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On Business Cycle Asymmetries in G7 Countries

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Abstract

We investigate whether business cycle dynamics in seven industrialized countries (the G7) are characterized by asymmetries in conditional mean. We provide evidence on this issue using a variety of time series models. Our approach is fully parametric. Our testing strategy is robust to any conditional heteroskedasticity, outliers, and / or long memory that may be present.

Our results indicate fairly strong evidence of nonlinearities in the conditional mean dynamics of the GDP growth rates for Canada, Germany, Italy, Japan, and the US. For France and the UK, the conditional mean dynamics appear to be largely linear.

Our study shows that while the existence of conditional heteroskedasticity and long memory does not have much affect on testing for linearity in the conditional mean, accounting for outliers does reduce the evidence against linearity.

Key phrases: business cycles; asymmetries; nonlinearities; conditional heteroskedasticity; long memory; outliers; real GDP; stable distributions

JEL codes: B22, B23, C32, E32

1 Introduction

The possible existence of asymmetries in economic fluctuations is being tested extensively using aggregate macroeconomic data. While studies such as Neftci (1984), Brunner (1992, 1997), Beaudry and Koop (1993), Potter (1995), and Ramsey and Rothman (1996) conclude that there are significant asymmetries, others (Falk (1986), Sichel (1989), DeLong and Summers (1986), and Diebold and Rudebusch (1990)) have either failed to confirm these findings or have found only weak evidence supporting them.

Detecting any nonlinearities that may be present in business cycles is important for several reasons. Nonlinearities imply that the effects of expansionary and contractionary monetary policy shocks on output are not symmetric. Any nonlinearities would invalidate measures of the persistence of monetary policy and other shocks on GNP that are based on linear models, including those derived from vector autoregressions (VARs). Nonlinearities would necessitate that in order to validate theories of business cycles, such as the real business cycle (RBC) theories, one would need to go beyond merely matching the first and second moments of data with the moments implied by the theories in question.

Granger (1995) recommends testing for linearity using heteroskedasticity-robust tests. French and Sichel (1993) and Brunner (1992, 1997) show the existence of conditional heteroskedasticity in real GNP data. Scheinkman and LeBaron (1989) report a weakening of evidence against linearity after accounting for conditional heteroskedasticity in this series.

There is a growing perception that the evidence of nonlinearity reported in several studies so far may be due to the presence of outliers. Tsay (1988) demonstrates that linearity could be rejected by the presence of outliers. Blanchard and Watson (1986) demonstrate the existence of outliers in GNP data. Balke and Fomby (1994) and Scheinkman and LeBaron (1989) report weakened evidence against linearity in US real GNP data once outliers are taken into account.

Several macroeconomic time series data have been characterized as fractionally integrated processes in a number of studies (Sowell, 1992b). While investigating the possible existence of asymmetries in economic fluctuations it is important to use time series models that describe both the long and short run properties of the data accurately.

Most of the studies on business cycles test for asymmetries without taking into account conditional heteroskedasticity, outliers, and / or long memory. An exception is Bidarkota (2000). This study finds robust evidence of non-linearities in the conditional mean dynamics of the chain-weighted quarterly US GNP growth rates.

The present study seeks to extend the analysis in Bidarkota (2000) to an examination of several developed countries. There are compelling reasons why such an extension is a worthwhile undertaking. For instance, it is of interest to know whether business cycles in different countries are all alike. If they are, then this provides a serious challenge to macroeconomic theorists to develop theories of business cycles that can explain fluctuations in economic activity in different countries without relying on country specific institutional features. Documenting business cycle characteristics in different countries is a first step in this task.

Also, in understanding spillover and contagion effects in international business cycles, macroeconometricians traditionally rely on linear vector autoregressions (VARs). These are convenient, simple to work with, well understood, and widely used in the literature. However, there are some new studies that attempt to build nonlinear multivariate models for studying business cycle linkages across countries (Anderson and Vahid (1998), Anderson and Ramsey (2002)). Since multivariate nonlinear modeling is complex, it is important that we document compelling evidence against linearity in international data first before venturing to build these more complicated models.

This study is organized as follows. Section 2 outlines the various empirical models that are used in our investigations. Section 3 discusses some important issues related to the estimation of the models. Section 4 provides details on the data sources and reports briefly on specification search. Statistical tests for asymmetries and other hypotheses of interest are reported in detail in Section 5. Important conclusions that can be drawn from the results in the study are summarized in Section 6.

2 Non-Linear Time Series Models

In this study we use three classes of models to detect the possible presence of asymmetries in real output series for the countries under investigation. These are termed CDR-Augmented, CDR-Switching, and SETAR-Switching models. Within each of these three classes of models, we entertain four different versions of the models. Model 1 incorporates stable distributions, conditional heteroskedasticity, and fractional differencing. Imposing no fractional differencing (i.e., only integer differencing) on Model 1 yields Model 2. Imposing homoskedasticity on Model 2 gives Model 3. Finally, restricting the errors in Model 3 to come from Gaussian distributions gives Model 4. Model 1 is the most general and it nests all other Models 2 through 4.

Each of the three classes of models listed above is described in detail in the following three sub-sections.

2.1 CDR Augmented Models

Beaudry and Koop (1993) estimated a standard autoregressive moving average (ARMA) model, augmented with an ad hoc nonlinear term for capturing asymmetries. This nonlinear term labeled the current depth of a recession measures the gap between the current level of output and the economy's historical maximum level.

Bidarkota (1999, 2000) extended this model by incorporating stable distributions, conditional heteroskedasticity, and long memory. In this study here we use this model.

In this class of models the most general model (Model 1) can be described as follows:

$$\Phi(L)(1-L)^{d}(\Delta y_t - \mu) = [\Omega(L)-1]CDR_t + \varepsilon_t \qquad (1a)$$

$$\varepsilon_t \mid I_{t-1} \sim z_t c_t, \qquad z_t \sim \text{i.i.d.} S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 \mid \varepsilon_{t-1} \mid^\alpha \tag{1b}$$

Here, $\Delta y_t \equiv 100 * \Delta(\ln \text{GDP}_t)$ is the growth rate of GDP, its unconditional mean is μ, d is the differencing parameter that takes on real values, $\Omega(.)$ and $\Phi(.)$ are polynomials of orders r and p respectively in the lag operator L, with $\Omega(0) = \Phi(0) = 1$. The term CDR_t is the current depth of recession that permits recessions to be less or more persistent than expansions depending on the parameter estimates. It is defined as $\text{CDR}_t = \max\{y_{t-j}\}_{j\geq 0} - y_t$.

A random variable X is said to have a symmetric stable distribution $S_\alpha(\delta, c)$ if its log characteristic function can be expressed as $\ln E \exp(iXt) = i\delta t - \mid ct \mid^\alpha$. $\delta \in [-\infty, \infty]$ is the location parameter that shifts the distribution either to the left or the right along the real line, $c \in [0, \infty]$ is the scale parameter that expands or contracts the distribution about δ, and $\alpha \in [0,2]$ is the characteristic exponent governing tail behavior. Smaller values of this exponent indicate thicker tails. When $\alpha = 2$ we obtain the normal distribution.

Equation 1b shows the evolution of the scales of the conditional distribution. When we set α to 2 in the model, we obtain a normal GARCH (1,1) process for the conditional variance in the volatility specification. Our Model 1b is analogous to the power ARCH model introduced by Ding, Granger, and Engle (1993), with the exception that in the latter specification the distribution of z_t does not depend on the characteristic exponent.

α. Dependence on α emerges here naturally because we are allowing for disturbances to be drawn from the stable family. Liu and Brorsen (1995) also modeled volatility of the daily foreign currency returns using stable errors. Similarly, McCulloch (1985) fitted a GARCH-stable model to bond returns using absolute values instead of α powers.

When $d = 0$ we get a unit root in y_t, but with $d = -1$ we end up with y_t being integrated of order zero $I(0)$. ARFIMA models with long memory are defined in terms of the rate of decay of their autocovariances, so the extension of these models to infinite variance stable shocks is not immediate. Kokoszka and Taqqu (1995) contributed significantly to the theory of fractionally differenced ARMA models with infinite-variance stable innovations.

According to Brockwell and Davis (1991), a stationary casual and invertible solution to an ARFIMA model with Gaussian errors requires that $|d| < 0.5$. Kokoszka and Taqqu (1995) showed that the existence of a unique casual MA (∞) representation to an ARFIMA model with stable shocks requires $\alpha(d-1) < -1$. This implies then that d be positive when $\alpha > 1$. Moreover, for the ARFIMA model to be a solution to an AR(∞) process requires that $\alpha > 1$ and $|d| < (1 - 1/\alpha)$. In order to force our estimated models to possess casual and invertible representations, we restrict α and d in Equation 1 to satisfy these constraints.

Although we have an ad hoc non-linear CDR_t term within an otherwise standard AR framework with fractional differencing, this model is simple and parsimonious. When

$\Omega(L) = 1$, Equation 1a reduces to an autoregressive (AR) model with non-integer differencing. Since it nests AR models, we can use the standard *t*-statistic or the likelihood ratio (LR) statistic to test the statistical significance of the non-linear term governing the conditional mean dynamics.[1] With $\Omega(L) \equiv 1 + \omega_1 L + \omega_2 L^2 + ... + \omega_r L^r$, when the autoregressive lag order p is 0 and r is 1, $\omega_1 = 0$ yields a random walk with drift. However, a positive ω_1 implies that negative shocks are less persistent whereas a negative ω_1 implies that positive shocks are less persistent.

The existence of asymmetries essentially means that either the innovations are asymmetric but the impulse transmission mechanism is linear, or that the innovations are symmetric but the impulse transmission mechanism is nonlinear, or that the innovations are asymmetric and the impulse transmission mechanism is nonlinear. However, it would be hard to disentangle the asymmetric innovations from the nonlinear propagation mechanism, if they both exist in a data series.

Although asymmetric α-stable distributions exist and are well defined, to determine whether asymmetries in the conditional mean dynamics of the real GDP growth rates are caused by asymmetric impulses being propagated linearly or symmetric impulses being propagated nonlinearly or asymmetric impulses being propagated nonlinearly is beyond

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[1] However, the asymptotic distribution of the *t*-test for the significance of the non-linear CDR_t term in the model given by Equation (1a) is non standard (Hess and Iwata, 1997), both when the dependent variable is non-stationary [i.e. integrated of order one I(1)], and when it is stationary [I(0)].

the scope of this study. Here, we are merely investigating whether asymmetries exist in the conditional mean regardless of how they can best be characterized.

2.2 CDR-Switching Models

Autoregressive models with time varying parameters (Tucci, 1995), threshold autoregressive models (TAR), regime-switching models (Hamilton, 1989), and many other nonlinear models have been used to capture asymmetries. TAR models (Tong and Lim, 1980) are piecewise linear autoregressions. They are not only capable of approximating a general nonlinear time series model of the form $y_t = f(y_{t-1}) + \varepsilon_t$ and capturing jump phenomenon, but they also admit limit cycles. Tsay (1988) introduces a procedure for building and testing TAR models.

Beaudry and Koop (1993) also estimated a switching autoregressive moving average model with the switch governed by a restriction defined in term of the current depth of recession CDR_t. In this section we use this switching model as modified by Bidarkota (2000).

The most general model estimated within this class of models is as follows:

In Regime 1:

$$(1-\phi_1 L-\phi_2 L^2)(1-L)^d(\Delta y_t-\mu_1)=\varepsilon_t \qquad (2a)$$

$$\varepsilon_t \mid I_{t-1} \sim z_t c_t, \qquad z_t \sim i.i.d.S_\alpha(0,1)$$

$$c_t^{\alpha} = b_1 + b_2 c_{t-1}^{\alpha} + b_3 \mid \varepsilon_{t-1} \mid^{\alpha} \tag{2b}$$

In Regime 2:

$$(1 - \phi_3 L - \phi_4 L^2)(1-L)^d (\Delta y_t - \mu_2) = \varepsilon_t \tag{2c}$$

$$\varepsilon_t \mid I_{t-1} \sim z_t \gamma c_t, \qquad z_t \sim \text{i.i.d.} S_{\alpha}(0,1)$$

$$c_t^{\alpha} = b_1 + b_2 c_{t-1}^{\alpha} + b_3 \mid \varepsilon_{t-1} / \gamma \mid^{\alpha} \tag{2d}$$

When $CDR_{t-1} = 0$, we get regime 1 and when $CDR_{t-1} > 0$ we obtain regime 2. The unconditional mean of the process and the AR coefficients in the two regimes are different. The parameter γ in regime 2 shows that the model has different scales in the two regimes as well.

2.3 SETAR-Switching Models

This is identical to the CDR-Switching model, except for what governs switching between the regimes. When a switch is governed by restrictions that are defined in terms of the observed series y_t, these models are called self-exciting threshold autoregressive (SETAR) models. Potter (1995) estimated this type of model with a single restriction for the log real GNP. The restriction is defined in terms of whether, $\Delta y_{t-d} > r$, where y_t is log real GNP, d is the delay and r is the threshold parameter.

The most general model estimated within this class of models is as follows:

In Regime 1:

$$(1-\phi_1 L-\phi_2 L^2)(1-L)^d(\Delta y_t-\mu_1)=\varepsilon_t \qquad (3a)$$

$$\varepsilon_t \mid I_{t-1} \sim z_t c_t, \qquad z_t \sim \text{i.i.d.} S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 \mid \varepsilon_{t-1} \mid^\alpha \qquad (3b)$$

In Regime 2:

$$(1-\phi_3 L-\phi_4 L^2)(1-L)^d(\Delta y_t-\mu_2)=\varepsilon_t \qquad (3c)$$

$$\varepsilon_t \mid I_{t-1} \sim z_t \gamma c_t, \qquad z_t \sim \text{i.i.d.} S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 \mid \varepsilon_{t-1} / \gamma \mid^\alpha \qquad (3d)$$

When $\Delta y_{t-2} > 0$, we get regime 1 and when $\Delta y_{t-2} \le 0$, we get regime 2.

2.4 Discussion of the Models

The purpose of fitting three different classes of nonlinear models is to see whether one can reject linear (in the conditional mean) models versus at least one of the three alternative classes of nonlinear models. As an anonymous referee has thoughtfully pointed out, the three different classes of models introduced above capture different types of asymmetries. For instance, the CDR term will be greater than zero after a trough until the level of output has reached its historical maximum and would, by definition, classify this period as a recession. On the other hand, during this same period, Δy_{t-2} could be positive and hence classify this period as an expansion even while the CDR term is still positive.

Furthermore, for countries such as Japan, that have been in a prolonged period of recession towards the end of the sample, the CDR term would be consistently positive and hence produce no signals of recovery. While a more local version of the CDR term may arguably be desirable for Japan, the SETAR switching model would in fact produce signals of recovery similar to a local version of the CDR variable and hence serve our purpose here of trying to reject linearity.

3 Estimation Issues

As mentioned earlier, Beaudry and Koop (1993) simply included an additive nonlinear term in a standard autoregressive moving average (ARMA) model to capture asymmetries in business cycles with the assumption that the shocks are normally distributed. Although addition of a nonlinear term in a standard autoregressive moving average framework is ad hoc, it does not impose any estimation problems. Bidarkota (1999, 2000) used this model without any moving average terms but with errors having a more general stable distribution, conditional heteroskedasticity, and long memory.

As in Bidarkota (1999, 2000), we do not consider any moving average (MA) terms in the specification of the model. Maximum likelihood estimation of mixed ARMA models with stable errors poses a challenge, although the Whittle estimator (Mikosch et al, 1995) and minimum dispersion estimators (Brockwell and Davis, 1991) have been used in this context.

We restrict ourselves to symmetric stable distributions here for a technical reason. We use the computational algorithm due to McCulloch (1996) to obtain stable densities for maximum likelihood estimation of our models. This algorithm works only when errors are symmetric. However, Nolan (undated) has developed computational routines for maximum likelihood estimation of models with stable distributions.

For the estimation of ARFIMA models the exact full information maximum likelihood (ML) method of Sowell (1992a) may be adopted if the errors are iid normal. But for the more complicated non-normal conditionally heteroskedastic models here, we use the conditional sum of squares (CSS) estimator.

Baillie et al (1996) also used the conditional sum of squares (CSS) method, originally proposed in the context of ARFIMA models by Hosking (1984), to estimate their ARFIMA-GARCH models, with normal and Student-*t* errors. The CSS procedure is equivalent to the full information MLE asymptotically. Baillie et al (1996) discussed some properties of the CSS estimator in the context of ARFIMA models, particularly with respect to its bias. They noted that not only does the CSS estimator do well when they compared with it Sowell's (1992a) exact MLE but also that it is computationally feasible for more complex models.

Essentially we fit an ARMA model to the series $(1-L)^d(\Delta y_t - \mu)$ that is obtained by expanding the differencing operator $(1-L)^d$ with the binomial expansion, and truncating the infinite series at the first available observation. The CSS estimator is discussed for ARMA models in Box and Jenkins (1976).

4 Empirical Analysis

4.1 Data Sources

We obtained quarterly GDP data from the International Financial Statistics (IFS) CD-ROM (September 2001) for the G7 countries that comprise of Canada, France, Germany, Italy, Japan, the United Kingdom (UK), and the United States of America (USA). The dataset spans the period from 1957:1 to 2000:4 for all countries except for France, Germany, and Italy for which the data begin in 1970:1, 1960:1, and 1970:2, respectively. Table 1.1 provides further details on the dataset used for each country. Figure 1 plots the annualized quarterly GDP growth rates for the seven countries.

For Germany, $(\Delta y_t - \mu)$ in Equation (1a) is replaced with $(\Delta y_t - \mu - \delta D_t)$ and $(\Delta y_t - \mu_1)$ in Equations (2a) and (3a) is replaced with $(\Delta y_t - \mu_1 - \delta D_t)$, where D_t is an indicator term that takes the value one at the time of re-unification of Germany, and zero otherwise.

4.2 Specification Search and Parameter Estimates

An extensive specification search for each country was conducted for the four versions (Model 1 through Model 4) of each of the three classes of models described in section 2. For the CDR-Augmented class of models, the specification search was done over all parameterizations with lag orders for the autoregressive and CDR_t terms of three or less for parsimony. For the two classes of switching models, namely the CDR-Switching and SETAR-Switching models, the search was done with the autoregressive lag polynomials in the two regimes restricted to be of orders (3,3), (2,2), (1,1), or (0,0).[2] The best parameterizations for each version within each class of models are selected for each country by the minimum Schwarz Bayesian criterion. Details on specification search as well as on the parameter estimates for the best parameterizations for each of the four versions within the three classes of models are omitted from the paper here in the interest of conserving space on the advice of the editor.

5 Hypotheses Tests

We performed four types of hypotheses tests on the estimated models. The first is a test for normality. The second is a test for homoskedasticity. The third tests for the possible presence of long memory. The last is a test for linearity in the conditional mean. The following sub-section describes the various tests and sub-section 5.2 provides empirical results on the hypotheses tests. Discussion of the inferences on hypotheses tests follows in sub-section 5.3, followed by a brief exploration into the nature of the asymmetries in sub-section 5.4.

5.1 Description of Various Tests

5.1.1 Test for Normality

[2] With the switching regime models (both CDR and SETAR switching models), as discussed in section 5.1.4 below, a test for linearity is formulated as a test for a single regime. The test is executed by testing for equality of the corresponding coefficients in the two regimes. Therefore, in order to make this test feasible, we restrict the AR orders in the two regimes to belong to (j, j) where $0 \le j \le 3$ rather than restrict them to (i, j) where $0 \le i, j \le 3$.

We performed a normality test based on the value of α. If α equals 2, normality results. If the value of α is less than 2, then the model is non-normal stable. This test compares the likelihood ratio (LR) statistic for two models with identical parameterizations.

Since the null hypothesis for this test lies on the boundary of admissible values for α, the LR test statistic does not have the usual χ^2 distribution asymptotically. Therefore, the test here is based on small sample critical values generated by Monte Carlo simulations reported in McCulloch (1997, Table 4, panel b).

5.1.2 Test for Homoskedasticity

The second test is a test for homoskedasticity. Under the null of homoskedasticity, the GARCH parameters $b_2 = b_3 = 0$. The test is based on the likelihood ratio test statistic. Since the null hypotheses lie on the boundary of admissible values for b_2 and b_3, the standard distribution theory (that is, the LR test statistic being asymptotically chi-squared distributed) does not go through. Andrews (2001) develops the appropriate asymptotic distribution theory applicable in such a situation. Here, we simply base our statistical inference on the critical value derived conservatively from the χ_3^2 distribution. This is discussed below further in a footnote to the table reporting the empirical results.

5.1.3 Test for Long Memory

The null hypothesis is $d = 0$ implying that the logarithm of GDP has a unit root. The alternative of $d \neq 0$ implies that the logarithm of GDP is described as a fractionally differenced series. Depending on the estimates of d that are obtained, the logarithm of

GDP could be a stationary or a non-stationary series, with or without long memory. See Baillie (1996) for an extensive survey on fractionally differenced processes. The test is once again based on the likelihood ratio. Model 1 is the unrestricted model and Model 2 is the restricted model with d restricted to zero. The test statistic is distributed as the χ_1^2 since we have only one restriction here.

5.1.4 Test for Linearity in Conditional Mean

The last is a test for linearity in the conditional mean. While the three tests above are performed alternatively using the various versions within all the three classes of models, the test for linearity in the conditional mean is done using different versions of the models within only the two classes of switching models. This is because the minimum SBC criterion ends up selecting among the four different versions of the CDR-Augmented class of models a version and a parameterization that does not include the additive CDR term for any of the countries except France. Footnote 1 reports some added difficulty in testing for the significance of the CDR term with a standard *t*-test or an LR test.

Depending on the versions of the two classes of switching models used, we end up testing for linearity successively using homoskedastic Gaussian models, homoskedastic stable models, GARCH stable models, and long memory GARCH stable models. Under the null hypothesis of linearity in conditional mean, the unconditional means in the two regimes μ_1 and μ_2 are equal, the scale ratio γ is equal to one, and the corresponding autoregressive coefficients in the two regimes, if present in the specific parameterizations of the model versions used in the test, are equal. If the null hypothesis is not rejected then we only have one regime (linear conditional mean dynamics). If not, then we have two distinct regimes describing the GDP growth rates.

In general SETAR models, the switch between regimes is determined by whether $\Delta y_{t-l} > s$, and *l* and *s* are estimated along with other parameters of the model. In such a case, under the null hypothesis of a single regime, the parameters *l* and *s* are not identified. Standard asymptotic distribution theory does not go through (Hansen, 1996). In our paper, since we do not estimate the parameters *l* and *s* but instead set $l = 2$ and $s = 0$ in accordance with the findings in previous studies, our tests do not suffer from this problem. Our test here is therefore carried out with the LR test statistic and critical values are drawn from the χ^2 distribution with appropriate degrees of freedom.

5.2 Empirical Results on Hypotheses Tests

Hypotheses tests for normality, homoskedasticity, long memory, and linearity in conditional mean were performed as elaborated in the earlier sub-section. The empirical results of the various hypotheses tests listed above are reported in Tables 2.1, 2.2, and 2.3, respectively, for the CDR-Augmented, CDR-Switching and SETAR Switching models. All tests are based on the likelihood ratio (LR) test statistic. For each of the three tables, a different test is reported in the various rows of the first column. For each such test, the numbers in the five rows in the other columns for that test are the LR test statistics for the five countries arranged alphabetically in ascending order. *P*-values are reported in parentheses. All statistical inferences are drawn at the five percent significance level.

5.2.1 Results on Normality

The test for normality easily rejects for the UK and the US. The evidence is somewhat weaker for France, Germany, Italy, and Japan. For Canada, however, the test fails to reject normality overwhelmingly. The test results are largely unchanged when we account for conditional heteroskedasticity and long memory.

5.2.2 Results on Homoskedasticity

From the three tables, it is clear that when we test for homoskedasticity using Model 2, all countries except France show strong evidence against homoskedasticity. France strongly fails to reject homoskedasticity. The statistical inferences remain unchanged when we go from 5 to 10 percent significance level. Also, there are almost no changes in the inferences when we do the tests after accounting for long memory with Model 1.

5.2.3 Results on Long Memory

The test for $d = 0$ strongly rejects in all countries with all three classes of models. The only exceptions are Germany, Italy, and the UK in some instances. The statistical inferences are the same at the 5 and 10 percent significance level. However, it is important to note that Sowell's (1992b) Monte Carlo simulations suggest that the true small sample *p*-values may be higher than the asymptotic χ_1^2 *p*-values. Typically, GDP data tend to be largely uninformative about the nature of their long run properties and it is generally hard to rule out trend stationarity, unit roots, as well as fractional differencing from these data series.

5.2.4 Results on Linearity in Conditional Mean

Canada, Germany, Italy, Japan, and the US show fairly strong evidence against linearity in the conditional mean dynamics of the GDP growth rates. Accounting for conditional heteroskedasticity and long memory seems to be relatively unimportant when testing for linearity. However, accounting for outliers decreases the evidence against linearity. Statistical inferences are not affected much when we change the significance level of the test from 10 to 5 percent.

5.3 Discussion of Results on Hypotheses Tests

Our results on nonlinearity in the conditional mean for the US are in line with Bidarkota (2000). This shows that the evidence against linearity in mean for the US is robust to changes in the sample period.

Koop and Potter (2001) investigate whether nonlinearities could arise from structural instability. Blanchard and Simon (2001) show a slowdown in the variance of US economic activity suggesting a possible structural change in the early 1980s. We do not account for this possibility in this study.

Diebold and Inoue (2001) demonstrate that spurious evidence of long memory may be found in a series that undergoes occasional structural change. This is analogous to the spurious evidence of unit roots in a series with occasional breaks (Perron, 1989). Our results on long memory are subject to this caveat.

5.4 Nature of the Asymmetries

Since our inferences on linearity in the conditional mean are largely unaffected by accounting for long memory and conditional heteroskedasticity for the data sets under consideration here, and since accounting for outliers reduces evidence against linearity, we simulate Model 3 (a version that incorporates stable distributions) to obtain generalized impulse response functions and measures of persistence (Koop, 1996; Koop et al. (1996).

Also, for space considerations, since the CDR-augmented model does not select a parameterization that includes the CDR-term, and since a local version of CDR may be desirable at times (as for Japan), we briefly present business cycle stylized facts below based only on SETAR-switching models. This choice also aids comparison with studies such as Potter (1995), where stylized facts are presented for the US based on SETAR-switching models as well.

Our simulations (not reported for brevity) show that the marginal densities implied by the estimated models are similar at different horizons, except for Japan for which it takes about 10 years for the effects of shocks to stabilize. Most shocks have an effect of less than 3 percent in absolute value for all countries except Japan. For Germany and the UK, there are occasionally large effects triggered by large shocks (due to low estimated values of α). Shocks in one regime tend to have larger effects than shocks occurring in another regime, except for Canada and the US.

6 Conclusions

We used three classes of models, namely, the CDR-Augmented models, CDR-Switching models, and SETAR-Switching models for testing asymmetries in the conditional mean dynamics of GDP data in seven industrialized countries (the G7 countries). Our approach is fully parametric. Our time series models account for the possibility of conditional heteroskedasticity, outliers, and long memory in the data series.

Our results indicate fairly strong evidence of nonlinearities in the conditional mean dynamics of the GDP growth rates for Canada, Germany, Italy, Japan, and the US. For France and the UK, the conditional mean dynamics seem to be largely linear.

Our study shows that the Switching models capture non-linearity better than the CDR-Augmented models. Accounting for conditional heteroskedasticity and long memory is relatively unimportant when testing for linearity. However, accounting for outliers decreases the evidence against linearity.

Our results also indicate that the GDP series for all countries excepting France reject homoskedasticity. Long memory appears to be pervasive. While normality is strongly rejected for the US and the UK, the evidence is somewhat weaker for France, Germany, Italy, and Japan, and overwhelmingly in favor of normality for Canada.

Figure 1 Annualized Quarterly GDP Growth Rates

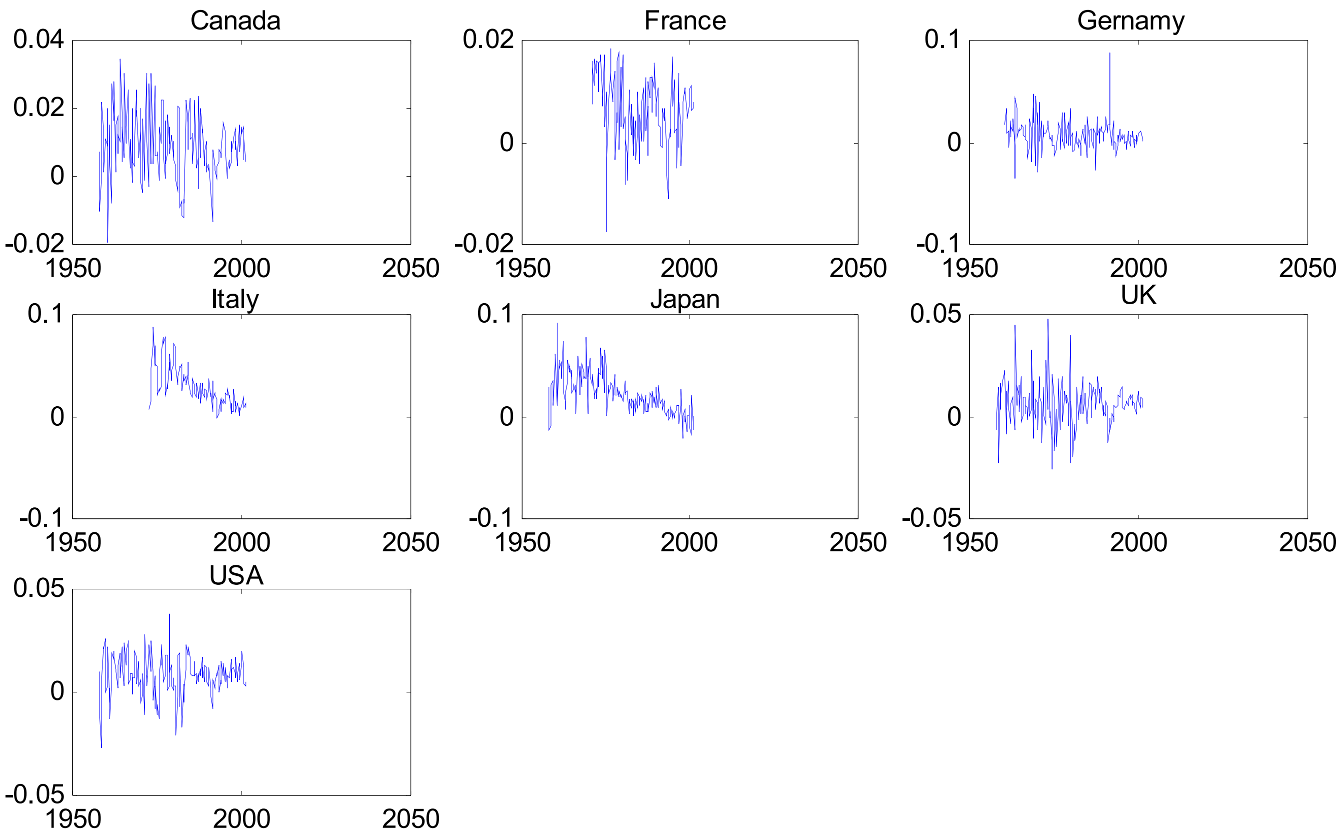

Table 1: Data Description

	Canada	France	Germany	Italy	Japan	UK	USA
Data Series	Quarterly Real GDP	Quarterly Real GDP	Quarterly Real GDP	Quarterly Real GDP	Quarterly Nominal GDP	Quarterly Real GDP	Quarterly Real GDP
Sample Period	1957:1-2000:4	1970:1-2000:4	1960:1-2000:4	1970:2-2000:4	1957:1-2000:4	1957:1-2000:4	1957:1-2000:4
Sample Length	176	124	164	123	176	176	176

Notes on Table 1.1

1. We obtained the quarterly seasonally adjusted GDP data for all countries from the September 2001 edition of the International Financial Statistics (IFS) CD-ROM.

2. We used nominal GDP for Japan because seasonally adjusted data was only available for the nominal series and not for the real series on the IFS CD-ROM.

TABLE 2.1: Hypotheses Tests – CDR-Augmented Models

	Model 4	Model 3	Model 2	Model 1
LR ($\alpha = 2$)		0.06	0.00	0.00
		2.06	0.54	0.00
		1.98	0.00	5.24
		10.00	5.48	5.40
		12.58	6.72	16.28
		23.36	20.34	18.62
		8.26	4.96	4.38
LR (no GARCH)			24.3 (0.00)	21.16 (0.00)
			0.02 (0.99)	1.32 (0.72)
			31.41(0.00)	28.60(0.00)
			13.50(0.00)	8.3(0.00)
			40.20 (0.00)	35.78 (0.00)
			41.16 (0.00)	41.24 (0.00)
			83.74 (0.00)	15.96 (0.00)
LR (d = 0)				17.88 (0.00)
				34.10 (0.00)
				6.00(0.11)
				1.16(0.76)
				55.66 (0.00)
				17.22 (0.00)
				4.60 (0.03)

Notes on Table 2.1

1. The table presents likelihood ratio (LR) test statistics and their associated *p*-values in parentheses.

2. For each item reported in column 1, row one in column 2 and subsequent columns for that item presents statistics for Canada, row two for France, row three for Japan, row four for the UK, and row five for the USA.

1. LR ($\alpha = 2$) is a test for normality. The distribution of the test statistic in this instance is not standard χ^2 because the null hypothesis is on the admissible boundary of α. However, critical values at the 10% and 5% significance level are available through Monte Carlo simulations from McCulloch (1997, Table 4, panel b). These are 0.243 and 1.120, respectively.

2. LR (no GARCH) is a test for homoskedasticity. The null hypothesis is $b_2 = b_3 = 0$. Under this null, b_1 and c_0 are trivial transformation of one another. As such, it is not clear whether the LR statistic has asymptotic χ^2 distribution with two or three degrees of freedom. We report conservative χ_3^2 *p*-values in parentheses.

3. LR ($d = 0$) is a test for the absence of long memory.

TABLE 2.2: Hypotheses Tests - CDR-Switching Models

	Model 4	Model 3	Model 2	Model 1
LR (α = 2)		0.46	4.12	0.00
		0.16	2.56	5.22
		0.40	30.4	45.02
		0.02	33.76	0.44
		15.44	0.78	0.00
		21.74	58.18	22.52
		6.84	4.80	11.44
LR (no GARCH)			31.68 (0.00)	12.66 (0.01)
			0.48 (0.92)	0.00 (1.00)
			61.64(0.00)	42.84(0.00)
			0.62(0.89)	121.28(0.00)
			50.98 (0.00)	55.72 (0.00)
			40.66 (0.00)	44.92 (0.00)
			15.80 (0.00)	--
LR (d = 0)				22.42 (0.00)
				19.84 (0.00)
				3.01(0.08)
				11.46(0.00)
				35.68 (0.00)
				36.32 (0.00)
				19.07 (0.00)
LR (one regime)	5.86 (0.00)	6.22 (0.10)	13.12 (0.00)	10.68 (0.00)
	2.15 (0.14))	1.30 (0.72)	1.46 (0.69)	0.10 (0.95)
	12.36 (0.00)	20.62 (0.00)	10.82 (0.00)	23.80 (0.00)
	8.24 (0.00)	7.3 (0.01)	7.44 (0.01)	0.56 (0.45)
	16.36 (0.00)	13.56 (0.00)	24.34 (0.00)	14.20 (0.00)
	2.72 (0.26)	0.98 (0.61)	4.46 (0.11)	4.16 (0.00)
	7.5 (0.06)	11.30 (0.01)	0.00 (1.00)	13.44 (0.00)

Notes on Table 2.2

1. The table presents likelihood ratio (LR) test statistics and their associated *p*-values in parentheses.

2. For each item reported in column 1, row one in column 2 and subsequent columns for that item presents statistics for Canada, row two for France, row three for Japan, row four for the UK, and row five for the USA.

3. LR (one regime) is a test for linear conditional mean dynamics. The null hypothesis is $\mu_1 = \mu_2$, $\gamma = 1$, and the corresponding autoregressive coefficients in the two regimes are equal.

4. See notes 3-5 in Table 2.1.

5. The symbol "--" denotes missing numbers because the numerical algorithm for maximization of the log likelihood function failed to converge.

TABLE 2.3: Hypotheses Tests – SETAR-Switching Models

	Model 4	Model 3	Model 2	Model 1
LR ($\alpha = 2$)		0.00	0.00	0.00
		2.80	0.00	4.74
		-5.48	12.16	3.96
		0.56	12.88	10.66
		12.66	0.38	0.00
		14.44	13.76	46.20
		2.98	--	12.12
LR (no GARCH)			11.50 (0.17)	18.74 (0.00)
			0.22 (0.97)	0.00 (1.00)
			31.22 (0.00)	14.6 (0.00)
			15.18 (0.00)	19.22 (0.00)
			48.74 (0.00)	75.82 (0.00)
			40.20 (0.00)	41.26 (0.00)
			--	--
LR (d = 0)				22.22 (0.00)
				12.96 (0.00)
				9.78 (0.00)
				0.08 (0.78)
				56.38 (0.00)
				2.40 (0.12)
				--
LR (one regime)	7.74 (0.01)	7.76 (0.02)	1.28 (0.53)	0.14 (0.93)
	5.58 (0.23)	4.46 (0.22)	4.34 (0.23)	1.06 (0.59)
	3.42 (0.06)	5.05 (0.00)	16.60 (0.00)	12.18 (0.00)
	0.76 (0.38)	1.16 (0.28)	16.02 (0.00)	5.1 (0.02)
	3.54 (0.62)	2.96 (0.71)	17.28 (0.00)	46.54 (0.00)
	9.00 (0.00)	4.74 (0.32)	5.46 (0.14)	3.52 (0.32)
	6.80 (0.08)	9.92 (0.01)	--	14.58 (0.00)

Notes on Table 2.3

1. The table presents likelihood ratio (LR) test statistics and their associated *p*-values in parentheses.

2. For each item reported in column 1, row one in column 2 and subsequent columns for that item presents statistics for Canada, row two for France, row three for Japan, row four for the UK, and row five for the USA.

3. LR (one regime) is a test for linear conditional mean dynamics. The null hypothesis is $\mu_1 = \mu_2,\ \gamma = 1$, and the corresponding autoregressive coefficients in the two regimes are equal.

4. See notes 3-5 in Table 2.1.

5. The symbol "--" denotes missing numbers because the numerical algorithm for maximization of the log likelihood function failed to converge.

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