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**ITEMISED DEDUCTIONS: A DEVICE TO REDUCE TAX
EVASION**

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Itemised deductions: a device to reduce tax evasion*

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Abstract

Direct incentives and punishments are the most common instruments to fight tax evasion. The theoretical literature disregarded indirect schemes, such as itemised deductions, in which an agent has an interest in that other agents declare their revenue. Itemised deductions provide an incentive for consumers to declare their purchases, and this forces sellers to do the same. I show that, for any level of taxation, it is possible to increase tax proceeds by choosing the proper level of itemised deduction; the cost for the government on the consumers' side is more than compensated by the extra proceeds on the sellers' side.

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“When there is an income tax, the just man will pay more and the unjust less on the same amount of income” (Plato (nd))

1 Introduction

For most countries tax evasion is a major issue and tax proceeds are strongly affected. By its nature, it is hard to estimate tax evasion. Franzoni (1999) estimates that the US federal tax gap¹ is about 17%. Slemrod (2007) proposes an updated and detailed description of tax compliance in the United States. In the 80’s, the black economy in western countries represented 5% to 15% of GDP (Cowell (1985)). More recently, McKay (1998) and Schneider (2005) estimated that the black economy roughly ranges from 6% (Switzerland) to 27% (Italy) of GDP.

Allingham and Sandmo (1972) and (1991) are amongst the most well known works on tax compliance. Many surveys are available of the recent literature on tax avoidance and/or evasion:² among the most complete, Slemrod and Yitzhaki (2002) focus on tax avoidance and it is more theory oriented while Andreoni, Erard, and Feinstein (1998) concentrate more on empirical works.

Social welfare reasons exist for not fighting tax evasion: it is generally expensive to reduce evasion; furthermore, taxes are distortive and evasion may partially overcome this distortion.³ Finally, if evasion is negatively correlated with income, tax evasion will have redistributive effects. Section 7 of Cowell (1985) offers other arguments for and against fighting tax evasion. For an analysis of the welfare consequences of tax evasion, see Davidson, Martin, and Wilson (2007).

A politician may want to reduce evasion nevertheless, so as to increase tax proceeds; to promote the country’s image on an international ground; or simply because tax evasion is illegal. Note also that black money is more likely to finance other illegal activities (Fortin, Lacroix, and Montmarquette (2000)): by reducing evasion, the government reduces funds spent on illicit activities. Finally, in the presence of equality/fairness concerns, some agents may have a disutility from knowing that other people evade.

The literature on tax evasion is mainly devoted to income taxation.⁴ Most of the analysis of tax evasion concentrates on how evasion can be deterred through

¹Tax gap, according to the United States Department of Treasury, measures the extent to which taxpayers do not file their tax returns and pay the correct tax on time.

²Tax evasion consists of not declaring some earnings that, by law, an agent is supposed to declare, while tax avoidance consists of abusing some laws or, often, using the lack of detail in some laws, to reduce the tax burden.

³In the presence of information asymmetries and different revenue elasticities, the (second-best) choice of government may distort sectors differently, and evasion may have a counterbalancing effect.

⁴Cremer and Gahvari (1993) and Marelli (1984) are notable exceptions.

detection and sanctions (Franzoni (1999)); rational consumers decide whether to evade or not on a “cost-benefit” analysis.⁵ The legislator disposes of a wide set of instruments to fight tax evasion (e.g., auditing, fines, incentives not to evade, direct taxes to make it harder to evade); the cost of auditing and agents’ limited liability strongly limit the legislator’s policy space. People receive direct incentives not to evade (such as decreasing tax rates), and evading becomes more costly (through higher fines or by using more sophisticated audit systems).

I instead focus on the indirect mechanism of itemised deductions. Deductions are not a widely used policy instrument: in Italy and Spain allowed deductions represent at most 5% of taxable income, in the UK and Ireland up to 9%; in all OECD countries itemised deductions are below 15%, excepted for France and the Scandinavian countries, where deductions range between 25% and 30% (OECD (1990)).

I show that giving incentives to consumers to declare their purchases reduces tax evasion by forcing sellers to declare their revenue. I consider a market with a consumption good available both legally and on the black market. Profits on the legal market and consumers’ income are both taxed. Tax rates (on profits and on income) are exogenous (possibly optimally chosen). Itemised deductions allow consumers to reduce their tax base by declaring their purchases; this force sellers to declare their earnings too.⁶

The itemised deduction that maximises tax proceeds is always strictly positive, which means that, *ceteris paribus*, introducing deductions implies: a) that the public budget constraint is relaxed (higher tax proceeds and/or lower tax rates) and b) that the size of the underground market shrinks. My model shows that partial deductibility of expenditures is sufficient enough of an incentive for the consumers to declare their purchases without negatively affecting tax proceeds. The reduction in the amount of collected taxes from consumers is less substantial than the increase in tax proceeds deriving from the reduction in the evasion of the tax on profits.

Section 2 presents the framework of the model. In section 3 I solve the model and find the optimal level of deduction. Section 4 analyses the results and provides a numerical example. The last section concludes.

⁵Note that evasion may also derive from the willingness to hide illegal activities: even with a 0% tax rate, the underground economy would still represent about 4% of GDP, because of illegal activities and agents willing to avoid regulation laws (McKay (1998)).

⁶A similar idea has been used in France over the last decades: the government partially subsidises the rent of the poorest citizens through the “Aide au Logement”. Landlords are forced to declare their renting income or to reduce the rent by the value of the subsidy.

2 The model

I describe here the main elements of the model: I start (subsection 2.1) by describing the goods in the economy and the agents' behaviour; then I analyse the organisation of the market and the firms' behaviour (subsection 2.2); finally I present the problem of the central authority (politician/social planner) (subsection 2.3).

2.1 The agents

A representative consumer can choose between the numeraire good M and a consumption good available both legally (X) and illegally (Y): x denotes the quantity of good exchanged on the legal market at price p , and y is the quantity purchased on the black market at price q .

For a consumer, X and Y are versions of a same good. It is natural to assume (in terms of the utility function) that the two goods are substitute. The degree of substitutability may range from perfect substitutability (e.g., a shop that may register or not the transaction) to very low substitutability (e.g., an illegal copy of a technological product, possibly with very different technical characteristics). Regardless of the possible differences between the two versions of the good, a consumer may have a preference for legal purchases over the illegal ones; for that reason, I introduce a parameter in the utility function allowing to consider for the aversion towards illegality (the parameter accounts for the depreciation, in terms of utility, of a good that is sold on the black market, compared to the same good sold on the legal market).

I assume that the representative consumer has the standard utility function (U) for imperfect substitute goods; with no restrictions on the (positive) quantities x and y that can be consumed, as far as the consumer's budget constraint is satisfied.

$$U(x, y, M) = (x - x^2) + \theta(y - y^2) + 2\sigma xy + M \quad (1)$$

where $\sigma \in [0, 1]$ is the degree of substitutability between X and Y (the two goods are perfect substitutes if $\sigma = 1$), and $\theta \in [0, 1]$ is the aversion-towards-illegality parameter.⁷ When $\theta = 1$, the consumer has no prejudice against the illegal good.

The consumer has an exogenous income I , on which he pays an income tax, which rate is t . An itemised deduction is a reduction in the tax base depending on the number of units of good that are (legally) purchased. The deduction is per-unit (opposed to per-value), this has the nice property of avoiding unnecessary inflation. The deduction is of amount a for any purchased unit exceeding the threshold x_0

⁷While σ depends on the real characteristics of the goods, θ is the type of the consumer/society. The larger θ , the more likely are people to accept to break the law.

(and 0 otherwise). The (exogenous) threshold x_0 is equal to the number of units that are purchased when itemised deductions are not allowed. Introducing the threshold allows to reduce the cost for the government of implementing this policy, without affecting its effectiveness.

The consumer's problem can be written as:

$$\begin{aligned} \max_{x,y,M} \quad & U(x, y, M) \\ \text{s.t.} \quad & px + qy + M = I - t(I - a(x - x_0)) \end{aligned} \tag{2}$$

From the first order conditions, we derive the inverse demand function respectively for X and Y :

$$p(x, y) = 1 - 2x - 2\sigma y + ta \tag{3}$$

and

$$q(x, y) = \theta - 2\theta y - 2\sigma x \tag{4}$$

2.2 The market

The organisation of the industry heavily depends on the type of good that we consider. The common factor is that we have two sub-markets, one for the legal and one for the illegal good. The simplest market that we can imagine is a duopoly, with one firm operating in each sub-market. This structure can apply, for example, to the case of a regulated legal market, in presence of an illegal cartel. In that case, one single agent is choosing the quantity and price on the legal market (the regulator) and one on the black market (the cartel). It seems reasonable to consider that the legal firm sets its own quantity and price, anticipating the behaviour of the illegal firm. For that reason, the natural way to model this framework is the Stackelberg competition with the legal firm acting as the leader.

For expositional convenience, I restrain my attention, in the core of the paper, to the Stackelberg duopoly case, that seems to me the best compromise between simplicity of the model and realism. The reader should not think that results are driven by this assumption; results hold (qualitatively) for a much larger set of market configurations. In the appendix I solve the problem for the case of Cournot competition, with n firms operating on the legal market and m firms on the underground market.

X and Y are produced by two independent firms competing à la Stackelberg and facing the same marginal cost of production c . The legal firm is the leader and chooses the quantity first, and the illegal one follows. By definition, only the legal firm pays the tax T on its profits. The maximisation problem of the follower,

given the demand function $q(x, y)$, is

$$\begin{aligned} \max_y & (q(x, y) - c)y \\ \text{s.t.} & q(x, y) = \theta - 2\theta y - 2\sigma x \end{aligned} \quad (5)$$

from which we obtain the follower's reaction function

$$y = \frac{\theta - 2\sigma x - c}{4\theta} \quad (6)$$

The Stackelberg leader maximisation problem is therefore:

$$\begin{aligned} \max_x & (1 - T)(p(x, y) - c)x \\ \text{s.t.} & p(x, y) = 1 - 2x - 2\sigma y + ta \\ & y = \frac{\theta - 2\sigma x - c}{4\theta} \end{aligned} \quad (7)$$

2.3 The government

The government is concerned by total tax proceeds (TP), which are given by the sum of consumers' income tax proceeds and firms' profits tax proceeds, and which are equal to

$$TP(a) = T(p(x, y) - c)x + t(I - a(x - x_0)) \quad (8)$$

The focus of this research is on itemised deductions; for that reason, I assume that both tax rates are exogenous (that is, the government has already chosen the optimal rates) and I focus on the optimal level of deduction. Therefore, the government only controls the level of a , which represents the reduction in the tax base for each unit of x purchased above the threshold x_0 to which the representative consumer is entitled.

The optimal level of deduction is defined, according to this maximisation problem, as the one that maximises tax proceeds. Clearly, the objective function of the social planner/politician may be different, ranging from minimising the size of the underground market to the reduction of the tax rate for some agents in the economy. These questions are beyond the scope of this work.

3 Market equilibrium and optimal deduction

In this section, I compute the market equilibrium, finding the optimal quantities and prices both for the legal and illegal firm. Afterwards, I compute the optimal

level of deduction a .

Solving equation (7), from the first order condition we obtain that

$$x^*(a) = \frac{\theta(2 - \sigma) - c(2\theta - \sigma)}{4(2\theta - \sigma^2)} + \frac{\theta}{2(2\theta - \sigma^2)}ta \quad (9)$$

Replacing x^* in equation (6), we obtain that

$$y^*(a) = \frac{\theta(4\theta - \sigma^2 - 2\sigma) - c(4\theta - \sigma^2 - 2\theta\sigma)}{8\theta(2\theta - \sigma^2)} - \frac{\sigma}{4(2\theta - \sigma^2)}ta \quad (10)$$

and finally, using (9) and (10) we can also compute prices:

$$p^*(a) = \frac{\theta(2 - \sigma) + c(2\theta + \sigma)}{4\theta} + \frac{1}{2}ta \quad (11)$$

$$q^*(a) = \frac{\theta(4\theta - 2\sigma - \sigma^2) + c(4\theta + 2\theta\sigma - 3\sigma^2)}{4(2\theta - \sigma^2)} - \frac{\theta\sigma}{2(2\theta - \sigma^2)}ta \quad (12)$$

As we are interested in a market with tax evasion, when the itemised deduction is not allowed (i.e., $a = 0$) we want the quantity of good purchased on the underground market to be positive ($y^*(0) > 0$), which occurs if and only if

$$c < \frac{\theta(4\theta - \sigma^2 - 2\sigma)}{(4\theta - \sigma^2 - 2\theta\sigma)} \quad (13)$$

Assuming also that

$$\theta > \sigma \quad (14)$$

is sufficient to guarantee that condition (13) is not requiring a negative marginal cost.

Conditions (13) and (14) are sufficient to ensure that both prices and quantities are positive in equilibrium and that both the consumer's and the firms' problems are strictly concave.⁸

Proposition 1 *The equilibrium quantities $x^*(a)$ and $y^*(a)$ are respectively defined by equations (9) and (10), and the corresponding prices are $p^*(a)$ and $q^*(a)$, as defined by equations (11) and (12).*

Proposition 2 *The legally purchased quantity and its price ($x^*(a)$ and $p^*(a)$) are both increasing in the level of deduction a ; the reverse is true for the quantity bought illegally and for its price ($y^*(a)$ and $q^*(a)$), which are decreasing in a .*

⁸By the strict concavity of the problem, the previously shown results (derived from the first order conditions) represent the unique maxima of the corresponding maximisation problems

Using the previous results, we can solve the government problem, consisting in finding, for an exogenous level of taxation, the itemised deduction a that maximises tax proceeds $TP(a)$.

$$\begin{aligned} \max_a \quad & T(p(a) - c)x + t(I - a(x - x_0)) \\ \text{s.t.} \quad & x = x^*(a) \end{aligned} \tag{15}$$

From the first order condition of the problem, we obtain that the optimal level of itemised deduction is:

$$a^* = \frac{T(\theta(2 - \sigma) - c(2\theta - \sigma))}{2\theta t(2 - T)} \tag{16}$$

Under the previous assumptions (13) and (14), the problem is strictly concave and a^* is always positive.

Proposition 3 *Equation (16) defines the optimal level of the per-unit itemised deduction a^* when the government's aim is to maximise tax proceeds for a given level of tax rates. The value a^* is always positive, meaning that it is always beneficial to allow for the deduction.*

3.1 Comparative statics

For its policy implications, a policy maker may be also interested in knowing the effect of the different parameters on the optimal value a^* . This subsection briefly analyses the impact of T , t , θ , σ and c on the optimal value of a .

$$\frac{\partial a^*}{\partial T} = \frac{(\theta(2 - \sigma) - c(2\theta - \sigma))}{\theta t(2 - T)^2} > 0 \tag{17}$$

$$\frac{\partial a^*}{\partial t} = -\frac{a^*}{t} < 0 \tag{18}$$

$$\frac{\partial a^*}{\partial \theta} = -\frac{c\sigma T}{2\theta^2 t(2 - T)} < 0 \tag{19}$$

$$\frac{\partial a^*}{\partial \sigma} = \frac{T(c - \theta)}{2\theta t(2 - T)} < 0 \tag{20}$$

$$\frac{\partial a^*}{\partial c} = -\frac{2\theta - \sigma}{2\theta t(2 - T)} < 0 \tag{21}$$

From the previous derivatives, we observe that the optimal deduction increases in the tax rate on profits T : when it is larger, reducing the size of the underground market is more profitable, therefore the public authority accepts larger losses on

the consumer side. The opposite reasoning applies for a larger tax rate t : the larger the tax on income, the more costly is to implement a deduction policy.

Whenever the legal firm's profits decrease, it becomes less profitable to introduce deductions, therefore the optimal value for a decreases. Both θ and c reduce the profits of the legal firm. The marginal cost of production, c , has a direct effect on profits; θ measures how likely citizens are to buy on the illegal market: a change in θ modifies the marginal rate of substitution between X and Y . When θ increases, X is replaced by Y and the legal firm's profits decrease. Consequently, both an increase in θ and in c induce a reduction in the optimal value of a .

Also an increase in σ implies a reduction in the value of a . The reason is that a larger value of σ means that X and Y are better substitutes. Consumers replace Y by X more easily, deductions are more effective and a smaller a is enough to obtain an increase in the consumption of the legal good.

3.2 Deductions and auditing

Readers certainly noticed that I do not explicitly considered the possibility of deterring evasion through auditing and fining, a diffused technique that has been largely studied in the theoretical, empirical and experimental literature.

Fining and auditing is a costly but also effective way of reducing evasion and it can and should be used together with itemised deductions and not as a substitute. I implicitly assumed that, given costs of auditing and limited liability, the government was already choosing the pair auditing-fine that maximises tax proceeds. The fine affects the final expected profit of the illegal firm, but (unless it determines the exit of the firm from the market) it does not affect the decision about quantities and prices. In this subsection, I study the main interaction between auditing and the optimal deduction.

Considering auditing (and fining), the maximisation problem of the underground firm (equation (5)) becomes

$$\begin{aligned} \max_y \quad & -\alpha F + (1 - \alpha T)(q(x, y) - c)y & (22) \\ \text{s.t.} \quad & q(x, y) = \theta - 2\theta y - 2\sigma x \end{aligned}$$

where α is the probability of being audited and F is the fine that the firm should pay when audited. The value of α is affecting profits but not the reaction function, which remains the same as in equation (6).

Denote by $\Delta(\alpha)$ the cost of auditing, assumed to be increasing and convex; the

maximisation problem of the government becomes

$$\begin{aligned} \max_{a,\alpha} \quad & T(p(a) - c)x + t(I - a(x - x_0)) + \alpha(F + T(q(a) - c)y) - \Delta(\alpha) \quad (23) \\ \text{s.t.} \quad & x = x^*(a) \\ & y = y^*(a) \end{aligned}$$

when deductions are allowed; while, for the case of no itemised deductions (i.e., $a = 0$), it is

$$\begin{aligned} \max_{a,\alpha} \quad & T(p(0) - c)x + tI + \alpha(F + T(q(0) - c)y) - \Delta(\alpha) \quad (24) \\ \text{s.t.} \quad & x = x^*(0) = x_0 \\ & y = y^*(0) = y_0 \end{aligned}$$

Using equation (24), the derivative with respect to α defines the optimal level of audit when deductions are not allowed $\alpha^*(0)$, which is

$$F + T(q(0) - c)y_0 = \frac{\partial \Delta(\alpha)}{\partial \alpha} \quad (25)$$

from which we have that

$$\alpha^*(0) = (\Delta')^{-1}(F + T(q(0) - c)y_0) \quad (26)$$

From $q(a)$ and $y(a)$ being decreasing functions of a follows that α^* is also decreasing in a . The optimal audit level is the one that equates the expected benefits (i.e., the increase in tax proceeds) and its cost; when introducing deductions, the size of the underground market shrinks, making it less profitable to audit. When the expected benefits are reduced, the optimal audit level decreases.

Analogously, starting from when audits are not performed ($\alpha = 0$), the level of deduction decreases if the government introduces audit. The optimal deduction equalises the increase in tax proceeds on the firm side with the losses generated by the deduction on the consumer side. An extra loss appears in the presence of audit, generated by the reduction in the proceeds derived from fines, equal to $\alpha \left(\frac{\partial(q(a)-c)y(a)}{\partial a} \right)$.

Proposition 4 *Both itemised deductions and audit are instruments to reduce tax evasion. A crowding out effect exists among the two, but it does not imply that the two measures are incompatible. An increase in deductions reduces the profitability of audits, and vice versa. If the audit cost function is continuous and well behaved, (absent fix costs of audit) the optimal levels α^* and a^* are both positive.*

4 Analysis of the results

I derived the equilibrium prices and quantities for a consumption good that is sold both on the legal and the underground market, under the assumption that the firm operating on the legal market and the one on the black market compete à la Stackelberg, with the legal firm representing the leader.⁹ Consumers perceive the two goods as substitutes, with σ as a parameter of how substitute the two goods are; and may have an aversion to illegality, represented by θ , that implies that, *ceteris paribus*, a consumer prefers to buy the good legally.

Imposing only a condition on c , θ and σ that ensures some tax evasion, I derived the optimal value for a , which is a per-unit itemised deduction that allows consumers to reduce their tax base for any purchased unit of the legal good above an exogenous threshold.

The deduction is independent of prices, otherwise we would observe an unnecessary artificial increase in the price p (inflation). The itemised deduction is a cost for the government (the tax proceeds collected from consumers decrease); but the consumption of legal good increases, generating extra proceeds from the tax T . This second effect offsets the previous one as long as $a \leq a^*$.

The optimal value a^* is positive for any combination of the parameters: for the government it is always beneficial (in terms of tax proceeds) to allow for itemised deductions. Furthermore, a induces a profits reduction for the underground firm. It is likely that profits from the underground economy are invested in illegal activities, therefore itemised deductions can also be used to fight illegal activities in the country. Depending on the social welfare function, therefore, the government may prefer a larger than optimal level of deduction (representing a loss in terms of tax proceeds but inducing a reduction in crime).

Tables 1 and 2 propose some numerical examples of the results of the model, for the case of $t = T = 0.3$, and $c = 0.2$; in table 1 $\theta = 1$ and $\sigma = 0.9$, while $\theta = 0.7$ and $\sigma = 0.5$ in table 2.¹⁰

	$a = 0$	$a = a^* = 0.259$	$a = 2.059$
p	0.42	0.459	0.729
q	0.434	0.404	-
x	0.185	0.217	0.444
y	0.117	0.102	0
TP	$0.3I+0.012$	$0.3I+0.014$	$0.3I-0.09$

Table 1: Numerical example 1.

⁹In the appendix, I show the results for the Cournot case, which are qualitatively analogous.

¹⁰Values are cut after the 3rd decimal.

	$a = 0$	$a = a^* = 0.365$	$a = 3.405$
p	0.511	0.566	1.02
q	0.355	0.339	-
x	0.189	0.223	0.5
y	0.111	0.099	0
TP	$0.3I+0.018$	$0.3I+0.021$	$0.3I-0.194$

Table 2: Numerical example 2.

The first column represents the market equilibrium when deductions are not allowed; the middle one is for the optimal level of deductions. When $a = a^*$, the size of the underground economy shrinks, but it does not disappear. The right column corresponds to a level of deduction large enough for the underground market to disappear. The bottom row (i.e., TP) represents tax proceeds. You can notice that the largest TP is attained when $a = a^*$.

5 Conclusions

This study investigates the effectiveness of itemised deductions as a device to reduce tax evasion, when both sides of the market (sellers and consumers) pay some taxes (possibly with different rates).

Consumers use their income to buy a good on the legal or underground market (or both), to consume the numeraire good and to pay a tax on their exogenous income. The legal and underground firms compete on quantities; the legal firm pays a tax on profit. Treating the tax rates as exogenous, the government chooses the level of itemised deductions that maximises tax proceeds.

I show that the optimal level of deductions is always positive, meaning that it is always beneficial to allow for deductions in a market characterised by tax evasion. Part of the effect of deductions is offset by the use of fines and audit to discourage evasion. The cost of auditing and limited liability nevertheless make fines an imperfect instrument; itemised deductions should not be considered as a substitute of auditing systems but rather as a complement. The cost of deductions consists in the drop in tax proceeds from consumers, but I show that this is more than compensated by a fall in tax evasion of firms and by the subsequent increase in tax proceeds.

Although I do not study welfare implications in my model, it is clear that the reduction in tax evasion does not represent an increase in efficiency but only a different redistribution of welfare (from the underground firm to the government, that will use it either to provide more public goods or to reduce tax rates). Certainly we do not observe a Pareto improvement; depending on the social welfare function, we can expect an increase in welfare deriving from a better arbitrage

amongst marginal utilities of money for the firm and consumers.

Provided that profits on the black market are likely to be used for financing illegal activities, if the social welfare function includes the crimes rate in the country, or the country's reputation on the international ground, the optimal level of deduction would be above the one that maximises tax proceeds. Using this model results, it would be interesting to study the consequences on the underground economy of a long term policy of deductions above the optimal level.

Another possible extension of this model would be to consider the implications in terms of optimal behaviour for a politician. In particular, depending on how profits from the underground firm are used and/or distributed among shareholders, would it be in the interest of a politician aiming to be elected to propose such a policy?

Finally, this model can be applied to a labour supply-demand model; in order to determine the conditions under which a reduction in the taxes on the labour force would push firms to legally hire their employees (forcing them to declare their income). In particular, it would be interesting to consider a more complex general equilibrium model, in which agents income is endogenous and an analogous scheme is used to convince workers to declare their income.

Appendix

A The Cournot case

I solve here the model for the case of Cournot competition, with n legal and m illegal firms being active on the market. The optimal value a^* is quantitatively different, but it remains positive; all the previous results hold under Cournot competition.

Each legal firm produces a quantity x_i , with $i \in [1, n]$; the total quantity is then $x = \sum_1^n x_i$. The firms on the underground market produce a quantity y_j , with $j \in [1, m]$. The maximisation problem of the a legal firm is

$$\begin{aligned} \max_{x_i} \quad & (1 - T)(p(x, y) - c)x_i & (27) \\ \text{s.t.} \quad & p(x, y) = 1 - 2x - 2\sigma y + ta \end{aligned}$$

while firms on the underground market face the following maximisation problem:

$$\begin{aligned} \max_{y_j} \quad & (q(x, y) - c)y_j & (28) \\ \text{s.t.} \quad & q(x, y) = \theta - 2\sigma x - 2\theta y \end{aligned}$$

The two corresponding first order conditions are respectively

$$4x_i = 1 - 2\sigma y - c + ta - 2 \sum_{h \neq i} x_h \quad (29)$$

$$4\theta y_j = \theta - 2\sigma x - c - 2\theta \sum_{k \neq j} y_k \quad (30)$$

Assuming that firms are symmetric, and that therefore $x_i = x_h$ for any $i, h \in [0, n]$, and that $y_j = y_k$ for any $j, k \in [0, m]$, we have that $\sum_{h \neq i} x_h = (n - 1)x_i$ and $\sum_{k \neq j} y_k = (m - 1)y_j$.

Solving the system of the equations (29) and (30) we obtain the optimal Cournot quantities:

$$x_i^c(a) = \frac{(m + 1)(1 - c)\theta - m\sigma(\theta - c)}{2((m + 1)(n + 1)\theta - mn\sigma^2)} + \frac{(m + 1)\theta}{2((m + 1)(n + 1)\theta - mn\sigma^2)}ta \quad (31)$$

$$y_i^c(a) = \frac{(n + 1)(\theta - c) - n\sigma(1 - c)}{2((m + 1)(n + 1)\theta - mn\sigma^2)} - \frac{n\sigma}{2((m + 1)(n + 1)\theta - mn\sigma^2)}ta \quad (32)$$

and the Cournot prices

$$p^c(a) = \frac{\theta[1 + m(1 - \sigma)] + c[mn(\theta - \sigma^2) + n\theta + m\sigma]}{(m + 1)(n + 1)\theta - mn\sigma^2} + \frac{(m + 1)\theta}{(m + 1)(n + 1)\theta - mn\sigma^2}ta \quad (33)$$

$$q^c(a) = \frac{\theta[n(\theta - \sigma) + \theta] + c[mn(\theta - \sigma^2) + \theta(m + n)\sigma]}{(m + 1)(n + 1)\theta - mn\sigma^2} - \frac{n\theta\sigma}{(m + 1)(n + 1)\theta - mn\sigma^2}ta \quad (34)$$

For the Cournot case with many firms, the assumptions that guarantee the presence of tax evasion when deductions are not available become

$$c < \frac{n(\theta - \sigma) + \theta}{n(1 - \sigma) + 1} \quad (35)$$

and

$$\theta \geq \frac{n}{n + 1}\sigma \quad (36)$$

The assumption that $\theta \geq \sigma$ is still sufficient for all the maximisation problems to be concave and the results (quantities and prices) to be always positive.

Remembering that the aggregate legal quantity $x^c(a)$ is defined as $x^c(a) = nx_i^c(a)$, let's now rename the quantity and price in the following way:

$$x^c(a) = n(x_0^c + x_v^c ta) \quad (37)$$

$$p^c(a) = p_0^c + p_v^c ta \quad (38)$$

where $x_0^c = n \frac{(m+1)(1-c)\theta - m\sigma(\theta-c)}{2((m+1)(n+1)\theta - mn\sigma^2)}$, $x_v^c = n \frac{(m+1)\theta}{2((m+1)(n+1)\theta - mn\sigma^2)}$, $p_0^c = \frac{\theta[1+m(1-\sigma)] + c[mn(\theta-\sigma^2) + n\theta + m\sigma]}{(m+1)(n+1)\theta - mn\sigma^2}$, and $p_v^c = \frac{(m+1)\theta}{(m+1)(n+1)\theta - mn\sigma^2}$.

The maximisation problem for the government is

$$\max_a T(p^c(a) - c)x^c(a) + t(I - a(x^c(a) - x^c(0))) \quad (39)$$

$$(40)$$

Solving the problem, we obtain the optimal level of deduction a^c , which is now defined by the following equation:

$$a^c = \frac{T[p_1^c x_0^c + (p_0^c - c)x_1^c]}{2tx_1^c(1 - p_1^c T)} \quad (41)$$

It is easy to check that a^c is always positive: $(p_0^c - c) > 0$ is a necessary condition to have a market in the case of no deduction, and $(1 - p_1^c T)$ is always positive.

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