CentER

Discussion Paper

No. 2010–53

# ONE-MODE PROJECTION ANALYSIS AND DESIGN OF COVERT AFFILIATION NETWORKS

By Roy Lindelauf, Peter Borm, Herbert Hamers

May 2010

ISSN 0924-7815

Tilburg University

# One-mode projection analysis and design of covert affiliation networks

Roy Lindelauf [a,b]  Peter Borm [b]  Herbert Hamers [b]

May 20, 2010

**Abstract**

Decision makers in the field of national security, counterterrorism and counterinsurgency are faced with an uncertain, adaptive and asymmetrical threat. It should come as no surprise that a great need exists to understand covert organizations, the structure of which becomes known partially only after an attack or operation has occurred. What is known however is that many covert networks are organized in compartmentalized cellular structures. To better understand these cellular structures we model and analyze these cells as a collection of subsets of all participants in the covert organization, i.e., as hypergraphs or affiliation networks. Since terrorist cells can be viewed as graph theoretical cliques, i.e., everybody in the cell knows everybody else, such a covert affiliation network structure is analyzed by evaluating the one-mode projection of the corresponding hypergraph. First we provide a characterization of the total distance in the one-mode projection using its corresponding cell-shrinked version. Secondly we evaluate the one-mode projection with respect to the secrecy versus information tradeoff dilemma every covert organization has to solve. We present and analyze affiliation networks representing common covert organizational forms: star, path and semi-complete hypergraphs. In addition we evaluate an example of a covert organization wishing to conduct an attack and compare its performance to that of the common covert organizational forms. Finally we investigate affiliation networks that are optimal in the sense of balancing secrecy and information and we prove that among covert organizational forms in the class of hypertrees with the same number of cells uniform star affiliation networks are optimal.

*Subject classifications:* Terrorism; Counterinsurgency; Intelligence; Defense; Covert networks; Affiliation networks.

*Area of review:* Military and Homeland Security.

*JEL Classifications:* C50, C78

## 1 Introduction

In decision making aimed at confronting covert organizations managers are faced with high-level, long-term planning issues characterized by an uncertain and complex networked environment. The

[a]Military Operational Art & Science, Netherlands Defense Academy, P.O.Box 90002, 4800 PA Breda, The Netherlands. E-mail: rha.lindelauf.01@nlda.nl

[b]CentER and Department of Econometrics and OR, Tilburg University, P.O.Box 90153, 5000 LE Tilburg, The Netherlands.

amalgam of opponents in Afghanistan that confront ISAF and Operation Enduring Freedom for instance should be viewed as interdependent rather than independent, autonomous units. They exchange information via communication networks, diffuse weapons through trafficking networks and their Shura councils meet in affiliation networks. Understanding the effects of such a complex operational environment and evaluating its social aspects thus becomes extremely important in launching a successful counterinsurgency campaign, a fact recognized by the U.S. counterinsurgency doctrine (Petreaus et al. 2007).

Knowledge of the structure of a covert organization is often obtained only after operations or attacks have occurred. Additional empirical analysis of covert networks is difficult as the nature of data available on these systems is sparse, and even if such data exists it often is unstructured, messy, inaccurate, incomplete and out of date (cf. Carley 2006). Therefore it is desirable to understand and develop models of covert organizations that can function as a guide in pinpointing strengths and weaknesses of such organizations. We argue that a combination of OR/MS tools related to social network organization and decision analysis can be valuable in this respect.

Some attention has already been given to the use of OR/MS tools and experiments in the domain of anti-terrorism planning, for instance in how to best respond to an anthrax attack (Craft et al. 2005) or on using queuing theory to analyze scheduling policies in a surveillance system to detect terrorists in time (Lin et al. 2009). Other examples include studies into the costs and disruptions that might arise if U.S. domestic airlines adopted an antiterrorist measure aimed at preventing baggage unaccompanied by passengers from traveling in aircraft luggage compartments (Barnett et al. 2001) and models that identify resource-limited interdiction actions that maximally delay the completion time of a nuclear's weapons project (Brown et al. 2009). What is clear from the current war on terror is that many decision makers in law enforcement, the military and other security organs face opponents of a nature quite different than they were used to: asymmetrical, irregularly operating groups and organizations. In this paper we present a new OR/MS related tool that can function as a guide and benchmark for a specific class of such hybrid organizations: covert affiliation networks.

Traditional models of organizations do not fully apply to organizations such as Al Qaeda which is said to have transformed from a hierarchical terrorist organization to a multifaceted 'network of networks' (Tucker 2001). Similarly Hamas abandoned its centralized, leadership structure and developed a compartmented organizational structure of sparsely overlapping cells (Gambill 2002). More generally many covert organizations today, be they criminal, terrorist or insurgent, have profited from the shift to networked organizational forms (Arquilla and Ronfeldt 2001, Asal et al. 2007). These covert organizations assign tasks to cells to complete an operation. Furthermore there is coordination and control among these cells to ensure operational succes. Even in case of autonomous cell formation those cells need to be directed, i.e., they need strategic guidance (Cruickshank and Hage Ali 2007). This covert organizational form has been studied mostly from a qualitative perspective (see for instance Asal et al. 2007, Mishal 2005). Since it is important to develop a more general framework in which the structure of a covert network can be predicted and analyzed several formal models have been developed (McAllister 2004, McCormick and Owen 2000, Enders and Su 2007). What is recognized in this regard is the fact that the requirement for secrecy distinguishes the covert organization from the overt organization (Baker and Faulkner 1993). Taking this dilemma explicitly into account Lindelauf et al. (2009) analyzed the problem of covert network structure design from a multi objective optimization perspective. In this paper we build upon this research by extending the analysis to the case of covert *affiliation* networks. What we adopt from Lindelauf et al. (2009) is the method of measuring secrecy and information in networks. Fundamentally different and new is the restriction to the domain of covert cells modeled by affiliation networks. We focus on affiliation in cells because covert organizations employ cells consisting of several indiviuals needed to complete a task. Furthermore, these cells have to be coordinated and controlled to better guarantee mission success. Common types of such cells are for instance a command and control cell, a tactical operations cell, an intelligence cell and a logistics cell (Nance, 2008).

Overt affiliation networks have been studied abundantly. Examples include interlocking boards of directors (Levine, 1972; Mariolis, 1975; Mintz and Schwartz, 1981a,b; Allen, 1982; Bearden and Mintz, 1987), club memberships (Bonacich, 1978) and social gatherings (Davis et al., 1941; Breiger 1974). However very few, if any, affiliation network analysis has been done in the important domain of covert networks taking the aspect of secrecy explicitly into account. In this paper we will present a general framework to analyze covert cells by evaluating them on the basis of the one-mode projection of the corresponding affiliation network.

Analyzing cell structured affiliation topologies is of twofold importance: it increases the understanding of their structure and henceforth helps to improve strategies to counter them, and it enables military organizations to optimize covert operations. Perhaps the best known example of a covert operation conducted according to cell structured affiliations is provided by Al Qaeda's 9/11 operation. The organizational structure of the covert group conducting that operation equalled 4 cells of 19 people (Zwikael 2007). Additionally there was a command and control 'cell' guiding the operation, consisting of Khalid Sheikh Mohammed, Mohammed Atef and Osama Bin Laden. A more historical, nation-state, example is the case of Israeli's operation Susannah (Johnson 2007, Golan 1978). The belief among Israel's defense chiefs was that by conducting underground operations in Egypt its military regime could be shown to be insufficiently reliable. Consequently it was hoped for that the British decision to leave Egypt would be reconsidered. The covert network tasked with conducting the attacks in Egypt consisted of two operational cells: one cell in Alexandria and another one in Cairo. Command and control of these cells came from Israeli emissaries which can be viewed as a third cell. The covert affiliation network conducting this operation can therefore be seen to consist of three cells of varying size. After some initial operations the Alexandria cell was detected by Egyptian intelligence and through observation and interrogation the Cairo cell members were also uncovered and arrested. This incident illustrates the importance of being able to evaluate several different possible cell structures *before* conducting and creating an underground network. We will analyze an explicit but hypothetical example of a covert organization wishing to conduct an attack (cf. Frantz et al 2005).

Many current covert organizational structures can be seen to consist of cells organized in one of several standard forms: a star, path or hybrid structure (Arquilla and Ronfeldt 2001). For instance Mishal et al. (2005) present several topological examples in case of Islamic terrorist organizations such as Hamas star like compartmentalization and Hizballah's infiltration of operatives into Israel according to path like structures. More formally Frantz et al. (2005) discusses a characterization of cellular networks. It is argued that often each cell in the network forms a clique, i.e., everybody in the cell is connected to everybody else in the cell. The choice of adopting cellular structures clearly is derived from maintaining secrecy and informational scrutiny. Assigning cell leaders and selecting their interaction topology reflects the desired span of control: central in case of a star topology and becoming more decentralized in case of a path, ending up in a hybrid structures. In this paper we will formalize these basic organizational structures of covert affiliation networks as they can serve as a starting point for the analysis of more advanced affiliation networks. Thus we will explicitly define the star and path structures consisting of cells that are cliques. In addition we analyze a hybrid structure, called a semi-complete network, consisting of a ring of cells whose leaders are all interconnected. First we will characterize the total distance for the one-mode projections of such affiliation networks. Subsequently we evaluate the secrecy, information and total performance measure for the one-mode projection of these three standard covert affiliation structures. Based on these covert affiliation network indicators we discuss optimality within the class of hypertrees and present a procedure to restructure the affiliation network structure while improving the trade-off performance. Using this procedure we prove that uniform star affiliation networks are optimal in balancing secrecy and information.

Section 2 discusses graph theoretical preliminaries and provides measures that capture the notions of secrecy and information in covert organizations. In addition an example of a covert organization is presented to illustrate the mathematical notation. Section 3.1 studies total distance of one-mode projections of several basic hypergraphs. The computation of the total distance is simplified by use of a proposition relating the total distance in a covert affiliation network to its cell-shrinked version. The performance with regard to the information versus secrecy tradeoff of the star, path and a hybrid affiliation structure is analyzed in section 3.2, and we compare their performance to that of the example introduced previously. In addition we will show in section 4 that among all hypertrees of given order and size organizing the affiliation network according to a star is optimal in balancing information and secrecy.

## 2 Mathematical Preliminaries

For a general overview of (hyper-)graph theory we refer to Bollobas (1986, 1998). Note that the words *graph* and *network* will be used interchangeably throughout the text as well as the words *hypergraph* and *affiliation network*.

A graph $g$ is an ordered pair $(N, E)$, where $N$ represents the finite set of players[a] and the set of edges $E$ is a subset of the set of all unordered pairs of players. An edge $\{i, j\}$ connects the players $i$ and $j$ and is also denoted by $ij$. The order of a graph is the number of players $|N| = n$ and the size equals its number of edges $|E| = m$. The set of connected graphs of order $|N|$ is denoted

[a] A player is modeled as a node in a graph and represents an individual terrorist, insurgent or criminal engaged in a covert organization.

by $\mathbb{G}(N)$. For $V \subset N$, the $V$-induced subgraph of $g$ is the graph $g' = (V, E')$ whose edge set $E'$ consists of all the edges $ij \in E$ of the original graph $g$ that connect players $i, j \in V$. The set of neighbors of player $i \in N$ in graph $g = (N, E)$ is indicated by $\Gamma_i(g) = \{j \in N | ij \in E\}$. We denote the degree of player $i$ in a graph $g$ by $d_i(g) = |\Gamma_i(g)|$. The shortest distance (measured by the number of edges) between player $i$ and $j$ in a graph is called the geodesic distance between $i$ and $j$. The geodesic distance between players $i, j$ in $g \in \mathbb{G}(N)$ is denoted by $l_{ij}(g)$. Clearly, $l_{ij}(g) = l_{ji}(g)$. We will write $l_{ij}$ instead of $l_{ij}(g)$ if there can be no confusion about the graph under consideration. We set $l_i(g) = \sum_{j \in N} l_{ij}(g)$. The total distance $T(g)$ in the graph $g = (N, E)$ is defined by $T(g) = \sum_{i \in N} l_i(g)$.

A hypergraph or affiliation network $H$ is a pair $(N, X)$, where $N$ is a finite player set and $X \subset 2^N$ is a collection of subsets of $N$. Elements of $X$ are called *events* or *cells*. We denote the set of cells a coalition of players $S \subset N$ is engaged in by $X(S) = \{A \in X | A \cap S \neq \emptyset\}$. A player $i \in N$ that is a member of more than one cell, i.e., such that $|X(\{i\})| \geq 2$, is called a cell leader. We define the set of cell leaders in $H$ by $L(H)$. The order of a hypergraph is the number of players $|N| = n$ and the size equals its number of cells $|X| = c$. The set of all subsets of $N$ of size $r$ is denoted by $N^r$. An $r-$uniform hypergraph on $N$ is a pair $(N, X)$ where $X \subset N^r$. The hypergraph $(N, X)$ is connected if for every $i, j \in N$ there exists a sequence $A_1, ..., A_s$ of cells with $s \geq 1$, $A_l \in X$, for all $l \in \{1, ..., s\}$, such that $i \in A_1$, $j \in A_s$ and $A_t \cap A_{t+1} \neq \emptyset$ for $t = 1, ..., s-1$. The class of all connected hypergraphs with player set $N$ is denoted by $\mathbb{C}(N)$. A cycle in a hypergraph $H = (N, X)$ is a sequence $A_1, ..., A_s$ with $s \geq 3$ of $s-1$ different cells $A_l \in X$, for all $l \in \{1, ..., s\}$, such that $A_i \cap A_{i+1} \neq \emptyset$ for $i = 1, ..., s-1$, $A_1 = A_s$ and $A_i \cap A_j = \emptyset$ otherwise. A connected hypergraph is a hypertree if it contains no cycles. The class of connected affiliation networks in which each two cells have at most one player in common is denoted by $\mathbb{H}(N) = \{(N, X) \in \mathbb{C}(N) | \quad |A \cap B| \leq 1 \text{ for all } A, B \in X\}$. We denote the class of all r-uniform hypergraphs in $\mathbb{H}(N)$ of size $c$ by $\mathbb{H}_r^c(N)$, the class of all hypertrees in $\mathbb{H}(N)$ of size $c$ by $\mathbb{H}_{tree}^c(N)$ and the class of all r-uniform hypertrees in $\mathbb{H}(N)$ of size $c$ is denoted by $\mathbb{H}_{r-tree}^c(N)$. We define the *one-mode* projection graph $g_{\perp}(H) = (N, E_H) \in \mathbb{G}(N)$ corresponding to the affiliation network $H = (N, X) \in \mathbb{H}(N)$ by letting $ij \in E_H$ if and only if there exists an $A \in X$ such that $i, j \in A$.

**Example 2.1 (cf. Frantz et al.)**

Consider an organization wishing to carry out an attack with an explosive device. In addition assume that the organization has 16 individuals available to prepare for and conduct such an attack. In preparing the attack several tasks have to be conducted, such as bomb building, delivery of materials and finances, target reconnaissance, target site preparation, etc. The organization adopts a cellular structure by having each cell conduct one such task. We present a *possible* affiliation structure for the preparation and planning of the attack as follows: we label the players 1 through 16 and assume that player 1,2,...,6 constitute the attack cell, player 7,8,...,12 the bomb building cell, player 1 and 7 coordinate between the attack cell and the bomb building cell, player 13 coordinates the finances with player 7, player 16 delivers the materials to player 10, player 14 conducts reconnaissance and delivers information on the target to player 12, and finally player 15 prepares the target site and coordinates this with player 11. Note that this organizational structure corresponds to an example of a covert network as introduced by Frantz et al. (2005). The hypergraph corresponding to this organization is denoted by $H_{ex} = (N, X)$ with $N = \{1, 2, ..., 16\}$ and,

$$X = \{A_1, A_2, ..., A_7\}$$

with cells $A_1 = \{7, 8, 9, 10, 11, 12\}$, $A_2 = \{7, 13\}$, $A_3 = \{10, 16\}$, $A_4 = \{12, 14\}$, $A_5 = \{11, 15\}$, $A_6 = \{1, 7\}$ and $A_7 = \{1, 2, 3, 4, 5, 6\}$. Clearly $H_{ex} \in \mathbb{H}^7_{tree}$ and $L(H_{ex}) = \{1, 7, 10, 11, 12\}$. The corresponding one-mode projection $g_\perp(H_{ex})$ is presented in Figure 1.

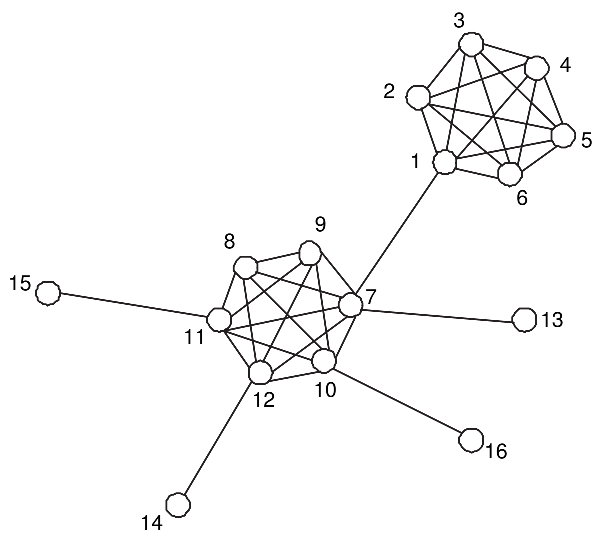


Figure 1: One-mode projection of the affiliation network of example 2.1.

Next we formally define several standard affiliation networks representing basic covert network organizational designs. In particular we model a cell in a covert organization as a cell in a hypergraph (cf. Frantz et al. 2005), and we consider a star, a path and a hybrid topology (cf. Arquilla and Ronfeldt 2001).

Let $H = (N, X) \in \mathbb{H}^c_r(N)$ be a hypergraph such that $X = \{A_i\}_{i=1}^c$, $c \geq 2$.
The hypergraph $H$ is called an *r-star*, denoted by $H^c_{r-star}$, if there is a $l \in N$ such that $A_i \cap A_j = \{l\}$ for all $i, j \in \{1, ..., c\}$ with $i \neq j$. Observe that $|L(H^c_{r-star})| = 1$.

The hypergraph $H$ is called an *r-path*, denoted by $H^c_{r-path}$, if $|A_i \cap A_j| = 1$ if and only if $j = i+1$ with $i \in \{1, ..., c-1\}$. Obviously $|L(H^c_{r-path})| = c - 1$.

For $c \geq 3$, the hypergraph $H$ is called an *r-ring*, denoted by $H^c_{r-ring}$, if $|A_i \cap A_j| = 1$ if and only if $j = i + 1$ with $i \in \{1, ..., c-1\}$ or $i = c$ and $j = 1$. Observe that $|L(H^c_{r-ring})| = c$.

Finally we introduce a hybrid affiliation network in which cell leaders have an active coordinating role. We do this by considering a ring structure where all cell leaders connect in one additional cell.

Consider $H = (N, X) \in \mathbb{C}(N)$ of size $c + 1, c \geq 3$, with $X = \{A_i\}_{i=1}^{c+1}$. The hypergraph $H$ is called *r-semicomplete*, denoted by $H^{c+1}_{r-semicomp}$, if it satisfies the following two properties:

(i) $(N, \{A_i\}_{i=1}^c)$ is an r-ring,

(ii) $L((N, \{A_i\}_{i=1}^c)) = A_{c+1}$.

Note that typically $H^{c+1}_{r-semicomp} \notin \mathbb{H}(N)$.

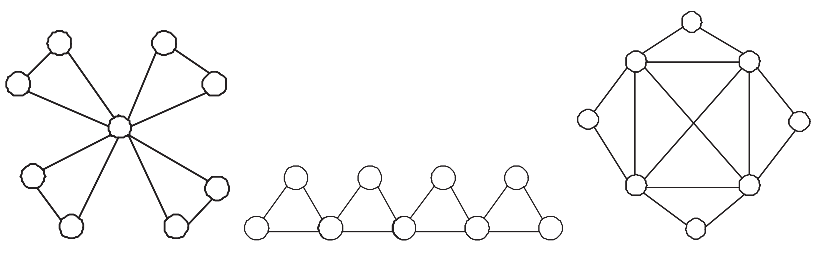

Figure 2: One-mode projections of $H^4_{3-star}$ (left), $H^4_{3-path}$ (middle) and $H^5_{3-semicomp}$(right).

In Table 1 we indicate the order and size of the one-mode projection graphs for the three standard affiliation networks.

| $g_\perp(H) = (N, E)$ | $\lvert N\rvert = n$ | $\lvert E\rvert = m$ |
|---|---|---|
| $H = H^c_{r-star}$ | $c(r-1)+1$ | $\frac{cr(r-1)}{2}$ |
| $H = H^c_{r-path}$ | $c(r-1)+1$ | $\frac{cr(r-1)}{2}$ |
| $H = H^{c+1}_{r-semicomp}$ | $c(r-1)$ | $\frac{c(r(r-1)+c-3)}{2}$ |

Table 1: Order and size of the three one-mode projections of standard hypergraphs.

Finally we recall the definitions of measures specifically designed for the analysis of the interaction structure of covert networks as introduced in Lindelauf et al. (2009). The average information performance $I(g)$ of a network $g \in \mathbb{G}(N)$ with $|N| = n$ is defined as the normalized reciprocal of the total distance $T(g)$,

$$I(g) = \frac{n(n-1)}{T(g)}. \tag{1}$$

It follows that $0 \leq I(g) \leq 1$. The secrecy performance $S(g)$ of a network $g \in \mathbb{G}(N)$ with $|N| = n$ and size $m$ is given by

$$S(g) = \frac{n^2 - n - 2m}{n^2} \tag{2}$$

with $0 \leq S(g) \leq 1$. Lindelauf et al. (2009) show that $S(g)$ represents the expected fraction of the network that 'survives' given an uniform exposure probability distribution and the assumption that upon exposure of individual $i$ all individuals with which he is connected are also exposed. Moreover it was argued on the basis of multi-objective optimization and bargaining theory that a covert organization that wishes to balance the tradeoff between secrecy and information does best by adopting a network $g$ that maximizes the performance measure $\mu$, defined by

$$\mu(g) = S(g)I(g). \tag{3}$$

# 3 One-mode Projection Analysis

## 3.1 Total distance

Computing the total distance of the one-mode projections $g_\perp(H)$ corresponding to a hypergraph $H$ can be cumbersome. We will prove that to compute the total distance in the one-mode projection of r-uniform hypertrees one only needs to compute the total distances of a certain subset of its players. This subset arises from the so-called corresponding 'cell-shrinked' version of the hypertree.

The *cell shrinked* graph $g^{-}(H)$ corresponding to an r-uniform hypertree $H = (N, X) \in \mathbb{H}^{c}_{r-tree}(N)$ is defined as follows. For each $A \in X$ such that $|A \cap L(H)| = 1$ we take exactly one *representative* $j_A \in A \setminus L(H)$ and define $R(H) = \{j_A | A \in X, |A \cap L(H)| = 1\}$. We set $LR = L(H) \cup R(H)$ and define $g^{-}(H)$ as the $LR$-induced subgraph of $g_{\perp}(H)$. See Figure 3 below for an illustration.

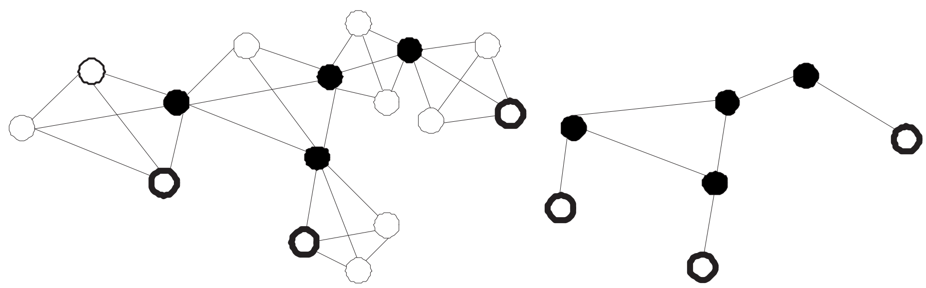

Figure 3: One-mode projection $g_{\perp}(H)$ (left) and its cell shrinked version $g^{-}(H)$ (right), the leaders are represented by solid dots and the representatives by bold line dots.

We relate the total distance in an r-uniform hypertree $H = (N, X)$ to the total distance of the players in its corresponding cell-shrinked version. For this aim define $n_A = |A \cap LR|$ for all $A \in X$ and let

$$w_k(H) = 1 + \sum_{A \in X : k \in A} \frac{r - n_A}{n_A}$$

for all $k \in LR$. Note that $n_A = 2$ if $|L(H) \cap A| = 1$, and that $n_A \geq 2$ otherwise.

**Proposition 3.1** *Let $H = (N, X) \in \mathbb{H}^{c}_{r-tree}$. Set $g^{-}(H) = (LR, E)$. Then*

*(i) For all $j \in N \setminus LR$ and $A \in X$ the unique event such that $j \in A$ it holds that $l_j(g_{\perp}(H)) = \frac{n-r}{n_A} + \frac{1}{n_A} \sum_{k \in LR \cap A} l_k(g_{\perp}(H))$,*

*(ii) $T(g_{\perp}(H)) = \sum_{k \in LR} w_k(H) l_k(g_{\perp}(H)) + (n-r) \sum_{A \in X} \frac{r-n_A}{n_A}$.*

**Proof:**

*(i)* Consider $j \in N \setminus LR$ and let $A \in X$ be the unique event such that $j \in A$. Let $k \in LR \cap A$ and define $N_k(j) = \{z \in N \setminus \{k\} | l_{kz}(g_{\perp}(H)) < l_{jz}(g_{\perp}(H))\}$. It readily follows that

$$l_j(g_{\perp}(H)) = l_k(g_{\perp}(H)) + |N_k(j)|.$$

Therefore,

$$l_j(g_\perp(H)) = \frac{1}{n_A}\sum_{k\in A\cap LR}(l_k(g_\perp(H)) + |N_k(j)|) = \frac{1}{n_A}\sum_{k\in A\cap LR} l_k(g_\perp(H)) + \frac{n-r}{n_A}.$$

$(ii)$ Note that

$$\begin{aligned}
T(g_\perp(H)) &= \sum_{k\in LR} l_k(g_\perp(H)) + \sum_{j\in N\setminus LR} l_j(g_\perp(H)) \\
&= \sum_{k\in LR} l_k(g_\perp(H)) + \sum_{A\in X}\sum_{j\in A\setminus LR} l_j(g_\perp(H)) \\
&= \sum_{k\in LR} l_k(g_\perp(H)) + \sum_{A\in X}(r-n_A)[\frac{n-r}{n_A} + \frac{1}{n_A}\sum_{k\in LR\cap A} l_k(g_\perp(H))] \\
&= \sum_{k\in LR} l_k(g_\perp(H)) + (n-r)\sum_{A\in X}\frac{r-n_A}{n_A} + \sum_{A\in X}\frac{r-n_A}{n_A}\sum_{k\in LR\cap A} l_k(g_\perp(H)) \\
&= \sum_{k\in LR} l_k(g_\perp(H)) + (n-r)\sum_{A\in X}\frac{r-n_A}{n_A} + \sum_{k\in LR} l_k(g_\perp(H))\sum_{A\in X, k\in A}\frac{r-n_A}{n_A} \\
&= \sum_{k\in LR}\{1 + \sum_{A\in X, k\in A}\frac{r-n_A}{n_A}\} l_k(g_\perp(H)) + (n-r)\sum_{A\in X}\frac{r-n_A}{n_A} \\
&= \sum_{k\in LR} w_k(H) l_k(g_\perp(H)) + (n-r)\sum_{A\in X}\frac{r-n_A}{n_A}.
\end{aligned}$$

Where the third equality follows from $(i)$. □

**Proposition 3.2** *Let $H = (N, X) \in \mathbb{H}^c_{r-tree}$ be such that $n_A = 2$ for all $A \in X$. Then*

$$T(g_\perp(H)) = (r-1)\sum_{k\in LR} w_k(H) l_k(g^-(H)) + (n-r)\sum_{A\in X}\frac{r-n_A}{n_A}. \tag{4}$$

**Proof:**

Since $n_A = 2$ for all $A \in X$ it holds that $g^-(H)$ is a tree. Hence, since every cell of $g_\perp(H)$ contains $r$ players it follows that

$$l_i(g_\perp(H)) = (r-1) l_i(g^-(H)) \quad \text{for all} \quad i \in LR.$$

and the result follows from Proposition 3.2(ii). □

For the three standard types of hypergraphs the total distances of their one-mode projections are provided in the lemma below.

**Lemma 3.1**

*(i)* $T(g_\perp(H^c_{r-star})) = c(r-1)[r + 2(c-1)(r-1)]$

*(ii)* $T(g_\perp(H^c_{r-path})) = c(r-1)(cr + \frac{1}{3}c^2r + \frac{4}{3} - c - \frac{1}{3}r - \frac{1}{3}c^2)$

*(iii)* $T(g_\perp(H^{c+1}_{r-semicomp})) = c(r-1)(3cr + 7 - 5c - 4r)$

**Proof:**

$(i)$ Consider $H = H^c_{r-star}$ and let $i \in N$ be the unique leader of $H$. Clearly $i$ has distance 1 to all other $c(r-1)$ nodes, i.e.,

$$l_i(g_\perp(H)) = c(r-1).$$

The nodes $j \in N \setminus \{i\}$ have distance 1 to each other member of the cell they belong to and distance 2 to the remaining nodes, hence

$$l_j(g_\perp(H)) = (r-1) + 2(c-1)(r-1)$$

for all $j \in N \setminus \{i\}$. Consequently

$$T(g_\perp(H)) = c(r-1) + c(r-1)[(r-1) + 2(c-1)(r-1)]$$

and the result follows.

$(ii)$ Consider $H = H^c_{r-path} = (N, X)$. Since $n_A = 2$ for all $A \in X$ we can use the result in Proposition 3.2 to determine $T(g_\perp(H))$. Let $g = g^-(H) = (LR, E)$ with $LR = L(H) \cup R(H)$.

Clearly $|L(H)| = c - 1$ and $|R(H)| = 2$. Then

$$
\begin{aligned}
T(g_{\perp}(H)) &= (r-1)\sum_{k\in LR} w_k(H)l_k(g) + (n-r)\sum_{A\in X}\frac{r-n_A}{n_A} \\
&= (r-1)\sum_{k\in L(H)} w_k(H)l_k(g) + (r-1)\sum_{k\in R(H)} w_k(H)l_k(g) + (n-r)c(\frac{r-2}{2}) \\
&= (r-1)\sum_{k\in L(H)} (r-1)l_k(g) + (r-1)\sum_{k\in R(H)} \frac{r}{2}l_k(g) + \frac{1}{2}c(c-1)(r-2)(r-1) \\
&= (r-1)^2\sum_{k\in L(H)} l_k(g) + \frac{r(r-1)}{2}\cdot 2\cdot\frac{c(c+1)}{2} + \frac{1}{2}c(c-1)(r-2)(r-1) \\
&= (r-1)^2[\frac{c(c+1)(c+2)}{3} - c(c+1)] + \frac{1}{2}rc(r-1)(c+1) + \frac{1}{2}c(c-1)(r-2)(r-1)
\end{aligned}
$$

and the result follows. Note that the last equality follows from the fact that $T(g) = \frac{c(c+1)(c+2)}{3}$ as is derived in Lemma 2.1 of Lindelauf et al. (2009).

(*iii*) Consider $H = H^{c+1}_{r-semicomp} = (N, X)$. Take $i \in L(H)$. Clearly

$$
\begin{aligned}
l_i(g_{\perp}(H)) &= 1\cdot(2(r-2)+c-1) + 2(c(r-1)-1-(2(r-2)+c-1)) \\
&= 2(c+1)r + 6 - 4r - 3(c+1).
\end{aligned}
$$

Now take $j \in N \setminus L(H)$. Then

$$
\begin{aligned}
l_j(g_{\perp}(H)) &= 1\cdot(r-1) + 2(c-2+2(r-2)) + 3(c(r-1)-1-(r-1+c-2+2(r-2))) \\
&= 3(c+1)r - 4(c+1) - 7r + 9.
\end{aligned}
$$

Then

$$
\begin{aligned}
T(g_{\perp}(H)) &= \sum_{i\in L(H)} l_i(g_{\perp}(H)) + \sum_{j\in N\setminus L(H)} l_j(g_{\perp}(H)) \\
&= c[2(c+1)r + 6 - 4r - 3(c+1)] + c(r-2)[3(c+1)r - 4(c+1) - 7r + 9]
\end{aligned}
$$

and the result follows. □

## 3.2 Covert affiliation network performance

In analyzing covert affiliation networks their one-mode projection graphs can be seen to represent the interaction structure among the members of the organization. In this section we analyze the performance measure $\mu$ for the one-mode projections of the three basic covert affiliation networks $H^c_{r-star}$, $H^c_{r-path}$ and $H^{c+1}_{r-semicomp}$.

From the definition of the information performance measure $I$ as given in equation (1) together with Table 1 and Lemma 3.1 one readily derives

**Lemma 3.2**

*(i)* $I(g_\perp(H^c_{r-star})) = \frac{c(r-1)+1}{2cr-2c-r+2}$

*(ii)* $I(g_\perp(H^c_{r-path})) = \frac{3(c(r-1)+1)}{3cr+c^2r+4-3c-r-c^2}$

*(iii)* $I(g_\perp(H^{c+1}_{r-semicomp})) = \frac{c(r-1)-1}{3cr-5c-4r+7}$

From the definition of the secrecy measure $S$ in (2) and Table 1 together with Lemma 3.1 we find

**Lemma 3.3**

*(i)* $S(g_\perp(H^c_{r-star})) = \frac{c(c-1)(r-1)^2}{(c(r-1)+1)^2}$

*(ii)* $S(g_\perp(H^c_{r-path})) = \frac{c(c-1)(r-1)^2}{(c(r-1)+1)^2}$

*(iii)* $S(g_\perp(H^{c+1}_{r-semicomp})) = \frac{(r-2)(r(c-1)-2)}{c(r-1)^2}$

In addition we present an asymptotic analysis in Table 2.

| $H$ | $H^c_{r-star}$ | $H^c_{r-path}$ | $H^{c+1}_{r-semicomp}$ |
|---|---|---|---|
| $\lim_{r\to\infty} I(g_\perp(H))$ | $\frac{c}{2c-1}$ | $\frac{3c}{c^2+3c-1}$ | $\frac{c}{3c-4}$ |
| $\lim_{c\to\infty} I(g_\perp(H))$ | $\frac{1}{2}$ | $0$ | $\frac{r-1}{3r-5}$ |
| $\lim_{r\to\infty} S(g_\perp(H))$ | $\frac{c-1}{c}$ | $\frac{c-1}{c}$ | $\frac{c-1}{c}$ |
| $\lim_{c\to\infty} S(g_\perp(H))$ | $1$ | $1$ | $\frac{r(r-2)}{(r-1)^2}$ |

Table 2: Asymptotic analysis of the information and secrecy performance measure.

From Lemma 3.2(i) and (ii) it follows that for sufficiently many cells, the star affiliation network outperforms the path with regard to information performance. Intuitively this is clear: the distance between cells in a star affiliation network is maximally 2 whereas it becomes increasingly more difficult to reach other cells in case of a path affiliation structure. However, it can also be seen that in case of a small number of cells the semi-complete hypergraph may outperform the star. From Lemma 3.3 it can be seen that in case of a low value of $r$, i.e., small cells, the star affiliation network outperforms the path and semi-complete hypergraph with regard to secrecy. From Table 2 it can also be seen that the star network outperforms the other networks asymptotically.

From the definition of the performance measure in (3), Lemma 3.2 and Lemma 3.3 we find

**Theorem 3.1**

*(i)* $\mu(g_{\perp}(H^{c}_{r-star})) = \frac{c(c-1)(r-1)^2}{(c(r-1)+1)(2cr-2c-r+2)}$

*(ii)* $\mu(g_{\perp}(H^{c}_{r-path})) = \frac{3c(c-1)(r-1)^2}{(c(r-1)+1)(3cr+c^2r+4-3c-r-c^2)}$

*(iii)* $\mu(g_{\perp}(H^{c+1}_{r-semicomp})) = \frac{(c(r-1)-1)(r-2)(r(c-1)-2)}{c(r-1)^2(3cr-5c-4r+7)}$

We compare the total performance of the star, path and semi-complete covert network affiliation structures in Figure 4.

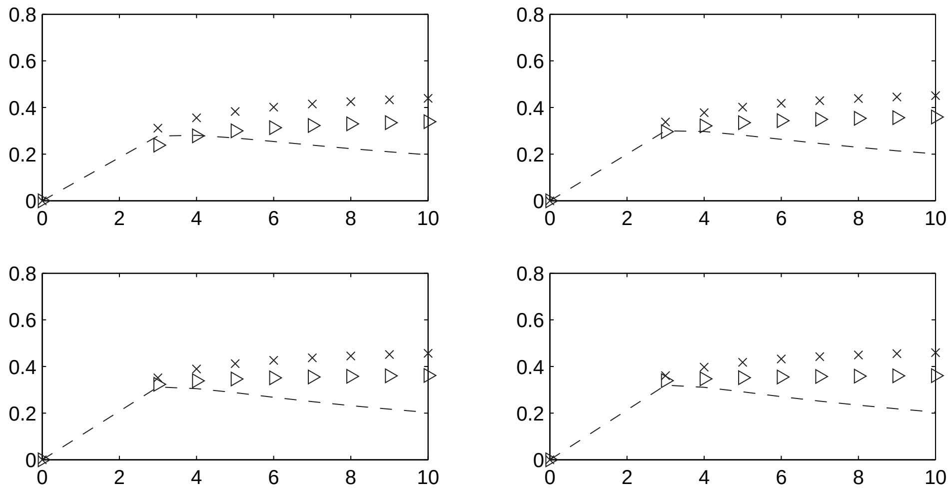

Figure 4: Performance measure $\mu$ of $H^{c}_{r-path}$ (–), $H^{c}_{r-star}$ (x) and $H^{c+1}_{r-semicomp}$ ($\Delta$) as a function of the number of cells $c$ (horizontal axis) and the number of nodes $r$ per cell. Top left: r=3, top right: r=4, down left: r=5, down right: r=6.

It can be seen that the star affiliation network outperforms the other basic affiliation networks.

**Example 3.1**

In example 2.1 we introduced an organization wishing to carry out an attack. Seven tasks were divided among as many cells. We compare the information, secrecy and trade-off performance of the affiliation network $H_{ex}$ as presented in example 2.1 with that of comparable basic affiliation networks. For this purpose we consider a star and path network consisting of 7 cells, i.e., $H^7_{3-star}$ and $H^7_{3-path}$, and since semi-complete networks have an additional cell of leaders, $H^8_{3-semicomp}$.

| | $I(g_\perp(H))$ | $S(g_\perp(H))$ | $\mu(g_\perp(H))$ |
|---|---|---|---|
| $H = H^7_{3-star}$ | 0.56 | 0.75 | 0.42 |
| $H = H^7_{3-path}$ | 0.32 | 0.75 | 0.24 |
| $H = H^8_{3-semicomp}$ | 0.56 | 0.57 | 0.32 |
| $H = H_{ex}$ | 0.44 | 0.66 | 0.29 |

Table 3: A comparison of the information, secrecy and total trade-off performance in the setting of example 2.1.

Both the star and semi-complete affiliation structures outperform the actual structure, whereas the path affiliation network performs worse. This leads to the conclusion that, assuming that secrecy and information are the most decisive parameters in conducting such a covert operation, the organizational structure could be improved upon.

## 4 On optimal affiliation networks

The results in section 3 indicate that an r-star hypergraph is a good affiliation network for covert organizations in terms of secrecy and information performance. This leads us to investigate the performance of the star hypergraph affiliation network $H^c_{r-star}$ in more detail. In this section we will show that the r-star outperforms all comparable hypertrees with the same number of cells and of the same order. Before we formally state and prove this assertion in Theorem 4.1 we first describe a 'tree-to-star' transformation procedure.

Consider a hypertree $H = (N, X) \in \mathbb{H}^c_{tree}$, consisting of $c$ cells, of possibly different size. With $j, k \in N$, $k \neq j$, define

$$N_k(j) = \{i \in N | l_{ki}(g_\perp(H)) < l_{ji}(g_\perp(H))\}$$

as the set of nodes closer to $k$ than to $j$ in the one-mode projection of $H$. The 'tree-to-star' transformation consists of the following five steps.

(1) Select $A \in X$ such that $|A \cap L(H)| > 1$. Note if $H$ is not a star, this is possible.

(2) Set $A \cap L(H) = \{a_1, ..., a_t\}$.

(3) Set $X_1 = X$.

(4) For $i = 2$ to $t$ do

(i) set $B_i = X(\{a_i\}) \setminus \{A\}$,

(ii) for all $C \in B_i$ let $\bar{C} = (C \setminus \{a_i\}) \cup \{a_1\}$,

(iii) let $\bar{B}_i = \{\bar{C} | C \in B_i\}$,

(iv) set $X_i = (X_{i-1} \setminus B_i) \cup \bar{B}_i$.

(5) Set $X = X_t$. If $\{A \in X || A \cap L(H)| > 1\} \neq \emptyset$ return to step 1, otherwise stop[b].

This procedure results in a hypergraph whose one-mode projection equals a star graph, possibly with cells of different sizes. We illustrate this procedure by an example.

**Example 4.1:** Let $H = (N, X)$ with $N = \{1, 2, ..., 16\}$ and

$$X = \{A_1, ..., A_5\}$$

with cells $A_1 = \{1, 2, 3, 15, 16\}$, $A_2 = \{3, 4, 5, 11, 14\}$, $A_3 = \{5, 6, 7\}$, $A_4 = \{7, 8, 9, 10\}$ and $A_5 = \{11, 12, 13\}$ (see Figure 5 top left for $g_\perp(H)$). Clearly $L(H) = \{3, 5, 7, 11\}$. In step 1 select $A = A_2$ with $A_2 \cap L(H) = \{5, 11, 3\}$ and set $a_1 = 5$, $a_2 = 11$ and $a_3 = 3$. Since $X(\{11\}) = \{A_2, A_5\}$ it follows that $B_2 = \{A_5\}$ (step 4i) and we obtain $\bar{A}_5 = \{5, 12, 13\}$, $\bar{B}_2 = \{\{5, 12, 13\}\}$ and $X_2 = \{A_1, A_2, A_3, A_4, \{5, 12, 13\}\}$ (in Figure 5 top right the one-mode projection of this intermediate hypergraph is presented ). Similarly we find $\bar{B}_3 = \{\{1, 2, 5, 15, 16\}\}$ and $X_3 = \{\{1, 2, 5, 15, 16\}, A_2, A_3, A_4, \{5, 12, 13\}\}$. Now $A_3 \cap L(H) = \{5, 7\}$, hence we return to step 1 and repeat. Choosing $a_1 = 5$ and $a_2 = 7$ results in the star $H' = (N, X')$ with

$$X' = \{\{3, 4, 5, 11, 14\}, \{1, 2, 5, 15, 16\}, \{5, 6, 7\}, \{5, 8, 9, 10\}, \{5, 12, 13\}\}.$$

In Figure 5 bottom the resulting one-mode projection $g_\perp(H')$ is presented.

[b]The algorithm stops since each iteration the number of cell leaders is reduced.

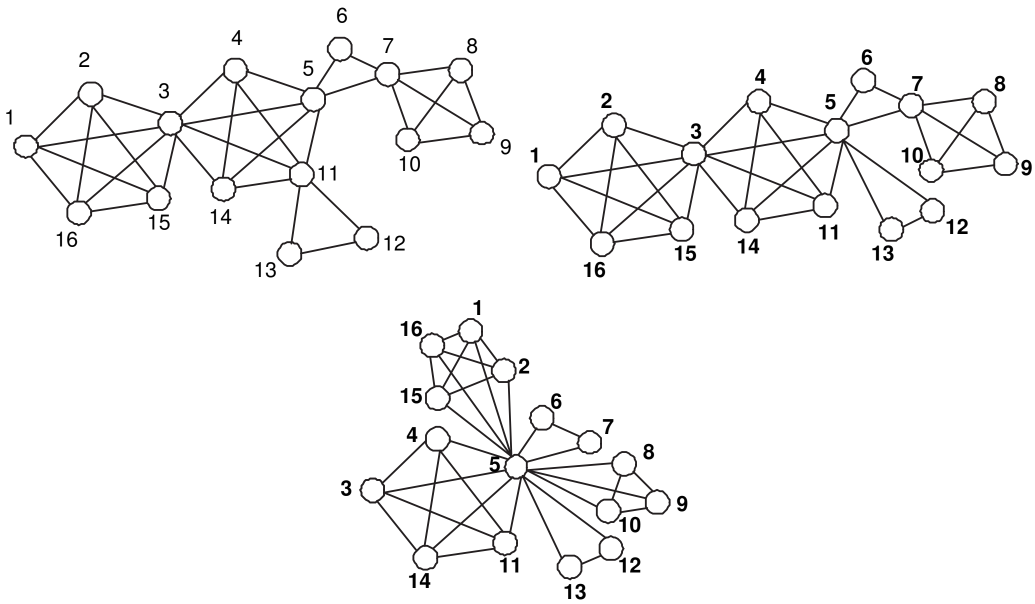


Figure 5: Illustration of the 'tree-to-star' transformation procedure in Example 4.1.

We now show that the r-star hypergraph maximizes the performance $\mu$ among all fixed size and order hypertrees by first showing that each iteration in the 'tree-to-star' transformation procedure increases the value of $\mu$ for the corresponding hypergraph and second that among all star affiliation networks with cells of different sizes r-uniform ones are optimal.

**Theorem 4.1** *$\mu(g_\perp(H^c_{r-star})) \geq \mu(g_\perp(H))$ for all $H \in \mathbb{H}^c_{tree}$ of order $n = (r-1)c+1$.*

**Proof:**
Let $H = (N, X) \in \mathbb{H}^c_{tree}$ with $|N| = (r-1)c+1$ and apply the 'tree-to-star' transformation procedure. Denote the resulting star hypergraph by $H' = (N, X')$. Note that every iteration of steps 4 reduces the total distance in the corresponding one-mode projections (in fact within each iteration $i$ the total distance reduces by $2|N_{a_i}(a_1)| \cdot |N_{a_1}(a_i)|$). Since the size and order of the one-mode projection remain constant during the transformation it follows that $\mu(g_\perp(H')) > \mu(g_\perp(H))$.

Let $g_\perp(H') = (N, E)$ with $|E| = m$. Note that there are exactly $m$ pairs $ij$ such that $l_{ij}(g_\perp(H')) = 1$ and hence $\binom{n}{2} - m$ pairs $ij$ with $l_{ij}(g_\perp(H')) = 2$. Then,

$$T(g_\perp(H')) = 2(m + 2(\binom{n}{2} - m)) = 2n(n-1) - 2m.$$

Therefore

$$\mu(g_\perp(H')) = \frac{(n^2-n)(n^2-n-2m)}{n^2(2n^2-2n-2m)}$$

and consequently $\frac{\partial\mu}{\partial m} = -\frac{1}{2}\frac{(n-1)^2}{(n^2-n-m)^2} < 0$. Since $m$ is minimal for $H^c_{r-star}$ it follows that $\mu(g_\perp(H^c_{r-star})) \geq \mu(g_\perp(H'))$ □

Theorem 4.1 shows that organizing cells in a r-star topology does well in balancing the trade-off between information and security. Clearly $H^c_{r-star}$ does not exist if $\frac{n-1}{c}$ is not integer. However Theorem 4.1 can easily be extended to these cases by considering the star hypergraph 'closest' to $H^c_{r-star}$: stars consisting of cells which differ at most one in size.

**References:**

ALLEN, M.P. 1982. The identification of interlock groups in large corporate networks: Convergent validation using divergent techniques. *Social Networks* **4** 349-366.

ARQUILLA, J. and RONFELDT, D. 2001. *Networks and Netwars.* RAND, Santa Monica, Calif.

ASAL, V., NUSSBAUM, B. and HARRINGTON, D.W. 2007. Terrorism as Transnational Advocacy: An Organizational and Tactical Examination. *Studies in Conflict & Terrorism* **30** 15-39.

BAKER, W.E. and FAULKNER, R. 1993. The Social Organization of Conspiracy: Illegal Networks in the Heavy Electrical Equipment Industry. *American Sociological Review* 58(12): 837-860.

BARNETT, A. ET AL. 2001. Safe at home? An experiment in domestic airline security. *Operations Research* **49** 181-195.

BEARDEN, J. and MINTZ, B. 1987. The structure of class cohesion: The corporate network and its dual. In: M.S. Mizruchi and M. Schwartz, *Intercorporate relations: The structural analysis of business.* Cambridge University Press, Cambridge, 187-207.

BOLLOBAS, B. 1986. *Combinatorics: Set Systems, Hypergraphs, Families Of Vectors And Probabilistic Combinatorics.* Cambridge University Press, Cambridge.

BOLLOBAS, B. 1998. *Modern Graph Theory.* Springer-Verlag: New York.

BONACICH, P. 1978. Using Boolean algebra to analyze overlapping memberships. In: K.F. Schuessler, ed., *Sociological methodology.* Jossey-Bass, San Francisco, 101-115.

BREIGER, R.L. 1974. The duality of persons and groups. *Social Forces* **53** 181-190.

BROWN, G.G., CARLYLE, W.M., HARNEY, R., SKROCH, E. and WOOD, R.K. 2009. Interdicting A Nuclear Weapons Project. *Operations Research* **57** 866-877.

CARLEY, K. M. 2006. Destabilization of Covert Networks. *Computational & Mathematical Organization Theory* 12(1): 51-66.

CRAFT, D.L. ET AL. 2005. Analyzing bioterror response logistics: the case of anthrax. *Management Science* **51** 679-694.

CRUICKSHANK, P. and HAGE ALI, M. 2007. Abu Musab Al Suri: Architect of the new Al Qaeda. *Studies in Conflict & Terrorism* **30** 1-14.

DAVIS, A., GARDNER, B. and GARDNER, M.R. 1941. *Deep South*, Chicago: University of Chicago Press..

ENDERS, W. and SU, X. 2007. Rational Terrorists and Optimal Network Structure. *Journal of*

*Conflict Resolution* **51**(1): 33-57.

Frantz, T.L. and Carley, K.M. 2005. A Formal Characterization of Cellular Networks. Carnegie Mellon University, School of Computer Science (SCS), Institute for Software Research (ISR), Center for Computational Analysis of Social and Organizational Systems (CASOS) Technical Report CMU-ISRI-05109.

Gambill, G. 2002. Sponsoring terrorism: Syria and Hamas. *Middle East Intelligence Bulletin* **4**.

Golan, A. 1978. *Operation Susannah.* Harper & Row, Publishers, Inc., New York.

Levine, J. 1972. The sphere of influence. *American Sociological Review* **37** 14-27.

Johnson, L.K. (ed.) 2007. *Strategic Intelligence: Covert action-beyond the veils of secret foreign policy.* Vol. 3. Praeger Security International: Westport.

Lin, K.Y., Kress, M. and Szechtman, R. 2009. Scheduling policies for an antiterrorist surveillance system. *Naval Research Logistics* **56** 113-126.

Lindelauf, R.H.A. and Borm, P.E.M. and Hamers, H.J.M. 2009. The influence of secrecy on the communication structure of covert networks. *Social Networks* **31** 126-137.

Mariolis, P. 1975. Interlocking directorates and control of corporations: the theory of bank control. *Sociological Quarterly* **56** 425-439.

McAllister, B. 2004. Al Qaeda and the Innovative Firm: Demythologizing the Network. *Studies in Conflict & Terrorism* 27: 297-319.

McCormick, G.H. and Owen, G. 2000. Security and Coordination in a Clandestine Organization. *Mathematical and Computer Modelling* 31: 175-192.

Mishal, S. and Rosenthal, M. 2005. Al Qaeda as a Dune Organization: Toward a Typology of Islamic Terrorist Organizations. *Studies in Conflict & Terrorism* 28: 275-293.

Mintz, B. and Schwartz, M. 1981a. The structure of intercorporate unity in American business. *Social Problems* **29** 87-103.

Mintz, B. and Schwartz, M. 1981b. Interlocking directorates and interest group formation. *American Sociological Review* **46** 851-869.

Nance, M.W. 2008. *Terrorism recognition handbook: a practitioner's manual for predicting and identifying terrorist activities.* Taylor & Francis Group, Boca Raton.

Petraeus, D.H. et al. 2007. *The U.S. Army Marine Corps Counterinsurgency Field Manual.* University of Chicago Press, Chicago.

Tucker, D. 2001. What is new about the new terrorism and how dangerous is it? *Terrorism and*

*Political Violence* **13** 114.

Zwikael, O. 2007. Al Qaeda's Operations: Project Management Analysis *Studies in Conflict & Terrorism* **30** 267-280.