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DOES POACHING DISTORT TRAINING?

by

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Does poaching distort training?*

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Abstract

We analyse the efficiency of the labour market outcome in a competitive search equilibrium model with endogenous turnover and endogenous general human capital formation. We show that search frictions do not distort training decisions if firms and their employees are able to coordinate efficiently, for instance, by using long-term contracts. In the absence of efficient coordination devices there is too much turnover and too little investments in general training. Nonetheless, the number of training firms and the amount of training provided are constrained optimal, and training subsidies therefore reduce welfare.

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1 Introduction

The positive relationship between wages and experience is well documented in the empirical labour literature. This stylised fact indicates that on-the-job training

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is one essential determinant of worker productivity. Accordingly, the extent to which the market induces firms to invest in general and specific training is crucial for economic welfare. In addition, turnover is important for allocational efficiency, to ensure that workers are optimally allocated to firms at any given time. It is well known from Becker (1964) that perfect competition leads to an efficient market outcome with respect to investment in training and turnover, provided that there are no credit constraints or minimum wages regulations.

This paper analyses the conditions under which the labour market outcome is efficient in a model with endogenous human capital formation and endogenous turnover in the presence of *search frictions*. To this end, we develop a directed search model in which turnover is necessary to obtain an efficient allocation of workers. More precisely, there exists two types of firms; training firms which have a comparative advantage in providing general training, and poaching firms which have a comparative advantage in utilising general human capital. Workers with different productivities are assumed to search in different submarkets. Within this setting we analyse whether training firms have the right incentives to enter the market and to provide the optimal amount of general training. In contrast to the existing literature, we treat both the worker's on-the-job search intensity and the number of poaching firms as endogenous variables.

Our first main result is that internal efficiency is a sufficient condition for an efficient resource allocation in this economy, both with respect to the allocation of workers to firms and with respect to the investments in general training. Internal efficiency refers to the resolution of co-ordination problems within each firm such that employer and his employees maximise their joint expected income. Internal efficiency may come about if workers and firms are able to write long-term binding contracts, or if they are able to bargain efficiently.

This efficiency result contrasts sharply with Acemoglu (1997). He finds that turnover in the presence of search frictions creates positive training externalities for future employers. As a result, there is underinvestment in general training even though firms and workers can write long-term contracts. He attributes the inefficient outcome to the workers' inability to contract with future employers. As we argue below, Acemoglu's result hinges (among other things) on his assumption that workers with different productivities search in the same search market. As a result, low-productivity workers create congestion effects for high-productivity workers, thereby reducing the return from training investments.

Our efficiency result also serves as a convenient benchmark when introducing imperfections other than search frictions and clarifies why such imperfections may give rise to inefficiencies. We focus on the case where internal efficiency does not hold because training firms set wages for experienced workers so as to maximise their *ex post* profit. In this case, wages for experienced workers in training firms are too low, the equilibrium turnover rate is too high, and investment in general training tends to be too low compared with the socially optimal level.

Our second main result is that this amount of human capital formation is still constrained efficient. Given the search behaviour of workers and the entry behaviour of poaching firms, the social and the private returns from general training coincide. Thus, subsidising general training reduces welfare. More complex policy measures may, however, increase welfare.

This second result also contrasts with the existing literature. Stevens (1994) argues that poaching creates a wedge between the social and the private returns from general training, as long as wages are set below worker productivity. For similar reasons, Booth and Snower (1995, page 345) propose that market failures caused by poaching should be mitigated by subsidising general training, for instance by letting the government pay a fixed proportion of the firms' training expenditures. Acemoglu and Pischke (1999a) are also sympathetic to training subsidies. Moreover, this view influences the policy debate. For instance, the OECD (1995, Chapter 7) argues that poaching externalities lead to underinvestment in general training, thereby providing a rationale for government subsidies, such as tax breaks for training expenses. Another example is the Swedish parliamentary investigation on individual human capital formation (Sveriges Riksdag, Direktiv 1999:106) which explicitly refers to the poaching externality as a rational for subsidising investments in general training. Our paper questions this widely held policy recommendation.

The paper is organised as follows. Section 2 describes the model. Section 3 analyses the equilibrium outcome with internal efficiency. Section 4 examines the case when wages for experienced workers are set so as to maximise *ex post* profits. Section 5 discusses robustness issues, and section 6 concludes. Mathematical proofs are provided in the appendix.

2 The model

In this section we describe the basic structure of the model and discuss the wage formation in some detail. The model is set in continuous time. Workers enter the labour market as unemployed and leave at an exogenous rate s. New workers enter the market at the same rate, keeping the total measure of workers constant.

There are two types of firms in the economy, training firms and poaching firms. Each firm hires at most one worker. Since only training firms invest in general training all workers start their career in a training firm. A worker that is hired by a training firm stays inexperienced for a period until he eventually becomes experienced. Within a continuous-time framework the natural way to model a period of time is to let the period length be stochastic: an inexperienced worker (a novice) employed in a training firm becomes experienced at a rate γ. The investment is made when the worker is a novice, and the return accrues once the worker is experienced. The structure of the model is illustrated in figure 1.

The productivity of a novice is y^n. The productivity of an experienced worker with human capital level h in a training firm is y^e and in a poaching firm y^p. We assume that $y^n < y^e < y^p$ for all $h \geq 0$. Hence, a poaching firm can utilise experienced workers better than training firms. The main implication of this assumption is that turnover is necessary for an efficient allocation of workers.[1] If instead $y^e = y^p$ were to hold, turnover had no social value. This, however, would not invalidate our efficiency results (see section 5 for details). The costs of creating a training vacancy and a poaching vacancy are K^t and K^p, respectively.

There are two distinct search markets in the model, one for employed workers and one for unemployed workers. In both markets, the number of matches between searching workers and firms is determined by a constant return to scale matching function $x(eu, v)$. This matching function maps a measure of workers u who search with an average intensity e for a measure of v vacancies into a flow x of new matches. Let p denote the probability rate that a worker finds a (new) job per unit of search intensity and q denote the probability rate that a firm with a vacancy finds a worker. The arrival rates p and q are interrelated, as both depend on the labour market tightness v/eu. Due to constant returns to scale, the matching function can be summarised as $q = q(p)$.[2] The equilibrium values of q and p are derived in the next sub-sections.

2.1 Asset values

Let W^u and W^n denote the expected discounted income, or "asset value" of an unemployed and of an inexperienced worker (novice), respectively. The asset value of an unemployed worker is given by

$$(r+s)W^u = e^u p^u (W^n - W^u) - c(e^u).$$

Here r denotes the discount rate, and $c(e^u)$ is the search effort cost of the worker. The latter is increasing, convex and $c(0) = c'(0) = 0$. We normalise the value of leisure to zero. The asset value of a novice is given by

$$(r+s)W^n = w^n - \mu ah + \gamma(W^e - W^n), \tag{1}$$

where w^n is the wage of a novice, μ is the share of the training costs that is paid by the worker, ah is the flow training cost, and W^e is the asset value of an experienced worker in a training firm with human capital level (training level) h. For expositional ease the dependence on h is surpressed. Analogously, W^e is given by

$$(r+s)W^e = w^e + e^e p^e (W^p - W^e) - c(e^e), \tag{2}$$

[1] This assumption is stronger than necessary.. It is sufficient that *some* workers are more efficient in poaching firms than in training firms, and that these are the workers engaged in on-the-job search.

[2] The probability rates p and q can be written as $p = x(eu, v)/eu = x(1, \theta) = \widetilde{p}(\theta)$ and $q = x(eu, v)/v = x(1/\theta, 1) = \tilde{q}(\theta)$, where $\theta = v/eu$ is the labour market tightness. The matching technology can thus be summarised by a function $q = \tilde{q}(\theta) = \tilde{q}(\widetilde{p}^{-1}(p)) = q(p)$.

where w^e is the wage of an experienced worker with human capital h in a training firm, p^e the probability rate that an experienced worker with human capital h finds a job in a poaching firm per unit of search intensity e^e, and W^p the expected income to the worker in a poaching firm. The expected income in a poaching firm is given by

$$(r+s)W^p = w^p,$$

where w^p is the wage in a poaching firm for a worker with human capital level h.

Turning to the asset value equations of firms, J^i , $i \in \{n, e, p\}$ denotes the expected discounted lifetime of a firm with an employee. A firm that is abandoned by its employee has no value. The corresponding asset value equations are

$$\begin{aligned}
(r+s)J^n &= y^n - w^n - (1-\mu)ah + \gamma[J^e - J^n], \\
(r+s)J^e &= y^e - w^e - e^e p^e J^e, \\
(r+s)J^p &= y^p - w^p.
\end{aligned} \tag{3}$$

Subsequently, we focus on the joint expected discounted income of a worker-firm pair. Denote the joint expected income of a firm and its employee by $Y^i \equiv W^i + J^i$, $i \in \{n, e, p\}$. The joint asset values are

$$(r+s)Y^n = y^n - ah + \gamma(Y^e - Y^n), \tag{4}$$

$$(r+s)Y^e = y^e + e^e p^e (W^p - Y^e) - c(e^e), \tag{5}$$

$$(r+s)Y^p = y^p. \tag{6}$$

Finally, the asset value equations for training vacancies (V^n) and poaching vacancies (V^p), are given by

$$rV^n = q(p^u)(Y^n - W^n - V^n), \tag{7}$$

$$rV^p = q(p^e)(Y^p - W^p - V^p). \tag{8}$$

2.2 Competitive search equilibrium

Competitive search equilibrium combines competitive price determination and search frictions and is thus a useful benchmark when analysing the impact of search frictions. As workers are assumed to know the wages in all firms prior to searching, the frictions are due to other aspects of the search process than collecting information on wages. Examples are the costs and time delays associated with writing and processing applications, with identifying firms with vacancies, or with testing applicants.

A core element of the competitive search equilibrium concept is the unique relationship between the advertised wage and the expected rate at which the vacancy is filled. The relationship can be derived in several settings.[3] Moen (1997) considers an economy in which a market maker creates submarkets, each characterised by a single wage. Workers and firms are free to choose which submarket to enter. As shown by Moen, wage advertisements by firms, or reputation about their wages, is sufficient to ensure that the same equilibrium wage prevails. In this paper we follow this wage advertisement approach. Mortensen and Pissarides (1999, section 4.1) give a similar interpretation to the one of the market maker, by assuming that a "middle man" (like a job centre) sets the wage. In Acemoglu and Shimer (1999 a) and b)) the labour market is divided into regional or industrial submarkets offering potentially different wages. Alternatively, the matching technology may be derived from the urn-ball process (Montgomery (1991), Peters (1991), and Burdett et al. (2001)).

In our model, searching workers are heterogeneous along (potentially) several dimensions: they may be experienced or inexperienced, and experienced workers may differ with respect to the level of human capital investments (although in equilibrium, all experienced workers choose the same level of training). Workers with different characteristics search, by assumption, in different submarkets. The number of matches in each submarket depends only on the number of workers and firms in that submarket. Separation into submarkets may be due to the production technology, say because a worker's training level determines what kind of tasks he can do (and will do in his next job). If training increases productivity, without affecting the range of job tasks that the worker can perform, the production technology by itself does not create separation. Still, firms may separate workers into different submarkets by advertising the required human capital level for their position (in addition to wages), thus mimicing a market maker that separates workers with different productivity into different submarkets.

Inderst (2000) shows that it is indeed optimal for a market maker to separate agents with different characteristics into different submarkets. The point is that the optimal labour market tightness (ratio of vacancies to workers) increases with worker productivity. Loosely speaking, letting a low-productivity worker into a search market for high-productivity workers implies that too much resources are spent in order to provide him with a job. The issue of separate search markets is discussed further in section 5.

Technical definition of competitive search equilibrium

We derive the competitive search equilibrium for the on-the-job search market. Analogous results can be derived for the unemployed search market (Moen and Rosén (2001)). For the moment the inflow of workers into the on-the-job search market, their human capital level h, and their wage during search are treated

[3] In this paragraph we borrow some arguments from Acemoglu and Shimer (1999b).

as exogenous. Although these variables are endogenous in the model, they are predetermined at the time at which on-the-job search takes place. Furthermore, we assume that a worker's on-the-job search is non-contractible, hence the wage during search cannot be made contingent on the worker's search behaviour.

From equations (2) and (8) it follows that we can write the asset value of an experienced worker as $W^e = W^e(W^p, p^e, e^e; h)$, and the asset value of a poaching firm as $V^p = \widetilde{V}^p(W^p, q(p^e); h) = V^p(W^p, p^e; h)$. Below, we surpress the dependence on h. The equilibrium in this search market is a vector (W^{p*}, p^{e*}, e^{e*}) that satisfies the three following conditions.

1. Optimal search effort
$$e^{e*} = \arg\max_{e^e} W^e(W^{p*}, p^{e*}, e^e).$$

2. Profit maximisation
$$(W^{p*}, p^{e*}) = \arg\max_{W^p,\, p^e} V^p(W^p, p^e) \quad \text{subject to} \quad W^e(W^p, p^e, e^{e*}) \geq W^{e*}.$$

3. Zero profit condition
$$V^p(W^{p*}, p^{e*}) = K^p.$$

The profit maximisation condition can be given the following interpretation: All submarkets (or firms) that attract workers must offer these workers their equilibrium expected income W^{e*}. There typically is only one wage advertised in equilibrium (see below). Nonetheless, when setting the wage, firms expect that the arrival rate of workers to their firm $\widehat{q}^e$ for out-of equilibrium wage offers will be given by $\widehat{q}^e(W^p) = q(p^e(W^p))$, where $p^e(W^p)$ satisfies
$$W^e(W^p, p^e; e^{e*}) = W^{e*}.$$
Firms choose W^p so as to maximise profits given these expectations. This yields the profit maximisation condition. Note that the expectations are rational in the following sense. Suppose that a small set of firms deviates and advertises an out-of equilibrium wage W'. Applications would then flow to these firms up to the point at which the applicants obtain exactly their equilibrium expected income W^{e*}, in which case $q^e(W') = \widehat{q}^e(W')$ holds (see Moen (1997) and Acemoglu and Shimer (1999a) for details).

The free entry condition and equation (8) imply that $W^p = Y^p - \frac{r+q^e}{q^e} K^p$. Inserted into equation (2) this gives
$$(r+s)W^e = w^e + e^e p^e (Y^p - \frac{r + q(p^e)}{q(p^e)} K^p - W^e) - c(e^e). \tag{9}$$
The competitive search equilibrium allocation is such that V^p is maximised given W^e, while free entry ensures that $V^p = K^p$. It is straightforward to show that in equilibrium W^e is maximised given that $V^p = K^p$. To be more precise, define the *feasible set* of pairs (W^p, p^e) as $\Phi^p = \{(W^p, p^e) | V^p(W^p, p^e) \geq K^p\}$.

Lemma 1 *In the competitive search equilibrium,* $W^e(W^p, p^e, e^e)$ *is maximised given that* $(W^p, p^e) \in \Phi^p$.[4]

Moen (1997) shows that the model may have multiple equilibria which all yield the same value of W^e. To avoid uninteresting technicalities, we assume that the equilibrium is unique. It follows that the competitive search equilibrium vector (W^{p*}, p^{e*}, e^{e*}) can be defined as the solution to the maximisation problem

$$\max_{W^p, p^e, e^e} W^e(W^p, p^e, e^e) \quad \text{given that} \quad (W^p, p^e) \in \Phi^p,$$

where Φ^p depends on h, i.e., $\Phi^p = \Phi^p(h)$. An equivalent result holds for the unemployed search market (Moen and Rosén (2001)).

Role of the wage during search

In this section we examine the effect that changes in w^e in an individual firm have on the search behaviour of its employee. In what follows we argue as if the training firms set the wage for experienced workers.

The competitive search equilibrium derived above depends on w^e, since W^e depends on w^e. As will become clear shortly, all workers are offered the same wage w^{e*} in equilibrium, and consequently all poaching firms offer the same wage w^{p*}. Still, the search behaviour of workers that receive out-of equilibrium wages may be important. Consider a small set of training firms that deviates and offers the experienced worker a wage $w' \neq w^{e*}$. This affects their workers' search behaviour, that is, both the search intensity e^e and the trade-off between W^p and p^e. The associated competitive search equilibrium in this new submarket, $(W^{p\prime}, p^{e\prime}, e^{e\prime}; w')$, solves

$$\max_{W^p, p^e, e^e} W^e(W^p, p^e, e^e; w') \text{ given that } (W^p, p^e) \in \Phi^p, \tag{10}$$

where w' is included as an argument to highlight the dependence of W^e on the wage during search.

In what follows, we assume that firms (and workers), when considering a wage $w' \neq w^{e*}$ form expectations about the resulting arrival rate of job offers and about the search intensity of the worker that are consistent with (10). The expectations are rational in the sense that if a small set of firms deviates and offer this wage, the expectations are fulfilled.[5]

[4] A similar result is derived in Acemoglu and Shimer (1999a).

[5] An alternative way to rationalise this assumption is to assume that some firms "tremble" and offer wages other than the equilibrium training wage. Non-empty submarkets for other training wages than the equilibrium wage then exist. The competitive search equilibrium can be defined as the limit obtained when the measure of deviating firms converges to zero (Moen (1994)).

Efficiency

We now derive the welfare properties of the equilibrium allocation in the on-the-job search market under the following assumptions and discuss later under which conditions these assumptions indeed hold.

Assumption 1: The experienced worker's income flow while searching is equal to his marginal productivity, i.e., $w^e = y^e$.

Assumption 2: The social and the private value of a match coincide.

We say that a search market is efficient if the net value created in the search market is maximised, for a given inflow of workers. This value is equal to the product of the number of matches times the value of each match less the cost of vacancy creation. The number of matches in the market is given by $e^e p^e N^e$, and the value of each match is Y^p. In steady state, the value creation in the market is thus $e^e p^e N^e Y^p -$ $e^e p^e N^e K^p$. Furthermore, as it takes time before vacancies are filled, the cost of having a stock v^p of vacancies is $v^p K^p r$. Total hiring costs can thus be written as $e^e p^e N^e \frac{r+q^e}{q^e} K^p$, and the planner's objective function is

$$R(N^e) = \int_0^\infty [y^e N^e + e^e p^e N^e Y^p - e^e p^e N^e \frac{r + q(p^e)}{q(p^e)} K^p - c(e^e) N^e] e^{-rt} dt. \quad (11)$$

The social planner maximises this function with respect to p^e and e^e, subject to the constraint

$$\dot{N}^e = b - (s + e^e p^e) N^e,$$

where b is the (for now) exogenous inflow of workers to the search market.[6]

Lemma 2 *Given assumptions 1 and 2, the following holds:*
a) The socially optimal allocation maximises W^e *given that* $(W^p, p^e) \in \Phi^p$.
b) The social value of an additional worker entering the search market is equal to his equilibrium expected income W^e.
c) Property b) still holds when the number of firms in the market is exogenous.

Together with Lemma 1, part a) immediately implies that the competitive search equilibrium outcome is optimal. Part b) implies that the private gain from entering the search market coincides with the social value. Part c) says that this property does not hinge on entry by poaching firms. With Assumptions1 and 2 adjusted accordingly equivalent results (part a and b) hold also for the unemployed search market (Moen and Rosén (2001))

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[6]Result a) is stated and proved in Acemoglu and Shimer (1999b)

3 Internal efficiency

In this section, we define internal efficiency and then derive and evaluate the equilibrium of the model, provided that internal efficiency holds. We also show how internal efficiency can be implemented through various contractual arrangements. Finally, we discuss whether the firm has an incentive to pay for the worker's general training.

In a training firm, the choice of the training level h and the worker's on-the-job search behaviour influence their joint income Y^n. We refer to a training firm as internally efficient if its co-ordination problems are resolved, such that the joint expected income Y^n is maximized. Internal efficiency requires that the following two conditions are satisfied:

1. Internal efficiency *ex post*: The on-the-job search behaviour that maximises W^e also maximises Y^e.

2. Internal efficiency *ex ante*: The training level is set so as to maximise Y^n.

One way to implement internal efficiency *ex post* is to set the wage of an experienced worker equal to his productivity in which case $Y^e \equiv W^e$ holds. Since the on-the-job search behaviour that maximizes Y^e also maximizes Y^n, it follows directly that internal efficiency holds if and only if there is internal efficiency ex post and ex ante.

3.1 Equilibrium with internal efficiency

A worker enters a search market twice during his career, once as an unemployed worker searching for his first job and once as an experienced worker searching for a job in a poaching firm. Clearly, these markets are interrelated. The opportunities for an experienced worker in the on-the-job search market influences his expected income when applying for jobs as an unemployed worker as well as his return from human capital investment. Furthermore, the prospect that workers may quit affects the incentives of a firm to enter the market as a training firm.

The model is solved backwards. We have already analysed the equilibrium in the on-the-job search market, for a given level of human capital h. Poaching firms advertise wages and required human capital level. For a given h, the equilibrium in this market solves the problem

$$\max_{W^p, p^e, e^e} Y^e(W^p, p^e, e^e; h), \text{ given that } (W^p, p^e) \in \Phi^p(h).$$

Hence, we can write the equilibrium value of Y^e as $Y^e = \widehat{Y}^e(h)$.

We now turn to the human capital investment decision. Given Y^e, the value of Y^n is determined by (4). *Ex ante* internal efficiency thus requires that h maximises Y^n. As will become clear soon, only one investment level is chosen in equilibrium. Nonetheless, when considering alternative values of h, the worker-firm pair expects that the associated expected incomes when the worker becomes experienced are given by $Y^e = \widehat{Y}^e(h)$ and chooses h so as to maximise Y^n given these expectations. These expectation formations are analogous to the expectation formations by firms considering out-of-equilibrium wages. Two further comments regarding these expectations formations are warranted.

First, as hazard rate expectations, these expectations are rational in the following sense: Suppose a small set of agents deviates and chooses an out-of equilibrium value h'. As a result, a new submarket would open up for these workers, and the workers would obtain an expected income equal to $\widehat{Y}^e(h)$. Second, if workers were heterogenous, the problems of "empty submarkets" could be avoided. In an earlier version of this paper (Moen and Rosén (2001)) we derive the equilibrium when the investment cost a has a discrete distribution and the training levels are discrete. In this case, all possible training levels are actually chosen in equilibrium. The equilibrium in this paper can be derived as the limit when the distribution of training costs converges to a mass point (without reducing the support) and when the difference between two adjacent investment levels becomes arbitrarily small.

The equilibrium value of h, h^*, thus solves the problem

$$\max_h Y^n(h) \quad \text{given that} \quad Y^e = \widehat{Y}^e(h).$$

In the unemployed-search market, firms advertise wage contracts, which may include provided training level and wages for experienced workers. The exact form of the advertised contract is discussed below. From the workers' point of view, the attractiveness of a wage contract is measured by the associated value of W^n (the expected lifetime income when becoming employed). As in the on-the-job search market (Lemma 1), the equilibrium in the unemployed-search market solves the problem

$$\max_{W^n, p^u, e^u} W^u(W^n, p^u, e^u), \quad \text{given that } (W^n, p^u) \in \Phi^t(h^*),$$

where $\Phi^t = \{(W^n, p^u) | V^n(W^n, p^u; h^*) \geq K^t\}$ and where V^n is defined by equation (7). In the appendix we prove the existence of the equilibrium under the conditions that $y^p - y^e > (r+s)K^p$ and $\frac{(r+s)(y^n - ah^*) + \gamma y^e(h^*)}{r+s+\gamma} > (r+s)K^t$. We assume that the equilibrium is unique.

We now turn to the welfare properties of this equilibrium. Solving the social planner's maximisation problem in full is rather complex, and is therefore deferred to the appendix. Here we give a heuristic explanation and the intuition for the results.

Consider first an on-the-job search market in which the workers' human capital level is h. For any inflow rate of workers into this market, the social planner maximises net value creation given by (11). From Lemma 2 we know that the competitive search equilibrium allocation solves this problem for all h. From the same Lemma we also know that $\frac{dR(N^e)}{dN^e} = Y^e$. Thus, the social and the private value of an additional experienced worker with this training level coincide.

Now consider the training level h. When the worker-firm pairs decide on h, they do so on the basis of their expectations $\widehat{Y}^e(h)$, which equal the social value of investing this amount. Since the relationship between Y^e and Y^n is mechanical (for a given h), the social and (perceived) private value of $Y^n(h)$ coincides, for all h. Accordingly, the planner solves the same maximisation problem as the agents in the market when choosing training level. This implies that $\max Y^n(h)$ reflects both the social and the private value of a match between an unemployed worker and a training firm. But then we know from Lemmata 1 and 2 (part a) that the unemployed-search market is efficient as well.

Proposition 1 *Given internal efficiency holds, the labour market equilibrium outcome is efficient. In particular,*

a) The level of general human capital investments is socially optimal.

b) The numbers of training firms and of poaching firms entering the market are socially optimal.

c) When the number of poaching firms is exogenous the equilibrium allocation is still efficient in the sense that aggregate output is maximised given the number of poaching firms.

To gain intuition, suppose *ex post* efficiency is obtained by paying experienced workers in training firms a wage equal to their productivity (see the next subsection for details). In this case, the workers' search behaviour has no externality on their employers. Since experienced workers do not generate any profits for training firms, these firms do not care whether their experienced workers stay or leave.

The entry decision of a firm with a vacancy gives rise to search externalities, a positive externality for workers and a negative externality for other firms with vacancies. In the competitive search equilibrium, these externalities offset each other (as the Hosios condition is met), and therefore an optimal number of vacancies exists in the market.

For a given number of poaching firms the same argument holds for workers. A worker that enters the on-the-job search market creates a negative externality for other workers and a positive externality for poaching firms. In the competitive search equilibrium, these two externalities exactly cancels out, and the social and private value of entering the market coincide. This is true in the search markets associated with all possible values of h (although only one of them is active in equilibrium). Thus the training choice of worker-firm pairs has no net search externalities on other agents. As training is determined so as to maximise joint surplus, the training decision is socially optimal. Furthermore, by Lemma 2 the social and the private value of a worker-training firm match coincide. But then we know that the unemployed search market is efficient as well.

With free entry of poaching firms, the number of workers entering the on-the-job search market, or their level of training, have no externalities for other workers or for poaching firms, as p^e is independent of the measure of workers in the submarket.

3.2 Implementing internal efficiency

In this subsection, we explore various arrangements that lead to internal efficiency and hence to an optimal amount of training.

Firms advertise long-term wage contracts

Suppose firms are able to advertise and commit to long-term contracts. Several types of wage contracts may then ensure internal efficiency.

The first thing to note is that it is in the firms' interest to advertise and commit to internally efficient arrangements. Consider a set of contracts that gives the workers an expected income W^n, arbitrarily chosen. The firm has then an incentive to offer the contract that maximises $Y^n - W^n$, i.e., to choose the contract that leads to internal efficiency.

One set of contracts that ensure internal efficiency is a long-term contingent contract $(w^n, w^e(h))$ such that the wage when the worker is experienced equals his productivity, $w^e(h) = y^e(h)$. As such a wage schedule makes the worker residual claimant on the return from human capital, the efficient investment is undertaken if the worker bears the entire investment cost. The wage w^n is set so as to scale the worker's total compensation. Trivially, the same outcome can be implemented by having only two levels for w^e, a low level if investment is below h^*, and a wage equal to $y^e(h^*)$ if investment is at or above h^*, where h^* is the optimal training level.

If wage contracts in which w^e is contingent on h are difficult to enforce, internal efficiency can be obtained by a non-contingent wage contract $w^e = y^e(h^*)$. In order to achieve internal efficiency, h^* could then be advertised. Alternatively, the firm can advertise a share μ of the investment costs that the worker has to bear. For a given w^e, the firm receives the increase in y^e associated with a higher h, while the worker gains by increasing his prospects in the on-the-job search market. By the envelope theorem (on $W^e(h^*)$), the worker's share of the total gain is

$$\mu = \frac{\frac{e^{e*}p^{e*}}{r+s+e^{e*}p^{e*}} \frac{dY^p(h^*)}{dh}}{\frac{dY^e(h^*)}{dh}}. \tag{12}$$

Thus, if the worker and the firm finance shares μ and $(1 - \mu)$ of the costs, the first best investment level is reached. Again, w^n should be adjusted to scale the worker's total compensation.

Firms sell off jobs

The simplest way to obtain internal efficiency is to let the firms advertise a "price" P that the worker has to pay in order to become residual claimant. The expected income to the worker is then $W^n = Y^n - P$. As a residual claimant, the worker is induced to behave in an internally efficient manner. Analogously to before, a firm trades off a high price P against a low arrival rate of workers.

Quitting fees

Alternatively, the firm can uses *quitting fees* to ensure optimal on-the-job search behaviour. Suppose the wage to an experienced worker w^e is less than his productivity y^e. In the absence of quitting fees, the worker engages in too much on-the-job search, thereby reducing joint expected income. It is straightforward to show that the worker can be induced to adopt the optimal on-the-job search behaviour (both in terms of search effort and the correct trade-off between wages and job finding rates) if he pays a fee $F = (y^e - w^e)/(r + s)$ when quitting.

With quitting fees and wages below marginal product, a worker is not residual claimant on the return from the human capital investment. Efficient investments may be achieved either if firms and workers bargain (see below), or if firms advertise training level h^*, or if the firm contributes to the investment costs as described above.

Efficient bargaining

Given symmetric information between the worker and the employee, standard Nash bargaining leads under quite general assumption to an internally efficient outcome. As long as the efficient outcome is in the opportunity set and utility (income) is transferable, internal efficiency prevails.

Let us give an example. Suppose that the firm advertises an unconditional wage $\overline{w}$ only, and that the worker and the firm bargain over the wage contract and the training level. For simplicity the firm is assumed to have all the bargaining power and makes a take-it-or-leave-it offer to the worker. The worker accepts any arrangement which gives him a payoff greater than or equal to the payoff that he receives with the initial contract. Denote this payoff by $W^n(\overline{w}, 0)$. The payoff to the firm is then $Y^n(h) - W^n(\overline{w}, 0)$. Being the residual claimant, the firm chooses an arrangement that maximises Y^n. Thus, if the firm has the contractual instruments available to propose an internally efficient contract it does so, and adjust w^n so as to scale total compensation. This may be a wage contract of the form (w^n, w^e), where w^e is set equal to the productivity of the worker when he is experienced, or a contract with quitting fees that ensures optimal on-the-job search behaviour.

Internal efficiency also emerges when both the worker and the firm have some bargaining power, provided that there is bargaining over both wages and human capital investment. Efficiency in the unemployed search market obtains if the firm can manipulate the worker's total compensation through its advertised contract $(\overline{w}, \overline{h})$.

Finally, suppose wages are renegotiated once the worker is experienced. Efficient on-the-job search behaviour can still be implemented as long as we allow for quitting fees hence *ex post* efficiency obtains. Knowing this, efficient bargaining over h is enough to ensure *ex ante* efficiency, and thus internal efficiency.

3.3 Who pays for training?

From Becker (1964) we know that when the labour market is perfectly competitive, workers pay the full cost of general training. Several recent papers address the issue of why and when firms have incentives to invest in general training.[7] One finding is that when the wages increase less than productivity, e.g. due to search frictions and wage determination by bargaining, firms have incentives to invest in general training.

In our model with internal efficiency, the extent to which firms pay for training depends on the contractual arrangement that is used for obtaining internal efficiency. Workers pay the full cost if firms advertise long-term contract with wages for experienced workers contingent on the amount of training that the worker undertakes as a novice. Since the worker as a novice also provides a profit margin sufficient to capitalise the firm's search costs, the wage for a novice may be low.

The more interesting case is when firms advertise long-term contracts in which the wage for experienced workers is not conditioned on h. Ultimately, the wage (net of human capital investments) is the same as with conditional wage contracts. At the margin, the firm finances, however, a share $1-\mu$ of the training, where μ is given by equation (12). Using that $Y^e(h) = W^e(h) + \frac{y^e(h)-w^e}{r+s+e^e p^e}$, that $\frac{\partial W^e}{\partial e^e} = \frac{\partial W^e}{\partial p^e} = 0$, and that $y^e(h^*) - w^e = 0$, it follows that we can write the expression for μ as

$$\mu = \frac{e^{e*} p^{e*} \frac{dY^p(h^*)}{dh}}{e^{e*} p^{e*} \frac{dY^p(h^*)}{dh} + \frac{dy^e(h^*)}{dh}}.$$

Hence, if the search frictions are large (measured as a low optimal turnover rate $e^e p^e$ for a given Y^p), μ is small and approaches zero as the turnover rate goes to zero. Thus, if the frictions are large, the firm finances the entire training cost at the margin. By contrast, if the turnover rate is large, μ is large as well and converges to 1 as turnover becomes immediate. The reason is that the longer the worker stays in the firm, the larger is the share of the return on training that accrues to the firm at the margin. This finding resembles the findings in Acemoglu and Pischke (1999b) that firms pay a larger share of the training costs

[7] For a survey see Acemoglu and Pischke (1999a).

when frictions are high. Though their mechanism differs from the one in the present paper. In Acemouglu and Pischke (1999b) all separations are exogenous, and higher frictions are tantamount to a lower exit rate from unemployment. A lower exit rate increases the share paid by the firm because it implies a more distorted (compressed) wage-schedule. In our model, the driving force is not through the degree of distortion in the wage schedule but the actual expected time the worker stays in the firm.

If wages for experienced workers are set below their productivity and internal efficiency is obtained by quitting fees, this may also induce firms to finance training. The share of the training costs that is born by the firm depends on the exact formulation of the bargaining game.

4 *Ex post* determination of wages

In this section we address the common concern in the literature that there may be excessive turnover and too little investment in general training because wages are for various reasons below the workers' productivity (e.g., Stevens (1994), OECD (1995, Chapter 7), and Booth and Chatterji (1998)).

4.1 Equilibrium with *ex post* wage setting

We modify our framework and assume that firms cannot commit *ex ante* to the wage that they will pay a worker once he is experienced. Thus, training firms set wages for experienced workers so as to maximise *ex post* profit. We do not allow for quitting fees. When setting the wage for an experienced worker, a firm trades off a low wage bill against a high turnover rate.[8] A high wage reduces the turnover rate for two reasons. First, it implies that the worker applies for jobs offering high wages with long job queues, thereby reducing p^e. Second, the worker reduces his on-the-job search effort e^e.

Formally, the firm chooses w^e so as to solve the problem (from equation (3))

$$\max_{w^e} y^e - w^e - e^e p^e J^e,$$

given that p^e, e^e solves

$$\max_{W^p, p^e, e^e} W^e(W^p, p^e, e^e; w^e), \qquad \text{given that } (W^p, p^e) \in \Phi^p, \tag{13}$$

where Φ^p is defined as in section 2.2 and $W^e(W^p, p^e, e^e; w^e)$ denotes the asset value of a searching worker with income flow w^e while searching, (see equation (10) and

[8]The trade-off between turnover and wage costs has been studied by several authors, (e.g. Salop (1979), Stiglitz (1985), Burdett and Mortensen (1998)). Our paper differs from these papers in several respect, most notably, in our choice of a directed search model, and in our focus on the efficiency of the level of general training provided by the market.

the following discussion). We are not able to find a closed form solution to this problem, even when we parameterise the matching function. It is, however, clear that the firm always sets $w^e < y^e$. At $w^e = y^e$ the firm earns zero profit while it obtains a strictly positive profit for all $w^e < y^e$. The maximisation problem given by (13) defines the equilibrium in the on-the-job search market $(W^{p*}, p^{e*}, e^{e*}; w^e)$.

We assume that the worker-firm pairs are able to obtain internal efficiency *ex ante.* Hence, the training level is set so as to maximise the joint income $Y^n(h)$, taking into account that the wages of experienced workers are set as described above.

Lemma 3 *Compared to the equilibrium with internal efficiency, the following holds in the equilibrium with ex post wage determination*

1. *For a given h:*

   (a) *Too many poaching firms enter the market relative to the number of training firms (p^e is higher).*

   (b) *The on-the-job search intensity is higher (e^e is higher).*

2. *Fewer training firms enter the market.*

While $Y^n(h)$ is maximised for any h with internal efficiency *ex post* this does not hold when wages are set so as to maximise *ex post* profit. Thus, Y^n is lower which implies that fewer training firms enter the market.

With respect to the amount of training in each firm, the impact of *ex post* wage determination is by no means clear cut. The reason is that we have no control over the relationship between w^e and h, it may even be discontinuous. If a small increase in h leads to a large increase in w^e, investments in h may be considered as a *commitment device.* By increasing h by a small amount the firm may find it in its own interest *ex post* to set a substantially higher wages, thereby reducing the inefficiencies created by excessive turnover. We can therefore not rule out that *ex post* wage determination actually increases the amount of training undertaken compared to the first best.

In order to derive more clear-cut results, further restrictions must be imposed on the model. As an example, assume that there are only two levels of human capital, zero and one and that only workers with human capital $h = 1$ engage in on-the-job search. In this case, excessive turnover due to *ex post* wage setting reduces $Y^e_{h=1}$ but has no effect on $Y^e_{h=0}$, and the joint private return from investing in human capital unambiguously falls. Furthermore, assume that the workers differ with respect to the cost of acquiring human capital, a, where each worker's a is independently drawn from a known distribution and the draw takes place after the worker is hired but prior to the investment decision. Then there exists a cut-off value a^* such that all workers with $a < a^*$ invest in training. This cut-off level may then be compared with the corresponding socially optimal cut-off level. The following result then obtains (proof omitted):

Lemma 4 *Given* $h \in \{0, 1\}$*,* $e^e_{h=0} = 0$*, and* $e^e_{h=1} > 0$ *with internal efficiency, the amount of training (the cut-off level of* a*) with ex post wage setting is lower than the first best level obtained with internal efficiency.*

Similar results have often been used in the literature to rationalise training subsidies.

4.2 Training subsidies

We begin the analysis of how training subsidies affect the outcome by defining the constrained efficient level of training.

Definition of Constrained Efficiency: *Suppose the social planner determines the number of training firms that enter the market and the level of training per worker* h*, while the decisions of experienced workers, training firms, and poaching firms are determined in the market according to (13). The level of training and the number of training firms entering the market are then constrained efficient if the social planer chooses the same outcome as the one that prevails in the market.*

We thus do a similar exercise as in Stevens (2001), where it is assumed that the planner can overrule only the investment decision of firms.

In general, the training level with *ex post* wage setting differs from the training level with internal efficiency, and the number of training firms that enter is unambiguously lower. As the next Proposition shows, the training level is, however, still constrained efficient.

Proposition 2 *The training level and the number of poaching firms are constrained efficient.*

Since training levels are constrained efficient, a training subsidy (if the training level is too low), regulations of training, or subsidised entry of training firms do not improve welfare.

The point is that although excessive turnover reduces the private returns from training, it also reduces the social returns from training by the same amount. Thus, the social and private gains from training coincide, and a training subsidy or regulation of training levels is inefficient.

Let us look at this point more closely. Suppose a small group of firms deviates from the equilibrium value h' and instead chooses a training level h''. We want to look at the consequences for the other agents in the economy. Fewer poaching firms will enter the submarket for h'-workers, and the equilibrium values of p^e, W^e, and e^e stay constant. Thus, workers in the h' -submarket are not affected, and since p^e and e^e are unaffected, neither are training firms.

When the worker and the firm choose h they do that so as to maximise their joint expected income Y^n, taking into account that the worker engages in excessive on-the-job search once he is experienced. But since there are no net externalities for other agents in the market, the level of h is constrained efficient.

The entry decision by training firms is constrained efficient since the unemployed search market maximizes the unemployed workers' welfare, given that training firms break even and since there is no net externalities on other workers or firms in the on-the-job search market.

As mentioned in the introduction, the literature tends to conclude that subsidies or regulation is welfare improving if there is underinvestment due to turnover. In fact, much of the literature only establishes circumstances for underinvestment in general training and infers from this finding that subsidies/regulation (in the absence of governmental failures) are welfare improving.

We want to compare our findings with those in Stevens (2001). In her model, both the number of firms and the number of workers trained in each firm are endogenously determined. Due to turnover, firms train too few workers and the equilibrium is not constrained efficient. Consequently, the government can improve welfare by forcing firms to train more workers. Stevens' model differs from ours in several respects. For instance, search frictions are not explicitly modelled and there is no free entry of firms in the on-the-job search market. This latter feature makes her model similar to our model with free entry of training firms but with a fixed number of poaching firms. With an exogenous number of poaching firms, the equilibrium of our model may not be constrained efficient either. To see this, consider the above example with only two training levels $h = 0$ and $h = 1$ and with no on-the-job search when $h = 0$. If firms train more workers (higher cut-off value a^*), the arrival rate p^e for trained workers falls, which may affect the incentives to invest in training in the first place.

In Stevens' model, first best (although achievable with direct regulation of training) cannot be achieved by subsidies alone, because a training subsidy distorts the *entry decision* of training firms. In order to reach efficiency, a tax on firms has to be imposed. As will become clear below, we are also able to obtain first best by a mix of taxes on poaching firms and training subsidies. The taxes in our model play, however, a different role than in Stevens' model. In our model, taxes are used to avoid excessive turnover, rather than to reduce the profitability of entering the market for training firms.

4.3 Combined policy measures

While a training subsidy or regulations of training alone cannot improve welfare, they may do so if combined with policy measures aimed at reducing the turnover rate, hereafter referred to as *ex post* policy measures. Also other policies, such as taxes on poaching firms, may by themselves improve welfare. We therefore analyse the effects of other policy instruments, e.g., profit and pay-roll taxes, alone and in combination with training subsidies. This also sheds light on the effectiveness of former and present policy programs targeted at promoting training in firms. For instance, the Australian government imposed an pay-roll tax of 1% for firms that provide insufficient amount of training (OECD (1995), Chapter 7), which corresponds to a pay-roll tax for poaching firms. In France and earlier also in U.K, a pay-roll tax for all firms is coupled with a subsidy for training (Steven (2001)).

Since the underinvestment in training is ultimately caused by the too low wages for experienced workers in training firms, the most direct policy is to enforce higher wages for these workers, say through collective wage agreements or wage subsidies. As such radical measures, however, distort the economy along other dimensions (not modelled here), we focus on measures aimed at changing the behaviour of poaching firms.

The trade-off that training firms face when setting the *ex post* wage is rather complex. This makes it extremely difficult to characterise the impact of *ex post* policy measures on wages in training firms. For instance, training firms may respond to less entry of poaching firms by reducing or increasing the wages, depending on the functional form of the matching function. In order to simplify the discussion, we therefore first assume that the wage for experienced workers is constant and independent of any policy intervention. Furthermore, we initially keep the amount of training in each training firm and the number of training firms fixed when analysing the effects of taxes on poaching firms.

Taxes on poaching firms

Turnover is created by both on-the-job search of workers and by entry of poaching firms, and more than one policy measure is needed to control both. Below we consider three kinds of taxes on poaching firms; entry tax, profit tax, and pay-roll tax.

Entry tax

An entry tax T for poaching firms increases the entry cost from K^p to $K^p + T$. Thus, free entry implies from equation (9) that $W^p = Y^p - \frac{r+q^e}{q^e}(K^p + T)$. Accordingly, the equilibrium values p^e and e^e solve the problem

$$\max_{p^e, e^e} e^e p^e [Y^p - \frac{r + q(p^e)}{q(p^e)}(K^p + T) - W^e] - c(e^e). \tag{14}$$

In the appendix we show that both p^e and e^e decrease with T and go to zero as $K^p + T$ approaches Y^p. Under the assumption that p^e and e^e are continuous functions of T, it is possible to choose T in such a way that either p^e or e^e is equal to its first best level. As shown in the appendix, the effect of T on p^e is stronger than that on e^e in the following sense: Setting the entry tax T such that p^e is reduced to its first-best level p^{e*}, also reduces e^e but keeps e^e above its first best level e^{e*}. The intuition is that an entry tax on poaching firms increases the cost of creating turnover through vacancies, while leaving the cost of creating turnover through on-the-job search unaltered.

Tax on profit

In a regime with a profit tax for poaching firms, $T = t(Y^p - W^p)$, free entry implies that $W^p = Y^p - t(Y^p - W^p) - \frac{r+q^e}{q^e}K^p$. Re-arranging gives $W^p = Y^p - \frac{r+q^e}{q^e}\frac{1}{1-t}K^p$. The competitive search equilibrium thus solves the problem.

$$\max_{p^e,e^e} e^e p^e [Y^p - \frac{r+q(p^e)}{q(p^e)}\frac{1}{1-t}K^p - W^e] - c(e^e).$$

This problem is equivalent to the problem given in (14) with $T = \frac{t}{1-t}K^p$. Thus, a profit tax is equivalent to an entry tax.

Pay-roll taxes

With pay-roll taxes, $T = tW^p$, free entry implies that $W^p = Y^p - tW^p - \frac{r+q^e}{q^e}K^p$. Thus, the competitive search equilibrium solves the problem (after multiplying with $(1+t)$)

$$\max_{p^e,e^e} e^e p^e [Y^p - \frac{r+q(p^e)}{q(p^e)}K^p - (1+t)W^e] - (1+t)c(e^e).$$

In the appendix we show that the pay-roll tax reduces both p^e and e^e. In contrast to entry taxes, a pay-roll tax has a larger effect on e^e than on p^e. A tax rate t' that lowers e^e to its optimal level e^{e*}, also reduces p^e but leaves p^e above its optimal level p^{e*}.

Combination of pay-roll and entry taxes

Both pay-roll taxes and entry taxes reduce entry of poaching firms and the search effort of workers in training firms, though their relative impact on p^e and e^e differs. This leads us to believe that there exists a combination of a pay-roll tax and an entry tax that implements the first best values of p^e and e^e (for a given h). In what follows we assume that this is indeed the case.

Training and entry subsidies to training firms

We now assume that pay-roll and entry taxes are set so as to induce first best levels of p^e and e^e, and analyse the training and entry decisions of training firms.

The reduction in turnover, created through taxes on poaching firms, has an ambiguous effect on the training level h even when wages are assumed to be fixed exogenously. For a given wage, lower turnover increases the return from investment for the training firms, but reduces the return for the worker, and *a priori* either effect may dominate.

Furthermore, the constrained efficiency result derived above no longer holds, as investments in training now gives rise to a tax externality. An increase in h influences the number of poaching firms entering the market and the wage that they offer, and thus also the tax collected by the government. For a fixed wage

w^e, an increase in h increases both the number of poaching firms and the wage w^p, provided that y^p increases proportionally at least as much as y^e. In this case, a training subsidy is warranted.

If we endogenise the *ex post* wage w^e, this may no longer hold. Since we cannot determine how w^e responds to changes in h, we cannot rule out that investment in training reduces turnover at the margin. If this effect dominates, there is a negative tax externality from training, and the subsidy should be negative (a tax).

How is the entry decision of training firms affected? Consider the situation where taxes on poaching firms and training subsidies are such that they implement first best training levels and turnover rates. Taxes on poaching firms tend to decrease the joint expected income Y^n (as taxes decreases the value of turnover to the worker), while a training subsidy (if positive) tends to increase Y^n. Hence, we cannot determine whether it is optimal to subsidise or to tax entry of training firms.

As stated above, the training enhancing policy in Australia may be interpreted as a pay-roll tax on poaching firms. It is clear that such a policy alone does not implement the first best solution. With policy invariant wages such a policy, however, tends to reduce turnover, which *ceteris paribus*, tends to increase welfare. The effect on the training level is ambiguous. With endogenous wages, we are not able to make robust predictions regarding the effects of this policy measure.

Taxing training and poaching firms

In practice, it may be difficult for the government to distinguish between training and poaching firms. In this subsection, we therefore analyse the effect of taxes on both, taking the training level and the entry of training firms as given.

Profit tax

A profit tax on training firms does not influence their wage setting for experienced workers. Thus, a profit tax on both training and poaching firms has the same effect on turnover as a profit tax on poaching firms only.

To see this, note that the profit flow of a training firm is given by $(1-t)(y^e - w^e)$, and it follows from (3) that

$$(r+s)J^e = (1-t)\frac{y^e - w^e}{r+s+e^e p^e}.$$

Firms choose w^e so as to maximise J^e, and it follows that the tax does not influence the wage that the training firm sets.

Pay-roll tax

Compared to the situation in which only poaching firms are taxed, pay-roll taxes on both training and poaching firms reduces the effect dramatically. In the appendix we show that for e^e exogenous and zero on-the-job search costs (or alternatively, deductible search costs), a pay-roll tax on both training and poaching firms has no effect on worker turnover. The reason is that such a tax does not influence any of the trade-offs that either poaching firms, or training firms, or workers face. Free entry ensures that the tax burden is borne by the workers, and the wage costs including the pay-roll tax remain unchanged.

With endogenous e^e, the model becomes more complicated. As the gain from on-the-job search is reduced by a factor $1/(1+t)$, the on-the-job search intensity falls, thus reducing turnover.

In our framework, the aforementioned training policy in France and U.K. is similar to pay-roll taxes and training subsidies in our framework. For a given level of e^e and h, a pay-roll tax does not affect turnover. Taxes, however, decreases the joint private value of a match in a training firm Y^n for a given h. They also imply that the joint private value of h is lower than its social value (provided that higher values of h imply higher wages and hence taxes). Accordingly, entry of training firms and/or the level of h would have to be subsidised. The best such a subsidizing policy can achieve is to bring welfare back to the level that welfare reaches in the absence of taxes and subsidies (as the latter is constrained efficient). With endogenous e^e, a pay-roll tax in combination with a training subsidy may improve welfare (for a given wage), as turnover is reduced.

Assessment of combined policy measures

We have argued that it is possible to obtain the first best resource allocation by a (rather complex) mixture of taxes and subsidies. If the authorities cannot discriminate between training firms and poaching firms, a tax on firm profits is still effective in reducing turnover. Pay-roll taxes are less effective, even though they reduce the on-the-job search intensity. These findings support the result in Stevens (2001) that training subsidies should be coupled with taxes on profits and not with pay-roll taxes.

5 Discussion

In this section, we discuss some important features and assumptions in our model, with particular focus on the matching technology and the wage determination process. We also contrast our efficiency results with the existing findings in the literature on general human capital investments, most notably Acemoglu (1997).

Before we do that, we like to point out that our paper is also a contribution to the literature on the broader issue of search and efficiency. In this literature, Acemoglu and Shimer (1999b) is closest related to this paper. They study the firms' incentives to invest in physical capital within a search context. Our model differs from theirs, most importantly in this context is which side of the market that undertakes the investment. In Acemoglu and Shimer (1999b), the agents that invest also advertise the wages, while in our model the agents at the other side of the market invests. Furthermore, in our model a third party (the incumbent firm) may influence the search process through the wage it sets for searching workers.

Also related, is the literature on efficient investments in a matching context without search frictions (e.g., Cole et al. (2001)). Cole et al. (2001) find that even when the parties cannot contract with each other before the investment is undertaken, an equilibrium with efficient investments can be sustained. They do, however, abstract from the workers' search behaviour, from firm entry and from turnover, which are all key in our analysis.

Finally, there exists a literature that relates training and asymmetric information. Asymmetric information tends to reduce turnover.[9] If the current employer has superior information concerning a worker's productivity, this may reduce the amount of turnover and thereby also the inefficiency that may be created by excessive turnover. On the other hand, asymmetric information may create inefficiencies along other dimensions.

5.1 Wage bargaining under the Hosios condition

In this subsection we study to what extent our results remain valid when agents search in separate markets but wages are determined by wage bargaining. We assume internal efficiency.

With wage bargaining, the search market is generally inefficient even with homogeneous workers, due to search externalities. The equilibrium outcome is efficient only if the sharing rule is such that the Hosios condition is met (Hosios 1990). The Hosios condition is satisfied whenever the absolute value of the elasticity of q (the arrival rate of workers to firms with a vacancy) with respect to the labour market tightness θ is equal to the worker's bargaining power (share of bargaining surplus), and when the parties' outside option in the bargaining is their "asset value" (expected net present value of future income) prior to the match. Thus, the Hosios condition implies efficient on-the-job search markets if the worker's outside option in the wage bargaining with the poaching firm is his current employment in the training firm.

In the search and matching literature, it is commonly assumed that the outside option for the worker is unemployment rather than his previous employment (see for instance Pissarides (1994)). The rationale for this assumption is that wages are frequently renegotiated. With this assumption, wages in poaching firms under the Hosios are too low compared to the level necessary to achieve the first best outcome. As wages in poaching firms are too low, too many poaching firms enter, and W^e is below its maximum. Welfare certainly falls, and our conjecture is that

[9] For references, see Greenwald (1986) and Acemoglu and Pischke (1998) and the refererences therein.

both the number of training firms and the training level in each training firm fall. If wages are set *ex post*, we conjecture that wage bargaining may exacerbate the inefficiencies created by increased turnover (although this depends on the relationship between p^e and w^e).

If the relevant disagreement point for a worker bargaining with a poaching firm is to remain in the training firm, our conjecture is that the Hosios condition ensures an efficient allocation, as long as internal efficiency holds. The Hosios condition ensures that the negative search externality for agents at the same side and the positive search externalities for agents at the opposite side of the market exactly balance in all submarkets. Wages and labour market tightness are the same as in a competitive search market in all on-the-job search markets (for all training levels). Hence, the expected income for a trained worker and thus the incentives to invest are the same as in a competitive search equilibrium model. The efficient outcome of the on-the-job search markets implies that the unemployment search market is also efficient (given that the Hosios condition holds).

Acemoglu (1997) also considers investments in on-the-job training in a setting with enforceable long-term contracts and bargaining. In his model, turnover is a result of an exogenous job destruction process after which the worker becomes unemployed and starts searching for a new job. Acemoglu identifies a positive externality from training on future employers, and as a result there is underinvestment in training. Within his model, we conjecture that efficiency can be obtained if one allows for separated search markets combined with wage advertisements or bargaining under the Hosios condition.

5.2 Matching technology

Crucial for our efficiency result (Proposition 1) is the assumption that workers with different characteristics search in different submarkets. If workers with different characteristics were searching in the same submarket, efficiency would no longer prevail. Suppose a subset of workers improve their training. As long as wages increase less than their productivity, more vacancies enter this market. If the search markets are not separated, this benefits all workers in the market. Thus, there exists a positive externality from training (the firms, by definition, earns zero profit in any case), and underinvestment in training results (see Acemoglu and Shimer (1999b) for a similar result with physical investments by firms).

The critical issue is therefore to what extent our assumption that different worker types search in separated search markets is plausible. To be clear, we do not necessarily argue that *complete* market separation is the most accurate description of the real world. Still we believe that this is an interesting benchmark, as is the complete-market competitive model without search frictions. Furthermore, there are compelling reasons that market separation takes place at least to some extent. As discussed in section 2, workers are separated into submarkets if, in addition to wages, firms advertise the human capital level required for the job. We have also noticed that a market maker finds it optimal to separate the market into submarkets. Furthermore, a somewhat counter intuitive implication of a non-separated search market is that workers with different productivities have the same probability of finding a job in a poaching firm.

In a setting where firms invest in physical capital, Acemoglu and Shimer (1999b) argue that even if firms cannot advertise wages, workers have an incentive to direct their search towards firms with high investments, as they anticipate that the bargaining outcome in such a firm will be attractive. Thus, even if wages are determine by wage bargaining, the market may endogenously separate into submarkets. This mechanism seems less realistic in our setting with investments in human capital. Firms usually hire a large number of workers, and it is therefore more plausible to assume that workers know the capital level in firms rather than the other way around.

The discussion concerning separated search markets points at a weakness of the Diamond-Mortensen-Pissarides search framework, namely the exogenity of the matching process. It would therefore be of interest to analyse the training decision in a framework in which the matching process is explicitly modelled. A natural starting point is the urn-ball process (Montgomery (1991), Peters (1991), Burdett, et al. (2001)). In this matching process, each firm posts one vacancy, and workers choose to which firm to apply, using mixed strategies. If a firm obtains more than one applicant, it selects one randomly. Moen (1999) analyses the workers' incentives to invest in education before entering the labour market. He finds that workers may over-invest in education in order to speed up their job-finding process. Because Moen assumes wage bargaining, his over-investment result is not directly comparable to the results in this paper. His analysis, however, shows that within the urn-ball matching framework, low productivity workers do not create congestion effects (search externalities) for high productivity workers. The reason is that if two workers with different productivities apply for the same job, the most productive worker is always hired in equilibrium (provided that the wage differential is smaller than the productivity differential). This strongly contrasts with the properties of the Diamond-Mortensen-Pissarides matching technology, which implies that an inflow of low productivity workers into the market reduces the contact rate between high productivity workers and firms. This leads us to conjecture that within the urn-ball matching framework, a sufficient condition for an optimal training decision is that firms are able to advertise wages contingent on worker productivity.

5.3 Production technology

Throughout the paper we assume that there are two types of firms, poaching and training firms and that trained worker are more productive in poaching firms than in training firms. As firms operate at different scales in different parts of the value chain or in different product markets it seems likely that they have different comparative advantages: some firms are in a better position to train workers than others. The coexistence of different types of firms then implies that the other firms must have advantages as well. In our two-dimensional case this means that they must be more efficient in utilising trained labour.

Our analysis is also applicable if trained workers were equally productive in training and poaching firms. With internal efficiency there would be no turnover in equilibrium. This would also be the efficient solution, as turnover has no social value. With *ex post* wage setting, the analysis presented here would be directly applicable. There would be turnover in equilibrium, implying a waste of resources from a social point of view.

As each firm hires at most one worker, it may be natural to interpret a firm as a job and firms as consisting of many jobs. An alternative interpretation of the model would then be that all firms have access to the same production technology, and faces the choice of hiring a trained or an untrained worker, i.e., of opening a poaching vacancy or a training vacancy. We then require an alternative explanations as to why workers in "poaching jobs" are more efficient than workers in "training jobs". One explanation may be that an experienced worker's productivity also depends on a worker-firm specific component which is unknown at the time of the training decision (or, equivalently, that an experienced worker's disutility from working differs across firms, for instance, due to locational issues). On-the-job searching workers would then be workers who made a bad "draw" and thus have a low worker-firm specific productivity component. In expected terms, these workers would be more productive in other firms. With this interpretation, our analysis also answers the question as to whether firms have sufficient incentives to train their workers themselves rather than to poach already trained workers from other firms. We answer this question in the affirmative, given that training firms operate internally efficient.

6 Conclusion

This paper analyses the incentives to invest in general training in a matching model with endogenous worker turnover and with wages being set in a competitive fashion. As long as employers and employees are able to resolve within-firm co-ordination problems (internal efficiency), search frictions do not induce inefficiencies and the resulting resource allocation is optimal. In the absence of internal efficiency, there may be underinvestment in training as a result of excessive turnover. As the excessive turnover reduces both the private and the social return from training, the level of investment in training is, however, still constrained efficient. A training subsidy alone therefore reduces welfare. In combination with additional policy measures aimed at reducing turnover, a subsidy may increase welfare.

Appendix

A. Proof of Lemma 1

Suppose Lemma 1 does not hold. Then there exists a triple $(W^{p\prime}, p^{e\prime}, e^{e\prime})$ such that $W^e(W^{p\prime}, p^{e\prime}, e^{e\prime}) > W^{e*}$ and $V^p(W^{p\prime}, p^{e\prime}) \geq K^p$. By continuity of the problem, there exists another triple $(W^{p\prime\prime}, p^{e\prime\prime}, e^{e\prime\prime})$ such that $W^e(W^{p\prime\prime}, p^{e\prime\prime}, e^{e\prime\prime}) > W^{e*}$ and $V^p(W^{p\prime\prime}, p^{e\prime\prime}) > K^p$. This implies that the profit maximisation condition is not satisfied, violating an equilibrium condition.

B. Proof of Lemma 2

We use optimal control theory to solve the problem. The associated current-value Hamiltonian is given by

$$H = y^e N^e + e^e p^e N^e Y^p - e^e p^e N^e \frac{r + q(p^e)}{q(p^e)} K^p - c(e^e)N^e + \lambda(b - (s + e^e p^e)N^e),$$

where λ is the associated adjunct function. First order conditions for the maximum are as follows

1. p^e and e^e maximise H
2. $r\lambda - \dot{\lambda} = \frac{\partial H}{\partial N^e}$

Condition 1 implies that p^e and e^e solve

$$\max_{p^e,\, e^e} e^e p^e (Y^p - \frac{r + q(p^e)}{q(p^e)} K^p - \lambda) - c(e^e). \tag{15}$$

In steady state, condition 2 implies that

$$(r + s)\lambda = y^e + e^e p^e (Y^p - \frac{r + q(p^e)}{q(p^e)} K^p - \lambda) - c(e^e). \tag{16}$$

The comparison of (9) and (16) shows that the expressions for λ and W^e are equivalent. Furthermore, as the maximisation problem (15) is equivalent to maximising λ in (16), the planner maximises W^e, as in the competitive search equilibrium. This proves part a). Moreover, as $\frac{dR}{dN^e} = \lambda$, the social value of a worker entering the market is equal to W^e, proving part b).

To prove part c), suppose that the number of poaching firms is exogeneously given. Consider the associated (steady state) competitive search equilibrium, and denote by V' the equilibrium value of a poaching vacancy. Compare this equilibrium with the equilibrium of a model in which poaching firms may enter at an entry cost V'. By construction, the equilibrium without entry is also an equilibrium with entry. Furthermore, as the equilibrium of the model is unique it follows that the two equilibria coincide. Hence, the asset value of a searching worker W^e with and without entry must also coincide.

We now want to show that the social value of a searching worker in the economy is the same with and without entry by firms. Let j' denote the associated number of jobs in the steady state equilibrium (which is initially equal to the exogenous number of jobs without entry) and write the aggregate discounted income net of entry- and search costs (welfare) as a function $G(N, j')$, where N is the number of searching workers in the economy. Without entry, the shadow price of a worker in this economy is $g_n = \frac{\partial G}{\partial N}$. With entry, the corresponding price is $g_e = \frac{\partial G}{\partial N} + \frac{\partial G}{\partial j'} \frac{dj'}{dN}$. Since the last term is zero due to the envelope theorem $g_n = g_e$ and $g_n = g_e = W^e$. This completes the proof of part c).

C. Proof of existence of equilibria

Assume that $y^p - y^e > (r+s)K^p$ and that $\frac{(r+s)(y^n - ah^*) + \gamma y^e(h^*)}{r+s+\gamma} > (r+s)K^t$. As regards the on-the-job search market, we have to show that maximising $Y^e(W^p, p^e)$ implies $p^e > 0$ given that $(W^p, p^e) \in \Phi^p$. The maximisation problem is independent of the measure of workers working in training firms (provided that it is strictly positive). Suppose first that the optimal solution requires $p^e = 0$. It follows that $Y^e = y^e/(r+s)$. Consider next the pair $(y^e/(r+s) + \varepsilon, \varepsilon)$ where $\varepsilon > 0$ is arbitrarily close to zero. Obviously, this yields a higher value of Y^e. We have to show that $(y^e/(r+s) + \varepsilon, \varepsilon) \in \Phi^p$. We know that $\lim_{p^e \to 0} q(p^e) = \infty$. Since $c(0) = c'(0) = 0$, q^e can be made arbitrarily large by choosing ε arbitrarily small. Thus,

$$\lim_{\varepsilon \to 0} V(y^e/(r+s) + \varepsilon, \varepsilon) = \frac{y^p - y^e}{r+s}$$

which is by assumption greater than K^p. It thus follows that if there is a positive measure of workers working in training firms, there is a positive measure of poaching firms entering the market.

Consider now the unemployed search market. We show by contradiction that $p^u = 0$ cannot be a solution to maximising $W^u(W^n, p^u)$ given that $(W^n, p^u) \in \Phi^t$. Suppose that $p^u = 0$ which implies that $W^u = 0$. Consider the pair $(\varepsilon, \varepsilon)$ where $\varepsilon > 0$ is arbitrarily close to zero. Obviously, $W^u(\varepsilon, \varepsilon) > W^u(W^n, 0)$ for any W^n. Since $\lim_{p^u \to 0} q^u(p^u) = \infty$,

$$\lim_{\varepsilon \to 0} V(\varepsilon, \varepsilon) = Y^n.$$

We know that $Y^n \geq Y^n(h^*, p^e = 0) = \frac{(r+s)(y^n - ah^*) + \gamma y^e(h^*)}{(r+s)(r+s+\gamma)}$. Hence $(\varepsilon, \varepsilon) \in \Phi^t$ for sufficiently small values of ε, given that $\frac{(r+s)(y^n - ah^*) + \gamma y^e(h^*)}{(r+s)(r+s+\gamma)} > (r+s)K^t$. Thus, $p^u = 0$ is inconsistent with equilibrium.

D. Proof of Proposition 1

Denote the number of unemployed workers by N_0, the number of novice workers by N_1, the number of experienced workers in training firms by N_2, and the number of workers in poaching firms by N_3. We normalize $N_0 + N_1 + N_2$ to one. The planner's objective function is then given by

$$\begin{aligned} R(N_0, N_1, N_2, N_3) &= \int_0^\infty [N_1(y^n - ah) + N_2 y^e + N_3 y^p - e^u p^u N_0 \frac{r + q(p^u)}{q(p^u)} K^t \\ &\quad - e^e p^e N_2 \frac{r + q(p^u)}{q(p^u)} K^p - N_0 c(e^u) - N_2 c(e^e)] e^{-rt} dt, \end{aligned}$$

which has to be maximised with respect to h, e^u, e^e, p^u and p^e subject to the following constraints:

$$\begin{aligned} \dot{N}_0 &= s - (e^u p^u + s) N_0, \\ \dot{N}_1 &= e^u p^u N_0 - (\gamma + s) N_1, \\ \dot{N}_2 &= \gamma N_1 - (e^e p^e + s) N_2, \\ \dot{N}_3 &= e^e p^e N_2 - s N_3, \end{aligned}$$

We first derive the solution for a given h. The associated current-value Hamiltonian can be written as

$$\begin{aligned} H &= N_1(y^n - ah) + N_2 y^e + N_3 y^p \\ &\quad -[e^u p^u N_0 \frac{r + q(p^u)}{q(p^u)} K^t + e^e p^e N_2 \frac{r + q(p^e)}{q(p^e)} K^p] - N_0 c(e^u) - N_2 c(e^e) \\ &\quad + \lambda_0 (s - (e^u p^u + s) N_0) \\ &\quad + \lambda_1 (e^u p^u N_0 - (\gamma + s) N_1) \\ &\quad + \lambda_2 (\gamma N_1 - (e^e p^e + s) N_2) \\ &\quad + \lambda_3 (e^e p^e N_2 - s N_3). \end{aligned}$$

The first order conditions for maximium are:

1. The Hamiltonian is maximised with respect to e^u, e^e, p^u and p^e.
2. For all i, $r\lambda_i = \frac{\partial H}{\partial N_i}$ (assuming that we are in steady state).

From condition 1 it follows that p^u and e^u solve

$$\max_{p^u, e^u} e^u p^u [\lambda_1 - \lambda_0 - \frac{r + q(p^u)}{q(p^u)} K^t] - c(e^u), \tag{17}$$

and that p^e and e^e solve

$$\max_{p^e, e^e} e^e p^e [\lambda_3 - \lambda_2 - \frac{r + q(p^e)}{q(p^e)} K^p] - c(e^e). \tag{18}$$

From condition 2 it follows that

$$(r+s)\lambda_0 = e^u p^u[\lambda_1 - \lambda_0 - \frac{r+q(p^u)}{q(p^u)}K^t] - c(e^u), \tag{19}$$

$$(r+s)\lambda_1 = y^n - ah + \gamma(\lambda_2 - \lambda_1), \tag{20}$$

$$(r+s)\lambda_2 = y^e + e^e p^e[\lambda_3 - \lambda_2 - \frac{r+q(p^e)}{q(p^e)}K^p] - c(e^e), \tag{21}$$

$$(r+s)\lambda_3 = y^p. \tag{22}$$

We now compare the optimal solution with the market solution. With internal efficiency and for a given value of h, the expressions for λ_0-λ_3 are identical to the corresponding expressions for W^u, Y^n ,Y^e, and Y^p. Furthermore, (17) and (18) imply that (p^u, e^u) maximises λ_0, and that (p^e, e^e) maximises λ_2, just as the competitive search equilibrium maximises W^u and Y^e. Thus, for a given value of h the equilibrium and the planner's solution coincide, proving part b).

We know from optimal control theory that the adjoint variables are equal to the marginal value of the associated state variables. The planner therefore chooses h so as to maximise the value of an additional worker entering the market. That is, he chooses h so as to maximise λ_0. From (19) it follows that this is equivalent to maximising λ_1. Since h is set so as to maximise Y^n in equilibrium, the planner and the agents in the market solve the same maximisation problem, and the equilibrium value of h is socially optimal, proving part a).

The proof of part c is analogous to the proof of Lemma 2 part c. Suppose the number of poaching firms is given exogeneously, and consider the corresponding equilibrium. Suppose the asset value of a poaching vacancy in this equilibrium is V'. Then consider the equilibrium that emerges with free entry of firms and a cost of creating poaching vacancies equal to V'. We know from the proof of Lemma 2 that this equilibrium will be identical to the equilibrium without entry of poaching firms (as all the asset values and thus also the investments in training will be the same). We want to show that the social value of training is the same in the two equilibria as well. Suppose a small subset of worker-firm pairs deviate and increase their investments in training. The optimal response with free entry will then be to increase the number of poaching firms as well. However, due to the envelope theorem the effect of the latter is of second order. Thus, the marginal social value of level of training is the same in the two equilibria. Thus, since the training level is optimal in the equilibrium with entry it follows that this will also be the case in the equilibrium without entry. The same argument holds for entry of training firms.

E. Proof of Lemma 3

Part (1a): Using equation (8), free entry by poaching firms implies that $W^p = Y^p - \frac{r+q^e}{q^e}K^p$. Hence, for a given w^e, p^e maximises

$$(r+s)W^e(w^e) = w^e + e^e p^e(Y^p - \frac{r+q(p^e)}{q(p^e)}K^p - W^e(w^e)) - c(e^e). \qquad (23)$$

The above equation implies that the equilibrium value p^{e*} maximises $p^e(Y^p - \frac{r+q^e}{q^e}K^p - W^e(w^e)) \equiv f(W^e(w^e), p^e)$ and that the cross derivative $f_{p^e,W^e} < 0$. As the second-order conditions for the maximum are always satisfied locally, $\frac{dp^{e*}}{dW^e} < 0$. From the envelope theorem it follows that $\frac{dW^e(w^e)}{dw^e} = 1/(r+s+e^e p^e) > 0$. Thus, p^{e*} decreases in w^e.

Part (1b): We know that p^e maximises W^e, and from equation (23) that p^e therefore maximises $p^e(Y^p - \frac{r+q^e}{q^e}K^p - W^e(w^e))$. Hence, we can write $W^e(w^e)$ as

$$(r+s)W^e(w^e) = \max_{e^e}\left\{w^e - c(e^e) + e^e \max_{p^e}[p^e(Y^p - \frac{r+q(p^e)}{q(p^e)}K^p - W^e(w^e))]\right\}.$$

Hence, the first order condition for e^e is

$$c'(e^e) = \max_{p^e} p^e(Y^p - \frac{r+q(p^e)}{q(p^e)}K^p - W^e(w^e)).$$

From the envelope theorem it follows that the derivative of the right hand side with respect to w^e is equal to $-p^e\frac{\partial W^e}{\partial w^e} = -p^e/(r+s+e^e p^e) < 0$. Hence, $\frac{de^e(w^e)}{dw^e} = \frac{-p^e/(r+s+e^e p^e)}{c''(e^e)} < 0$.

Part (2): Using equation (7), free entry by training firms implies that $W^n = Y^n - \frac{r+q^u}{q^u}K^t$. Hence, in the unemployed search

$$(r+s)W^u = e^u p^u(Y^n - \frac{r+q(p^u)}{q(p^u)}K^t - W^u) - c(e^u)$$

is maximised with respect to e^u, and p^u. The above equation implies that the equilibrium value p^u maximises $p^u(Y^n - \frac{r+q^u}{q^u}K^t - W^u) \equiv f(Y^n, p^u)$, and that the cross derivative $f_{p^u,Y^n} > 0$. As the second-order conditions for the maximum are always satisfied locally, $\frac{dp^u}{dY^n} > 0$. Since, Y^n is strictly less with ex post wage setting than with internal efficiency fewer training firms are created.

F. Proof of Proposition 2

We first show that the social and the private value of an additional experienced worker entering the market coincide, given the workers' search behaviour and entry decisions of firms in the on-the-job search market.

The joint private value an experienced worker in a training firm is given by $Y^e = \frac{y^e - w^e}{r+s+e^e p^e} + W^e(w^e)$, where the first term denotes profits and the second the expected discounted income to workers. From Lemma 2 it follows that the social value of an experienced worker with *productivity* w^e in the training firm and y^p in a poaching firm is equal to $W^e(w^e)$. When the productivity exceeds the wage the difference $(y^e - w^e)$ is allocated to the firm. The social value of one more experienced worker is thus $\frac{y^e - w^e}{r+s+e^e p^e} + W^e(w^e) = Y^e(w^e)$. That is, the social and the private value coincide.

In each submarket, the equilibrium solves (10). Because a training subsidy does not influence this maximisation problem, the equilibrium values of w^e and p^e are independent of the training subsidy.

It follows that at the stage at which human capital investments are made, the social and the private returns from training coincide. As the training firms behave by assumption internally efficient at this stage, it follows that the training levels undertaken by the agents are equal to the investment levels undertaken by the planner, i.e., the equilibrium is constrained efficient. Finally, this implies that the social value of hiring an inexperienced worker coincides with the private value. Thus, the unemployed search market is efficient as well, and the optimal number of training firms enter the market. As the market is constrained efficient at the stage where the entry decision of training firms and their investment decision in training are undertaken, a training subsidy reduces the allocative efficiency of the economy.

G. Proofs of effects of policy measures

i) Entry tax

We first show that p^e and e^e are decreasing in T. Substituting $W^p = Y^p - \frac{r+q^e}{q^e}(K^p + T)$ into equation (2) yields

$$(r+s)W^e = w^e + e^e p^e (Y^p - \frac{r+q(p^e)}{q(p^e)}(K^p + T) - W^e) - c(e^e). \qquad (24)$$

Let $f(p^e, T)e^e \equiv (r+s)\frac{dW^e}{dp^e}$. For any T, the optimal value of p^e is thus given by $f(p^e, T) = 0$. Using equation (24) we find that

$$\begin{aligned} f(p^e, T) &= [Y^p - \frac{r+q(p^e)}{q(p^e)}(K^p + T) - W^e] - [p^e \frac{d}{dp}\frac{r+q(p^e)}{q(p^e)}(K^p + T)] \quad (25) \\ &= A(p^e, T) - B(p^e, T). \end{aligned}$$

The function $f(p^e, T)$ only depends on e^e through W^e. Since e^e is chosen optimally the envelope theorem implies that $\partial W^e / \partial e^e = 0$ around the equilibrium point. Accordingly we can analyse changes in p^e independently of e^e. It is straightforward to show that for a given p^e, $\frac{dA(p^e,T)}{dT} = -\frac{r+q^e}{q^e}\frac{r+s}{r+s+e^e p^e}$. Hence,

$f_T(p^e, T) < 0$. Since the second order conditions must be satisfied locally, $f_{p^e} < 0$, and hence $\frac{dp^e}{dT} < 0$.

The first order condition for e^e is given by

$$c'(e^e) = p^e A(p^e, T). \tag{26}$$

Thus, e^e increases with T if and only if $p^e A(p^e, T)$ increases with T. It follows from the envelope theorem and equation (24) that $\frac{dW^e}{dT} = -\frac{r+q^e}{q^e}\frac{1}{r+s+e^e p^e} < 0$. At the same time, $\frac{dW^e}{dT} < 0$ if and only if $p^e A(p^e, T)$ is strictly decreasing in T (the envelope theorem implies, again, that we can ignore the effects of e^e on W^e). Thus, e^e is strictly decreasing in T.

We now show that $e^e(T') > e^{e*}$ if $p^e(T') = p^{e*}$. Suppose that p^e is at its first best level p^{e*}, at a tax T'. Let $\widetilde{A}$ and $\widetilde{B}$ denote the values of A and B that emerges in the (first-best) equilibrium with internal efficiency (and no taxes), in which case $p^e = p^{e*}$ and $e^e = e^{e*}$. By definition, $A(p^{e*}, T') - B(p^{e*}, T') = 0$ and since $B(p^{e*}, T') > \widetilde{B}$, it follows that $A(p^{e*}, T') > \widetilde{A}$. But from the first order condition for e^e it then follows that $e^e(T') > e^{e*}$.

Pay-roll tax

Substituting $(1+t)W^p = Y^p - \frac{r+q^e}{q^e}K^p$ into equation (2) yields

$$(r+s)(1+t)W^e = w^e + e^e p^e (Y^p - \frac{r+q(p^e)}{q(p^e)}K^p - (1+t)W^e) - (1+t)c(e^e). \tag{27}$$

Let $f(p^e, t)e^e \equiv (r+s)(1+t)\frac{dW^e}{dp^e}$. For any t, the optimal value of p^e is given by $f(p^e, t) = 0$. Using equation (27) we find that

$$\begin{aligned} f(p^e, t) &= [Y^p - \frac{r+q(p^e)}{q(p^e)}K^p - W^e(1+t)] - [p^e(\frac{d}{dp}\frac{r+q(p^e)}{q(p^e)})K^p] \\ &= A(p^e, t) - B(p^e) = 0 \end{aligned} \tag{28}$$

An increase in t reduces $A(p^e, t)$ but has no effect on $B(p^e)$ for a given p^e. Since $A_{p^e} < 0$ it thus follows from equation (28) that p^e decreases with t.

The equilibrium value of e^e maximsies W^e, and is thus given by (analoguous to equation (26))

$$(1+t')c'(e^{e*}) = p^e(t')A(p^e(t'), t') \tag{29}$$

and by using the same argument as in that subsection we find that e^e is strictly decreasing in t.

To prove that the effect on e^e is stronger than that on p^e, suppose t' is such that $p^e(t') = p^{e*}$, where p^{e*} denote the first best value of p^e. Define $\widetilde{A}$ and $\widetilde{B}$ as in the subsection above (entry tax). It follows that $B(p^{e*}) = \widetilde{B}$ and hence from equation (28) that $A(p^{e*}, t') = \widetilde{A}$. Equation (29) can thus be written as

$$(1+t')c'(e^e(t')) = p^e(t')\widetilde{A}$$

Since e^{e*} by definition is given by $c'(e^{e*}) = p^e(t')\widetilde{A}$, it follows that $c'(e^e(t')) < c'(e^{e*})$ and hence that $e^e(t') < e^{e*}$.

Pay-roll tax on both training firms and poaching firms.

Let w^e denote the wage net of the pay-roll tax. We want to show that the training firms set the wage in such a manner that their wage costs including taxes, $\widetilde{w}^e = w^e(1+t)$, is fixed and independent of the tax rate as is p^e. Suppose the wage costs are independent of taxes. In this case $W^{et} = W^{e0}/(1-t)$, where W^{et} and W^{e0} are the expected income to experienced workers with and without taxes. Free entry by poaching firms implies that $W^{pt} = Y^p - tW^{pt} - \frac{r+q^e}{q^e}K^p$. It follows that W^{et} is given by

$$(r+s)W^{et} = \frac{w^{e0}}{1+t} + p^e[\frac{Y^p - \frac{r+q(p^e)}{q(p^e)}K^p}{1+t} - W^{et}],$$

or that $W^{et} = \frac{W^{e0}}{1+t}$ for any p^e. Since competitive search equilibrium maximises W^{et}, it follows that p^e is independent of the tax rate. Thus, given that the training firms set wages such that the wage cost is unaffected by taxes, pay-roll taxes on both training firms and poaching firms have no effect on p^e. Finally, the training firms maximise

$$J^e = \frac{y^e - \widetilde{w}^e}{r+s+p^e}.$$

We have just shown that for any $\widetilde{w}^e$, p^e is independent of t. Thus, the trade-off between wage costs $\widetilde{w}^e$ and the workers' quit-rate that training firms face is independent of t. Hence, for a given h, $\widetilde{w}^e$ is independent of t. Since, p^e is independent of t, the free entry condition implies that so is $\widetilde{w}^p$. Given our assumptions regarding e^e and $c(e^e)$, it thus follows that, a pay-roll tax on both training and poaching firms has no effect on turnover, for a given h.

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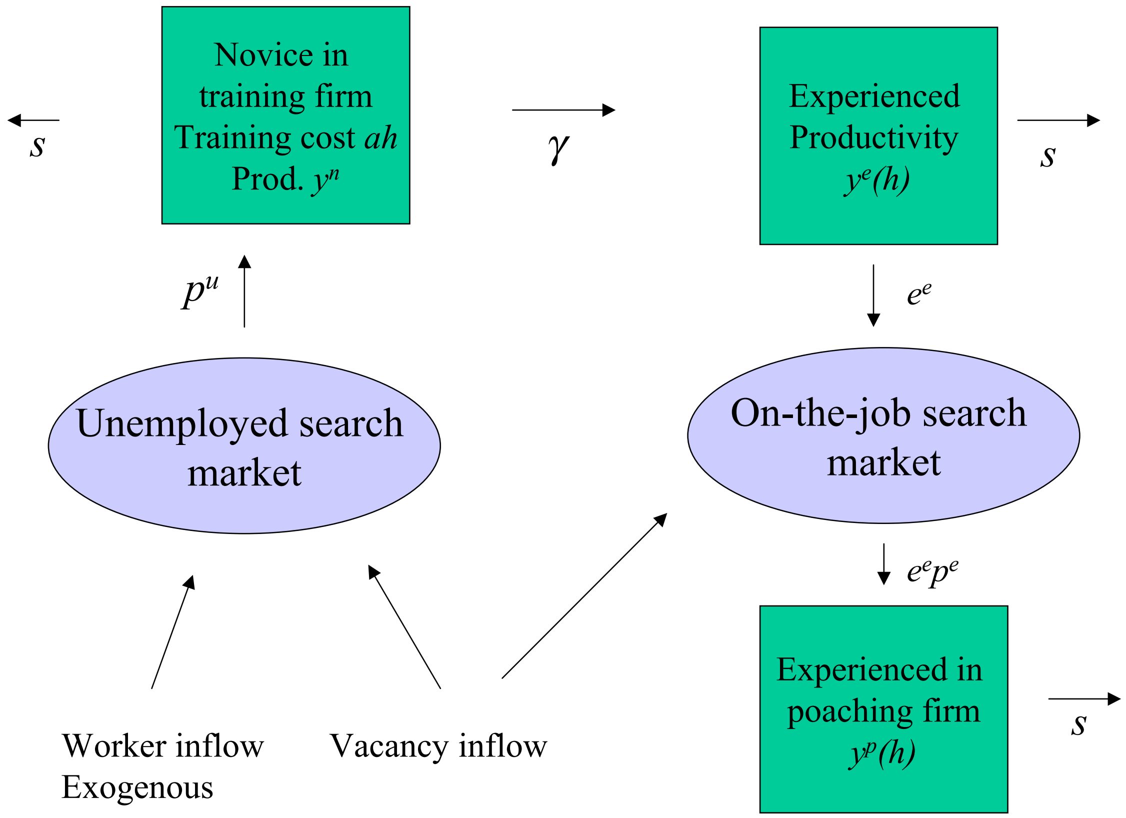

Figure 1:

Figure 1. Worker flows in the economy.