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Impossibility of Stable and Non-damaging bossy Matching Mechanism

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Abstract

In this paper we prove the impossibility of stability rules that satisfy a concept weaker than nonbossiness. Stability and nonbossiness are essential to matching theory. However, Kojima Fuhito(2010) shows that a matching mechanism that is both stable and nonbossy dose not exist. We define a new concept that is weaker than nonbossiness and consider whether or not stability and the new concept are compatible. Unfortunately, we show that these properties are incompatible.

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1. Introduction

Matching is one of the important functions of markets. Decisions regarding the person to be selected for a job, the applicant to be admitted to a school, the person someone chooses to marry, etc. are problems that are very important for people who are involved in them. This is because the outcomes of these problems have a great effect on their lives and careers. The theory of two-sided matching, introduced by Gale and Shapley (1962), analyzes such matching problems between two types of agents, such as between workers and firms, applicants and universities, and men and women.

One of the most attractive applications of the matching theory is the designing of mechanisms that determine a matching in any market. Empirical studies have shown that mechanisms satisfying some properties often succeed and those not satisfying the properties often fail in real world applications. See Roth (2002) for evidence.

When designing mechanisms in matching markets, stability plays a central role in the theory. A mechanism is stable if no agent and no pair of agents have the incentive to deviate from the outcome produced by the mechanism. If some agents deviate from the outcome, the other agents may not participate in the market; thus, it is important to consider this property at the time of desgining mechanisms.

The concept of nonbossiness was introduced by Satterthwaite and Sonnenschin (1981). This property is important in many allocation problems. A mechanism is nonbossy if an agent cannot change allocation of other agents without changing her own allocation. Kojima Fuhito (2010)insists that nonbossiness requires an aspect of fairness, because it may be unfair for an agent to be affected by changes of reported preferences of someone else even though the change has no consequence on the allocation of the latter. Further, if a mechanism is not nonbossy, it may invite strategic manipulation. [1]

Although stability and nonbossiness are important properties when a designer considers a mechanism, Kojima Fuhito (2010) shows that these properties are incompatible in matching markets.

We study the possibility of a stable mechanism that an agent makes the allocation of other agents better off even if she influences the allocation of other agents without changing her own allocation. To do so, we define a weaker concept than nonbossy called non-damaging bossy. Thus, nonbossiness implies non-damaging bossy. A mechanism is non-damaging bossy if an agent cannot make the allocation of other agents worse off unless doing so also changes her own allocation.

[1] See Kojima Fuhito (2010) with regard to this discussion.

Unfortunately, we show that there does not exist such a mechanism. Thus, the stable mechanism cannot avoid the situation where an agent makes the allocation of some agent worse off without changing her own allocation.

2. Model

We consider a (one to one) matching problem with tuple (S, C, P). Let S and C be finite and disjoint sets of students and colleges, respectively. Each student $s \in S$ has a strict, transitive, and complete preference P_s over $C \cup \{s\}$; each college $c \in C$ has a strict, transitive, and complete preference P_c over $S \cup \{c\}$. For each $s \in S$, let $\mathcal{P}_s$ be a set of all possible preferences over $C \cup \{s\}$ and similarly, for each $c \in C$, let $\mathcal{P}_c$ be a set of all possible preferences over $S \cup \{c\}$. We write $P \in \mathcal{P} = \prod_{i \in S \cup C} \mathcal{P}_i$. A *matching* μ is a mapping from the set $S \cup C$ onto itself and satisfying (i) for all $s \in S$, $\mu(s) \in C \cup \{s\}$; (ii) for all $c \in C$, $\mu(c) \in S \cup \{c\}$; and (iii) for all $i \in S \cup C$, $\mu(\mu(i)) = i$.

A matching μ is *individually rational* at preference profile P if $\mu(i) P_i i$ or $\mu(i) = i$ for all $i \in S \cup C$. A *blocking pair* of μ at preference profile P is a pair $\{s, c\} \in S \times C$ such that $c P_s \mu(s)$ and $s P_c \mu(c)$. A matching μ is *stable* at preference profile P if it is individually rational and there exists no blocking pair of μ.

Let $\mathcal{M}$ be the set of all possible matchings on $S \cup C$. A *mechanism* is a procedure used to determine a matching for each matching market, that is, a mechanism on $\mathcal{P}$ is a mapping ϕ from $\mathcal{P}$ to $\mathcal{M}$. A *stable mechanism* is a mechanism that selects a matching that is stable with respect to the submitted preference profile. Gale and Shapley (1962) show the existence of a stable mechanism. They propose deferred acceptance algorithms, which find stable matchings for all preference profiles.

3. Result

Kojima Fuhito (2010) proves that there does not exist a mechanism that is stable and nonbossy. The concept of nonbossoiness was introduced by Satterthwaite and Sonnenschin (1981). A mechanism is nonbossy if an agent cannot change the allocation of other agents without changing her own allocation. This concept is formally defined as follows.

Definition 3.1. A mechanism ϕ is nonbossy if, for any P and P'_i, $\phi_i(P'_i, P_{-i}) = \phi_i(P)$ implies $\phi(P'_i, P_{-i}) = \phi(P)$.

Remark 3.2 (Theorem 1. (Kojima Fuhito (2010))). There dose not exist a mechanism that is stable and nonbossy.

We study the possibility of a mechanism that is stable and a concept that is weaker than nonbossiness. The weaker concept is defined as follows.

Definition 3.3. A mechanism ϕ is non-damaging bossy if, for any P and P'_i, $\phi_i(P'_i, P_{-i}) = \phi_i(P)$ implies $\phi(P'_i, P_{-i}) R \phi(P)$.

A mechanism is non-damaging bossy if an agent does not make the allocation of other agents worse off without changing her own allocation.

Remark 3.4. Nonbossy implies a Non-damaging bossiness.

Unfortunately, we show that there does not exist a mechanism that satisfies stable and non-damaging bossy. We prove the statement using a example borrowed from Kojima Fuhito (2010).

Theorem 3.5. *There does not exist a mechanism that is stable and non-damaging bossy.*

Proof. Consider a market with three students and colleges with preferences P given by

$$\begin{aligned} P_{s_1} &: c_2, c_3, c_1, s_1; \\ P_{s_2} &: c_2, c_3, c_1, s_2; \\ P_{s_3} &: c_1, c_2, c_3, s_3; \\ P_{c_1} &: s_1, s_2, s_3, c_1; \\ P_{c_2} &: s_3, s_2, s_1, c_2; \\ P_{c_3} &: c_3; \end{aligned}$$

where P_{s_1} indicates that her first choice is to be matched to college c_2, her second choice is to be matched to college c_1, her third choice is to be matched to college c_3, and her forth choice is to remain unmatched, for example. In this market, there exists unique stable matching $\phi(P)$ given by

$$\phi(P) = \begin{pmatrix} c_1 & c_2 & c_3 & - \\ s_1 & s_3 & - & s_2 \end{pmatrix}$$

which means that c_1 is matched to s_1, c_2 is matched to s_3, and c_2 and s_2 are unmatched. Consider P'_{s_2} given by $P'_{s_2} : s_2$. Then, there are two stable matching, μ and μ', given by

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & - \\ s_3 & s_1 & - & s_2 \end{pmatrix}$$

and

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & - \\ s_1 & s_3 & - & s_2 \end{pmatrix}$$

respectively. Therefore, any stable mechanism must choose one of μ or μ' when the preference $P' = (P'_{s_2}, P_{-s_2})$ is stated.

Suppose the mechanism chooses μ. Thus, $\phi(P'_{s_2}, P_{-s_2}) = \mu$. In this case, we have $\phi_{s_2}(P'_{s_2}, P_{-s_2}) = \phi_{s_2}(P)$. However, for example, agent c_1 prefers $\phi_{c_1}(P)$ to $\phi_{c_1}(P'_{s_2}, P_{-s_2})$. Thus, ϕ is non-damaging bossy.

On the other hand, suppose the mechanism chooses μ'. Thus, $\phi(P'_{s_2}, P_{-s_2}) = \mu'$. Now, consider $P''_{c_3} : s_1, s_3, c_3$. Then, a stable mechanism $\phi(P'_{s_2}, P''_{c_3}, P_{-s_2,c_3})$ is induced by

$$\phi(P'_{s_2}, P''_{c_3}, P_{-s_2,c_3}) = \begin{pmatrix} c_1 & c_2 & c_3 & - \\ s_3 & s_1 & - & s_2 \end{pmatrix}$$

Therefore, we have that $\phi_{c_3}(P'_{s_2}, P''_{c_3}, P_{-s_2,c_3}) = \phi_{c_3}(P'_{s_2}, P_{-s_2})$. However, an agent c_1 prefers $\phi_{c_1}(P'_{s_2}, P_{-s_2})$ to $\phi_{c_1}(P'_{s_2}, P''_{c_2}, P_{-s_2,c_2})$. Thus, ϕ is not non-damaging bossy. □

Kojima Fuhito (2010) proved the impossibility of mechanism that is stable and nonbossy. Thus, he showed that stable mechanisms cannot avoid the situation where an agent influences allocation of other agents without changing her own allocation. Moreover, we prove the impossibility of mechanism that is stable and non-damaging bossy. Thus, stable mechanism cannot avoid the even situation where an agent makes allocation of some agent worse off without changing her own allocation. We interpret that this result is more negative result than the one derived by Kojima Fuhito (2010) because there does exist a stable mechanism where an agent makes allocation of other agents worse off without changing her own allocation.

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