Socially Beneficial Mergers: A New Class of Concentration Indices*

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Abstract

The most prominent industry concentration index, the Herfindahl-Hirschman index (HHI), yields a higher concentration level in response to any merger between firms, thus implying that any merger will decrease the social welfare. Although HHI is the index used by the Anti-trust Division of the U.S. Department of Justice, its merger implications are not fully embraced by the anti-trust authorities; in practice, the Anti-trust Division allows many mergers - especially among

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smaller firms - which clearly cannot be justified on the basis of HHI. We propose a class of concentration indices that is in line with the spirit of the Anti-trust Division's merger policies and consider different theoretical models which indicate that the Anti-trust Division is justified in allowing such mergers, as they counter the market power of dominant firms.

1 Introduction

In 1982, the Department of Justice replaced the standard four firm concentration index ($C4$) with the Herfindahl-Hirschman index (HHI) as its primary tool in scrutinizing potential mergers.[1] Whereas $C4$ adds up the market shares of the top four firms to calculate industry concentration, HHI is more complete and elaborate in that it uses a weighted average of market shares of all firms. Nevertheless, HHI yields a higher concentration level in response to *any* merger between firms, thus implying that any merger will decrease the social welfare. Realizing this shortcoming of HHI, the Department of Justice (DoJ) has continuously adjusted its merger criteria to allow for mergers that are deemed to increase efficiency without causing a significant reduction in competition.[2]

[1] The Anti-trust Division of the merger policy stands out as the most active area of the U.S. anti-trust policy. Between 1994 and 2003, the U.S. Department of Justice conducted about 160 merger investigations annually, while conducting about 130 other investigations such as actions against price-fixing cartels and against firms engaging in anti-competitive practices; e.g., the case against Microsoft (Anti-trust Division Workload Statistics 1994-2003).

[2] The Anti-trust Division of DoJ allows some mergers as long as the HHI index stays under .18 after the merger *and* the merger does not cause an increase of more than .01. Further, mergers involving any financially troubled firms (failing firms) are treated more leniently. Thus, the merger implications of HHI are not fully embraced by the anti-trust authorities. The 1992 Merger Guidelines of the Anti-trust Division in a sense stress this fact by stating that "[w]hile challenging competitively harmful mergers, the

No better example of DoJ's departure from HHI can be found than its approval of the October 2001 merger of Chevron and Texaco which followed the footsteps of the British Petroleum/Amoco/Arco and Exxon/Mobil mergers, none of which involves any small firms. Many industry practitioners such as the former CEO of British Petroleum, John Cross, supported the approval of this merger by DoJ. Cross, referring to the Chevron and Texaco as "mid-size" firms, added that "they were not big enough to take on the big guys" (Worthen, 2002). That is, in his estimation, the merger was necessary for the survival of the firms and was thus good for competition.[3]

The basic idea behind DoJ's departure from HHI involves the trade-off between reducing the number of firms in an industry versus increasing efficiency and/or market share symmetry in the market. Theoretical results and empirical findings in the industrial organization literature almost invariably support DoJ's departure from HHI.[4] For example, Levin's (1990) extension

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Agency seeks to avoid unnecessary interference with the larger universe of mergers that are either competitively beneficial or neutral" (Anti-trust Division (1992, p. 3)). One of the additional reasons for the half-hearted use of HHI by the anti-trust authorities seems to be that, especially in a contestable market or in one that approaches contestability, HHI has even less significance. Another seems to be that, restrictions on mergers can adversely affect entry by reducing subsequent liquidity that the disposition of assets via sale through merger offers. (We thank Michael Gort for raising this point.)

[3]Interestingly, this merger was deemed necessary by industry experts as early as by March 1999. Then, Fadel Gheit, an oil analyst with Fahnestock & Co said: "BP's purchase of Amoco and Arco and Exxon's takeover of Mobil, all within the last eight months, sends a very clear wake-up call to the unattached that you have to act quickly or it's going to be too late. In my view, Texaco will be the first to go, then Chevron soon after, unless they merge as a single company" (BBC News, 1999).

Equally interestingly, Lopez (2000) states that "the deal has already received support from the Clinton administration despite ... the concentration of the oil industry among a few major players. US Energy Secretary Bill Richardson told reporters last week: 'My initial view is positive ... I think this is an inevitable result of the global economy'. A former director of the FTC's Bureau of Competition, William Baer, explained that the Chevron-Texaco deal would come under scrutiny, but that in order to compete against their bigger rivals they were left with no other option."

[4]In a notable theoretical exception, Daughety (1990) presents a model where asymmetry can lead to welfare increases. However, unlike the model that we pursue in Section

of the Salant, Switzer, and Reynolds' (1983) Cournot model shows that mergers of firms with less than 50% of the total market share enhance welfare even if the merged firms do not continue to engage in Cournot competition. Likewise, Akgun (2004) uses a model initially developed by McAfee and Williams (1992) to show that if competition entails neither a Bertrand or Cournot setting, but rather a hybrid of them in which firms compete based on supply schedules, then symmetric markets are socially optimal. Hence, although a merger may reduce the number of competitors, welfare may increase if the merger results in a more symmetric industry. Gugler et al. (2003) provide empirical support for these models by presenting strong evidence that among mergers that increase profits, those involving larger firms achieve these profits by increasing their market power, while mergers involving smaller firms achieve higher profits by increasing efficiency.

Given that HHI is clearly no longer in line with DoJ's goals, we propose an alternative class of indices to measure an industry's concentration. Specifically, our class of indices yields a higher concentration level when mergers involve the largest firm(s), but a lower concentration level when mergers occur among smaller firms.

We also consider theoretical models that illustrate the potential benefits of mergers among smaller firms and compares their predictions to the welfare implications of our index and HHI. In particular, we first consider a model in which one large "national" firm competes in separated markets against regional firms. Not surprisingly it turns out that, although mergers between regional firms will decrease the number of firms in the industry, overall wel-

3 below, his notion of asymmetry is based on the classification of firms as leaders and followers rather than on market share asymmetries caused by the firms' production costs.

fare is enhanced if the mergers generate synergies. Particularly appealing, however, is the fact that these mergers are welfare enhancing under diverse forms of competition (Cournot and Bertrand). In addition, we examine simulations based on Akgun's (2004) model of supply-schedule competition and provide the welfare implications of mergers that do not involve the largest firm.

The paper is organized as follows. Section 2 introduces the proposed class of indices and analyzes its features especially concerning various merger possibilities. Section 3 provides the theoretical models and simulations regarding socially beneficial mergers. Section 4 concludes.

2 Concentration Indices

Let the market shares of n firms be listed as $v_1 \geq v_2 \geq ... \geq v_n > 0$. As mentioned above, the most prominent concentration index in the industrial organization literature is the Herfindahl-Hirschman index (HHI): $HHI = (a_1 v_1 + ... + a_n v_n)$, where $a_i = v_i$. Observe that the weights, $a_1, ..., a_n$, sum to one. The other notable concentration index, the four-firm concentration ratio, $C4$, does not depend on the market shares of firms which are not the largest four firms. Neither does it assign different weights to different market shares of the firms.

As alluded to in a footnote above, the Anti-trust Division of DoJ uses HHI mainly in the following way: If HHI increases by less than .01 as a result of a horizontal merger *and* if the industry's HHI measure does not exceed .18 after the merger, a horizontal merger is allowed. It is likely to be challenged otherwise.

There are two other notable concentration indices. The one proposed by Hall and Tideman. (1967) stresses the need to include the number of the firms in the calculation when measuring the concentration level of an industry (the number of firms measures the ease of entry into that particular industry). Let n firms be ranked such that the firm with largest market share v_1 is Firm 1, the firm with next largest market share v_2 is Firm 2 and so on. Let i denote each firm's rank. The Hall-Tideman concentration index HTI is $\frac{1}{(2\sum_{i=1}^{n} iv_i)-1}$. Like HHI, HTI also increases in any mergers.

Another notable concentration index is an index of entropy, $E = -\sum_{i=1}^{n} v_i \log v_i$ (see Hart (1967, p. 78) for a discussion). Unlike the other indices considered so far, it does not have a range of 0 to 1; it takes the value 0 when the market structure is a monopoly and takes a value far exceeding 1 when the market structure is perfect competition. Any merger increases the industry concentration according to this index too.

Before introducing our index, we will first consider a property proposed by Hall and Tideman (1967) (Property 5, p. 164), and then a stronger version of it.

- *Weak Symmetry* ($WSym$): Suppose there are n identical-sized firms. Then, the industry concentration decreases in n for any index I.

- *Strong Symmetry* ($SSym$): Suppose there are n identical-sized firms. Then, for any index, $I = \frac{1}{n}$.

No doubt, a symmetric market with n firms is socially more beneficial than a symmetric market with n' firms, such that $n > n' \geq 1$. The above symmetry properties capture this idea at differing degrees. Observe that

$HHI(\frac{1}{n}, ..., \frac{1}{n}) = n(\frac{1}{n^2}) = \frac{1}{n}$. Clearly, $HHI(\frac{1}{n}, ..., \frac{1}{n})$ decreases in n. Note also that HTI satisfies both $SSym$ and $WSym$ while E violates $SSym$, yet satisfies $WSym$ in its own way (recall that the value of E increases as n increases). Likewise, $C4$ does not satisfy $SSym$ but satisfies $WSym$.[5]

Our proposed class of indices has different concentration implications than these other indices when horizontal mergers do not include the largest firm(s). Consider each Firm i's market share relative to that of the largest firm, $\frac{v_i}{v_1}$. Then the total of number of firms in that industry, relative to the largest firm's market share, becomes $\frac{v_1}{v_1} + \frac{v_2}{v_1} + ... + \frac{v_n}{v_1}$. We first consider a concentration index where market shares of firms are measured in terms of the largest firm's market share.

$$\begin{aligned} I^*(1) &= \frac{1}{(\frac{v_1}{v_1})^2 + (\frac{v_2}{v_1})^2 + ... + (\frac{v_n}{v_1})^2} \\ &= \frac{1}{[(v_1)^2 + (v_2)^2 + ... + (v_n)^2]\frac{1}{(v_1)^2}} \\ &= \frac{(v_1)^2}{(v_1)^2 + (v_2)^2 + ... + (v_n)^2}. \end{aligned}$$

Observe that, $I^*(1) = \frac{(v_1)^2}{HHI}$. Further, observe that $I^*(1)(\frac{1}{n}, ..., \frac{1}{n}) = \frac{1}{n}$. Thus,

[5]Dansby and Willig (1979) studied industry performance indices that measure the potential social gains from appropriate government interventions (such as anti-trust, regulatory, and deregulatory actions). Their performance indices establish welfare theoretic basis for indices such as $C4$, HHI, and so on. Essentially, Dansby-Willig versions of these indices incorporate a weight assigned to them. This weight is the inverse of the price elasticity of the industry demand. Note that Dansby-Willig versions of these indices do not satisfy the Strong Symmetry property but do satisfy the Weak Symmetry property. Some indices (such as the one by Blackorby, Donaldson and Weymark (1982) who use a Cobb-Douglas functional form) assign weights to not only on firms' market shares but also on the total output. Not surprisingly, such indices too fail to satisfy the Strong Symmetry property but typically do satisfy the Weak Symmetry property.

$I^*(1)$ satisfies $SSym$ (and thus $WSym$).

Table 1 provides a few examples to illustrate the stark differences between HHI and $I^*(1)$.

Table 1: A comparison of HHI and I^* Under Different Market Structures

Market	Shares						HHI	$I^*(1)$
70	5	5	5	5	5	5	.50	.97
70	30						.58	.84
51	26	11	11	1			.35	.74
51	37	11	1				.41	.64
40	37	11	11	1			.32	.5
40	37	9	9	5			.32	.51
33	33	33					.33	.33
50	50						.5	.5
55	45						.52	.6
55	35	10					.44	.7
55	25	20					.41	.75
40	40	20					.36	.44
40	40	10	10				.34	.47
40	20	20	20				.28	.57

$I^*(1)$ measures the industry concentration when there is a single dominant firm. Although there may be industries in which increasing the market share of the second largest firm may cause a reduction in the industry price, in many industries a reduction in price cannot be achieved until a higher critical number of large firms is reached. Lamm (1981, p. 75) reports empirical findings from the food retailing industry that in many urban markets "growth in the 3 largest firms' shares have a significant positive effects on prices ... In contrast, an increase in the market share of the fourth largest firm causes a reduction in food prices." This clearly indicates that the number of dominant firms in a market may be greater than one. This prompts us to consider the m-dominant firms version of $I^*(1)$, where $1 < m < n$.

$$\begin{aligned} I^*(m) &= \frac{1}{\frac{v_1^2}{v_1^2+...+v_m^2} + \frac{v_2^2}{v_1^2+...+v_m^2} + ... + \frac{v_n^2}{v_1^2+...+v_m^2}} \\ &= \frac{1}{[(v_1)^2 + (v_2)^2 + ... + (v_n)^2]\frac{1}{v_1^2+...+v_m^2}} \\ &= \frac{v_1^2 + ... + v_m^2}{(v_1)^2 + (v_2)^2 + ... + (v_n)^2}. \end{aligned}$$

Observe that, when $m > 1, I^*(m) = \frac{v_1^2+...v_m^2}{HHI}$. Further, observe that $I^*(m)(\frac{1}{n}, ..., \frac{1}{n}) = \frac{m}{n}$. Thus, $I^*(m)$ does not satisfy $SSym$ but satisfies $WSym$ instead. The following proposition describes how $I^*(m)$ behaves in mergers that do and do not involve the largest firm.

Proposition 1 *(1) Suppose a merger does not involve m largest firms and does not make the new firm one of the m largest firms. Then $I^*(m)$ decreases. (2) Suppose a merger involves one of the m largest firms. Then $I^*(m)$ increases.*

Proof: (1) Since $[(v_1)^2 + (v_2)^2 + ... + (v_n)^2]$ increases in any merger, $I^*(m) = \frac{v_1^2+...+v_m^2}{(v_1)^2+(v_2)^2+...+(v_n)^2}$ must decrease in any merger that does not involve any of $v_1, v_2, ..., v_m$. (2) Consider $I^*(m) = \frac{v_1^2+...+v_m^2}{(v_1)^2+...+(v_m)^2+...+(v_n)^2}$. Suppose Firm i and Firm j merge such $i \leq m$ and $j > m$. Thus, after the merger $I^{*\prime}(m) = \frac{v_1^2+...+v_{i-1}^2+v_{i+1}^2+...v_m^2+(v_i+v_j)^2}{v_1^2+...+v_{i-1}^2+v_{i+1}^2+...+v_m^2+(v_i+v_j)^2+...+(v_{j-1})^2+(v_{j+1})^2+...+(v_n)^2}$. Let $A = (v_1)^2 + ... +$

$(v_m)^2$ and $B = (v_1)^2 + ... + (v_m)^2 + ... + (v_n)^2$; hence, $B > A$. Thus, $I^*(m)$ becomes $\frac{A}{B}$ and $I^{*\prime}(m)$ becomes $\frac{A+(v_i+v_j)^2-v_i^2}{B+(v_i+v_j)^2-v_i^2-v_j^2}$.

Then $I^{*\prime}(m) \gtreqless I^*(m)$ reduces to $Bv_j^2 + Bv_iv_j \gtreqless A2v_iv_j$. Since $B > A$, we obtain $I^{*\prime}(m) > I^*(m)$. ■

The next proposition describes how much $I^*(m)$ increases when a Firm j merges with Firm i instead of with Firm i' where $v_i > v_{i'} > v_m > v_j$.

Proposition 2 *Consider a merger M between Firm i and Firm j, and a merger M' between Firm i' and Firm j, where $v_i > v_{i'} > v_m > v_j$. Then $I^*(m, M) > I^*(m, M')$.*

Proof: $I^*(m, M) = \frac{v_1^2+...+v_{i-1}^2+v_{i+1}^2+...v_m^2+(v_i+v_j)^2}{v_1^2+...+v_{i-1}^2+v_{i+1}^2+...+v_m^2+(v_i+v_j)^2+...+(v_{j-1})^2+(v_{j+1})^2+...+(v_n)^2}$ and

$I^*(m, M') = \frac{v_1^2+...+v_{i'-1}^2+v_{i'+1}^2+...v_m^2+(v_{i'}+v_j)^2}{v_1^2+...+v_{i'-1}^2+v_{i'+1}^2+...+v_m^2+(v_{i'}+v_j)^2+...+(v_{j-1})^2+(v_{j+1})^2+...+(v_n)^2}$.

Let $A = (v_1)^2 + ... + (v_m)^2$ and $B = (v_1)^2 + ... + (v_m)^2 + ... + (v_n)^2$. Thus, $I^*(m, M)$ becomes $\frac{A+(v_i+v_j)^2-v_i^2}{B+(v_i+v_j)^2-v_i^2-v_j^2}$ and $I^*(m, M')$ becomes $\frac{A+(v_{i'}+v_j)^2-v_{i'}^2}{B+(v_{i'}+v_j)^2-v_{i'}^2-v_j^2}$. Then $I^*(m, M) \gtreqless I^*(m, M')$ reduces to $\frac{A+v_j^2+2v_iv_j}{B+2v_iv_j} \gtreqless \frac{A+v_j^2+2v_{i'}v_j}{B+2v_{i'}v_j}$. Since $B > A + v_j^2$ and $2v_iv_j > 2v_{i'}v_j$, we obtain $I^*(m, M) > I^*(m, M')$. ■

The implication of the preceding proposition is that, according to $I^*(m)$, a merger between a small firm and a relatively large dominant firm will increase the concentration in that industry more than a merger between the same small firm and a relatively small dominant firm will. The table below considers several different market settings in an attempt to gauge how HHI

and I^* respond to proposed mergers. Table 2 considers several different market settings and allows for various combinations of merging firms. In each row, merging firms' pre-merger market shares are denoted with a box around them. For instance, Rows 1-5 entail a situation in which there is one dominant firm and six identical smaller firms. All of the smaller firms merge in Row 1, five of the smaller firms merge in Row 2, and so on. The last two columns furnish the predicted changes in the two indices. Observe that HHI increases while I^* decreases for each merger.

Table 2: Changes in HHI and I^* Resulting from Mergers

Market	Shares						HHI	$I^*(1)$	Change in HHI	Change in $I^*(1)$
70	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	.50	.97	.08	-.13
70	5	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	.50	.97	.06	-.09
70	5	5	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	.50	.97	.04	-.06
70	5	5	5	$\boxed{5}$	$\boxed{5}$	$\boxed{5}$	.50	.97	.02	-.03
70	5	5	5	5	$\boxed{5}$	$\boxed{5}$	.50	.97	.01	-.01
70	$\boxed{10}$	$\boxed{10}$	$\boxed{10}$				.52	.94	.06	-.10
70	10	$\boxed{10}$	$\boxed{10}$				.52	.94	.02	-.03
51	$\boxed{25}$	$\boxed{12}$	$\boxed{12}$				.35	.74	.15	-.22
51	$\boxed{25}$	$\boxed{24}$					.38	.68	.12	-.16
60	$\boxed{20}$	$\boxed{10}$	$\boxed{10}$				.42	.86	.10	-.17
60	20	$\boxed{10}$	$\boxed{10}$				.42	.86	.02	-.04
60	$\boxed{20}$	$\boxed{20}$					.44	.82	.08	-.13

3 Theoretical Examples

While not technically demanding, the following theoretical models provide support (in addition to that already found in the literature) for the idea that mergers not involving the largest firm need not decrease the social welfare. The first two models consider Cournot and Bertrand competition between one dominant firm and several smaller rival firms in separate markets. The basic idea is that when smaller firms merge across regions, not only is it profitable for them, but it also increases welfare in the market. This is not surprising since costs are reduced in each market and the mergers do not reduce the number of firms competing in each market; yet it provides a plausible example that will serve as a benchmark when comparing our index to HHI. The third theoretical setup considered is that of Akgun (2004) where the equilibrium is in supply schedules.

3.1 Localized Cournot Competition

Let there be Z identical locations. A national firm N competes at each of these locations against a local firm L.[6] At each location, the inverse demand is given by $P(Q) = a - bQ$ where $a, b > 0$ and $Q_i = q_N + q_L$ with q_j denoting firm j's output $(i = 1, ..., Z)$. Let the cost function of firm j be $C_j(q_j) = c_j q_j$ where $c_j = \frac{1}{k_j}$ with $k_j > 0$ being the overhead of firm j. The standard asymmetric Cournot equilibrium gives firm j profits of

$$\frac{1}{9b} \left[a + c_i - 2c_j\right] \left[a - 2c_i + c_j\right]$$

yielding equilibrium consumer surplus of

$$\frac{1}{9b} \left(2a - c_i - c_j^2\right)^2,$$

[6]We assume that there is no monopoly involved in any market either pre- or post-merger.

resulting in total welfare in the pre-merger market of

$$\frac{1}{3b}\left[2a^2 - 2ac_N - c_N^2\right] + \frac{1}{3b}\left[4c_N c_L - 2ac_L - c_L^2\right]. \tag{1}$$

Let m of the local firms merge. The post-merger cost function of this newly merged firm becomes $C_M(q_M) = c_M q_M$ where $c_M = \frac{1}{\sum_{L=1}^{m} k_L}$. Clearly, welfare in markets with non-merging firms does not change. However, welfare in markets with newly merged firms becomes

$$\frac{1}{3b}\left[2a^2 - 2ac_N - c_N^2\right] + \frac{1}{3b}\left[4c_N c_M - 2ac_M - c_M^2\right] \tag{2}$$

where c_M denotes the new, lower marginal cost of the merged firm. Summing Eqs. (1) and (2) over the M locations involved in the merger, and subtracting Eq. (1) from Eq. (2) gives a welfare change of

$$\frac{m}{3b}\left[4c_N c_M - 2ac_M - c_M^2\right] - \sum_{L=1}^{m} \frac{1}{3b}\left[4c_N c_L - 2ac_L - c_L^2\right]. \tag{3}$$

Due to the assumption that merging firms create synergies by combining capital stocks, Eq. (3) is positive. Hence, any merger that does not result in a monopoly situation will increase welfare. It should, however, be noted that we have tilted the scales in favor of mergers since mergers do not result in fewer firms in any market. While this is true, it points to the inadequacy of HHI as a guideline for mergers. Figure 1 examines the relationship between welfare changes, I^*, and HHI. The horizontal axis measures the size of the national firm in term of its capital stock, k_N. The starting point is one where all firms are of identical size and the national firm grows larger than the regional firms as we move along the x-axis.

Figure 1: Localized Cournot Competition

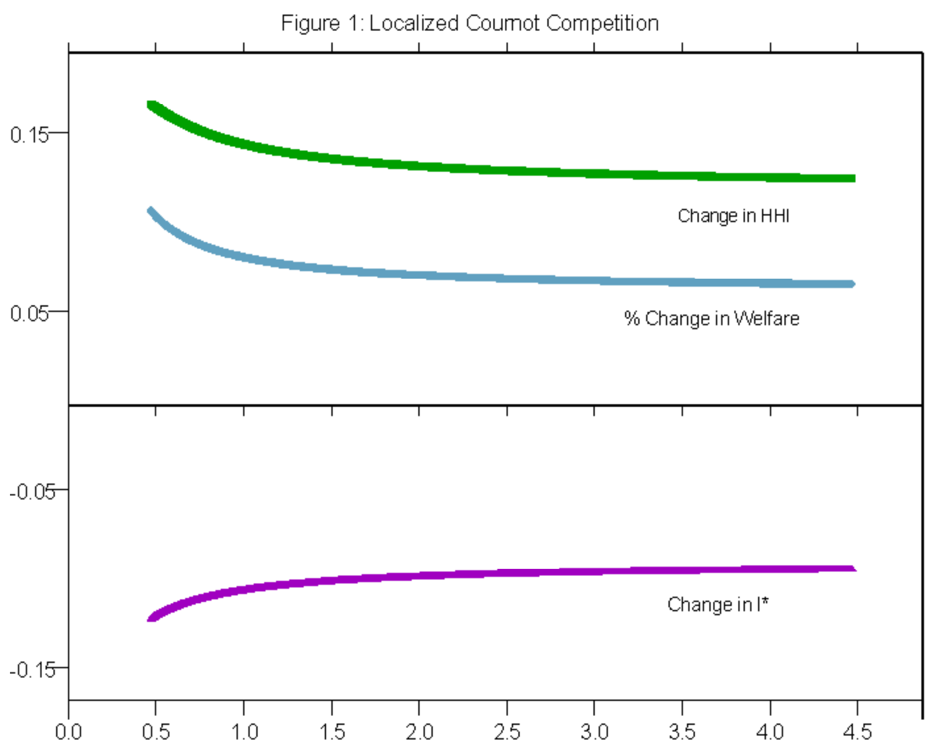


Figure 1 shows that all mergers result in an increase in welfare. This is not surprising since costs go down but the number of firms in each region stays the same. Notice that the change in I^* is always negative, indicating that the merger should be allowed and is thus in line with the welfare changes. The change in HHI, on the other hand, is always positive indicating that the merger, although welfare enhancing, should not be allowed. Clearly, I^* provides a better criterion for examining mergers within this framework than does HHI.

3.2 Localized Differentiated Bertrand Competition

As in the previous section, assume that there is a National firm, N, with $C_N(q_N) = c_N q_N = \frac{1}{k_N}$. Imagine that this firm is located at a hub with unit lines extending in Z directions. At the end of each unit line is a rival firm, L, with $C_L(q_L) = c_L q_L = \frac{1}{k_L}, L = 1, ..., Z$. Firm N engages in differentiated Bertrand competition with each of its Z rivals. Let the position of the hub firm along the unit line be zero and the location of the local firm be one. Consumers are located uniformly along each unit line. A consumer at location y receives a surplus of $u_0 - ty - p_N$ if they buy from the hub firm and $u_0 - t(1-y) - p_L$ if they buy from the local firm, where $t > 0$ represents the lack of desirability for a firm's service stemming from exogenous differentiation of the goods. By equalizing utilities, it follows that the central consumer is located at $\widetilde{y} = \frac{p_L - p_N + t}{2t}$.

In this standard Hotelling game, firms N and L have equilibrium profits of

$$\pi_N = \frac{1}{18t}(3t + c_L - c_N)^2 \tag{4}$$

$$\pi_L = \frac{1}{18t}(3t + c_N - c_L)^2, \tag{5}$$

consumer receive surplus of

$$\int_0^{\tilde{y}} [u_0 - ty - p_L]dy + \int_{\tilde{y}}^1 [u_0 - t(1-y) - p_N]dy,$$

where the equilibrium prices are

$$p_N = \frac{1}{3}(3t + c_L + 2c_N) \tag{6}$$

$$p_L = \frac{1}{3}(3t + c_N + 2c_L). \tag{7}$$

Thus, pre-merger welfare is

$$u_0 - \frac{t}{4} - \frac{(c_n + c_L)}{2} + \frac{5(c_L - c_N)^2}{36t}. \tag{8}$$

Post-merger welfare in a single market is

$$u_0 - \frac{t}{4} - \frac{(c_n + c_M)}{2} + \frac{5(c_M - c_N)^2}{36t}. \tag{9}$$

Taking the difference between post- and pre-merger welfare levels and summing over the m markets from which the merging firms originate gives a change in total welfare of

$$\left[\frac{c_L - c_M}{36t}\right][18t + 5c_L + 5c_M - 10c_N].$$

Under the assumption that mergers create synergies, the first term is positive and hence the sign of the welfare change is simply the sign of

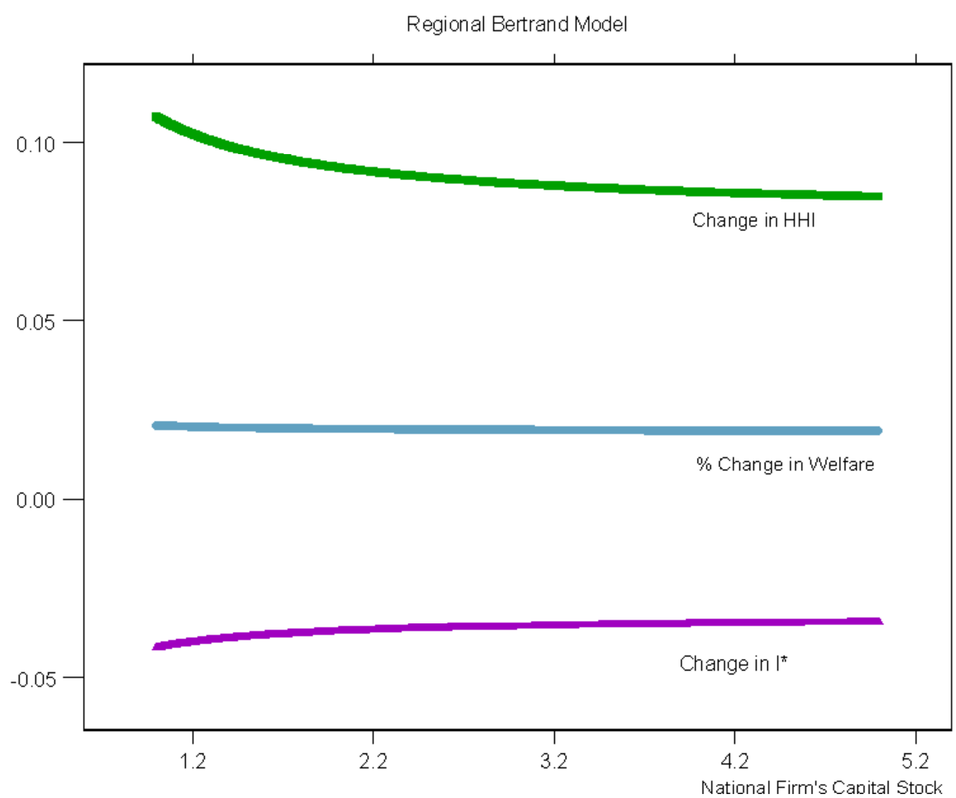
$$m(18t - 10c_N) - \sum_{L=1}^{m} 5(c_L + c_M). \tag{10}$$

Unlike the Cournot example, the welfare changes do not always favor mergers. If the merging firms are "too" small (high c, low k) then the convexity of the profit functions dictates that the gains in consumer surplus and profits of the merged firms are not large enough to offset the decrease in the national firm's profits. In that case, the second term will outweigh the first term in (10) and the change in welfare will be negative. However, in non-"extreme" cases, the first term will outweigh the second term producing a net increase in welfare after the merger.

Figure 2 examines the relationship between welfare changes, I^*, and HHI. The horizontal axis measures the size of the national firm in terms of its capital stock, k_N. The starting point is one where all firms are of identical size and the national firm grows larger than the regional firms as we move along the x-axis.

Notice that all mergers considered in this simulation result in an increase in welfare. As in the Cournot model, this is not surprising since costs go down but the number of firms in each region stays the same. Once again I^* is always negative, inferring that the merger should be allowed and is thus in line with the welfare changes. Also, the change in HHI is always positive indicating that the merger, although welfare enhancing, should not be allowed. Once again, I^* provides a better criterion for examining mergers within this framework than does HHI.

Figure 2: Localized Bertrand Competition



3.3 Competition in Supply Functions

The model considered here is from Akgun (2004) which follows from Klemperer and Meyer (1989). The model is a hybrid of Cournot and Bertrand settings in the sense that firms' equilibrium behavior is determined by strategically chosen supply functions with both price and quantity components. The basic assumptions of the model entail a linear demand curve $D(p) = a - bp$ and a cost function for each firm of $C_i(q_i) = \frac{q_i^2}{2k_i}$ where k_i is firm i's capital stock. Firms produce according to their submitted supply functions at the market clearing price p^* defined by $D(p^*) = \sum_{i=1}^{n} S_i(p^*)$.

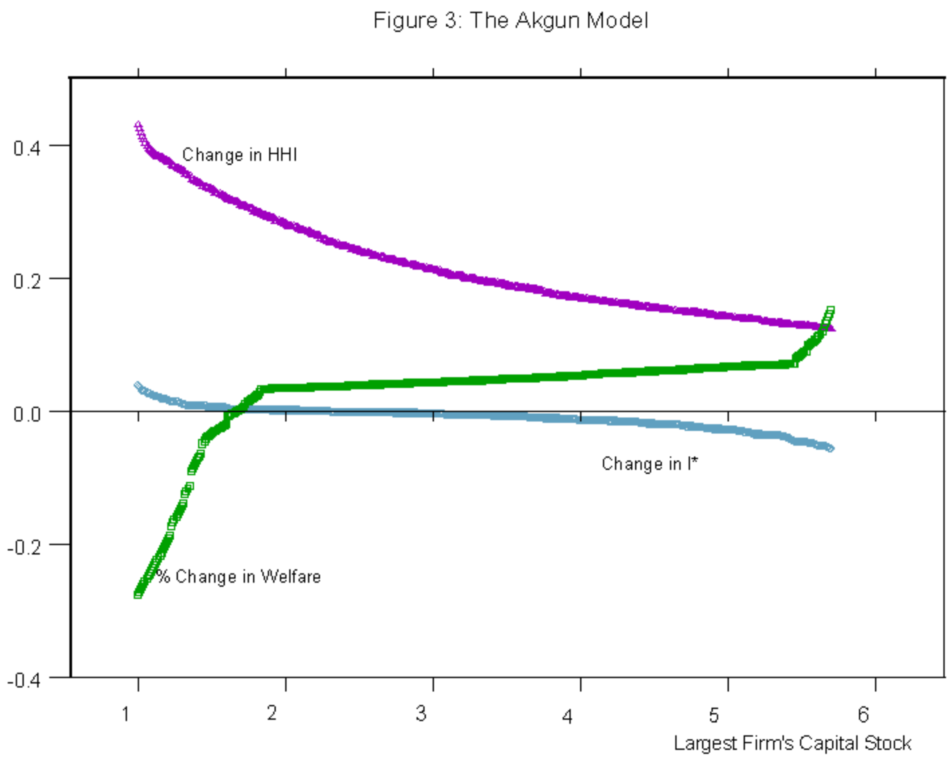
In equilibrium, each firm i submits a linear supply schedule with slope β_i such that

$$\beta_i = \frac{k_i(b + \beta_{-i})}{k_i + b + \beta_{-i}}, i = 1, ..., N \tag{11}$$

where β_{-i} is the sum of i's competitors' βs. Akgun shows that the equilibrium supply schedule submissions based on this system of equations can lead to welfare increasing mergers. Under his formulation any merger is more likely to increase welfare if it makes the firm size distribution more "symmetric" in the industry - the only decreasing effect on welfare comes from having fewer firms in the industry.

The primary difficulty with Akgun's model is that the system of equations defined by Eqs. (11) must be solved numerically in all but the most simple cases. In order to provide a link between the game in supply schedules and the prominent Cournot and Bertrand settings, he is forced to resort to numerical simulations. We too use simulations based on his setup. However, our simulations are aimed at measuring the welfare improvements generated

Figure 3: The Akgun Model



when small firms merge to compete against a dominant firm. As in previous parts of this section, we offer a graphical analysis of the relation between welfare changes, I^*, and HHI. Unlike, the Cournot and Bertrand localized competition settings, however, Akgun's setup generates some instances where welfare increases and others where welfare decreases as a result of a merger. The following graph examines the relationship between welfare changes, I^*, and HHI. The simulations start with six equally sized firms. The x-axis measures the growth of one of these firms in term of its capital stock, k.

In the beginning (the left-hand side of the graph) all firms are symmetric in size. Hence, a merger reduces the symmetry of the industry and results in a reduction in welfare as revealed by the change in welfare plot which is below zero. However, when the dominant firm in the industry is large enough (the right-hand side of the graph) a merger between two relatively small firms forces the dominant firm to be more competitive and thus the welfare begins to increase. In the beginning, both I^* and HHI increase with the prospect of a merger. However, as the size of the dominant firm increases, I^* begins to decrease, implying that the merger should be allowed, while the change in HHI remains positive no matter how large the dominant firm becomes. However, we must point out that there is a range where I^* increases along with HHI, yet welfare actually increases. That is, while I^*s merger implications are in line with welfare changes in the vast majority of the cases we have considered in this section, clearly its merger implications will not be flawless in all possible setups.

3.4 Additional Simulations

The above parts of this section presented examples and simulations where the size of one firm was allowed to grow, but all other variables were held constant. To provide a more comprehensive comparison of I^* and HHI, we now examine a number of different industry structures and report the changes in welfare predicted by each of the three models considered above. Table 2 presents results for mergers not involving the largest firm. The columns on the left-hand side of the table yield the market shares of the firms (e.g. the largest firm controls 70% of the market and there are six small firms, each controlling 5% of the market). A box around a small firm's market share indicates that the firm is part of the merger (e.g. all six small firms merge in the first case, only five of the small firms merge in the second case, and so on). The columns on the right-hand side of the table report the percentage increase in welfare under each of the three set-ups considered in this section.

Table 3: Simulations for Various Market Settings

Market	Shares						Cournot	Bertrand	Supply Functions
70	[5]	[5]	[5]	[5]	[5]	[5]	26.4%	24.6%	.20%
70	5	[5]	[5]	[5]	[5]	[5]	20.9%	19.6%	.41%
70	5	5	[5]	[5]	[5]	[5]	15.3%	14.7%	.48%
70	5	5	5	[5]	[5]	[5]	9.9%	9.7%	.47%
70	5	5	5	5	[5]	[5]	4.6%	4.8%	.41%
70	[10]	[10]	[10]				19.7%	19.4%	.52%
70	10	[10]	[10]				9.1%	9.6%	.43%
51	[25]	[12]	[12]				19.3%	28.7%	.19%
51	[25]	[24]					6.6%	3.7%	.05%
60	[20]	[10]	[10]				11.7%	17.5%	.23%
60	20	[10]	[10]				12.9%	16.3%	.25%
60	[20]	[20]					11.5%	13.2%	.15%

It is clear from Table 2 that all mergers considered result in increases in welfare. It is not surprising that the Cournot and Bertrand models predict a larger increase in welfare as those models do not require a reduction in the number of firms competing in a given market. Yet, Akgun's setup reveals that even when there is such a reduction in the number of firms as a result of a merger, welfare will still increase. Recall from Table 1 that in all these mergers HHI increases whereas $I*$ decreases. That is, $I*$'s merger implications are fully in line with the welfare increases in those cases whereas those of HHI are not.

4 Concluding Remarks

As alluded to in the Introduction, the Anti-trust Division of the U.S. Department of Justice does not fully embrace the merger implications of the concentration index (HHI) it has officially adopted. The class of indices proposed in this paper is substantially more in line with the Anti-trust Division's guidelines and its recent practices regarding merger attempts. The ideal market structure would consist of many symmetric firms. When the distribution of firm sizes is significantly asymmetric, such that some dominant firms have significantly greater market power than the rest of the firms, a merger between two smaller firms can be welfare enhancing. As our theoretical examples and simulation results reveal, the basic intuition behind such welfare increases of mergers between smaller firms is that although such mergers will decrease the number of firms, they will increase the symmetry of the industry thereby increasing welfare. Therefore, the Anti-trust Division is justified in allowing mergers especially among smaller firms to counter the market power of dominant firms. As such, the new class of concentration indices that we propose in this paper provides a much better fit with the Anti-trust Division's intentions and, compared to the existing concentration indices, it is much more relevant in assessing the welfare implications of potential mergers.

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