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Abstract

Increasing inter-bank lending has an ambiguous impact on financial stability. Two opposing effects have been identified: promoting stability through risk sharing and providing a channel through which contagion may spread. In this paper we identify the conditions under which each relationship holds. In response to large economy-wide shocks, greater numbers of inter-bank lending relationships are shown to worsen systemic events, however, for smaller shocks the opposite effect is observed. As such there is no optimal inter-bank market structure which maximizes stability under all conditions. In contrast, deposit insurance costs are always reduced under greater numbers of inter-bank lending relationships.

Keywords: Systemic risk, Inter-bank lending, Contagion, Regulation, Network

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1 Introduction

The financial regulation of banks has primarily focused on ensuring that individual institutions have sufficient funds to protect themselves from the risk of their own investments. The events of 2007 and 2008 demonstrated the shortcomings of this approach. Problems in a small number of banks spread throughout the financial system resulting in the collapse of institutions which, according to regulatory requirements, were adequately protected. The inter-bank market was supposed to provide insurance and stability by allowing banks to access liquidity and share risk. Instead, it served as a mechanism by which problems could spread between institutions. In this paper we examine how the structure of the inter-bank lending market effects the stability of the financial system[1]. We consider a model of heterogenous banks within a closed economy. Households deposit money in banks who invest those funds in risky projects proposed by firms. The proceeds of these investments are circulated in the real economy and in future periods paid into banks as deposits, as such the money multiplies. Banks interact with each other through an inter-bank market, obtaining funds but exposing themselves and other banks to counter-party risk and potentially contagion.

It is found that the structure of the inter-bank market has a significant effect on the ability of the system to resist contagion in response to system-wide macroeconomic shocks. The optimal structure, however, is dependent on the magnitude of the shock faced. For small shocks a highly connected inter-bank market provides a risk-sharing effect reducing the probability of a contagious failure. In contrast, for larger systemic shocks, rather than reducing risk, inter-bank connections act to propagate the effects of failures: markets with more inter-bank connections become the most vulnerable. Regardless of the size of shock the cost to the deposit insurer is minimized for the most connected markets as more of the cost of failures is borne by surviving banks. The effect of regulatory changes are investigated. Higher equity and reserve ratio's are both found to decrease the market's susceptibility to contagion by reducing the number of banks who cause a second bank to

[1] These are not the only inter-bank linkages which can propagate distress. For instance Allen and Carletti (2006), Markose et al. (2010) and Mendoza and Quadrini (2010) demonstrate alternative mechanisms.

fail. Constraining the size of inter-bank loans, is found to be able to reduce the number of bankruptcies whilst increasing the quantity of loans given to firms. Care, however, must be taken with this regulatory change, if the regulation is set too tight to too loosely it inhibits the economy or has no effect. Finally if banks condition their confidence of being repaid on recent bankruptcies the economy becomes less stable and efficient. Whilst if banks condition their lending on the financial position of the borrower stability increases.

The paper is structured as follows: the next section will review related literature. Section 3 will describe the model. Section 4 will consider the systems susceptibility to contagion under varying shocks and market structures. Section 5 examines the effect of regulation whilst Section 6 demonstrates parameter stability and extends the model. Section 7 concludes.

2 Literature review

The inter-bank lending market allows financial institutions to lend funds or borrow money to meet liquidity or investment requirements. In their influential work, Allen and Gale (2001) show that in equilibrium banks will optimally insure themselves against liquidity risk by holding deposits in other banks. This protection, however, makes them potentially vulnerable to counter-party risk. As such the failure of a signal bank may spread if its creditors are unable to recover lent funds. This may potentially cause severe contagious events (Gai and Kapadia, 2010), resulting in a loss of equity (Eisenberg and Noe, 2001) and may justify government or regulatory intervention (Kahn and Santos, 2010)[2].

The majority of trading in the inter-bank-market happens over-the-counter (OTC), directly between pairs of banks, as opposed to through a central counter-party. Banks borrow funds and repay them over a length of time which can range from overnight, up to periods of several years. At any point a particular bank may be involved in multiple lending or borrowing relationships and as such may be connected to multiple counter-parties. Across all banks these linkages form a structure which may be described by a

[2] Also see Giesecke and Weber (2006), Elsinger et al. (2006) and Brusco and Castiglionesi (2007) for alternative views.

weighted, directed graph in which nodes are financial institutions and edges are lending relationships of a specific value (e.g. Iori et al. (2008)).

If a single bank fails initially only those banks to which it owes money suffer directly, the remainder of the system is unaffected[3]. The direct impact, however, may cause one or more of the banks counter-parties to fail which can harm further institutions within the system. The structure of inter-bank markets, the numbers and distribution of linkages together with their size, has a large effect on how shocks spread and the markets potential susceptibility to systemic events (Haldane and May, 2011). Muller (2006) and Upper and Worms (2004), by analyzing data from the Swiss and German banking systems respectively, show that in both cases there is significant potential for contagion. Highly centralized markets, those with a few large hub banks like the UK (Becher et al., 2008), being particularly susceptible to this risk. In contrast Angelini et al. (1996), Boss et al. (2004) and Furfine (2003) find that there is relatively little danger of systemic events. Only a very small number of banks could cause other banks to fail if they themselves defaulted.

The difference in conclusions is driven in part by differences in the inter-bank markets, e.g. trade volume (Angelini et al., 1996). However, theoretical models present a similarly ambiguous picture of the effect (e.g. Leitner, 2005). Vivier-Lirimont (2006) finds that increasing the number of inter-bank connections worsens contagion. This is partially supported by Brusco and Castiglionesi (2007) who show that increasing cross-holdings increase the extent of contagion but reduces the effect on individual institutions. In contrast Giesecke and Weber (2006), in line with Allen and Gale (2001) find that more connections reduce contagion. Using simulation techniques Nier et al. (2007) show that a small increase in connectivity increases systemic risk but beyond a certain point the degree of systemic risk decreases. In contrast, Lorenz and Battiston (2008) and Battiston et al. (2009) find the opposite relationship, the scale of bankruptcies is minimized for intermediate levels of connectivity. Georg (2011) examines different types of inter-bank

[3]For the present we ignore issues regarding market confidence and beliefs. In reality, a bank that is not directly effected may still fear for their investments and alter their portfolio to limit the possibility of future losses, e.g. Lagunoff and Schreft (2001).

networks finding that those which resemble random graphs suffer more from contagion than those with scale free or small world properties.

The models above give apparently contradictory results regarding the effect of the inter-bank market. Some show increasing connectivity as providing stabilization, others as increasing the potential for contagion whilst a few give non-monotone relationships. The mixed results are due to the interaction of the two effects of inter-bank relationships discussed by Allen and Gale (2001), risk sharing versus contagious vulnerability. Whilst sparser networks limit the ability of shocks to spread, reducing contagion, they also reduce the risk sharing capacity of the market and so increase the risk of individual banks failing. As Iori et al. (2006) show the inter-bank market may permit crisis to spread, however, it also provides stabilization meaning the overall effect is ambiguous. The model presented in the next section will aim to shed light on this ambiguity by identifying the conditions under which each effect is dominant and so the conditions under which the various conclusions regarding inter-bank markets hold. Using this model the effect of regulation in limiting financial fragility will be considered.

3 Model

We consider a model of a closed economy containing N banks, M households and Q firms. Households, banks and firms each occupy locations on the circumference of a unit circle. This circle represents a dimension, not necessarily physical, on which the households, firms and banks differ. Banks are equidistantly spaced with bank 1 being located at the top of the circle and the remaining banks arrayed in index order clockwise around the circumference. The same arrangement is followed by households and firms with the agent with index 1 being at the top of the circle. The distance between a bank and another economic agent affects the banks ability to attract that agent as a potential borrower or depositor.

The model operates in discrete time and repeats for an infinite number of time steps. The actions and investments of each bank in each time step effect their financial position in future periods. The following sub-sections describe the behavior of the banks, firms and households during each period.

3.1 Households

Each household, j, holds depositable funds (d_j), the quantity of which is determined exogenously. Households attempt to maximize the return from these funds by placing them in the bank which maximizes its expected return:

$$\arg\max_{i \in N} d_j(r_i^{deposit} - g(i, j)) \tag{1}$$

Where $g(i, j)$ is the distance between i and j[4]. and $r_i^{deposit}$ is bank i's deposit interest rate. If no i exists such that Equation 1 is positive the household retains its funds and earns no interest. Banks do not refuse any household deposits. Full deposits insurance is provided by an agent outside of the system who guarantees that households will be repaid the full value of their deposits in the event of bank failure. Households are, therefore, not concerned with the risk of bank default and so select the bank offering the highest return[5].

3.2 Firms

Each time period, each firm is presented with a single limited liability investment opportunity, l_j^t. Each opportunity requires an initial investment of a single unit of currency at time t and provides a payoff to the firm at time $t + 2$ of μ with probability $\theta_{l_j^t}$. With probability $1 - \theta_{l_j^t}$ the investment provides zero payoff. Values of μ are fixed across loans whilst $\theta_{l_j^t}$ is drawn from a distribution. In order to invest in the opportunity firms are required to borrow the full amount from a bank. Each firm, k, approaches the single bank which maximizes the firms expected return:

[4]In line with the previous hotelling literature(e.g. Salop, 1979) we model transaction costs as linear in the distance between two actors. Alternative functions were tested and had little qualitative effect on the results.

[5]We model households as being highly active in their management of deposits, however, in reality deposits tend to be sticky. Individuals are slow to respond to changes in interest rates, frequently maintaining their deposits in institutions paying suboptimal rates, rather than switching. Experiments were performed in which deposits were sticky - depositors moved with a fixed probability. Probabilities of switching greater than 2% produced no significant difference in results.

$$\arg\max_{i \in N} \theta_{l_k^t}(\mu - (1 + r_i^{loan})^2) - g(i,k) \tag{2}$$

Where r_i^{loan} is bank i's per period lending interest rate. If no i exists such that Equation 2 is positive the opportunity goes unfunded. If bank i funds an investment opportunity, l_k^t, with probability $\theta_{l_j^t}$ the bank receives $(1+r_i^{loan})^2$ at time $t+2$ whilst with probability $1-\theta_{l_j^t}$ the bank receives nothing.

3.3 Banks

Each bank, i, has a balance sheet comprising equity (E_i), deposits (D_i), cash reserves (R_i), loans to the non-bank sector (L_i) and loans to the other banks (I_i)[6]. Each time step, each bank, i, attempts to maximize its expected return, $E(r_i)$ given by:

$$(\sum_{k_i^t=1}^{K_i^t} \theta_{k_i^t}(1 + r_i^{loan})^2 - 1) + I_i^t((1 + r^{interbank})^2 f(I_i^t) - 1) - D_i r_i^{Deposit} \tag{3}$$

Where $\theta_{k_i^t}$ is the repayment probability for loan k_i^t (each loan is of unit size) and $f(I_i^t)$ is a function giving an estimate of the probability of inter-bank lending being repaid:

$$f(I_i^t) = \begin{cases} \theta_i^{interbank}, & \text{if } I_i^t > 0 \\ 1, & \text{if } I_i^t \leq 0 \end{cases} \tag{4}$$

Here $\theta_i^{interbank}$ is bank, i's estimate of the probability of being repaid in the inter-bank market. The failure to repay inter-bank lending results in the bankruptcy of the defaulting bank. In calculating their expected return banks, therefore, assume that they always repay their own inter-bank borrowings so the probability is 1.

This maximization is subject to the following constraints:

$$L_i + R_i + I_i = E_i + D_i \tag{5}$$

[6]Positive values correspond to lending, negative to borrowing.

$$D_i = \sum_{j=1}^{M} S(i,j) d_j \text{ where } S(i,j) = \begin{cases} 1, & \text{if } i = \arg\max_{i \in N} d_j(r_i^{deposit} - g(i,j)) \\ 0, & \text{Otherwise} \end{cases} \tag{6}$$

$$R_i \geq max(\alpha_g, \alpha_i) D_i \tag{7}$$

$$E_i \geq max(\beta_g, \beta_i)(L_i + max(I_i, 0)) \tag{8}$$

$$L_i = \|K_i^t\| + \|K_i^{t-1}\| \tag{9}$$

The first constraint states that each bank's balance sheet must balance; i.e. assets are equal to liabilities. The second constraint specifies that the bank's holding of deposits is equal to the sum of deposits placed in that bank by households. The bank may neither refuse deposits nor gain access to additional deposits outside of those contributed by households. The third constraint governs the level of liquid cash reserves which the bank holds. It is the maximum of the banks preferred level, α_i and a minimum level imposed by regulation α_g. The fourth constrain specifies the maximum equity to risky assets ratio. Where β_i is the bank's preferred equity ratio and β_g is a minimum value imposed by regulation. The second *max* operator means only positive values, i.e. inter-bank lending and not inter-bank borrowing are considered. Note, whilst reserves are assets, they are not included in the equity ratio as they are risk-less. In this model inter-bank lending and firm lending are equally weighted in the risk calculation. The fifth constraint states that the amount invested in loans is equal to the total funds invested in individual projects. Here, K_i^t is the set of investments funded by bank i in period t and we define $\|.\|$ to be the sum of the values of loans in the included set. Importantly since loans last for two periods, this constraint includes all projects funded at time t but also those that were funded at time $t-1$.

We consider this maximization problem to proceed in two stages. First, at the start of each time period each bank publicly declares its deposit interest rate, $r_i^{deposit}$, and lending interest rate, r_i^{loan}. Households and firms respond to these rates, placing deposits and submitting lending request to the appropriate banks. Banks then determine the allocation of assets and liabilities on their balance sheets to maximize the expected return.

Money is distributed from household deposits and inter-bank borrowing to fund loans to firms, inter-bank lending and to save as cash reserves. The banks equity is the result of its previous investment decisions up to the current time period. This together with the above constraints mean that in any given period at the point returns are maximized the level of Equity, Deposits and Reserve are all known. Additionally the bank still has positions in loans to firms and inter-bank lending form the previous time step which it may not change. The maximization problem it therefore the distribution of the remaining funds between new inter-bank lending and borrowing and new loans to firms. In making this decision bank i determines the composition of K_i^t the set of funded investment opportunities. The loans are selected from P_i^t, the set of investment opportunities presented to bank i by firms at time t, i.e. $K_i^t \subseteq P_i^t$. Bank's invest in zero or more loans in decreasing order of expected return until the expected return falls below the inter-bank lending rate or the bank runs out of funds. If the bank runs out of suitable loan opportunities whilst it still has available funds the bank may lend to other institutions subject to the expected return of the loan being positive. Alternatively if a bank has excess loan opportunities it may borrow money from other banks to fund these investments.

3.4 Inter-bank market

Inter-bank lending occurs through an over-the-counter market. The inter-bank rate is dependent on the lending and borrowing preferences of individual banks which, as shown in the above maximization, are themselves dependent on the inter-bank rate. There is no closed form solution for the equilibrium, so in order to identify the interest rate it is necessary to use an iterative numerical approach. To simplify the initial analysis we assume a single inter-bank interest rate at which all transactions are conducted[7]. In section 6 we relax this assumption.

In over-the-counter markets, transactions are bilateral, when a bank lends money it lends to one (or more) specific counter-parties. The pattern of inter-bank lending connec-

[7]During non-crisis periods, both in reality and this model, the rate at which banks fail is very low and in a steady state there should be little difference in the offered inter-bank rates between banks. Empirically Iori et al. (2008) show that even in an OTC market there is sufficient information exchange for efficient pricing of bilateral transactions

tions is determined exogenously allowing a range of market structures to be investigated[8]. We consider random graphs where the probability that a particular inter-bank lender lends money to a particular borrower is λ. As λ increases the density of inter-bank connections increases such that at λ equal to 1 each lender will lend to all borrowers.

The inter-bank connections are constructed as follows. Initially the population of banks is partitioned into three sets by their desired inter-bank positions: lenders, borrowers and those with no position. Each member of the set of lenders is considered in turn in decreasing order of the magnitude of funds offered. Let the set of borrowers to which lender i lends money be C_i. For each borrower, b, in the population with probability λ, b is added to C_i. If the total amount of funds requested by the members of C_i is less than the amount i wishes to lend, further banks are added to C_i in decreasing order of magnitude of requested funds until this is no longer the case. Doing this ensures the minimum number of banks are added to the set which, is particularly important in generating minimally connected markets. The lender, i, lends money to each member of C_i in proportion to their requested funds. By ensuring a minimum level of requested funds by set C_i we model an efficient financial market where lenders and borrowers are able to find each other. The variation of parameter λ therefore effects the degree of diversification in inter-bank lending. If this minimum were not imposed the market would not be efficient and banks would be constrained in the amount they could lend or borrow by the identities of their potential partners. Evidence suggests e.g. Iori et al. (2008) that in developed markets for large financial institutions during non-crisis periods the market is efficient and the chosen representation is appropriate.

In the next section we will show that this method produces networks which match many features observed in reality. Other mechanisms for determining the allocation of connections were considered, however, they produced results similar to those generated with this mechanism for the same number of connections[9]. Future work will consider alternative classes of network, e.g. those considered by Cossin and Schellhorn (2007).

[8] A future development would be to make connection decisions endogenous.

[9] We also considered λ as an endogenous variable set by each bank. It was found that there was no significant difference to the results presented below.

The two period nature of investments is important in capturing the structure of the inter-bank market. In any period each bank may be either an inter-bank lender or a borrower, they may not be both. Consequently if investments and the inter-bank borrowing funding it, lasted only a single period the network would be bipartite. This would limit the potential for contagion to the failed banks direct creditors. Two period loans allow a bank to be both a lender and borrower in subsequent periods, allowing failures to spread and therefore, richer and potentially more realistic contagious events.

3.5 Model Operation

This section details the order of events within each time period. At the start of period t, interest is paid by banks to households on the deposits established during period $t-1$ (Equation 1). After interest is paid, loan success is evaluated for loans established in period $t-2$ and banks repaid by firms. The inter-bank lending from time $t-2$ which funded these investments is then repaid. If after interest payments and loan success have been evaluated the bank has negative equity, or if a bank has insufficient cash reserves to repay its inter-bank debts, it is declared bankrupt. In the event of a bank failure sufficient assets are retained, if available, to cover the value of deposits, any remaining liquid assets are used to repay creditors in proportion to the size of their debt. If a creditor bank is not fully repaid it suffers a loss in equity which may, potentially cause it to go bankrupt. If this occurs any inter-bank borrowing on its balance sheet is resolved in the same manner. As such the failure of one bank may spread to counter-parties and then further within the system. A bankrupt bank is removed from the financial system and takes no further actions.

If a bank fails to which a firm or bank owes money, the borrower is still required to repay its loan at the appropriate due date. This is consistent with an administrator ensuring creditors of a bank meet their requirements. Any funds arising from such repayments are considered to either be absorbed by the administrators of the failed bank or to go to the deposit insurer (Equation 4). After loans and bankruptcies have been resolved the deposits each household possess at time t are set such that:

$$d_j^t = \frac{\sum_{i=1}^{N} L_i^{t-1}}{M} \tag{10}$$

The cash holdings of households available for deposits at the current time step are equal to the total loans from the previous time step. Money is transferred from firms to households as part of the operation of the real economy. When funds are lent to a firm to invest, goods or services are purchased and individuals receive wages, resulting in monetary transfers. In this paper we do not consider the detail and distribution of these interactions and so we assume that funds are distributed uniformly[10].

At this point households place their deposits in banks. Banks then allocate their funds as described above and the inter-bank rate is calculated along with the lending and borrowing relationships. Finally at the end of each period an inflationary process is applied to all values (including cash, loans, reserves etc.) at the following rate:

$$F^t = \frac{\sum_{i=1}^{N} E_i^t}{N} - 1 \tag{11}$$

The effect of the inflationary process is to maintain a fixed value of equity. Without this the level of equity within the model could potentially grow to infinity and prevent a solution being found.

3.6 Parameters and Learning

Banks' allocation of funds is determined by several endogenous parameters. These are: reserve ratio (α_i), equity ratio (β_i), lending interest rate (r_i^{loan}), deposits interest rate ($r_i^{deposit}$) and their estimate of being repaid in the inter-bank market ($\theta_i^{interbank}$). There is no closed form solution for assigning optimal values to these parameters within this model with time varying heterogenous banks and under different regulatory frameworks. Instead the values of these parameters are optimized by a genetic algorithm[11].

Here we optimize the parameters such that the profitability of banks in maximized,

[10] Alternative mechanisms including having uneven redistribution and time varying distributions were tested but had little effect on the results.

[11] See Arifovic (1996) and Noe et al. (2003) for examples of GA's used in economics.

i.e. we find those parameters which lead to higher equity. The genetic algorithm functions as follows. Each parameter for each bank is initially randomly drawn from $U(0, 1)$. Each time period two banks from the population (including those which are bankrupt) are selected at random with probability proportionate to $1 + E_i$. The parameters of the bank with lower equity are replaced by the values of those of the richer bank subject to a small perturbation drawn from $U(-0.01, 0.01)$. If the poorer bank is bankrupt it is reintroduced to its previous location on the unit circle with $E = 1$, $R = 1$ and no other assets or liabilities. As such this process also replaces failed banks. This process ensures the parameter space is explored whilst the population of banks converges to an optimal parameter set.

4 Results

In order to evaluate the behavior of the model in response to shocks it is necessary to first consider the steady state. All experiments in this paper use the parameters presented in Table 1 unless otherwise stated. An analysis of robustness to parameters and assumptions is provided in Section 6.

The first two parameter values are chosen based on real world equivalents. US banking regulation defines a minimum reserve requirement of 10% and a minimum capital requirement for a bank to be adequately capitalized of 8%. In making this calculation we count both inter-bank and firm loans as having a risk weighting of 1 whilst reserves are risk-less.

At the start of the simulation $E_i = 1$, $R_i = 1$ for all banks whilst all other assets and liabilities are set to zero. The model was run with 500 different random seeds for each of 11 different values of λ. Each simulation was run for 10000 time steps. To test convergence the average values of market parameters during periods $8000 - 8999$ and $9000 - 9999$ were calculated and a T-Test performed to ensure the parameters were stable. At this point market statistics were recorded.

4.1 Steady state analysis

In this section we present statistics describing the state of the converged simulations. In order to support the conclusions of the model we show that key ratios and quantities are of same magnitudes as those observed empirically. We do not match exactly the balance sheets of a particular country. To do so precisely would require a considerably more complex model with many more parameters. In this paper it is sufficient that the magnitudes are correct so that the conclusions drawn from this model hold for a range of financial systems. At the same time a simpler model allows the mechanisms driving the results to be more clearly identified.

Table 2 shows the average asset and liability holdings of all banks within the model economy, together with the balance sheets of all American commercial banks in 2006. Here pre-financial crisis data were chosen to compare to pre-shock model data. Model balance sheet terms are matched to their closest equivalents. Some terms on the real balance which have no model equivalents are omitted. In this, and all subsequent tables, inter-bank loans are the total funds lent within the system, the sum of positive positions. The sum of all positions within the market would be 0 as inter-bank lending is equal to inter-bank borrowing within this closed economy[12].

Crucially the level of inter-bank lending within the model is close to the value calculated within the real economy. Inter-bank loans are the mechanism by which failures spread. If the level of loans in the model were of a different magnitude to that seen in reality then bank failures would either spread much more easily or much less frequently than in reality. Consequently the model would not provide a reasonable estimation of the role of inter-bank relationships on systemic stability. Importantly the level of cash reserves in both cases is almost identical, indicating both system have similar amounts of liquidity available in the event of failures.

The ratio of loans to deposits is very similar in both the model and empirical data. Relative to equity, however, both of these values are smaller in the model. This is a

[12]During this period American banks were net borrowers, the figure for Borrowing (including both national and international relationships) is therefore a better estimate than that of American Inter-bank Lending.

consequence of the inflationary process. In order to maintain a fixed level of equity for computational tractability a relatively high rate of inflation (on average 13%) is necessary. This reduces the value of loans and deposits each time step and is cumulative as loans at time t are used to calculate deposits at time $t + 1$. When inflation and reserve requirements are taken into account the maximum value of loans possible within the model is approximately 401 which is very close to the observed value.

Bank's preferred equity and reserve ratios (Table 3) are both less than the values specified by the regulations i.e. 8% and 10%. This means that the regulated values are used in all cases and the banks are maximally leveraged. If the banks adopted this behavior without the inflationary effect, the value of deposits and loans within the model would be very similar to the empirical data. The banks therefore, behave in a very similar manner to those in reality. Whilst the absolute value is low the relationship and behaviors are correct.

The deposit and loan rates within the model of 6.9% and 2.8% (Table 3) are empirically plausible. The inter-bank rate of 5.8% is high compared to historical values, however, it is necessary to remember that within this model there is no other source of funds so this rate rises due to demand for funds to lend to firms rather than risk.

The model does a good job of matching the magnitudes and key ratios observed in empirical data. A more complex model with more parameters could do this even more closely. We emphasize, however, that the purpose of the model is not to exactly reproduce empirical values from a particular economy. Rather this section shows that the model replicates the magnitudes and key relationships important in determining systemic stability. The model, using a relatively simple framework, may therefore be used to identify relationships and draw conclusions regarding financial stability for a range of financial system.

4.2 Market Structure

The structure of the inter-bank market is determined by a combination of the endogenous bank behavior and exogenous structure. In particular the number of lenders and borrowers, their size and distribution is determined endogenously by the supply and demand of funds and loan opportunities. This section will describe and characterize the endogenously determined features of the inter-bank market and compare them to empirical observations.

Table 3 shows that in line with the empirical results of Iori et al. (2008), for the Italian inter-bank market, there are more lenders than borrowers and that the majority of banks act as either sources or sink for loanable funds, relatively few both lend and borrow. Examination of the average equity of banks within these groups shows agreement with the findings of Cocco et al. (2009) and Iori et al. (2008) that large banks are net borrowers whilst small banks are net lenders and that large lenders have many small creditors (Muller, 2006). This is because within the model there are only a small number of large banks, typically around 15%, that are constrained by the amount of funds they are able to raise through deposits. These banks have high equity and so in order to be maximally leveraged they must borrow on the inter-bank market. In contrast small banks do not need to borrow. They are constrained by their level of equity and would be unable to invest borrowed funds in risky projects. This implies that banks would tend to lend or borrow from banks of a different size to them self who are not constrained in the same manner. Empirically this is demonstrated by Cocco et al. (2009) who examines the distribution of loans between banks, finding that the most common links are between large and small banks whilst the least common are between pairs of small banks. Table 4 shows a similar relationship in the model when the population is partitioned around the median wealth.

The endogenous structure of the inter-bank market replicates key structural features observed in reality. It is important to highlight that these features were not specified in the model, rather they were endogenously determined as the optimal behavior of banks within this financial setting. The results suggest that this model may have a strong basis to provide insight into the behavior of real financial systems.

4.3 Individual Bankruptcy

As discussed in the introduction, opinion is divided on the effect of the structure of the inter-bank market on the probability and severity of contagion. The stabilizing effect and the contagion spreading effect appear to be in direct contradiction. To examine these forces we first consider the failure of a single bank and its impact on the rest of the financial system. Similar analysis has been conducted in other studies both analytically and empirically for a range of inter-bank markets with mixed results[13]. Elsinger et al. (2006), using Austrian data, show that systemic failures from the collapse of a single bank only occur in about 1% of cases. Further, only a small proportion of banks are able to cause systemic crisis (Boss et al., 2004) or are susceptible should a partner institution fail (Angelini et al., 1996). The effect of contagion when it occurs, however, can be very large (Gai and Kapadia, 2010). For instance Humphrey (1986) shows that the collapse of a large American bank could potentially bankrupt 37% of banks in the market.

The converged economies presented in the previous section serve as a basis for this analysis. The state of the market, the bank positions and inter-bank loans, is recorded and a single bank made bankrupt by setting its equity and reserves to zero. The effect of this bankruptcy on the rest of the economy is analyzed before the state of the market is reset. This is repeated for each bank in turn.

Table 5 shows that as the market becomes more connected the effect of a bankruptcy, as measured by the average number of subsequent failures, is reduced, in agreement with the findings of Allen and Gale (2001), Giesecke and Weber (2006) and Freixas et al. (2000). This is because fewer banks are able to cause another to fail (decreasing Probability in Table 5). As Brusco and Castiglionesi (2007) argue, whilst more banks may be touched by contagion, if the market is more heavily connected the probability that any of them will fail is reduced. The higher level of connectivity provides diversification of credit risk for the banks. When a bank fails the impact is more spread and the effect on each individual lender is reduced.

The same table also shows that for more connected markets whilst contagious events

[13]For example: Boss et al. (2004), Upper and Worms (2004), Nier et al. (2007), Gai and Kapadia (2010) Vivier-Lirimont (2006) and Allen and Gale (2001).

are rarer when they do occur the number of banks which go bankrupt increases. This suggest a greater vulnerability, however, this is not the case. The average equity of the banks which cause contagion increases with connectivity i.e. only the larger banks are able to cause contagious failures. The impact of smaller banks is sufficiently well spread that in many cases they do not cause other banks to fail. The average equity of failing banks for all market structures in approximately 0.75 which is less than the market average of one, indicating that smaller banks are more vulnerable to contagious failure.

An alternative measure of a market's potential susceptibility to contagion is the maximum number of bankruptcies a failure may cause. The sizes of the largest failures are of the same magnitude as those seen in reality. Upper and Worms (2004) find within the German Banking system a single bankruptcy may cause at most 15% of the other banks to fail whilst Humphrey (1986) shows that the collapse of a major US bank could lead to 37% of banks defaulting. The relationship with connectivity differs from that of average contagion. Here the most vulnerable markets are those with an intermediate level of connectivity ($\lambda = 0.4$). Whilst not, on average, the most susceptible to contagion these markets are particularly vulnerable to the failure of crucial banks. Banks within these markets are sufficiently poorly connected that if one fails, the shock is strong enough to drive other banks to failure. At the same time the market is sufficiently well connected that a single bankruptcy could potentially affect many other banks. The combination of large shocks and wide spread make these markets particularly vulnerable if the wrong large bank fails.

4.4 Systemic Shocks

The results presented in the previous section are important in understanding the vulnerability of the financial system to a single failure. In reality, however, the failure of a bank is often not an isolated event. Instead a failure may be caused by a shock which affects the whole financial system. Macroeconomic events may affect multiple institutions simultaneously, weakening balance sheets and potentially causing several unconnected banks to fail at the same time (Gorton, 1988). This section will consider the effect of such a shock.

Few studies have examined the effect of the inter-bank market during a systemic shock. It is not clear whether contagion in the inter-bank market will be significant or if it will be secondary to the financial shock itself (e.g. Giesecke and Weber, 2006). At the same time it is unclear how the risk-bearing and contagion spreading effects interact as equity is eroded. A market in which each bank is connected to a greater number of counter-parties may allow system liquidity to be better utilized reducing the impact. Alternatively, as the market becomes more connected the weakest banks may be more likely to be effected and fail. One of the few papers to comment on this issue is Lorenz and Battiston (2008), they find that increasing inter-bank connectivity at first reduces the incidence of bankruptcy but for more connected markets it increases. Battiston et al. (2009), whilst not explicitly modeling a systemic shock, find a similar pattern when they permit multiple bankruptcies to occur in the same period.

In this section we examine the effect of systemic shocks on the model economy. Each converged market suffers a macro-economic shock during the first time step after the converged state. This shock is implemented by changing the probability of project success for projects which finish in the shock time step from θ_i^{t-2} to θ^{shock}. All projects ending in other time periods are left unchanged. We perform the experiment for a range of values of θ^{shock} showing how different macroeconomic shock severities effect the stability of the financial system.

Figure 1 presents results showing the average number of bankruptcies across different market architectures and for different shock severity's. As θ^{shock} decreases fewer projects are completed successfully, leading to higher losses for banks and more failures. Market connectivity has a non-linear effect on this relationship. For small shocks a more highly connected market reduces bankruptcies, limiting the spread of contagion by spreading the impact of failures. In contrast for larger shocks the pattern is reversed, more sparsely connected markets are less susceptible to contagion. For intermediate shock sizes, moderately connected markets may be the most vulnerable, for example $\theta^{shock} = 0.775$.

The severity of contagion is highly dependent on the degree to which failures spread.

As connectivity increases each bankruptcy affects more counter-parties. At the same time a lender splits the same amount of funds between more banks meaning loans are smaller and so the probability that the non-repayment of an inter-bank loan causes the lender to go bankrupt is reduced, i.e. they diversify some of the credit risk. A systemic shock reduces the equity of all banks. For small shocks equity is only slightly damaged. In highly connected markets if a bank fails the impact is sufficiently well spread that the bankruptcy rarely cause a counter-party to fail as a result. The diversifying effect of increased inter-bank connectivity reduces risk. As connectivity decreases the average loan size to counter-parties increases and failures becomes more likely. Larger systemic shocks result in much reduced bank equities and so smaller counter-party losses may cause bankruptcy. Consequently banks in more connected markets start to be at risk from the failure of their counter-parties. For the largest systemic shocks bank equities are damaged to such an extent that regardless of connectivity the failure of any counter-party is sufficient to cause a lender to fail. Instead of spreading the impact the higher connectivity results in more banks being affected and failing. At the same time the diversification effect from many inter-bank connections is weakened as the failure of banks becomes increasingly correlated. In less well-connected markets banks fail but the scope of contagion is reduced as each bank failure effects a smaller subset of the population.

For $\theta^{shock} = 0.775$ the point at which the liklihood of a bank failing and spreading a shock is maximized at intermediate levels of connectivity. At this level of shock, more connected markets spread impacts sufficiently well that relatively few banks fail whilst less connected markets spread the shock to too few partners, limiting the spread.

The results support a range of empirical and analytical findings which previously appeared contradictory. They agree with Giesecke and Weber (2006) that for small shocks, connections reduce contagion whilst they also support the finding of Vivier-Lirimont (2006) that more connected markets result in more banks in the contagion process. Similarly they support the results of Georg (2011) and Iori et al. (2006) that contagion may be more significant when the market is more connected. Whilst the results for the largest shocks agree with Allen and Gale (2001), the inter-bank market is of little use when there is a system wide shortage of liquidity.

The pattern of failures shown in this paper differs from that of Lorenz and Battiston (2008) and Battiston et al. (2009). Both of these papers find that failures are minimized for intermediate levels of market connectivity. In each case the authors examine different mechanisms to those employed here. The model of Lorenz and Battiston (2008) differs in that it does not permit cascades, a mechanism central to our findings. The results of Battiston et al. (2009), in contrast, are driven by an inter-temporal *financial accelerator*. This mechanism does not have an equivalent within our model as we focus on the short term (within period) effects. If this mechanism is removed, the authors find a similar pattern of results to that seen in this paper for smaller shocks i.e. increasing connectivity reducing contagion.

To examine the importance of contagion we separate the failures in the banking system into two groups (Figure 1) in a similar manner to Martinez-Jaramillo et al. (2010); the casualties of the initial shock and those caused by the failure of counter-parties. In line with Elsinger et al. (2006), for all but the smallest shock in the most connected markets over half of the bankruptcies are caused by contagion. The systemic shock plays a major role in weakening the banks' equity, however, it is the failure of counter-parties which induces bankruptcy in the majority of cases and so is a significant aspect of systemic risk.

The number and size of banks which fail in the face of a systemic crisis is only one measure of the severity of the impact. If a bank fails the deposit insurer has to step in to compensate depositors. The insurer may therefore be concerned with the cost of repaying deposits rather than the number of bank failures in judging the optimal inter-bank market structure and whether rescuing banks would be appropriate. Figure 2 shows that as the size of the shock increases the cost to the depositor insurer also increase. The market architecture, however, has a very different effect on the cost of failures from that observed for the number of bankruptcies. As connectivity decreases the cost to the insurer increase regardless of the size of shock. This is because the more connected a market is the more of the cost of failures are born by the surviving banks. When a bank fails in a weakly connected market it has a large impact on a relatively small number of creditors. The impact heavily damages their balance sheets resulting in a large loss in equity and little left to pay depositors. In contrast, in a strongly connected market the failure of each bank affects many more counter-parties. This may result in more bankruptcies, however, the smaller impacts mean that those banks which fail may still have some assets on their balance sheet and be able to partially repay depositors. The surviving affected banks effectively bear some of the cost of the failure in terms of reduced equity. For the insurer increased connectivity is beneficial as it reduces costs, even if it potentially increases the number of bank failures[14].

5 Regulation

The previous section highlighted the effects of the market structure on contagion under both individual and systemic shocks. Here we consider mechanisms for limiting the impact of these events and their wider effect on the market.

5.1 Equity and Reserve ratio

A key proposal put forward in the Basel III reforms requires banks to hold a higher percentage of capital relative to their risky assets. As a result, banks are more tightly constrained in the degree to which they can leverage their positions and so should be less at risk of failure through poor investment outcomes. An alternative proposal has been made to tighten banks minimum reserve ratios. This change would force banks to hold a higher proportion of liquid reserves which would provide them with increased protection against liquidity shocks. Both of these mechanism are tested within this model. The equity and reserve ratios are varied independently and 500 further experiments conducted for each parameter combination. We consider increases of each requirement to 12% and 15% for the equity and reserve ratios respectively. We focus our analysis on the case of systemic shocks as the effect of these regulatory changes on individual bank failures has already received much attention. Nier et al. (2007), Iori et al. (2006) and Gai and Kapadia (2010)

[14]There may be additional social costs due to damage to the payment system if many banks fail.

all find that increasing the amount of reserves which banks hold reduces the number of bankruptcies.

Figure 3 shows that increasing the equity ratio results in a large reduction in failures in nearly all cases. The decreased level of leverage reduces the impact of the macro-economic shock. At the same time there is a reduction in inter-bank lending which limits the effect of failing banks on their counter-parties. Together these two factors combine to reduce the total impact of a shock. Increasing the reserve ratio has a relatively small effect on the markets susceptibility to contagion which is generally only significant for very large shocks. This is because contagion is primarily driven by banks failing through lack of equity. The increased reserve ratio means banks hold more liquid funds which may allow a bank to repay a loan when one of its own loans is not repaid. In the model market, as in real markets, there are relatively few banks which both lend and borrow (Iori et al., 2008) so increasing liquidity has a limited effect. Whilst both of the regulations reduce the number of bankruptcies the mechanism by which they do so, restricted lending to firms and banks, has a negative effect on the economy as a whole. The average value of loans to firms reduces by 8% to 361.3 for the change in reserve ratio and 12% to 345.1 for the change in equity ratio. The overall effect of these regulatory changes is therefore ambiguous, they reduce bankruptcies but at the same time reduce lending.

5.2 Borrowing Constraints

An alternative to constraining the total lending or borrowing is instead to constrain the maximum funds a bank may lend to a single counter-party. This forces banks to diversify their inter-bank lending, making them less susceptible to the failure of a single debtor. Here we implement this regulation by limiting the maximum a particular lender may lend to a particular borrower to be no more than a multiple η of the borrowers equity.

Table 6 presents the results of 500 simulation for three different borrowing constraints. For $\eta = 10$ it can be seen that the constraint does not effect the steady state results, there is no significant change in any of the market statistics. As η is decreased the constraint becomes binding. For $\eta = 5$ the effect of the regulation is beneficial, the number of systemic bankruptcies is significantly reduced. The regulatory change limits the size of the inter-bank connections reducing the probability of a bank failing due to the collapse of one of its creditors. The regulatory change also has a broader beneficial effect. There is a reduction in the demand for inter-bank loans which, reduces the total volume of loans and the interest rate in the market. As a result the volume of loans to firms increases and there is more competition between banks forcing down the firm borrowing rate.

If the borrowing constraints are too tight, however, there can be substantial negative effects. For $\eta = 2$ there is still a significant reduction in bankruptcies. The function of the inter-bank market, however, is severely impaired, meaning funds are no longer efficiently allocated and the total value of loans to firms is heavily reduced. By regulating too heavily the economy is severely restricted.

6 Model Sensitivity

The model presented above provides a framework which captures the key behaviors of banks, firms and households. Assumptions were made in forming the model, which whilst making it more transparent, simplified important aspects of real world behavior. Here we relax several of these assumptions to move the model closer to reality whilst also permitting a greater degree of heterogeneity within the system.

6.1 Parameter sensitivity

The results presented above are based on one parameter combination. Here we demonstrate the robustness of the results and how behavior changes if parameters are varied. Table 1 details the models six key parameters. Of these six, changes to α_g and β_g have already been considered as regulatory actions. Further simulations were run in which the remaining four parameter values were changed and the key affects reported[15].

Varying the payoff from investments, μ, affects the loan, deposit and inter-bank interest rates. Greater returns from investments allow banks to charge firms higher interest rates

[15]Tables of results demonstrating the relations are available from the author upon request.

which in turn allows banks to pay higher rates for funds from both depositors and on the inter-bank market. The model is robust to a wide range of values. $\mu = 1.15$ was chosen as it produced deposit and loans rates comparable to reality.

The parameters controlling the probability of a successful investment, θ, and the number of firms, Q, are closely linked. Together they control the supply of potentially fundable loan requests. A decrease in firms results in fewer loan requests per time-period, whilst a decrease in θ reduces the expected return of projects making fewer profitably fundable[16]. The results of the model are robust across a wide range of parameter values ($0.9 < \theta < 0.999$, $Q > 20N$), if either or both values are too low there may be insufficient profitable investment proposals resulting in unallocated funds and potentially no inter-bank lending. $Q = 10000$ and $\theta = 0.99$ provided sufficient supply of funding requests whilst maintaining computational tractability. Increasing Q beyond this point leads to significantly slower program execution without changing the results.

While θ and Q describe the supply of investment projects, N, the number of banks, controls the demand. The model produces qualitatively similar results for a wide range of values ($N > 40$). $N = 100$ was chosen as it is of the same magnitude as the number of banks in many of the worlds inter-bank markets, though some are much larger or smaller. M the number of households has very little effect on the behavior of the model as households simply pay deposits into the banks. The number of households was set equal to the number of firms, however, for numbers of households $500 < M < 1000000$ there was little quantitative effect.

6.2 Bank confidence

One of the key features of the recent financial crisis was the loss of liquidity within inter-bank markets. Banks observed the failures of other financial institutions and became reluctant to lend due to the fear of not regaining their funds. The loss of confidence resulted in a shortage of liquidity and an exacerbation of the crisis. In the model presented above the failure of a bank may cause other banks to fail. Banks, however, do not take

[16]Note this parameter also interacts with μ. The larger the value of μ the lower θ may be whilst maintaining a profitable project.

this into account, they do not become more reluctant to lend even though the probability of funds not being returned is potentially increased. To capture this effect the model is modified. Equation 4 is changed such that:

$$f(I_i^t) = \begin{cases} \theta_i^{interbank} - \kappa_i B^t, & \text{if } I_i^t > 0 \\ 1, & \text{if } I_i^t \leq 0 \end{cases} \tag{12}$$

Where B^t is the number of bank failures in the current time step t and κ_i is a parameter controlling the size of bank i's reaction to bankruptcies. A larger value of κ_i means that bank i reacts more strongly to a bankruptcy with a greater loss of confidence in the inter-bank market and so a greater reduction in the banks estimate of the liklihood of being repaid. The value of κ_i is assigned randomly at the start of the simulation and is optimized in the same way as deposit and loan interest rates. B^t is set each time period based on the number of bank failures.

Allowing banks to react to failures negatively affects the efficiency of the economy. There are fewer loans to firms and fewer inter-bank loans, both quantities also have a higher standard deviation (Table 7). This reduction in lending, however, does reduce the number of bankruptcies in systemic shocks. Less inter-bank lending means fewer banks fail due to contagion. Unfortunately this reduction is accompanied by a much larger fall in the amount of loans to firms. Banks react to the failure of counter-parties by stopping lending on the inter-bank market. As a consequence funds are less efficiently allocated and the economy as a whole suffers.

6.3 Credit Worthiness

In the base model it was assumed that there existed a single inter-bank interest rate. It was argued that this was a reasonable assumption if banks have limited information about each others states, the probability of systemic events is low, and the market is efficient. In a crisis, however, banks vary their inter-bank rates depending on the counter-party. More credit worthy banks, those thought less likely to fail, pay lower rates. At the same time banks tend to interact repeatedly with the same counter-parties (Cocco et al., 2009) potentially allowing more attractive interest rates due to improved information.

Each time period each bank has associated with it a risk premia, ζ_i drawn from $|N(0, 1/E_i)|$ which is the market estimation of the necessary compensation to lenders for the risk of it failing. This is a simplification of a potentially very complex effect. In reality a banks risk premia is dependent on its own situation and the risk attitudes of all other market participants. This mechanism, however, matches the empirical findings of Akram and Christophersen (2010) that larger banks receive more favorable inter-bank interest rates. I also agrees with our earlier observation that larger banks are less likely to fail (e.g. Section 4.3). This rate is added to the inter-bank rate bank i pays when it borrows. When a bank lends money it calculates its lending preferences using the base inter-bank rate. The recipients premia is not included as the additional value received over the base inter-bank rate is considered to be fair compensation for the additional risk. As such the bank does not have a preference between potential borrowers.

The addition of a risk premia reduces inter-bank lending and increases stability. As a result the market is less volatile, there are fewer bankruptcies and more funds are allocated to firms (Table 7). The system as a whole is more resilient, in response to systemic shocks, the number of failures is reduced whilst the amount of lending to firms is less effected. This agrees with Park (1991), who shows that historically the availability of solvency information regarding individual banks reduces the severity of panics. Here the risk premia is conditional on bank equity and so is equivalent to giving banks this information. The introduction of the risk premia makes it relatively more expensive for smaller and potentially more vulnerable banks to borrow. As a consequence inter-bank borrowing along with the potential for systemic risk are both reduced. This occurs in a consistent and stable manner, as opposed to the effects of bank confidence set out above, resulting in the increased stability and lending to firms.

7 Conclusion

The structure of the inter-bank lending market has a major effect on the stability of the financial system. Previous work has shown two contradictory relationships, both an increasing and decreasing liklihood of failures with increasing market connectivity. The model presented here demonstrates the conditions under which each effect is dominant. For systemic shocks the optimal inter-bank market connectivity varies with shock size. Under small shocks higher connectivity helps to resist contagion but for larger shocks it has the opposite effect. As a consequence there is no single best market architecture able to limit contagion from systemic shocks. There is, however, an optimal market structure for reducing the costs of these shocks. The more connected a market is, the more the costs of failures are internalized reducing the cost to an insurer. In response to an individual bankruptcy more inter-bank connections reduce the expected number of failures. Despite this relationship it is found that intermediately connected markets potentially suffer the largest contagious effects. These markets share risk less well than those better connected yet are potentially susceptible to the failure of a single bank spreading and affecting the whole market making them particularly vulnerable to the failure of the largest banks.

In order to limit the effects of contagion several regulatory actions were examined. Changes to both the reserve and equity ratios were considered but were found to have ambiguous results. In both cases increasing the ratios resulted in a decreased size of contagion but also decreased lending, though both effects are more marked for changes in the equity ratio. Loan constraints that limit the amount a lender may lend to a particular borrower, were also considered. For intermediate levels of regulation bankruptcies were reduced and more loans given to firms, suggesting this could be a promising mechanism for limiting systemic risk. It was also shown that if banks react to the bankruptcies of their peers the economy is destabilized and funds are allocated less efficiently. In contrast if banks condition their lending rates on the credit worthiness of their counter-parties this reduces risk and makes the market less susceptible to contagion.

The model is sufficiently general that it invites further extension. The architecture of the market considered in this paper was imposed exogenously, banks had no choice about their counter-parties. A richer model would relax this constraint, allowing lenders to select and decline potential borrowers and to offer different interest rates based on the counter-parties financial position. This would allow issues such as the characterization of the optimal market structure to be addressed. Even without making this endogenous there are other market structures which could be investigated, for instance hierarchical networks as seen in the UK inter-bank market or scale-free topologies.

The regulatory changes considered in this paper were of a static nature, regulations were changed and the model simulated to find the new equilibrium. This does not have to be the case. There is scope to investigate the application of regulations dynamically, for instance changing capital or reserve requirement or providing banks with additional liquidity at particular points in time. The role of the central bank was also not considered. Allen et al. (2009) have shown how a central bank may limit volatility through open market operations whilst Georg (2011) examines the ability of a central bank to stabilize a financial network. Central bank intervention, in the form of bail outs or quantitative easing could also be examined with this model.

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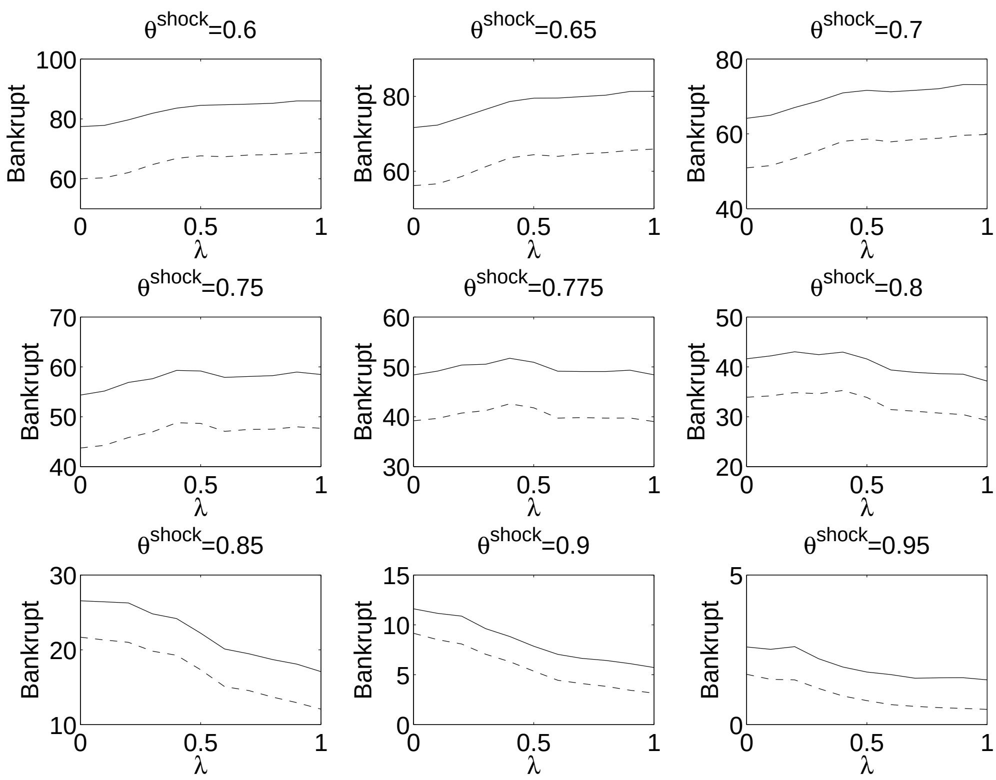

Figure 1: Total number of bankruptcies occurring on shock period (solid line) and the number of bankruptcies which were caused by contagion (dashed line), for different values of θ^{shock} and λ. Note the scale on the Y axis changes to illustrate the effect of λ. All shocks conducted at period 10000 and averaged over 500 repetitions.

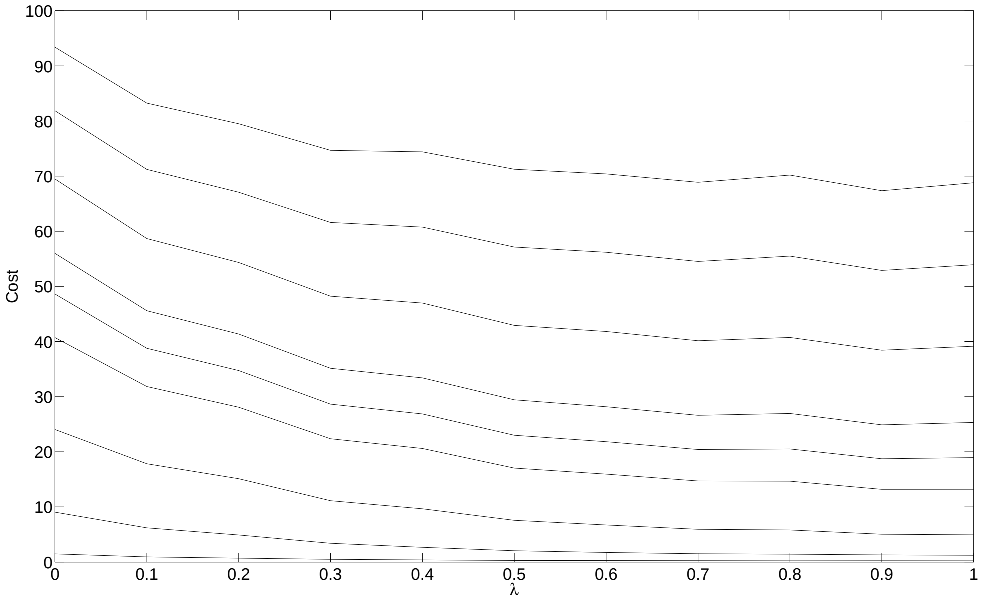

Figure 2: Total cost of repaying depositors of failed banks for different values of θ^{shock} and λ. The top line corresponds to the largest shock ($\theta^{shock} = 0.6$) the lines below are for shocks of decreasing size. All shocks conducted on period 10000 and averaged over 500 repetitions.

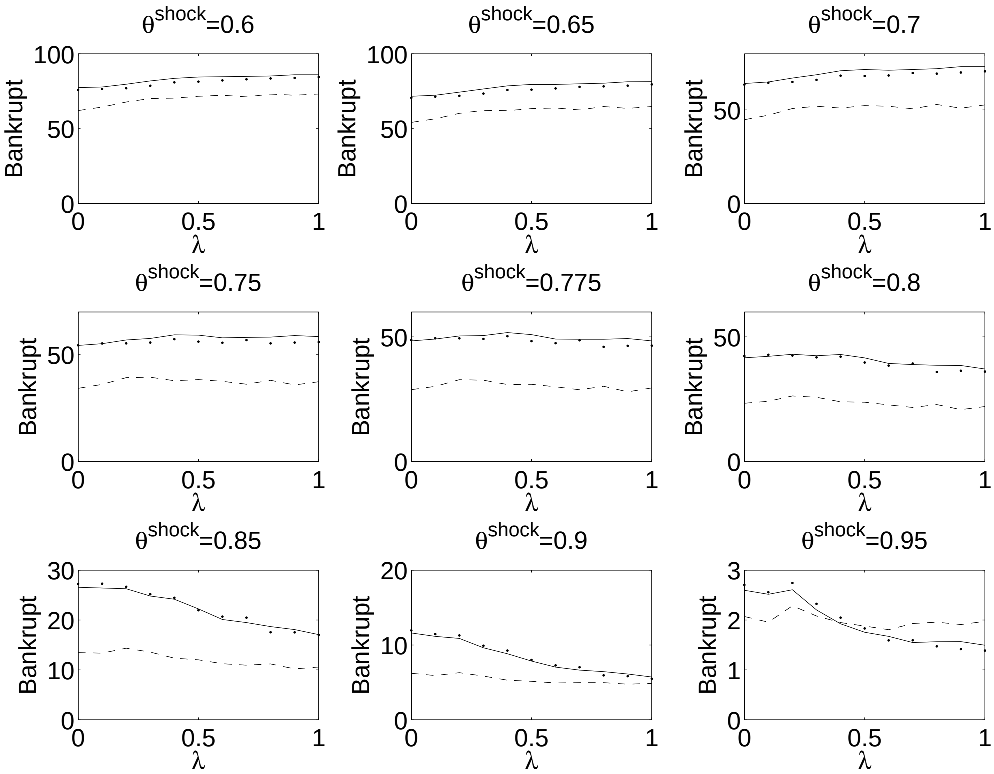

Figure 3: Total number of bankruptcies occurring on shock period for the base model (solid line), increased equity ratio (dashed line) and increased reserve ratio (dotted line), for different values of θ^{shock} and λ. Note the changing scale on the Y axis to illustrate changes with λ. All shocks conducted at period 10000 and averaged over 500 repetitions in each case.

Parameter	Meaning	Value
α_g	Reserve Requirement	0.10
β_g	Capital Requirement	0.08
N	Banks	100
M	Households	10000
Q	Firms	10000
θ_j^t	Project success probability	$U(0.99, 1.0)$
μ	Project payoff	1.15

Table 1: Parameters used for all simulations (unless otherwise stated).

Model Type	Value	SD	Empirical Type	Normalized	Real
Loans	391.5	(32.6)	Loans	986.7	8281.9
Inter-bank Loans	283.3	(36.9)	Inter-bank Borrowing	233.4	1958.8
Reserves	34.8	(3.4)	Cash Assets	35.9	301.0
Unused capital	14.3	(6.8)			
Deposits	341.3	(31.1)	Deposits	751.0	6303.2
Equity	99.1	(5.1)	Residual	99.1	831.8

Table 2: Assets and liabilities of model data along with data for commercial banks in the USA (billions of Dollars), December 2006, source: H.8 statement, Board of Governors of the Federal Reserve System. The left hand side of the table presents the model data whilst the right hand side presents empirical data normalized such that the Residual is equal to the model Equity. Unused capital is capital placed in reserves above that which the banks reserve ratio specifies due to the bank being unable to find a profitable way to allocate the funds. The level of inter-bank lending in the model is the sum of all positive positions. By definition the sum of all positions, positive and negative is 0. Items in the H.8 statement with no equivalent in the model are omitted

Term	Value	SD	Term	Value	SD
Loan Rate	0.069	(0.011)	Inter-bank Rate	0.058	(0.01)
Deposit Rate	0.028	(0.006)	Inflation Rate	0.13	(0.02)
Lenders	77.6	(6.1)	Average Lender Equity	0.83	(0.08)
Borrowers	21.1	(4.9)	Average Borrower Equity	1.67	(0.61)
Both	4.57	(2.79)	Average Both Equity	0.87	(0.29)
Bankrupt	0.18	(0.81)	α_i	0.06	(0.03)
$\theta_i^{interbank}$	0.99	(0.05)	β_i	0.06	(0.04)

Table 3: Aggregate model statistics at period 10000 averaged over 500 runs. Standard deviations in parenthesis. Values calculated prior to inflation/consumption effect. ‘Both’ in the table refers to those banks in the system who were lenders in one period and borrowers in the next (or vise versa).

λ	Connections		Large to Large		Large to Small		Small to Small	
0.0	180.0	(26.7)	65.4	(9.4)	97.5	(21.2)	17.1	(13.2)
0.1	386.5	(55.0)	123.3	(13.8)	210.4	(44.6)	52.7	(29.1)
0.2	684.2	(109.4)	207.2	(26.2)	364.3	(89.3)	112.7	(57.6)
0.3	1017.7	(154.2)	307.8	(39.6)	537.2	(124.8)	172.7	(81.5)
0.4	1307.4	(204.0)	408.9	(56.8)	694.5	(165.7)	204.0	(104.6)
0.5	1643.0	(253.3)	517.4	(69.9)	875.5	(205.6)	250.1	(130.5)
0.6	1965.0	(298.9)	627.4	(83.1)	1054.7	(244.0)	282.9	(151.3)
0.7	2298.5	(339.4)	727.2	(95.1)	1227.1	(272.5)	344.2	(178.5)
0.8	2598.6	(394.2)	829.6	(111.2)	1391.5	(314.7)	377.4	(209.7)
0.9	2984.0	(440.6)	942.2	(123.0)	1597.3	(359.6)	444.6	(222.9)
1.0	3298.9	(494.8)	1049.1	(137.4)	1778.5	(403.6)	471.2	(251.2)

Table 4: Statistics describing the structure of the inter-bank market network for variation in λ. Statistics collected at day 10000 and averaged over 500 runs. Standard deviations in parenthesis. The last three columns give the number of lending relationships between large banks (above median size) and small banks (below median size).

λ	Failures		Probability		Size		Cause Equity		Largest	
0.0	1.62	(0.61)	0.226	(0.059)	7.16	(3.98)	2.08	(3.20)	19.8	(10.5)
0.1	1.59	(0.45)	0.213	(0.049)	7.45	(2.87)	1.84	(1.15)	24.6	(11.7)
0.2	1.43	(0.47)	0.183	(0.036)	7.82	(3.30)	1.92	(0.83)	28.9	(13.1)
0.3	1.17	(0.52)	0.144	(0.029)	8.10	(3.90)	2.15	(0.90)	28.8	(14.3)
0.4	0.96	(0.60)	0.105	(0.029)	9.15	(4.88)	2.52	(1.05)	29.8	(16.5)
0.5	0.71	(0.75)	0.074	(0.030)	9.58	(6.06)	2.81	(1.06)	27.5	(18.1)
0.6	0.57	(0.93)	0.052	(0.029)	10.89	(8.06)	3.15	(1.31)	27.2	(20.3)
0.7	0.43	(1.18)	0.036	(0.026)	11.75	(10.90)	3.28	(1.74)	25.8	(23.3)
0.8	0.35	(1.42)	0.026	(0.024)	13.46	(14.56)	3.34	(2.29)	26.0	(26.5)
0.9	0.26	(1.77)	0.018	(0.022)	13.93	(18.42)	3.24	(2.94)	23.4	(28.7)
1.0	0.22	(2.10)	0.013	(0.019)	16.79	(25.70)	3.13	(3.51)	23.1	(32.4)

Table 5: Statistics showing the effects of single bankruptcies on the economy for variation in λ. 'Failures' is the average number of banks which fail as a consequence of a single bank being made bankrupt (excluding the initial bank). 'Probability' is the chance that contagion will occur. 'Size' is the average number of banks which go bankrupt conditional on contagion occurring. 'Cause Equity' is the average equity of the banks which cause contagion. 'Largest' is the size of the largest contagion. Data collected using market states saved at period 10000 and averaged over 500 runs.

Shock Size	η							
	∞		10		5		2	
0.6	82.9	(12.17)	82.5	(12.49)	66.1	(15.48)**	37.1	(13.51)**
0.7	69.9	(18.03)	69.7	(18.12)	50.5	(17.39)**	26.3	(10.57)**
0.8	40.6	(22.43)	40.6	(22.40)	26.1	(15.95)**	17.4	(8.10)**
0.9	8.4	(9.37)	8.4	(9.35)	7.8	(5.41)**	9.4	(4.55)**
Loans	391.5	(32.62)	392.9	(35.44)	404.0	(75.58)**	303.3	(86.72)**
I-bank Loans	283.3	(36.91)	282.2	(39.98)	189.3	(63.64)**	66.1	(27.25)**
Lending Rate	0.069	(0.008)	0.068	(0.006)	0.050	(0.008)**	0.025	(0.001)**
I-bank Rate	0.058	(0.010)	0.058	(0.013)	0.045	(0.018)**	0.016	(0.011)**

Table 6: Statistics showing the effects of systemic shocks on the economy for different borrowing constraints averaged across λ. All shocks conducted at period 10000 and averaged over 500 repetitions in each case. $\eta = \infty$ corresponds to the base case where there is no constraint. The market statistics at the bottom are pre-crash values. 1% significance indicated by **, 5% significance by *.

	Inter-bank Confidence			Credit Worthiness		
	Bankrupt	Loans	I-B Loans	Bankrupt	Loans	I-B Loans
Steady	0.26	341.0	246.6	0.04	410.7	247.7
state	(1.11)	(89.2)**	(80.4)**	(0.26)**	(20.3)**	(28.8)**
0.6	66.37	58.4	16.5	79.65	84.8	14.9
	(30.80)**	(87.7)	(55.8)	(8.99)**	(81.9)**	(27.4)
0.7	56.79	83.9	25.9	64.74	141.1	40.7
	(30.10)**	(102.0)**	(61.6)*	(15.86)**	(112.0)**	(64.9)
0.8	35.46	126.8	53.7	33.71	227.8	109.4
	(26.00)**	(109.5)**	(75.6)**	(19.32)**	(91.9)**	(80.9)*
0.9	7.57	199.2	113.9	6.56	347.4	205.0
	(10.00)	(93.9)**	(77.6)**	(6.27)**	(37.7)**	(35.6)**

Table 7: Statistics showing the steady state and effect of systemic shocks for two different model cases. Values averaged over λ, collected at period 10000 for 500 repetitions in each case. 1% significance indicated by **, 5% significance by *.