

Fiscal Policy in a Two-Sector Economy with Public Capital and Congestion

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Abstract

This paper focuses on the role of government capital as a critical productive input when the level of services that the agent derives from it is subject to congestion. I develop a two-sector “non-scale” production model in which there are two types of firms, conventional profit-maximizing private firms, and “public firms”, whose objective is to produce a specified quantity of government investment goods – determined by government policy – at minimum cost. Furthermore, the production functions of the two sectors need not in general coincide. Using this two-sector production set-up I assume that the positive externality of the public capital is associated with two types of congestion, *proportional* and *aggregate*. A variety of fiscal disturbances are analyzed. Because of the complexity of the model the analysis is carried out using simulations of a calibrated economy. The effects of tax policies are remarkably robust with respect to the relative capital intensities of the two productive sectors. In contrast, the effects of government investment are much more sensitive to this aspect. The introduction of congestion decreases the steady state growth rate of the economy. The relative congestion has stronger effects when the variation in the government investment is analyzed, whereas the absolute congestion is more relevant in the analysis of the change in the tax on capital income. The papers highlight the intertemporal dimensions of fiscal policy and the tradeoffs these involve for economic performance, especially growth and welfare.

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1. Introduction

This paper focuses on the role of government capital as a critical productive input when the level of services that the agent derives from it is subject to congestion. The analysis of the productive role of private capital has received significant attention in the recent years, but in most cases this has been carried out using a one-sector model of production, in which public capital enters the aggregate production function together with private capital and labor; see e.g. Arrow and Kurz (1970), Baxter and King (1993), Futagami, Murata, and Shibata (1993), Turnovsky (2003). This implicitly assumes that all production occurs in the private sector. The government then enters the private market and makes its purchases, in competition with private agents, using the resources it generates from tax revenues and from borrowing.

While this is a reasonable description one could argue that the governments effectively conduct their own productive operations. Consider the following stylized description. A government passes legislation to invest in an airport facility, say. It sets out precise specifications of the project, which it then puts out for bids to private contractors who will hire labor and employ private capital to carry out the project. But instead of being free to hire productive factors to maximize his profit -- and thereby determine his output level (the size of the airport) endogenously -- the contractor is constrained (to win the contract) to construct the project, as specified by the government, most efficiently. Moreover, there is no reason to assume that the technology employed in the public project need be the same as that in the private sector. Indeed, one can plausibly argue that government investment projects that involve the nation's infrastructure may well be more capital intensive (in private capital) than is the average technology employed in the private sector.

In Turnovsky and Pintea (2003) we develop a two sector model in which private output is produced in one sector by profit-maximizing firms. The production function in that sector depends upon the stocks of both private and public capital, as well as upon endogenously supplied labor. Public capital introduces a positive externality in production, so that the complete production function is one of overall increasing returns to scale in these three productive factors. Government capital is produced in a second sector by "public firms", hiring labor and private capital, with public capital also

providing an externality. In hiring their productive factors these firms compete with the private sector and therefore need to pay competitive factor returns. The objective of the public firm is to provide the new public capital, as determined by government policy, at minimum cost. Thus in contrast to the usual model in which the government uses its resources to finance its direct purchases of new investment, the government uses its resources to purchase the services of the productive factors that it employs. These resources are obtained by taxing capital income, labor income, and consumption, or by imposing non-distortionary lump-sum taxation.

I use this model as a base model in my analysis. However in this paper I assume that the agent can not chose how much labor to provide. Since labor is exogenously determined, the tax on consumption and labor income are going to behave very similarly to a lumps sum tax, without influencing the evolution of the real variables in our economy.

The main contribution of this paper is to introduce congestion effects in the two production functions. The congestion effects in a one sector economy have been extensively researched in the context of AK models; see Barro and Sala-i-Martin (1992, 1995), Turnovsky (1997), Fisher and Turnovsky (1998) and Eicher and Turnovsky (2000). This paper introduces two types of congestion in a context of a “non-scale” growth model following Eicher and Turnovsky (2000). The positive externality of the public capital is associated with both *relative* and *aggregate congestion*¹. The relative or proportional congestion refers to the fact that the level of services, derived by an individual from the provision of a public good depends of the usage of his capital stock relative to aggregate capital stock. Highway usage constitutes an example. Unless you drive your car, you do not derive any service from the public good. The aggregate congestion refers to the fact that the level of the services derived by an individual depends on how much that public good is used on the aggregate. Medicare and hospital services are a good example. If these services are used by many people then the level of services that one individual enjoys is reduced (crowding). To use the previous example of the public good being a airport facility, relative congestion refers to the fact that you do not benefit from it unless you take the plane, and the more times you take the plane

¹ see also Edwards (1990) for different specifications of production functions with congestion

the more you are going to benefit from it. Absolute congestion refers to the fact that as there are more and more planes that want to use it, they will be more restricted in their usage and one individual plane is going to derive less of the services provided by the airport.

The model I employ is a “non-scale” growth model of the type introduced by Jones (1995a, 1995b), Kortum (1997), Segerstrom (1998), and others. This model that can be viewed as an extension of the neoclassical model (generalized to allow for non-constant returns to scale) has the property that the long-run equilibrium growth rate is determined by the interaction between the population growth rate, technological and congestion production parameters and is independent of government policy parameters.

The non-scale model typically yields slow asymptotic speeds of convergence². This implies that policy changes can affect growth rates for sustained periods of time so that their impacts during the transition from one equilibrium to another will eventually accumulate to potentially large influences on steady-state levels. Thus, the fiscal policy has important effects on the *levels* of key economic variables, such as the per capita stock of capital and output. These considerations suggest that attention should be directed to analyzing the impact of fiscal policy on the transitional dynamics and this has been the focus of a number of previous studies; see e.g. Auerbach and Kotlikoff (1987), Baxter and King (1993), King and Rebelo (1993), Devereux and Love (1994), Turnovsky (2003).

The goal of this paper is to analyze the effects of government investment and changes in the alternative tax rates in this two-sector productive economy with congestion effects. I set out the dynamic equilibrium of this economy and show how the stable adjustment is characterized by a two dimensional locus in terms of the two stationary variables, referred to as “scale-adjusted” per capita stocks of private and public capital. Due to the complexity of the model the effects of various policy shocks are analyzed numerically. Different fiscal shocks have different impacts on government revenues, and to preserve comparability I normalize the shocks in terms of their impacts on the intertemporal government deficit. Both the transitional adjustments and the eventual long-run equilibrium responses are considered. A significant share of the

² see Eicher and Turnovsky (1999).

analysis is devoted to the welfare of the representative agent, the instantaneous utility and intertemporal welfare, as represented by the present value of the accumulated benefits.

The major focus of the paper is on highlighting the intertemporal dimensions of fiscal policy and the tradeoffs these involve for economic performance, paying particular attention to the role of the more general production structure and different congestion effects.

The analysis focuses on positive shocks to the economy from the long run welfare point of view, an increase in the share of public capital and a decrease in the tax on capital income. Public investment increases consumption and welfare in the short run, even as resources are attracted towards government investment. In our model doubling the rate of public investment from 4% to 8% involves short-run welfare increase of between 2.5% - 4.3% and intertemporal welfare gains of between 8.3% - 13%, depending upon the relative sectoral capital intensities. Normalized government investment will be significantly more expansionary as the capital intensity of the public sector increases.

The effects of a decrease in the capital income tax vary less with the intensity of capital in the public production function. The strongest effect is on the allocation of the resources across the two sectors towards the sector less intensive in capital (private sector if the public production function is more intensive in capital and public sector in the opposite case). The analysis of the normalized cut on the tax on capital income reveals a decrease in the consumption to output ratio of about 2% which leads also to a decrease in the output to capital ratio of 10% and an increase in the overall output level of 25%. In the short run the normalized reduction in this tax leads to immediate welfare losses of about 2.8%, as resources are diverted away from consumption toward private capital accumulation. In the long run the accumulation of more capital and output generate however intertemporal welfare gains of 1.5% which are significantly lower than the gains that are generated by the increase in g . An increase in absolute congestion causes a significant decrease in the welfare in the short run. As the absolute congestion increases, the long run welfare gains steadily diminish until the gains derived from the decrease of the tax on capital income can transform into losses.

In general the introduction of congestion decreases the positive effects brought by changes in the share of government spending and the decrease in the capital income tax. It also decreases the steady state growth rate of the economy. The introduction of relative congestion has stronger negative effects when I analyze the increase in g , whereas the absolute congestion has stronger negative effects if I analyze the decrease in the tax on capital income. As relative congestion increases, the reallocation of resources from one sector to the other diminishes.

The remainder of the paper proceeds as follows. Section 2 sets out the structure of the model, while its equilibrium dynamics are characterized in Section 3. Section 4 calibrates the model and considers the numerical effects of different policy changes. Section 5 concludes.

2. The Model

2.1 Private Firms

I shall assume that there are N private firm indexed by i . A private firm produces output Y_i in accordance with the production function:

$$Y_i = \mathbf{a}_Y (K_{j,i})^b [L_{j,i}]^{1-b} \left[K_g \left(\frac{K_{j,i}}{K} \right)^{1-q_1} K^{-s_1} \right]^s \quad 0 \leq q_1 \leq 1, \quad 0 \leq s_1 \leq 1, \quad (1a)$$

where $K_{j,i}$ represents the capital employed by the firm i , $L_{j,i}$ denotes the labor employed by the firm, K_g denotes the aggregate stock of public capital and K the aggregate stock of private capital. $1-q_1$ measures the degree of relative congestion and $-s_1$ the degree of absolute congestion. If $1-q_1$ is 0 then there is no relative congestion effect, how much capital the agent has relative to the aggregate stock of private capital does not influence the level of services that he derives from the provision of public goods. As q_1 varies between 0 and 1, the relative weight of the individual agent's private capital influences

his benefit of the public good. If s_I is different from 0 one observes a “crowding effect”, the level of services that one individual derives from the provision of the public goods decreases as the aggregate private capital increases.

I can rewrite the production function as:

$$Y_i = \mathbf{a}_Y (K_{j,i})^{b+s(1-q_1)} [L_{j,i}]^{1-b} K^{-s(s_1+(1-q_1))} K_g^s, 0 \leq q_1 \leq 1, 0 \leq s_1 \leq 1 \quad (1b)$$

I assume that the private firm maximizes profit, so that the rates of return satisfy:

$$r = \frac{\partial Y_i}{\partial K_{j,i}} = (b + s(1 - q_1)) \frac{Y_i}{K_{j,i}} \quad ; \quad (2a)$$

$$w = \frac{\partial Y_i}{\partial L_{j,i}} = (1 - b) \frac{Y_i}{L_{j,i}} \quad (2b)$$

2.2. Public Firms

I shall assume that there are N public firms, indexed by P , that hire labor and private capital, paying the market wage and return to capital, so as to produce some prescribed level of output, J , to be used for public investment, at minimum cost. The output of this public firm is given by the production function

$$J_i = \mathbf{a}_J (K_{p,i})^d [L_{p,i}]^{1-d} \left[K_g \left(\frac{K_{p,i}}{K} \right)^{1-q_p} K^{-s_p} \right]^h, 0 \leq q_1 \leq 1, 0 \leq s_1 \leq 1 \quad (3a)$$

similar with (1a), where $1-q_p$ measures the degree of relative congestion and $-s_p$ the degree of absolute congestion. The different exponents and degree of congestion allow for potentially different factor intensities in the public sector from those in the private sector. I assume that the production of output in the private sector involves a different technology than does the production of infrastructure produced in the public sector.

I can rewrite the public production function as

$$J_i = \mathbf{a}_J (K_{p,i})^{d+\mathbf{h}(1-q_p)} [L_{p,i}]^{1-d} K^{-\mathbf{h}(s_p+(1-q_p))} K_g^{\mathbf{h}}, 0 \leq q_1 \leq 1, 0 \leq s_1 \leq 1 \quad (3b)$$

The individual public firm's optimization problem is to produce a specified output J , chosen by the policy maker, in accordance with the production function (3b), at a minimum cost:

$$\min rK_{p,i} + wL_{p,i} \quad (4a)$$

$$\text{subject to the constraint } \dot{K}_{G,i} = J_i - \mathbf{d}_G K_{G,i} \quad (4b)$$

I write the Lagrangean as

$$L = rK_{p,i} + wL_{p,i} + \mathbf{I}_i (J_i - \mathbf{d}_G K_{G,i}) \quad (4c)$$

where \mathbf{I}_i measures the marginal cost of a unit of public investment. Performing the optimization yields:

$$r = \mathbf{I}_i \frac{\partial J_i}{\partial K_{p,i}} = \mathbf{I}_i (d + \mathbf{h}(1 - q_p)) \frac{J_i}{K_{p,i}} \quad (5a)$$

$$w = \mathbf{I}_i \frac{\partial J_i}{\partial L_{p,i}} = \mathbf{I}_i (1 - d) \frac{J_i}{L_{p,i}} \quad (5b)$$

2.3. Representative Agent

The representative agent supplies one unit of labor, a fraction θ of which is to employment in the private sector and $1-\theta$ to employment in the public sector. Agent i also owns K_i units of capital, which he rents out a fraction \mathbf{f} to the private sector and $1-\mathbf{f}$ to the public sector, both at the rental rate, r

The representative agent's optimization problem is

$$\max \int_0^{\infty} \frac{1}{\mathbf{g}} C_i^g e^{-bt} dt, \quad -\infty < \mathbf{g} \leq 1; \quad (6a)$$

where C_i denotes the agent's consumption, $1/(1-g)$ equals the intertemporal elasticity of substitution and β is the time discount factor. The agent's objective is to maximize (6a) subject to his accumulation equation:

$$\dot{K}_i = [(1-t_k)r - n - d_k]K_i + (1-t_w)w - (1+t_c)C_i - T_i \quad (6b)$$

where t_k =tax on capital income, t_w =tax on wage income, t_c =consumption tax and $T_i=T/N$ =agent's share of lump sum taxes (transfers). Equation (6b) assumes that private capital depreciates at the rate d_k , so that with the growing population, the net after tax private return to capital is $(1-t_k)r - n - d_k$.

Performing the optimization yields:

$$C_i^{g-1} = m_i(1+t_c) \quad (7a)$$

$$(1-t_k)r - n - d_k = b - \frac{\dot{m}_i}{m_i} \quad (7b)$$

$$(1-t_k)(b+s(1-q_1))\frac{Y_i}{fK_i} - n - d_k = b - \frac{\dot{m}_i}{m_i} \quad (7c)$$

Equation (7a) equates the marginal utility of consumption to the individual's tax adjusted shadow value of wealth, μ_i , while (7b) equates the marginal utility of leisure to its opportunity cost, the after tax real wage, valued at the shadow value of wealth. The third equation is the Keynes-Ramsey consumption rule, equating the rate of return on consumption to the after-tax rate of return on capital where the return on capital incorporates the relative congestion effects. Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality condition must be imposed:

$$\lim_{t \rightarrow \infty} m_i K_i e^{-bt} = 0 \quad (7d)$$

2.4. Aggregate Relationships

Letting the time allocation of labor to the private sector be denoted by \mathbf{q} :

$$1 = L_{j,i} + L_{p,i}; \quad L_{j,i} = \mathbf{q}, \quad L_{p,i} = (1 - \mathbf{q}) \quad \text{and aggregating over } N \text{ agents:}$$

$$NL_{j,i} = N\mathbf{q} = L_j; \quad NL_{p,i} = N(1 - \mathbf{q}) = L_p \quad (8a)$$

Likewise, letting \mathbf{f} denote the allocation of capital, we have in equilibrium

$$K_i = K_{i,j} + K_{i,p}, \quad K_{i,j} = \mathbf{f}K_i, \quad K_{i,p} = (1 - \mathbf{f})K_i$$

$$NK_{i,j} = \mathbf{f}NK_i = \mathbf{f}K; \quad NK_{i,p} = (1 - \mathbf{f})NK_i = (1 - \mathbf{f})K \quad (8b)$$

Equation (8a) asserts that the total supply of labor must be allocated either to one of the private firms or to the public firm, while (8b) describes an analogous allocation condition for private capital. Using these relationships, the aggregate production function for the private sector and the public production function may be expressed as:

$$Y = \mathbf{a}_y \mathbf{f}^{b+s(1-q_1)} \mathbf{q}^{1-b} N^{1-b-s(1-q_1)} K^{b-sq_1} K_g^s \quad (9a)$$

$$J = \mathbf{a}_j (1 - \mathbf{f})^{d+h(1-q_p)} (1 - \mathbf{q})^{1-d} N^{1-d-h(1-q_p)} K^{d-hs_p} K_g^h \quad (9b)$$

The government runs a balanced budget in accordance with

$$\mathbf{t}_K rK + \mathbf{t}_w Nw + \mathbf{t}_c C + T = rK_p + wL_p + \mathbf{s}(1 - q_1)Y \quad (10)$$

i.e. the government pays out of tax revenues, its production costs and a subsidy to the firms that incur losses due to the negative externality imposed by the relative congestion effect.

T represents the amount of lump-sum taxation (or transfers) necessary to finance the primary deficit and is therefore a measure of current fiscal imbalance. Recalling the expressions for the returns to capital and labor derived in (2a, b), it can be expressed as

$$T = \left[\frac{b}{f}(1-f) + \frac{1-b}{q}(1-q) - t_k \frac{b}{f} - t_w \frac{1-b}{q} - t_c \frac{C}{Y} \right] Y \quad (11a)$$

Defining $V = \int_0^{\infty} T(t) e^{\int_0^t -s(u)(1-t_k) du}$ it is derived:

$$V = \int_0^{\infty} Y \left[\frac{b}{f}(1-f) + \frac{1-b}{q}(1-q) - t_k \frac{b}{f} - t_w \frac{1-b}{q} - t_c \frac{C}{Y} \right] e^{\int_0^t -s(u)(1-t_k) du} dt \quad (11b)$$

where $s(1-t_k) = r(1-t_k) - d_k$ is the implied equilibrium of interest, V measures the present discounted value of the lump sum taxes or transfers necessary to balance the government budget over time, and thus is a measure of the intertemporal fiscal imbalance.

I shall also assume that government investment is tied to aggregate private output by

$$J = gY \quad (12)$$

Using the aggregate production functions, (9a) and (9b) and recalling the budget constraint and the returns to capital and labor, implies goods market clearing:

$$\dot{K} = Y - C - d_k K \quad (13a)$$

this equation asserts that private output can be costlessly transformed into productive private capital or consumption. Also,

$$\dot{K}_g = gY - d_G K_g \quad (13b)$$

2.5. Macroeconomic Equilibrium

Our objective is to analyze the dynamics of the aggregate economy about a stationary growth path. Along such an equilibrium path, aggregate output, private capital stock, and public capital (due to the fact that it is a constant fraction of the private output) are assumed to grow at the same constant rate, so that the output-capital ratio and the ration of public capital to private capital remain constant, while the fraction of time devoted to leisure also remains constant. Taking percentage changes of the aggregate production functions, the long run equilibrium growth of output, private and public capital is:

$$y = \frac{1-b-s(1-q_1)}{1-b-s(1-s_1)}n \quad (14a)$$

with the requirement that:

$$\frac{1-b-s(1-q_1)}{1-b-s(1-s_1)} = \frac{1-d-h(1-q_p)}{1-d-h(1-s_p)} \quad (14b)$$

Equations (14a, b) are necessary in order for the policy to be sustainable. (14b) imposes a weak condition on the externality. (14a) shows that the growth rate decreases if there is an increase in either relative or absolute congestion.

3. Transitional Dynamics

To analyze the transitional dynamics of the economy about its balanced growth path, I express the system of the stationary variables in the scale-adjusted per capita quantities:

$$k = \frac{K}{N^{\frac{1-b-s(1-q_1)}{1-b-s(1-s_1)}}}; k_G = \frac{K_G}{N^{\frac{1-b-s(1-q_1)}{1-b-s(1-s_1)}}}; y = \frac{Y}{N^{\frac{1-b-s(1-q_1)}{1-b-s(1-s_1)}}}; j = \frac{J}{N^{\frac{1-b-s(1-q_1)}{1-b+s_1-s}}}$$

Using this notation, the scale-adjusted private and respectively public production functions can be written as:

$$y = \mathbf{a}_y \mathbf{f}^{b+s(1-q_1)} \mathbf{q}^{1-b} k^{b-s} k_g^s \quad (15a)$$

$$j = \mathbf{a}_j (1-\mathbf{f})^{d+h(1-q_p)} (1-\mathbf{q})^{1-d} k^{d-hs_p} k_g^h \quad (15b)$$

The optimality conditions then enable the dynamics to be expressed in terms of these scale adjusted variables as follows. First by using the no arbitrage condition which implies the equality of return to capital and labor in the private and public sector (2a), (2b), (5a) and (5b) and the labor and private capital allocations (8a) and (8b), together with (12) one can derive that:

$$\frac{b+s(1-q_1)}{d+h(1-q_p)} \frac{(1-\mathbf{f})}{\mathbf{f}} = \frac{1-b}{1-d} \frac{(1-\mathbf{q})}{\mathbf{q}} \quad (16a)$$

Then, substituting (15a) and (145) into (12) yields:

$$\mathbf{a}_j \frac{(1-\mathbf{f})^{d+h(1-q_p)} (1-\mathbf{q})^{1-d}}{\mathbf{f}^{b+s(1-q_1)} \mathbf{q}^{1-b}} = g \mathbf{a}_y k^{b-s_1-d+h s_p} k_g^{s-h} \quad (16b)$$

In principle, one can solve these two equations for the allocation of labor and private capital across the production of private and public capital:

$$\mathbf{q} = \mathbf{q}(k, k_G) \quad (17a)$$

$$\mathbf{f} = \mathbf{f}(k, k_G) \quad (17b)$$

In the Appendix I show how the equilibrium dynamics can be expressed as the following system in the stationary variables k , kg and c :

$$\frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k} - \mathbf{d}_k - \mathbf{y} \quad (18a)$$

$$\frac{\dot{k}_g}{k_g} = g \frac{y}{kg} - \mathbf{d}_g - \mathbf{y} \quad (18b)$$

$$\frac{\dot{c}}{c} = \frac{1}{\mathbf{g}-1} \left(\mathbf{b} + n\mathbf{g} + \mathbf{d}_k - \mathbf{y}(\mathbf{g}-1) - (1-\mathbf{t}_k)(\mathbf{b} + \mathbf{s}(1-q_1)) \frac{y}{\mathbf{f}k} \right) \quad (18c)$$

$f(\cdot)$ and $q(\cdot)$ are determined by (17a), and respectively (17b).

This third order system has two sluggish variables, k and k_G , and one jump variable, c . To yield a well behaved dynamic behavior I require that the eigenvalues of this system consist of two stable and one unstable root, a property that I found to prevail over all of our wide-ranging simulations.

3.1 Steady State

The steady state of this economy, denoted by “ \sim ”, is obtained by setting $\dot{k} = \dot{k}_G = \dot{c} = 0$ in (18a) – (18c) and can be summarized by:

$$(\tilde{y} - \tilde{c}) = \tilde{k}(\mathbf{d}_k + \mathbf{y}) \quad (19a)$$

$$g\tilde{y} = \tilde{k}_g(\mathbf{d}_g + \mathbf{y}) \quad (19b)$$

$$(1 - \mathbf{t}_k)(b + \mathbf{s}(1 - q))\frac{\tilde{y}}{\tilde{f}k} = \mathbf{b} + n\mathbf{g} + \mathbf{d}_k - \mathbf{y}(\mathbf{g} - 1) \quad (19c)$$

together with the allocation conditions (16a, b).

Equation (19a) describes the growth of private output, given consumption, necessary to provide the private capital to equip the growing labor force and replace depreciation. Equation (19b) is an analogous condition for public capital while equation (19c) equates the long-run net return to private capital to the rate of return on consumption.

3.2 Centrally Planned Economy

The central planner possesses complete information and chooses all quantities directly, taking into account the congestion caused by all agents. The formal optimization is to maximize per capita utility in the economy:

$$\max_{\mathbf{g}} \int_0^{\infty} \frac{1}{\mathbf{g}} (C/N)^{\mathbf{g}} e^{-bt} dt \quad (20a)$$

$$\text{s.t. } \dot{K} = Y - C - \mathbf{d}_k K \text{ and } \dot{K}_G = J - \mathbf{d}_G K_G \quad (20b)$$

The optimality conditions for such an economy comprise the efficient allocation (16a), the scale adjusted production functions (15a, b), the equilibrium growth conditions $(\tilde{y} - \tilde{c}) = \tilde{k}(\mathbf{d}_k + \mathbf{y})$ and $\tilde{j} = \tilde{k}_g(\mathbf{d}_g + \mathbf{y})$ together with:

$$\frac{1-b}{\tilde{q}} = \tilde{q} \frac{1-d}{1-\mathbf{q}} \frac{\tilde{j}}{\tilde{y}} \quad (21a)$$

$$(b - \mathbf{s}_{s_1}) \frac{\tilde{y}}{\tilde{k}} - \mathbf{d}_k + \tilde{q}(d - \mathbf{h}_{s_p}) \frac{\tilde{j}}{\tilde{k}} = \mathbf{b} + \mathbf{y}(1 - \mathbf{g}) + n\mathbf{g} \quad (21b)$$

$$(b - \mathbf{s}_{s_1}) \frac{\tilde{y}}{\tilde{k}} - \mathbf{d}_k + \tilde{q}(d - \mathbf{h}_{s_p}) \frac{\tilde{j}}{\tilde{k}} = \frac{\mathbf{s}}{\tilde{q}} \frac{\tilde{y}}{\tilde{k}_g} + \mathbf{h} \frac{\tilde{j}}{\tilde{k}_g} - \mathbf{d}_g \quad (21c)$$

where q denotes the shadow price of public capital in terms of private capital. These 8 equations determine the steady state solutions for \hat{c} , \hat{y} , \hat{k} , \hat{k}_g , \hat{j} , $\hat{\mathbf{f}}$, $\hat{\mathbf{q}}$ and \hat{q} . Note that the implied investment share, \hat{g} is obtained from the ratio $\hat{g} = \hat{j} / \hat{y}$. In contrast to the decentralized economy, the after tax private rates of return to capital are replaced by the corresponding social rate of returns, which take into consideration the congestion effects imposed by the accumulation of private capital. Thus in (21b) the after tax return to capital which incorporates the effect of relative congestion that affects only the individual agent is replaced by the social return to a unit of capital which incorporates the effect of absolute congestion that affects the whole economy, and ignores the relative congestion effects which is irrelevant in the aggregate. Similarly (21a) determines the socially optimal ratio of public capital to private capital. (21b) and (21c) equate the long run social rates of return to investment in the two types of capital to the rate of return to consumption, again taking into consideration the effect of absolute congestion affecting them.

4. Numerical Analysis of Transitional Paths

Further insight into the effects of fiscal policy can be obtained by carrying out numerical analysis of the model. I begin by characterizing a benchmark economy, calibrating the model using the parameters representative of the US economy (Table 1.a)

The elasticity on capital implies that approximately 35% of output accrues to private capital and the rest to labor, which grows at an annual rate of 1.5%. The elasticity $s=0.2$ on public capital implies that public capital generates a significant externality in production. This parameter lies within the range of the consensus estimates (see Gramlich 1994). For the benchmark economy I consider that the private and the public production functions are identical. However, in my analysis I consider the cases where the public production is either more capital intensive or labor intensive than the private production function and evaluate how this change in the production parameters modify our results.

I vary the congestion parameters from 0 to 1, from a pure public good to varying degree of rivalry for both types of congestion. The analysis performs permutation of these models, i.e. $1-q=0.5$ and $s=1$ means that there is some degree of relative congestion, that the individual increasing its relative possession of private capital derives a higher level of services from the provision of the public good. At the same time since $s=1$ the increase in aggregate capital diminishes the level of services that each individual derives due to the crowding effect. I also experiment with different degree of congestion in the two production functions, i.e. $1-q_I=0.5$, $1-q_p=1$, $s_I=1$ that I did not report in the paper. In this case, keeping β constant s_p is derived endogenously within the model. The reason for which I prefer to limit the analysis to identical levels of congestion in the two production function is that even if there valid arguments for having different production function for the public and private capital, it is difficult to argue that identical factors of production (labor and private capital) are subject to different degree of congestion. Also, if the public congestion function experiences a higher degree of relative congestion than the private production function, the resulting degree of absolute congestion is negative, which means that there are positive aggregate capital spillovers.

The value of $\sigma=1.5$ implies an intertemporal elasticity of substitution in consumption of 0.4, consistent with the estimation by Ogaki and Reinhardt (1998). The benchmark tax on wage income, $t_w=0.28$ reflects the average marginal personal income tax rate in the US. Given the complex nature of capital income taxes, part of which may be taxed at a lower rate than wages and part of which at a higher rate, I have chosen the common rate, $t_k=0.28$ as the benchmark. The annual depreciation rates $d_g=0.035$ and $d_k=0.05$ approximate the average depreciation rates for public and private capital for the US during recent years.

These parameters lead to the benchmark equilibria reported in Table 1.b. Focusing on $d = 0.35$ and no congestion, the total consumption-output ratio = 0.84, the private output-private capital ratio = 0.46, and the ratio of public to private capital = 0.32. The benchmark equilibrium also implies that over 96% of labor and private capital is employed in the final output sector, which is also consistent with the available data. Finally, the steady-state growth rate, which by the non-scale nature of the economy is independent of policy, equals 2.17%. Table 1.b reports the equilibria when the public investment sector is labor intensive relative to the private sector ($d = 0.2$) and relatively capital intensive ($d = 0.5$). These fairly large changes in relative sectoral intensities have relatively small effects on the equilibrium. Table 1.b also reports the equilibria when we introduce different level of congestion. Congestion reduces the steady-state growth rate to 1.5%. The relative congestion reduces dramatically the consumption to output ratio to 0.73, the public to private capital ratio to 0.19 and the private output-private capital ratio to 0.24. At the same time a higher proportion of the resources are allocated towards the production of public sector. The absolute congestion has a negative effect on the steady state growth, but it has a very mild effect on the ratios mentioned above. The economic welfare is the optimized utility of the representative agent.

$$W = \int_0^{\infty} Z(t)e^{-bt} dt = \int_0^{\infty} \frac{1}{g} (C/N)^g e^{-bt} dt \quad (22)$$

where $Z(t)$ denotes instantaneous utility and C/N is evaluated along the equilibrium path. The welfare gains are calculated as the percentage change in the flow of income necessary to equate the initial level of utility to what it would be following a policy

change. The short run impact is measured by the changes in $Z(t)$, while the long run impact is summarized by the change in the overall intertemporal index W .

The other key measure of economic performance, the measure of the intertemporal fiscal balance has been defined previously in equation (11b). For the assumed tax rates and expenditure parameters, current tax revenues exceed government expenditure on the inputs necessary to produce the public good.

Table 2, 3 and 4 report the percentage changes in the intertemporal fiscal surplus, with a negative change implying a reduction relative to the base measure.

4.1. Normalized Fiscal Changes

Table 2, 3 and 4 describe various basic policy changes from the benchmark economy. These are uncompensated, meaning that they lead to changes in the government's fiscal deficit. To be able to make comparisons between them these changes have been standardized so that the tax decrease and the increase in g lead to the same increase in the present value of the government deficit, V (-16.8).

4.1.1. Increase in government investment

Public investment increases consumption and welfare in the short run, even as resources are attracted towards government investment. The consumption output ratio increases upon impact and there are gains in the short run welfare. Since more resources are allocated towards the public sector the private capital slightly decreases in the short run, but increases over time as the investment bears fruit and productivity is enhanced. In the long run the consumption to output ratio decreases and as output increases thanks to the increase in productivity enhancing government capital, the long run welfare increases further. In this model doubling the rate of public investment from 4% to 8% involves short-run welfare increase of between 2.5% - 4.3% and intertemporal welfare gains of between 8.3% - 13%, depending upon the relative sectoral capital intensities

(see table 2, 3 and 4).

Intuitively, on one hand the increase in output in the future would imply through consumption smoothing that consumption increases upon impact, but on the other hand this increase in output is due to the increase in government capital which implies that more resources should be allocated to production. The simulations show that consistently the consumption output ratio increases upon impact and therefore the welfare effects in the short run are positive and continue to increase in the long run.

As either/both relative and absolute congestion of the public good increase, the level of services that one derives from an increase in the government capital declines and thus the benefits on both the short run and long run welfare decrease. The relative congestion effect is far more damaging from the welfare point of view than the absolute congestion effect. As relative congestion increases, the agents perceive the private capital as being more productive, since it increases the level of services that they derive from the public good and implicitly their private marginal return to capital. Therefore they over-accumulate capital, the public good becomes congested and the increased productivity and welfare effects brought about by the increase in g are significantly diminished.

If $1-q=1$, the short run welfare effect of increasing g becomes negative when the two production functions are identical or the public production function is less intensive in capital (i.e. it varies from -0.15% to -0.22% for identical production functions and from -0.35% to -0.37% if the public production function is more intensive in capital, depending on the degree of absolute congestion). If the public production function is more intensive in capital the positive short run effects are still present, but significantly diminished (i.e. it decreases from 4.33% to 0.61% as the proportional congestion indicator increases from 0 to 1). The consumption to output ratio increases upon impact

in all cases, but if the public production function is more intensive in capital there is more capital attracted towards the production of the public good and relatively less output and consumption goods produced in the private sector. In general if the public production function is more intensive in capital the positive effects associated with the increase in g are significantly higher than in the case when it is less intensive in capital (see table 3 and 4). The increase in output is 36.32% compared to 25.11% which translates into an increase in the long run welfare of 12.94% compared to 8.28%. This result is robust to the introduction of different degree of congestion such that even when both congestion effects are present simultaneously the increase in g has a steadily positive welfare effect in both short and long run.

At the same time the normalization that is used determines significant different changes in g depending on the introduction of relative or aggregate congestion. To derive the same present value of the fiscal deficit, the increase in g is much lower in the case when only relative congestion is present compared to the case when there is no congestion or only absolute congestion is present. The explanation lays in the fact that the agents are paid their marginal return to capital. Relative congestion translates into losses for the productive firms, losses that are financed from the government budget. Therefore as relative congestion increases the agents who ignore the social returns to capital over accumulate private capital and the fiscal deficit increases very fast. If I perform the analysis imposing the same increase in g for different degree of congestion, and ignoring the effect that these changes will have on the government budget, the welfare effects improve when the relative congestion is concerned. The relative congestion is still more welfare damaging than the absolute congestion, but by a lesser degree.

4.2.1. Decrease in tax on capital income

The analysis of the normalized cut on the tax on capital income reveals a decrease in the consumption to output ratio of about 2% which leads also to a decrease in the output to capital ratio of 10% and an increase in the overall output level of 25% (see Table 2). In the short run the normalized reduction in this tax leads to immediate welfare losses of about 2.8%, as resources are diverted away from consumption toward private capital accumulation. In the long run the accumulation of more capital and output generate intertemporal welfare gains of 1.5% which are however significantly lower than the gains that are generated by the equivalent increase in g .

As the congestion of the public good increases, the benefits of decreasing the tax on capital income decrease. In contrast with the increase in the share of the government investment g , the absolute congestion has a more damaging effect than the relative congestion, due to the increase in the aggregate capital brought by the decrease in the capital income tax. These results hold even when it is normalized with respect to the amount of tax being cut (i.e a decrease of tk from 0.28 to 0.2 determines long run welfare losses of 0.63% if I consider the relative congestion and of 1.18% if I consider the absolute congestion effect, both independently; if both effects are considered simultaneously the welfare loss is 3.47%). Similar to the analysis concerning the increase in g , it is noticeable that the normalization affects more significantly the change in the level of taxes in the case of relative congestion due to the losses incurred by firms who have to pay the agents the private marginal return to capital. As I increase the relative congestion the decrease in the level of taxes necessary to reach the benchmark present value of the fiscal deficit is lower, therefore the short run welfare losses diminish.

The increase in absolute congestion causes a significant decrease in the welfare in

the short run. As the absolute congestion increases, the positive long run welfare steadily diminishes, such that when $s=1$ the change in long run welfare becomes negative. The cut of the tax on capital income determines an increase in the aggregate private capital. As the congestion effect becomes more prevalent, the increase in output brought by a more intensive investment in private capital is lower. Since the consumption-output ratio decreased to allow for a higher accumulation of capital, the new steady state consumption is lower than before the shock.

The results of the cut in income tax do not differ significantly based on the capital intensity of the public production function. The highest short run welfare losses (-4.19%) are derived when the public production function is more intensive in capital and the only congestion effect is due to aggregate congestion. At the same time if the public production function is more intensive in capital, the cut in capital income tax brings on the highest increase in the long run welfare (1.52%) when there are no congestion effects.

The allocation of resources is determined by the capital intensity of the public production function. A decrease in the tax on capital income determines the reallocation of resources (private capital and labor) towards the sector less intensive in private capital (private sector if the public production function is more intensive in capital and public sector in the opposite case).

5. Conclusions

This paper focuses on the role of government capital as a critical productive input when the level of services that the agent derives from it is subject to congestion. I develop a two sector model in which private output is produced in one sector by profit-maximizing firms and government capital is produced in a second sector by cost minimizing firms. The two sectors compete in hiring labor and private capital, and the

public capital provides a positive externality. The two production functions can differ in their sectoral intensities. We introduce two types of congestion associated with the positive externality of the public capital, *relative* and *aggregate congestion*.

We analyze the growth and welfare effects of an increase in government investment and equivalent decrease in the tax on capital income.

Public investment in general increases consumption and welfare in the short run and long run. (i.e. an increase in the rate of public investment from 4% to 8% involves short-run welfare increase of between 2.5% - 4.3% and intertemporal welfare gains of between 8.3% - 13%, depending upon the relative sectoral capital intensities. Normalized government investment will be significantly more expansionary as the capital intensity of the public sector increases.

The introduction of congestion decreases the positive effects brought by the fiscal policies that I analyze. It also decreases the steady state growth rate of the economy.

The introduction of relative congestion has stronger negative effects when one analyzes the increase in g , whereas the absolute congestion has stronger negative effects if one analyzes the decrease in the tax on capital income.

The normalized decrease of the tax on capital income determines a decrease in the consumption to output ratio of about 2% and an increase in the overall output level of 25%. In the short run it leads to welfare losses of about 2.8%, while in the long run it generates intertemporal welfare gains of 1.5%. The increase in absolute congestion causes a significant decrease in the welfare in the short run. As the absolute congestion increases, the long run welfare gains steadily diminish until they reach negative values.

The allocation of resources is determined by the capital intensity of the public production function. A decrease in the tax on capital income determines the reallocation of resources towards the sector less intensive in private capital. As relative congestion increases, the reallocation of resources from one sector to the other diminishes.

Table 1.a
Base Parameters Values

<i>Production parameters</i>	$a_J=1, s=0.2, b=0.35, d_G=0.035, d_K=0.05, a_Y=1, n=0.015,$ $d_0=0.35, \tau_0=0.2, d_1=0.2, \tau_1=0.246, d_2=0.5, \tau_2=0.1538$
<i>Congestion parameters</i>	$1-q_1=1-q_p=0, 0.5, 1$ $s_1=s_p=0, 0.5, 1$
<i>Preference parameters</i>	$\alpha=-1.5, \beta=1.75, \beta=0.04$
<i>Fiscal parameters</i>	$g=0.04, t_k=0.28, t_w=0.28, t_c=0.0$

Table 1.b

Base equilibria

			c/y	Kg/k	y/k	T	F	?	PV Govt Def
D=0.35	Q=1, S=0	No cong	0.84	0.32	0.464	96.1	96.1	2.166	-7.11
	Q=0 S=0	Rel. Cong	0.73	0.19	0.24	93.59	93.59	1.5	-4.23
	Q=1 S=1	Abs. Cong	0.83	0.32	0.4	96.15	96.15	1.5	-6.69
D=0.2	Q=1, S=0	No cong	0.84	0.33	0.47	94.77	97.5	2.166	-6.87
	Q=0 S=0	Rel. Cong	0.73	0.197	0.24	89.91	93.12	1.5	-2.81
	Q=1 S=1	Abs. Cong	0.839	0.324	0.405	94.6	97.4	1.5	-6.49
D=0.5	Q=1, S=0	No cong	0.84	0.32	0.46	97.57	95.59	2.16	-7.43
	Q=0 S=0	Rel. Cong	0.74	0.2	0.25	96.44	94.61	1.5	-5.56
	Q=1 S=1	Abs. Cong	0.83	0.31	0.39	97.65	95.73	1.5	-6.95

Table 2
Uncompensated Fiscal Changes Normalized with respect to Present Value of Fiscal Deficit V (Identical Production Function for Private and Public Capital)

b=0.35, d=0.35, s=0.2; ?=0.2,

Percentage change in the PV of budget deficit: -16.8

	%?c/y	%?y/k	%?kg/k	%?ck	%?kg	%?y	%?T	Short run welf. gains	Long run welf. gains
No congestion effect: q=1, s=0									
Increase in g: 0.04 to 0.08	-0.702	-3.7	92.59	33.81	157.7	28.86	-14.7	2.89	9.67
Decrease in tk: 0.28 to 0.2	-2	-9.88	-9.88	20.33	8.43	8.43	-11.4	-2.77	1.46
Congestion effects: q=0.5, s=0									
Increase in g: 0.04 to 0.0621	-0.648	-2.44	51.46	19.6	81.15	16.68	-14.94	2.89	9.67
Decrease in tk: 0.28 to 0.2336	-1.6	-6.05	-6.05	11.74	4.97	4.97	-13.28	-2.32	0.32
Congestion effects: q=0, s=0									
Increase in g: 0.04 to 0.0493	-0.43	-1.2	21.76	8.56	32.2	7.25	-15.85	-0.15	1.6
Decrease in tk: 0.28 to 0.2602	-0.975	-2.67	-2.67	4.94	2.132	2.132	-15.13	-1.33	0.13
Congestion effects: q=1, s=0.5									
Increase in g: 0.04 to 0.0805	-0.73	-3.74	93.7	27.17	146.3	22.41	-14.92	2.51	8.71
Decrease in tk: 0.28 to 0.1993	-2.11	-10	-10	16.71	4.94	4.94	-11.69	-3.48	0.080
Congestion effects: q=0.5, s=0.5									
Increase in g: 0.04 to 0.0624	-0.67	-2.47	52.14	15.95	76.42	13.09	-15.14	0.57	4.28
Decrease in tk: 0.28 to 0.2333	-1.74	-6.12	-6.12	9.63	2.91	2.91	-13.46	-2.76	-0.52
Congestion effects: q=0, s=0.5									
Increase in g: 0.04 to 0.0493	-0.44	-1.2	21.76	6.95	30.23	5.66	-15.85	-0.18	1.44
Decrease in tk: 0.28 to 0.26	-1.01	-2.7	-2.7	4.065	1.25	1.25	-15.28	-1.53	-0.51
Congestion effects: q=1, s=1									
Increase in g: 0.04 to 0.081	-0.76	-3.79	94.99	22.81	139.48	18.14	-15.12	2.2	7.81
Decrease in tk: 0.28 to 0.198	-2.2	-10.22	-10.22	14.19	2.52	2.52	-11.88	-4.18	-1.21

Congestion effects: $q=0.5, s=1$									
Increase in g: 0.04 to 0.0602	-0.64	-2.42	51.23	19.52	80.76	16.62	-14.88	0.7	4.68
Decrease in tk: 0.28 to 0.2328	-1.8	-6.15	-6.15	8.12	1.47	1.47	-13.51	-3.17	-1.29
Congestion effects: $q=0, s=1$									
Increase in g: 0.04 to 0.0495	-0.46	-1.22	22.22	5.96	29.52	4.66	-16.18	-0.22	1.29
Decrease in tk: 0.28 to 0.26	-1.04	-2.7	-2.7	3.42	0.63	0.63	-15.28	-1.71	-0.85

Table 3
Uncompensated Fiscal Changes Normalized with respect to Present Value of Fiscal Deficit V (Public Production Function more intensive in capital)

b=0.35, d=0.5, s=0.2; $\theta=0.1538$,

Percentage change in the PV of budget deficit: -16.8

	% θ c/y	% θ y/k	% θ kg/k	% θ ck	% θ kg	% θ y	% θ T	Short run welf. gains	Long ru welf. gains
No congestion effect: q=1, s=0									
Increase in g: 0.04 to 0.0882	-0.97	-5.04	109.3	43.56	200.6	36.32	-14.48	4.33	12.94
Decrease in tk: 0.28 to 0.1995	-2.03	-9.96	-9.96	20.77	8.68	8.68	-11.44	-2.81	1.52
Congestion effects: q=0.5, s=0									
Increase in g: 0.04 to 0.07	-0.85	-3.19	69.4	27.77	116.4	23.68	-14.5	2.03	7.66
Decrease in tk: 0.28 to 0.2306	-1.75	-6.35	-6.35	12.53	5.37	5.37	-13.16	-2.43	0.394
Congestion effects: q=0, s=0									
Increase in g: 0.04 to 0.0553	-0.61	-1.71	35.87	14.68	55.83	12.71	-14.98	0.61	3.638
Decrease in tk: 0.28 to 0.2554	-1.18	-3.26	-3.26	6.169	2.699	2.699	-14.84	-1.60	0.115
Congestion effects: q=1, s=0.5									
Increase in g: 0.04 to 0.0899	-1.02	-5.11	113.2	35.38	188.7	28.54	-14.68	3.83	11.7
Decrease in tk: 0.28 to 0.198	-2.13	-10.12	-10.12	16.96	5.11	5.11	-11.63	-3.51	0.13
Congestion effects: q=0.5, s=0.5									
Increase in g: 0.04 to 0.0707	-0.89	-3.22	71.28	22.7	110.1	18.74	-14.6	1.81	7.01
Decrease in tk: 0.28 to 0.2299	-1.84	-6.45	-6.45	10.2	3.18	3.18	-13.33	-2.89	-0.49
Congestion effects: q=0, s=0.5									
Increase in g: 0.04 to 0.0556	-0.64	-1.74	37.06	12.21	53.81	10.25	-15.17	0.55	3.4
Decrease in tk: 0.28 to 0.255	-1.24	-3.32	-3.32	5.1	1.6	1.6	-14.99	-1.86	-0.58
Congestion effects: q=1, s=1									
Increase in g: 0.04 to 0.0919	-0.64	-1.74	37.06	12.21	53.81	10.25	-15.71	0.55	3.4
Decrease in tk: 0.28 to 0.1968	-2.22	-10.26	-10.26	14.38	2.64	2.64	-11.78	-4.19	-1.17

Congestion effects: $q=0.5, s=1$									
Increase in g: 0.04 to 0.0724	-0.94	-3.35	74.93	19.71	109.4	15.7	15.12	1.63	6.48
Decrease in tk: 0.28 to 0.229	-1.91	-6.55	-6.55	8.79	1.66	1.66	-13.51	-3.35	-1.33
Congestion effects: $q=0, s=1$									
Increase in g: 0.04 to 0.0564	-0.675	-1.79	38.47	10.59	53.14	8.61	-15.52	0.47	3.13
Decrease in tk: 0.28 to 0.2548	-1.28	-3.34	-3.34	4.33	0.843	0.843	-15.04	- 2.098	-1.01

Table 4
Uncompensated Fiscal Changes Normalized with respect to Present Value of Fiscal Deficit V (Public Production Function less intensive in Capital)

b=0.35, d=0.2, s=0.2; ?=0.246,

Percentage change in the PV of budget deficit: -16.8

	%?c/y	%?y/k	%?kg/k	%?ck	%?kg	%?y	%?T	Short run welf. gains	Long ru welf. gains
No congestion effect: q=1, s=0									
Increase in g: 0.04 to 0.0773	-0.41	-2.27	88.85	28.02	141.78	25.11	-15.32	2.54	8.28
Decrease in tk: 0.28 to 0.202	-1.95	-9.82	-9.82	19.91	8.132	8.132	-11.22	-2.74	1.4
Congestion effects: q=0.5, s=0									
Increase in g: 0.04 to 0.0575	-0.45	-1.72	41.27	14.27	61.438	12.30	-15.63	0.101	3.033
Decrease in tk: 0.28 to 0.2368	-1.56	-5.71	-5.71	10.81	4.48	4.48	-13.27	-2.19	0.24
Congestion effects: q=0, s=0									
Increase in g: 0.04 to 0.04525	-0.25	-0.71	12.32	4.46	17.33	3.72	-16.62	-0.35	0.557
Decrease in tk: 0.28 to 0.2659	-0.708	-1.94	-1.94	3.46	1.45	1.45	-15.36	-0.97	-0.13
Congestion effects: q=1, s=0.5									
Increase in g: 0.04 to 0.077	-0.437	-2.3	88.07	22.18	129.79	19.37	-15.53	2.23	7.45
Decrease in tk: 0.28 to 0.201	-2.05	-9.94	-9.94	16.2	4.67	4.67	-11.51	-3.42	0.026
Congestion effects: q=0.5, s=0.5									
Increase in g: 0.04 to 0.0573	-0.468	-1.72	40.77	11.38	56.8	9.46	-15.78	0.028	2.72
Decrease in tk: 0.28 to 0.2367	-1.61	-5.72	-5.72	8.78	2.55	2.55	-13.37	-2.58	-0.54
Congestion effects: q=0, s=0.5									
Increase in g: 0.04 to 0.0451	-0.26	-0.69	11.96	3.53	15.92	2.8	-16.65	-0.35	0.48
Decrease in tk: 0.28 to 0.26623	-0.71	-1.9	-1.9	2.76	0.8	0.8	-15.29	-1.09	-0.39
Congestion effects: q=1, s=1									
Increase in g: 0.04 to 0.0771	-0.45	-2.33	88.24	18.46	123	15.69	-15.82	2.01	6.74
Decrease in tk:	-2.13	-10.05	-10.05	13.74	2.3	2.3	-11.67	-4.09	-1.25

0.28 to 0.2									
Congestion effects: $q=0.5, s=1$									
Increase in g: 0.04 to 0.0575	-0.49	-1.76	41.21	9.62	54.8	7.68	-16.19	- 0.037	2.44
Decrease in tk: 0.28 to 0.2365	-1.66	-5.75	-5.75	7.41	1.24	1.24	-13.5	-2.97	-1.27
Congestion effects: $q=0, s=1$									
Increase in g: 0.04 to 0.045	-0.26	-0.69	11.72	2.91	14.98	2.2	-16.7	-0.35	0.39
Decrease in tk: 0.28 to 0.2664	-0.72	-1.87	-1.87	2.3	0.38	0.38	-15.32	-1.2	- 0.625

Appendix

In this Appendix I derive the linear approximation to the macro-dynamic equilibrium employed in our simulations. I begin with the four equilibrium equations (18a) – (18c) written as

$$\dot{k} = y - c - k(\mathbf{d}_k + \mathbf{y}) \quad (\text{A.1a})$$

$$\dot{k}_G = gy - k_G(\mathbf{d}_G + \mathbf{y}) \quad (\text{A.1b})$$

$$\frac{\dot{c}}{c} = \frac{1}{\mathbf{g}-1} \left(\mathbf{b} + n\mathbf{g} + \mathbf{d}_k - \mathbf{y}(\mathbf{g}-1) - (1-t_k)(b+s(1-q_1)) \frac{y}{fk} \right) \quad (\text{A.1c})$$

Given (17a) and (17b) one can find the time derivatives :

$$\dot{\mathbf{q}} = \frac{\partial \mathbf{q}}{\partial k} \dot{k} + \frac{\partial \mathbf{q}}{\partial k_g} \dot{k}_G \text{ and } \dot{\mathbf{f}} = \frac{\partial \mathbf{f}}{\partial k} \dot{k} + \frac{\partial \mathbf{f}}{\partial k_g} \dot{k}_G \quad (\text{A2})$$

Substituting these expressions into (A.1 a-c) one can derive the linearized equilibrium:

$$\begin{aligned} \dot{k} = & \left[(b-s_1) \frac{y}{\mathbf{f}} + (b+s(1-q_1)) \frac{y}{k} \frac{\partial \mathbf{f}}{\partial k} + (1-b) \frac{y}{\mathbf{q}} \frac{\partial \mathbf{q}}{\partial k} - (\mathbf{d}_k + \mathbf{y}) \right] (k - \tilde{k}) \\ & + \left[(b+s(1-q_1)) \frac{y}{\mathbf{f}} \frac{\partial \mathbf{f}}{\partial k_g} + (1-b) \frac{y}{\mathbf{q}} \frac{\partial \mathbf{q}}{\partial k} + s \frac{y}{k_g} \right] (k_g - \tilde{k}_g) - (c - \tilde{c}) \end{aligned} \quad (\text{A.3a})$$

$$\begin{aligned} \dot{k}_G = & \left[g(b+s(1-q_1)) \frac{y}{\mathbf{f}} \frac{\partial \mathbf{f}}{\partial k} + g(b-s_1) \frac{y}{k} + g(1-b) \frac{y}{\mathbf{q}} \frac{\partial \mathbf{q}}{\partial k} \right] (k - \tilde{k}) \\ & + \left[g(b+s(1-q_1)) \frac{y}{\mathbf{f}} \frac{\partial \mathbf{f}}{\partial k_g} + g(1-b) \frac{y}{\mathbf{q}} \frac{\partial \mathbf{q}}{\partial k_g} + g s \frac{y}{k_g} - (\mathbf{d}_G + \mathbf{y}) \right] (k_g - \tilde{k}_g) \end{aligned} \quad (\text{A.3b})$$

$$\dot{c} = -\frac{c}{\mathbf{g}-1}(1-\mathbf{t}_k)(b+\mathbf{s}(1-q_1))\frac{y}{\mathbf{f}k} \left\{ \left[\frac{1}{k}(b-1-\mathbf{s}s_1) + \frac{(b+\mathbf{s}(1-q_1)-1)}{\mathbf{f}} \frac{\partial \mathbf{f}}{\partial k} + \frac{(1-b)}{\mathbf{q}} \frac{\partial \mathbf{q}}{\partial k} \right] (k-\tilde{k}) \right. \\ \left. + \left[\frac{(b+\mathbf{s}(1-q_1)-1)}{\mathbf{f}} \frac{\partial \mathbf{f}}{\partial k_G} + \frac{(1-b)}{\mathbf{q}} \frac{\partial \mathbf{q}}{\partial k_g} + \frac{\mathbf{s}}{k_g} \right] (k_G-\tilde{k}_G) \right\} \quad (\text{A.3c})$$

Note that these expressions all involve the partial derivatives

$$\frac{\partial \mathbf{q}}{\partial k}, \frac{\partial \mathbf{f}}{\partial k}, \frac{\partial \mathbf{q}}{\partial k_G} \text{ and } \frac{\partial \mathbf{f}}{\partial k_G}$$

To evaluate these, I proceed in the following sequential manner. Taking differentials of the sectoral allocation equations (16a) and (16b) I obtain:

$$\mathbf{x}(1+\mathbf{h}(1-q_p)) \left(-\frac{\partial \mathbf{f}}{\partial k} \right) (1-\mathbf{f})^{h(1-q_p)} \left[\mathbf{f}(1-b)(d+\mathbf{h}(1-q_p)) + (1-\mathbf{f})(1-d)(b+\mathbf{s}(1-q_1)) \right]^{d-b} \\ + \mathbf{x}(1-\mathbf{f})^{1+h(1-q_p)} (d-b) \left[\frac{\partial \mathbf{f}}{\partial k} (1-b)(d+\mathbf{h}(1-q_p)) - \frac{\partial \mathbf{f}}{\partial k} (1-d)(b+\mathbf{s}(1-q_1)) \right] * \quad (\text{A.4.a})$$

$$\left[\mathbf{f}(1-b)(d+\mathbf{h}(1-q_p)) + (1-\mathbf{f})(1-d)(b+\mathbf{s}(1-q_1)) \right]^{d-b-1} = \\ = [1+\mathbf{s}(1-q_1)] \frac{\partial \mathbf{f}}{\partial k} \mathbf{f}^{\mathbf{s}(1-q_1)} k^{b-\mathbf{s}s_1-d+\mathbf{h}s_p} k_g^{\mathbf{s}-\mathbf{h}} + \mathbf{f}^{1+\mathbf{s}(1-q_1)} (b-\mathbf{s}s_1-d+\mathbf{h}s_p) k^{b-\mathbf{s}s_1-d+\mathbf{h}s_p-1} k_g^{\mathbf{s}-\mathbf{h}}$$

$$\mathbf{x}(1+\mathbf{h}(1-q_p)) \left(-\frac{\partial \mathbf{f}}{\partial k_g} \right) (1-\mathbf{f})^{h(1-q_p)} \left[\mathbf{f}(1-b)(d+\mathbf{h}(1-q_p)) + (1-\mathbf{f})(1-d)(b+\mathbf{s}(1-q_1)) \right]^{d-b} + \\ \mathbf{x}(1-\mathbf{f})^{1+h(1-q_p)} (d-b) \left[\frac{\partial \mathbf{f}}{\partial k_g} (1-b)(d+\mathbf{h}(1-q_p)) - \frac{\partial \mathbf{f}}{\partial k_g} (1-d)(b+\mathbf{s}(1-q_1)) \right] * \quad (\text{A.4.b})$$

$$* \left[\mathbf{f}(1-b)(d+\mathbf{h}(1-q_p)) + (1-\mathbf{f})(1-d)(b+\mathbf{s}(1-q_1)) \right]^{d-b-1} = \\ = [1+\mathbf{s}(1-q_1)] \frac{\partial \mathbf{f}}{\partial k_g} \mathbf{f}^{\mathbf{s}(1-q_1)} k^{b-\mathbf{s}s_1-d+\mathbf{h}s_p} k_g^{\mathbf{s}-\mathbf{h}} + \mathbf{f}^{1+\mathbf{s}(1-q_1)} (\mathbf{s}-\mathbf{h}) k^{b-\mathbf{s}s_1-d+\mathbf{h}s_p} k_g^{\mathbf{s}-\mathbf{h}-1}$$

$$\text{where } \mathbf{x} = \frac{\mathbf{a}_j (1-d)^{1-d} (b + \mathbf{s}(1-q_1))^{1-d}}{g\mathbf{a}_y (1-b)^{1-b} (d + \mathbf{h}(1-q_p))^{1-b}}$$

$$\frac{\partial \mathbf{q}}{\partial x} = \frac{\partial \mathbf{f}}{\partial x} \frac{(d + \mathbf{h}(1-q_p))(1-b)(1-d)(b + \mathbf{s}(1-q_1))}{\left[\mathbf{f}(1-b)(d + \mathbf{h}(1-q_p)) + (1-\mathbf{f})(1-d)(b + \mathbf{s}(1-q_1)) \right]^2}, \text{ where } x = k, k_g \text{ (A.4.c)}$$

Expressions (A.4 a-c) yield all the partial derivatives and substituting these partial derivatives into the elements (A3.a-c) yields the linearized dynamics of the equilibrium system.

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