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DEPARTMENT OF ECONOMICS

DO CHILDLESS HOUSEHOLDS SUPPORT LOCAL PUBLIC PROVISION OF EDUCATION

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Do childless households support local public provision of education *

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January 20, 2007

Abstract

Empirical and theoretical studies show that the local provision of public education affects the well being of individuals through two channels: the first reflects the direct use of the good, whereas the second runs through the value of the housing. The second effect leans on the idea that the quality of public education is capitalized into the value of the own housing.

Empirical evidence finds that in a multi-community model childless households support local public spending in education because of the capitalization effect. I study the behavior of childless households, not necessarily elderly, in a two community model and show that the capitalization effect may not be a sufficient condition for middle aged households without children to support local public spending in education by a majority voting.

JEL numbers: H52, H72, I22, R2

1   Introduction

Empirical and theoretical studies show that the local provision of public education affects the well being of individuals through two channels: the first reflects the direct use of the good, whereas the second runs through the value of the housing. The second effect leans on the idea that the quality of public education is capitalized into the value of the own housing.

In this paper I try to show whether, in a model where the local provision of public education is financed by a tax on residents, the capitalization of public education into the value of housing is a sufficient condition for childless households to support local taxation.

The pioneer in capitalization study is Oates (1969). He analyzes a 1960 sample of northern New Jersey communities and finds that the value of housing increases in the public expenditure in the school system. Sonstelie and Portney (1980a and 1980b), Heinberg and Oates (1970), Orr (1968) and Hamilton (1979) confirm Oates's results of the capitalization of school quality and per pupil expenditure into the value of the housing. It is also shown that a property tax is the most effective channel the local public expenditure runs through. Yinger et al. (1988) survey all the main papers dealing with the capitalization of a property tax into the value of the housing.

A wide literature analyzes the capitalization effect within a framework where the level of provision is decided by a voting system. This literature shows that the capitalization effect may allow households to support local public spending in education. Yinger (1981) shows that in multi-communty model the capitalization of a local tax into the housing values is a sufficient condition for the median voter to support local public spending. Fischel (2001) invented the term "homevoter" to represent the homeowners whose voting tends to maximize the value of their housings. The main idea is that when the government allows the residents to decide the tax by a majority voting, each voter internalizes the capitalization of public education into the value of his housing, and then votes for a positive tax.

I introduce the childless voters into a model where the local provision of public education is financed by a tax decided in a majority voting.

Empirical evidence finds that in a multi-community model childless households vote for a positive tax because of the capitalization effect. Benson and O'Halloran (1987) find that childless voters in California support school spending because of its positive effect on their property's value. Baldosn and Brunner (2003) find that in California elderly generally vote to decreases the state spending but are much more willing to support local spending. Cutler et al. (1993), Hoxby (1998), Goldin and Katz (1997, 1999), Alesina et al. (1999), Bergstrom et al. (1982), Harris et al. (2001), Hilber and Mayer (2006) show that percent of elderly may be associated with higher local school spending. However, all these studies identify childless households only as elderly by abstracting from any evaluation about the behavior of childless households whose members are middle aged.

I study the behavior of childless households, not necessarily elderly, in a two

community model and show that the capitalization effect may not be a sufficient condition for middle aged households without children to vote for a positive tax.

A wide branch of urban economics literature deals with the local provision of education by a majority voting. Tiebout (1956) is the first to theoretically model an economy composed of many independent communities where public good is provided by the local government by a local tax. Many studies attempted to refine Tiebout (1956) by introducing the local provision of public education. Although these attempts strongly extend this literature in the last decade, surprisingly the direct introduction of childless households in a theoretical model is still missing. These studies abstract from childless voters and show that households vote for a positive tax because they benefit from the educational provision trough their school aged children[1].

My results are in line with the theoretical model of Brueckner and Joo (1991). They consider a model where households live in their community only for two periods. The local government finances the provision of public good with a property tax. At the beginning of the first period, the residents are allowed to vote on the level of public good, which remains fixed over the two periods. Since all the households are assumed identical, this paper only deals with the behavior of a representative voter by abstracting from the existence of the voting equilibrium. When voting, the households forecast to leave the community in the future. In this model households are more willing to vote for maximizing the value of their housing the nearer is the date of their departure or the higher is the probability of an early departure in presence of uncertainty about the future.

Following Brueckner and Joo (1991) I allow the capitalization effect to run through the sale of the housing, within a context in which public school is locally provided. Differently from Brueckner and Joo, I introduce the analysis of the voting equilibrium. I consider a two period model analyzing a metropolitan area composed of two communities whose boundaries are exogenously fixed. The area is inhabited by a continuum of households both with and without a child. The public education is provided by the local government through a tax decided in a majority voting. In the first period households sort into communities and buy a housing in a competitive market. Once the allocation process is ended, households can send their child, if any, only to the school belonging to community where they live. At the end of the first period, households vote for the tax. Since I assume that voting takes place only once, the tax remains fixed over the two periods. In the second period with a certain probability households must leave and resell their housing. New households come into the area, buy a housing from the leaving households and sort into the communities. The new residents can not modify the local provision of education. In this model the capitalization effect consists in the positive correlation between the reselling price and the tax. The reselling price is higher the higher is the tax decided in the first period. In this framework childless households vote for a positive tax only

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[1]Neychba (2003) stresses the necessity of introducing the childless households in a multi-community model, but he also points out the complications arising from this refinement in terms of voting equilibrium.

for a sufficiently high probability of leaving and reselling their housing. The idea is that the probability of reselling the housing could be considered as a weight given to the capitalization effect. Only for a sufficiently high probability the marginal benefit from the higher tax capitalized into the housing price allows childless households to vote for a positive tax.

I extend the capitalization effect to a two-community theoretical model in which the childless households are directly introduced. The aim of my paper is to show whether the capitalization of the tax into the housing price is a sufficient condition for the childless households to vote for a positive tax, by ruling out any consideration on whether such a positive tax effectively maximizes the value of the housing. Hence, my paper does not properly belongs to the literature (i.e. Brueckner and Joo 1991, and Fischel, 2001) dealing with the maximization of the housing value by a voting on the local public spending. My work can be thought as a pioneering study attempting to stress the necessity of introducing the voting behavior of the childless households in the standard urban economics theoretical models in which the local tax is decided by a majority voting.

I extend the capitalization effect to a dynamic contest to better represent the behavior of the middle aged households. The probability of changing location may reflect some labour market issues as labour mobility or turnover. The use of such a probability may help to differentiate the childless households according to the age of their members. The childless households whose members are young may be identified by a high probability of changing location, whereas elderly are supposed to be retired and in general less willing then youth to leave their own housing. In this sense, my result shows that only for young childless households the capitalization effect may be a sufficient condition to support a local public spending in education.

2   The model

I consider a two periods $(t = 1, 2)$ model analyzing a metropolitan area divided into 2 communities (a, b). The area is inhabited by a continuum of households whose measure is normalized to 1. In the metropolitan area the boundaries and land of the two community are exogenously determined. The amount of housings is the same in each community and is denoted by H. Houses are homogenous and each household consumes one unit of housing. This implies that H also denotes the number of households living in each community. We assume that $H = 1 \backslash 2$, this means that housing capacity in both communities is just enough to contain the population living in the metropolitan area. Households can move between the two communities without mobility and residence costs.

In each community the local government imposes a tax to fund the provision of education. This tax is decided by the residents in a majority voting.

The sequence of the events is illustrated in fig. 1. At the beginning of time 1, households vote on the tax and the child, if any, goes to school. The voting takes place only at time 1, therefore tax remains fixed over the two periods.

At time 2, a shock occurs and with probability q households must emigrate.

Fig. 4.1 The sequence of the events
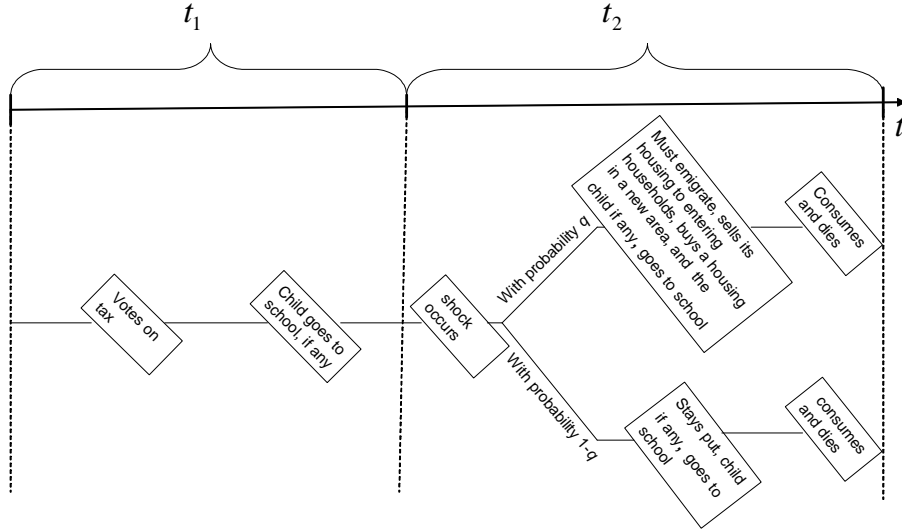
They sell their current housing and go in another area. Once bought a housing in the new area, these households send the child, if any, to school, consume and die at the end of the period.

With probability $(1 − q)$ households stay put. They send their child to school, consume and die at the end of the period.

At time 2, new residents move into the metropolitan area, buy a housing from the leaving households and sort into communities. Without loss of generality we assume that the measure of entering households is equal to $2\,(qH)$, this means that housings supplied by leaving households are just enough to contain all entering households. Since each households buy only one housing and the at time 1 all the housings are inhabited, then qH represents both the fraction of households leaving and entering one community. New residents send the child if any, to school, consume and die at the end of the period. Since at time 2 voting does not take place, the new entrants cannot modify local provision of education. In both periods, the housings price in each community is determined in a competitive market.

The objective of this model is to show whether the capitalization of local pubic education into the housing price is a sufficient condition for childless households to vote for a positive tax.

3   Households

At time 1, households differ in income $y \in \left[\underline{y}, \overline{y}\right]$ and whether or not they have a child. In particular, we denote c and n respectively household with and without

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a child. We assume that income is distributed according to the density function $f(y)$. Households consume three goods: private good z, housing h and public education per student E.

Public school is locally provided and each child is enrolled at the school belonging to the community where he is resident. The private good z is considered as numeraire and its price is normalized to 1.

The inter-temporal utility function for households with a child at time 1 is defined as follows:

$$U_c(E, z; q) = v(z_1) + E_1 + (1-q)(v(z_2) + E_2) + qv(y^c + p_2) \qquad (1)$$

Housing does not appear in the utility function because households consume just one unit of this and it is homogenous.

The first two terms denote the utility at time 1: the household with a child consumes the numeraire z_1 and receives the educational expenditure per student E_1. The third and fourth terms represent the expected utility at time 2. The discount factor is normalized to 1. In the second period, with probability $(1-q)$ households stay put. Since they do not leave the community, their consumption of numeraire is z_2 and the provision of public education per student is E_2. With probability q, households must leave. They sell their housing at price p_2 and emigrate in another metropolitan area. Once in the new area, these households buy a housing, pay the tax and send their child to school. Without loss of generality, the gross of tax price and the public education per student in this new area are normalized to zero. Therefore, the household's utility in the new area simply depends on the sum of second period income and the reselling price p_2. The household's income is given by y^c and it is assumed to remain the same in both periods.

Utility function for household without a child is:

$$U_n(z; q) = v(z_1) + (1-q)v(z_2) + qv(y^n + p_2) \qquad (2)$$

Households without a child do not receive public education, therefore utility function 2 does not directly depend on E. The childless household's income is given by y^n and, as for the households with a child, it is assumed to remain the same in both periods.

The budget constraints at time 1 and 2 are the following:

$$z_1 = y^i - \bar{p}_1^j \qquad (3)$$

$$z_2 = y^i - T^j \qquad (4)$$

where $j = a, b$, denotes the community, and y^i, with $i = c, n$ denotes the income of the households with and without a child.

Constraint 3 gives the consumption of numeraire in the first periods. The gross of tax price paid by each household in community j is $\bar{p}_1^j = p_1^j - T^j$. At time 1, household pays net of tax housing price p_1^j to buy housing in community j. The local government imposes a tax T^j. In this period, households do not save and the consumption of numeraire z_1 is given by the difference between income and gross of tax price. At time 2, the consumption of numeraire is given by the difference between the second period income and the tax, as defined in constraint 4.

Let λ be the probability that entering household has no children. Therefore the fraction of new childless residents is given by $\lambda 2qN$. The new residents come from another metropolitan area and they live in community j just at time 2.

The utility function for new residents with a child is:

$$U_c (E, z) = v (z_2) + E_2 \tag{5}$$

and for new residents without a child:

$$U_n (z) = v (z_2) \tag{6}$$

Utility of new residents without a child depend on numeraire z only.

Entering households consider the local tax as a constant because they do not vote. Each new resident pays T^j to finance the educational expenditure per student E^j.

All new residents face the following budget constraint:

$$z_2 = y - T^j - p_2^j \tag{7}$$

They do not save, and all income they have left once bought housing and paid the tax is used to consume z.

In the rest of the model we use the following assumption:

Assumption 1 *Households utility function is linear in E and increasing in all its arguments. $v(z_i)$ is twice continuously differentiable, strictly concave in z_i and $v'''(z_i) > 0$, with $i = 1, 2$.*

Assumption 1 explains that the utility functions for all types of households positively depend on numerarire and public education, but the marginal utility with respect to z is decreasing. The positive third partial derivative of the utility with respect to z is a technical assumption and determines our results in a determinant way. This implies that the risk adversion of households is increasing in the tax.

4 Local government and education production

In each community the local government imposes a tax to collect resources and provide public education (E). In this model education is considered as a private good and it is produced from the numeraire according to a constant returns to scale technology with respect to the number of student and the quantity provided.

The budget constraints for the local government in the two periods are the following:

$$n_1^j E_1^j = T^j N \tag{8}$$

$$\left((1 - q)\, n_1^j + n_2^j \right) E_2^j = T^j N \tag{9}$$

Constraint 9 and 10 respectively represent the budget constraint for the local government at time 1 and at time 2. We assume that the government cannot transfer resources between periods and that within each community households receive the same educational expenditure per student.

N is the number of households living in each community. The assumption that community a and b have the same capacity implies that the number of residents is the same in each community. Since at time 2 each leaving household sells its housing to one new resident, the number of leaving households is equal to the number of entering ones. This implies that N remains fixed in both periods. n_1^j is the number of households with a child living in community j at time 1[2], while E_i^j is the educational expenditure per student in community j at time i.

The right hand side of 8 represents all the resources collected by the local government to finance the provision of public education in the first period. At time 1, each household pays T^j, therefore the tax revenue is $T^j N$. The total level of education provided in community j is $n_1^j E_1^j$.

At time 2, each household living in community j pays T^j for the provision of E_2^j. In this period, the tax revenue in community j is $T^j N^j$. Since the new residents leave tax just the way it is, the government collects the same tax revenue in both periods. The total provision of education provided in community j in the second period is $\left((1 - q)\, n_1^j + n_2^j \right) E_2^j$, where $(1 - q)\, n_1^j$ is the number of households with a child remaining in community j once the shock has occurred and n_2^j is the number of households with a child entering the community j in the second period.

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[2] n_1^j is exogenously given. It is possible to think that nature a time 1 decides the allocation of households across the two communities. I use this assumption because the aim of this paper is only to show whether the childless households support a local public spending in education by a voting equilibrium, by abstracting from any housing market equilibrium at time 1.

5   The equilibrium of the model

In this section we characterize the equilibrium of the model by solving it backwards. We start by the housing market at time 2 and solve the maximization problems of the new residents. Then, we proceed by solving the maximization problems of the residents at time 1.

5.1   Housing market and sorting into communities at time 2

At time 2, once the shock has occurred, the housing supply in community j is equal to qH, i.e. the number of leaving households[3]. Each entering household buys one unit of housing at price p_2^j. In this period, new residents choose community given T^j and $(1-q)\, n_1^j$. For expositional convenience and without loss of generality we assume $T^a < T^b$.

To characterize the housing market equilibrium at time 2 we need the net of tax price p_2^j paid by new residents to buy a housing. Since in each community the housings market is competitive, the net of tax equilibrium prices are given by the market clearing condition. We restrict the analysis to the case of $\lambda > 1\backslash 2$[4], that is, the fraction of entering childless households is higher than the fraction of the entering households with a child. Furthermore, we use the technical assumption $(1 - \lambda) > \frac{n_1^a + n_1^b}{N}$, whose meaning will be clear in the next section.

At time 2, the housing supply function in community j is vertical at the number of housings sold by leaving households; it is given by:

$$S^j = qH \tag{10}$$

The housing demand is derived by solving the location problem of the new residents.

Entering households take decision according to their indirect utility.

The new residents' indirect utility functions in community j are[5]:

$$V_c^j \left(T^j, y^c\right) = v\left(y^c - \bar{p}_2^j\right) + \frac{T^j N}{(1-q)\, n_1^j + n_2^j} \tag{11}$$

$$V_n^j \left(T^j, y^n\right) = v\left(y^n - \bar{p}_2^j\right) \tag{12}$$

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[3] The housing supply is vertical at qH

[4] When $\lambda > 1\backslash 2$ we have $qN < 2\left(\lambda qN\right)$, therefore the housings available in one community at time 2 are not enough to allocate all entering childless households (whose measure is given by $2\left(\lambda qN\right)$)

[5] Since $U\left(E, z\right)$ is assumed to be continuously differentiable function, then $V\left(.\right)$ has the same properties.

where $j = a, b$, $\bar{p}_2^j = p_2^j + T^j$ is the gross of tax price in the second period and y^c, y^n denote the income respectively of households with and without a child. We recall that from the government's budget constraint we have $E_2^j = \frac{T^j N}{(1-q)n_1^j + n_2^j}$.

Entering households choose to live in the community in which their indirect utility is maximized. Therefore their maximization problems are:

$$\max \left\{ v\left(y^c - \bar{p}_2^a\right) + \frac{T^a N}{(1-q)\,n_1^a + n_2^a}, v\left(y^c - \bar{p}_2^b\right) + \frac{T^b N}{(1-q)\,n_1^b + n_2^b} \right\} \quad (13)$$

$$\max \left\{ v\left(y^n - \bar{p}_2^a\right), v\left(y^n - \bar{p}_2^b\right) \right\} \quad (14)$$

The allocation decisions of childless households at time 2 is explained in the Proposition 1.

Proposition 1 *New childless residents choose the community with the lowest gross of tax price for every income $y^n \in \left[\underline{y}, \bar{y}\right]$*

Proof. The partial derivative of the indirect utility function with respect to the gross of tax price $\bar{p}_2^j$ is:

$$\frac{\partial V_n^j\left(T^j, y^n\right)}{\bar{p}_2^j} = -v'\left(y^n - \bar{p}_2^j\right) < 0 \quad \forall\ y^n \in \left[\underline{y}, \bar{y}\right] \quad (15)$$

∎

The entering childless households prefer to live in the community in which the gross of tax price is lower because they neither send their child to school nor benefit from the capitalization of public education into the reselling housing price.

The allocation decisions of households with a child also depend on the educational expenditure per student E_j^i.

Since $2qN$ is the number of households entering the area at time 2, and λ is the probability that an entrant has no child, then the number of households with a child entering the area is $(1 - \lambda)\,2qN$[6]. Hence, we can define the number of households with a child choosing community b at time 2 as $n_2^b = (1 - \lambda)\,2\,(qN) - n_2^a$.

Definition 1 shows how the level of educational expenditure per student in the community a at time 2 (E_2^a) depends on the number of entering households with a child.

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[6] With $n_2^b + n_2^a = (1 - \lambda)\,q\,(2N)$

Definition 1 *Given $E_2^a = \frac{T^a N}{(1-q)n_1^a + n_2^a}$, $E_2^b = \frac{T^b N}{(1-q)n_1^b + n_2^b}$, and $n_2^b = (1-\lambda) 2 (qN) - n_2^a$, there exists a number of households with a child choosing community a at time 2, denoted by $n_2^a = n^* (T^a, T^b)$, such that $E_2^a \gtreqqless E_2^b$ if $n_2^a \lesseqqgtr n^*$; it is defined as follows:*

$$n^* (T^a, T^b) = \frac{(1-q) (n_1^b T^a - n_1^a T^b) + (1-\lambda) q (2N) T^a}{(T^b + T^a)} \qquad (16)$$

Definition 1 helps to compare the indirect utilities that a household with a child obtain in both the communities. It says that the educational expenditure per student in each community is decreasing in the number of households with a child. In particular, for a given number of households with a child remaining in the community a after the shock $((1-q)n_1^a)$, the higher is the number of households with a child entering the community a at time 2 lower is the educational expenditure per student. When n_2^a is sufficiently small the education per student in a is higher than the level provided in b.

Furthermore, Lemma 1 completes the Definition 1 and shows the effect that the number of households with a child remaining in the community a after the shock has on n^*.

Lemma 1 *n^* is decreasing in the number of households with a child remaining in the community a after the shock, for every $q \in [0,1]$*

Lemma 1 shows that if a relatively high number of households with a child remains in the community a after the shock, then it is enough just a small number of new residents with a child to make the education expenditure per student in the community a lower than the level provided in the community b. Formally, the effect of an increase in $(1-q) n_1^a$ pushes n^* down and widens the interval of the values of n_2^a such that $E_2^a < E_2^b$.

Given definition 1, Proposition 2 defines the equilibrium in the housings market at time 2.

Proposition 2 *When $\lambda > 1\backslash 2$, at time 2 there exists an unique housings market equilibrium. In this equilibrium $\bar{p}_2^a = \bar{p}_2^b$, $n_2^a = n^*$, and both communities are inhabited by new residents both with and without a child.*

Proof. Given the assumption $\lambda > 1\backslash 2$, we proceed by contradiction and sketch the proof over two steps. **1)** Consider the scenario in which $\bar{p}_2^a \neq \bar{p}_2^b$, $n_2^a \gtreqless n^*$, and the housings market is in equilibrium. When $\bar{p}_2^a \neq \bar{p}_2^b$ the indirect utility functions of childless household differ between communities, then at least one childless household has an incentive to chose one community for every $n_2^a \gtreqless n^*$. Since $\lambda > 1\backslash 2$, the capacity of each community available at time 2 is not enough to allocate all childless households, then condition $\bar{p}_2^a \neq \bar{p}_2^b$ does not characterize a housing market equilibrium. **2)** Consider the scenario in which $\bar{p}_2^a = \bar{p}_2^b$, $n_2^a > n^*$, and the housing market is in equilibrium. From Definition 1 we know

that $n_2^a > n^*$ implies $E_2^a < E_2^b$. Therefore, at least one household with a child has an incentive to go in community b because it would benefit more. Hence, if $T^a < T^b$ and $\lambda > 1\backslash 2$, then $\bar{p}_2^a = \bar{p}_2^b$ and $n_2^a > n^*$ do not characterize an equilibrium. The same argument holds for the case $\bar{p}_2^a = \bar{p}_2^b$ and $n_2^a < n^*$. ■

Proposition 2 says that at time 2, if $\lambda > 1\backslash 2$, there exists a unique housings market equilibrium. In this equilibrium community a and b have the same gross of tax prices, the same educational expenditure per student and are inhabited by new residents both with and without a child. In particular, the number of new residents with a child living in community a is n^*.

Now, given n^*, we are able to define the educational expenditure per student provided in the community a at time 2.

Definition 2 *Let* $E_2^{a*}\left(T^a, n^*\left(T^a, T^b\right)\right)$ *be the educational expenditure per student in the community a when $n_2^a = n^*$. It is defined as follows:*

$$E_2^{a*}\left(T^a, n^*\left(.\right)\right) = \frac{N\left(T^b + T^a\right)}{(1-q)\left(n_1^a + n_1^b\right) + (1-\lambda)\,q\,(2N)} \tag{17}$$

where:

$$\frac{\partial E_2^{a*}\left(T^a, n^*\left(.\right)\right)}{\partial T^a} = \frac{N}{(1-q)\left(n_1^a + n_1^b\right) + (1-\lambda)\,q\,(2N)} > 0 \tag{18}$$

Definition 2 shows that the educational expenditure per student increases in the tax *ceteris paribus*.

To fully characterize the housing market equilibrium at time 2, we need to find the net of tax equilibrium prices. To do that we use the following assumptions:

1) The utility function is:

$$U_c\left(E, z\right) = \frac{1}{1-\sigma}z^{1-\sigma} + E \tag{19}$$

2) The indirect utility of the lowest income household with a child sorting into community a^7 at time 2 is normalized to $\underline{V}$.

Therefore, by inserting budget constraints (9) and (7) into (29), we have:

$$\frac{1}{1-\sigma}\left(\underline{y}^c - \bar{p}_2^j\right)^{1-\sigma} + \frac{T^j N}{(1-q)\,n_1^j + n_2^j} = \underline{V} \tag{20}$$

after simple algebra:

$$p_2^j = \underline{y}^c - T^j - \frac{\underline{V}_0 \left((1-q)\, n_1^j + n_2^j \right)}{T^j N}, \tag{21}$$

where $\underline{V}_0 = \underline{V} \exp (\sigma - 1)$

Housing equilibrium prices in community a is:

$$p_2^{a*} \left(T^a, n^* \left(T^a, T^b \right) \right) = \underline{y}^c - T^a - \frac{\underline{V}_0 \left((1-q)\, n_1^a + n^*(.) \right)}{T^a N}8, \tag{22}$$

Since proposition 2 shows that in the housing market equilibrium we have $\overline{p}_2^a = \overline{p}_2^b$, then the net of tax housing price a time 2, in the community b, solves that identity $p_2^b = p_2^a + T^a - T^b$; it is defined as follows:

$$p_2^{b*} \left(T^b, n^* \left(T^a, T^b \right) \right) = \underline{y}^c - T^b - \frac{\underline{V}_0 \left((1-q)\, n_1^a + n^*(.) \right)}{T^a N}, \tag{23}$$

putting 16 in 23 and 24 we obtain:

$$p_2^{a*} \left(T^a, n^* \left(T^a, T^b \right) \right) = \underline{y}^c - T^a - \frac{\underline{V}_0}{N} \left(\frac{(1-q)\left(n_1^a + n_1^b \right) + (1-\lambda)\, q \, (2N)}{\left(T^b + T^a \right)} \right) \tag{24}$$

and:

$$p_2^{b*} \left(T^b, n^* \left(T^a, T^b \right) \right) = \underline{y}^c - T^b - \frac{\underline{V}_0}{N} \left(\frac{(1-q)\left(n_1^a + n_1^b \right) + (1-\lambda)\, q \, (2N)}{\left(T^b + T^a \right)} \right) \tag{25}$$

Fig. 4.2 shows the market equilibrium given $\lambda > 1\backslash 2$. The net of tax prices are p_2^{a*} and p_2^{b*}, and the number of new residents with a child choosing a is n^*.

Since the gross of tax prices are equal in both community and $T^a < T^b$, the net of tax equilibrium price in community a is higher than the net price in community b.

After an exercise of comparative static on the prices in 24 and 25, we obtain the following proposition:

Proposition 3

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[8]We recall that $n^* = \frac{(1-q)\left(n_1^b T^a - n_1^a T^b \right) + (1-\lambda) q (2N) T^a}{\left(T^b + T^a \right)}$

Case: $T^a < T^b, p_2^{a*} = p_2^{b*} + T^b - T^a, n_2^a = n^*$
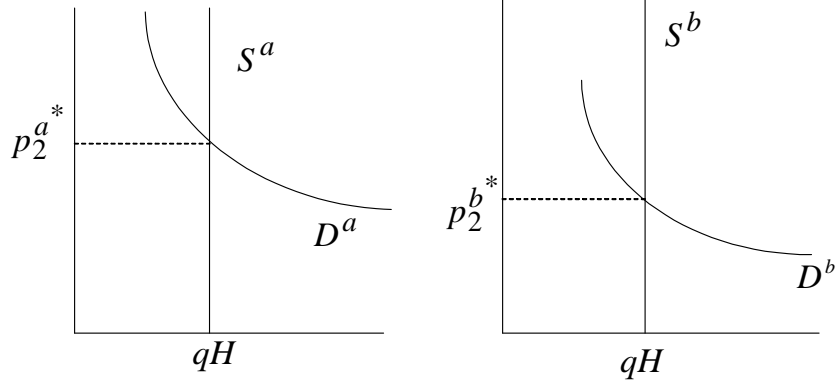

Fig 2. The housings market equilibrium

i) At time 2, there exists a tax $\widehat{T}^a$ such that the net of tax equilibrium price in community a is increasing and concave in T^a for every $0 \leq T^a < \widehat{T}^{a9}$.

ii) At time 2, there exists a tax $\widehat{T}^b$ such that the net of tax equilibrium price in community b is increasing and concave in T^b for every $0 \leq T^b < \widehat{T}^{b10}$.

Proof. We differentiate 24 with respect to T^a and have:

$$\frac{\partial p_2^{a*}}{\partial T^a} = -1 + \frac{V_0}{N} \frac{(1-q)\left(n_1^a + n_1^b\right) + (1-\lambda)\, q\,(2N)}{(T^b + T^a)^2} > 0 \ if \ 0 \leq T^a < \widehat{T}^a \quad (26)$$

$$\frac{\partial^2 p_2^{a*}}{(\partial T^a)^2} < 0 \qquad \forall T^a, T^b > 0 \qquad\qquad (27)$$

Then, we differentiate 25 with respect to T^b and have:

$$\frac{\partial p_2^{b*}}{\partial T^b} = -1 + \frac{V_0}{N} \frac{(1-q)\left(n_1^a + n_1^b\right) + (1-\lambda)\, q\,(2N)}{(T^b + T^a)^2} > 0 \ if \ 0 \leq T^b < \widehat{T}^b \quad (28)$$

$$\frac{\partial^2 p_2^{b*}}{(\partial T^b)^2} < 0 \qquad \forall T^a, T^b > 0 \qquad\qquad (29)$$

∎

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[9] Where after some albegra we have: $\widehat{T}^a = -T^b + \sqrt{\frac{V_0\left(n_1^a+n_1^b\right)(1-q)}{N} + (1-\lambda)\, 2\underline{V}_0 q} > 0$

[10] Where after some albegra we have: $\widehat{T}^b = -T^a + \sqrt{\frac{V_0\left(n_1^a+n_1^b\right)(1-q)}{N} + (1-\lambda)\, 2\underline{V}_0 q} > 0$

Proposition 3 introduces the capitalization effect and shows that the level of tax may be capitalized into the housing prices. In our model there exists a range of taxes such that the value of the housing, given by its selling price, is increasing in the tax imposed by the local government to finance public education. In particular, the higher tax set at time 1 higher is the selling price at time 2. This implies that for households with a child the marginal benefit of a higher tax runs through the educational expenditure and the reselling price, whereas childless households only may benefit from a higher tax through capitalization.

5.2 Voting at time 1

The aim of this section is finding the optimal tax chosen by a majority voting. In particular, in the section 5.2.1 we show that the preferences of households with and without a child may be single peaked. In the section 5.2.2 we show that a majority voting equilibrium exists and the median income voter may be pivotal. For the rest of the paper we restrict the analysis on the voting problem in the community a when the capitalization effect exists, that is $0 \leq T^a < \widehat{T}^a$. A relatively small range of taxes the government allows to vote on is politically realistic, because it avoids issues of dramatic structural changes that might be contested by the populations.

5.2.1 Single peaked preferences

In this framework, voting takes place once the households have already bought the housing, then the housing prices at time 1 are given[11]. The voters know the local government's budget constraints and forecast p_2^{j*} and n^*.

Voters in the community a choose the tax T^a maximizing the following indirect utility functions:

$$V_c^a\left(T^a, y; q\right) = v\left(y^c - p_1^a - T^a\right) + \frac{T^a N}{n_1^a} + (1-q)\left(v\left(y^c - T^a\right) + E_2^{a*}\right) + qv\left(y^c + p_2^{a*}\right){}^{12},$$
(30)

$$V_n^a\left(T^a, y; q\right) = v\left(y^n - p_1^a - T^a\right) + (1-q)v\left(y^n - T^a\right) + qv\left(y^n + p_2^{a*}\right). \quad (31)$$

Therefore the maximization problems are:

$$\max_{T^a} v \left(y^c - p_1^a - T^a\right) + \frac{T^a N}{n_1^a} + (1-q) \left(v \left(y^c - T^a\right) + E_2^{a*}\right) + qv \left(y^c + p_2^{a*}\right) \quad (32)$$

and

$$\max_{T^a} v \left(y^n - p_1^a - T^a\right) + (1-q) v \left(y^n - T^a\right) + qv \left(y^n + p_2^{a*}\right) \quad (33)$$

The first order condition for households with and without a child are respectively:

$$-v'\left(.\right) + \frac{N}{n_1^a} - (1-q) v'\left(.\right) + (1-q) \frac{\partial E_2^{a*}}{\partial T^a} + \frac{\partial p_2^{a*}}{\partial T^a} qv'\left(.\right) = 0 \quad (34)$$

$$-v'\left(.\right) - (1-q) v'\left(.\right) + \frac{\partial p_2^{a*}}{\partial T^a} qv'\left(.\right) = 0 \quad (35)$$

The second order condition with respect to the tax is satisfied for both types of households; in fact we have:

$$v''\left(.\right) + (1-q) v''\left(.\right) + q \left(\frac{\partial^2 p_2^{a*}}{\partial T^{a2}} v'\left(.\right) + v''\left(.\right) \frac{\partial p_2^{a*}}{\partial T^a}\right) < 0^{13} \quad (36)$$

$$v''\left(.\right) + (1-q) v''\left(.\right) + q \left(\frac{\partial^2 p_2^{a*}}{\partial T^{a2}} v'\left(.\right) + v''\left(.\right) \frac{\partial p_2^{a*}}{\partial T^a}\right) < 0 \quad (37)$$

Since the indirect utilities 30 and 31 are strictly concave in the range of taxes we are interested in (that is, $0 \leq T^a < \widehat{T}^a$), then these utilities reach the maximum at unique value of the tax. This implies that the preferences of both households are single peak.

Let T_c^{a*} and T_n^{a*} be the taxes satisfying the first order conditions 34 and 35. They are defined as follows:

$$T_c^{a*} = \arg\max_{T^a} V_c^a \left(T^a, y^c; q\right) \quad (38)$$

$$T_n^{a*} = \arg\max_{T^a} V_n^a \left(T^a, y^n; q\right) \quad (39)$$

Corollary 1 *The optimal taxes are such that $T_n^{a*} < T_c^{a*}$ for every $0 \leq T^a < \widehat{T}^a$.*

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[13] We recall that $\frac{\partial^2 p_2^{a*}}{\partial T^{a2}} < 0$, and by easy computation is possible to see that $\frac{\partial^2 E_2^{a*}}{\partial T^{a2}} = 0$

Proof. Given the first order condition in 34-35, the assumption $v''(.) < 0$ is a sufficient condition for T_n^{a*} to be lower than T_c^{a*}. ∎

The next proposition enable us to show whether the indirect utility of childless households reaches a single peak at a positive tax.

Proposition 4 *Given $(1 - \lambda) > \frac{n_1^a + n_1^b}{N}$, when at time 2 the housing price in community a is increasing in the tax, then there exists a critical value $\widehat{q}$ such that the childless household's most preferred tax is unique and positive for every $q > \widehat{q}$, whereas it is never positive for every $q \leq \widehat{q}$.*

Proof. The proof proceeds along two steps. In particular, the second step bases on two Lemmas. Firstly, we show that the slope of the indirect utility function $V_n^a(T^a, y)$ valued at tax T^a sufficiently close to $\widehat{T}^a$ is negative. In the second step we show that the slope of the indirect utility function $V_n^a(T^a, y)$ valued at $T_n^a = 0$ is increasing in q, positive for every probability lower than a critical value $\widehat{q}$, and negative for every probability lower than $\widehat{q}$. Hence, concavity of $V_n^a(T^a, y)$ over $0 \leq T^a < \widehat{T}^a$ gives the following results: i) when the probability q is sufficiently high, the function $V_n^a(T^a, y)$ reaches a unique peak at a positive tax, ii) when q is sufficiently low, the function $V_n^a(T^a, y)$ reaches its peak at a non positive tax .

First step. In this step we show that the slope of the indirect utility $V_n^a(T^a, y)$ as defined in the FOC (35) is negative at T^a sufficiently close to $\widehat{T}^a$. Let the slope of $V_n^a(T^a, y)$ be defined as follows:

$$\frac{\partial V_n^a(T^a, y)}{\partial T^a} = -v'(y^n - p_1^a - T^a) - (1-q)v'(y^n - T^a) + \frac{\partial p_2^{a*}}{\partial T^a}qv'(y^n + p_2^{a*})$$

By 26 and 27 we know that $\frac{\partial p_2^{a*}}{\partial T^a} = 0$ for $T^a = \widehat{T}^a$; therefore there exists an ε sufficiently small ($\varepsilon \simeq 0$) and a level of tax $T_\varepsilon^a = \widehat{T}^a - \varepsilon$ such that $\frac{\partial p_2^{a*}}{\partial T^a} \simeq 0$. Hence, given assumption 1 (implying $v'(.) > 0$) and expressions 26 and 27, we have that the slope of the indirect utility function is negative when $T^a = T_\varepsilon^a$.

Second step. Given the value of $\frac{\partial p_2^{a*}}{\partial T^a}$ in 26, we rewrite the slope of the indirect utility at zero tax as follows:

$$\frac{\partial V_n^a(T^a, y)}{\partial T^a} \Big|_{T^a=0} = -v'(y^n - p_1^a) - (1-q)v'(y^n) + \left(\frac{\partial p_2^{a*}}{\partial T^a} \Big|_{T^a=0} \right) qv'(y^n + p_2^{a*} |_{T^a=0})^{14}$$

$$(40)$$

The following Lemmas enable use to study the effect of the probability q on the value of $\frac{\partial V_n^a(T^a,y)}{\partial T^a} \Big|_{T^a=0}$

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[14] Where $\frac{\partial p_2^{a*}}{\partial T^a} \Big|_{T^a=0} = -1 + \frac{V_0}{N} \frac{(1-q)\left(n_1^a + n_1^b\right) + (1-\lambda)q(2N)}{(T^b)^2}$ and $p_2^{a*} \Big|_{T^a=0} = \underline{y}^c - \frac{V_0}{N} \left(\frac{(1-q)\left(n_1^a + n_1^b\right) + (1-\lambda)q(2N)}{T^b} \right)$

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Lemma 2 $\frac{\partial^2 V_n^a(T^a,y)}{\partial T^a \partial q} \mid_{T^a=0} > 0$ and $\frac{\partial^3 V_n^a(T^a,y)}{\partial T^a \partial q \partial q} \mid_{T^a=0} > 0$ for every $(1-\lambda) > \frac{n_1^a + n_1^b}{N} \parallel$.

Lemma 2 says that the slope of indirect utility function valued at zero tax is always increasing in the probability of leaving community. Until now, the condition on $(1-\lambda)$ is a technical assumption employed for exponential convenience.

Lemma 3 When $q = 0$, we have $\frac{\partial V_n^a(T^a,y)}{\partial T^a} \mid_{T^a=0} < 0 \parallel$

Lemma 3 shows that when the probability of reselling housing is zero, the slope of the indirect utility function valued at zero tax is negative.

Now from Lemma 2 and 3 we know that there exists a critical positive value $\widehat{q}$ such that the slope of the indirect utility is zero when $T^a = 0$; The critical value $\widehat{q}$ is defined as follows:

$$-v'(y^n - p_1^a) - (1-\widehat{q})\,v'(y^n) + \left(\frac{\partial p_2^{a*}}{\partial T^a} \mid_{T^a=0,q=\widehat{q}}\right) \widehat{q} v'(y^n + p_2^{a*} \mid_{T^a=0,q=\widehat{q}}) = 0^{15}$$

(41)

Given $\widehat{q}$, by Lemma 2 and Lemma 3, we have the following results: $i)$ $\frac{\partial V_n^a(T^a,y)}{\partial T^a} \mid_{T^a=0} \leq 0$, for every $q \leq \widehat{q}$, and $ii)$ $\frac{\partial V_n^a(T^a,y)}{\partial T^a} \mid_{T^a=0} > 0$ for every $q > \widehat{q}$.

Hence, by concavity of $V_n^a(T^a,y)$ over $0 \leq T^a < \widehat{T}^a$, we conclude the proof by showing that: $i)$ the indirect utility of childless households is maximized at negative tax for every $q < \widehat{q}$, $ii)$ it reaches a peak at zero tax when $q = \widehat{q}$, and $iii)$ it presents a unique peak at positive tax for every $q > \widehat{q}$. ∎

Proposition 4 says that the childless household's most preferred tax is positive only for sufficiently high probability of leaving community and reselling housing. The reason is that this probability can be considered as the weight given to the capitalization effect. When childless household does not leave community $(q = 0)$, the benefit from capitalization disappears, whereas when households must leave the community for sure, then the benefit from the capitalization of a higher tax is totally gained. Figure 4.3 illustrates the indirect utility functions of childless households[16].

Given the result in Proposition 4 we are able to show that when the probability of reselling is sufficiently high, then also the most preferred tax of the households with a child is positive.

---

[15] Where $\frac{\partial p_2^{a*}}{\partial T^a} \mid_{T^a=0,q=\widehat{q}} = -1 + \frac{V_0}{N} \frac{(1-\widehat{q})\left(n_1^a + n_1^b\right) + (1-\lambda)\widehat{q}(2N)}{(T^b)^2}$ and $p_2^{a*} \mid_{T^a=0,q=\widehat{q}} = \underline{y^c} - \frac{V_0}{N}\left(\frac{(1-\widehat{q})\left(n_1^a + n_1^b\right) + (1-\lambda)\widehat{q}(2N)}{T^b}\right)$

[16] For exponential convenience we restrict the graphical representation to the case in which $V_n^a(T^a,y)$ is concave even for $T^a < 0$. Actually, we found concavity only for a positive value of the tax, nevertheless we remark that the only aim of the figure is showing that when the probability of leaving community is sufficiently low, then there exists at least one peak at a non positive tax .
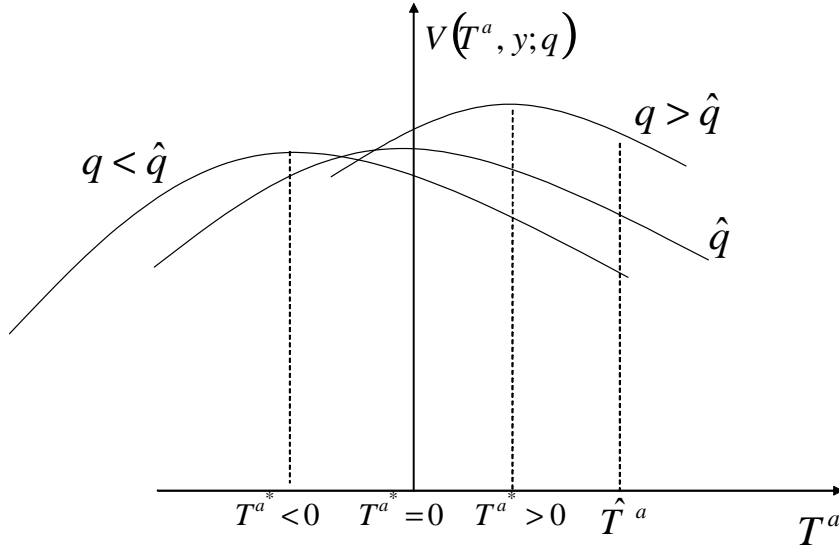
Fig Indirect utility of childless households

Proposition 5 *Given* $(1 - \lambda) > \frac{n_1^a + n_1^b}{N}$, *when at time 2 the housing price in community a is increasing in the tax, then the tax most preferred by the households with a child is unique and positive for every* $q > \widehat{q}$.

Proof. The first order conditions 34 and 35 show that the slope of the indirect utility function of the households with a child with respect to T^a is higher than the slope of the indirect utility of the childless households. This implies that the argument of proposition 4 can be also applied to show that the indirect utility of the households with a child has a single peak at positive tax. Hence, by the proof of the proposition 4 and the concavity of $V_c^a(T^a, y)$ in T^a, we have that when $q \geq \widehat{q}$ and $0 \leq T^a < \widehat{T}^a$, the indirect utility of households with a child reaches a single peak at a positive tax. ∎

The concavity of the indirect utility function of both households with and without a child makes the preferences of all the voters single peaked. This enables us to characterize the majority voting equilibrium by the median voter theorem.

5.2.2 The Voting Equilibrium

Now we characterize the voting equilibrium in community a at time 1 and show whether the median income voter is pivotal. We deal with the interesting case of $\frac{N}{2} > n_1^a$, that is, the number of childless households living in the community a is higher than the number of households with a child[17].

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[17]Since the number of childless households living into community a at time 1 is $N - n_1^a$, the sufficient condition for the childless households to be more than the households with a child

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To show whether the median income voter is pivotal we firstly need to see how the level of income drives the voting behavior of households. To do that we need the sign of the cross derivative of the indirect utility with respect to the income and the tax, that is $\frac{\partial^2 V_i(T^a,y;q)}{\partial y \partial T^a}$, with $i = c, n$. According to the standard theory of public provision of education by a majority voting, when $\frac{\partial^2 V_i(T^a,y;q)}{\partial y \partial T^a} > 0$ ($\frac{\partial^2 V_i(T^a,y;q)}{\partial y \partial T^a} < 0$) the utility of higher income households increases more when the tax is higher (lower), then higher income households prefer higher (lower) tax.

Hence, by differentiate the indirect utility $V_i(T^a, y; q)$ with respect to y^i and T^a we obtain:

$$\frac{\partial^2 V_i(T^a, y; q)}{\partial y \partial T^a} = -v''\left(y^i - p_1^a - T^a\right) - (1-q)\, v''\left(y^i - T^a\right) + qv''\left(y^i + p_2^{a*}\right) \frac{\partial p_2^{a*}}{\partial T^a} \tag{42}$$

It is possible to see that for the households both with and without a child the sign of 42 is ambiguous and depends on q and T^a.

The following proposition provides the conditions such that the sign of 42 is not ambiguous, then we are able to study the effect of household income on the voting behavior.

Proposition 6 *There exists a strictly increasing function defined over $0 \leq T^a < \widehat{T}^a$, denoted by $\widetilde{q}(T^a)$, and defined the* indifference locus*, such that a high income household with and without a child prefers higher tax if and only if the probability of reselling housing is lower than $\widetilde{q}(T^a)$.*

Proof. A high income household is indifferent between preferring high or low tax if:
$$\frac{\partial^2 V_i(T^a, y; q)}{\partial y \partial T^a} = 0 \tag{43}$$

Where $i = c, n$ denotes households with and without a child.

Lemma 4 $\frac{\partial^2 V_i(T^a,y)}{\partial y \partial T^a}$ is decreasing in q for every $0 \leq T^a < \widehat{T}^a$. $\|$

Lemma 4 shows that there exists a unique value of $q \in [0,1]$ which solves the 42 for any $0 \leq T^a < \widehat{T}^a$. This required value is $\widetilde{q}(T^a)$, it is called indifference locus

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is $N - n_1^a > \frac{N}{2}$, that can be written as $n_1^a < \frac{N}{2}$. Furthermore, we stress that $n_1^a < \frac{N}{2}$ is a necessary condition for the inequality $1 - \lambda > \frac{n_1^a + n_1^b}{N}$ to be satisfied, therefore $n_1^a < \frac{N}{2}$ is also a necessary condition for proposition 4 to hold.

and is defined as follows:

$$-v'' \left(y^i - p_1^a - T^a\right) - (1 - \widetilde{q}(T^a)) \, v'' \left(y^i - T^a\right) + \widetilde{q}(T^a) \, v'' \left(y^c + p_2^{a*}\right) \frac{\partial p_2^{a*}}{\partial T^a} = 0^{18}$$
(44)

Now by totally differentiating the (44), the slope of the indifference locus is:

$$\frac{d\widetilde{q}(T^a)}{dT^a} = -\frac{v''' \left(y^i - p_1^a - T^a\right) + (1 - q) v''' \left(y^i - T^a\right) + q \left[v''' \left(y^c + p_2^{a*}\right) \left(\frac{\partial p_2^{a*}}{\partial T^a}\right)^2 + \frac{\partial^2 p_2^{a*}}{\partial T^{a2}} v'' \left(y^c + p_2^{a*}\right)\right]}{q v'' \left(y^i - T^a\right) + v'' \left(y^c + p_2^{a*}\right) \frac{\partial p_2^{a*}}{\partial T^a} + q \left[v''' \left(y^c + p_2^{a*}\right) \frac{\partial p_2^{a*}}{\partial q} \frac{\partial p_2^{a*}}{\partial T^a} + \frac{\partial p_2^{a*}}{\partial T^a \partial q} v'' \left(y^c + p_2^{a*}\right)\right]}$$
(45)

That is positive, given assumption 1, expression (26) and the condition $(1 - \lambda) > \frac{n_1^a + n_1^b}{N}$. Figure 4.4 [19] illustrates the indifference locus $\widetilde{q}(T^a)$ [20].

Hence, given $\widetilde{q}(T^a)$ and the results in Lemma 4, we have that: $i)$ $\frac{\partial^2 V_i(T^a, y; \widetilde{q}(T^a))}{\partial y \partial T^a} > 0$ for every $q < \widetilde{q}(T^a)$, and $ii)$ $\frac{\partial^2 V_i(T^a, y; \widetilde{q}(T^a))}{\partial y \partial T^a} < 0$ for every $q > \widetilde{q}(T^a)$. $\blacksquare$

The result in proposition 6 implies that the indirect utility functions of both households with and without a child evaluated at different level of taxes cross at most once in the plane (U, y). In particular, for sufficiently high probability the utility with higher tax cross the other from above, whereas for sufficiently low probability the utility with higher tax cross the other from below. The fig. 4.4 illustrates that for any given probability, high income households prefer higher tax only when they are allowed to choose within a range of taxes sufficiently high. When the households can choose only within a range of taxes relatively low, then high income households prefer higher tax only if the probability of reselling housing is sufficiently low. This result is in line with Kenny (1978), Denzau and Grier (1984), Fischer (1988) and Epple and Romano (1996) according to which high income households are more willing to bear a tax rise for increased public education[21]. My results also show that when the voting is over a range of taxes relatively low and the probability of reselling the housing is high, then

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[18] We stress that in this definition we have
$:p_2^{a*}\left(T^a, n^*\left(T^a, T^b\right)\right) = \underline{y}^c - T^a - \frac{V_0}{N} \left(\frac{(1 - \widetilde{q}(T^a))\left(n_1^a + n_1^b\right) + (1 - \lambda)\widetilde{q}(T^a)(2N)}{(T^b + T^a)}\right)$, and $\frac{\partial p_2^{a*}}{\partial T^a} = -1 + \frac{V_0}{N} \frac{(1 - \widetilde{q}(T^a))\left(n_1^a + n_1^b\right) + (1 - \lambda)\widetilde{q}(T^a)(2N)}{(T^b + T^a)^2}$

[19] In the fig. 4.4 we draw the indifference locus as convex function in T^a. Actually, we do not show that the indifference locus is convex in T^a because it would imply a lot of complex computations. By deriving the 44 with respect to T^a we would find whether the the slope of the indifference locus increases in the tax or not. Moreover, even though the indifference locus was concave, the qualitative results providing by the figure would not change.

[20] From 44 it is possible to see that when the value of the tax is sufficiently close to $\widehat{T}^a$, then $\frac{\partial^2 V_i(T^a, y; q)}{\partial y \partial T^a} > 0$ for every $0 \leq q \leq 1$. This implies that the indifference locus does not cross the vertical line at $\widehat{T}^a$. Moreover, by looking at the expression 44, it is also possible to see that when the probability is sufficiently close to zero, then we have $\frac{\partial^2 V_i(T^a, y; q)}{\partial y \partial T^a} \big|_{q=0} > 0$ for every $0 \leq T^a < \widehat{T}^a$. This implies that the indifference locus has positive value when $T^a = 0$, that is $\widetilde{q}(0) > 0$.

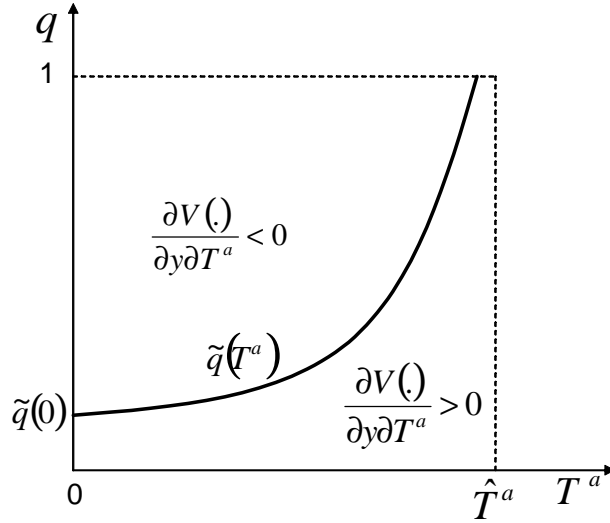[21] Epple and Romano (1996) explain that the estimation about the effect of household

Fig. 4.4 The indifference locus

low income households prefer higher tax. Hence, the higher the weight given in the utility function to the future capitalization effect (higher q) lower will be the willingness of high income households to bear a tax rise for increased public education today.

The following corollary completes the proposition 6 and helps to study the voting behavior when the most preferred tax of households both with and without a child is positive.

Corollary 2 *Given* $(1 - \lambda) > \frac{n_1^a + n_1^b}{N}$, *when* $q > \widehat{q}$, *then high income household with and without a child prefers higher positive tax if and only if the probability of reselling housing is lower than* $\widetilde{q}(T^a)$.

Proof. The proof comes from the result of proposition 4, 5 and 6. ■

Corollary 2 helps to better explain the voting behavior of the childless households by using the indifference locus. It confirms that for every given level of probability such that all households prefer positive tax, high income childless households prefer higher tax only when they are allowed to choose within a range of taxes sufficiently high.

Figure 4.5 illustrates the voting behavior when $q > \widehat{q}$, assuming that $\widehat{q} > \widetilde{q}(0)$. When $q > \widetilde{q}(T^a)$ and $q > \widehat{q}$ (the dotted area), then high income childless households prefer lower tax, whereas when $q < \widetilde{q}(T^a)$ and $q > \widehat{q}$ (the dashed area), then high income childless households prefer higher tax. Hence, high

income on the willingness to bear such a rise tax is still controversial and, for this reason, they consider the theoretical cases in which high income households prefer both a higher and a lower tax.
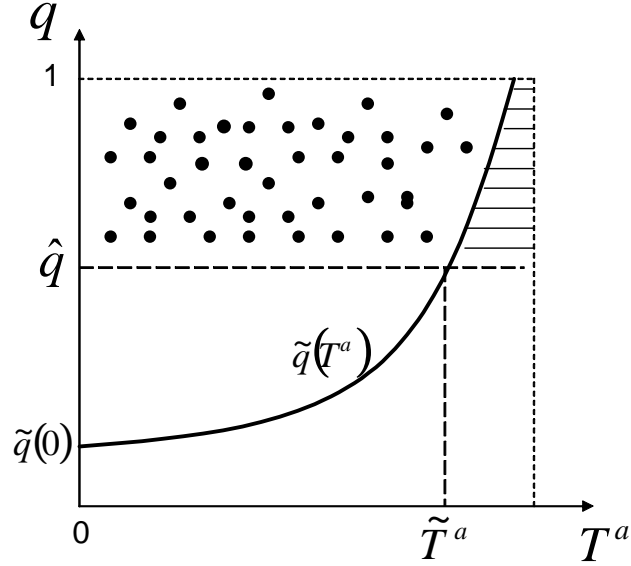
income childless households are more willing to bear a tax rise for increased public education only because of the capitalization effect. In particular, when the weight (q) given to the capitalization effect is sufficiently high, then high income childless households are more willing to bear a tax rise only when they are allowed to vote within a range of taxes sufficiently high. The same analysis can be applied to the case $\widehat{q} < \widetilde{q}(0)$ and $q > \widehat{q}$

By summarizing, with Proposition 6 and Corollary 2 I show that for whatever weight given to the capitalization effect (or equivalently for every probability of reselling the housing), when voters are allowed to choose within a range of tax sufficiently high, then only high income voters are willing to bear a tax rise for increased public education.

The behavior of households given in Proposition 6 and Corollary 1 enable us to characterize the voting equilibrium and check whether the median income is pivotal.

Definition 3 Let $\overline{y}$ be the income such that $F(\overline{y}) = \frac{N}{2}$. Let $\overline{T}_c^a$ and $\overline{T}_n^a$ be respectively the preferred tax by the voter with and without a child whose income is $\overline{y}$.

Now we can state the following proposition:

Proposition 7 *Given* $\lambda > 1\backslash 2$ *, when* $\frac{N}{2} > n_1^a$ *a majority voting equilibrium in the community* a *at time* 1 *exists and the income of median voter is different from the median income for every* $q \in [0, 1]$.

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Proof. We focus on the community a, but without loss of generality the proof holds also for the community b. We proceed by dividing the rest of the proof in three steps. Since the indirect utilities of voters both with and without a child reach a unique peak then the preferences of all voters over the tax $0 \leq T^a < \widehat{T}^a$ are single peaked. Consequently, by the median voter theorem (Black, 1958), a unique majority votig equilibrium exists and the median voter is pivotal. In the following steps we show that the median voter has income different from the median income.

i) Consider the scenario with $\frac{N}{2} > n_1^a$, $\widehat{q} < q < \widetilde{q}(T^a)$, such that both households vote for a positive tax, and higher income households prefer a higher tax. We following Epple and Romano (1996) and show that the median income voter is not decisive. Let $\overline{V}(\overline{T})$ be the median-income voter's indirect utility function. We procede by showing that $\overline{T}_c$ and $\overline{T}_n$ can not characterize a majority voting equilibrium because there existes a coalition composed of at least half the households preferring a different tax.

Lemma 5 When $\widehat{q} < q < \widetilde{q}(T^a)$, $\overline{T}_n$ is also the most preferred tax by the voters with a child whose income $\widetilde{y}^c < \overline{y}$, and $\overline{T}_c$ is also the most preferred tax by the childless voters with income $\widetilde{y}^n > \overline{y} \parallel$

Consider $\overline{T}_n$, Proposition 6 and Corollary 1 imply that childless voters with income $y \geq \overline{y}$ and voters with a child and income $y \geq \widetilde{y}^c$ prefer a tax $T^a > \overline{T}_n$. Since $\overline{y}$ is the median-income, then at least half the voters prefers a tax $T^a > \overline{T}_n$.

Consider $\overline{T}_c$, Proposition 6, Corollary 1 and lemma 5 imply that voters with a child whose income is $y \leq \overline{y}$ and childless voters with income $y \leq \widetilde{y}^c$ prefer a tax $T^a < \overline{T}_c$. Since $\overline{y}$ is the median-income, then at least half the voters prefers a tax $T^a < \overline{T}_c$.

ii) Consider the scenario with $\frac{N}{2} > n_1^a$, $\widehat{q} < q$ and $q > \widetilde{q}(T^a)$, such that both households vote for a positive tax, and higher income households prefer a lower tax. We follow the case i) and show that the median voter's income is different from the median income.

Lemma 6 When $\widehat{q} < q$ and $q > \widetilde{q}(T^a)$, then $\overline{T}_n$ is also the most preferred tax by the voters with a child whose income is $\widetilde{y}^c > \overline{y}$, and $\overline{T}_c$ is also the most preferred tax by the the childless voter with income is $\widetilde{y}^n < \overline{y}. \parallel$

Consider $\overline{T}_n$, Proposition 6 and Corollary 1 imply that childless voters with income $y \leq \overline{y}$ and voters with a child and income $y \leq \widetilde{y}^c$ prefer a tax $T^a > \overline{T}_n$. Since $\overline{y}$ is the median-income, then at least half the voters prefers a tax $T^a > \overline{T}_n$.

Consider $\overline{T}_c$, Proposition 6, Corollary 1 and Lemma 6 imply that voters with a child whose income is $y \geq \overline{y}$ and childless voters with income $y \geq \widetilde{y}^c$ prefer a tax $T^a < \overline{T}_c$. Since $\overline{y}$ is the median-income, then at least half the voters prefers a tax $T^a < \overline{T}_c$.

iii) Consider the scenario $\frac{N}{2} > n_1^a$ and $q \leq \widehat{q}$. In this case, all childless households vote for zero tax, then there will be always a coalition of childless households composed of more than half the population of community a blocking any proposed positive tax. Hence, a majority voting exists, but the median

voter's income is different from the median income.

Hence, steps (i), (ii) and (iii) show that the tax most preferred by the voter with the median income does not characterize a majority voting equilibrium. ∎

Proposition 7 says that when at time 1 the community a is inhabited by a majority of childless households, then a majority voting equilibrium exists only if the probability of reselling housing is sufficiently high. In this equilibrium the median voter's income is different from the media income. When the probability of reselling housing is sufficiently low, most of the population in the community would prefer zero tax, then a majority voting still exists but and the median voter still has income different from the median income.

In our model the expenditure in education is capitalized into the housing price, that is, the higher tax higher is the reselling housing price. When the probability of leaving the community is high, then the benefit from higher tax capitalized into the housing price is sufficiently high to allow childless households to prefer positive tax even though they do not have school aged children. When the probability of leaving is low, then the marginal cost from reducing consumption overcomes the marginal benefit from higher reselling price, then childless households do not vote for a positive tax.

The probability of leaving may help to dicriminate households according to the age of the members. The young households may be characterized by higher probability of changing location over time, whereas elderly may be considered less willing then young to leave their own housing[22]. Hence, the results of this model would show that only for young childless households the capitalization effect may be a suffcient condition to support local public spending in education.

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[22]Frenette, Picot and Sceviour (2004) find that older households are more likely than young households to spend long time in low income neighborhoods

6   Conclusion

Empirical evidence and theoretical studies show that when a local government finances the local provision of public education by a tax set by a majority voting, then homeowner households with school aged children support local spending for two main reasons: children benefit from a higher provision of education, and a higher public spending in education is capitalized into the value of the housing. In this context, capitalization means that the higher is the provision of local public education higher is the value of the housing.

Empirical evidence finds that even childless households support local public spending in education. Most of this literature explains this result with the capitalization effect.

The current theorethical model shows that the capitalization effect may not be a sufficient condition for the childless households to support local public spending in education.

I study a two period model composed of a metropolitan area diveded in two communities. Households living in the metropolitan area may or not have a child. Each local government provides public education by a tax imposed on the residents. At the beginning of the first period, households vote on the tax and the child, if any, goes to school. The voting takes place only once, therefore the tax remains fixed over the two periods. In the second period, with a certain probability households must sell their housing and emigrate in another area. New residents move into the metropolitan area, buy a housing from the leaving households and sort into communities. The new entrants cannot modify the local provision of education. Following Brueckner and Joo (1991) the capitalization effect employed in the current model leans on the relation between the tax and the reselling housing price: the higher is the tax set in the first period higher will be the resilling housing price in the second period.

My result crucially depends on the probability of leaving the community. Childless households vote for a positive tax only for a sufficiently high probability of leaving.

The presence of the childless households differentiates my model from the previous theoretical literature which analyzes only the behavior of households with children. Furthermore, the introduction of the probability of leaving may help to differentiate old (whose probability could be sufficiently low) from young childless households (whose probability could be assumed sufficently high). In this sense, my result may show that only for young childless households the capitalization effect may be a sufficient condition to support local public spending in education.

The current model tries to provide a theoretical base to the empirical literature confirming that the childless households support local public spending in education. This work stresses the necessity of introducing childless voters in a multy community model in which the local public education is financed by a tax set by a majority voting.

The qualitative result of this model depends on the channels the capitalization effect runs through. Further reserach could model the capitalization effect by allowing some externality, such as rich people with no children preferring to live near rich people who, if they have children vote for high tax.

7 Appendix

Proof of Lemma 1, 2, 3, and 4 are just first and second derivatives.

Proof of Lemma 1

Let $\alpha = (1 - q) n_1^a$ be the number of households with a child remaining in the community a after the shock, then we simply have:

$$\frac{\partial n^* (.)}{\partial \alpha} = \frac{-T^b}{(T^b + T^a)} < 0 \tag{46}$$

■

Proof of Lemma 2

The cross derivative of the function $V_n^a (T^a, y)$ with respect to T^a and q is as follows:

$$\frac{\partial^2 V_n^a (T^a, y)}{\partial T^a \partial q} = v' (y^n - T^a) + \frac{\partial^2 p_2^{a*}}{\partial T^a \partial q} q v' (y^n + p_2^{a*}) + \frac{\partial p_2^{a*}}{\partial T^a} v' (y^n + p_2^{a*}) + q \frac{\partial p_2^{a*}}{\partial T^a} \frac{\partial p_2^{a*}}{\partial q} v'' (y^n + p_2^{a*}) q \tag{47}$$

where:

$$\frac{\partial^2 p_2^{a*}}{\partial T^a \partial q} = \frac{V_0}{N (T^b + T^a)^2} \left((1 - \lambda) 2N - (n_1^a + n_1^b) \right) > 0 \Leftrightarrow 1 - \lambda > \frac{(n_1^a + n_1^b)}{2N} \text{ and for every } \forall \, T^a, T^b \tag{48}$$

$$\frac{\partial p_2^{a*}}{\partial q} = \frac{V_0}{N (T^b + T^a)} \left((n_1^a + n_1^b) - (1 - \lambda) 2N \right) < 0 \Leftrightarrow 1 - \lambda > \frac{(n_1^a + n_1^b)}{2N} \, \forall \, T^a, T^b \tag{49}$$

Then, given $0 \leq T^a < \widehat{T}^a$, we obtain that $\frac{\partial^2 V_n^a (T^a, y)}{\partial T^a \partial q} > 0$ for every $(1 - \lambda) > \frac{(n_1^a + n_1^b)}{2N}$.

Now, the second derivative of the slope of the indirect utility function with respect to q is as follows:

$$
\begin{aligned}
\frac{\partial^3 V_n^a (T^a, y)}{\partial T^a \partial q \partial q} &= \frac{\partial^2 p_2^{a*}}{\partial T^a \partial q} \left(v' (y^n + p_2^{a*}) + q v'' (y^n + p_2^{a*}) \frac{\partial p_2^{a*}}{\partial q} \right) + \frac{\partial^2 p_2^{a*}}{\partial T^a \partial q} v' (y^n + p_2^{a*}) + \\
&\quad + v'' (y^n + p_2^{a*}) \frac{\partial p_2^{a*}}{\partial q} \frac{\partial p_2^{a*}}{\partial T^a} + \left(\frac{\partial p_2^{a*}}{\partial T^a} v'' (y^n + p_2^{a*}) + v''' (y^n + p_2^{a*}) \frac{\partial p_2^{a*}}{\partial q} \frac{\partial p_2^{a*}}{\partial T^a} \right) \frac{\partial p_2^{a*}}{\partial q} q + \\
&\quad + \left(\frac{\partial^2 p_2^{a*}}{\partial q \partial q} q + \frac{\partial p_2^{a*}}{\partial q} \right) \frac{\partial p_2^{a*}}{\partial T^a} v'' (y^n + p_2^{a*})
\end{aligned} \tag{50}
$$

Since $v''' (.) > 0$, then $\frac{\partial^3 V_n^a (T^a, y)}{\partial T^a \partial q \partial q} \mid_{T^a = 0} > 0$

■

Proof of Lemma 3

Since the slope of the indirect utility function of childless households with respect to T^a, valued at $T^a = 0$ is:

$$\frac{\partial V_n^a(T^a, y)}{\partial T^a}\mid_{T^a=0} = -v'(y^n - p_1^a) - (1-q)v'(y^n) + \left(\frac{\partial p_2^{a*}}{\partial T^a}\mid_{T^a=0}\right)qv'(y^n + p_2^{a*}\mid_{T^a=0})$$

(51)

then for $q = 0$, we have:

$$\frac{\partial V_n^a(T^a, y)}{\partial T^a}\mid_{T^a=0} = -v'(y^n - p_1^a) - v'(y^n) < 0$$

(52)

∎

Proof of Lemma 4.

The cross derivative of the indirect utility function of households both with and without a child with respect to the income and the tax is given by:

$$\frac{\partial^2 V_i(T^a, y)}{\partial y \partial T^a} = -v''(y^i - p_1^a - T^a) - (1-q)v''(y^i - T^a) + qv''(y^c + p_2^{a*})\frac{\partial p_2^{a*}}{\partial T^a}$$

(53)

Now differentiating (61) with respect to q, we obtain:

$$\frac{\partial^3 V_i(T^a, y)}{\partial y \partial T^a \partial q} = v''(y^i - T^a) + \left[v''(y^c + p_2^{a*})\frac{\partial p_2^{a*}}{\partial T^a}\right] + q\left[v'''(y^c + p_2^{a*})\frac{\partial p_2^{a*}}{\partial q}\frac{\partial p_2^{a*}}{\partial T^a} + \frac{\partial p_2^{a*}}{\partial T^a \partial q}v''(y^c + p_2^{a*})\right] < 0$$

(54)

That given $v''(.) < 0, v'''(.) > 0$, and the condition $(1-\lambda) > \frac{(n_1^a + n_1^b)}{2N}$ (implying $\frac{\partial p_2^{a*}}{\partial q} < 0$, $\frac{\partial p_2^{a*}}{\partial T^a} > 0$ and $\frac{\partial^2 p_2^{a*}}{\partial T^a \partial q} > 0$) is negative.

Furthermore, the function $\frac{\partial^2 V_i(T^a, y)}{\partial y \partial T^a}$ is always increasing in T^a, in fact we have:

$$\frac{\partial^3 V_i(T^a, y)}{\partial y (\partial T^a)^2} = v'''(y^i - p_1^a - T^a) + (1-q)v'''(y^i - T^a) + q\left[v'''(y^c + p_2^{a*})\left(\frac{\partial p_2^{a*}}{\partial T^a}\right)^2 + \frac{\partial^2 p_2^{a*}}{\partial T^{a2}}v''(y^c + p_2^{a*})\right] > 0$$

(55)

That given $v''(.) < 0, v'''(.) > 0, \frac{\partial^2 p_2^{a*}}{\partial T^{a2}} < 0$ is always positive.

∎

Proof of Lemma 5

Proposition 6 implies that when $\widehat{q} < q < \widetilde{q}(T^a)$ the optimal tax of both voters with and without a child is increasing in income. Since Corollary 1 implies that $\overline{T}_c > \overline{T}_n$, then there exists: i) an income $\widetilde{y}^n$ higher than $\overline{y}$ such that childless voters with income $\widetilde{y}_n$ prefer $\overline{T}_n = \overline{T}_c$, and ii) an income $\widetilde{y}^c$ lower than $\overline{y}$ such that voters with a child and income $\widetilde{y}^c$ $\overline{T}_c = \overline{T}_n$.INSERT GRAPH

∎

Proof of Lemma 6
Assuming $\widehat{q} < q$ and $q > \widetilde{q}(T^a)$, the proof follows the proof of Lemma 5.
∎

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