Incomplete Information
In a Long Run Risks Model of Asset Pricing

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Abstract

We study the effects of incorporating incomplete information in the recently developed long run risks model of asset pricing. Studying the effects of incomplete information in such a setting is tractable, especially in the homoskedastic case with no fluctuating economic uncertainty. The incomplete information model is solved using approximate analytical methods as in the complete information framework analyzed in the literature. Model implications on moments of endogenous variables of interest including rates of return are compared in the long run risks model with and without incomplete information.

Key phrases: asset pricing; long run risks; incomplete information; Kalman filter; equity returns; riskfree returns

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1. INTRODUCTION

Recent work on long run risks and their implications for asset pricing pioneered by Bansal and Yaron (2004) shows a great deal of promise in resolving several empirical puzzles in asset pricing. An abundance of research attention is currently focused on exploring implications of long run risks in a variety of contexts. Bansal (2007) provides a brief review of this literature. See also Hansen, Heaton and Li (2005).

The structure of the long run risks model in Bansal and Yaron (2004) is well suited for an exploration of the effects of incomplete information in asset pricing context. The potential importance of the effects of incomplete information in asset pricing has been recognized in early work by Dothan and Feldman (1986), Detemple (1986,1989), Gennotte (1986), and more recently, by Brennan and Xia (2001). This literature studies incomplete information in a linear Gaussian setting wherein the classic Kalman filter provides an optimal Bayesian updating rule for learning about the unobservable dividend drift rate with the arrival of new information on dividends each period. More recently, David (1997) and Veronesi (2004) study asset pricing with incomplete information in a non-Gaussian setting where the unobservable dividend growth rate undergoes jumps, driven either by a Markov switching or Poisson arrival process.

A linear Gaussian conditionally homoskedastic model (the no fluctuating economic uncertainty case in Bansal and Yaron 2004) with incomplete information is particularly amenable to analysis. This is because, with conditional homoskedasticity, the variance of the filter density of the state vector becomes time invariant once the Kalman filter has reached steady state. The approximate analytical solution method in Bansal and Yaron (2004) can then be used to solve the model.

In this paper we explore the implications of incorporating incomplete information in the long run risks model in a linear Gaussian homoskedastic framework. We derive approximate analytical solution to the model in this setting using the methods in Bansal and Yaron (2004). We quantify the implications of incomplete information on the moments of risk premium and risk free rate by making a detailed comparison with the complete information case.

The paper is organized as follows. We describe the economic environment, the specification of the exogenous stochastic process for consumption and dividend growth rates, as well as solution to the asset pricing model in section 2. We parameterize the model and analyze its empirical implications in section 3. We conclude in section 4 with some key observations derived from the study.

2. THE MODEL

Is this section we lay out the economic environment, including specification of exogenous stochastic processes and information structure, and present solution to the asset pricing model.

2.1 Model Economy

Our model economy is very similar to the one studied in Bansal and Yaron (2004). The first order condition for a representative agent with Epstein and Zin (1989) and Weil (1989) recursive preferences satisfies:

$$E_t\left[\delta^\theta G_{t+1}^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}\right] = 1 \tag{1}$$

for any asset with gross returns $R_{i,t+1}$. Here in this equation, G_{t+1} is the aggregate

gross growth rate of consumption, $0 < \delta < 1$ is the time discount factor, $\psi \geq 0$ is the

elasticity of intertemporal substitution (IES), and $\theta \equiv \dfrac{1-\gamma}{1-\dfrac{1}{\psi}}$ with $\gamma \geq 0$ being the risk

aversion parameter. The intertemporal marginal rate of substitution (IMRS)

$\delta^{\theta} G_{t+1}^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)}$ is alternatively referred to in the literature as the pricing kernel or the

stochastic discount factor.

The above first-order condition applies in turn to an asset with unobservable gross

returns $R_{a,t+1}$ that pays aggregate consumption as dividends each period, to an asset

with observable gross returns $R_{m,t+1}$ that pays aggregate dividends on the market

portfolio, as well as an asset with observable risk free returns $R_{f,t}$ that pays one unit of

aggregate consumption.

Let $z_t \equiv \ln\left(P_{a,t}/C_t\right)$ be the log price-consumption ratio and $r_{a,t+1} \equiv \ln\left(R_{a,t+1}\right)$

be the continuous return. From the definition of gross returns, one can utilize first-order

Taylor's series approximation to write:

$$r_{a,t+1} \cong k_0 + k_1 z_{t+1} - z_t + g_{t+1} \tag{2}$$

where k_0 and k_1 are approximating constants that depend only on the average level of

z. Similarly, we define analogous quantities, namely $z_{m,t} \equiv \ln\left(P_{m,t}/D_t\right)$ and

$r_{m,t+1} \equiv \ln(R_{m,t+1})$ on the market portfolio. The following equation then holds approximately for the continuous returns on the market portfolio:

$$r_{m,t+1} \cong k_{0,m} + k_{1,m}z_{m,t+1} - z_{m,t} + g_{d,t+1} \qquad (3)$$

where the approximating constants $k_{0,m}$ and $k_{1,m}$ now depend only on the average level of the log price-dividend ratio z_m.

The logarithm of the IMRS is easily seen to be:

$$m_{t+1} = \theta\ln(\delta) - \frac{\theta}{\psi}g_{t+1} + (\theta-1)r_{a,t+1} \qquad (4)$$

where $g_{t+1} \equiv \ln(G_{t+1})$ is the log of gross consumption growth rate.

2.2 Consumption and Dividends Growth Rate Processes under Complete Information

We assume that consumption and dividend growth rates stochastically evolve according to the following process:

$$g_{t+1} = x_t + \sigma\eta_{t+1} \qquad (5a)$$

$$x_{t+1} - \mu = \rho(x_t - \mu) + \varphi_e\sigma e_{t+1} \qquad (5b)$$

$$g_{d,t+1} = \phi x_t + \varphi_d\sigma u_{t+1} \qquad (5c)$$

$$\eta_{t+1}, \; e_{t+1}, \; u_{t+1} \sim \text{iid } N(0,1)$$

with the three shocks η_{t+1}, e_{t+1}, and u_{t+1} being mutually independent. This is very similar to the homoskedastic process used in Bansal and Yaron (2004). The only difference is in the treatment of the non-zero mean term. Unlike above, in Bansal and Yaron (2004), x_t is a zero mean process while the equations for g_{t+1} and $g_{d,t+1}$ have

non-zero μ and μ_d respectively appearing as additive terms on the r.h.s. Our process above is a constrained version of this case, where we restrict $\mu_d = \phi\mu$.

Here, x_t is interpreted as the conditional growth rate of consumption assumed known at time t and σ^2 is its conditional variance. We refer to this as the complete information case.

Under complete information the expected growth rate of consumption x_t is the state variable. The endogenous solution to the price dividend ratios on claims to aggregate consumption and the market returns can be fully characterized in terms of this single state variable.

Following the solution method in Bansal and Yaron (2004) closely, we conjecture the following linear solution forms for z_t, the log price-dividend ratio on a claim to aggregate consumption and for $z_{t,m}$, the log price-dividend ratio on a claim to market returns:

$$z_t = A_0 + A_1 x_t \tag{6a}$$

$$z_{t,m} = A_{0,m} + A_{1,m} x_t . \tag{6b}$$

One can now easily derive the solution coefficients for the price-dividend ratios. These are given by:

$$A_1 = \frac{1 - 1/\psi}{1 - k_1\rho}, \qquad\qquad A_{1,m} = \frac{\phi - 1/\psi}{1 - k_{1,m}\rho} . \tag{7}$$

In the complete information case the formulae for the unconditional mean and variance of the risk premium on the market portfolio and of the risk free rate, as well as the unconditional variance of the log price-dividend ratio on the market portfolio can be

worked out as special (homoskedastic) cases of the more general formulae derived in the time-varying uncertainty case in the appendix to Bansal and Yaron (2004).

2.3 Consumption and Dividends Growth Rate Processes under Incomplete Information

We assume that consumption and dividend growth rates stochastically evolve according to the following process:

$$g_t = x_t + \sigma\eta_t \tag{8a}$$

$$x_t - \mu = \rho(x_{t-1} - \mu) + \varphi_e\sigma e_t \tag{8b}$$

$$g_{d,t} = \phi x_t + \varphi_d\sigma u_t \tag{8c}$$

$$\eta_t,\ e_t,\ u_t \ \sim \text{iid } N(0,1)$$

with the three shocks η_t, e_t, and u_t being mutually independent.

We interpret the model above as a state space model, with x_t being the unobservable state variable driven by the dynamics in Equation (8b) with the signal shock e_t. Equations (8a) and (8c) are the measurement equations, with consumption and dividend growth rates g_t and $g_{d,t}$ being observable. Here, x_t can only be inferred probabilistically through a Bayesian filtering process. Since the model is linear Gaussian, the conditional filter density of x_t is itself Gaussian and hence is completely characterized by its mean and variance. These are recursively given by the classical Kalman filter recursions. We refer to this as the incomplete information case.

In the rest of the paper, for the sake of analytical simplicity, we assume that investors ignore the measurement Equation (8c) while trying to learn about the unobservable x_t. Therefore, Bayesian filtering on the investors' part is done treating

Equation (8a) as the measurement equation and Equation (8b) as the state transition equation. For all other purposes, namely in trying to derives rates of return and risk premia on the market portfolio, investors are assumed to fully take the process for the dividend growth rates specified in Equation (8c) into account.

Let $Y_t \equiv \{g_1, g_2, ..., g_t\}$ denote the history of consumption growth rates up to and including the value at time t. Then the filter density of x_t, $p(x_t \mid Y_t) \sim N(a_t, P_t)$ can be characterized by the filter mean a_t and the filter variance P_t. In this homosekdastic model, once the filter reaches steady state, the filter variance stabilizes to a constant value $\overline{P}$. The Kalman filter updating formula for the filter mean can then easily be shown to be (see, for instance, Harvey 1992, Ch.3):

$$a_t = (1-\rho)\mu + \rho a_{t-1} + \left(\frac{\rho^2 \overline{P} + \varphi_e^2 \sigma^2}{\rho^2 \overline{P} + \varphi_e^2 \sigma^2 + \sigma^2} \right) \{\rho(x_{t-1} - a_{t-1}) + \varphi_e \sigma e_t + \sigma \eta_t\}. \qquad (9a)$$

The steady state filter variance can be obtained by solving the following Riccati equation for $P_t = P_{t-1} = \overline{P}$:

$$P_t = \frac{\left(\rho^2 P_{t-1} + \varphi_e^2 \sigma^2\right)\sigma^2}{\rho^2 P_{t-1} + \varphi_e^2 \sigma^2 + \sigma^2}. \qquad (9b)$$

The resulting quadratic equation for $\overline{P}$ has only one positive root which we take to be the steady state filter variance.

Under incomplete information the filter mean of x_t is the relevant state variable. The endogenous solution to the price dividend ratios on claims to aggregate consumption and the market returns can be fully characterized in terms of the filter mean.

We conjecture the following linear solution forms for z_t, the log price-dividend ratio on a claim to aggregate consumption and for $z_{t,m}$, the log price-dividend ratio on a claim to market returns:

$$z_t = A_0 + A_1 a_t \tag{10a}$$

$$z_{t,m} = A_{0,m} + A_{1,m} a_t. \tag{10b}$$

Following the methods in Bansal and Yaron (2004), one can easily derive the solution coefficients for the price-dividend ratios. These are given by:

$$A_1 = \frac{\rho(1 - 1/\psi)}{1 - k_1 \rho}, \qquad A_{1,m} = \frac{\rho(1 - 1/\psi)}{1 - k_{1,m} \rho}. \tag{11}$$

In the incomplete information case, using methods identical to those in Bansal and Yaron (2004) but recognizing that the relevant state variable is now, instead of the unobservable x_t, its filter mean a_t, one can derive expressions for the relevant moments of endogenous variables of interest. These are given in Appendix B.

3. ANALYSIS OF MODEL IMPLICATIONS

In this section we report parameter values used to specify preferences and exogenous stochastic process for consumption and dividend growth rates. We then present model implications for various endogenous quantities of interest, including rates of return.

3.1 Model Parameterization

The asset pricing model has three preference parameters δ, γ and ψ, with θ being determined by the latter two. It also has six parameters describing the joint stochastic processes for consumption and dividend growth rates. In what follows, we choose parameter values given in Bansal and Yaron (2004). These are presented in Table

1. Subsequent empirical implications of the model will be explored for various alternative values of the risk aversion parameter γ and the IES parameter ψ in addition to those listed in Table 1.

The preference and stochastic process parameters are identical for the complete and incomplete information cases. In the latter case, the steady state filter variance obtained by solving the Riccati Equation (9b) works out to be $1.683e - 6$.

The approximating constants k_0, k_1, $k_{0,m}$, and $k_{1,m}$ appearing in Equations (2)-(3) depend on the average levels of z and z_m which are themselves endogenous. Therefore, these constants can be solved for endogenously, by solving an equation such as Equation (10) in Bansal, Kiku and Yaron (2006). For our chosen parameter values, the approximating constants k_1 and $k_{1,m}$ work out to be 0.9990 and 0.9967.[1] In the incomplete information case, these constants work out to be 0.9981 and 0.9997 respectively.

3.2 Analysis of Unconditional Moments of Model-Implied Variables of Interest

Table 2 reports the unconditional means and variances of the risk premium and the risk free rate, as well as the unconditional variance of the log price-dividend ratio on the market portfolio for various values of γ and ψ in both the complete and incomplete information cases. For ease of comparison we also reproduce these statistics estimated from data from Table IV in Bansal and Yaron (2004).

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[1] I thank Ravi Bansal for bringing this point to my attention and Dana Kiku for providing implied values of the approximating constants for the model parameterization used in Bansal and Yaron (2004).

Results reported in the complete information case are comparable to those in Table II of Bansal and Yaron (2004) with no fluctuating economic uncertainty. Differences between the results can be directly attributed to the slightly different specification of our stochastic process for consumption and dividend growth rates as noted in the paragraph immediately following Equations (5) above. The most significant difference between the results is that our complete information model ends up generating about 0.30 percent per annum higher risk premium on average across the various parameter specifications considered. This is because our stochastic process and our parameterization implies that the average dividend growth rate is $\phi\mu$ which from Table 1 works out to be 5.4 percent per annum rather than 1.8 percent per annum implied by the process in Bansal and Yaron (2004).

Adding incomplete information to the long risks model shows that all the statistics reported in Table 2 uniformly decline in magnitude. Specifically, the risk premium is either negative or very close to zero and the risk free rate declines by about 0.6 to as much as 1.4 percent per annum. The volatility of the risk premium declines to about 12.5 percent per annum for all values of γ and ψ, while the volatility of the risk free rate reduces to about 0.24 or 0.73 percent per annum depending on the parameter values. Finally, the volatility of the log price dividend ratio reduces to 0.05 percent per annum or less.

The decline in volatility of the risk free rate and the log price dividend ratio is easily rationalized. Equations (B1) and (B2) provide formulae for these volatilities in the incomplete information case. In the complete information case, these formulae have $\sigma^2(x_t)$ rather than $\sigma^2(\mu_t)$ terms appear on the rhs, since x_t, not μ_t, is the relevant

state variable in this instance.[2] For our model parameterization, $\sigma^2(x_t)$ is 2.834e-6 while $\sigma^2(\mu_t)$ is only 1.152e-6.

Overall, it appears that adding incomplete information to the long run risks model moves the mean risk free rate (and for some parameter values its volatility) in the right direction to match up with the corresponding statistic in the data. However, along all other dimensions (i.e. in terms of the mean and volatility of the risk premium and the volatility of the log price-dividend ratio), the model performance deteriorates.

4. CONCLUSIONS

We explored the effects of incomplete information in the benchmark long run risks model of asset pricing due to Bansal and Yaron (2004). We compared the homoskedastic (no fluctuating economic uncertainty) complete information model analyzed in the above study with an incomplete information version. We solved the asst pricing model in the incomplete information case using the approximate analytical methods exposited in the above study.

We analyzed the implications of incorporating incomplete information on the means and volatilities of model-implied rates of return on risk free asset and the market portfolio, as well as the volatility of the log price dividend ratio. Our results show that incomplete information helps to move the model implications on the risk free rate closer to the data (at least for some parameter values) but that the model performance deteriorates significantly in terms of all other moments.

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[2] Also, for the volatility of the risk free rate in the complete information case, ρ^2 is replaced by 1 on the rhs but this has little impact since ρ is calibrated to be 0.979.

APPENDIX A

In this appendix we provide expressions for some quantities of interest, namely the innovations to the return on the aggregate consumption claim and the IMRS, as well as the risk premium on the aggregate consumption claim in the incomplete information case. The derivations rely on methods identical to those in Bansal and Yaron (2004) but recognizing that the relevant state variable in the incomplete information case is now, instead of the unobservable x_t, its filter mean a_t. The steady state variance of the filter density is given by $\overline{P}$. We use the notation $K \equiv \dfrac{\left(\rho^2\overline{P} + \varphi_e^2\sigma^2\right)\sigma^2}{\rho^2\overline{P} + \varphi_e^2\sigma^2 + \sigma^2}$ in what follows.

The innovation to the return on the consumption claim can be shown to be:

$$r_{a,t+1} - E_t\left(r_{a,t+1}\right) = \left[1 + k_1 A_1 K\right]\left[\rho\left(x_t - a_t\right) + \varphi_e\sigma e_{t+1} + \sigma\eta_{t+1}\right]. \quad (A1)$$

It then easily follows that $\mathrm{var}_t\left(r_{a,t+1}\right) = \left[1 + k_1 A_1 K\right]^2\left[\rho^2\overline{P} + \varphi_e^2\sigma^2 + \sigma^2\right]$.

The innovation to the IMRS can be shown to be:

$$m_{t+1} - E_t\left(m_{t+1}\right) = \left[\frac{-\theta}{\psi} + (\theta - 1)\{1 + k_1 A_1 K\}\right]\left[\rho\left(x_t - a_t\right) + \varphi_e\sigma e_{t+1} + \sigma\eta_{t+1}\right]. \quad (A2)$$

The risk premium for holding the aggregate consumption claim is determined by the conditional covariance of its return with the IMRS. Thus,

$$E_t\left(r_{a,t+1} - r_{f,t}\right) = -\mathrm{cov}_t\left[m_{t+1} - E_t\left(m_{t+1}\right), r_{a,t+1} - E_t\left(r_{a,t+1}\right)\right] - 0.5\,\mathrm{var}_t\left(r_{a,t+1}\right).$$

This can be shown to be:

$$E_t\left(r_{a,t+1} - r_{f,t}\right) = -\left[1 + k_1 A_1 K\right]\left[\frac{-\theta}{\psi} + (\theta - 0.5)\{1 + k_1 A_1 K\}\right]\left[\rho^2\overline{P} + \varphi_e^2\sigma^2 + \sigma^2\right]. \quad (A3)$$

APPENDIX B

In this appendix we provide expressions for the unconditional mean and variance of the risk premium on the market portfolio and of the risk free rate, as well as the unconditional variance of the log price-dividend ratio on the market portfolio in the incomplete information case. The derivations rely on methods identical to those in Bansal and Yaron (2004) but recognizing that the relevant state variable in the incomplete information case is now, instead of the unobservable x_t, its filter mean a_t. The steady state variance of the filter density is given by $\overline{P}$. We use the following notation in what follows. Let $F \equiv \rho^2 \overline{P} + \varphi_e^2 \sigma^2 + \sigma^2$ and $K \equiv \dfrac{\left(\rho^2 \overline{P} + \varphi_e^2 \sigma^2\right)\sigma^2}{\rho^2 \overline{P} + \varphi_e^2 \sigma^2 + \sigma^2}$.

Then one can show that $\sigma^2(x_t) = \dfrac{\varphi_e^2 \sigma^2}{1 - \rho^2}$, $\sigma^2(a_t) = \dfrac{K^2 F}{1 - \rho^2}$, and

$$\mathrm{cov\,ar}(x_t, a_t) = \frac{K \varphi_e^2 \sigma^2}{1 - \rho^2}.$$

The unconditional variance of the log of the price-dividend ratio on the market portfolio can be shown to be:

$$\sigma^2(z_{m,t}) = A_{1,m}^2 \sigma^2(a_t). \tag{B1}$$

The unconditional variance of the risk free rate can be shown to be:

$$\sigma^2(R_f) = \frac{\rho^2}{\psi^2} \sigma^2(a_t). \tag{B2}$$

Let $A = 1 + k_1 A_1 K$ and $B = \left(\dfrac{-\theta}{\psi} + (\theta - 1)A\right)$. Then unconditional mean risk free rate can be shown to be:

$$E\left(r_{f,t}\right) = -\ln(\delta) + \frac{\mu}{\psi} - \frac{(1-\theta)AF}{\theta}\left[\frac{-\theta}{\psi} + (\theta - 0.5)A\right] - \frac{0.5FB^2}{\theta}. \qquad \text{(B3)}$$

Let $C = k_{1,m}A_{1,m}K + \phi$. Then unconditional mean risk premium on the market portfolio can be shown to be:

$$E\left(r_{m,t+1} - r_{f,t}\right) = -BCKF - k_{1,m}A_{1,m}KB\sigma^2 - 0.5C^2\rho^2\overline{P} - 0.5C^2\varphi_e^2\sigma^2$$
$$-0.5\left(k_{1,m}A_{1,m}K\right)^2\sigma^2 - 0.5\varphi_d^2\sigma^2 \qquad \text{(B4)}$$

Let $D_0 \equiv -\left[A_{1,m}\left(k_{1,m}\rho - 1\right) + \phi\rho\right]\mu$, $D_1 \equiv C\rho$, $D_2 \equiv -\left[k_{1,m}A_{1,m}K\right]\rho$, $D_3 \equiv C$, and $D_4 \equiv k_{1,m}A_{1,m}K$. Then, unconditional variance of return on the market portfolio can be shown to be:

$$\sigma^2\left(r_{m,t+1}\right) = D_1^2\sigma^2\left(x_t\right) + D_2^2\sigma^2\left(a_t\right) + 2D_1D_2\,\mathrm{covar}\left(x_t, a_t\right) + D_3^2\varphi_e^2\sigma^2 + D_4^2\sigma^2 + \varphi_d^2\sigma^2.$$

$$\text{(B5)}$$

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Table 1. Calibrated Parameter Values

Preference Parameters	
δ	0.999
γ	10
ψ	1.5
Parameters of Stochastic Process for Consumption	
μ	0.0015
ρ	0.979
σ	0.0078
φ_e	0.044
Parameters of Stochastic Process for Dividends	
ϕ	3
φ_d	4.5

This table reports calibrated parameter values for the consumption and dividend growth rates processes g_t and $g_{d,t}$, respectively. In the complete information case, the stochastic process is written as:

$$g_{t+1} = x_t + \sigma \eta_{t+1}$$
$$x_{t+1} - \mu = \rho(x_t - \mu) + \varphi_e \sigma e_{t+1}$$
$$g_{d,t+1} = \phi x_t + \varphi_d \sigma u_{t+1}$$
$$\eta_{t+1}, \; e_{t+1}, \; u_{t+1} \sim \text{iid } N(0,1)$$

with the three shocks η_t, e_t, and u_t being mutually independent. These parameter values are identical to those used in Bansal and Yaron (2004). In the incomplete information case, the equations for g_{t+1} and $g_{d,t+1}$ have x_{t+1} rather than x_t appearing on the r.h.s.

Table 2. Asset Pricing Implications

			$E(R_m - R_f)$	$E(R_f)$	$\sigma(R_m)$	$\sigma(R_f)$	$\sigma(p-d)$
Data		Estimate	6.33	0.86	19.42	0.97	0.29
		SE	2.15	0.42	3.07	0.28	0.04
Complete Information	γ	ψ					
	7.5	0.5	0.69	4.79	13.43	1.17	0.08
	7.5	1.5	3.02	1.66	17.82	0.39	0.18
	10	0.5	1.42	4.85	13.43	1.17	0.08
	10	1.5	4.71	1.40	17.82	0.39	0.18
Incomplete Information							
	7.5	0.5	-2.05	3.83	12.68	0.73	0.05
	7.5	1.5	-0.17	1.23	12.37	0.24	0.02
	10	0.5	-2.53	3.46	12.68	0.73	0.05
	10	1.5	0.04	0.81	12.37	0.24	0.02

The table reports unconditional moments implied by the long run risks model, with complete and incomplete informational assumptions. Moments for the equity premium $(R_m - R_f)$ and the risk free rate (R_f) are expressed in percent per annum. Model-implied moments are reported for a range of values for the risk aversion parameter γ and the intertemporal elasticity of substitution (IES) ψ as in Bansal and Yaron (2004). Estimates using data reported in the first row are reproduced from Table IV of Bansal and Yaron (2004).