

Leisure Externalities: Implications for Growth and Welfare

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Abstract

This paper develops a neoclassical growth model with leisure externalities. Ignoring positive (negative) leisure externalities leads to equilibrium consumption, labor and capital that are too high (low) and leisure that is too low (high). The government should tax (subsidize) labor income according to whether the leisure externality is positive or negative. The level of this tax (subsidy) depends on the elasticity of individual and average leisure and the consumption tax. Equilibrium dynamics are characterized, and two shocks to the economy are analyzed – an increase in the growth rate of labor productivity, and an increase in the tax on labor income – by simulating a calibrated economy. Adjustment processes of key variables in a competitive and centrally planned economy with and without leisure externalities are also compared.

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A growing economic literature is concerned with the effects that average consumption has on individual consumption. The phrases “keeping up with the Joneses” and “habit formation” are now found everywhere and are symptomatic of this trend. Theoretical models were developed to explain some of the asset pricing literature puzzles and the empirical literature followed to test their implications. However, even though leisure is an important component of an individual’s well being and utility function, it has not yet generated the same level of interest as average consumption has. Economists are only paying scant attention to any type of externalities that leisure might provide.

We argue that there are significant complementarities in the enjoyment of leisure at the community level and that leisure can represent one’s social status as much as conspicuous consumption does. There is a lot of anecdotal evidence that many of the leisure activities are more enjoyable if they are done with others (sports, trips, shopping and even watching TV). Using the British Household Panel Survey, Jenkins and Osberg (2003) show that spouses synchronize their working time so that they can spend their leisure time together. They also show that individuals’ participation in associational activity depends on the leisure time and activity of others in their community. Hamermesh (2002) and Hunt (1998) found similar effects of working hours synchronization between spouses using United States and German data respectively. The concentration of working hours to 9-5, Monday-Friday, and the tradition of European August vacation despite the disadvantages due to crowded infrastructure also show that people have a preference to rest when others rest and work when others work. Alesina et al. (2005) notice significant differences in labor force participation across demographic subgroups within areas and suggest that leisure complementarities is an explaining factor, along with the tax rates and labor market regulations. They also argue that leisure complementarities are an important explanatory factor for the difference in working hours in Europe and the US starting in 1970s. In Europe unions managed to impose a philosophy of “work less, work all” and reduce the amount of hours worked by each individual in the hope that the work will be spread across more workers and reduce the unemployment rate. This has led to the development of a culture where people are used to working less and enjoy more leisure even though it has not reached its primal objective

of reducing unemployment. The US have not experienced this push towards fewer hours and consequently 30 years later the numbers of hours worked per capita decreased by about 12% in EU-15, whereas US experienced an increase of about 20% (OECD). Alesina et al. (2005) conjectured that leisure complementarities have led to a “social multiplier” that intensified the effects of tax differences in the two regions.

The sociology literature gives significant evidence that along consumption, work and leisure are powerful symbols of social status. The amount of leisure of others, more specifically the dominant class has an important social status and significant effects on the leisure of individuals. This concept can be traced back to Veblen’s Theory of Leisure Class (1899).

Throughout most of the human history leisure was the symbol of the dominant social status. The dominant class in the feudal system was represented by hunters and warriors who thought working the land or in trade was degrading. The middle class men (bourgeoisie) who had to work for a living were considered second class and tried to assert their status by having at least their wives and daughters in a state of “wasteful idleness” (Gershuny 2000). The use of time in a non productive manner was due to a sense of the unworthiness of productive work, and leisure was used as a mean to gain the respect of others (Veblen 1899). For the working class the income effect dominated the substitution effect, demonstrating once again a preference for leisure. At the beginning of the industrial revolution, increasing workers’ salaries had the perverse effect of discouraging people from working. Employees worked as much as they needed to earn a certain amount of money and spent the rest of their time in a leisurely manner (Schor 1991, Veal 2004).

Later, the “bored wives” of the 1890s American middle class (Gershuny 2000), or even more recently, the housewives of the 1950s were symptomatic of men’s attempt to mirror the habits of the upper class by being the sole providers for their families and by maintaining their wives in a state of at least apparent leisure.

How significant leisure is as a symbol of social status can be measured by changes in leisure’s perception when leisure habits of the dominant class evolve. At the start of the 21st century, work is the new symbol of dominant social status. Rajan and Zingales (2003) point out the difference between the “idle rich” of 1929 when 70 percent

of the income of the top .01 percent of income earners in the United States came from holding of capital and the “working rich” of 1998 when wages and entrepreneurial income made up 80 percent of the income of the top .01 percent of income earners in the United States. They also notice that in the 1890s the richest 10 percent of the population worked fewer hours than the poorest 10 percent, whereas the opposite is true today. Also, over the 1961-2001 period, higher human capital groups have increased their work time relative to the lower human capital groups from 0.94 (1) to 1.034 (1.094) for men (women). A similar pattern applies to Canada, France, the Netherlands, the US (Changing times, p. 177, table 7.9). The reason for which higher human capital groups have increased their work load is that the industrialization process has continuously increased the importance of human capital in the production process. Goldin and Katz (2001) and Abramowitz and David (2000) show that the contribution of human capital accumulation to the growth process nearly doubled since 1890. Since human capital is embedded in people, the increased importance of human capital for the production process implies that the ones who possess high levels of human capital and potential for high earning have to work long hours to capitalize on that potential.

As a consequence of these changes in the distribution of who works more hours, having a paid job, besides the income that it would bring has increased in desirability because it improves one’s social status. Thus “busyness” becomes the new “badge of honor” (Gershuny 2005). Stay-at-home moms have to defend their choice of not also having a career outside the household. Rich heiresses make declarations about how busy they are designing purses for a living. Overall, the society frowns upon not being involved in paid work and not having an active, goal oriented leisure time.

We develop a neoclassical general equilibrium model that takes into consideration these leisure externalities. In the case of a negative leisure externality (i.e. crowding of the leisure facilities like parks and public swimming pools), we show that equilibrium consumption, labor and capital are lower than their respective socially optimal equilibrium values and therefore equilibrium leisure is too high. In the case of a positive leisure externality (i.e. people enjoy leisure more when others do as well) equilibrium consumption, labor and capital are shown to be too high and leisure suboptimal.

We estimate the tax rates necessary to bring the economy to a social optimum. We show that the optimal capital tax is equal to 0 and we determine the tax on labor income required to rectify the source of distortions. This tax depends on the elasticity of individual and average leisure and the consumption tax. The government should tax (subsidize) labor income according to whether the leisure externality is positive or negative.

We focus on the case when leisure externalities are *positive*. The higher the effect that average leisure has on individual utility, the higher is the tax necessary to bring the competitive economy to a social optimum. Imposing this tax leads to significant welfare increases both in the short and in the long run.

We use a constant elasticity of substitution (CES) utility function with different degrees of leisure externalities and a Cobb-Douglas production function to calibrate the model numerically and analyze the transitional dynamics and implications for welfare given by different exogenous shocks to the economy: an increase in the growth rate of labor productivity and an increase of the tax on labor income.

The transition paths are sensitive to the presence of leisure externalities both in the centrally planned and competitive economy. Even though leisure externalities have no impact on the choice of leisure and consumption in the long run in the competitive economy they affect the transition paths. They reduce the immediate impact that productivity and fiscal shocks have on leisure and affect the speed of convergence. The presence of leisure externalities has different effects on the speed of convergence in the centrally planned and competitive economy. In the centrally planned economy, leisure externalities speed up the convergence rate whereas the opposite is true in the competitive economy.

An increase in the growth rate of labor productivity leads to a permanent increase in the equilibrium growth rate, so that quantities per capita grow forever at a higher rate. It leads to an immediate decrease in both the investment rate and the equilibrium labor, so that the agent enjoys both more leisure and more consumption and thus higher welfare. The centrally planned economy where leisure externality is present enjoys the highest level of leisure and the smallest relative increase in leisure in the long run.

An increase in the tax on labor income leads to an increase in leisure as the return from labor is affected. Short run welfare increases as the increase in leisure compensates for the decrease in consumption. An increase in the labor income tax leads to losses in intertemporal welfare when there are no leisure externalities, but has a positive effect in the opposite case.

Even if the agent from the competitive economy does not take into consideration the effects of leisure externality when maximizing utility, these effects are apparent ex post when the welfare is estimated. Since both the increase in productivity growth and in tax on labor income lead to increases in the leisure level in the long run, as leisure externalities increase this has a stronger positive influence on welfare.

The remainder of the paper proceeds as follows. Section 2 sets out the structure of the model, outlines the steady-state equilibrium in the competitive and centrally planned economy and derives the optimal tax levels. Section 3 characterizes the equilibrium dynamics in the competitive and centrally planned economy. Section 4 calibrates the model and considers the numerical effects of an increase in the growth rate of labor productivity and an increase in the tax on labor income. Section 5 provides concluding remarks, while technical details of the solution are provided in the Appendix

2. The Model

2.1 Representative Consumer

The representative consumer is endowed with one unit of time that can be allocated to leisure, l_i , leaving $(1-l_i)$ available for work. The economy is populated by N agents, all identical, and the population growth is n . Let \hat{l} denote the average leisure in the economy $\hat{l} = \sum l_i / N$. In equilibrium since all agents are identical $\hat{l} = l_i$. Agent i owns K_i units of private capital and C_i is the consumption of the representative household.

Given the externalities in leisure we assume that the representative agent's welfare depends not only on his own consumption and leisure, but also on the average leisure in the economy and is specified by the intertemporal isoelastic utility function:

$$\max \int_0^{\infty} u(C_i, l_i, \hat{l}) e^{-\rho t} dt \quad (1)$$

The key issue is the externality imposed by average leisure on the well being of the individual agent. The household's utility is positively influenced ($u_{\hat{l}} > 0$) if an increase in average leisure give one the opportunity to “play” with more people or reduces the stigma associated with being a slacker. The average leisure can negatively influenced the household's utility ($u_{\hat{l}} < 0$) i.e. due to crowding of leisure amenities. The utility function which includes the leisure externalities in this model is similar with specifications promoted in the literature devoted to consumption externalities (Gali 1994, Ljungqvist and Uhlig 2000, Dupor and Liu 2003, Liu and Turnovsky 2004). This specification treat the utility functions as time separable as opposed to the habit formation models adopted by Carroll et al. (1997, 2000), Ljungqvist and Uhlig (2000), Alvarez-Cuadrado et al. (2004). The steady state properties of the two models are not very different given that the reference stocks converge to stationary models, however the transition dynamics would be different. The question whether leisure can be also habit forming is an interesting topic that has not been addressed sufficiently in the economic literature even though there are some studies dealing with its implications for growth (Gurdgiev 2004, Karayalcin 2003) We choose the time separable utility function due to the plethora of evidence presented in the introduction in which leisure is complementary across individuals at distinct moments in time.

We impose that u possess continuous first and second order partial derivatives: $u_c > 0$, $u_{cc} < 0$, $u_l > 0$, $u_{ll} < 0$ and $u_{cc}u_{ll} - u_{cl}^2 > 0$. Further conditions are imposed on the strength of the external leisure effects to ensure that either the externality augments the direct effect or if it is offsetting it is dominated by the direct effect. $u_l + u_{\hat{l}} > 0$, $u_{ll} + u_{\hat{l}\hat{l}} < 0$ and $u_{cl} + u_{c\hat{l}} > 0$.

The agent's objective is to maximize (1) subject to his accumulation equation,

$$\dot{K}_i = F(K_i, 1 - l_i) - (n + \delta_k)K_i - C_i \quad (2)$$

where $F(K_i, 1-l_i)$ is the production function and δ_k is the depreciation of the capital.

The optimality conditions are:

$$u_c = \lambda_{i,d} \quad (3a)$$

$$u_l = \lambda_{i,d} F_{1-l} \quad (3b)$$

$$F_k - n - \delta_k = -\frac{\dot{\lambda}_{i,d}}{\lambda_{i,d}} + \rho \quad (3c)$$

where $\lambda_{i,d}$ is the shadow value of an additional unit of capital in the competitive economy. Eq (3a) equates the marginal utility of consumption with the shadow value of capital, eq. (3b) equates the marginal utility of leisure with the marginal utility derived from the additional output if labor increases by one unit. Finally, eq. (3c) is the standard Keynes-Ramsey consumption rule, equating the rate of return on consumption to the rate of return on capital.

2.2 Central planner

In deriving the optimal allocation of resources in the competitive economy, the individual agent neglects the effects that her own leisure has on the utility that the others derive from their own leisure. Therefore the equilibrium optimum might diverge from the socially optimal level. To derive this socially optimal allocation of resources we consider a **central planner** who takes into account the externality imposed by average leisure when maximizes (1) subject to the budget constraint (2).

The optimality conditions in this case are

$$u_c = \lambda_{i,c} \quad (4a)$$

$$u_l + u_l = \lambda_{i,c} F_{1-l} \quad (4b)$$

$$F_k - (n + \delta_k) = -\frac{\dot{\lambda}_{i,c}}{\lambda_{i,c}} + \rho \quad (4c)$$

where $\lambda_{i,c}$ is the social shadow value of an additional unit of capital in the economy. The interpretation of equations (4a) - (4c) mirrors that of (3a)-(3c) with the comment that (4b) reflects the externality imposed by the average leisure on the marginal utility of leisure.

Imposing that $\dot{\lambda}_{i,c} = \dot{\lambda}_{i,d} = 0$ we can derive the steady state values K_i, C_i and l_i in the competitive and the centrally planned economy and the following properties can be inferred (Appendix A):

In the case of a negative leisure externality ($u_l < 0$) equilibrium consumption, labor and capital are lower than their respective long run equilibrium values and therefore equilibrium leisure is too high.

In the case of a positive leisure externality ($u_l > 0$) equilibrium consumption, labor and capital are too high and leisure is suboptimal.

Externality in leisure leads to a divergence between the long run competitive equilibrium and the socially optimal one. In the following section we derive the taxes that the social planner can impose to achieve the first best optimum allocation in the economy in a competitive setting.

2.3 Optimal tax rates

Let τ_k be the tax rate imposed on the return to capital, τ_w the tax rate imposed on labor income, τ_c the tax on consumption and T_i lump sum taxes.

Consider again the competitive economy populated by identical agents. The budget constraint in this case is

$$\dot{K}_i = [(1 - \tau_k)r - n - \delta_k]K_i + (1 - \tau_w)(1 - l_i)w - (1 + \tau_c)C_i + T_i \quad (5)$$

where $r = F_k$ is the return to capital, $w = F_{1-l}$ is the labor income and all taxes are remitted back to the agent in the form of lump sum taxes T_i .

The individual agent maximizes (1) subject to the budget constraint (5).

The optimality conditions in this case are

$$u_c = \lambda_{i,do}(1 + \tau_c) \quad (6a)$$

$$u_l = \lambda_{i,do}w(1 - \tau_w) \quad (6b)$$

$$(1 - \tau_k)r - n - \delta_k = \rho - \frac{\dot{\lambda}_{i,do}}{\lambda_{i,do}} \quad (6c)$$

where $\lambda_{i,do}$ is the shadow value of an additional unit of capital in the competitive economy with taxes. (6a) equates the marginal utility of consumption to the individual's tax adjusted shadow value of wealth, while (6b) equates the marginal utility of leisure to its opportunity cost, the after tax real wage, valued at the shadow value of wealth. The third equation is the standard Keynes-Ramsey consumption rule, equating the rate of return on consumption to the after-tax rate of return on capital.

The optimal taxes are chosen such that the time path of K_i, C_i and I_i are identical in the competitive and centrally planned economy. Replicating the dynamic path of the centrally planned economy requires $\dot{\lambda}_{i,c} / \lambda_{i,c} = \dot{\lambda}_{i,do} / \lambda_{i,do}$ which implies that $\lambda_{i,c} = \varepsilon \lambda_{i,do}$, where ε is an arbitrary constant.

Using (4a) and (6a) the optimal tax on consumption can be derived:

$$\lambda_{i,c} = \lambda_{i,do}(1 + \tau_c) \quad \therefore \quad \tau_c = 1 - \varepsilon = \bar{\tau}_c \text{ constant} \quad (7a)$$

Using (4c), (6c) and (7a), the optimal tax on capital income is shown to be zero, in line with results of Chamley (1986) and Judd (1985)

$$\tau_k = 0 \quad (7b)$$

The key distortion of the model comes from the difference in the willingness to substitute consumption for leisure between the competitive and socially optimal economy. The marginal rate of substitution of consumption for leisure:

$$MRS^d = -\frac{u_c}{u_l} > (<) -\frac{u_c}{u_l + u_{\dot{l}}} = MRS^c \quad \text{if } u_{\dot{l}} > (<) 0 \quad (7c)$$

It shows that the agent from the competitive economy undervalues (overvalues) leisure, if leisure externalities are positive (negative) and its willingness to substitute consumption for leisure is too high (low) relative to the socially desirable level.

Tax on labor income corrects for this distortion. Using (4b), (6b) and (7a) we estimate it to be:

$$\tau_w = 1 - \frac{u_l}{u_l + u_{\dot{l}}}(1 + \bar{\tau}_c) \quad (7d)$$

It depends on the elasticity of individual leisure, the extent of leisure externalities and the consumption tax.

Tax on consumption and capital are fixed, whereas tax on labor income is time-varying and converges to a constant level as capital, consumption and labor converge to their respective steady state equilibrium values. These results are similar with those from the literature analyzing optimal taxation when consumption externalities are present (Fisher and Hof 2000, Liu and Turnovsky 2005) with the remark that due to the different nature of distortions in their models it is the tax on consumption that is time-varying.

If there are no leisure externalities then (7c) becomes $\tau_w = -\bar{\tau}_c$. Since $\bar{\tau}_c$ raises the price of consumption in terms of leisure, in the absence of leisure externality τ_w moves into the opposite direction to correct for this distortion.

Given that the wage tax corrects the distortions from the labor market, consumption tax can be set to 0. Then, the government should tax (subsidize) labor income according to whether the leisure externality is positive or negative.

3. Dynamics

Externalities lead to a divergence between the long run competitive equilibrium and the socially optimal one. They also affect transitional dynamics. We illustrate these differences in the context of an exogenous growth model:

$$F(K_i, 1-l_i) = Y_i = \alpha(A(1-l_i))^\sigma K_i^{1-\sigma} \quad (8)$$

Where labor productivity grows at an exogenous constant rate g (i.e. $\dot{A}/A = g$).

Consider the aggregate production function: $Y = NY_i = \alpha(AN(1-l_i))^\sigma K^{1-\sigma}$ which is constant returns to scale in labor and capital and where $K = NK_i$; $L = NL_i = N(1-l_i)$;

Taking percentage changes, assuming that the ratio K/Y is constant and therefore the growth rates of output and capital are equal $\psi_K = \psi_Y$, we obtain $\psi_Y = \sigma g + \sigma n + (1-\sigma)\psi_Y$ which implies that the growth rate of the economy is constant and exogenous $\psi_Y = n + g$.

We define the scale adjusted variables that are constant in steady state:

$$k = \frac{K}{NA} = \frac{K_i}{A}; c = \frac{C}{NA} = \frac{C_i}{A}; y = f(k, 1-l) = \frac{Y}{NA} = \frac{Y_i}{A} = \alpha(1-l)^\sigma k^{1-\sigma} \quad (9b)$$

It is also assumed that the government maintains a balanced budget:

$$\tau_k Kr + \tau_w N(1-l)w + \tau_c C = T \quad (10)$$

and since the marginal product of capital and labor respectively are

$$r = (1-\sigma)\frac{Y_i}{K_i} \text{ and } w = \sigma\frac{Y_i}{1-l_i} \text{ the budget constraint can be rewritten in aggregate form as:}$$

$$\dot{K} = Y - C - \delta_k K \quad (5')$$

To express the dynamics and the difference between the transition paths in the centrally planned and competitive economies we use a constant elasticity of substitution (CES) utility function:

$$u(C_i, l_i, \hat{l}) = \frac{1}{\gamma} \left[C_i l_i^\phi \hat{l}^\beta \right]^\gamma \quad (11)$$

where the conditions imposed on the general utility function (1) are reflected in $\gamma(1+\phi) < 1$, $\phi > 0$, $\gamma < 1$, $\phi + \beta > 0$, $\gamma(\phi + \beta) < 1$.

Given this utility we can compare the intertemporal elasticity of substitution of consumption and leisure in the case when leisure externalities are present and when they are not:

$$IES_c^{\beta \neq 0} = IES_c^{\beta = 0} = -\frac{u_c}{cu_{cc}} = \frac{1}{1-\gamma} \quad (12a)$$

$$IES_l^{\beta \neq 0} = -\frac{u_l}{lu_{ll}} = \frac{1}{1-\gamma(\phi + \beta)} \text{ and } IES_l^{\beta = 0} = \frac{1}{1-\gamma\phi} \quad (12b)$$

Leisure externalities do not affect the intertemporal elasticity of substitution of consumption, but they affect intertemporal elasticity of substitution of leisure. Assuming that the intertemporal elasticity of substitution of both consumption and leisure is less than 1, then $\gamma < 0$ and $IES_l^{\beta \neq 0} < (>) IES_l^{\beta = 0}$ if $\beta > (<) 0$. Willingness to shift leisure over time is lower (higher) when leisure externalities are positive (negative), and thus the speed of adjustment of the economy is lower (higher).

Substituting in (7c) and remembering (7a) the optimal tax on labor income

$$\text{becomes } \tau_w = 1 - \frac{\phi}{\phi + \beta} (1 + \bar{\tau}_c) \text{ and depends on the effect that own leisure and average}$$

leisure have on the utility of individual consumer and the arbitrary consumption tax. We

also note that using the CES utility function the optimal tax on labor income is constant over time. Keeping consumption tax, $\bar{\tau}_c$, constant, the higher is β (magnitude of leisure externalities), the higher is the tax on labor income necessary to bring the competitive economy to a social optimum.

$$\frac{\partial \tau_w}{\partial \beta} = \frac{\phi}{(\phi + \beta)^2} (1 + \bar{\tau}_c) > 0$$

The equilibrium dynamics of the competitive economy can be expressed in terms of the stationary variables by the following system:

$$(\gamma - 1) \left(\frac{\dot{c}}{c} + g \right) + \gamma (\phi + \beta) \frac{\dot{l}}{l} = \rho + n + \delta_k - (1 - \tau_k)(1 - \sigma) \frac{y}{k} \quad (13a)$$

$$\frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k} - \delta_k - n - g \quad (13b)$$

$$\phi \frac{c}{y} = \sigma \frac{l}{1-l} \frac{(1 - \tau_w)}{(1 + \tau_c)} \quad (13c)$$

Equation (13a) is obtained by taking the time derivative of (6a), combining it with (6c) and (9b) and noting that $r = (1 - \sigma) \frac{y}{k}$. Equation (13b) is the accumulation equation for scale-adjusted capital and is obtained by combining (9b) with (5'). Equation (13c) is obtained by dividing (6a) by (6b), using the CES utility function (11), the equilibrium real wage $w = \sigma \frac{yA}{1-l}$ and the scale adjusted variables (9b). It implies that the consumption to output ratio, given leisure, is increasing with a decrease in the tax on labor income and tax on consumption.

Similarly, the equilibrium dynamics of the centrally planned economy can be expressed, using the optimality conditions (4a)–(4c), the budget constraint (5'), the utility function (11) and stationary variables (9b) by the following equations:

$$(\gamma - 1) \left(\frac{\dot{c}}{c} + g \right) + \gamma(\phi + \beta) \frac{\dot{l}}{l} = \rho + n + \delta_k - (1 - \sigma) \frac{y}{k} \quad (14a)$$

$$\frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k} - \delta_k - n - g \quad (14b)$$

$$\frac{c}{y} = \sigma \frac{l}{1 - l} \frac{1}{(\phi + \beta)} \quad (14c)$$

Equation (14c) implies that the consumption to output ratio, given leisure, is increasing with a decrease in β .

In Appendix B we show how for both the centrally planned and competitive economy, these systems can be reduced to autonomous sets of differential equations in k and l , which then form the basis for our subsequent numerical work. These two order systems have one sluggish variables, k and one jump variable, l . We can not solve formally these systems of differential equations and we linearize around the steady state to derive numerically the time path of leisure, capital and consumption in two cases: as a reaction to an exogenous increase in the productivity level g and an exogenous increase in the tax on labor income. To yield a well behaved dynamic behavior we require that the determinants of the linearized systems are negative, a property that we found to prevail over all of our wide-ranging simulations.

4. Numerical analysis of the transition paths

Further insight into the effects of different economic shocks can be obtained by carrying out numerical analysis of the model. We begin by characterizing a benchmark economy, calibrating the model using the parameters representative of the US economy (Table 1). Most of these parameters are standard and non-controversial.

The labor share of income is $\sigma=0.65$ and population grows at an annual rate of 1.5%. The value of $\gamma=-1.5$ implies an intertemporal elasticity of substitution in consumption of 0.4, consistent with the estimation by Ogaki and Reinhardt (1998). The annual depreciation rates $\delta_k=0.05$ approximate the average depreciation rates for private capital for the US during recent years. The elasticity of private leisure $\phi = 1.75$ and the rate of time preference $\rho=0.04$ accord with standard values in the business cycle

literature. All these values are well documented, the only parameter for which we do not have an estimate is β which represents the effect of average leisure on individual utility.

In the numerical analysis we focus on positive leisure externalities¹. We use $\beta=0$ (no leisure externality) as our benchmark model and choose $\beta = 1.5$ as the magnitude of the positive leisure externalities. Even though we do not have an estimate for the magnitude of leisure externalities from the empirical literature, it is plausible that average leisure would have a lower effect on individual utility than own leisure. Therefore we choose $\beta < \phi$, lower than the elasticity of private leisure.

These parameters lead to the benchmark equilibria reported in Table 2.a. and 2.b. The models yield a consumption-output ratio of about 0.8, and an output-capital ratio that increases from 0.3 to 0.44, as the productivity growth is 2% higher. An increase in the productivity rate also leads to an increase in leisure and an increase in the speed of convergence as measured by the stable (negative) eigenvalues associated with the linear approximation of the dynamic systems in k and l derived in Appendix B. All these values are in line with empirical evidence on OECD economies. The central planner from Table 2.b. incorporates leisure externality in her optimization problem such that when this externality is present the leisure ratio increases by 17% (17.43%) when $g=2\%$ (0%). Taking into consideration the effect of leisure externality in the optimization problem leads to significant welfare increases both in the short and in the long run (Table 3). An increase in the tax level from 0 to 46.15% which based on the parameters of the model is estimated to bring the competitive economy to a social optimum when leisure externality is present, would lead to an impressive increase in the short run welfare of about 33% and an increase in the intertemporal welfare of about 16%. The optimal tax level depends on the elasticity of leisure and the consumption tax, and is independent of the production parameters, including the growth rate of labor productivity g . The growth rates of the economy are independent of policy and leisure externalities and equal 1.5% when the growth rate of the productivity of labor is 0, and 3.5% when g is 2%. Leisure externalities reduce the instantaneous impact that productivity and fiscal shocks have on leisure. The presence of leisure externality has different effects on the speed of convergence in the centrally planned and competitive economy. In the competitive economy it slows down

¹ The evidence given in the introduction favors the idea of positive leisure externalities.

the convergence rate from 6.69% to 6.42% if $g=0$ and from 9.73% to 9.3% if $g=2\%$. In the centrally planned economy, where the externality effect is incorporated into the maximization problem it has the opposite effect, of increasing the convergence rate to 6.77% for $g=0$ and 9.87% for $g=2\%$. The intuition for that is provided by the analysis of the intertemporal rate of substitution of leisure (12b) and the marginal rate of substitution of consumption for leisure (7c). First, from (12b) we note that willingness to shift leisure over time is lower when positive leisure externalities are present, and thus the speed of adjustment of the economy is lower for both the competitive and centrally planned economy. The competitive economy is only affected by this intertemporal rate of substitution effect, and thus in the presence of positive leisure externalities slows down the convergence rate of this economy. Second, (7c) shows that, in the presence of positive leisure externalities, the central planner values leisure more, and its willingness to substitute consumption for leisure is lower than if there are no leisure externalities. Therefore the central planner chooses more leisure and less capital and the economy reaches its steady state faster. The second effect dominates the first effect in the centrally planned case and thus the presence of positive leisure externalities end up increasing the convergence rate. As the productivity growth rate, g increases, the speed of convergence increases as well².

In the following sections we will analyze the dynamic response to two shocks: a 2% increase in the growth rate of productivity of labor and a 10% increase in the tax on labor income.

4.1 Increase in rate of growth of labor productivity

An increase in the growth rate of labor productivity by 2% leads to dramatic structural changes in the economy (Table 4.a). The shock leads to a permanent increase in the equilibrium growth rate, so that quantities per capita grow forever. After 50 years output per capita increase by about 116%, consumption by 123% and capital by 47%. In steady

² The empirical evidence on the rate of convergence is mixed. Barro and Sala-i-Martin (1992, 1995) estimated it at around 2-3% per year, Casselli et al. (1996) at 10%, and Islam (1995) obtains an estimate of about 4.7% for non-oil countries and 9.7% for OECD countries. This model's estimates are therefore within the range of values established in the empirical literature.

state consumption -output ratio increases by 3.15% and output-capital by 46.6% and these increases are constant across our different specifications of leisure externality. However different degrees of leisure externality affect the transition paths and thus the long run levels of per capita quantities even in the competitive economy.

An increase in g leads to an immediate decrease in both the investment rate and employment. However as income grows at a higher rate the agent can enjoy both more leisure and more consumption. Upon impact leisure increases, overshooting its steady state level and then decreases such that in the long run leisure increase is about 0.6-1%.

The immediate effect of an increase in leisure is the decrease in output per capita. Upon impact capital growth becomes negative, but recovers as the positive technological shock leads to increases in the output growth of 2.3% upon impact and 3.5% in the long run. The instantaneous increase in the consumption to output ratio of 12-13.6% compensates for the decrease in output such that even in the short run the consumption per capita increases.

The presence of leisure externalities reduces the instantaneous increase in leisure and per capita consumption for both the centrally planned and competitive economy. In the competitive economy the increased accumulation of capital in the initial stages leads to higher per capita levels of output, consumption and capital in the long run. In the centrally planned economy, the *relative* long run increases in per capita output, consumption, capital and leisure are however lower when leisure externalities are present as the initial steady state quantities are different. Figure 1 shows that in the centrally planned economy where leisure externality is present, the absolute level of leisure starts and stays at the higher level, even though the relative increase in leisure is smaller.

Table 4.b. shows that such increases in consumption and in leisure lead to dramatic increases in the short run welfare varying from 14.49% when leisure externalities are not present to 19.49% (15.11%) when leisure externalities are present and the economy is competitive (centrally planned).

The long run increase in consumption and leisure brought by the increase in growth rate of labor productivity has evidently significant effects on the long run welfare as well. The increase in long run welfare varies from 37% when there are no leisure

externalities to 41.2 % (37.2%) when leisure externalities are present and the economy is competitive (centrally planned).

The percent increase in welfare is higher in the competitive case both in the short and long run. However as the competitive economy has a non-optimal consumption and leisure structure, its initial welfare level is significantly lower than in the centrally planned economy. The absolute gains in welfare are lower in the competitive economy and its welfare stays below the optimum.

Increase in welfare is higher when leisure externality is present in both the centrally planned and competitive economy. Even if the agent from the competitive economy does not take into consideration the effects of leisure externality when maximizing utility these effects are apparent ex post when the welfare is estimated. An increase in productivity growth leads to an increase in leisure level which as leisure externalities increase will have a stronger positive influence on welfare.

4.2 Increase in tax on labor income τ_w

An increase in the tax on labor income from 0 to 10% leads to an increase in leisure as the return from labor is affected. As the supply of labor decreases, the output-capital ratio decreases by about 5% and consumption-output ratio increases slightly. The immediate increase in consumption-output ratio mitigates some of the effect that the decrease in output has on per capita consumption.

Figure 2 shows that the increase in leisure leads to an immediate decrease in the growth rate of capital and output which then recover in the long run. Even though the long run growth rates are not affected in the long run, the long transition path assures that the steady state levels of output (capital) are about 6.9-7% (6.7-7%) lower than before the increase in taxes. If the growth rate of labor productivity is positive, the levels of output, capital and consumption increase by about 153% in 50 years. However these levels are about 7% lower than they would have been if taxes had not changed.

In the short run as the increase of about 3.5% in leisure compensates for the decrease in consumption there are slight welfare gains of about 1.78 (1.71%) if there are no leisure externalities and of about 7.1 (6.96%) when leisure externalities are present for

$g=0$ (2%). Leisure overshoots its long run level and then slowly decreases such that in the long run leisure increases by 3.25% -3.3% depending on the rate of productivity growth. In the long run consumption decreases by about 7% relative to the consumption level that would have prevailed if taxes had not changed. Thus the intertemporal welfare gains are about 4.8% if leisure externalities are present and transform into losses of about 0.3% if there no externalities to leisure.

The presence of leisure externalities leads to a higher negative jump in per capita consumption and has a mitigating effect on the positive jump in leisure. These effects lead to an increased accumulation of capital in the initial stages which translates in slightly lower decreases in per capita levels of output, consumption and capital after 50 years. The differences in the transition paths of the economies with and without leisure externalities are not very big as the agent from the competitive economy is not aware of the effect that its own leisure has on others' utility. However, from (12b), the willingness to shift leisure over time is higher when leisure externalities are present, and therefore the speed of adjustment of the economy is lower. Thus even though upon impact the agent from the economy with leisure externalities is closer to the steady state it does not reach it faster than the agent from the economy without leisure externalities.

In the long run, as the agent ignores leisure externalities, different levels of β do not affect her choice of consumption and leisure, nor the output level, but they affect her welfare. An increase in the labor income tax is ill advised when there are no leisure externalities, but has a positive effect on the welfare in the opposite case.

5. Conclusions

Recent research in the economic literature is concerned with the effects that average consumption has on individual consumption. This paper argues that leisure has significant externalities as well and develops a neoclassical growth model that incorporates these effects. Ignoring the positive (negative) externalities provided by leisure leads to equilibrium consumption, labor and capital that are too high (low) and leisure that is too low (high). The government should tax (subsidize) labor income if

leisure externalities are positive (negative) to rectify the source of distortions. These taxes or subsidies depend on the elasticity of individual and average leisure and the consumption tax. Imposing them lead to significant welfare increases both in the short and in the long run.

We focus on positive leisure externalities in our numerical analysis. The numerical analysis of transitional dynamics and implications for welfare given by different exogenous shocks to the economy bring out a series of interesting results. The dynamics are sensitive to the presence of leisure externalities even in the competitive economy. Leisure externalities reduce the immediate impact that productivity and fiscal shocks have on leisure, but have no impact on the magnitude of relative leisure changes in the long run. The presence of leisure externality has different effects on the speed of convergence in the centrally planned and competitive economy. In the competitive economy it slows down the convergence rate whereas it has the opposite effect in the centrally planned economy

An increase in the growth rate of labor productivity leads to a permanent increase in the equilibrium growth rate, so that quantities per capita grow forever. The adjustment processes of key variables in a competitive and centrally planned economy with leisure externalities are compared. The agent from the centrally planned economy enjoys the highest level of leisure and the smallest relative increase in leisure. Even if the agent from the competitive economy does not take into consideration the effects of leisure externality when maximizing utility these effects are apparent ex post when the welfare is estimated. An increase in productivity growth leads to an increase in the leisure level in the long run which as leisure externalities increase has a stronger positive influence on welfare.

An increase in the tax on labor income leads to an increase in leisure as the return from labor is affected. Since the agent ignores leisure externalities, their different levels do not affect the choice of consumption and leisure, nor the output level in the long run, but they affect the overall welfare and the transition paths. An increase in the labor income tax leads to losses in intertemporal welfare when there are no leisure externalities, but has a positive effect in the opposite case. The presence of leisure externalities therefore has profound consequences on the optimal tax level of labor income and the measurement of welfare.

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Appendix A

From (3a)-(3c), the steady state in the competitive economy is characterized by the following equations:

$$u_c(C_i^*, l^*, \hat{l}^*) = \lambda_d^*$$

$$u_l(c^*, l^*, \hat{l}^*) = \lambda_d^* F(k^*, 1-l^*)$$

$$F_k(K_i^*, 1-l^*) - n - \delta_k = \rho$$

$$F(k^*, 1-l^*) - (n + \delta_k)K_i^* - C_i^* = 0$$

From (4a)-(4c), the steady state in the economy with a central planner that takes into consideration the leisure externalities to derive the socially optimal equilibrium is characterized by the following equations

$$u_c(\tilde{C}_i, \tilde{l}, \tilde{l}) = \tilde{\lambda}_{i,c}$$

$$u_l(\tilde{C}_i, \tilde{l}, \tilde{l}) + u_l(\tilde{C}_i, \tilde{l}, \tilde{l}) = \tilde{\lambda}_{i,c} F_{1-l}(\tilde{K}_i, 1-\tilde{l})$$

$$F_k(\tilde{K}_i, 1-\tilde{l}) - (n + \delta_k) = \rho$$

$$F(\tilde{K}_i, 1-\tilde{l}) - (n + \delta_k)\tilde{K}_i - \tilde{C}_i = 0$$

In order to compare the values of the variables in the two steady states we construct the matrix:

$$\begin{pmatrix} u_{cc} & u_{cl} + u_{c\hat{l}} & 0 & -1 \\ u_{lc} & u_{ll} + u_{l\hat{l}} + \lambda F_{1-l-l} & -\lambda F_{1-lk} & -F_{1-l} \\ 0 & -F_{k1-l} & F_{kk} & 0 \\ -1 & -F_{1-l} & F_k - n - \delta_k & 0 \end{pmatrix} \begin{pmatrix} c^* - \tilde{c} \\ l^* - \tilde{l} \\ k^* - \tilde{k} \\ \lambda_d^* - \tilde{\lambda}_c \end{pmatrix} = \begin{pmatrix} 0 \\ u_{\hat{l}} \\ 0 \\ 0 \end{pmatrix}$$

$$Det = (F_k - n - \delta)F_{1-lk}(F_{1-l}u_{cc} - F_{1-lk}) - \lambda(F_{kk}F_{1-l-l} - F_{1-lk}^2) + F_{kk}F_{1-l}(u_{c\hat{l}} + 2u_{cl} - F_l u_{cc}) < 0$$

$$Det_c = u_{\hat{l}}(F_{kk}F_{1-l} - F_{k1-l}(F_k - n - \delta_k))$$

$$Sign(Det_c) = -sign(u_{\hat{l}})$$

$$Sign(c^* - \tilde{c}) = sign(u_{\hat{l}})$$

$$Det_l = -u_l F_{kk}$$

$$sign(Det_l) = sign(u_l)$$

$$sign(l^* - \tilde{l}) = -sign(u_l)$$

$$Det_k = -u_l F_{k1-l}$$

$$sign(Det_k) = -sign(u_l)$$

$$sign(k^* - \tilde{k}) = sign(u_l)$$

$$Det_\lambda = -u_l \left(F_{1-l} F_{kk} u_{cc} - F_{kk} (u_{cl} + u_{cl}) - (F_k - n - \delta_k) u_{cc} F_{k1-l} \right)$$

$$sign(Det_\lambda) = -sign(u_l)$$

$$sign(\lambda^* - \tilde{\lambda}) = sign(u_l)$$

Appendix B

The equilibrium dynamics of the competitive economy are expressed in terms of the stationary variables by the (13a)-(13c).

$$(\gamma - 1) \left(\frac{\dot{c}}{c} + g \right) + \gamma(\phi + \beta) \frac{\dot{l}}{l} = \rho + n + \delta_k - (1 - \tau_k)(1 - \sigma) \frac{y}{k} \quad (13a)$$

$$\frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k} - \delta_k - n - g \quad (13b)$$

$$\phi \frac{c}{y} = \sigma \frac{l}{1-l} \frac{(1 - \tau_w)}{(1 + \tau_c)} \quad (13c)$$

In steady state we impose $\frac{\dot{k}}{k} = \frac{\dot{l}}{l} = \frac{\dot{c}}{c} = 0$ and we can find out the long run equilibrium

values of c , k , l and y .

We can not solve formally these systems of differential equations and we linearize around the steady state to determine the transitional dynamics:

From (13c)

$$c = \frac{\sigma}{\phi} \frac{y l}{1-l} \frac{(1 - \tau_w)}{(1 + \tau_c)} = \frac{\sigma}{\phi} \alpha \frac{(1 - \tau_w)}{(1 + \tau_c)} l (1-l)^{\sigma-1} k^{1-\sigma}$$

Thus:

$$\frac{\partial c}{\partial l} = \frac{\sigma}{\phi} \alpha \frac{(1-\tau_w)}{(1+\tau_c)} k^{1-\sigma} \left((1-l)^{\sigma-1} - (\sigma-1)l(1-l)^{\sigma-2} \right) = \frac{\sigma}{\phi} \frac{(1-\tau_w)}{(1+\tau_c)} \frac{y}{1-l} \left(1 - (\sigma-1) \frac{l}{1-l} \right)$$

$$\frac{\partial c}{\partial k} = \frac{\sigma}{\phi} \alpha \frac{(1-\tau_w)}{(1+\tau_c)} l(1-l)^{\sigma-1} (1-\sigma) k^{-\sigma} = \frac{\sigma}{\phi} (1-\sigma) \frac{(1-\tau_w)}{(1+\tau_c)} l \frac{y}{k(1-l)}$$

We can express:

$$\dot{c} = \sigma \phi \alpha \frac{(1-\tau_w)}{(1+\tau_c)} \left(\dot{l}(1-l)^{\sigma-1} k^{1-\sigma} + (1-\sigma)l\dot{k}(1-l)^{\sigma-1} k^{-\sigma} - l\dot{l}(\sigma-1)(1-l)^{\sigma-2} k^{1-\sigma} \right) \text{ and}$$

$$\frac{\dot{c}}{c} = \frac{\dot{l}}{l} + (1-\sigma) \frac{\dot{k}}{k} + (1-\sigma) \frac{\dot{l}}{1-l}$$

Plug $\frac{\dot{c}}{c}$ into (13a) and

$$(\gamma-1) \left(\frac{\dot{l}}{l} + (1-\sigma) \frac{\dot{k}}{k} + (1-\sigma) \frac{\dot{l}}{1-l} + g \right) + \gamma(\phi + \beta) \frac{\dot{l}}{l} = \rho + n + \delta_k - (1-\tau_k)(1-\sigma) \frac{y}{k}$$

$$\left(\gamma - 1 + (1-\sigma)(\gamma-1) \frac{l}{1-l} + \gamma(\phi + \beta) \right) \frac{\dot{l}}{l} + (\gamma-1)(1-\sigma) \frac{\dot{k}}{k} = \rho + n + \delta_k - (1-\tau_k)(1-\sigma) \frac{y}{k} - (\gamma-1)g$$

$$\left(\frac{1}{l}(\gamma-1 + \gamma(\phi + \beta)) + (1-\sigma)(\gamma-1) \frac{1}{1-l} \right) \dot{l} + \frac{(\gamma-1)(1-\sigma)}{k} \dot{k} = \rho + n + \delta_k - (1-\tau_k)(1-\sigma) \frac{y}{k} - (\gamma-1)g$$

From (13b): $\dot{k} = y - c - k(n + g + \delta_k)$

Linearize around the SS:

$$\dot{k} = \left[(1-\sigma) \frac{y}{k} - (n + g + \delta_k) - \frac{\partial c}{\partial k} \right] (k - \tilde{k}) + \left[-\sigma \frac{y}{1-l} - \frac{\partial c}{\partial l} \right] (l - \tilde{l})$$

$$\dot{k} = a11(k - \tilde{k}) + a12(l - \tilde{l})$$

$$\dot{l} = \frac{1}{\Delta} \left[\rho + n + \delta_k - (\gamma-1)g - (1-\tau_k)(1-\sigma) \frac{y}{k} - \frac{(\gamma-1)(1-\sigma)}{k} \dot{k} \right]$$

where $\Delta = \frac{1}{l}(\gamma-1 + \gamma(\phi + \beta)) + (1-\sigma)(\gamma-1) \frac{1}{1-l}$

Linearizing this is a little more complex, but in doing so, we evaluate the expression at the steady state, where $\dot{k} = 0$. Following this procedure yields:

$$\dot{l} = \frac{1}{\Delta} \left[\sigma(1-\tau_k)(1-\sigma) \frac{y}{k^2} - \frac{(\gamma-1)(1-\sigma)}{k} a_{11} \right] (k - \tilde{k}) + \frac{1}{\Delta} \left[\sigma(1-\tau_k)(1-\sigma) \frac{y}{(1-l)k} - \frac{(\gamma-1)(1-\sigma)}{k} a_{12} \right] (l - \tilde{l})$$

$$\dot{l} = a_{21}(k - \tilde{k}) + a_{22}(l - \tilde{l})$$

Centrally planned economy

The equilibrium dynamics of the centrally planned economy are expressed in terms of the stationary variables by the (14a)-(14c)

$$(\gamma-1) \left(\frac{\dot{c}}{c} + g \right) + \gamma(\phi + \beta) \frac{\dot{l}}{l} = \rho + n + \delta_k - (1-\sigma) \frac{y}{k} \quad (14a)$$

$$\frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k} - \delta_k - n - g \quad (14b)$$

$$\frac{c}{y} = \sigma \frac{l}{1-l} \frac{1}{(\phi + \beta)} \quad (14c)$$

In steady state we impose $\frac{\dot{k}}{k} = \frac{\dot{l}}{l} = \frac{\dot{c}}{c} = 0$ and we can find out the long run equilibrium values of c , k , l and y .

We can not solve formally these systems of differential equations and we linearize around the steady state to determine the transitional dynamics:

$$\text{Use (14c) and: } c = \frac{\sigma}{\phi + \beta} \frac{yl}{1-l} = \frac{\sigma}{\phi + \beta} \alpha l (1-l)^{\sigma-1} k^{1-\sigma}$$

Thus:

$$\frac{\partial c}{\partial l} = \frac{\sigma}{\phi + \beta} \alpha k^{1-\sigma} \left((1-l)^{\sigma-1} - (\sigma-1)l(1-l)^{\sigma-2} \right) = \frac{\sigma}{\phi + \beta} \frac{y}{1-l} \left(1 - (\sigma-1) \frac{l}{1-l} \right)$$

$$\frac{\partial c}{\partial k} = \frac{\sigma}{\phi + \beta} \alpha l (1-l)^{\sigma-1} (1-\sigma) k^{-\sigma} = \frac{\sigma}{\phi + \beta} (1-\sigma) l \frac{y}{k(1-l)}$$

Therefore:

$$\dot{c} = \frac{\sigma}{\phi + \beta} \alpha \left(\dot{l}(1-l)^{\sigma-1} k^{1-\sigma} + (1-\sigma)l\dot{k}(1-l)^{\sigma-1} k^{-\sigma} - l\dot{l}(\sigma-1)(1-l)^{\sigma-2} k^{1-\sigma} \right)$$

$$\frac{\dot{c}}{c} = \frac{\dot{l}}{l} + (1-\sigma)\frac{\dot{k}}{k} + (1-\sigma)\frac{\dot{l}}{1-l}$$

Plug $\frac{\dot{c}}{c}$ into (14a)

$$(\gamma-1) \left(\frac{\dot{l}}{l} + (1-\sigma)\frac{\dot{k}}{k} + (1-\sigma)\frac{\dot{l}}{1-l} + g \right) + \gamma(\phi + \beta)\frac{\dot{l}}{l} = \rho + n + \delta_k - (1-\sigma)\frac{y}{k}$$

$$\dot{l} \left(\frac{1}{l}(\gamma-1 + \gamma(\phi + \beta)) + (\gamma-1)(1-\sigma)\frac{1}{1-l} \right) + (\gamma-1)(1-\sigma)\frac{\dot{k}}{k} = \rho + n + \delta_k - (1-\sigma)\frac{y}{k} - g(\gamma-1)$$

Use (14b): $\dot{k} = y - c - k(n + g + \delta_k)$

Linearize around the SS:

$$\dot{k} = \left[(1-\sigma)\frac{y}{k} - (n + g + \delta_k) - \frac{\partial c}{\partial k} \right] (k - \tilde{k}) + \left[-\sigma\frac{y}{1-l} - \frac{\partial c}{\partial l} \right] (l - \tilde{l})$$

$$\dot{k} = a^c 11(k - \tilde{k}) + a^c 12(l - \tilde{l})$$

$$\dot{l} \left(\frac{1}{l}(\gamma-1 + \gamma(\phi + \beta)) + (\gamma-1)(1-\sigma)\frac{1}{1-l} \right) + (\gamma-1)(1-\sigma)\frac{\dot{k}}{k} = \rho + n + \delta_k - (1-\sigma)\frac{y}{k} - g(\gamma-1)$$

$$\dot{l} = \frac{1}{\Delta} \left[\rho + n + \delta_k - (\gamma-1)g - (1-\sigma)\frac{y}{k} - \frac{(\gamma-1)(1-\sigma)}{k} \dot{k} \right]$$

$$\text{where } \Delta = \frac{1}{l}(\gamma-1 + \gamma(\phi + \beta)) + (1-\sigma)(\gamma-1)\frac{1}{1-l}$$

Linearizing this is a little more complex, but in doing so, we evaluate the expression at the steady state, where $\dot{l} = \dot{k} = 0$. Following this procedure yields:

$$\dot{l} = \frac{1}{\Delta} \left[\sigma(1-\sigma)\frac{y}{k^2} - \frac{(\gamma-1)(1-\sigma)}{k} a^c 11 \right] (k - \tilde{k}) + \frac{1}{\Delta} \left[\sigma(1-\sigma)\frac{y}{(1-l)k} - \frac{(\gamma-1)(1-\sigma)}{k} a^c 12 \right] (l - \tilde{l})$$

$$\dot{l} = a^c 21(k - \tilde{k}) + a^c 22(l - \tilde{l})$$

Leisure Externalities: Implications for Growth and Welfare

Tables

Table 1. Base parameter values

Preference and population parameters	$\gamma=-1.5, \phi = 1.75, \rho=0.04, n=0.015$
Production parameters	$A(0)=1, \sigma=0.65, \delta_K=0.05, \alpha=1, g=0$

Table 2.a Competitive economy (Base equilibria)

No leisure externality $\beta = 0$						Leisure externality $\beta = 1.5$					
y/k	c/y	Leisure	Growth	Optimal τ_w	Stable eigenvalue	y/k	c/y	Leisure	Growth	Optimal τ_w	Stable eigenvalue
$g = 0\%$											
0.3	0.783	0.678	1.5%	0	-0.06691	0.3	0.783	0.678	1.5%	0.4615	-0.0642
$g = 2\%$											
0.44	0.8	0.685	3.5%	0	-0.097342	0.44	0.8	0.685	3.5%	0.4615	-0.093022

Table 2.b. Centrally planned economy (Base equilibria)

No leisure externality $\beta = 0$					Leisure externality $\beta = 1.5$				
y/k	c/y	Leisure	Growth	Stable eigenvalue	y/k	c/y	Leisure	Growth	Stable eigenvalue
$g = 0\%$									
0.3	0.783	0.678	1.5%	-0.06691	0.3	0.783	0.796	1.5%	-0.06775
$g = 2\%$									
0.44	0.8	0.685	3.5%	-0.097342	0.44	0.8	0.801	3.5%	-0.0987208

Table 3: Welfare analysis of a move to a socially optimal economy

		No leisure externality $\beta = 0$	Leisure externality $\beta = 1.5$
Optimal tax level		$\tau_w = 0$	$\tau_w = 46.15$
$g = 0\%$	SR welfare	0	33.314
	LR welfare	0	16.245
$g = 2\%$	SR welfare	0	32.5245
	LR welfare	0	15.8734

Table 4.a. Increase in the rate of technological change g from 0 to 2%

Per capita quantities (change %)									
		Upon impact				After 50 years			
		K_i	C_i	Y_i	l	K_i	C_i	Y_i	l
Competitive	$\beta = 0$	0	7.07	-5.42	3.9	47.14	123.2	116.21	0.99
	$\beta = 1.5$	0	6.51	-5.00	3.6	47.37	123.32	116.32	0.99
Centrally planned	$\beta = 0$	0	7.07	-5.42	3.9	47.14	123.2	116.21	0.99
	$\beta = 1.5$	0	6.25	-6.46	2.49	46.54	122.33	115.38	0.62
Growth rates themselves , not percentage changes									
		Upon impact				Steady State			
		\hat{K}	\hat{C}	\hat{Y}	\hat{l}	\hat{K}	\hat{C}	\hat{Y}	\hat{l}
Competitive	$\beta = 0$	-0.9	1.4	2.3	-0.27	3.5	3.5	3.5	0
	$\beta = 1.5$	-0.7	1.5	2.3	-0.2	3.5	3.5	3.5	0
Centrally planned	$\beta = 0$	-0.9	1.4	2.3	-0.27	3.5	3.5	3.5	0
	$\beta = 1.5$	-1.0	1.4	2.4	-0.18	3.5	3.5	3.5	0
Ratios (changes %)									
		Upon impact				Steady State			
		y/k		c/y		y/k		c/y	
Competitive	$\beta = 0$	-5.42939		13.22		47.619		3.15717	
	$\beta = 1.5$	-5.00827		12.125		47.619		3.15717	
Centrally planned	$\beta = 0$	-5.42939		13.22		47.619		3.15717	
	$\beta = 1.5$	-6.46861		13.6027		47.619		3.15717	

Table 4.b. Welfare consequences of an increase in the rate of technological change g from 0 to 2%

		No leisure externality $\beta = 0$	Leisure externality $\beta = 1.5$
Competitive	SR welfare	14.49	19.49
	LR welfare	37.11	41.20
Centrally planned	SR welfare	14.49	15.11
	LR welfare	37.11	37.178

Table 5.a. Increase in tax on labor income τ_w from 0 to 10%

Per capita quantities (change %)									
		Upon impact				After 50 years			
		K_i	C_i	Y_i	l	K_i	C_i	Y_i	l
$g = 0\%$	$\beta = 0$	0	-4.23	-4.92	3.54	-6.76	-6.91	-6.93	3.33
	$\beta = 1.5$	0	-4.2787	-4.89	3.52	-6.73	-6.9	-6.92	3.33
$g = 2\%$	$\beta = 0$	0	-4.23	-5.01	3.5	152.7	152.6	152.6	3.25
	$\beta = 1.5$	0	-4.27	-4.98	3.47	152.7	152.6	152.6	3.25
Growth rates themselves , not percentage changes									
		Upon impact				Steady State			
		\hat{K}	\hat{C}	\hat{Y}	\hat{l}	\hat{K}	\hat{C}	\hat{Y}	\hat{l}
$g = 0\%$	$\beta = 0$	1.02	1.308	1.35	-.014	1.5	1.5	1.5	0
	$\beta = 1.5$	1.045	1.31822	1.36	-.012	1.5	1.5	1.5	0
$g = 2\%$	$\beta = 0$	2.8	3.21	3.29	-0.02	1.5	1.5	1.5	0
	$\beta = 1.5$	2.8	3.22	3.29	-0.02	3.5	3.5	3.5	0
Ratios (changes %)									
		Upon impact				Steady State			
		y/k		c/y		y/k		c/y	
$g = 0\%$	$\beta = 0$	-4.95		0.72		0		0	
	$\beta = 1.5$	-4.89333		0.646253		0		0	
$g = 2\%$	$\beta = 0$	-5.01		0.82		0		0	
	$\beta = 1.5$	-4.98		0.74		0		0	

Table 5.b. Welfare effects of an increase in tax on labor income τ_w from 0 to 10%

		No leisure externality $\beta = 0$	Leisure externality $\beta = 1.5$
$g = 0\%$	SR welfare	1.7800	7.11928
	LR welfare	-0.283649	4.8455
$g = 2\%$	SR welfare	1.71223	6.96975
	LR welfare	-0.299298	4.74782

Figure 1: Transitional dynamics after an increase in the rate of technological change g from 0 to 2%. Comparison of the centrally planned vs. competitive economy in the presence of leisure externalities ($\beta=1.5$)

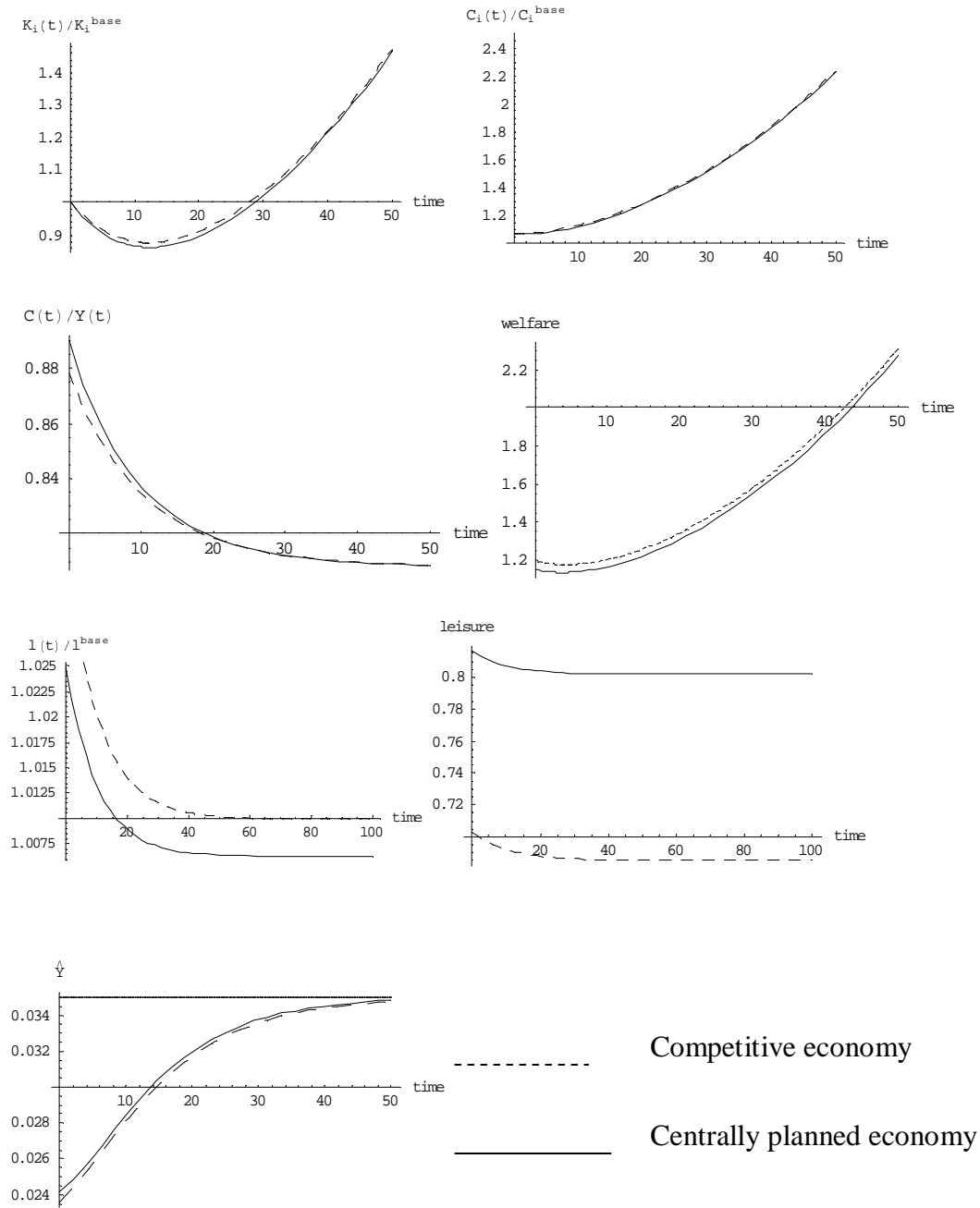


Figure 2. Transitional dynamics after an increase in tax on labor income τ_w from 0 to 10%. Comparative analysis with leisure and no leisure externalities

