willingness of consumers to switch brands together

[1] Sharing of data on customers is addressed in the banking literature, but this strand focuses on default risk of customers, whereas we consider data on consumer preferences.

with the portfolio of data that firms hold, which determines whether or not information sharing takes place. In contrast to the cited literature we find that information sharing may occur even with firms having perfectly symmetric market shares, depending on the consumer data firms hold. Similar to our analysis, Esteves (2009) considers price discrimination in a two-dimensional setting where firms have access to partial information on brand and product preferences of consumers. The author presents a two-dimensional Hotelling model with consumers located on a unit square, where the axes represent the two dimensions of consumer preferences. With partial information, firms can observe a consumer's location in only one of two dimensions and discriminate accordingly. Her main result is that price discrimination increases industry profits, if firms have information about the locations of consumers in the less differentiated dimension and ignore information about the more differentiated one.

This article is also related to the literature on competitive price discrimination. Earlier articles in this strand of literature focus on the question whether competition eliminates price discrimination. Borenstein (1985) presents a spatial model of monopolistic competition and shows that price discrimination prevails in a duopoly environment. He treats consumers as being heterogeneous along three dimensions: their reservation prices and brand preferences as well as the strength of the latter. The author relies on numerical simulation to determine which sorting strategy is more profitable: price discrimination based upon reservation prices or strength of brand preferences. Several newer articles on competitive price discrimination focus on consumer targetability, see e.g. Liu and Serfes (2005, 2007), Chen and Zhang (2009).

Thisse and Vives (1988) apply a standard Hotelling model, where firms may or may not observe the location of each consumer in the market. The authors show that price discrimination tends to intensify competition for each consumer and that discriminatory prices are usually lower than uniform prices. A similar insight is derived from a model of competitive couponing by Bester and Petrakis (1996) who analyze the sellers' incentives to offer rebates to their customers in two distinct regions. They find that offering rebates to consumers in form of coupons tends to intensify competition, which leads to lower prices and profits. In their survey on price discrimination Armstrong (2006) and Stole (2007) summarize the competitive effects of price discrimination and use the notion of best-response symmetry and asymmetry originally introduced by Corts (1998). We will rely on this concept to explain our results and discuss it in greater detail later on.

3 The Model

We present a duopoly pricing game between two differentiated firms, A and B, each selling a variety of the same product. Firms are situated at the two ends of a Hotelling line of unit length with firm A located at point 0 and firm B at point 1. Every consumer is characterized by an address $x \in [0, 1]$ corresponding to his brand preference for the ideal product. If the consumer buys from a firm, which does not provide the ideal product, he incurs linear transportation costs proportional to the distance to the firm. We depart from the standard Hotelling setup by introducing heterogeneity in consumer transportation costs per unit distance, which we denote by $t \in [\underline{t}, \overline{t}]$. Consumers are distributed uniformly and independently on a rectangle, where the horizontal (vertical) axis represents consumer brand preference (transportation cost parameter). The mass of consumers is normalized to one and every consumer is uniquely described by a pair (t, x). With t and x being uniformly and independently distributed, we have the following density functions: $f_t = 1/(\overline{t} - \underline{t})$, $f_x = 1$, $f_{t,x} = 1/(\overline{t} - \underline{t})$. We distinguish between two versions of the model based on the distribution of transportation cost parameters. In the first version we call consumers *relatively mobile* and assume that $\underline{t} = 0$. In the second version we assume that $\underline{t} > 0$ and $\overline{t}/\underline{t} \leq 2$ and label consumers as *relatively immobile*. When consumers are relatively mobile, switching brands is costless for some consumers (those with $\underline{t} = 0$). In the model with relatively immobile consumers, switching involves costs for every consumer, and the difference between the highest and lowest transportation cost parameter is not too large.[2] The utility of a consumer from buying at firm $i \in \{A, B\}$ is given by

$$U_i(p_i, t, x) = v - t\,|x - x_i| - p_i(t, x),$$

where v is a basic utility from consuming the product, which is the same across all consumers, x_i is firm i's address with $x_A = 0$ and $x_B = 1$ and $p_i(t, x)$ is the price firm i offers to consumer (t, x). A consumer buys from the firm delivering higher utility. Firm A provides a strictly higher utility if the following condition holds:

$$t(1 - 2x) + p_B > p_A. \tag{1}$$

[2] Transportation costs are closely related to switching costs as both capture how sensitive consumers react to price changes. There is evidence that switching rates vary in different industries as well as among consumers (European Commission 2009).

Assumption 1 states our tie-breaking rule.

Assumption 1: *In case both firms offer equal utilities, i.e.,*

$$t(1-2x)+p_B=p_A, \tag{2}$$

the consumer chooses the firm closer in the brand preference space (if $x=1/2$, then w.l.o.g. the consumer visits firm A).

In case of a price tie, consumers behave in the socially optimal manner and choose the closest firm. We say a consumer (t,x) is on *firm i's turf,* if he chooses firm i over firm j if prices are equal. The turf of firm A (B) is given by consumers with $x \leq 1/2$ ($x > 1/2$). Depending on the available data, firms can adopt the following strategies: If a firm has information on both consumer locations and transportation cost parameters, it can offer individual prices for each consumer. With information on either locations or transportation cost parameters, a firm can discriminate across groups of consumers. Without customer data, a firm sets uniform prices. Marginal costs are assumed to be zero. Firms set prices $p_i(t,x)$ to maximize their profits,

$$\Pi_i = \int_{X_i}\int_{T_i} f_{t,x} p_i(t,x)dtdx,$$

with X_i and T_i denoting the domains of locations and transportation cost parameters for consumers who buy from firm i. Next, we explain the way firms may acquire, hold and share customer data and describe the game played.

Customer Data and Timing

Let X and T be two sets containing information about the brand preferences and transportation cost parameters of *all* consumers, respectively. We refer to X and T as *datasets*. We define the union of datasets firm i holds as firm i's *information set* and denote it by I_i. Each firm may either hold information only about transportation cost parameters ($I_i = T$), only about brand preferences ($I_i = X$), complete information about consumer preferences ($I_i = X \cup T$), or no information ($I_i = \emptyset$). To simplify the notation, we write $I_i = XT$ to denote the case where firm i has complete information on consumers. We use the term *information scenario* to describe the datasets held by both firms in a pricing game, $\{I_A, I_B\}$. The superscript $I_A|I_B$ indexes values of functions and variables in the information scenario $\{I_A, I_B\}$. For example,

$\Pi_A^{XT|XT}$ denotes the profit function of firm A when both firms have full information on consumers: $\{I_A, I_B\} = \{XT, XT\}$. We refer to the cases, where $I_A = I_B$ as *symmetric information scenarios*. Cases, where $I_A \neq I_B$ are referred to as *asymmetric information scenarios*. Throughout the article, we assume that firms can acquire and exchange datasets X and/or T in their entirety. We rule out the case, where data is acquired or shared for only a subset of consumers.

Firms may engage in two types of cooperation involving customer data: i) *joint information acquisition*, and ii) *information sharing*. We analyze these cooperation types separately.

In the case of joint information acquisition (JIA), the following game is played:

Stage 1 (JIA): Firms decide cooperatively whether or not to acquire dataset X or T or both from an external source, in addition to the data they already hold. Simultaneously, they decide on a distribution rule for profits realized in the next stage. Apart from this transfer, the acquisition of data is assumed to be costless. After the data is acquired, it becomes available to both firms.

Stage 2 (JIA): Firms compete in prices and realize profits, which are distributed according to the rule agreed upon in stage 1.

When firms cooperate in information sharing (IS), the game unfolds as follows:

Stage 1 (IS): The firm holding more datasets decides whether and which dataset to sell to the rival. Simultaneously, the firms decide on a distribution rule for profits realized in the next stage, which determines the price of the dataset sold. After the sale of a dataset, it is available to both firms.

Stage 2 (IS): Firms compete in prices and realize profits, which are distributed according to the rule agreed upon in stage 1.

We do not model the rule for profit distribution in Stage 1 of both games. Instead, we analyze whether a necessary condition for both types of cooperation is fulfilled, which is a strict increase of joint profits. The following Assumption relates to the timing of pricing decisions in stage 2 of both games.

Assumption 2: *In symmetric information scenarios, firms set prices simultaneously. In asymmetric information scenarios, the firm with less information moves first and the other firm follows.*

The timing structure specified in Assumption 2 is consistent with most literature on competitive price discrimination, where firms choose their targeted offers after setting uniform prices (e.g., Thisse and Vives 1988, Shaffer and Zhang 2000, and Liu and Serfes 2006). Furthermore, it corresponds to the observation that prices are adjusted slower, if they are applied to a larger group of consumers. In particular, it is more difficult to adjust a firm's regular (uniform) price to a large customer group compared to changing discounts (by coupons) and targeted offers to smaller groups. For the remainder of this article we assume that firm A (B) is the firm with more (less) information. To solve the pricing game in stage 2, we seek for subgame-perfect Nash equilibria in asymmetric information scenarios and Nash equilibria in symmetric information scenarios. We restrict our attention to pure strategies.

The case when firms have no data constitutes a useful benchmark for further analysis. The following lemma shows that a symmetric Nash equilibrium in pure strategies in the information scenario, where firms do not hold any customer data exists when consumers are relatively immobile and does not exist when consumers are relatively mobile.

Lemma 1. *When firms have no information about consumers (i.e. $\{I_A, I_B\} = \{\emptyset, \emptyset\}$),* then
i) no symmetric pure-strategy Nash equilibrium exists if consumers are relatively mobile, and
ii) there is a unique pure-strategy Nash equilibrium if consumers are relatively immobile: Both firms' prices equal the harmonic mean of the range of transportation cost parameters.

With relatively mobile consumers, for any strictly positive price of the competitor, a firm finds it profitable to undercut the rival: a small advantage in price allows to attract new consumers. Zero prices can not constitute an equilibrium either: by increasing its price slightly, a firm can attract the closest consumers with the highest transportation cost parameters and make positive profits. With relatively immobile consumers, undercutting the competitor does not constitute a profitable strategy in the equilibrium as consumers do not easily switch brands.

We next consider equilibria in other information scenarios. Proposition 1 states our results.

Proposition 1. *Equilibrium prices and profits in each information scenario are as stated in Tables 1 and 2, respectively.*

Proof. See Appendix.

Note that in equilibrium firms use all available customer data for price discrimination and do not ignore any data. The equilibrium prices in Table 1 are functions of the available data that firms hold. In symmetric information scenarios, a firm's best-response function specifies the profit-maximizing price to any given price of the competitor. In this case, the only effect of not using all available customer data is to decrease the degrees of freedom in pricing. The

Table 1: Equilibrium Prices in Different Information Scenarios

I_A	I_B	p_A^*	p_B^*
		Relatively Mobile Consumers	
X	X	$\begin{cases} 2\bar{t}\left(1-2x\right)/3,\ x \leq 1/2 \\ \bar{t}\left(2x-1\right)/3,\ x > 1/2 \end{cases}$	$\begin{cases} \bar{t}\left(1-2x\right)/3,\ x \leq 1/2 \\ 2\bar{t}\left(2x-1\right)/3,\ x > 1/2 \end{cases}$
T	T	t	t
XT	XT	$\begin{cases} t\left(1-2x\right),\ x \leq 1/2 \\ 0,\ x > 1/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ t\left(2x-1\right),\ x > 1/2 \end{cases}$
X	$\emptyset$	$\begin{cases} \bar{t}(0.73-x),\ 0 \leq x < 0.27 \\ 0.465\bar{t},\ 0.27 \leq x \leq 0.5 \\ \bar{t}(1.47-2x),\ 0.5 < x < 0.62 \\ 0.24\bar{t},\ 0.62 \leq x \leq 1 \end{cases}$	$0.47\bar{t}$
T	$\emptyset$	$\begin{cases} 0.85\bar{t}-t,\ t < 0.28\bar{t} \\ (0.85\bar{t}+t)/2,\ t \geq 0.28\bar{t} \end{cases}$	$0.85\bar{t}$
XT	$\emptyset$	$\max\{0, 0.28\bar{t}+t(1-2x)\}$	$0.28\bar{t}$
XT	X	$\begin{cases} t\left(1-2x\right),\ x \leq 1/2 \\ (2x-1)\left(\bar{t}/2-t\right),\ x > 1/2,\ t < \bar{t}/2 \\ 0,\ x > 1/2,\ t \geq \bar{t}/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ \bar{t}(x-1/2),\ x > 1/2 \end{cases}$
XT	T	$\max\{0, t/2+t(1-2x)\}$	$t/2$
		Relatively Immobile Consumers	
$\emptyset$	$\emptyset$	$H(\underline{t},\bar{t})$	$H(\underline{t},\bar{t})$
X	X	$\begin{cases} \underline{t}(1-2x),\ x \leq 1/2 \\ 0,\ x > 1/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ \underline{t}(2x-1),\ x > 1/2 \end{cases}$
T	T	t	t
XT	XT	$\begin{cases} t(1-2x),\ x \leq 1/2 \\ 0,\ x > 1/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ t\left(2x-1\right),\ x > 1/2 \end{cases}$
X	$\emptyset$	$\begin{cases} \widetilde{H}(\underline{t},\bar{t})+\underline{t}(1-2x),\ x \leq \frac{1}{2} \\ \widetilde{H}(\underline{t},\bar{t})-\bar{t}(2x-1),\ \frac{1}{2} < x < \frac{1}{2}+\frac{\widetilde{H}(\underline{t},\bar{t})}{2(2\bar{t}-\underline{t})} \\ \frac{\widetilde{H}(\underline{t},\bar{t})-\underline{t}(2x-1)}{2},\ \frac{1}{2}+\frac{\widetilde{H}(\underline{t},\bar{t})}{2(2\bar{t}-\underline{t})} \leq x \leq \frac{\underline{t}+\widetilde{H}(\underline{t},\bar{t})}{2\underline{t}} \\ 0,\ x > \frac{1}{2}+\frac{\widetilde{H}(\underline{t},\bar{t})}{2\underline{t}} \end{cases}$	$\widetilde{H}(\underline{t},\bar{t})$
T	$\emptyset$	$t/2+3H(\underline{t},\bar{t})/4$	$3H(\underline{t},\bar{t})/2$
XT	$\emptyset$	$\max\{0, H(\underline{t},\bar{t})/2+t(1-2x)\}$	$H(\underline{t},\bar{t})/2$
XT	X	$\begin{cases} t(1-2x),\ x \leq 1/2 \\ 0,\ x > 1/2 \end{cases}$	$\begin{cases} 0,\ x \leq 1/2 \\ \underline{t}(2x-1),\ x > 1/2 \end{cases}$
XT	T	$\max\{0, t/2+t(1-2x)\}$	$t/2$

$A(\underline{t},\bar{t}) = (\bar{t}+\underline{t})/2$, $H(\underline{t},\bar{t}) = (\bar{t}-\underline{t})/\ln(\bar{t}/\underline{t})$, $\widetilde{H}(\underline{t},\bar{t}) = (\bar{t}-\underline{t})/\ln(2\bar{t}/\underline{t}-1)$

same is true for the firm with more datasets in asymmetric information scenarios, which moves after observing the competitor's price. Perhaps less obviously, the firm with fewer datasets also maximizes its profit by using all the available customer data. Although firms maximize profits by using all the available customer data, higher profits could be reached by committing not to use some data sets. In particular, they could enjoy higher individual and joint profits by committing not to use data on consumer brand preferences.

Table 2: Equilibrium Profits in Different Information Scenarios

I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$
		Relatively Mobile Consumers		Relatively Immobile Consumers	
$\emptyset$	$\emptyset$			$H(\underline{t},\bar{t})/2$	$H(\underline{t},\bar{t})/2$
X	X	$\bar{t}/8$	$\bar{t}/8$	$\underline{t}/4$	$\underline{t}/4$
T	T	$\bar{t}/4$	$\bar{t}/4$	$A(\underline{t},\bar{t})/2$	$A(\underline{t},\bar{t})/2$
XT	XT	$\bar{t}/8$	$\bar{t}/8$	$A(\underline{t},\bar{t})/4$	$A(\underline{t},\bar{t})/4$
X	$\emptyset$	$0.32\bar{t}$	$0.12\bar{t}$	$5\widetilde{H}(\underline{t},\bar{t})/8+\underline{t}/4$	$\widetilde{H}(\underline{t},\bar{t})/4$
T	$\emptyset$	$0.53\bar{t}$	$0.23\bar{t}$	$21H(\underline{t},\bar{t})/32+A(\underline{t},\bar{t})/8$	$9H(\underline{t},\bar{t})/16$
XT	$\emptyset$	$0.32\bar{t}$	$0.05\bar{t}$	$5H(\underline{t},\bar{t})/16+A(\underline{t},\bar{t})/4$	$H(\underline{t},\bar{t})/8$
XT	X	$5\bar{t}/32$	$\bar{t}/16$	$A(\underline{t},\bar{t})/4$	$\underline{t}/4$
XT	T	$9\bar{t}/32$	$\bar{t}/16$	$9A(\underline{t},\bar{t})/16$	$A(\underline{t},\bar{t})/8$
$A(\underline{t},\bar{t})=(\bar{t}+\underline{t})/2$, $H(\underline{t},\bar{t})=(\bar{t}-\underline{t})/\ln(\bar{t}/\underline{t})$, $\widetilde{H}(\underline{t},\bar{t})=(\bar{t}-\underline{t})/\ln(2\bar{t}/\underline{t}-1)$					

To understand the differences in equilibrium profits in Table 2, it is useful to recall the concepts of *best-response symmetry* and *best-response asymmetry* discussed by Corts (1998). He refers to models, where both firms set higher prices for the same group of consumers as exhibiting best-response symmetry. In contrast, best-response asymmetry exists, where one firm sets lower (higher) prices for those consumers who have a higher (lower) willingness to pay for the other firm. Prices and profits tend to be higher with best-response symmetry and lower with best-response asymmetry. To illustrate this, we first consider symmetric information scenarios. In these cases, with both relatively mobile and relatively immobile consumers, profits are the highest when both firms have data only on consumer transportation cost parameters. In contrast, if both firms hold dataset X (either alone or together with dataset T), they realize lower profits.

We obtain best-response symmetry when firms only know transportation cost parameters. All other symmetric information scenarios give rise to best-response asymmetry. When firms only hold dataset T, both set higher prices for those consumers, who are less willing to switch brands (i.e., those with higher values of t) and lower prices to those, who are ready to switch brands. In our case, best-response functions take the form

$$p_i^{T|T}(p_j|\, t) = \begin{cases} (p_j + t)/2, & p_j < 3t \\ p_j - t, & p_j \geq 3t, \end{cases}$$

for $i, j \in \{A, B\}$ and $i \neq j$. Provided that $p_j < 3t$, $p_i^{T|T}(p_j|\, t)$ increases in t. In contrast, if firms have information only on brand preferences and consumers are relatively immobile, the best-response functions for $x < 1/2$ are

$$p_A^{X|X}(p_B|\, x \;\; < \;\; 1/2) = \begin{cases} \left[p_B + \bar{t}(1-2x)\right]/2, & p_B < \bar{t}(1-2x) \\ p_B, & p_B \geq \bar{t}(1-2x) \end{cases}$$

$$p_B^{X|X}(p_A|\, x \;\; < \;\; 1/2) = \begin{cases} \left[p_A - \underline{t}(1-2x)\right]/2, & p_A < 2\bar{t}(1-2x) \\ p_A - \bar{t}(1-2x), & p_A \geq 2\bar{t}(1-2x). \end{cases}$$

If $x > 1/2$, then the best-response functions take the form:

$$p_A^{X|X}(p_B|\, x \;\; > \;\; 1/2) = \begin{cases} p_B/2, & p_B < 2\bar{t}(2x-1) \\ p_B - \bar{t}(2x-1), & p_B \geq 2\bar{t}(2x-1) \end{cases}$$

$$p_B^{X|X}(p_A|\, x \;\; > \;\; 1/2) = \begin{cases} \left[p_A + \bar{t}(2x-1)\right]/2, & p_A < \bar{t}(2x-1) \\ p_A, & p_A \geq \bar{t}(2x-1). \end{cases}$$

Clearly, in the case where firms have data only on brand preferences, every firm sets a higher price for consumers, who prefer its brand and lower ones for those who like the competitor more. As both groups of consumers ($x < 1/2$ and $x > 1/2$) have different brand preferences, the best-response functions imply best-response asymmetry. Formally, $p_A^{X|X}(p_B|\, x < 1/2) > p_A^{X|X}(p_B|\, x > 1/2)$, whereas $p_B^{X|X}(p_A|\, x < 1/2) < p_B^{X|X}(p_A|\, x > 1/2)$. If both types of information are available to the firms, best-response asymmetry is preserved. The best-response functions in this case are

$$p_A^{XT|XT}(p_B|\,x) = \begin{cases} p_B + t(1-2x), & x \leq 1/2 \\ \max\{0, p_B + t(1-2x) - \epsilon\}, & x > 1/2 \end{cases}$$

$$p_B^{XT|XT}(p_A|\,x) = \begin{cases} \max\{0, p_A - t(1-2x) - \epsilon\}, & x \leq 1/2 \\ p_A - t(1-2x), & x > 1/2, \end{cases}$$

where ϵ is an infinitesimal, positive value. It is easily verified that $p_A^{XT|XT}(p_B|x \leq 1/2) > p_A^{XT|XT}(p_B|x > 1/2)$ whereas $p_B^{XT|XT}(p_A|x \leq 1/2) < p_B^{XT|XT}(p_A|x > 1/2)$, hence, the reaction functions imply best-response asymmetry.

In the asymmetric information scenarios, firms' profits are the highest in the information scenario $\{T, \emptyset\}$, in which case both firms set high prices to consumers. In contrast, profits are the lowest in the information scenario $\{XT, X\}$, which exhibits best-response asymmetry.

The concepts of best-response symmetry and asymmetry explain well why prices and profits are higher in some information scenarios than in others. In the following we demonstrate that the concepts, nevertheless, do not completely explain the incentives to jointly acquire and share customer data. In particular, they cannot be applied to situations when the market exhibits the same best-response property before and after cooperation.

By jointly acquiring customer data that neither firm holds beforehand or by making a proprietary database available to the rival, firms can influence the competitive environment. These decisions are the subject of the next sections.

4 Joint Acquisition of Customer Data

We now analyze the incentives of firms to cooperatively acquire customer data for price discrimination. We focus on symmetric information scenarios with firms holding identical datasets and analyze the incentives to jointly acquire additional information on consumer preferences, which (after acquisition) becomes available to both firms.

First, our results show that price discrimination may provide sufficient incentives for joint information acquisition. Only information on consumer transportation cost parameters can be jointly acquired, but not information on brand preferences. Second, incentives to jointly acquire data on transportation cost parameters depend on the consumer willingness to switch brands.

Although more information potentially allows firms to extract more rents from consumers, intensified price competition may reduce prices and profits. The competition effect dominates, if consumer mobility is relatively high. If consumers are relatively loyal to their brands, acquiring data on transportation cost parameters induces little additional competition. The following proposition summarizes our insights on joint data acquisition incentives and Table 3 illustrates our results for the case of relatively mobile consumers with $\bar{t} = 1$ and the case of relatively immobile consumers with $\underline{t} = 1$ and $\bar{t} = 2$.

Proposition 2. *Firms' incentives to jointly acquire information on consumer preferences depend on the distribution of transportation cost parameters.*

i) If consumers are relatively mobile and firms have partial information on consumers (either $\{I_A, I_B\} = \{X, X\}$ *or* $\{I_A, I_B\} = \{T, T\}$*), firms have no incentives to jointly acquire further information for price discrimination purposes. Profits across symmetric information scenarios are ranked as* $\Pi_i^{XT|XT} < \Pi_i^{X|X} < \Pi_i^{T|T}$.

ii) If consumers are relatively immobile, firms do not jointly acquire dataset X*, but acquire dataset* T*. Profits across information scenarios are ranked as* $\Pi_i^{X|X} < \Pi_i^{XT|XT} < \Pi_i^{\emptyset|\emptyset} < \Pi_i^{T|T}$.

Proof. See Appendix.

Firms do not jointly acquire information on brand preferences, but only acquire information on consumer transportation cost parameters. Since additional information on consumer brand preferences always induces best-response asymmetry, firms do not jointly acquire dataset X. If firms initially have no information on consumers and acquire dataset T, they switch to best-response symmetry, which increases industry profits.

When firms initially hold dataset X and cooperate on gathering dataset T, the concepts of best-response symmetry and asymmetry cannot be applied to explain incentives to acquire customer data. As mentioned above, both information scenarios $\{X, X\}$ and $\{XT, XT\}$ exhibit best-response asymmetry. Whether information acquisition takes place, depends on consumer mobility and is not driven by a change in the best-response property of the market. If consumers do not differ much in terms of the strength of their brand preferences (i.e., $\bar{t}/\underline{t} \leq 2$), acquiring dataset T is profitable. If, however, consumer mobility is relatively high, then complementing dataset X with T reduces industry profits.

A closer look at the two main effects at work reveals why firms do not acquire dataset T in addition to brand preference data with relatively mobile consumers and why they do acquire it

Table 3: Profits and Incentives for Joint Information Acquisition

Before Data Acquisition					Data Acquired	After Data Acquisition					Acquire?
I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Pi_A+\Pi_B$		I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Pi_A+\Pi_B$	
Relatively Mobile Consumers ($\underline{t}$= 0 and $\bar{t}$ = 1)											
X	X	.14	.14	.28	T	XT	XT	.13	.13	.26	No
T	T	.25	.25	.50	X	XT	XT	.13	.13	.26	No
Relatively Immobile Consumers ($\underline{t}$ = 1 and $\bar{t}$ = 2)											
$\emptyset$	$\emptyset$	.72	.72	1.44	X	X	X	.25	.25	.50	No
$\emptyset$	$\emptyset$	.72	.72	1.44	T	T	T	.75	.75	1.50	Yes
$\emptyset$	$\emptyset$	.72	.72	1.44	XT	XT	XT	.38	.38	.75	No
X	X	.25	.25	.50	T	XT	XT	.38	.38	.75	Yes
T	T	.75	.75	1.50	X	XT	XT	.38	.38	.75	No

if consumer mobility is low. First, the *rent-extraction effect*: more information on consumers enables firms to better target and segment consumers. Second, the *competition effect* takes account for the change in the strength of price competition between firms. Whether firms have incentives to acquire additional information on consumers depends on the sum of these two effects.

If consumers are relatively immobile, they visit the closest firm in both information scenarios $\{X, X\}$ and $\{XT, XT\}$, as shown in Figure 1. Additional information on transportation cost parameters allows firms to better target consumers. Although with the firms having both datasets X and T each consumer receives individual offers from both firms, as consumers are relatively immobile, the better targeting induces little competition and the rent-extraction effect dominates.

However, if consumers are mobile, firms will not complement their existing data on brand preferences with dataset T. Note that pricing strategies and, hence, equilibrium prices in the scenario where both firms have full information, do not depend on the distribution of transportation cost parameters. The reason for the altered incentives to acquire dataset T is that the pricing decisions of firms in the information scenario $\{X, X\}$ change depending on the mobility of consumers. Let us take a closer look at the strategies of the firms in this information scenario. Due to the symmetry of firms, it is sufficient to focus on the region with $x \leq 1/2$ and analyze competition on firm A's turf.

In information scenario $\{X, X\}$, if consumer mobility is low, for any given price by firm B to a group of consumers with brand preferences $x \leq 1/2$, firm A can keep all consumers in this group without significantly decreasing its price offered to them. Firm A's optimal strategy is to set a price for a group x, which allows to attract all members, even those who are most willing to switch, i.e., consumers with the lowest transportation cost parameters. The low willingness of consumers to switch brands and firm A's strategy to hold them all in turn induces firm B to price very aggressively on A's turf and to decrease its price to zero, putting a downward pressure on firm A's prices. In the end, firm A is able to keep all consumers on its own turf, but only by charging every group x a relatively low price. The same forces are at work on firm B's turf. With industry profits being relatively low, moving into the scenario with full customer data is attractive for the competitors, where they can extract more consumer surplus. If consumer mobility is high, it is expensive for firm A to hold all consumers with a given x. To achieve this, firm A must reduce its prices to prevent consumers with the lowest transportation costs from switching to firm B. It is more profitable for firm A to give up the most mobile consumers and set a price for every group x, which targets the consumers with higher values of t. Firm B is, hence, able to capture the most mobile consumers on A's turf, even with a relatively high price. In the emerging equilibrium firm A sets prices to every group x on its turf to target consumers with higher transportation cost parameters, while firm B targets those with lower values of t. With industry profits being relatively high in the information scenario $\{X, X\}$, firms do not want to acquire data on consumer transportation costs.

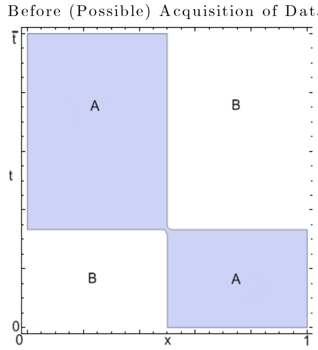
Our results show that best-response symmetry and asymmetry are not anchored in a particular type of information. The same type of information can induce both best-response symmetry and asymmetry depending on the additional data firms own. In particular, information on transportation cost parameters may induce different strategies, either best-response symmetry (if only dataset T is available) or best-response asymmetry (if dataset T is combined with dataset X). This extends the analysis in Armstrong (2006), who emphasizes that firms have an incentive to acquire information about their consumers, if firms can discriminate between consumers according to their transportation cost parameters. We show that this might not always be the case: It holds that industry profits are higher if firms can only discriminate based on T compared to the case when firms lack consumer data. However, depending on the distribution of transportation cost parameters, industry profits may either decrease or increase, when firms have access to both sets of information compared to the case, when they can only discriminate based on X.

Next, we compare consumer surplus and social welfare across information scenarios and draw

Figure 1: Demand Regions with Mobile and Immobile Consumers in X,X and XT,XT

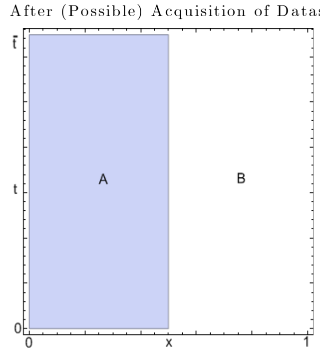
Relatively Mobile Consumers

Before (Possible) Acquisition of Dataset T



$I_A = X, I_B = X$

After (Possible) Acquisition of Dataset T

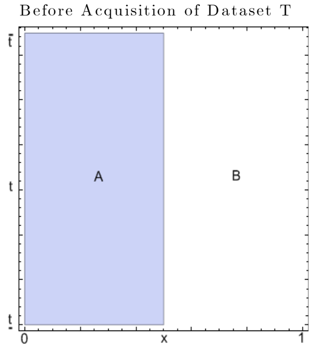


$I_A = XT, I_B = XT$

A (B) denotes demand region of firm A (B)

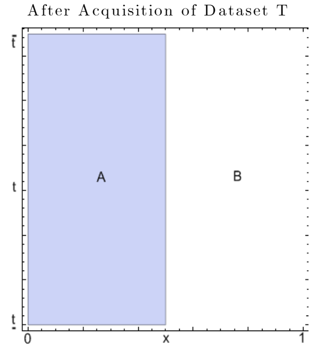
Relatively Immobile Consumers

Before Acquisition of Dataset T



$I_A = X, I_B = X$

After Acquisition of Dataset T



$I_A = XT, I_B = XT$

A (B) denotes demand region of firm A (B) conclusions about the welfare implications of joint information acquisition. The next proposition summarizes our results.

Proposition 3. *The ranking of consumer surplus (CS) and social welfare (SW) in symmetric information scenarios and welfare implications of joint customer data acquisition depend on the distribution of the transportation cost parameters.*

i) If consumers are relatively mobile, then consumer surplus and social welfare are ranked as $CS^{T|T} < CS^{X|X} < CS^{XT|XT}$ *and* $SW^{X|X} < SW^{T|T} = SW^{XT|XT}$.

ii) If consumers are relatively immobile, then consumer surplus is ranked as $CS^{T|T} < CS^{\emptyset|\emptyset} < CS^{XT|XT} < CS^{X|X}$ *and social welfare is same in all the symmetric information scenarios. Joint acquisition of dataset T reduces consumer surplus and is neutral to social welfare.*

Proof. See Appendix.

Two effects determine the ranking of consumer surplus along information scenarios: First, a competition effect capturing the level of prices, and second, an allocative effect related to the distribution of consumers between firms. Allocative efficiency requires that consumers choose the nearest firm. The only case, where allocative efficiency is distorted is the scenario where both firms hold dataset X and consumers are relatively mobile: consumers with the lowest transportation cost parameters ($t < \bar{t}/3$) then visit the firm further away, giving rise to allocative inefficiencies. When allocative efficiency is preserved, the ranking of consumer surplus is the opposite of the ranking of industry profits.

We conclude that price discrimination may provide sufficient incentives for firms to cooperatively acquire information on consumer transportation costs. With mobile consumers, firms do not acquire additional data if they already hold some, although doing so would be socially beneficial. With immobile consumers, firms cooperate on acquiring data on transportation cost parameters, regardless what data they already have. This is neutral to social welfare and decreases consumer surplus.

5 Sharing of Customer Data

We now analyze the incentive of a firm with more information to share it with the competitor. Our main question is under which conditions a firm possessing a particular dataset is willing to provide the competitor with access to it. The dataset(s) with information on brand preferences and/or on transportation cost parameters may be given to the rival. We call information exchange *partial*, if a firm has access to both datasets, but shares only one of them with its competitor. The following proposition summarizes our results on information sharing and Table 4 shows how information sharing alters profits using the examples with $\bar{t} = 1$ for relatively mobile consumers and $\underline{t} = 1$ and $\bar{t} = 2$ for relatively immobile consumers.

Proposition 4. *Incentives to share information depend on the distribution of consumer transportation cost parameters and the portfolio of data firms hold.*
i) With relatively mobile consumers, a firm with full information on consumers shares its data on transportation cost parameters with the competitor, if the latter holds data on customer brand preferences.
ii) If consumers are relatively immobile, then data on consumer transportation cost parameters is shared in two cases: First, if one firm has full information on consumers, whereas the other holds data on customer brand preferences, and second, if one firm has full information on consumers, whereas the other has no data.

Proof. See Appendix.

Table 4: Joint Profits and Incentives for Information Sharing

Before (Possible) Data Sharing					Data Shared	After (Possible) Data Sharing					Share?
I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Pi_A+\Pi_B$		I_A	I_B	$\Pi_A^{I_A\|I_B}$	$\Pi_B^{I_A\|I_B}$	$\Pi_A+\Pi_B$	
Relatively Mobile Consumers ($\bar{t} = 1$)											
X	$\emptyset$	.32	.12	.44	X	X	X	.14	.14	.28	No
T	$\emptyset$	.53	.23	.76	T	T	T	.25	.25	.50	No
XT	$\emptyset$	.32	.05	.37	X	XT	X	.16	.06	.22	No
XT	$\emptyset$	.32	.05	.37	T	XT	T	.28	.06	.34	No
XT	X	.16	.06	.22	T	XT	XT	.13	.13	.26	Yes
XT	T	.28	.06	.34	X	XT	XT	.13	.13	.26	No
Relatively Immobile Consumers ($\underline{t} = 1$ and $\bar{t} = 2$)											
X	$\emptyset$	.82	.23	1.05	X	X	X	.25	.25	.50	No
T	$\emptyset$	1.13	.81	1.94	T	T	T	.75	.75	1.50	No
XT	$\emptyset$	.83	.18	1.01	X	XT	X	.38	.25	.63	No
XT	$\emptyset$	.83	.18	1.01	T	XT	T	.84	.19	1.03	Yes
XT	X	.38	.25	.63	T	XT	XT	.38	.38	.75	Yes
XT	T	.84	.19	1.03	X	XT	XT	.38	.38	.75	No

A conventional explanation for the incentives of firms to share information is whether doing so induces best-response symmetry in the market (Armstrong 2006). For instance, data on consumer brand preferences is never shared in our model. The reason for this is that dataset X induces best-response asymmetry (and, hence, stronger competition) if both firms have it. This offsets any benefits arising from the possibility to better target consumers. Although dataset X is never shared, it plays a decisive role for the incentives of firms whether to share the dataset T. We call this interplay between the datasets X and T the *portfolio effect*. With this label we refer to the observation that the incentives to share a particular dataset depend on what other data both firms already hold. The same dataset may or may not be shared with the competitor depending on what additional data firms already hold. In particular, the necessary condition for sharing dataset T is that the firm with more information also holds dataset X. If one firm owns data only on transportation cost parameters (while the other has no data at all), information sharing does not take place.[3]

Our results highlight the importance of consumer transportation cost parameters on the incentives of firms to share customer data. With mobile consumers, a firm with full information does not share its dataset T with the competitor who holds no data, while in the same scenario with relatively immobile consumers this data is shared even without monetary transfers. Figure 2 presents the demand regions with relatively mobile and immobile consumers for the information scenarios $\{XT, \emptyset\}$ and $\{XT, T\}$. The differences in incentives to share dataset T in the scenario $\{XT, \emptyset\}$ depend on consumer mobility and originate from the differences in pricing strategies of the firm with less information (firm B) before potential data sharing. In the scenario after information sharing (i.e., in $\{XT, T\}$) regardless of the distribution of transportation cost parameters, firm B sets $p_B = t/2$ and firm A matches this price to leave consumers indifferent whenever it can with a non-negative price. Firm A pursues the same strategy in the information scenario before potential information sharing (i.e., in $\{XT, \emptyset\}$): it matches the price of the competitor and leaves consumers indifferent whenever it can set a non-negative price. The strategy of firm B, however, depends on the level of consumer mobility in information scenario $\{XT, \emptyset\}$. If consumers are mobile, firm B tailors its price to target only the most loyal consumers (i.e., those who are close to it and have high transportation costs). This relatively high price serves

[3] This result contrasts with Armstrong (2006), who shows that with simultaneous pricing decisions dataset T is shared in the information scenario, where one firm holds only dataset T, while the other does not have any customer data. It is easy to check that with simultaneous pricing decisions our model also predicts that the firm possessing dataset T shares it with the competitor both with mobile and immobile consumers.

as basis for firm A as well, resulting in high overall industry profits. In contrast, with relatively immobile consumers (given firm A's strategy), it is optimal for firm B to set a uniform price, which allows to attract some of the consumers even with the lowest transportation costs, close to firm B. The latter must decrease its price to avoid being undercut by firm A, resulting in a relatively low uniform price set by firm B. As firm A bases its prices on firm B's uniform price, all prices in the market are relatively low.

What changes, if firm B obtains database T? By being able to identify groups of consumers with the same transportation cost parameters, firm B sets lower (higher) prices to those with lower (higher) values of t. With relatively mobile consumers, firm B's uniform price is targeted at consumers with higher values of t. In this case the improved ability to price discriminate allows firm B to increase its price only for a few consumers (with nearly maximal values of transportation cost parameters), while it reduces the price for all consumers with lower t values. As firm A acts similarly, the additional information generally leads to a price decrease in the market. With relatively immobile consumers, the price of firm B is aimed to appeal even to consumers with low values of t. And with additional data on transportation cost parameters firm B can increase the price for most consumers, which drives up firm A's prices as well. Hence, with immobile consumers both firms profit from sharing dataset T.

Finally, we turn to the welfare implications of customer information sharing. Proposition 5 summarizes our insights.

Proposition 5. *Welfare implications of customer data sharing depend on the distribution of transportation cost parameters among consumers.*

i) With relatively mobile consumers, information sharing is neutral for consumer surplus and enhances social welfare.

ii) With relatively immobile consumers, information sharing always decreases consumer surplus and social welfare either decreases or does not change.

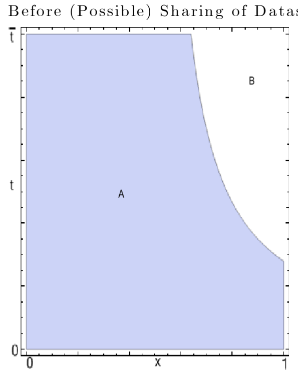
Proof. See Appendix.

Proposition 5 highlights the importance of consumer mobility for the welfare effects of information sharing. When consumers are relatively mobile, information sharing is Pareto-optimal: it increases profits and leaves consumer surplus unchanged. However, with relatively immobile

Figure 2: Demand Regions with Mobile and Immobile Consumers in XT,0 and XT,T

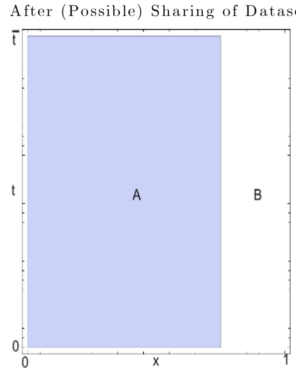
Relatively Mobile Consumers

Before (Possible) Sharing of Dataset T



$\{XT, \emptyset\}$

After (Possible) Sharing of Dataset T

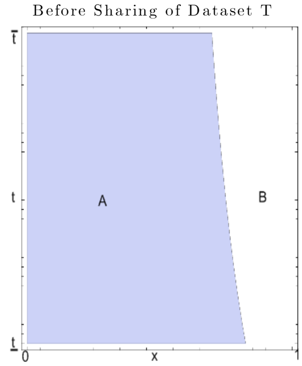


$\{XT, T\}$

A (B) denotes demand region of firm A (B)

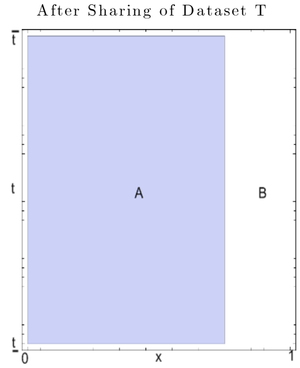
Relatively Immobile Consumers

Before Sharing of Dataset T



$\{XT, \emptyset\}$

After Sharing of Dataset T



$\{XT, T\}$

A (B) denotes demand region of firm A (B) consumers information sharing harms consumers and is at best neutral to social welfare. In our setup, social welfare can only decrease due to the misallocation of consumers, which occurs if consumers do not visit their closest firm.

When consumers are mobile and a firm with full information shares its dataset T with the rival holding dataset X, social welfare increases, because it leads to a more efficient allocation of consumers among the firms. In the resulting equilibrium all consumers are served by their most preferred firm. Consumers on firm B's turf with high transportation costs lose, because firm B uses its new dataset T to extract higher rents from them. However, consumers on firm B's turf with low transportation cost parameters gain, because they are served by their preferred firm. In our setting, these two effects cancel each other out, which renders information sharing neutral for consumer surplus.

When consumers are relatively immobile between brands, information sharing takes place in two cases: a firm with full information shares its dataset T with the rival either holding dataset X or holding no information. In the former case, sharing customer data does not affect social welfare as consumers choose the closest firm both before and after the transaction. Information sharing leads here solely to a redistribution of rents from consumers to firms due to the improved targeting ability. If consumers are relatively immobile and the firm with full information shares dataset T with its rival (who initially holds no data), social welfare decreases. This result is driven by the increased misallocation of consumers between firms: Some consumers with high values of t (which previously visited their most preferred firm, B) now choose firm A. This negative effect is not compensated by the improved allocation of some consumers with low values of t, which previously visited their less preferred firm, A. Since industry profits increase due to data sharing, consumer surplus declines.

6 Conclusions

It is increasingly observable that competitors in different information-intensive industries coordinate on information acquisition in terms of standardization or exchange profiles of their customers with each other. These activities have raised the suspicion of consumer advocates as well as regulatory authorities. We present a modified Hotelling model with first- and third-degree price discrimination and horizontally differentiated firms, which possess different sets of data on consumer preferences (that is brand preferences and transportation cost parameters).

Of particular interest to us are two kinds of agreements between rivals: joint acquisition and sharing of customer data.

We model cooperation with regard to customer data in a novel manner: We distinguish between two datasets firms may acquire and share, which encompass brand preferences and transportation cost parameters. We analyze how the incentives to engage in cooperation involving customer data depend on the type of information. Furthermore, we allow firms to hold asymmetric customer data. A firm with more datasets can decide to share its datasets with the competitor. With relatively mobile consumers, firms do not cooperate on acquiring customer data, if they already hold any of the two datasets. When consumers are immobile, firms cooperate to obtain the dataset on transportation cost parameters regardless of whether they possess data on brand preferences. In this case, information acquisition reduces consumer surplus and is neutral to social welfare. Incentives to share information depend on the portfolio of data the firms hold and the distribution of consumers with respect to their transportation cost parameters. Information sharing may arise with both relatively mobile and immobile consumers. Whereas information sharing is at best neutral for consumer surplus, it enhances social welfare with relatively mobile consumers.

Our results highlight that the evaluation of such agreements depends on the welfare standard adopted by a competition authority. Competition authorities pursuing a consumer surplus standard should be critical towards cooperation agreements between competitors involving customer data. Consumers are especially likely to be harmed, if their willingness to switch brands is low. Taking into account other potentially problematic issues such as privacy and collusion (which are not addressed herein), we are sceptical that consumers benefit overall from such agreements. However, under a social welfare standard information sharing is beneficial, if consumers are relatively mobile, in which case it improves allocative efficiency.

Appendix

Definitions and Notation. Before we proceed with the proofs, we introduce some definitions and notation. Let $t^c(p_A, p_B, x)$ denote the transportation cost parameters of those consumers with brand preference x, who are indifferent between firms A and B for given prices p_A and p_B: $t^c(\cdot) = (p_B - p_A)/(2x-1)$. It holds that $U_A(p_A, t^c(\cdot), x) = U_B(p_B, t^c(\cdot), x)$. For given p_A, p_B and x we have $\Pr\{t \geq t^c(\cdot)\} = 0$ if $t^c(\cdot) > \bar{t}$, $\Pr\{t \geq t^c\} = f(t)\left[\bar{t} - t^c(\cdot)\right]$ if $\underline{t} \leq t^c(\cdot) \leq \bar{t}$ and $\Pr\{t \geq t^c\} = 1$ if $t^c(\cdot) < \underline{t}$. As equilibrium strategies may differ on the intervals $x < 1/2$ and $x > 1/2$, it is useful to distinguish between $\underline{t}^c := t^c(\cdot, x < 1/2)$ and $\bar{t}^c := t^c(\cdot, x > 1/2)$.

Similarly, let $x^c(p_A, p_B, t)$ denote the brand preference of consumers with transportation cost parameter t indifferent between firms A and B for given prices p_A and p_B: $x^c(\cdot) = 1/2 - (p_A - p_B)/2t$. It holds that $U_A(p_A, t, x^c(\cdot)) = U_B(p_B, t, x^c(\cdot))$. For given p_A, p_B and t it holds that $\Pr\{x \geq x^c(\cdot)\} = 0$ if $x^c(\cdot) > 1$, $\Pr\{x \geq x^c(\cdot)\} = 1 - x^c(\cdot)$ if $0 \leq x^c(\cdot) \leq 1$ and $\Pr\{x \geq x^c(\cdot)\} = 1$ if $x^c(\cdot) < 0$. Let $\underline{x}(p_A, p_B, \underline{t})$ and $\bar{x}(p_A, p_B, \bar{t})$ denote the brand preferences of the indifferent consumers for given prices p_A and p_B with the lowest and highest transportation cost parameters, respectively. Formally, $t^c(p_A, p_B, \underline{x}) = \underline{t}$ and $t^c(p_A, p_B, \bar{x}) = \bar{t}$.

We introduce $A(\underline{t}, \bar{t}) := (\bar{t}+\underline{t})/2$ and $H(\underline{t}, \bar{t}) := (\bar{t}-\underline{t})/\ln(\bar{t}/\underline{t})$ to denote the arithmetic and the harmonic mean of the transportation cost parameters $t \in \left[\underline{t}, \bar{t}\right]$ when $\underline{t} > 0$, respectively. Note that for any $\underline{t}$, $\bar{t}$ it holds that $A(\underline{t}, \bar{t}) > H(\underline{t}, \bar{t})$. We also introduce $\widetilde{H}(\underline{t}, \bar{t}) := (\bar{t}-\underline{t})/\ln\left[(2\bar{t} - \underline{t})/\underline{t}\right]$. Moreover, if $\underline{t} > 0$ we denote the ratio of the highest and the lowest transportation cost parameters as $k := \bar{t}/\underline{t}$.

We will omit the notation of information scenarios for best-response functions and equilibrium prices, which should be clear from the context.

Proof of Lemma 1. We first prove part *i)* of Lemma 1. We show that a small deviation downwards from the competitor's price is always profitable. Without loss of generality we focus on the pricing of firm A. If firm A sets $p_A = p_B > 0$, it captures half of the consumers and realize profits $\Pi_A^{\emptyset|\emptyset}(p_A = p_B, p_B) = p_B/2$. If firm A deviates downwards by setting $p_A < p_B$, it captures all consumers on its own turf and some consumers with low transportation cost parameters on the competitor's turf. Solving $t^c(\cdot) = \bar{t}$ for x we obtain $\bar{x} = (p_B - p_A)/(2\bar{t}) + 1/2$. Firm A's profit if $p_A < p_B$ is $\Pi_A^{\emptyset|\emptyset}(p_A < p_B, p_B) = \int_0^{\bar{x}}\int_0^{\bar{t}} [f(t)p_A]\, dtdx + \int_{\bar{x}}^{1}\int_0^{t^c} [f(t)p_A]\, dtdx = p_A\left[(p_B - p_A)/\left(2\bar{t}\right) + 1/2 - (p_B - p_A)\ln\left((p_B - p_A)/\bar{t}\right)/2\right]$. It is helpful to introduce $\Delta = p_B - p_A$ with $\Delta \in (0, p_B]$ as the magnitude of firm A's downward deviation from firm B's price.

Comparing profits with and without deviation from $p_B > 0$, we obtain that deviation is not profitable if $p_B < \Delta + \bar{t}/\left[1 - \bar{t}\ln(\Delta/\bar{t})\right]$ for any $\Delta \in (0, p_B]$. We now show that there is no such price p_B, which fulfills the latter condition. Note that the RHS of this condition is increasing in Δ, hence, it is fulfilled for any $\Delta \in (0, p_B]$ if and only if it holds for the lowest possible value of Δ. As $\lim_{\Delta \longrightarrow 0}[\Delta + \bar{t}/[1 - \bar{t}\ln(\Delta/\bar{t})]] = 0$, the condition is always violated.

It remains to consider whether $p_A = p_B = 0$ constitutes an equilibrium. This is not the case as these prices yield zero profits to both firms. With a minimal deviation upward, firm A could attract the nearest consumers with the highest transportation cost parameters and make positive profit. This completes the proof of part *i)* in Lemma 1.

We now turn to the proof of part *ii)*. Assume that $\underline{t} > 0$ and $\bar{t}/\underline{t} \leq 2$. Since firms are symmetric, we focus without loss of generality on the pricing of firm B. Consider first the case where firm B sets a (weakly) higher price than firm A: $p_B \geq p_A$. Let $\bar{d} = p_B - p_A$. Depending on the level of $\bar{d}$, the demand regions may take two possible forms: One with $\underline{x} \leq 1$ $(0 \leq \bar{d} \leq \underline{t})$ and another with $\underline{x} > 1$ $(\underline{t} < \bar{d} < \bar{t})$. Let $0 \leq \bar{d} \leq \underline{t}$. In this case profits are $\Pi_A^{\emptyset|\emptyset}(p_A, p_B \geq p_A) = \int_0^{\bar{x}} \int_{\underline{t}}^{\bar{t}} [f(t)p_A]\, dtdx + \int_{\bar{x}}^{\underline{x}} \int_{\underline{t}}^{t^c} [f(t)p_A]\, dtdx$ and $\Pi_B^{\emptyset|\emptyset}(p_A, p_B \geq p_A) = \int_{\bar{x}}^{\underline{x}} \int_{t^c}^{\bar{t}} [f(t)p_B]\, dtdx + \int_{\underline{x}}^{1} \int_{\underline{t}}^{\bar{t}} [f(t)p_B]\, dtdx$. Maximization yields the reaction function $p_i(p_j) = \left[p_j + H(\underline{t}, \bar{t})\right]/2$ with $i, j \in \{A, B\}$ and $i \neq j$. The optimal prices are $p^* = H(\underline{t}, \bar{t})$. The corresponding profits are $\Pi_i^{\emptyset|\emptyset}(p^*, p^*) = H(\underline{t}, \bar{t})/2$. Note that these prices satisfy $\underline{x} \leq 1$. Assume next that $\underline{t} < \bar{d} < \bar{t}$, in which case $\Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}) + \bar{d}) = \int_{\underline{x}}^{1} \int_{t^c}^{\bar{t}} [f(t)p_B]\, dtdx = f(t)\left[\bar{d} + H(\underline{t}, \bar{t})\right]\left[\bar{t} - \bar{d} + \bar{d}\ln\left(\bar{d}/\bar{t}\right)\right]/2$. Taking the derivative with respect to $\bar{d}$ we get $\partial \Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}) + \bar{d})/\partial \bar{d} = f(t)\left[\bar{t} - \bar{d} + \left(2\bar{d} + H(\underline{t}, \bar{t})\right)\ln\left(\bar{d}/\bar{t}\right)\right]/2$, which is negative if $\bar{t}/\underline{t} \leq 2$. It follows that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}) + \bar{d}) < \Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}))$ for any $0 \leq \bar{d} \leq \bar{t}$, hence, firm B does not have an incentive to deviate upwards when firm A sets $p_A = H(\underline{t}, \bar{t})$.

We next analyze deviation downwards where firm B sets a (weakly) lower price than firm A: $p_B \leq p_A$. Let $\underline{d} = p_A - p_B$. Depending on the level of $\underline{d}$, the demand regions may take two possible forms: One with $0 \leq \underline{x} \leq 1/2$ $(0 \leq \underline{d} \leq \underline{t})$ and another with $\underline{x} < 0$ $(\underline{t} < \underline{d} < H(\underline{t}, \bar{t}))$. Let $0 \leq \underline{d} < \underline{t}$. Note that in this case the optimization problem of firm B mirrors that of firm A when $0 \leq \bar{d} \leq \underline{t}$, and it holds that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}) - \underline{d}) \leq \Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}))$, with equality if $\underline{d} = 0$. Assume next that $\underline{t} < \underline{d} < H(\underline{t}, \bar{t})$. Firm B realizes $\Pi_B^{\emptyset|\emptyset}(H(\underline{t}, \bar{t}), H(\underline{t}, \bar{t}) - \underline{d}) = \int_0^{\underline{x}} \int_{\underline{t}}^{t^c} [f(t)p_B]\, dtdx + \int_{\underline{x}}^{1} \int_{\underline{t}}^{\bar{t}} [f(t)p_B]\, dtdx = \left[\underline{d} - H(\underline{t}, \bar{t})\right]\left[2\underline{t} - \underline{d} - \bar{t} + d\ln\left(\underline{d}/\bar{t}\right)\right]/\left[2(\bar{t} - \underline{t})\right]$. Taking the derivative with respect to $\underline{d}$ we get $\partial\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}),H(\underline{t},\bar{t})-\underline{d})/\partial\underline{d} = -\left[\underline{d}+\bar{t}-2\underline{t}+(H(\underline{t},\bar{t})-2\underline{d})\ln(\underline{d}/\bar{t})\right]/\left[2(\bar{t}-\underline{t})\right]$. This expression is negative with $\bar{t}/\underline{t} \leq 2$. It follows that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}),H(\underline{t},\bar{t})-\underline{d}) < \Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}),H(\underline{t},\bar{t}))$. Hence, for any $0 \leq \underline{d} \leq H(\underline{t},\bar{t})$ we have that $\Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}),H(\underline{t},\bar{t})-\underline{d}) < \Pi_B^{\emptyset|\emptyset}(H(\underline{t},\bar{t}),H(\underline{t},\bar{t}))$, with equality if $\underline{d}=0$, hence, firm B does not have an incentive to deviate downwards when firm A sets $p_A = H(\underline{t},\bar{t})$. *Q.E.D.*

Proof of Proposition 1. We derive equilibrium prices and profits of the firms in different information scenarios. We first consider the symmetric information scenarios.

Claim 1. Let $\underline{t}=0$. *Consider the information scenario* $\{X,X\}$. *In equilibrium firm* i *sets* $p_i^*(x)=2\bar{t}\,|1-2x|/3$ *on its own turf and* $p_i^*(x)=\bar{t}\,|1-2x|/3$ *on the competitor's turf. Firm* i *serves consumers with* $t \geq \bar{t}/3$ *on its own turf and consumers with* $t < \bar{t}/3$ *on the competitor's turf and realizes profit* $\Pi_i^{X|X} = \bar{t}/8$.

Proof of Claim 1. As firms are symmetric, we only analyze pricing strategies on firm A' turf. Consider first $x < 1/2$. A consumer in this region chooses firm A if $t \geq t^c$. Both firms treat the consumer transportation cost parameter as a random variable and maximize their expected profits for a given value of x: $E\left[\Pi_A^{X|X}|x<1/2\right] = p_A \Pr\{t \geq t^c\}$ and $E\left[\Pi_B^{X|X}|x<1/2\right] = p_B \Pr\{t < t^c\}$. Solving the corresponding maximization problems yields equilibrium prices $p_A^*(x)=2\bar{t}(1-2x)/3$ and $p_B^*(x)=\bar{t}(1-2x)/3$ for $x<1/2$. Consider now $x=1/2$. It follows from Assumption 1 that $E\left[\Pi_B^{X|X}|x=1/2\right]=0$, whenever $p_B \geq p_A$. Firm B will always undercut firm A if $p_A^*(1/2)>0$, hence, it must be that $p_A^*(1/2)=p_B^*(1/2)=0$. From $p_A^*(x)$ and $p_B^*(x)$ when $x \leq 1/2$ we get $t^c=\bar{t}/3$. To compute firm A's equilibrium profit we sum up the revenues across the demand regions: $\Pi_A^{X|X} = \int_0^{1/2}\int_{t^c}^{\bar{t}}\left[f(t)2\bar{t}(1-2x)/3\right]dtdx + \int_{1/2}^{1}\int_0^{t_c}\left[f(t)\bar{t}(2x-1)/3\right]dtdx = 5A(\underline{t},\bar{t})/18$. Since firms are symmetric, $\Pi_B^{X|X} = \Pi_A^{X|X}$. This completes the proof of Claim 1.

Claim 2. Let $\underline{t}>0$ *and* $k \leq 2$. *Consider the information scenario* $\{X,X\}$. *In equilibrium firm* i *sets* $p_i^*(x)=\underline{t}\,|1-2x|$ *on its own turf and* $p_i^*(x)=0$ *on the competitor's turf. Every firm serves all consumers on its own turf and realizes profit* $\Pi_i^{X|X} = \underline{t}/4$.

Proof of Claim 2. As firms are symmetric, we only analyze firms' pricing strategies on firm A' turf. A consumer in this region chooses firm A if $t \geq t^c$. Both firms treat consumer transportation cost as a random variable and maximize their expected profits for a given value of x: $E\left[\Pi_A^{X|X}|x<1/2\right] = p_A \Pr\{t \geq t^c\}$ and $E\left[\Pi_B^{X|X}|x<1/2\right] = p_B \Pr\{t < t^c\}$. Solving the corresponding maximization problems yields equilibrium prices $p_A^*(x) = \underline{t}(1-2x)$ and $p_B^*(x) = 0$ for $x < 1/2$. Consider now $x = 1/2$. It follows from Assumption 1 that $E\left[\Pi_B^{X|X}|x = 1/2\right] = 0$, whenever $p_B \geq p_A$. Firm B will always undercut firm A if $p_A^*(1/2) > 0$, hence, it must hold that $p_A^*(1/2) = p_B^*(1/2) = 0$. On its turf firm A serves all consumers. Equilibrium profits are: $\Pi_A^{X|X} = \int\limits_0^{1/2}\int\limits_{\underline{t}}^{\bar{t}} [f(t)\underline{t}(1-2x)]\, dtdx = \underline{t}/4 = \Pi_B^{X|X}$. This completes the proof of Claim 2.

Claim 3. Consider the information scenario $\{T, T\}$. *In equilibrium firm* i *sets* $p_i^*(t) = t$ *and serves all consumers on its own turf. Firms realize profits* $\Pi_i^{T|T} = A(\underline{t}, \bar{t})/2$.

Proof of Claim 3. Both firms treat consumer brand preference as a random variable and maximize their expected profits: $E\left[\Pi_A^{T|T}|t\right] = f(t)p_A \Pr\{x \leq x^c\}$ and $E\left[\Pi_B^{T|T}|t\right] = f(t)p_B \Pr\{x > x^c\}$, which yields $p_A^*(t) = p_B^*(t) = t$ and $x^c = 1/2$. Firm A realizes the profit $\Pi_A^{T|T} = \int\limits_0^{x^c}\int\limits_{\underline{t}}^{\bar{t}} [f(t)t]\, dtdx = A(\underline{t}, \bar{t})/2$. It holds that $\Pi_A^{T|T} = \Pi_B^{T|T}$. This completes the proof of Claim 3.

Claim 4. Consider the information scenario $\{XT, XT\}$. *In equilibrium firm* i *sets* $p_i^*(x,t) = t\,|1-2x|$ *on its own turf and* $p_i^*(x,t) = 0$ *on the competitor's turf, and serves all consumers on its own turf. Firms realizes profits* $\Pi_i^{XT|XT} = A(\underline{t}, \bar{t})/4$.

Proof of Claim 4. As firms are symmetric, we only consider pricing decisions in the region $x \in [0, 1/2]$. Here firm A has a cost advantage, hence, its best-response to any price of firm B is to render consumers indifferent by setting $p_A(p_B) = p_B + t(1-2x)$. Firm B's best-response is to undercut firm A's price by setting $p_B(p_A) = p_A - t(1-2x) - \varepsilon$ whenever it is feasible (i.e., $p_A - t(1-2x) > 0$), with $\epsilon > 0$. Otherwise, firm B sets $p_B = 0$. As undercutting is not possible in equilibrium, we get $p_B^*(x,t) = 0$ and $p_A^*(x,t) = t(1-2x)$. Firm A's profit is $\Pi_A^{XT|XT} = \int\limits_0^{1/2}\int\limits_{\underline{t}}^{\bar{t}} [f(t)t(1-2x)]\, dtdx = A(\underline{t}, \bar{t})/4$. Due to the symmetry, $\Pi_A^{XT|XT} = \Pi_B^{XT|XT}$. This completes the proof of Claim 4.

We now turn to the asymmetric information scenarios.

Claim 5. Let $\underline{t} = 0$. *Consider the information scenario* $\{X, \emptyset\}$. *If* $x < 1/2 - p_B^*/(2\bar{t})$, *then in equilibrium firm* A *sets* $p_A^*(x) = \left(\bar{t}\,(1-2x) + p_B^*\right)/2$ *and serves consumers with* $\bar{t}/2 - p_B^*/\left[2(1-2x)\right] \leq t \leq \bar{t}$. *If* $1/2 - p_B^*/(2\bar{t}) \leq x \leq 1/2$, *then firm* A *sets* $p_A^*(x) = p_B^*$ *and serves consumers with* $t \leq \bar{t}$. *If* $1/2 < x < 1/2 + p_B^*/(4\bar{t})$, *then firm* A *sets* $p_A^*(x) = p_B^* - \bar{t}(2x-1)$ *and serves consumers with* $t \leq \bar{t}$. *If* $x \geq 1/2 + p_B^*/(4\bar{t})$, *then firm* A *sets* $p_A^*(x) = p_B^*/2$ *and serves consumers with* $t \leq p_B^*/\left[2(2x-1)\right]$. *Firm* B *sets* $p_B^* = 0.47\bar{t}$. *Firms realize profits* $\Pi_A^{X|\emptyset} = 0.32\bar{t}$

and $\Pi_B^{X|\emptyset} = 0.12\bar{t}$.

Proof of Claim 5. On its own turf firm A maximizes expected profit $E\left[\Pi_A^{X|\emptyset}|x < 1/2\right] = \Pr\{t \geq t^c\} p_A$, which yields reaction functions $p_A(p_B) = \left(\bar{t}(1-2x) + p_B\right)/2$ if $0 \leq p_B < \bar{t}(1-2x)$ and $p_A(p_B) = p_B$ if $p_B \geq \bar{t}(1-2x)$. Moreover, $p_A(p_B) = p_B$ if $x = 1/2$. The reaction functions give $\underline{t}^c(x, p_B) = \bar{t}/2 - p_B/\left[2(1-2x)\right]$. Solving $\underline{t}^c(x, p_B) = \underline{t} = 0$ we get $\underline{x}(p_B) = 1/2 - p_B/\left(2\bar{t}\right)$. If $x \leq \underline{x}(p_B)$, firm A captures consumers with $t \geq \underline{t}^c(x, p_B)$, while it gets all consumers if $\underline{x}(p_B) < x \leq 1/2$. On the competitor's turf firm A maximizes the expected profit $E\left[\Pi_A^{X|\emptyset}|x > 1/2\right] = \Pr\{t < t^c\} p_A$, which yields the reaction functions $p_A(p_B) = p_B - \bar{t}(2x-1)$ if $p_B \geq 2\bar{t}(2x-1)$ and $p_A(p_B) = p_B/2$ if $0 \leq p_B < 2\bar{t}(2x-1)$. These reaction functions give $\bar{t}^c(x, p_B) = p_B/\left[2(2x-1)\right]$. Solving $\bar{t}^c(x, p_B) = \bar{t}$ we get $\bar{x}(p_B, \bar{t}) = 1/2 + p_B/(4\bar{t})$. If $1/2 < x < \bar{x}(p_B, \bar{t})$, then firm A gets all consumers, while it captures consumers with $t < \bar{t}^c(x, p_B)$ if $x \geq \bar{x}(p_B, \bar{t})$. Given firm A's reaction functions, firm B's profit is $\Pi_B^{X|\emptyset} = \int_0^{\underline{x}}\int_0^{\underline{t}^c} \left[f(t)p_B\right] dt dx + \int_{\bar{x}}^1\int_{\bar{t}^c}^{\bar{t}} \left[f(t)p_B\right] dt dx$. Maximization of the latter profit yields $p_B^* = 0.47\bar{t} < \bar{t}$, which implies that, indeed, $0 < \underline{x}(p_B^*) < 1/2$ and $1/2 < \bar{x}(p_B^*) < 1$. Firm A's profit is computed as $\Pi_A^{X|\emptyset} = \int_0^{\underline{x}}\int_{\underline{t}^c}^{\bar{t}} \left[f(t)(\bar{t}(1-2x) + p_B^*)/2\right] dt dx + \int_{\underline{x}}^{1/2}\int_0^{\bar{t}} \left[f(t)p_B^*\right] dt dx + \int_{1/2}^{\bar{x}}\int_0^{\bar{t}} \left[f(t)\left(p_B^* - \bar{t}(2x-1)\right)\right] dt dx + \int_{\bar{x}}^1\int_0^{\bar{t}^c} \left[f(t)\left(p_B^*/2\right)\right] dt dx$. Firms realize profits $\Pi_A^{X|\emptyset} = 0.32\bar{t}$ and $\Pi_B^{X|\emptyset} = 0.12\bar{t}$. This completes the proof of Claim 5.

Claim 6. Let $\underline{t} > 0$ and $k \leq 2$. *Consider the information scenario* $\{X, \emptyset\}$. *In equilibrium, on its own turf firm A sets* $p_A^*(x) = p_B^* + \underline{t}(1-2x)$ *and serves all consumers. If* $1/2 < x < 1/2 + p_B^*/\left[2(2\bar{t} - \underline{t})\right]$, *then firm A sets* $p_A^*(x) = p_B^* - \bar{t}(2x-1)$ *and serves all consumers. If* $1/2 + p_B^*/\left[2(2\bar{t} - \underline{t})\right] \leq x \leq 1/2 + p_B^*/(2\underline{t})$, *then firm A sets* $p_A^*(x) = \left[p_B^* - \underline{t}(2x-1)\right]/2$ *and serves consumers with* $t < \underline{t}/2 + p_B^*/\left[2(2x-1)\right]$. *If* $x > 1/2 + p_B^*/(2\underline{t})$, *then firm A sets* $p_A^*(x) = 0$ *and serves no consumers. Firm B sets* $p_B^* = \widetilde{H}(\underline{t}, \bar{t})$. *Firms realize profits* $\Pi_A^{X|\emptyset} = 5\widetilde{H}(\underline{t}, \bar{t})/8 + \underline{t}/4$ *and* $\Pi_B^{X|\emptyset} = \widetilde{H}(\underline{t}, \bar{t})/4$.

Proof of Claim 6. On its own turf firm A maximizes the expected profit $E\left[\Pi_A^{X|\emptyset}|x < 1/2\right] = \Pr\{t \geq t^c\} p_A$, which yields the reaction function $p_A(p_B) = p_B + \underline{t}(1-2x)$. Moreover, if $x = 1/2$, then $p_A(p_B) = p_B$. Firm A captures all consumers on its own turf. On the competitor's turf firm A maximizes the expected profit $E\left[\Pi_A^{X|\emptyset}|x > 1/2\right] = \Pr\{t < t^c\} p_A$, which yields the reaction functions $p_A(p_B) = p_B - \bar{t}(2x-1)$ if $p_B \geq (2\bar{t} - \underline{t})(2x-1)$, $p_A(p_B) = \left[p_B - \underline{t}(2x-1)\right]/2$ if $\underline{t}(2x-1) < p_B < (2\bar{t} - \underline{t})(2x-1)$ and $p_A(p_B) = 0$ if $p_B \leq \underline{t}(2x-1)$. These reaction functions give $t^c(x, p_B) = \underline{t}/2 + p_B/\left[2(2x-1)\right]$. Solving $t^c(x, p_B) = \bar{t}$ we get $\overline{x}(p_B,\overline{t}) = 1/2 + p_B/\left[2(2\overline{t}-\underline{t})\right]$, while $t^c(x,p_B) = \underline{t}$ yields $\underline{x}(p_B,\underline{t}) = 1/2 + p_B/(2\underline{t})$. If $1/2 < x < \overline{x}(p_B,\overline{t})$, then firm A captures all consumers; if $\overline{x}(p_B,\overline{t}) \leq x \leq \underline{x}(p_B,\underline{t})$, then firm A serves consumers with $t < t^c(x,p_B)$; finally, firm A does not get any consumers if $x > \underline{x}(p_B,\underline{t})$. Given firm A's reaction functions, firm B's profit is $\Pi_B^{X|\emptyset} = \int_{\overline{x}}^{\underline{x}}\int_{t^c}^{\overline{t}} [f(t)p_B]\, dtdx + \int_{\underline{x}}^{1}\int_{\underline{t}}^{\overline{t}} [f(t)p_B]\, dtdx$. Maximizing with respect to p_B yields $p_B^* = \widetilde{H}(\underline{t},\overline{t})$. Under the constraint $1 < k \leq 2$ it holds that $\widetilde{H}(\underline{t},\overline{t}) < \underline{t}$, hence, indeed, $1/2 < \overline{x}(p_B^*,\overline{t}) < \underline{x}(p_B^*,\underline{t}) < 1$. Firm A's profit is computed as $\Pi_A^{X|\emptyset} = \int_{0}^{1/2}\int_{\underline{t}}^{\overline{t}} [f(t)\left(p_B^* + \underline{t}(1-2x)\right)]\, dtdx + \int_{1/2}^{\overline{x}}\int_{\underline{t}}^{\overline{t}} \left[f(t)\left(p_B^* - \overline{t}(2x-1)\right)\right] dtdx + \int_{\overline{x}}^{\underline{x}}\int_{\underline{t}}^{t^c} [f(t)\left(p_B^* - \underline{t}(2x-1)\right)/2]\, dtdx$. Firms realize profits $\Pi_A^{X|\emptyset} = 5\widetilde{H}(\underline{t},\overline{t})/8 + \underline{t}/4$ and $\Pi_B^{X|\emptyset} = \widetilde{H}(\underline{t},\overline{t})/4$. This completes the proof of Claim 6.

Claim 7. Consider the information scenario $\{T,\emptyset\}$*. If* $\underline{t} = 0$*, then in equilibrium firm* A *sets* $p_A^*(t) = p_B^* - t$ *and serves all consumers if* $t < p_B^*/3$*, if* $t \geq p_B^*/3$*, then it sets* $p_A^*(t) = (p_B^* + t)/2$ *and serves consumers with* $x < 1/4 + p_B^*/(4t)$*. Firm* B *sets* $p_B^* \approx 0.85\overline{t}$*. Firms realize profits* $\Pi_A^{T|\emptyset} \approx 0.53\overline{t}$ and $\Pi_B^{T|\emptyset} \approx 0.23\overline{t}$*. If* $\underline{t} > 0$ *and* $k \leq 2$*, then in equilibrium firms set* $p_A^*(t) = (t + p_B^*)/2$ *and* $p_B^* = 3H(\underline{t},\overline{t})/2$*. Firm* A *serves all consumers if* $x < 1/4 + p_B^*/(4\overline{t})$*, serves consumers with* $t < p_B^*/(4x-1)$ *if* $1/4 + p_B^*/(4\overline{t}) \leq x \leq 1/4 + p_B^*/(4\underline{t})$ *and serves no consumers when* $x > 1/4 + p_B^*/(4\underline{t})$*. Equilibrium profits are* $\Pi_A^{T|\emptyset} = 21H(\underline{t},\overline{t})/32 + A(\underline{t},\overline{t})/8$ *and* $\Pi_B^{T|\emptyset} = 9H(\underline{t},\overline{t})/16$*.*

Proof of Claim 7. Firm A takes p_B as given and maximizes its expected profit $E\left[\Pi_A^{T|\emptyset}|t\right] = f(t)p_A \Pr\{x \leq x^c\}$, which yields firm A's equilibrium strategies as $p_A(p_B) = (p_B + t)/2$ if $p_B \leq 3t$ and $p_A(p_B) = p_B - t$ if $p_B > 3t$. From these reaction functions we get $t^c(x,p_B) = p_B/(4x-1)$. Assume that $\underline{t} > 0$ and $1 < k \leq 2$. Solving $t^c(x,p_B) = \overline{t}$ and $t^c(x,p_B) = \underline{t}$ we get $\overline{x}(p_B,\overline{t}) = 1/4 + p_B/(4\overline{t})$ and $\underline{x}(p_B,\underline{t}) = 1/4 + p_B/(4\underline{t})$. Depending on the relation between $\underline{x}(p_B^*,\underline{t})$ and 1 two cases are possible in equilibrium: $\underline{x}(p_B^*,\underline{t}) \geq 1$ if $3\underline{t} \leq p_B^* < 3\overline{t}$ and $\underline{x}(p_B^*,\underline{t}) < 1$ if $p_B^* < 3\underline{t}$. We show that $3\underline{t} \leq p_B^* < 3\overline{t}$ does not emerge in equilibrium. Assume that $3\underline{t} \leq p_B^* < 3\overline{t}$. Firm B chooses its price to maximize the profit $\Pi_B^{T|\emptyset} = \int_{\overline{x}}^{1}\int_{t^c}^{\overline{t}} f(t)p_B dtdx$. The optimal price p_B solves equation $p_B\left[1 + \ln(9)\right] - 3\overline{t} - 2p_B \ln\left(p_B/\overline{t}\right) = 0$. There is no analytical solution to this problem, the value $p_B \approx 0.85\overline{t}$ is, however, a good numerical approximation which fulfills the second order condition. Note that $0.85\overline{t} < 3\underline{t}$ given that $1 < k \leq 2$, hence, $3\underline{t} \leq p_B^* < 3\overline{t}$ cannot hold in equilibrium. Assume further that p_B^* satisfies $p_B^* < 3\underline{t}$. Firm B maximizes the profit

$\Pi_B^{T|\emptyset} = \int_{\overline{x}}^{\underline{x}} \int_{t^c}^{\overline{t}} [f(t)p_B]\, dtdx + \int_{\underline{x}}^{1} \int_{\underline{t}}^{\overline{t}} [f(t)p_B]\, dtdx$, which yields $p_B^* = 3H(\underline{t},\overline{t})/2$. Under the constraint $1 < k \leq 2$ it holds that $3H(\underline{t},\overline{t})/2 < 3\underline{t}$, hence, $p_B^* = 3H(\underline{t},\overline{t})/2$ is, indeed, the equilibrium price. Firm A' profits are computed as $\Pi_A^{T|\emptyset} = \int_0^{\overline{x}} \int_{\underline{t}}^{\overline{t}} [f(t)\,(p_B^* + t)\,/2]\, dtdx + \int_{\overline{x}}^{\underline{x}} \int_{\underline{t}}^{t^c} [f(t)\,(p_B^* + t)\,/2]\, dtdx$. Equilibrium profits are $\Pi_A^{T|\emptyset} = 21H(\underline{t},\overline{t})/32 + A(\underline{t},\overline{t})/8$ and $\Pi_B^{T|\emptyset} = 9H(\underline{t},\overline{t})/16$. Consider now $\underline{t} = 0$. Maximization of $\Pi_B^{T|\emptyset} = \int_{\overline{x}}^{1} \int_{t^c}^{\overline{t}} [f(t)p_B]\, dtdx$ yields $p_B^* \approx 0.85\overline{t}$. Firm A' profits are computed as $\Pi_A^{T|\emptyset} = \int_0^1 \int_0^{\frac{p_B^*}{3}} [f(t)\,(p_B^* - t)]\, dtdx + \int_0^{\overline{x}} \int_{\frac{p_B^*}{3}}^{\overline{t}} [f(t)\,(p_B^* + t)\,/2]\, dtdx + \int_{\overline{x}}^{1} \int_{\frac{p_B^*}{3}}^{t^c} [f(t)\,(p_B^* + t)\,/2]\, dtdx$. Firms realize profits $\Pi_A^{T|\emptyset} \approx 0.53\overline{t}$ and $\Pi_B^{T|\emptyset} \approx 0.23\overline{t}$. This completes the proof of Claim 7.

Claim 8. Consider the information scenario $\{XT, \emptyset\}$. If $\underline{t} = 0$, then in equilibrium firms A and B set $p_A^*(x,t) = \max\{p_B^* + t(1-2x), 0\}$ and $p_B^* \approx 0.28\overline{t}$. If $x < 1/2 + p_B^*/(2\overline{t})$, firm A serves all consumers; if $x \geq 1/2 + p_B^*/(2\overline{t})$, firm A serves consumers with $t < p_B^*/\,(2x-1)$. Equilibrium profits are $\Pi_A^{XT|\emptyset} \approx 0.32\overline{t}$ and $\Pi_B^{XT|\emptyset} \approx 0.05\overline{t}$. If $\underline{t} > 0$ and $k \leq 2$, in equilibrium firms set $p_A^*(x,t) = \max\{p_B^* + t(1-2x), 0\}$ and $p_B^* = H(\underline{t},\overline{t})/2$. If $x < 1/2 + p_B^*/\left(2\overline{t}\right)$ firm A serves all consumers; if $1/2 + p_B^*/\left(2\overline{t}\right) \leq x \leq 1/2 + p_B^*/\,(2\underline{t})$ firm A serves consumers with $t < p_B^*/\,(2x-1)$; if $x > 1/2 + p_B^*/\,(2\underline{t})$ firm A serves no consumers. Equilibrium profits are $\Pi_A^{XT|\emptyset} = 5H(\underline{t},\overline{t})/16 + A(\underline{t},\overline{t})/4$ and $\Pi_B^{XT|\emptyset} = H(\underline{t},\overline{t})/8$.

Proof of Claim 8. Consider first $\underline{t} > 0$ and $k \leq 2$. Firm A maximizes its profit given p_B. Firm A's optimal strategy is $p_A(p_B) = \max\{0, t(1-2x) + p_B\}$, which gives $t^c(x, p_B) = p_B/(2x-1)$ and $\overline{x}(p_B, \overline{t}) = 1/2 + p_B/(2\overline{t})$ and $\underline{x}(p_B, \underline{t}) = 1/2 + p_B/(2\underline{t})$. Depending on the relation between $\underline{x}(p_B^*, \underline{t})$ and 1 two cases are possible in equilibrium: $\underline{x}(p_B^*, \underline{t}) \leq 1$ if $p_B^* \leq \underline{t}$ and $\underline{x}(p_B^*, \underline{t}) > 1$ if $\underline{t} < p_B^* < \overline{t}$. We show first that $\underline{t} < p_B^* < \overline{t}$ cannot characterize firm B's equilibrium price. Assume that $\underline{t} < p_B^* < \overline{t}$. Firm B sets p_B to maximize the profit $\Pi_B^{XT|\emptyset} = \int_{\overline{x}}^{1} \int_{t^c}^{\overline{t}} [f(t)p_B]\, dtdx$ given firm A's optimal strategy. The optimal price p_B solves the equation $p_B\left[2\ln\left(p_B/\overline{t}\right) - 1\right] + \overline{t} = 0$. There is no analytical solution to this problem, the value $p_B \approx 0.28\overline{t}$ is, however, a good numerical approximation, which fulfills the second order condition. Note that $0.28\overline{t} < \underline{t}$ given $1 < k \leq 2$, hence, $\underline{t} < p_B^* < \overline{t}$ is not possible in equilibrium. We show next that in equilibrium $p_B^* \leq \underline{t}$. Assume this is the case. Firm B sets p_B to maximize the profit $\Pi_B^{XT|\emptyset} = \int_{\overline{x}}^{\underline{x}} \int_{t^c}^{\overline{t}} [f(t)p_B]\, dtdx + \int_{\underline{x}}^{1} \int_{\underline{t}}^{\overline{t}} [f(t)p_B]\, dtdx = \left[p_B(\overline{t} - \underline{t} - p_B \ln(\overline{t}/\underline{t}))\right] / \left[2(\overline{t} - \underline{t})\right]$, which yields $p_B^* = H(\underline{t},\overline{t})/2$. Under the constraint $1 < k \leq 2$ it holds that $H(\underline{t},\overline{t})/2 <$

$\underline{t}$, hence, $p_B^* = H(\underline{t},\bar{t})/2$ is indeed the equilibrium price. Firm A's profit is computed as $\Pi_A^{XT|\emptyset} = \int_0^{\bar{x}}\int_{\underline{t}}^{\bar{t}} [f(t)(p_B^* + t(1-2x))]\, dtdx + \int_{\bar{x}}^{\underline{x}}\int_{\underline{t}}^{t^c} [f(t)(p_B^* + t(1-2x))]\, dtdx$. Equilibrium profits are $\Pi_A^{XT|\emptyset} = 5H(\underline{t},\bar{t})/16 + A(\underline{t},\bar{t})/4$ and $\Pi_B^{XT|\emptyset} = H(\underline{t},\bar{t})/8$. Consider finally $\underline{t} = 0$, in which case $\Pi_B^{XT|\emptyset} = \int_{\bar{x}}^{1}\int_{\bar{t}^c}^{\bar{t}} [f(t)p_B]\, dtdx$ and $p_B^* \approx 0.28\bar{t}$. Firm A's profit is computed as $\Pi_A^{XT|\emptyset} = \int_0^{\bar{x}}\int_0^{\bar{t}} [f(t)(p_B^* + t(1-2x))]\, dtdx + \int_{\bar{x}}^{1}\int_0^{t^c} [f(t)(p_B^* + t(1-2x))]\, dtdx$. Equilibrium profits are $\Pi_A^{XT|\emptyset} \approx 0.32\bar{t}$ and $\Pi_B^{XT|\emptyset} \approx 0.05\bar{t}$. This completes the proof of Claim 8.

Claim 9. Consider the information scenario $\{XT, X\}$. In equilibrium firm A sets $p_A^*(x,t) = t(1-2x)$ if $x \le 1/2$ and $p_A^*(x,t) = (2x-1)\max\{0, \bar{t}/2 - t\}$ if $x > 1/2$. Firm B sets $p_B^*(x) = 0$ if $x \le 1/2$ and $p_B^*(x) = (2x-1)t^m$ if $x > 1/2$ and serves consumers with $x > 1/2$ and $t \ge t^m$, where $t^m = \max\{\bar{t}/2, \underline{t}\}$. Firms realize profits $\Pi_A^{XT|X} = 5\bar{t}/32$ and $\Pi_B^{XT|X} = \bar{t}/16$ if $\underline{t} = 0$ and $\Pi_A^{XT|X} = A(\underline{t},\bar{t})/4$ and $\Pi_B^{XT|X} = \underline{t}/4$ if $\underline{t} > 0$ and $k \le 2$.

Proof of Claim 9. Firm B treats t as a random variable and maximizes its expected profits given firm A's equilibrium strategy separately in the regions $x \le 1/2$ and $x > 1/2$. In the region $x \le 1/2$ firm A can undercut any price set by firm B, hence, $p_B^*(x) = 0$ for $x \le 1/2$. In the region $x > 1/2$ firm A can undercut firm B as long as it can set a non-negative price, which is the case if $t(2x-1) < p_B(x)$ holds. Firm B's expected profit in the region $x > 1/2$ is $E\left[\Pi_B^{XT|X} | x > 1/2\right] = p_B \Pr\{t(2x-1) \ge p_B | x > 1/2\}$. Maximization of the latter profit yields the optimal price of firm B: $p_B^*(x) = \bar{t}(x - 1/2)$ if $\bar{t} > 2\underline{t}$ and $p_B^*(x) = \underline{t}(2x-1)$ if $\bar{t} \le 2\underline{t}$. If $\underline{t} = 0$, then $p_B^*(x) = \bar{t}(x-1/2)$, which yields $t^c = \bar{t}/2$ and firm B serves consumers with $t \ge t^c$ on its turf. Firms realize profits $\Pi_A^{XT|X} = \int_0^{1/2}\int_0^{\bar{t}} [f(t)t(1-2x)]\, dtdx + \int_{1/2}^{1}\int_0^{\frac{\bar{t}}{2}} \left[f(t)\left(\bar{t} - 2t\right)(x - 1/2)\right] dtdx = 5\bar{t}/32$ and $\Pi_B^{XT|X} = \int_{1/2}^{1}\int_{\frac{\bar{t}}{2}}^{\bar{t}} \left[f(t)\bar{t}(x-1/2)\right] dtdx = \bar{t}/16$. If $\underline{t} > 0$ and $k \le 2$, then $p_B^*(x) = \underline{t}(2x-1)$ and firm B serves all consumers on its turf. Firms realize profits $\Pi_A^{XT|X} = \int_0^{1/2}\int_{\underline{t}}^{\bar{t}} [f(t)t(1-2x)]\, dtdx = A(\underline{t},\bar{t})/4$ and $\Pi_B^{XT|X} = \int_{1/2}^{1}\int_{\underline{t}}^{\bar{t}} [f(t)\underline{t}(2x-1)]\, dtdx = \underline{t}/4$. This completes the proof of Claim 9.

Claim 10. Consider the information scenario $\{XT, T\}$. In equilibrium firm A sets $p_A^*(x,t) = \max\{t/2 + t(1-2x), 0\}$ and serves consumers with $x < 3/4$. Firm B sets $p_B^*(t) = t/2$. Firms realize profits $\Pi_A^{XT|T} = 9A(\underline{t},\bar{t})/16$ and $\Pi_B^{XT|T} = A(\underline{t},\bar{t})/8$.

Proof of Claim 10. Since firm A has full information, it can undercut the rival as long as it can set a non-negative price. This translates into firm A's equilibrium strategy as $p_A(p_B) = \max\{p_B + t(1-2x), 0\}$. Undercutting is possible whenever $t(2x-1) < p_B(t)$. Firm B treats x as a random variable and maximizes its expected profit given firm A's equilibrium strategy: $E\left[\Pi_B^{XT|T}|t\right] = f(t)p_B \Pr\{t(2x-1) \geq p_B\}$. Solving the maximization problem for p_B yields $p_B^*(t) = t/2$, which gives $p_A^* = \max\{t/2 + t(1-2x), 0\}$, such that $t/2 + t(1-2x)$ is positive whenever $x < x^c = 3/4$. Firms A and B realize profits $\Pi_A^{XT|T} = \int_0^{x^c}\int_{\underline{t}}^{\bar{t}} f(t)\left(t/2 + t(1-2x)\right) dtdx = 9A(\underline{t},\bar{t})/16$ and $\Pi_B^{XT|T} = \int_{x^c}^{1}\int_{\underline{t}}^{\bar{t}} f(t)(t/2)dtdx = A(\underline{t},\bar{t})/8$, respectively. This completes the proof of Claim 10.

The equilibrium prices and profits stated in Claims 1-10 are given in Tables 1 and 2. *Q.E.D.*

Proof of Proposition 2. With $\underline{t} = 0$ the comparison of profits across different information scenarios is straightforward and yields $\Pi_i^{XT|XT} < \Pi_i^{X|X} < \Pi_i^{T|T}$. Consider now $\underline{t} > 0$ and $k \leq 2$. It is straightforward that $\Pi_i^{X|X} < \Pi_i^{XT|XT}$ and $\Pi_i^{\emptyset|\emptyset} < \Pi_i^{T|T}$. By substituting in k into $\Pi_i^{\emptyset|\emptyset} - \Pi_i^{XT|XT}$ and reaaranging, we get $4\ln k(\Pi_i^{\emptyset|\emptyset} - \Pi_i^{XT|XT})/\left(\bar{t} + \underline{t}\right) = 4(k-1)/(k+1) - \ln k$. The second derivative of the RHS of the latter equality is negative on the interval $1 < k \leq 2$, while the first derivative is positive if $k = 2$, hence, the RHS increases on the interval $1 < k \leq 2$. As it approaches zero if $k \to 1$, we get that $\Pi_i^{XT|XT} < \Pi_i^{\emptyset|\emptyset}$. These comparisons yield the ranking $\Pi_i^{X|X} < \Pi_i^{XT|XT} < \Pi_i^{\emptyset|\emptyset} < \Pi_i^{T|T}$. *Q.E.D.*

Proof of Proposition 3. Consider first the case $\underline{t} = 0$. We use the demand regions and equilibrium prices as stated in the proof of Proposition 1 to find consumer surplus. As in the information scenarios $\{XT, XT\}$ and $\{T, T\}$ every firm serves only its own turf, we use the formula $\int_0^{1/2}\int_0^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_0^{\bar{t}} U_B(x,t)f(t)dtdx$ to compute $CS^{XT|XT} = v - 3A(\underline{t},\bar{t})/4$ and $CS^{T|T} = v - 5A(\underline{t},\bar{t})/4$. We also obtain $CS^{X|X} = \int_0^{1/2}\int_{t^c}^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_0^{t^c} U_A(x,t)f(t)dtdx + \int_0^{1/2}\int_0^{t^c} U_B(x,t)f(t)dtdx + \int_{1/2}^{1}\int_{t^c}^{\bar{t}} U_B(x,t)f(t)dtdx = v - 31A(\underline{t},\bar{t})/36$. The comparison is straightforward and yields the ranking $CS^{T|T} < CS^{X|X} < CS^{XT|XT}$. Social welfare follows immediately from adding up profits and consumer surplus as $SW^{I_A|I_B} = CS^{I_A|I_B} + \Pi_A^{I_A|I_B} + \Pi_B^{I_A|I_B}$, from where we get $SW^{XT|XT} = SW^{T|T} = v - A(\underline{t},\bar{t})/4$ and $SW^{X|X} = v - 11A(\underline{t},\bar{t})/36$.

The comparison is straightforward and yields the ranking $SW^{X|X} < SW^{XT|XT} = SW^{T|T}$.

Consider now $\underline{t} > 0$ and $k \leq 2$. Note that in all the symmetric information scenarios firms share the market equally, hence, social welfare is same and is given by $SW^{XT|XT} =$

$SW^{T|T} = SW^{X|X} = SW^{\emptyset|\emptyset} = v - 2\int_0^{1/2}\int_{\underline{t}}^{\bar{t}} txf(t)dtdx = v - A(\underline{t},\bar{t})/4$. We can use the formula $CS^{I_A|I_B} = SW^{I_A|I_B} - \Pi_A^{I_A|I_B} - \Pi_B^{I_A|I_B}$ to derive consumer surplus as $CS^{T|T} = v - 5A(\underline{t},\bar{t})/4$, $CS^{\emptyset|\emptyset} = v - H(\underline{t},\bar{t}) - A(\underline{t},\bar{t})/4$, $CS^{XT|XT} = v - 3A(\underline{t},\bar{t})/4$ and $CS^{X|X} = v - (\bar{t} + 5\underline{t})/8$. Since social welfare is same in all the symmetric information scenarios, the ranking of consumer surplus follows directly from the ranking of the profits as $CS^{T|T} < CS^{\emptyset|\emptyset} < CS^{XT|XT} < CS^{X|X}$. *Q.E.D.*

Proof of Proposition 4. The comparison of joint profits in the case of mobile consumers is straightforward and shows that only dataset T is shared, in the information scenario $\{XT, X\}$. We now turn to the case of immobile consumers. Many comparisons are straightforward using $H(\underline{t},\bar{t}) < A(\underline{t},\bar{t})$. We only consider the non-trivial cases. Let $\Pi_{A+B}^{I_A|I_B}$ denote the sum of profits in the scenario $\{I_A, I_B\}$. We first show that dataset X is not shared in the scenario $\{XT, \emptyset\}$. By substituting k into $\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|X}$ and rearranging we get $16\ln k(\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|X})/\underline{t} = 7(k-1) - 4\ln k$. The LHS of the latter equation increases on the interval $1 < k \leq 2$ and approaches zero when $k \to 1$, hence, $\Pi_{A+B}^{XT|\emptyset} > \Pi_{A+B}^{XT|X}$. We next show that both datasets together are not shared in this information either. Substituting k into $\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|XT}$ and rearranging yields $16\ln k(\Pi_{A+B}^{XT|\emptyset} - \Pi_{A+B}^{XT|XT})/\underline{t} = 7(k-1) - 2(k+1)\ln k$. The second derivative of the RHS of the latter equation is negative on the interval $1 < k \leq 2$ and the first derivative is positive at the point $k = 2$, hence, the LHS increases on the whole interval. Note, finally, that the RHS approaches zero when $k \to 1$, hence, $\Pi_{A+B}^{XT|\emptyset} > \Pi_{A+B}^{XT|XT}$. There is no information sharing in the scenario $\{T, \emptyset\}$. By substituting k into $\Pi_{A+B}^{T|\emptyset} - \Pi_{A+B}^{T|T}$ and rearranging we get $32\ln k(\Pi_{A+B}^{T|\emptyset} - \Pi_{A+B}^{T|T})/\underline{t} = 39(k-1) - 14(k+1)\ln k$. The second derivative of the RHS of the latter equation is negative on the interval $1 < k \leq 2$ and the first derivative is positive at the point $k = 2$, hence, the LHS increases on the whole interval. Note, finally, that the RHS approaches zero when $k \to 1$, it follows that $\Pi_{A+B}^{T|\emptyset} > \Pi_{A+B}^{T|T}$. Finally, we show that dataset X is not shared in the information scenario $\{X, \emptyset\}$. Substituting k into $\Pi_{A+B}^{X|\emptyset} - \Pi_{A+B}^{X|X}$ and rearranging yields $8\ln(2k-1)(\Pi_{A+B}^{X|\emptyset} - \Pi_{A+B}^{X|X})/\underline{t} = 7(k-1) - 2\ln(2k-1)$. The derivative of the RHS of the latter equation is positive on the interval $1 < k \leq 2$. Moreover, the RHS approaches zero when $k \to 1$, hence, it takes only positive values and $\Pi_{A+B}^{X|\emptyset} > \Pi_{A+B}^{X|X}$. *Q.E.D.*

Proof of Proposition 5. Consider first $\underline{t} = 0$. Consumer surplus in the information scenario $\{XT, X\}$ is $CS^{XT|X} = \int_0^{1/2}\int_0^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_0^{\bar{t}/2} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_{\bar{t}/2}^{\bar{t}} U_B(x,t)f(t)dtdx =$

$v-3\bar{t}/8$. As was shown in the proof of Proposition 3, $CS^{XT|XT}=v-3\bar{t}/8$, hence, $CS^{XT|X}=CS^{XT|XT}$. Social welfare follows immediately from adding up firms' profits and consumer surplus such that $SW^{XT|X}\approx v-0.16\bar{t}<SW^{XT|XT}=v-0.13\bar{t}$.

Consider now $\underline{t}>0$ and $k\leq 2$. Consumer surplus in the information scenario $\{XT,\emptyset\}$ is $CS^{XT|\emptyset}=\int_0^{\bar{x}}\int_{\underline{t}}^{\bar{t}}U_A(x,t)f(t)dtdx+\int_{\bar{x}}^{\underline{x}}\int_{\underline{t}}^{t^c}U_A(x,t)f(t)dtdx+\int_{\bar{x}}^{\underline{x}}\int_{t^c}^{\bar{t}}U_B(x,t)f(t)dtdx+\int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}}U_B(x,t)f(t)dtdx=v-\left[A(\underline{t},\bar{t})+H(\underline{t},\bar{t})\right]/2$ and social welfare is $SW^{XT|\emptyset}=v-\left[4A(\underline{t},\bar{t})+H(\underline{t},\bar{t})\right]/16$. Consumer surplus in the information scenario $\{XT,T\}$ is $CS^{XT|T}=\int_0^{x^c}\int_{\underline{t}}^{\bar{t}}U_A(x,t)f(t)dtdx+\int_{x^c}^{1}\int_{\underline{t}}^{\bar{t}}U_B(x,t)f(t)dtdx=v-A(\underline{t},\bar{t})$ and social welfare is $SW^{XT|T}=v-5A(\underline{t},\bar{t})/16$. Straightforward comparison yields that $CS^{XT|\emptyset}>CS^{XT|T}$ and $SW^{XT|\emptyset}>SW^{XT|T}$.

Consumers enjoy $CS^{XT|X}=\int_0^{1/2}\int_{\underline{t}}^{\bar{t}}U_A(x,t)f(t)dtdx+\int_{1/2}^{1}\int_{\underline{t}}^{\bar{t}}U_B(x,t)f(t)dtdx=v-(\bar{t}+3\underline{t})/8$ in the information scenario $\{XT,X\}$. We showed in the proof of Proposition 3 that $CS^{XT|XT}=v-3A(\underline{t},\bar{t})/4$, hence, $CS^{XT|X}>CS^{XT|XT}$. As in the information scenarios $\{XT,X\}$ and $\{XT,XT\}$ every firm serves consumers on the own turf, it follows that $SW^{XT|X}=SW^{XT|XT}$. *Q.E.D.*

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