A MULTI-PRODUCT ANALYSIS OF ENERGY DEMAND IN AGRICULTURAL SUBSECTORS

Adesoji Adelaja and Anwarul Hoque

Abstract

A multi-product cost function model was used to analyze energy demand in various agricultural subsectors. This approach has advantages over previously used approaches since it reduces aggregation bias, considers technological jointness, and provides various disaggregative measures related to energy input demand. When fitted to West Virginia county level data, labor and miscellaneous inputs in crop and livestock production were found to be substitutes for energy, while capital, machinery, and fertilizer were complementary to energy. Energy demand was inelastic and increases in machinery prices had the largest reduction effect on energy demand. Technological change was found to be capital, machinery, and fertilizer using, but it was labor and energy saving. Analyses indicated that the elasticity of demand for energy inputs with respect to livestock output was significantly larger than the elasticity with respect to crop output.

Key words: disaggregate analysis, energy demand, agriculture, multi-product cost function.

In recent years, a number of studies have estimated the demand for energy in various important sectors of the economy. Studies conducted for the manufacturing sector, for instance, include those by Berndt and Wood, Griffin and Gregory, Harper and Field, and those pertaining to agriculture include Miranowski and Mensah, and Lopez. Each of these studies estimated price elasticities of derived demand from aggregate production functions or from dual cost functions for the overall sector. Although aggregation is desirable because it allows simple estimation of sectoral energy demand, it has some inherent disadvantages. For instance, in agriculture, where the subsectors are mostly dissimilar, aggregation of agricultural products can intuitively be rejected on the basis of Solow's test of consistency in aggregation (Ray; Shumway).

An important implication of the rejection of aggregate models is the possibility that previous estimates of energy demand which are based on these aggregate models are biased. Since previous measures of energy demand elasticities for the agricultural sector are based on presumably misspecified aggregate production or dual cost functions, these estimates are subject to specification and/or aggregation error. The solution generally offered is the specification of separate and non-joint production functions for each commodity produced (Ball and Chambers; Hoque and Adelaja). Although such an approach allows disaggregate estimation of the energy demand structure for each commodity, it fails to account for the technological jointness that exists due to technical interdependencies in the production of several agricultural products (Just et al.; Shumway et al.). These technical interdependencies arise mainly due to the presence of fixed inputs in agricultural production and the fact that farmers' decisions on the production of individual commodities are sometimes interrelated (Shumway). Since non-joint models are not representative of the production behavior in a multi-product agricultural sector, an *a priori* assumption of non-jointness can significantly bias estimates of the coefficients of production technology. Besides, the non-joint models do not allow the estimation of several input-product and product-product relationships,

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such as the marginal rate of product transformation, which are sometimes useful in the analysis of energy demand.

An alternative solution to the problem is found in the specification of a dual multi-product cost function model for the agricultural sector. The multi-product cost function framework allows disaggregative analysis and consequently reduces aggregation bias while it takes technological jointness into consideration. It also can be used to derive several other important disaggregate measures such as individual product marginal costs and the elasticities of scale which are important indicators of the impact of output changes and energy price changes on energy demand. The multi-product cost function approach, therefore, seems to offer a solution in the attempt to reconcile the problems associated with excessive aggregation and those associated with estimating production functions by commodity.

Another advantage of the multi-product cost function approach comes from the general lack of data necessary to estimate production or cost functions by commodity in a sector. Information on the allocation of farm inputs to each commodity is usually not available for the agricultural sector because, at the farm level, such information is rarely recorded by farmers (Just et al.). This makes the estimation of individual production or cost functions difficult.

The purpose of this study is to present a multi-product translog cost function approach to the study of energy demand in a multi-product sector such as agriculture. The proposed model generates estimates which are free of the problems associated with aggregate models and the extremely disaggregate ones. The model is specified in such a way as to detect changes in the input mix, as well as the technology of production. Furthermore, because energy policies and problems are important at the state level, the model is applied to data from West Virginia. Such application of the model allows one to test the efficiency of the multi-product cost function model at an *extremum*; that is, in the analysis of small subsistent-like farming technology. In this paper, various other disaggregate measures of production technology that are related to energy demand in agriculture are also derived in order to present a comprehensive analysis of energy substitution and demand. Even though it was only applied to West Virginia data, the model has general applicability.

THE MULTI-PRODUCT COST FUNCTION MODEL

In an agricultural sector where m categories of products are jointly produced with n distinct categories of inputs, the joint technology can be implicitly represented by a production or transformation function. While the production function may not be expressible in explicit terms, its dual cost function can be expressed implicitly as follows (Diewert; Humphrey and Moroney):

$$(1)\ C = C(Q, P, t),$$

where C is total agricultural sector cost of production; Q is a vector of outputs (Q_r), r = 1, 2, ..., m; P is a vector of input prices (P_i), i = 1, 2, ..., n; and t is the time variable used to reflect technological change. The cost function in equation (1) must be linearly homogeneous, monotonic, continuous, and concave in input prices; non-negative and non-decreasing at all prices and output levels; and twice differentiable with respect to input prices and the products.

A form of the translog expansion of equation (1) is given as follows:

$$\begin{aligned}(2)\ \ln C = a_0 &+ \sum_{r=1}^{m} b_r \ln Q_r + \\ &\frac{1}{2} \sum_{r=1}^{m} \sum_{s=1}^{m} d_{rs} \ln Q_r \ln Q_s \\ &+ \sum_{i=1}^{n} e_i \ln P_i + \\ &\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} h_{ij} \ln P_i \ln P_j \\ &+ z_T t + \frac{1}{2} z_{TT} t^2 \\ &+ \sum_{r=1}^{m} \sum_{i=1}^{n} k_{ri} \ln Q_r \ln P_i \\ &+ \sum_{r=1}^{m} g_{Tr} t \ln Q_r \\ &+ \sum_{i=1}^{n} z_{Ti} t \ln P_i,\end{aligned}$$

where r, s = 1, 2, ..., m; i, j = 1, 2, ...,n; and T respectively denote output, input, and time. Since total cost will double if all input prices are simultaneously doubled, linear homogeneity of total cost in input prices must be imposed on the parameters in equation (2). The restriction implies that:

$$(3)\quad \sum_{i=1}^{n} h_{ij} = 0 \ (j = 1, 2, \ldots, n);$$

$$\sum_{r=1}^{m} k_{ri} = 0 \ (i = 1, 2, \ldots, n);$$

$$\sum_{i=1}^{n} e_i = 1; \text{ and } \sum_{i=1}^{n} z_{Ti} = 0.$$

Furthermore, since the Hessian matrix is symmetrical because $(\partial \ln C)^2/(\partial \ln P_i \partial \ln P_j) = h_{ij} = h_{ji}$ and $(\partial \ln C)^2/(\partial \ln Q_r \partial \ln Q_s) = d_{rs} = d_{sr}$, the restrictions $h_{ij} = h_{ji}$ and $d_{rs} = d_{sr}$ (for all i, j, r, and s) must be imposed.

Shephard's lemma allows one to derive the cost shares of each input as:

$$(4)\quad S_i = \frac{X_i P_i}{C} = \frac{\partial \ln C}{\partial \ln P_i} = e_i + \sum_{i=1}^{n} h_{ij} \ln P_j + \sum_{r=1}^{m} k_{ri} \ln Q_r + z_{Ti}\, t.$$

Furthermore, the assumption of price competition in all product and factor markets allows one to derive the revenue shares of each product as (Ray; Burgess):[1]

$$(5)\quad S_r = \frac{P_r Q_r}{C} = \frac{\partial C}{\partial Q_r}\frac{Q_r}{C} = \frac{\partial \ln C}{\partial \ln Q_r} = b_r + \sum_{s=1}^{m} d_{rs} \ln Q_s + \sum_{i=1}^{n} k_{ri} \ln P_i + g_{Tr}\, t.$$

In general, the cost function is usually estimated through the input and revenue share equations (equations (4) and (5)).

The Allen-Uzawa partial elasticity of substitution (σ_{ij}) for the agricultural sector is obtained as (McFadden; Ball and Chambers):

$$(6)\quad \sigma_{ij} = (h_{ij} + S_i S_j)/S_i S_j \text{ for all i and j, } i \neq j$$

and

$$\sigma_{ii} = (h_{ii} + S_i^2 - S_i)/S_i^2 \text{ for all i.}$$

From the elasticity of substitution, the price elasticity of input demand (E_{ij}) is obtained as (Binswanger):

$$(7)\quad E_{ij} = S_j \sigma_{ij} \text{ for all i and j, } i \neq j$$

and

$$E_{ii} = S_i \sigma_{ii} \text{ for all i.}$$

These price elasticities are likely to be more accurate than those derived from aggregate models and the extremely disaggregate ones for reasons suggested previously.

Although, price elasticities of input demand for each product category can not be derived from the multi-product cost function, other disaggregate (commodity specific) measures related to energy demand can be derived. These include the elasticity of scale for each product, the marginal cost of production for each product, marginal rates of product transformation, the annual rates of technical progress, the input biases of technical progress, input demand elasticities with respect to output, and other important measures. The partial elasticity of scale for product r (V_r) which measures returns to scale for each commodity is obtained as (Denny and Pinto; Ray):

$$(8)\quad V_r = (\partial \ln C/\partial \ln Q_r)^{-1}, \text{ for all r.}$$

The overall elasticity of scale (V) is also obtained as:

$$(9)\quad V = \left[\sum_{r=1}^{m} (1/V_r)\right]^{-1}.$$

The marginal cost of producing product r (MC_r) is obtained as:[2]

$$(10)\quad MC_r = P_r = C/V_r Q_r.$$

Estimates of product marginal cost can provide an indication of the impact of severe changes in energy prices on production costs in the subsectors. The marginal rate of product transformation between pairs of products ($MRPT_{rs}$) which can be obtained as:

$$(11)\quad MRPT_{rs} = MC_r/MC_s,$$

can also provide some insight into changes in the production cost structure.

The rate of technical progress (V_t) is derived as (Ball and Chambers):

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[1] If support payments are considered to be negligible, output prices can be assumed to be their marginal costs in the competitive market.

[2] Since $V_r = 1/(\partial \ln C/\partial \ln Q_r) = 1/(P_r Q_r/C)$, $P_r = C/(V_r Q_r)$. With the assumption of perfect competition, $MC_r = P_r = C/(V_r Q_r)$.

$$(12)\quad V_t = -(\partial \ln C/\partial t) = -(z_T + z_{TT}\, t + \sum_{r=1}^{m} g_{Tr} \ln Q_r + \sum_{i=1}^{n} z_{Ti} \ln P_i).$$

Also, the input and product biases of technical change are derived as:

$$(13)\quad Bias_i = \partial S_i/\partial t = z_{Ti}, \text{ for all } i$$

and

$$(14)\quad Bias_r = \partial S_r/\partial t = g_{Tr}, \text{ for all } r$$

such that if $z_{Ti} < 0$, technological change is input i saving and if $g_{Tr} > 0$, technological change is product r intensifying. On the other hand, if $z_{Ti} > 0$, technological change is input i using and if $g_{Tr} < 0$, technological change is product r reducing. Increased intensity of product r occurs when the revenue share of total cost from product r increases.

It is possible for the bias of each product to be negative since the revenue shares of total cost, unlike input cost shares, do not necessarily sum to unity in the short run under perfect competition. This is because many farmers produce items such as home consumed items which are not sold on the market. If the product bias is negative for all products, a better measure of product bias is the relative bias ($Bias_{rs}$) which is defined as:

$$(15)\quad Bias_{rs} = (Bias_r/Bias_s) = g_{Tr}/g_{Ts}.$$

If $Bias_{rs} > 1$, the product mix is shifting towards the production of product s. This means that the revenues from product s as a percentage of the revenues derived from products r and s is increasing. If, however, $Bias_{rs} < 1$, the product mix is shifting toward production of product r. These measures of technological bias can be used to interpret the effects of energy price changes.

Although, the energy demand structure for each commodity can not be obtained, following Burgess, the demand for input i with respect to increases in product r can be derived as:[3]

$$(16)\quad E_{ir} = (S_i S_r + k_{ri})/S_i.$$

The expression in equation (16) is, of course, based on the assumption that the technology of production is non-homothetic.[4] When used in conjunction with the price elasticity of demand for inputs, the measure of input demand with respect to output can provide an indication of the effects of energy prices on output.

APPLICATION TO WEST VIRGINIA DATA

The model was applied to agricultural sector data from West Virginia.[5] Farm inputs used in the sector were divided into six categories: labor (L); fertilizer inputs (F) which included fertilizer, chemicals, and lime; energy inputs (E) included gasoline, diesel, LP gas, fuel oil, electricity, kerosene, and natural gas; machinery inputs (M) which included machinery rental, custom work, machinery, and equipment; capital (C) which included land, buildings, and other fixed inputs such as livestock and poultry used in production; and miscellaneous inputs (N) which included all other inputs. The output of the sector was also divided into two major categories: (1) livestock products (l) which included all poultry, dairy, cattle, hog, feeder pigs, sheep, and lamb products and (2) crop products (c) which included all grain, seed, hay, forage, silage, tobacco, fruit, nut, berry, vegetable, melon, nursery, and greenhouse products.[6] Data required to fit the model were input prices, input shares, revenue shares, and output.

Pooled cross-section time-series data were used to estimate the model because cross-section data alone (county data) may not reveal all the time related production parameters while time-series data alone (yearly state aggregate data) may not fully capture all the scale effects. Besides, the use of time-series data alone could lead to multicollinearity problems due to the close relationship between the time factor and many of the other independent variables. Furthermore, since the cost function has a large number of parameters, the data set must be sufficient to fulfill the rank condition and the degree of freedom requirement. Neither the available time-series or cross-section data sets were large enough to meet these requirements. However, by combining data from each of West

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[3] E_{ir} is obtained as follows: since $S_i = (P_i X_i)/C$, $X_i = (S_i C)/P_i$, it can be shown that $(\partial \ln X_i/\partial \ln Q_r) = (\partial \ln S_i/\partial \ln Q_r) + (\partial \ln C/\partial \ln Q_r) - (\partial \ln P_i/\partial \ln Q_r)$. Since, however, $\partial \ln P_i = 0$ when prices are constant, $(\partial \ln X_i/\partial \ln Q_r) = (\partial \ln S_i/\partial \ln Q_r) + (\partial \ln C/\partial \ln Q_r = (\partial S_i/S_i \partial \ln Q_r) + (S_i \partial \ln C/S_i \partial \ln Q_r) = (k_{ri}/S_i) + (S_i S_r/S_i) = (S_i S_r + k_{ri})/S_i$.

[4] A separability test which is analogous to the test of non-homotheticity is provided.

[5] Because agricultural support payments to West Virginia farmers have been negligible over the years, the earlier assumption of perfect competition seems plausible.

[6] Use of West Virginia data limited the degree of disaggregation because data on the outputs of various subcategories of products such as dairy, fruit, and vegetable products were not consistently available.

Virginia's 55 counties, pooled over six time periods, over 300 observations were available. These were used to estimate the model based on the assumption that all West Virginia counties employ similar production technology and thus that production in each county in a given year represents one point along the aggregate production function for the state. However, the validity of this assumption could not be tested due to the fact that the degrees of freedom were not large enough to permit a test of overall homogeneity.

Input prices were obtained from *Agricultural Prices* (USDA). The indexes of prices paid by farmers for fertilizer, farm wage rates, prices paid for fuels and energy, and machinery prices were used as proxies for the price of fertilizer (P_F), price of labor (P_L), price of energy inputs (P_E), and the price of machinery inputs (P_M). The price index for capital (P_C) was calculated as the weighted average of the index of interest on indebtedness, the price index for livestock, and the price index for buildings. Since the miscellaneous input category contained items such as feed and seed, the price index for miscellaneous inputs was calculated as the weighted average of the price indexes for seed, feed, and all other inputs.

Input shares were obtained for each of West Virginia's 55 counties for 1959, 1964, 1969, 1974, 1978, and 1982 from the expense information available in *Census of Agriculture, West Virginia* (U.S. Department of Commerce). The expense on capital was calculated as the imputed user cost of land and buildings plus depreciation on livestock and poultry used in production (10 percent of the market value of livestock and poultry). The user cost of land and buildings was obtained as $(VLB_t)\ (R_t) + (Tx_t) - (VLB_t - VLB_{t-1})$, where VLB_t and VLB_{t-1} are the values of land and buildings in time periods t and t−1, respectively; R_t is the rate of return on or the opportunity cost of land and buildings in year t; and Tx_t is the total tax expenditures on land and buildings in year t (see Christensen and Jorgenson). The average interest rates on loans outstanding were obtained from *Agricultural Statistics* (USDA) and used as proxies for the rate of return (opportunity cost) on land and buildings. Total tax expenditure on land and buildings in each county was obtained as the county's share of the state's value of land and buildings times the total taxes levied on farm real estate in the state. The latter was obtained from *Agricultural Statistics* (USDA). The value of land and buildings was obtained from *Census of Agriculture, West Virginia* (U. S. Department of Commerce). $VLB_t - VLB_{t-1}$ represented reductions in the user cost of land and buildings which come from appreciation of land and building values. The expenses on miscellaneous inputs included animal health cost; the costs of feed, seeds, coal, wood, and coke; and other production costs.

Output data for each county for each of the six time periods were also obtained from *Census of Agriculture, West Virginia* (U.S. Department of Commerce) but had to be indexed by the Divisia indexing method. Revenue shares for livestock and crop products were also obtained from cost and revenue information from the same source. They were calculated as the market value of farm products in each product category divided by the total cost of production (Ray; Denny and Pinto; Burgess). To estimate all the coefficients of the dual cost function, the revenue share equations given in equation (5), the cost share equations given in equation (4), and the cost function given in equation (2) needed to be estimated simultaneously. Thus, to achieve stochasticity in the equations, error terms which presumably represent errors in optimization were added to them. The added error terms were presumed to be intertemporally independent and symmetrically distributed around zero with non-zero contemporaneous covariances which satisfy the requirements of the Zellner's seemingly unrelated regression model.

Since all the cost shares add to one, the sum of the error terms associated with the cost share equations at each observation point is zero and the variance-covariance matrix is singular. However, non-singularity was achieved by dropping the cost share equation for miscellaneous inputs, using the price of miscellaneous inputs as numeraire, and estimating the other equations simultaneously with the revenue share equations and the cost function by the Iterative Zellner's Efficient procedure (IZEF) (Barten; Ruble; Kmenta and Gilbert; Hoque and Adelaja; Griffin and Gregory; Ray; Humphrey and Moroney).

The symmetry and linear homogeneity conditions were met by imposing linear parametric restrictions within and across some of the equations. Similarly, since the kr_{ri} coefficients appear in both the cost share and the revenue share equations, their values were constrained across both sets of equations and the cost function. Constraints implying constant returns to scale were not imposed, however, to enable the estimation of the elasticity of scale.

EMPIRICAL RESULTS

More than 85 percent of the estimated parameters were significant. The R^2 values for the cost function and the revenue share equations were high; 89, 76, and 76 percent, respectively. However, R^2 measures for the cost share equations ranged from 16 percent for fertilizer inputs to 39 percent for machinery inputs. The estimated parameters of the cost function are reported in Table 1, while the estimated R^2 measures are reported

TABLE 1. ESTIMATED PARAMETERS OF THE COST FUNCTION, WEST VIRGINIA, 1959-1982

Parameter	Estimate	Standard error[a]
a0-intercept	2.3277	0.3082[b]
b1-livestock	0.5266	0.0889[b]
b2-crops	0.0253	0.0892[e]
d1-livestock/livestock	0.1335	0.0046[b]
d2-crops/crops	0.1179	0.0041[b]
d3-livestock/crops	−0.0828	0.0032[b]
e1-labor	0.1602	0.0152[b]
e2-fertilizer	0.0333	0.0083[b]
e3-energy	0.0526	0.0048[b]
e4-machinery	0.0438	0.0094[b]
e5-capital	0.3469	0.0345[b]
e6-miscellaneous	0.3632	—
h1-labor/labor	0.0099	0.0231[b]
h2-labor/fertilizer	0.046	0.0107[b]
h3-labor/energy	−0.0189	0.0064[b]
h4-labor/machinery	0.0389	0.0126[b]
h5-labor/capital	0.0530	0.0148[b]
h6-labor/miscellaneous	−0.1245	—
h7-fertilizer/fertilizer	0.0599	0.0321[d]
h8-fertilizer/energy	−0.0111	0.0136[e]
h9-fertilizer/machinery	−0.0469	0.0167[b]
h10-fertilizer/capital	0.0167	0.0072[c]
h11-fertilizer/miscellaneous	−0.0602	—
h12-energy/energy	0.0148	0.0080[d]
h13-energy/machinery	0.0305	0.0097[b]
h14-energy/capital	0.0044	0.0044[c]
h15-energy/miscellaneous	−0.0197	—
h16-machinery/machinery	−0.1279	0.0209[b]
h17-machinery/capital	0.0071	0.0087[e]
h18-machinery/miscellaneous	0.0983	—
h19-capital/capital	−0.1370	0.0313[b]
h20-capital/miscellaneous	0.0556	—
h21-misc./miscellaneous	0.0505	—
k2-crops/fertilizer	0.0046	0.0009[b]
k3-crops/energy	0.0009	0.0006[d]
k4-crops/machinery	0.0053	0.0011[b]
k5-crops/capital	0.0054	0.0053[e]
k6-crops/miscellaneous	0.0422	0.0246[d]
k7-livestock/labor	−0.0139	—
k8-livestock/fertilizer	−0.0046	—
k9-livestock/energy	−0.0009	—
k10-livestock/machinery	0.0053	—
k11-livestock/capital	−0.0054	—
k12-livestock/miscellaneous	0.0422	—
g1-time/livestock	−0.0052	0.0015[b]
g2-time/crops	−0.0064	0.0014[c]
z1-time/labor	−0.0056	0.0008[b]
z2-time/fertilizer	0.0001	0.0004[b]
z3-time/energy	−0.0008	0.0002[b]
z4-time/machinery	0.0025	0.0005[c]
z5-time/capital	0.0149	0.0019[b]
z6-time/miscellaneous	−0.0111	—
z7-time	0.0498	0.0118[b]
z8-time/time	−0.0002	0.0013[b]

[a] Standard errors are not obtained for miscellaneous input coefficients since they are computed using the homogeneity restrictions.
[b] Significance at the 1 percent level;
[c] Significance at the 5 percent level;
[d] Significance at the 10 percent level;
[e] Indicates insignificance.

TABLE 2. R-SQUARE MEASURES FOR THE ESTIMATED EQUATIONS, WEST VIRGINIA, 1959-1982

Equation	R-square
Cost function	0.8899
Labor cost share	0.2460
Fertilizer cost share	0.1639
Energy cost share	0.1802
Machinery cost share	0.3888
Capital cost share	0.3273
Livestock revenue share	0.7599
Crop revenue share	0.7568

TABLE 3. TESTS OF SEPARABILITY AND TECHNOLOGICAL NON-JOINTNESS

Test	Required restrictions	F-statistic	Critical value[a]
Separability[b]	$k_{ri}=0$	1.85	1.67
Separability[c]	$k_{ri}b_s=k_{si}b_r$	3.52	2.01
Non-jointness	$d_{rs}=-b_rb_s$	7.97	3.84

[a] The level of significance for all tests was 5 percent.

[b] This test corresponds to the separability test if the translog cost function is assumed to be an exact representation of the true cost function.

[c] This test corresponds to the separability test if the translog cost function is assumed to be an approximation of the true cost function.

in Table 2. The tests of separability of the cost function in output and input prices, technological non-jointness, and monotonicity as well as concavity of the cost function were conducted in order to determine the appropriateness of the specified model and the behavior of the estimated cost function. Results of some of these tests are reported in Table 3.

The null hypothesis for separability is contingent on whether or not the translog cost function is considered to be an exact representation or an approximation of the true cost function. If the former is assumed, the null hypothesis for separability is $k_{ri} = 0$ for all r and i (Denny and Fuss). According to Denny and Fuss, however, this null hypothesis is too restrictive and the test cannot be accepted as a general separability test. Denny and Pinto, and Denny and Fuss suggested that the appropriate null hypothesis for separability when the translog cost function is assumed to be an approximation of the true cost function is $k_{ri}\, b_s = k_{si}\, b_r$, for all i and r. In this study, both separability tests were conducted by imposing parametric restrictions implying the null hypotheses on the cost function and calculating the appropriate F-statistics (Adelaja). As shown in Table 3, both tests suggest rejection of separability of outputs from input prices for West Virginia agriculture. Such rejection implies that the production technology is non-homothetic and that the outputs cannot be consistently aggregated into a single output (Denny and Pinto; Ray). Results of the separability tests also offer empirical support for the specification of multi-product rather than aggregate functions.

The test of technological non-jointness is important because of the implications it has for the underlying technology and the appropriateness of the specified multi-product cost function. When the technology used in producing a commodity is joint, decisions about the production of that commodity are dependent on decisions about the production of other commodities. Thus, it would be more appropriate to specify a multi-product cost function rather than separate cost functions for each commodity. As suggested by Ray, the null hypothesis for the test of non-jointness is $d_{rs} = -b_r b_s$. This test was also conducted by imposing a parametric restriction on the cost function and calculating the appropriate F-statistic. As shown in Table 3, non-jointness of the production technology was strongly rejected. This suggests that the dual multi-product cost function approach is preferable to non-joint models specified for each commodity and that production decisions about livestock and crop products are interrelated. Thus, the multi-product cost function seems reasonable.

According to Binswanger, if the cost shares calculated by fitting the cost share equations with estimated coefficients are positive at each annual observation, the estimated cost function is monotonic. The cost shares obtained by applying Binswanger's method were not only positive, but were also very similar in magnitudes to the average shares of each input for each year as calculated from the data. The estimated cost function therefore passed the test of monotonicity.

Concavity of the cost function is satisfied if the Hessian matrix $(\partial \ln C)^2/(\partial \ln P_i\, \partial \ln P_j)$ is negative semidefinite within the range of input prices. As suggested by Burgess, this could be tested by examining the principal minors of successive order to see if they alternate in sign starting negative. The Hessian matrix based on the estimated parameters of the cost function proved to be negative semidefinite. Therefore, the estimated cost function also passed the test of concavity.

ENERGY DEMAND IN THE WEST VIRGINIA FARM SECTOR

As indicated in Table 4, energy is a substitute for labor and miscellaneous inputs but it is complementary to capital, machinery,

TABLE 4. ESTIMATED ALLEN-UZAWA PARTIAL ELASTICITIES OF SUBSTITUTION BETWEEN PAIRS OF FARM INPUTS, WEST VIRGINIA, 1959-1982

Input pairs	Estimate	Standard error
Labor/labor	-14.5470	8.5101[b]
Labor/fertilizer	27.9752	6.9383[a]
Labor/energy	8.6480	3.2670[a]
Labor/machinery	10.2521	2.9968[a]
Labor/capital	2.7822	0.4976[a]
Labor/miscellaneous	−9.4260	—
Fertilizer/fertilizer	-101.1500	36.6371[a]
Fertilizer/energy	−8.9732	12.2197[a]
Fertilizer/machinery	-18.6340	6.9912[a]
Fertilizer/capital	1.9884	0.4261[a]
Fertilizer/miscellaneous	-7.8234	—
Energy/energy	−15.1270	5.6587[a]
Energy/machinery	-11.0517	3.1968[a]
Energy/capital	-1.2050	0.2050[a]
Energy/miscellaneous	1.2859	—
Machinery/machinery	-31.0310	3.2092[a]
Machinery/capital	1.1541	0.1889[a]
Machinery/miscellaneous	6.3145	—
Capital/capital	−1.1724	0.0961[a]
Capital/miscellaneous	1.4250	—
Miscellaneous/miscellaneous	-2.4017	—

[a] Significance at the 1 percent level;
[b] Significance at the 10 percent level;

and fertilizer. These results are consistent with expectations since machinery and capital intensive technologies usually tend to be energy using. Also, since labor and machinery are usually substitutes, energy and labor are expected to be substitutes.

The price elasticity of demand for energy can provide some insight in analyzing the effects of the energy crisis of the 1970s on the input mix and on production, Table 5. The demand for energy was estimated to be price inelastic (−0.5688). Consequently, when energy prices go up, energy consumption falls slightly and the expenses for energy therefore increase. Energy inputs could thus be said to be essential inputs that could not be easily reduced when their prices rise. Furthermore, when energy prices increase, the demand for labor and miscellaneous inputs increase while the demand for machinery, capital, and fertilizer fall. Since machinery, capital, and fertilizer inputs are energy using inputs, these results are again consistent with expectations. It is further observed that when energy prices go up, the reduction in the demand for machinery is larger than the reduction in the demand for capital. This is due to the relative fixity of capital inputs (especially land) in the production process.

When compared to the profit function estimates obtained by Lopez for the Canadian agricultural sector, the present estimate of own-price elasticity of demand for energy inputs suggest that the demands for energy inputs are more inelastic than usually presumed. Lopez estimated a price elasticity of demand which is close to −1.0 while the estimate from this study is −0.5688. On the other hand, in a recent study by Hoque and Adelaja, a price elasticity of demand of −0.3535 for fuel inputs was estimated for the dairy industry. Therefore, there appears to be a possibility that the aggregate models tend to generate estimates of the elasticity of demand for energy which are biased upward while the disaggregate models tend to generate estimates which are biased downward. The differences, however, may be due to the differences in the data sets and the differences in underlying assumptions of profit and cost functions.

Another set of results was obtained regarding the changes in energy use levels when other input prices increase. When wage rates increase, energy demand increases because the demand for energy dependent inputs, which are substitutes for labor, increase. However, when the prices of energy dependent inputs (fertilizer, machinery, and capital) increase, energy demand falls. It is further observed that increases in machinery prices have the greatest reduction effect on energy demand. This is because the bulk of energy used in agriculture goes for powering machinery. Consequently, reductions in machinery use could lead to cutbacks in energy use.

TABLE 5. ESTIMATED PRICE ELASTICITIES OF DERIVED DEMAND FOR AGRICULTURAL INPUTS (E_{ij}), WEST VIRGINIA, 1959-1982

Elasticity (E_{ij})	Estimate	Standard error
E-Labor/labor	−0.7579	0.4433[b]
E-Fertilizer/labor	1.4575	0.3615[a]
E-Energy/labor	0.4506	0.1702[a]
E-Machinery/labor	0.5341	0.1561[a]
E-Capital/labor	0.1450	0.0259[a]
E-Miscellaneous/labor	−0.4911	—
E-Labor/fertilizer	0.8281	0.2054[a]
E-Fertilizer/fertilizer	−2.9940	1.0845[a]
E-Energy/fertilizer	−0.2656	0.3617[a]
E-Machinery/fertilizer	−0.5516	0.2069[a]
E-Capital/fertilizer	0.0589	0.0126[a]
E-Miscellaneous/fertilizer	−0.2331	—
E-Labor/energy	0.3252	0.1228[a]
E-Fertilizer/energy	−0.3374	0.4595[a]
E-Energy/energy	−0.5688	0.2128[a]
E-Machinery/energy	−0.4155	0.1202[a]
E-Capital/energy	−0.0453	0.0077[a]
E-Miscellaneous/energy	0.0484	—
E-Labor/machinery	0.8273	0.2418[a]
E-Fertilizer/machinery	−1.5038	0.5642[a]
E-Energy/machinery	−0.8919	0.2580[a]
E-Machinery/machinery	−2.5042	0.2590[a]
E-Capital/machinery	0.0931	0.0152[a]
E-Miscellaneous/machinery	0.5096	—
E-Labor/capital	1.5881	0.2841[a]
E-Fertilizer/capital	1.1350	0.2432[a]
E-Energy/capital	−0.6878	0.1170[a]
E-Machinery/capital	0.6588	0.1078[a]
E-Capital/capital	−0.6692	0.0548[a]
E-Miscellaneous/capital	0.8134	—
E-Labor/miscellaneous	−2.1604	—
E-Fertilizer/miscellaneous	−1.8046	—
E-Energy/miscellaneous	0.2947	—
E-Machinery/miscellaneous	−1.4473	—
E-Capital/miscellaneous	0.3266	—
E-Miscellaneous/miscellaneous	−0.5505	—

[a]Significance at the 1 percent level;
[b]Significance at the 10 percent level;

Responses of input demand to changing energy and labor prices have implications for manpower and energy policies in the state. Since labor and energy are substitutes, for example, any actions directed at stimulating increased farm wages will tend to make the farm sector more energy dependent. Conversely, higher energy prices will tend to make the agricultural sector more labor dependent. This implies that the energy conservation policies of the 1970s and 1980s may have slowed down the displacement of labor from agriculture.

Estimates of elasticities of scale, marginal costs, rates of technical progress, and marginal rates of product transformation are presented in Table 6. The elasticity of scale measures indicate that although the livestock and crop subsectors are both characterized by increasing returns to scale, significantly higher scale benefits can be derived from crop production. Furthermore, unusual measures of scale elasticities for crop production were observed for the period after 1974. Prior to 1974, the elasticity of scale for crops increased steadily. However, between 1974 and 1978, it remained fairly constant, but it declined after 1978. Since the leveling off and subsequent decline of the elasticity of scale for crops coincided

TABLE 6. PARTIAL AND OVERALL ELASTICITIES OF SCALE, MARGINAL COSTS, MARGINAL RATES OF PRODUCT TRANSFORMATION, AND THE RATES OF TECHNICAL PROGRESS IN WEST VIRGINIA, SELECTED YEARS, 1959-1982

	Partial elasticity of scale			Index of marginal cost			
Year	Livestock	Crops	Overall elasticity of scale	Livestock	Crops	Marginal rate of product transformation	Rate of technical progress
1959	1.8889	3.9747	1.2804	43.1	72.1	0.597	−0.0052
1964	2.0270	4.3976	1.3875	44.8	70.8	0.632	0.0095
1969	2.2928	4.5143	1.5202	55.3	82.6	0.669	0.0187
1974	2.6215	4.4912	1.7094	85.6	158.6	0.540	0.0331
1978	2.8966	5.0492	1.8407	103.7	204.3	0.508	0.0410
1982	3.3764	4.5899	1.9454	97.6	216.2	0.460	0.0506

with the energy crisis period, it is likely that the rising energy prices eroded some of the scale advantages available to crop farmers. Therefore, crop products are likely to be more severely affected by a changing energy environment than are livestock products. This might be due to the relatively high demand for energy in crop enterprises, as indicated by the high energy share of total cost for crop enterprises.

At first glance, the observed increasing returns to scale for each year appear to contradict the earlier assumption of perfect competition. This dilemma, however, can be explained. In the long run, the assumption of perfect competition implies that profit maximization is attained when product price is equal to marginal cost. This occurs at the minimum point of the longrun average total cost function where both the short and longrun elasticities of scale are unitary, the average and marginal costs are equal, and there is no economic profit in the industry. Thus, in the long run, marginal cost pricing is inconsistent with increasing returns to scale. In the short run, however, increasing returns to scale may be consistent with marginal cost pricing. It simply implies that the price may be below the average total cost but above the average variable cost of production. Since it is unlikely that farmers achieve longrun profit maximization at every annual observation, the observed increasing returns to scale suggest that West Virginia farmers may be experiencing losses in the short run but that they operate somewhere between the shut-down and the break-even points. They also suggest that by expanding production capacity, farmers in West Virginia could realize significant cost reduction. They do not imply that the farmers in the State do not pursue cost minimization or profit maximization. The observed increasing returns to scale may explain why the number of farms has been decreasing while the average farm size in acres has been increasing over the years.

Indices of marginal cost indicate that livestock and crop production costs have increased over the years but the most rapid increase occurred around 1974, the year following the beginning of the oil embargo period. Marginal cost between 1969 and 1974 almost doubled for crop products while it only increased by 40 percent for livestock products. Consequently, crop products were more seriously affected by the energy price increase. The marginal rate of product transformation also suggests that until 1969, the cost advantage was shifting towards crop production. However, the trend was reversed by 1974 by the rising energy prices. This made crop production relatively more expensive and thereby made livestock production relatively cheaper.

The reader may wonder why the cost of production is more adversely affected and why economies of scale are more easily eroded in the production of crops, compared to livestock, when energy prices rise. One possible explanation is that there is less flexibility to substitute other inputs for energy inputs in crop production. For example, since fuel is heavily used in crop production to power field machinery, higher fuel prices will result in an increase in the cost of producing crops because there is no alternative source of energy for field machinery. On the other hand, livestock producers are more flexible in their ability to substitute one form of energy input for another. For instance, many of the energy using livestock activities such as heating, can be done with a wider variety of energy inputs (fuel, natural gas, and electricity).

Significant technological progress was not realized by West Virginia farmers until the 1970s when the annual rates of technical progress exceeded 3 percent. Prior to that, technical change was rather slow (less than 2 percent). In fact, the estimated rate of technical progress for 1959 was about −0.5 percent which implies technical regression. However, as indicated in Table 7, technological change has been labor saving, which explains, in part, why farm employment and population have fallen over the years. Technological change has also been energy saving, suggesting that farmers in the state have responded to the energy crisis by employing energy saving technology. This may have been due to the various government incentive programs designed to encourage energy conservation. On the other hand, technological change has been capital, machinery, and fertilizer using. This suggests that like most other states, West Virginia agriculture is no exception to the increased use of mechanical and chemical inputs.

Estimated product biases of technological change also provide some useful results. Livestock and crop reducing technological improvements are observed for the sector. This suggests that although the newer technologies had a reducing effect on the revenue shares of total cost from both crop and livestock products, they may have favored the production of non-market products. Thus, farmers in West Virginia may be shifting towards producing home consumed goods and recreational goods, thus increasing non-market income at a faster rate than market income. This is not a surprising finding considering the fact that a large proportion of West Virginia farmers are part-time farmers.

The relative bias which measures the relative intensity of production suggests that the technological change in the sector was livestock intensifying. This implies that there is a more rapid decline in the revenue share of total cost for crop products and that the technological change is less favorable to crop production. However, this may also be due in part to the observed relative cost disadvantage in crop production brought about by the oil crisis.

Finally, measures of input demand with respect to output, depicted in Table 7, suggest that increased livestock production requires a higher percentage increase in energy use than increased crop production. When livestock production increases by 1 percent, energy demand increases by 0.4 percent. On the other hand, when crop production increases by 1 percent, energy demand increases by 0.17 percent. In view of the previous observations regarding energy demand, this finding suggests that although crop production involved relatively more energy in absolute terms, the marginal increase in energy use resulting from increased output tends to be larger when livestock output, as opposed to crop output, is to be increased. In other words, a higher percentage increase in energy inputs is required to stimulate increased livestock production than is required to stimulate increased crop production. The possibility that growth in the agricultural subsectors may have differential impacts on energy demand has been generally ignored in literature. Such information is useful in projecting future energy demand in agriculture.

SUMMARY AND CONCLUSIONS

The few studies that have focused on energy demand in agriculture have either been commodity specific or aggregate in scope. Since the commodity specific studies tend to ignore technological jointness, estimates of price elasticities of demand obtained from them are subject to errors arising from model mis-specification. Estimates from studies based on aggregate sector production or cost functions may also be subject to aggregation bias. Estimates of energy demand elasticities obtained via the multi-product cost function in this study, however, are less likely to be biased due to the disaggregate treatment of agricultural products and the fact that the model considers technological jointness. Therefore, in terms of accuracy, the multi-product cost function appears to be a more plausible specification than previous models.

Disaggregation enabled estimation of several disaggregate measures of energy demand elasticities which were previously unavailable. These measures allowed a more comprehensive analysis of energy demand in agriculture than were conducted in previous studies. For example, the elasticity of demand for energy with respect to livestock and crop outputs provided useful information on the relative impacts of changes in subsectoral output on energy use in West Virginia agriculture. Other measures such as the marginal costs of production and the rates of product transformation provided by the multi-product cost function were also useful in the analysis of energy demand.

It is concluded from the study that higher energy prices result not only in increased production costs but also in slowdowns in the rate of increase in mechanization. However, compared to livestock costs, crop production costs are more sensitive to changes in energy prices. As such, the energy crisis affected crop farmers more than it did livestock farmers. Also, marginal increases in livestock production require more energy, in

TABLE 7. ELASTICITIES OF INPUT DEMAND WITH RESPECT TO OUTPUT AND THE BIASES OF TECHNOLOGICAL CHANGE, WEST VIRGINIA, 1959-1982[a]

Input	Product		Input bias of technological change
	Livestock	Crops	
Labor	0.1537	0.4081	Saving
Fertilizer	0.2651	0.2966	Using
Energy	0.3966	0.1651	Saving
Machinery	0.3548	0.2069	Using
Capital	0.4110	0.1507	Using
Miscellaneous	0.2364	0.3253	Saving

[a] Technological change was also found to be crop reducing and livestock reducing. However, the relative bias indicated that technological change was livestock intensifying, relative to crops ($\text{Bias}_{rs} < 1$).

percentage terms, than increases in crop production but newer technologies adopted in West Virginia agriculture have been energy saving.

POLICY IMPLICATIONS

Prior to 1970, the energy share of production costs were generally negligible in West Virginia as well as in United States agriculture. However, by 1974, energy began to play an important role as energy prices rose sharply and farmers had to spend considerably more for this input. Thus, energy conservation became a major policy issue.

Information generated in this study is of value because it sheds some light on the role of energy in agriculture. The information is therefore useful to policymakers interested in reducing the burden faced by farmers in trying to cope with an economic environment where costs seem to be ever-increasing relative to product prices. Since much of the information generated in this study is descriptive of the structure of agriculture and the effect of energy on production, it is useful in predicting the impact of energy policy and changing energy environment on agriculture.

As observed from this study, energy is essential to agricultural production since its use can not be easily reduced. Furthermore, rapid increases in energy prices can seriously affect agricultural production and therefore farm incomes. The observed inelastic demand for energy suggests that if energy prices continue to increase, energy will play an even more important role in production unless effective substitutes for energy or alternative energy reducing technologies are found. The observed energy saving bias of technological change, however, suggests that farmers in West Virginia are already taking steps to reduce their energy dependency and will adopt energy saving technology as it becomes available. It appears therefore that policies that stimulate researchers to develop and introduce these alternatives could be beneficial to farmers.

The dependence of farming on energy arises primarily due to the machinery using nature of farm technology. Consequently, the solution to the dependency problem in West Virginia may have to take into consideration the patterns of investments in agriculture. Given the current lack of energy alternatives, arbitrary reduction in energy use can be expected to result in severe cutbacks in production unless capital investments are simultaneously encouraged.

Policymakers need to be aware of the effect of rising energy prices on the subsectors. In general, energy policies would affect the subsectors in different ways. Results of this study, for example, suggest that the crop subsector is more sensitive to energy price changes. It is therefore more likely to benefit from or be harmed by energy related policies. Since, however, growth in the livestock industry requires larger increases in energy demand, the industry will tend to be relatively more sensitive to energy prices as it grows.

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