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Competitive Charitable Giving and Optimal Public Policy with Multiple Equilibria*

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Abstract

Consider a large number of small individuals contributing to a charity or to a public good. We study the properties of a competitive equilibrium in giving and allow for multiple equilibria. Our proposed condition, aggregate strategic complementarity, is a necessary condition for multiple equilibria. Consider two equilibria with low (L) and high (H) levels of giving. Comparative statics at L could be perverse (subsidies reduce giving) while those at H could be normal (subsidies induce giving), which rules out the use of incentives at L. We demonstrate how public policy, in the form of temporary direct government grants to charity can engineer a move from L to H. We use a welfare analysis to determine the optimal mix of private and public contributions to charity. Our paper contributes to the broader and more fundamental question of using public policy to engineer moves between multiple equilibria.

Keywords: Multiple equilibria; privately supplied public goods; aggregate strategic substitutes and complements; competitive and non-cooperative equilibria; direct grants; charitable redistribution; voluntary contributions to public goods; optimal mix of public and private giving.

JEL Classification: D6, H2, H4.

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"*To construct an acceptable theory of philanthropy one must therefore jettison at least one of the three assumptions of the public good theory - publicness, utility maximization and Nash conjectures. One possibility is to drop the assumption of Nash conjectures while keeping the other two ... when I contribute to a large charity, I do not imagine that my action will have any significant effect, positive or negative, on other donors ... it may be possible to develop a theory of philanthropy along these lines.*" Robert Sugden (Economic Journal, 1982, page 348.)

1. Introduction

Charitable donations is a significant economic activity. For instance, in 2003 for the USA, 89 percent of all households gave to charity with the average annual gift being \$1620, which gives an aggregate total of about \$100 billion; see, Mayr et al. (2009). The most recent 'World Giving Index' published by the Charities Aid foundation used Gallup surveys of 195,000 people in 153 nations. It found that more than 70% of the population gave money to some sort of charity in Australia, Ireland, United Kingdom, Switzerland, Netherlands, Malta, Hong Kong, Thailand.[1]

We first highlight some relevant stylized facts, S1-S5, associated with charitable giving; see Andreoni (2006) for more details. S1: There is substantial heterogeneity in giving between countries.[2] S2: Individual private donors are the largest contributors.[3] S3: Government direct grants are significant in terms of magnitude.[4] S4: Contributions to charity are typically tax deductible. For instance, the rate of charitable deductions is 50% in the US and 17-29% for Canada. S5: Direct grants in the form of seed money or leadership contributions (which precede private giving) made by governments, foundations, the national lottery (as in the UK), or exceptionally rich individuals etc. are efficacious.[5]

1.1. A simple setup

To fix ideas, consider n consumers who are indexed by $i = 1, 2, ..., n$. Consumer i has income $m_i \geq 0$, consumes $c_i \geq 0$ and makes a contribution $g_i \geq 0$ to a charity. Let aggregate

[1] See https://www.cafonline.org/research/publications/2010-publications/world-giving-index.aspx for the index.

[2] As a percentage of GDP, for 1995-2000, non-religious philanthropic activity was in excess of 4% for the Netherlands and Sweden; 3-4% for Norway and Tanzania; 2-3% for France, UK, USA; and less than 0.5% for India, Brazil, and Poland; see Salamon et al. (2004).

[3] For US data, for 2002, individuals accounted for 76.3 percent of the total charitable contributions. Other givers are: foundations (11.2%), bequests (7.5%), corporations (5.1%); see Andreoni (2006).

[4] For non-US data, governments are typically the single most important contributors to charities. On average, in the developed countries, charities receive close to half their total budget directly as grants from the government, while the average for developing countries is about 21.6 percent; See the Johns Hopkins Comparative Nonprofit Sector Project (http://www.jhu.edu/~cnp/).

[5] See, for instance, Karlan and List (2007), Potters et al. (2007), and Rondeau and List (2008).

taxable income of all individuals be M. The government taxes individual incomes at the rate $t \in [0,1]$, makes a direct grant, $D \geq 0$, to the charity and subsidizes private giving to charity at the rate $s \in [0,1]$.[6] Charities typically perform two main functions. They either make direct transfers to individuals or they collect money to supply some public good. The charity in our model, which is a passive player, makes a direct transfer, $\tau_i \geq 0$, to the i^{th} consumer and possibly also finances a public good.[7]

The sum of public and private contributions to charities is $G = D + \sum_{i=1}^{n} g_i$. The utility function of the i^{th} giver to charity is $u^i(c_i, g_i, G)$; g_i appears in the utility function on account of a *warm glow* motive; u^i could depend positively on G either because G provides a public good or on account of *altruism*.

Consumer i chooses her level of giving, g_i, so as to maximize her utility, u^i, given her budget constraint and the government instruments s and t. The government chooses its instruments, s and t,[8] to maximize a social welfare function, $U(u^1, u^2, ..., u^n)$, correctly anticipating the public's choice of g_1, g_2, ... , g_n (in the manner of a Stackelberg leader).

The economics of charity has elicited notable scholarship but it relies on the following two essential features, which we relax in this paper. These two features are 'strategic giving' and 'unique Nash equilibrium'. We consider these in turn below.

1.2. Equilibrium: Strategic or Competitive?

When the consumers decide on their private giving, conditional on the choices made by the government and the charity, there are the following two possibilities.

1. *Strategic behavior*. Each giver, i, chooses own giving, g_i, so as to maximize own utility conditional on the vector of donations of all other givers, $\mathbf{g}_{-i}$. For tractability, many models of charity typically restrict attention to a symmetric Nash equilibrium.[9] The strategic approach assumes critically that when someone decides to contribute, say £10, to the Red Cross, they are engaged in a strategic game in charitable contributions with respect to all other givers. Certainly, no evidence of this is ever provided. There is a great deal of evidence that the predictions of a Nash equilibrium are often, and systematically, violated; see, Camerer (2003).[10] Indeed, in a charity context, in particular, Sugden (1982) has already argued that the predictions do not conform

[6] Hence, the government budget constraint is $tM = D + s\sum_{i=1}^{n} g_i$.

[7] This basic setup implies that the budget constraint of giver, i, is $c_i + (1-s)\, g_i \leq (1-t)\, m_i + \tau_i$, $i = 1, 2, ..., n$.

[8] D is then determined as a residual using the government budget constraint.

[9] But there are also several exceptions. For instance, Bergstrom et al. (1986) consider the case of heterogenous individuals. However, in order to obtain sharper results, even they consider the symmetric case in their section 4.

[10] For a more up to date reading list see the course page by Vincent Crawford at Oxford: http://weber.ucsd.edu/~vcrawfor/SecondYrAdvMicroBehaviouralEcon

with a Nash equilibrium.[11] In the absence of any compelling evidence, it stretches credulity to believe that individuals contributing relatively small amounts to charities are playing a strategic non-cooperative game vis-a-vis all other contributors.

2. *Competitive equilibrium*: In this view, which we subscribe to in this paper, there is a large number of contributors who are individually small. It would then seem that an analysis based on competitive equilibrium in giving is more compelling. In a competitive markets view, any g_i is sufficiently small compared to G, so that each giver, i, takes total giving, G, as exogenous.

1.3. Equilibrium: Unique or Multiple?

The literature has typically focussed on a *unique strategic equilibrium* in giving, ruling out *multiple equilibria* by assumption. In contrast, multiple equilibria are endemic in many important economic phenomena. This would seem particularly to be the case for economics of charity. It is quite conceivable that the uncoordinated giving of a large number of dispersed small donors gives rise to multiple equilibria, depending on the beliefs held by the donors.

Suppose that individual private giving, g_i, and aggregate giving, G (which is synonymous with the size of the charity in our paper), are *strategic complements*. This means that the marginal utility of contributing an extra unit of g_i is increasing in the level of G. Now, if contributors believe that G will be high (low), then they are also induced to contribute a large (small) amount, g_i. Ex post, G would be high (low), confirming the givers' expectations. Hence, there could be several, self fulfilling, equilibria. In some equilibria, giving is *high* (H), while in other equilibria, giving is *low* (L).

We allow for *multiple competitive equilibria* in this paper. In the non-cooperative Nash equilibrium framework of Cooper and John (1988) where all goods are private, *strategic complementarity* is a necessary condition for multiple equilibria. On the other hand, when some goods have the nature of a public good, we show that *aggregate strategic complementarity*, a new concept that is a modification of strategic complementarity, is a necessary condition for multiple equilibria.

1.4. Some implications of multiple equilibria

If multiple equilibria exist then one could rank them by the amount of aggregate giving and also possibly socially rank the equilibria. There are two immediate implications.

[11] Large donors, on the other hand, may behave in a strategic manner (say, to establish green credentials) but their share in the total contributions is small (5.1% for the US).

1. Multiple equilibria can potentially explain *heterogeneity in giving*. Among similar societies, some can achieve the high equilibrium, H, while others can be stuck at the low equilibrium, L.

2. At some equilibria one might obtain *perverse comparative static results* (i.e., an *increase* in subsidy, s, *reduces* contributions). At other equilibria, the comparative static results could be *normal*, in the sense that contributions respond positively to incentives. Policy makers could, understandably, be interested in engineering a move from a *low equilibrium with perverse comparative statics* (LP) to a *high equilibrium with normal comparative statics* (HN), if such equilibria exist.

Suppose that we have two, and only two, equilibria, LP and HN, but that the economy is stuck at LP. Consider a policy maker who desires to engineer a move from LP to HN. How should the policy maker proceed? This question is of fundamental importance to economics and has lacked satisfactory answers. We provide a concrete answer in the context of the economics of charity. Clearly incentives for charitable giving, in the form of higher subsidies, s, will not work at the equilibrium LP because of the perverse comparative static effects. Suppose, instead, that the government gives a *temporary* direct grant, D, to the charity, financed by an income tax.[12] Let D exceed the level of aggregate contributions, G, at LP. This makes the equilibrium at LP unfeasible (as shall show in greater detail below), leaving HN as the only feasible equilibrium.

Once the economy arrives at the equilibrium HN (where the comparative statics are *normal*), the government can withdraw the temporary direct grant, D, and successfully stimulate private giving through greater subsidies. This is welfare improving, because, unlike direct government grants, increased private giving confers warm glow on the contributors. The applicability of these ideas is illustrated by examples where voluntary giving contributes towards public redistribution or towards public goods (see Sections 3 and 7).[13]

1.5. A brief comparison with some other models of multiple equilibria

The rationale and implications of multiple equilibria in our paper are quite different from Andreoni's (1998) strategic-Nash framework, which requires a special type of non-convexity

[12] Empirical evidence shows that crowding-out of *private contributions* by direct government grants, if any, is quite small. See Andreoni (2006) for a discussion of the empirical evidence. Cornes and Sandler (1994, p.419) show that crowding out is always likely to be less than full whenever there is a private benefit (e.g., warm glow) that arises from the act of donating. It is also likely that some of the observed crowding-out is due to moral hazard issues on account of the fund-raising activities by the charity. These issues lie beyond the scope of our paper.

[13] These results also provide another explanation for the effectiveness of *seed money* or *leadership donations*. Seed money, leadership contributions and national lottery money in the UK play a role similar to the direct grant, D, in our framework. Existing models assume a unique equilibrium, hence, we provide an alternative explanation of this important phenomenon.

in production. Andreoni (1998) distinguishes between two different kinds of charities. (1) *Start-up charities* that require some initial amount of contributions $\overline{G}$ (the set-up or fixed costs) before the charity can even begin operations. (2) *Continuing charities* that have already invested $\overline{G}$ in the past and are now running concerns.

Andreoni's (1998) focus is only on start-up charities. There are two kinds of equilibria for start-up charities. In one equilibrium, beliefs are that the charity will fail to raise the amount $\overline{G}$, hence, the consistency of actions with beliefs requires that $g_i = 0$. In the second equilibrium, beliefs are that the charity will successfully raise the amount $\overline{G}$, and so positive individual contributions become optimal, fulfilling the original beliefs. Hence, if leadership contributions are equal to or greater than $\overline{G}$, then positive individual donations are induced.

The reader will immediately note that our analysis differs very significantly from Andreoni's (1998) important contribution. First, our analysis applies both, to start-up charities and to continuing charities, but particularly the latter where in Andreoni's framework there cannot be multiple equilibria. Second, Andreoni (1998) uses a non-cooperative symmetric Nash equilibrium analysis while we use a competitive analysis that is not restricted to symmetric equilibria. Third, the critical feature that generates multiple equilibria in our paper is not non-convexities in production but *aggregate strategic complementarity*, a new concept that we introduce below. Fourth, the mechanisms to engineer moves between various equilibria are different in the two models. Fifth, the issue of perverse comparative statics is critical in our case but not in Andreoni's (1998). Sixth, unlike Andreoni (1998), we perform a welfare analysis in a general equilibrium model with a government budget constraint where the fiscal parameters (such as s and t) are optimally determined.

The idea of engineering a move from a low equilibrium characterized by poverty traps to an equilibrium with greater prosperity is important in development economics but it relies on a different mechanism.[14]

1.6. Results

We make nine main contributions. (1) Our proposed condition *aggregate strategic complementarity* is a necessary condition for multiple equilibria in a *competitive equilibrium* in charitable contributions. (2) We show that a Nash equilibrium in giving converges to a competitive equilibrium as the number of givers increases. (3) For the case of our two examples, and for the parameters chosen, even for relatively small numbers of contributors, the competitive solution is close to the non-cooperative Nash equilibrium. (4) *Multiple equilibria* provide a possible explanation of heterogeneity in charitable giving.

[14] See Rosenstein-Rodan (1943) for an early idea. Murphy et al. (1989) focus on the mechanism of aggregate demand spillover arising from a coordinated increase in outputs in a range of imperfectly competitive industries. However, our mechanism for engineering moves between equilibria is very different.

(5) Using temporary direct grants, a policy maker can *engineer a move* from the low to the high equilibrium. This is particularly desirable when comparative statics at the low equilibrium are *perverse* and those at the high equilibrium are *normal*. (6) When comparative statics at the low equilibrium are *normal* and those at the high equilibrium are *perverse*, the government can do better than simply encouraging subsidy-induced giving at the low equilibrium. Indeed, once the government successfully engineers a move to the high equilibrium using temporary direct grants, the perverse comparative statics at the high equilibrium ensure that a reduction in subsides will induce even greater private giving. (7) By carrying out a welfare analysis, we give conditions that specify the *optimal mix* of public contributions and private contributions to charity. (8) We show that our results are equally applicable to *redistributive* and *public goods* contexts. (9) A key technical device is our proposed *aggregate desire to give* function; this may prove useful in other contexts.

Section 2 formulates the theoretical model. Section 3 gives two illustrative examples of voluntary private contributions to redistribution and public good provision, respectively. Section 4 derives the equilibria of the model and their comparative static results. Section 5 examines multiple equilibria in aggregate giving in more detail. Section 6 performs a welfare analysis and characterizes the normatively optimal public policy. Section 7 provides an explicit solution and numerical analyses of the two examples of Section 3. Section 8 discusses dynamic issues. Section 9 establishes the convergence of a Nash equilibrium in giving to a competitive equilibrium as the number of givers increases. Section 10 concludes. Most proofs are in the appendix.

2. Formal model

There are three types of players in the economy, (1) *consumers*, (2) a *fiscal authority* or *Government*, and (3) *charities*. There are n consumers indexed by $i = 1, 2, ..., n$. Consumer i has an exogenously fixed income of $m_i \geq 0$. The aggregate income is

$$M = \sum\nolimits_{i=1}^{n} m_i. \tag{2.1}$$

2.1. Fiscal instruments

The government exercises the following two fiscal instruments. (i) An income tax on individual incomes, m_i, at the rate $t \in [0, 1]$. (ii) A subsidy to private giving to charity at the rate, $s \in [0, 1]$. The direct public contribution to charity, $D \geq 0$, is determined as the residual using the government budget constraint.

2.2. Consumers

The utility function of consumer i is

$$u^i(c_i, g_i, G), \tag{2.2}$$

where c_i is private consumption expenditure of individual i, $g_i \geq 0$ is his contribution to charity, it reflects the *warm glow* or *prestige* (also known as *impure altruism*) from own contribution[15] and $G \geq 0$ is the aggregate level of giving to charity. G bears two interpretations. G could reflect *pure altruism* on the part of individuals (see Example 3.1, below) or G could be the aggregate level of public goods (see Example 3.2, below). There is overwhelming experimental/field evidence and growing neuroeconomic evidence that justifies such a formulation.[16]

Remark 1 *(Notation): Superscripts on the utility function denote the identity of individual givers (e.g., u^i is the utility function of the i^{th} giver). On the other hand, subscripts are used to denote partial derivatives (e.g., $u_2^i = \partial u^i / \partial g_i$).*

The assumptions on preferences are quite standard. c_i is bounded below by a constant, $\underline{c}_i \geq 0$ (possibly a subsistence level). We assume that u^i is a C^2 function (continuous, with continuous first and second partial derivatives) for $g_i > 0$ and $c_i > \underline{c}_i$. We also assume that $u_1^i > 0$, $u_{11}^i \leq 0$ (strictly positive but non-increasing marginal utility of consumption), $u_2^i \geq 0$, $u_3^i \geq 0$ (non-negative marginal utilities of own and aggregate giving) and $u_{22}^i \leq 0$ (concavity in own giving). In addition, we assume that, for some i, $u_2^i > 0$ and $m_i - \underline{c}_i > 0$. The last assumption guarantees that at least one consumer is willing and able to give to charity.

We make three technical assumptions. The first assumption guarantees the concavity of a transformed utility function (to be introduced),

$$(1-s)^2 u_{11}^i - 2(1-s) u_{12}^i + u_{22}^i < 0. \tag{2.3}$$

The second technical assumption ensures an interior solution for c_i,

$$u_1^i \uparrow \infty \text{ as } c_i \downarrow \underline{c}_i. \tag{2.4}$$

[15] The introduction of a warm glow motive was suggested by Cornes and Sandler (1984) and Andreoni (1989, 1990). The presence of a warm glow term reflects the fact that individuals no longer consider their contributions to be perfect substitutes for the contributions of others. Hence, there is extra utility from one's own contribution, which mitigates the free rider problem arising from purely altruistic considerations, i.e., from a utility function of the form $u^i(c_i, G)$. It also obviously implies that government grants to charities do not crowd out private donations completely because the two are imperfect substitutes from the point of view of givers.

[16] For the evidence on altruism, see Andreoni (2006). For experimental evidence on warm glow preferences, see Andreoni (1993, 2006), Palfrey and Prisbrey (1997) and Andreoni and Miller (2002). For the neuroeconomic evidence see, for instance, Harbaugh et al. (2007), Moll et al. (2006), and for a survey of the neuroeconomic evidence see Mayr et al. (2009).

In words, the marginal utility of consumption tends to infinity as consumption tends to its lower bound from above.

Our third technical assumption is that either

$$u^i \text{ is extended to the boundary, } g_i = 0, \text{ as a } C^1 \text{ function,} \tag{2.5}$$

or

$$u_2^i \uparrow \infty \text{ as } g_i \downarrow 0. \tag{2.6}$$

We require one of (2.5), (2.6) to hold. Thus, (2.5) holds for Example 3.1 and (2.6) holds for Example 3.2, below.

Examples that satisfy the above assumptions include $u = \sqrt{c} + g$ or $u = \ln(c-1) + \ln g$. The more complex Examples 1 and 2 in Section 3 also satisfy these assumptions.

The budget constraint of consumer i is given by

$$c_i + (1-s)\, g_i \leq (1-t)\, m_i + \tau_i. \tag{2.7}$$

The RHS of (2.7) is total income which comprises of the after-tax income plus an individual-specific transfer, $\tau_i \geq 0$, received from the charity. The LHS is total expenditure which is made up of private consumption plus the (net of subsidy) private charitable giving. The instruments of the government are s and t. The charity determines τ_i (see Remark 4 below).

If $s = 1$, then the budget constraint (2.7) of each consumer reduces to $c_i \leq (1-t)\, m_i + \tau_i$. If $u_2^i > 0$ (and we have assumed this to be the case for at least one consumer) then it would follow that $g_i = \infty$. This is, clearly, not feasible. Hence, $s < 1$. Thus warm glow is never a free good.

Furthermore, we assume that each g_i is a small fraction of G, so that each consumer takes the aggregate G as given[17]. Similarly, and we believe quite realistically, consumer i takes τ_i as given. Thus, in making her decision to allocate after-tax income between c_i and g_i, the consumer takes as given m_i, τ_i, s, t, G (as in the theory of competitive markets) and maximizes u^i given in (2.2) subject to the budget constraint (2.7).

Remark 2 : *We treat aggregate consumer expenditure, c_i, as a composite good whose relative price is unity. We treat, g_i, as a good whose relative price is $1-s$. It is standard in economics to postulate that consumers derive utility from the bundles of goods consumed, not the expenditure on them. Therefore, it is right that g_i enters the utility function in (2.2) and not $(1-s)\, g_i$. Thus, in particular, a giver can enjoy warm glow from giving even if its price, $1-s$, is very low.*

[17] Our approach can, of course, be made completely rigorous by adopting an appropriate measure-theoretic formulation with a continuum of consumers. We have found this to considerably complicate our paper, without adding anything to either our conclusions or the literature on the measure-theoretic approach to economics.

Remark 3 *(Warm glow, taxes and private giving): One might ask why in our model does direct private giving to charities yield a warm glow while paying taxes does not? One reason is that giving to charity is voluntary while paying taxes is not. Another reason is empirical. People do enjoy warm glow from giving but, generally, resent paying taxes. Also, in actual practice, when an individual contributes a Euro towards tax payments he does not know what fraction of that Euro will be used by the government towards support of his chosen charity. Hence, it is natural that individuals derive a relatively greater warm glow from direct giving to charity as compared to indirect giving by paying taxes. For pedagogical simplicity we have chosen to model warm glow arising only through direct giving. Our results also carry through if there is some warm glow arising through indirect giving, so long as the warm glow through direct giving is relatively greater.*

2.3. Government

The government collects income tax revenue equal to $\sum_{i=1}^{n} tm_i = t\sum_{i=1}^{n} m_i = tM$. This is used to finance subsidies for donations to charity, $s\sum_{i=1}^{n} g_i$, and on aggregate direct grants from the government to the charities, $D \geq 0$. Therefore, the (balanced) government budget constraint is

$$tM = D + s\sum_{i=1}^{n} g_i. \tag{2.8}$$

The Government chooses its instruments s and t to maximize a social welfare function

$$U = U\left(u^1, u^2, ..., u^n\right), \tag{2.9}$$

which is strictly increasing in the individual utility functions, $u^1, u^2, ..., u^n$.[18]

Since $s \geq 0$, $t \geq 0$, $D \geq 0$ and $g_i \geq 0$ we get, from (2.8), that

$$g_i \leq M, \text{ for each } i. \tag{2.10}$$

2.4. Charities

In order to focus on the simultaneous determinants of private giving and the influence of public policy, we assume that charities are passive players in the game. They merely collect all donations from private consumers, $\sum_{i=1}^{n} g_i$, and from the government, D. Let

$$G = D + \sum_{i=1}^{n} g_i. \tag{2.11}$$

The charity uses G to finance transfers to individuals, $\sum_{i=1}^{n} \tau_i$, while using the balance $G - \sum_{i=1}^{n} \tau_i$ to finance provision of public goods.[19] For feasibility we shall assume that for

[18] Note that the resulting social optimum is constrained by the available set of instruments $\{s, t\}$. In particular, an even better social optimum may be available if subsidies and taxes $\{s_i, t_i\}$ could be varied across individuals. We assume that this is either not desirable or not possible (although it is straightforward to extend our analysis to cope with the more general case).

[19] One example of each of these uses is given in Section 3.

each i, $\underline{c}_i < (1-t)\,m_i + \tau_i$, i.e., each consumer has enough disposable income for minimal consumption, with at least a bit left over.

Remark 4 *(Active and passive charities): In our paper charities are passive players. However, in actual practice, charities could act strategically to attract extra donations or respond to direct government grants by raising less money; see, for instance, Andreoni (2006). This could then endogenously determine the split into redistribution and public good provision and the determination of individual-specific redistribution (τ_1, τ_2, ... , τ_n). However, treating charities as active or passive players is not critical to our paper. Suppose charities were active players, and suppose that the government and the charities simultaneously or sequentially choose their respective policies (taxes, redistribution and even fund-raising) followed by private giving of individuals. So long as charities cannot completely negate the effect of direct government grants (an assumption that is supported by the evidence) then all our results go through. However, our approach is pedagogically simpler and clearer.*

Remark 5 *(The nature of redistribution): In our model, redistribution is carried out by charities only. One could extend the model to allow for direct public redistribution in addition to charitable redistribution. In this richer model, a part of tax revenues finance public redistribution, while the remaining part finances direct grants to charities and subsidizes private charitable giving. However, that adds nothing substantial to our framework or to the main insights that we offer. Hence, we have chosen the current level of abstraction to make our points as clearly as possible.*

Remark 6 *(A special case): Suppose that individuals make voluntary contributions towards the provision of a public good and that $\tau_i = 0$ for all i. Then our model reduces to the case of privately provided public goods, financed by voluntary contributions. Indeed over human history many important public goods have been provided in this manner at some point in time or the other.*[20]

2.5. Sequence of moves

The charity moves first, to announce the parameters $\tau_1, \tau_2, ..., \tau_n$. The government moves next to announce the parameters s, t and D.[21] Finally, the consumers move simultaneously. Consumer i, $i = 1, 2, ..., n$, chooses g_i so as to maximize her utility, $u^i(c_i, g_i, G)$, subject to her budget constraint, $c_i + (1-s)\,g_i \leq (1-t)\,m_i + \tau_i$, and given G, s, t, m_i and τ_i. In particular, consumers are 'G-takers', just as under general competitive equilibrium

[20] This includes many standard examples of public goods, defence, lighthouses, information on marine navigation, shipping intelligence, mail services, education, public works etc.

[21] We shall see below that announcing D can help coordinate private expectations of the level of G.

consumers are price takers. An equilibrium, G^*, is a value of G that equates supply and demand for charitable giving, just as in general competitive equilibrium an equilibrium price vector, $\mathbf{p}^*$, is a vector of prices that equates aggregate supply of each good to its aggregate demand.

We solve the model backwards. Consumer i, $i = 1, 2, ..., n$, chooses g_i so as to maximize her utility, $u^i(c_i, g_i, G)$, subject to her budget constraint, $c_i + (1-s) g_i \leq (1-t) m_i + \tau_i$, and given G, s, t, m_i and τ_i. The government chooses s and t given $\tau_1, \tau_2, ..., \tau_n$ correctly anticipating the consumers' choices of $g_1, g_2, ..., g_n$. The charity chooses $\tau_1, \tau_2, ..., \tau_n$ (through some unmodelled process because they are passive players) correctly anticipating s, t, $g_1, g_2, ..., g_n$.

2.6. Some preliminary results

Since $u_1^i > 0$, the budget constraint (2.7) holds with equality. Hence, we can use it to eliminate c_i from (2.2). Letting $U^i(g_i, G; s, t)$ be the result, we have

$$U^i(g_i, G; s, t) = u^i((1-t) m_i + \tau_i - (1-s) g_i, g_i, G). \tag{2.12}$$

From (2.3) and (2.12) it follows that

$$U_{11}^i < 0. \tag{2.13}$$

In a competitive equilibrium, consumers take as given the total contributions, G. Hence, the consumer's maximization problem is[22]

$$\text{Maximize}_{\langle g_i | s,t,G \rangle} \ U^i(g_i, G; s, t) \ \text{ subject to } 0 \leq g_i \leq \frac{1}{1-s}\left[(1-t) m_i + \tau_i - \underline{c}_i\right]. \tag{2.14}$$

The constraint follows from (2.7) and from the assumption that g_i, c_i are bounded below by zero and $\underline{c}_i$, respectively.

Proposition 1 *: Suppose $\underline{c}_i < (1-t) m_i + \tau_i$.*
(a) The consumer's maximization problem (2.14) has a unique solution, g_i^*.
(b) $0 \leq g_i^* < \frac{1}{1-s}\left[(1-t) m_i + \tau_i - \underline{c}_i\right]$.
(c) If, in addition, (2.6) holds, then $g_i^* > 0$.

Since g_i^* is unique, we can write it as a function of the parameters that are exogenous to the consumer's maximization problem (2.14). In particular, we write $g_i^*(s, t, G)$ explicitly as a function of the tax rate, t, the subsidy rate, s, and aggregate giving, G.[23]

---

[22] Recall that $s < 1$.

[23] We have suppressed other parameters such as τ_i and m_i to improve readability.

A simple calculation shows that Proposition 1(b) implies the following:

$$tM \leq tM + (1-s)\sum\nolimits_{i=1}^{n} g_i^*(s,t,G) < M + \sum\nolimits_{i=1}^{n} \tau_i - \sum\nolimits_{i=1}^{n} \underline{c}_i. \qquad (2.15)$$

On the left hand side of the second inequality in (2.15) we have the total amount paid in taxes and donations to charity (net of subsidy). On the right hand side of the inequality we have total pre-tax income minus total expenditure on subsistence consumption. Clearly, the former cannot exceed the latter. However, from (2.4) it follows that optimal consumption must be strictly higher than subsistence consumption. Hence, the strict inequality.

Lemma 1, below, gives technical results that are important for the paper.

Lemma 1 *: Suppose $g_i^* > 0$. Let $c_i^* = (1-t)m_i + \tau_i - (1-s)g_i^*$. Then, at g_i^*, c_i^*,* *(a)* $U_1^i = 0$, *(b)* $(1-s)u_1^i = u_2^i$, *(c)* $\frac{\partial g_i^*}{\partial G} = \frac{u_{23}^i - (1-s)u_{13}^i}{-(1-s)^2 u_{11}^i + 2(1-s)u_{12}^i - u_{22}^i}$, *(d)* $\frac{\partial g_i^*}{\partial s} = \frac{u_1^i + g_i^*\left[u_{12}^i - (1-s)u_{11}^i\right]}{2(1-s)u_{12}^i - (1-s)^2 u_{11}^i - u_{22}^i}$, *(e)* $\frac{\partial g_i^*}{\partial t} = \frac{m_i\left[(1-s)u_{11}^i - u_{12}^i\right]}{2(1-s)u_{12}^i - u_{22}^i - (1-s)^2 u_{11}^i}$.

2.7. Strategic complements and strategic substitutes

Following Bulow, Geanakoplos and Klemperer (1985), strategic complements and strategic substitutes can be defined as follows.

Definition 1 *: (Strategic complements and substitutes) g_i and G are strategic complements (substitutes) if, and only if,* $\frac{\partial^2 U^i}{\partial g_i \partial G} > 0 \ \ (\leq 0)$.

Thus, g_i and G are strategic complements (respectively, substitutes) if the marginal utility to individual i of making an extra unit of contribution, g_i, increases (respectively, decreases) with an increase in aggregate contributions, G.

Lemma 2 *: g_i and G are strategic complements (substitutes) if, and only if,*

$$u_{23}^i - (1-s)u_{13}^i > 0 \quad (\leq 0).$$

Lemma 3 *: g_i and G are strategic complements (substitutes) if, and only if,*

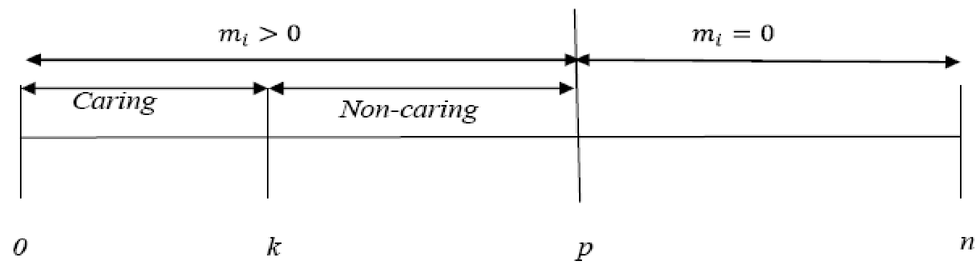
$$\frac{\partial g_i^*}{\partial G} > 0 \ \ (\leq 0).$$

3. Two examples

In this section we present two examples of the general theoretical model. In Example 1 (subsection 3.1) charitable contributions provide income to consumers who, otherwise, have no income. In Example 2 (subsection 3.2) charitable contributions finance public good provision. Section 7 will provide explicit solutions and numerical analysis of these two examples. Section 9 will establish that a Nash equilibrium in giving converges to a competitive equilibrium, and illustrates this in the context of our two examples.

3.1. Example 1: Charitable contributions as public redistribution

We consider an economy where some consumers have no income. Their consumption expenditure is financed entirely by either charitable donations, g_i, made by other 'caring' consumers with positive income and/or by tax financed direct government grants, D (which now have the interpretation of social welfare payments). This is illustrated in Figure 3.1. The assumptions are as follows.



1. There are n consumers. Of these, p consumers, $0 < p < n$, and indexed by $i = 1, 2, ..., p$, have positive income ($m_i > 0$). The other $n - p$ consumers, indexed by $i = p + 1, ..., n$, have no income ($m_i = 0$). All incomes are publicly observable.

2. The aggregate of all donations to charity (private and public), G, is divided among the consumers with no income. Hence, $\tau_i = 0$, $i = 1, 2, ..., p$ and $\sum_{i=p+1}^{n} \tau_i = G$.

3. Of the p consumers with positive incomes, k, $0 < k \leq p$, care about the plight of those with no income. Each of these *caring* consumers has the utility function

$$u^i (c_i, g_i, G) = \ln c_i + a_i g_i G,\ a_i > 0,\ i = 1, ..., k, \tag{3.1}$$

where the following technical condition holds:

$$\frac{1}{a_i G} < \frac{1-t}{1-s} m_i,\ i = 1, ..., k. \tag{3.2}$$

4. The other $p - k$ consumers have positive income but do not care about those with no income. The utility function of the latter two groups of consumers (the non-caring with positive income and those with no income) is given by

$$u^i = \ln c_i,\ i = k + 1, ..., n. \tag{3.3}$$

From (3.1) and Lemma 2, it follows that g_i, G are *strategic complements.*

3.2. Example 2: Voluntary contributions to a public good

Individuals often voluntarily contribute to, and directly use, several kinds of public goods such as health services and education[24]. Suppose that the utility function of consumer i, $i = 1, 2, ..., n$ is given by

$$u^i (c_i, g_i, G) = (1 - a_i) \ln \left(c_i - \frac{b_i}{G} \right) + a_i \ln g_i, \tag{3.4}$$

where

$$0 < a_i < 1,\ b_i > 0,\ \frac{b_i}{G} < (1 - t)m_i. \tag{3.5}$$

Condition (3.5) guarantees that consumer i has enough disposable income, $(1 - t)m_i$, to sustain a level of private consumption expenditure, c_i, greater than $\frac{b_i}{G}$ and also a positive level of donation to charity, g_i. It is straightforward to check that $u_1^i > 0$, $u_2^i > 0$, $u_3^i > 0$.

This example can be given the following interpretation. Private (voluntary) contributions to public goods, $\sum_{i=1}^{n} g_i$, plus public contribution, D, financed from income taxation, provide the necessary infrastructure for private consumption, c_i. An increase in aggregate expenditure on infrastructure, $G = D + \sum_{i=1}^{n} g_i$, leads to a higher level of utility for a given level of c_i. Using Lemma 2 and (3.4), g_i, G are *strategic complements.*[25]

4. Equilibrium giving and public policy

Let us begin with an analogy of competitive markets in an exchange economy. Suppose there are n consumers whose vectors of initial endowments are $\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, ..., \boldsymbol{\omega}_n$. Let the price vector be $\mathbf{p}$. Denote the utility maximizing demand vector of the i^{th} consumer by $\mathbf{x}_i (\mathbf{p}, \boldsymbol{\omega}_i)$. The aggregate demand vector is then $\sum_{i=1}^{n} \mathbf{x}_i (\mathbf{p}, \boldsymbol{\omega}_i)$. The aggregate supply is, of course, $\sum_{i=1}^{n} \boldsymbol{\omega}_i$. There is no guarantee that at an arbitrary price vector, $\mathbf{p}$, aggregate desire to consume, $\sum_{i=1}^{n} \mathbf{x}_i (\mathbf{p}, \boldsymbol{\omega}_i)$, will be equal to aggregate supply, $\sum_{i=1}^{n} \boldsymbol{\omega}_i$. A competitive equilibrium price vector is a price vector, $\mathbf{p}^*$, that equates the two:

$$\sum_{i=1}^{n} \mathbf{x}_i (\mathbf{p}^*, \boldsymbol{\omega}_i) = \sum_{i=1}^{n} \boldsymbol{\omega}_i.$$

The competitive equilibrium in charitable donations is determined in an analogous manner, which we turn to below.

[24] For the US, education, health and human services account for the greatest proportion of private giving after religion; see Table 3 in Andreoni (2006).

[25] One can check that

$$u_{23}^i - (1 - s)\, u_{13}^i = \frac{(1 - s) b_i (1 - a_i)}{(c_i G - b_i)^2} > 0.$$

4.1. The aggregate desire to give to charity[26]

When making their charity decision, consumers take as given aggregate donations to charity, G, and determine their optimal charitable contributions, $g_i^*(s,t,G)$. Just as the aggregate demand in competitive markets need not equal actual aggregate supply for all price vectors, the aggregate of all *desired* donations, $D+\sum_{i=1}^{n} g_i^*(s,t,G)$, need not equal *actual* aggregate contributions, G. Therefore, we introduce a new function, F, which represents the aggregate of all desires (public and private) to give to charity:

$$F = D + \sum_{i=1}^{n} g_i^*(s,t,G). \tag{4.1}$$

From the government budget constraint, (2.8), we get

$$D(s,t,G) = tM - s\sum_{i=1}^{n} g_i^*(s,t,G). \tag{4.2}$$

From (4.1) and (4.2) we get

$$F(s,t,G) = tM + (1-s)\sum_{i=1}^{n} g_i^*(s,t,G). \tag{4.3}$$

From (2.15), (4.3) it follows that

$$0 \leq F(s,t,G) < M + \sum_{i=1}^{n} \tau_i - \sum_{i=1}^{n} \underline{c}_i. \tag{4.4}$$

To reduce the length of formulae, let

$$F_{\max} = M + \sum_{i=1}^{n} \tau_i - \sum_{i=1}^{n} \underline{c}_i, \tag{4.5}$$

then (4.4) becomes

$$0 \leq F(s,t,G) < F_{\max}. \tag{4.6}$$

Recalling that individuals take s,t as given at this stage, hence, without loss of generality, we may view $F(s,t,G)$ as a mapping from $[0,F_{\max}]$ to $[0,F_{\max}]$.

The above discussion suggests the following definition.

Definition 2 : *By the aggregate desire to give we mean the mapping,* $F(s,t,G) : [0,F_{\max}] \rightarrow [0,F_{\max}]$, *defined by*

$$F(s,t,G) = tM + (1-s)\sum_{i=1}^{n} g_i^*(s,t,G).$$

From Definition 2, we get that

$$F_G(s,t,G) = \frac{\partial F(s,t,G)}{\partial G} = (1-s)\sum_{i=1}^{n} \frac{\partial g_i^*(s,t,G)}{\partial G}. \tag{4.7}$$

In general, g_i and G could be strategic complements for consumer i but strategic substitutes for consumer j, $j \neq i$. So, we might wish to ask if in some *aggregate sense*, g and G are strategic complements or substitutes. Lemma 3 and (4.7) suggest the following definition.

[26] Our solution method has some similarities with the techniques developed in Cornes and Hartley (2007).

Definition 3 *(Aggregate strategic complements and substitutes): g and G are aggregate strategic complements (substitutes) at G if, and only if,*

$$\sum_{i=1}^{n} \frac{\partial g_i^* (s,t,G)}{\partial G} > 0 \quad (\leq 0) .$$

From Definition 3, strategic complementarity (or substitutability) for *all* individuals is *sufficient but not necessary* for aggregate strategic complementarity (or substitutability). A global analogue of Definition 3 can also be given.

Definition 4 *(Global aggregate strategic complements and substitutes): g and G are global aggregate strategic complements (substitutes) if Definition 3 holds at all feasible levels of G.*

Lemma 4 *: g and G are aggregate strategic complements (substitutes) at G if, and only if,*

$$F_G (s,t,G) > 0 \quad (\leq 0) . \tag{4.8}$$

If condition 4.8 holds for all permissible levels of G then we get global aggregate strategic complements and substitutes.

4.2. Competitive Equilibria

Definition 5 *(Competitive equilibrium in giving): The economy is in a competitive equilibrium if, and only if, the aggregate of all desires to donate to charity, F, equals the aggregate of all donations, G, i.e., $G^* \in [0, F_{\max}]$ is an equilibrium if, and only if,*

$$G^* = F (s,t,G^*) .$$

Definition 6 *(Isolated equilibrium): An equilibrium, G^*, is isolated if there is a neighborhood of G^* in which it is the only equilibrium.*

Proposition 2 *: (a) An equilibrium, $G^* \in [0, F_{\max}]$, exists and satisfies $0 \leq G^* < F_{\max}$. (b) If $F_G < 1$ for all $G \in [0, F_{\max}]$ (and, in particular, if g and G are global aggregate strategic substitutes, i.e., $F_G \leq 0$ for all $G \in [0, F_{\max}]$), then an equilibrium, G^*, is unique. (c) If $[F_G]_{G^*} \neq 1$, then G^* is an isolated equilibrium.*

Proof of Proposition 2: (a) Recall that in a competitive equilibrium, $G^* = F (s,t,G^*)$. Let

$$H (s,t,G) = G - F (s,t,G) .$$

If $F (s,t,0) = 0$ then, clearly, 0 is an equilibrium. $F (s,t,0) < 0$ is not feasible. If $F (s,t,0) > 0$, then

$$H (s,t,0) = -F (s,t,0) < 0.$$

From (4.6), $F(s,t,G) < F_{\max}$, hence,

$$H(s,t,F_{\max}) = F_{\max} - F(s,t,F_{\max}) > 0.$$

Since $H(s,t,G)$ is continuous, it follows that $H(s,t,G^*) = 0$ for some $G^* \in [0, F_{\max})$, i.e.,

$$G^* = F(s,t,G^*) \text{ and } 0 \leq G^* < F_{\max}.$$

(b) From the definition of $H(s,t,G)$, we get

$$\frac{\partial H}{\partial G} = 1 - \frac{\partial F}{\partial G}.$$

If $\frac{\partial F}{\partial G} < 1$ for all $G \in [0, F_{\max}]$ then $\frac{\partial H}{\partial G} > 0$ for all possible values of G. Thus, in this case, the equilibrium is unique. In particular, if g and G are global aggregate strategic substitutes then, by Lemma 4, $\frac{\partial F}{\partial G} \leq 0$ for all $G \in [0, F_{\max}]$ and, hence, the equilibrium is unique.

(c) If $[F_G]_{G^*} \neq 1$ then $[H_G]_{G^*} \neq 0$ and, hence, G^* is an isolated solution of $H(s,t,G) = 0$. ■

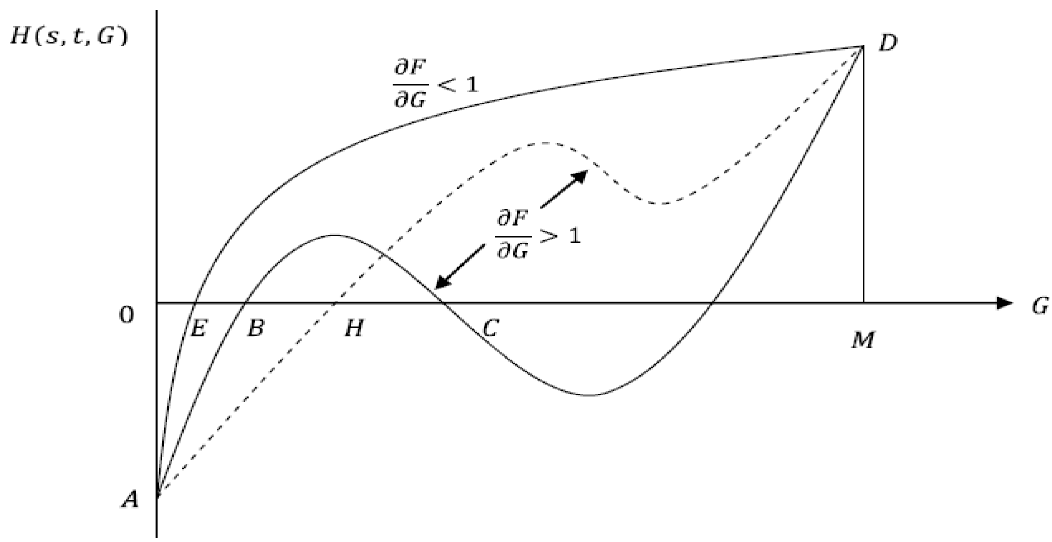


Figure 4.1: Unique and multiple equilibria.

Figure 4.1 illustrates the results in Proposition 2. Three possible shapes of the function $H(s,t,G)$ are shown. Along the curve AED, the sufficient condition for *uniqueness*, $F_G < 1$, for all $G \in [0, F_{\max}]$ holds, and we have a unique equilibrium at E. Along the two paths, ABCD and AHD, this sufficient condition is violated. In particular, along these curves we observe values of G for which $F_G > 1$ (i.e., g_i and G are strategic complements). In this case, we could have a *unique equilibrium* (as in the case of curve AHD) or *multiple equilibria* (as in the case of curve ABCD); see also our examples 3.1 and 3.2. All equilibria shown in Figure 4.1 satisfy $[F_G]_{G^*} \neq 1$, hence, they are all isolated equilibria.

4.3. Equilibrium analysis: Normal, neutral and perverse comparative statics

We now investigate how aggregate equilibrium giving to charity, G, responds to the policy instruments, s, t. We, therefore, consider an equilibrium, G^*, at which $F_G \neq 1$. By Proposition 2c, such an equilibrium is isolated. We can then regard G^* as a C^1 function, $G^*(s,t)$, of s and t in that neighborhood (this is a special case of the implicit function theorem). The following lemma gives a useful identity.

Lemma 5 : $\sum_{i=1}^{n}\left(\frac{\partial g_i^*}{\partial s}+\frac{\partial g_i^*}{\partial G}G_s^*\right)=\frac{1}{1-s}\left(G_s^*+\sum_{i=1}^{n}g_i^*\right)$.

Proposition 3 gives some comparative static results for an isolated equilibrium.

Proposition 3 : *Let G^* be an equilibrium at which $F_G(s,t,G^*) \neq 1$. Then G^* is isolated and*
(a) $G_s^*(s,t)=\frac{F_s}{1-F_G}$, *(b)* $G_t^*(s,t)=\frac{F_t}{1-F_G}$, *(c)* $G_{tt}^*(s,t)=\frac{(F_{tt}+F_{tG}G_t^*)(1-F_G)+F_t(F_{tG}+F_{GG}G_t^*)}{(1-F_G)^2}$.

We now define the critical concepts of *normal*, *neutral* and *perverse* comparative statics.

Definition 7 *(Normal, neutral and perverse incentives):*
(a) Comparative statics are normal *if $G_s^*>0$, i.e., if an increase in the subsidy, s, to private charitable giving increases aggregate equilibrium contributions.*
(b) Comparative statics are neutral *if $G_s^*=0$, i.e., if an increase in the subsidy, s, to private charitable giving leaves unchanged aggregate equilibrium contributions.*
(c) Comparative statics are perverse *if $G_s^*<0$, i.e., if an increase in the subsidy, s, to private charitable giving reduces aggregate equilibrium contributions.*

A glance at the statement of Proposition 3 will easily motivate the Corollary below.

Corollary 1 : *(a) Comparative statics are normal if (i) $F_s>0$ and $F_G<1$ or if (ii) $F_s<0$ and $F_G>1$.*
(b) Comparative statics are neutral if $F_s=0$.
(c) Comparative statics are perverse if (i) $F_s>0$ and $F_G>1$ or if (ii) $F_s<0$ and $F_G<1$.

For ease of reference and to facilitate the discussion in subsequent sections, we have found it helpful to organize the results in this subsection in Propositions 4 and 5 below.

Proposition 4 : *Let G^* be an equilibrium at which $F_G(s,t,G^*) \neq 1$. Then*
(a) Let $F_s>0$ at G^*.
(i) If $F_G<1$ at G^* then $G_s^*(s,t)>0$ (normal comparative statics). (ii) In particular, if

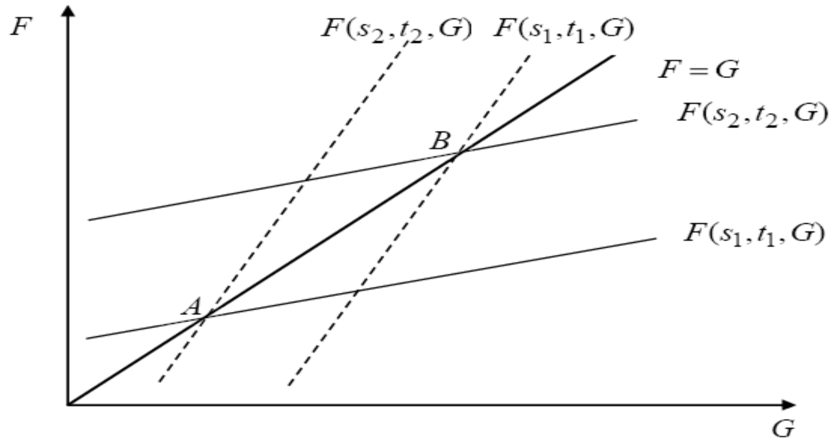


Figure 4.2: $F_s > 0$, $s_1 < s_2$. $0 < F_G < 1$ (continuous, light lines). $F_G > 1$ (dashed lines)

g, G are aggregate strategic substitutes ($F_G \leq 0$), then $G_s^*(s,t) > 0$. (iii) If $F_G > 1$ at G^* then $G_s^*(s,t) < 0$ (perverse comparative statics).
(b) Let $F_t > 0$ at G^*.
(i) If $F_G < 1$ at G^* then $G_t^*(s,t) > 0$. (ii) In particular, if g, G are aggregate strategic substitutes ($F_G \leq 0$), then $G_t^*(s,t) > 0$. (iii) If $F_G > 1$ at G^* then $G_t^*(s,t) < 0$.

Proposition 5 *: Let G^* be an equilibrium at which $F_G \neq 1$. Then*
(a) Let $F_s < 0$ at G^*.
(i) If $F_G < 1$ at G^* then $G_s^*(s,t) < 0$ (perverse comparative statics). (ii) In particular, if g_i, G are aggregate strategic substitutes ($F_G \leq 0$), then $G_s^*(s,t) < 0$. (iii) If $F_G > 1$ at G^* then $G_s^*(s,t) > 0$ (normal comparative statics).
(b) Let $F_t < 0$ at G^*.
(i) If $F_G < 1$ at G^* then $G_t^*(s,t) < 0$. (ii) In particular, if g_i, G are aggregate strategic substitutes ($F_G \leq 0$), then $G_t^*(s,t) < 0$. (iii) If $F_G > 1$ at G^* then $G_t^*(s,t) > 0$.

Figure 4.2 illustrates the two cases of *normal* and *perverse* comparative statics in Proposition 4(a). In Figure 4.2, $F_s > 0$ for all values of G and $s_1 < s_2$. The 45^o line, $F = G$, is shown as the dark line. The case $0 < F_G < 1$ is illustrated by the two, thin, continuous straight lines, while the other case, $F_G > 1$ is shown by the two dashed lines.[27] Thus, in each case, from Lemma 4, g and G are *aggregate strategic complements.* Figure 4.2 shows the outcome arising from an increase in subsidy from s_1 to s_2, financed by an increased in taxes from t_1 to t_2.

[27] The conditions in Proposition 4 are local conditions. For pedagogical purposes, Figure 4.2 is drawn when these conditions hold globally, i.e., the conditions $0 < F_G < 1$ and $F_G > 1$ hold for all values of G.

A. *Normal comparative statics* ($G_s^* > 0$): Figure 4.2 illustrates Proposition 4(ai) for the case $F_s > 0$, $0 < F_G < 1$ (although only $F_G < 1$ is required) by an upward shift of the continuous, light, curve $F(s_1, t_1, G)$ to $F(s_2, t_2, G)$. The equilibrium moves from A to B. Hence, the optimal level of aggregate giving, G, is increasing in the subsidy to individual giving, i.e., $G_s^* > 0$.

B. *Perverse comparative statics* ($G_s^* < 0$). This case is shown in Figure 4.2, by an upward shift of the dashed curve $F(s_1, t_1, G)$ to $F(s_2, t_2, G)$, which assumes $F_s > 0$, $F_G > 1$, as required in Proposition 4(aiii). The equilibrium moves from B to A. Equilibrium aggregate contributions, G, *decrease* as the price of giving reduces (larger s). Because $F_G > 1$, consumers over-react to an increase in G. Thus, paradoxically, G needs to *fall* in order to ensure equilibrium in the market for charity.[28] The surprising effects of the tax reforms of the 1980's on charitable giving in the US is a potential example of perverse comparative statics; see, Clotfelter (1990). The tax reforms increased the price of giving by reducing the tax preference for charitable donations. Contrary to the predictions, charitable contributions continued to rise in the following years.

4.4. D and t as instruments

In most of the paper, we assume that the government's instruments are the subsidy, s, to private giving and the tax rate, t, with the public grant to charity, D, determined as the residual using the government budget constraint. For Propositions 7 and 8, below, it is more convenient to view the government instruments as D and t; with s then determined by the government budget constraint. The following lemma will be useful in welfare analysis, below.

Lemma 6 : $\frac{\partial s}{\partial D} = -1/\sum_{i=1}^{n}\left[g_i^* + \frac{s}{1-s}\left(G_s^* + \sum_{j=1}^{n} g_j^*\right)\right]$.

5. Equilibrium analysis with multiple equilibria

Suppose we have two equilibria, $G^- < G^+$. Suppose that the economy is at G^- and we wish to move it to G^+. Clearly, if it is possible for the government to give a direct grant, D, $G^- < D < G^+$, then, from (2.11), we see that G^+ becomes the only feasible candidate. But we wish to go further. Does such a D exist? Once G^+ is established, can we phase

[28] To aid intuition, imagine an upward sloping demand curve that cuts the supply curve from below. Following an increase in income, we get the perverse result that price (and quantity) will fall. In demand theory this case requires atypical assumptions such as *giffen goods*. By contrast, in the charity context such cases arise naturally; aggregate strategic complementarity being a necessary condition.

out D? Would this cause G^+ to decline or increase further? Could the economy revert back to G^-? It is questions like these that we address below.

There is no reason to suppose that the equilibrium with perverse comparative statics is 'unstable' and that with normal comparative statics is 'stable'; see section 8 below.

We investigate one case in detail: $F_s > 0, F_t > 0, F_G > 0, F_{GG} < 0$ (subsection 5.2) The case $F_s = G_s^* = 0$ (neutral comparative statics) is considered in detail via Example 2 in subsection 7.2, below. All other cases can be dealt with in a similar fashion. We begin with an reasonable assumption which we call "stability of beliefs".

5.1. Stability of beliefs

Suppose one has two equilibria. In some sense, suppose that one of the two equilibria is 'good' and the other is 'bad'. Ideally, one would like to adopt an explicitly dynamic model in which beliefs endogenously evolve over time as individuals engage in learning.[29] In the absence of a satisfactory resolution of this problem, one needs to fall back on some ad-hoc (though hopefully plausible) assumption about the stability of beliefs. This is an issue in all static economic models.[30]

Consider a problem in coordination where people have to decide whether to drive on the left or the right of the road. Suppose that one lives in the UK. People drive on the left, safe in their beliefs that all others will also drive on the left. A similar observation applies to people in the US, except that expectations are to drive on the right. One could compare each of these countries at a point of time in the past when expectations were not coordinated on the good equilibrium (all drive on one side of the road) but rather were uncoordinated on a bad equilibrium (people drive on different sides of the road). What this example illustrates powerfully is the idea that when a good equilibrium gets established, then, with time people come to expect that the good equilibrium will prevail.

In other words once established, beliefs may exhibit inertia to change, i.e., they may be stable. In a charity context, once the Red Cross is established as a "large" charity then beliefs are that it will be large next period.

Weber (2006) asks if coordination can be grown in the lab as one gradually increases the group size. The main finding is that coordination in groups can be grown gradually from a small enough size of groups *where cooperation is already existent but not otherwise*. The norm of cooperation is then learnt by successive entrants, who also cooperate. These findings again illustrate the stability of beliefs on a good equilibrium when a good equilibrium is established.

[29] Most traditional learning models do not perform too well when taken to the evidence although some behavioral models of learning do better; see Camerer (2003).

[30] See Section 8, below for further discussion of these issues.

In the context of our specific model, we illustrate the stability of beliefs in Figure 5.1. Suppose that we have two equilibria, $G^{-}(s,t)$ and $G^{+}(s,t)$ such that $G^{-}(s,t) < G^{+}(s,t)$. Suppose that the comparative statics along G^{-} are perverse, i.e., $G_s^{-} < 0$, while those along G^{+} are normal, i.e., $G_s^{+} > 0$.[31] Figure 5.1 illustrates the nature of these comparative static results.

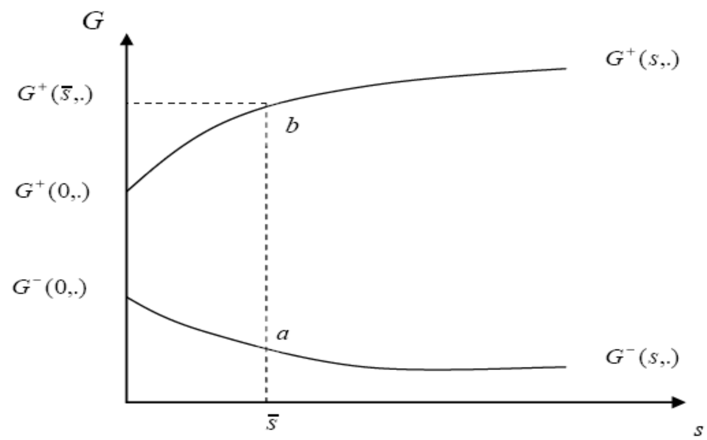


Figure 5.1: Persistence in beliefs

Suppose that we begin at point 'a' on the G^{-} locus, where $G_s^{-} < 0$. As we vary s we will simply trace out various points on the G^{-} locus. Now suppose that we can somehow engineer a jump to the G^{+} locus, say, to point 'b' where $G_s^{+} > 0$ (point b need not be directly above point a). Once the economy is on the G^{+} locus (at point, b, say) for a sufficient length of time then the stability of beliefs requires that as we adjust s the economy moves along the G^{+} locus.

5.2. Engineering moves between equilibria: The case $F_s > 0$, $F_t > 0$, $F_G > 0$, $F_{GG} < 0$

Suppose that the objective of the government is to move the economy from a low equilibrium with perverse comparative statics to a high equilibrium with normal comparative statics. We now discuss this critical question that is central to our paper. We shall consider in detail the case, $F_s > 0$, $F_t > 0$, $F_G > 0$, $F_{GG} < 0$. The case $F_s = G_s^{*} = 0$ (neutral comparative statics) is considered in detail via Example 2 in subsection 7.2, below. All other cases can be dealt with in a similar fashion, so we omit a discussion of these cases.

Figure 5.2, below, sketches the case $F_G > 0$, $F_{GG} < 0$. Thus F is strictly increasing and strictly concave in G. We assume that $s_1 < s_2$. since $F_s > 0$, it follows that the graph of

[31] From Proposition 4a, this occurs when $F_G > 1$ on G^{-} and $F_G < 1$ on G^{+}.

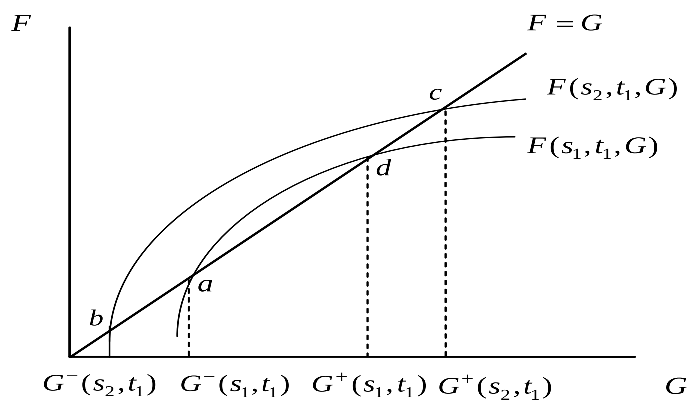


Figure 5.2: The case $s_1 < s_2$ with t_1 fixed

$F(s_2, t_1, G)$ is strictly above that of $F(s_1, t_1, G)$. There are four equilibria: a $(G^-(s_1, t_1))$ and d $(G^+(s_1, t_1))$ corresponding to the parameter values (s_1, t_1); and b $(G^-(s_2, t_1))$ and c $(G^+(s_2, t_1))$ corresponding to the parameter values (s_2, t_1). Furthermore, we see that $F_G > 1$ at points a $(G^-(s_1, t_1))$ and b $(G^-(s_2, t_1))$ but $F_G < 1$ at points c $(G^+(s_2, t_1))$ and d $(G^+(s_1, t_1))$.

Suppose now that the economy is at point a, with $G = G^-(s_1, t_1)$, in 5.2. Also suppose that (for whatever reason) the government wants to shift the economy from a to d, where the latter point corresponds to $G = G^+(s_1, t_1)$. Since $F_s > 0$ and $F_G > 1$ at point a it follows, from Corollary 1c(i), that comparative statics at a are perverse ($G_s^- < 0$). Hence, an increase is s would make things worse by reducing aggregate giving, G, as is clear from Figure 5.2. In particular, an *increase* is s, from s_1 to s_2, would *reduce* aggregate giving, from $G^-(s_1, t_1)$ to $G^-(s_2, t_1)$.

A *decrease* in s would not enable the economy to move to the G^+ locus either (recall the discussion around Figure 5.1). Thus the instrument, s, is ineffective in moving the economy from a to d. Changes in s simply move the economy along the locus $G^-(s, t_1)$. We will now show that, by contrast, the other instrument, t, can be effective.

The method of our proof follows the following steps. Starting from the policy parameters, s_1, t_1, at point a, we alter the policy parameters to $0, t_2$ and at the same time give a public grant equal to $D > G^-(0, t_2)$. Using (2.11), this leaves $G^+(0, t_2)$ as the only equilibrium. Once the economy is allowed to get established at this equilibrium, the stability of beliefs argument (see subsection 5.1) can be used to argue that the economy is now on the G^+ locus. One can then choose the policy parameters optimally. In particular, one can choose the parameters s_1, t_1 in which case the economy reaches the equilibrium $G^+(s_1, t_1)$ corresponding to point d. We now elaborate this method of proof.

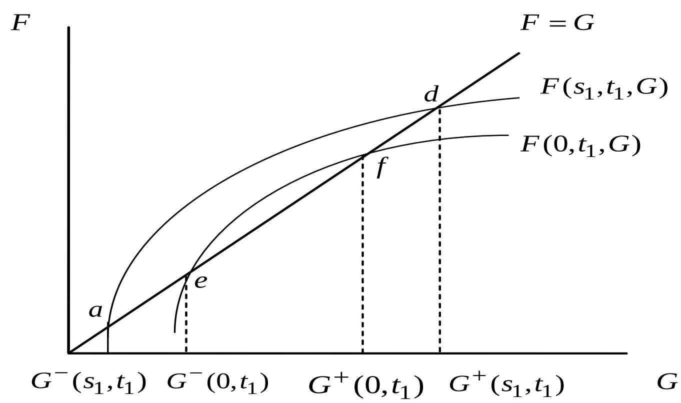


Figure 5.3: The case of decreasing subsidies from s_1 to 0 with t_1 fixed.

Figure 5.3 plots the locus of $F(s_1, t_1, G)$, which is exactly as in Figure 5.2. However, instead of sketching the graph of $F(s_2, t_1, G)$, $s_1 < s_2$, as we did in Figure 5.2, we now sketch the graph of $F(0, t_1, G)$ in Figure 5.3, which is strictly below the graph of $F(s_1, t_1, G)$, if $s_1 > 0$, because $F_s > 0$. There are four equilibria a, d, e and f. Equilibria a $(G^-(s_1, t_1))$ and d $(G^+(s_1, t_1))$, in Figure 5.3, are exactly the same as in Figure 5.2 and correspond to the parameter values (s_1, t_1). However, e $(G^-(0, t_1))$ and f $(G^+(0, t_1))$ correspond to the parameter values $(0, t_1)$.

Figure 5.4 plots the locus of $F(0, t_1, G)$, which is exactly as in Figure 5.3. However, instead of sketching the graph of $F(s_1, t_1, G)$, as we did in Figure 5.3, we now sketch the graph of $F(0, t_2, G)$, $t_1 < t_2$, in Figure 5.4, which is strictly above the graph of $F(0, t_1, G)$, because $F_t > 0$. There are four equilibria e, f, g and h. Equilibria $e(G^-(0, t_1))$ and $f(G^+(0, t_1))$, in Figure 5.4, are exactly the same as in Figure 5.3 and correspond to the parameter values $(0, t_1)$. However, $g(G^-(0, t_2))$ and $h(G^+(0, t_2))$ correspond to the parameter values $(0, t_2)$. We now consider a policy that moves the economy from a to d.

5.2.1. Moving the economy from a to d.

Suppose that the economy is at a $(G^-(s_1, t_1))$ of Figures 5.2, 5.3. The government changes the values of the instruments from (s_1, t_1) to $(0, t_2)$, where $t_2 = \frac{G^-(0,t_1)}{M}$, and makes a direct grant to the charity, $D = G^-(0, t_1)$, financed by the income tax levied at the rate t_2. If $s_1 > 0$ then $G^-(0, t_1) > G^-(s_1, t_1)$ (see Figure 5.3) because of the perverse comparative statics, $G_s^- < 0$, on the locus $G^-(s, t_1)$.

Since $G^-(0, t_1) = D(0, t_1, G^-(0, t_1)) + \sum_{i=1}^{n} g_i^*(0, t_1, G^-(0, t_1))$, it follows that $t_1 = \frac{D(0,t_1,G^-(0,t_1))}{M} < \frac{G^-(0,t_1)}{M} = t_2$, if some $g_i^* > 0$. Since $G_t^- < 0$ (because $F_t > 0$, $F_G > 1$ and $G_t^* = \frac{F_t}{1-F_G}$), it follows that $G^-(0, t_2) < G^-(0, t_1) = D$ as in Figure 5.4. Hence,

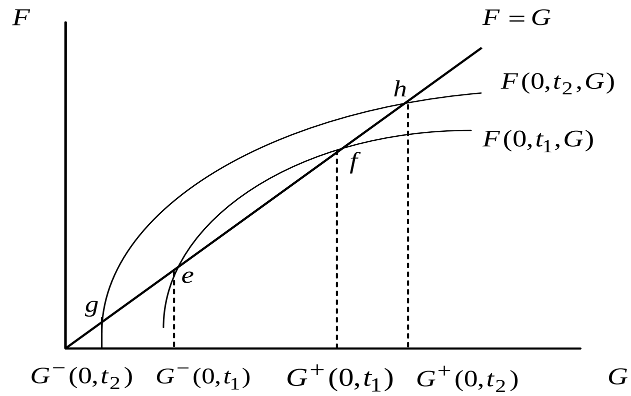


Figure 5.4: The case of increasing subsidies from t_1 to t_2 with $s = 0$ fixed.

the sole equilibrium, consistent with D, is now the *high* equilibrium, $G^{+}(0, t_2)$, point h of Figure 5.4. Once $G^{+}(0, t_2)$ is established, each individual will make her private giving decision conditional on $G^{+}(0, t_2)$, thereby, raising her own giving to a higher level such that equilibrium beliefs about $G^{+}(0, t_2)$ become self-fulfilling.

Invoking the *stability of beliefs* assumption, outlined in subsection 5.1 above, as beliefs about $G^{+}(0, t_2)$ become self-fulfilling, the economy moves along the G^{+} locus in Figure 5.1 as one varies the policy parameters. If desired (we consider these issues in section 6 below), the government can now reduce tax rate from t_2 back to its original value, t_1, and increase s back to its original value, s_1. The economy then moves to point d $(G^{+}(s_1, t_1))$, point d of Figure 5.3.

Garrett and Rhine (2007) report that in the US, the growth in private charitable giving over time has been paralleled by similar growth in expenditure by various levels of government. In particular, between 1965 and 2005, the most rapid growth in private giving (g_i in our set up) has been in charities associated with health, education and social services. These are precisely the areas in which there has been large increases in direct government expenditure (D in our set up). The fact that the Government has responded to an increased demand for these services by increasing the direct grant, rather than by increasing the subsidy to private giving, is consistent with our explanation.

5.2.2. Optimal public policy at the new equilibrium

Substantial field, experimental and neuroeconomic evidence supports the assertion that private charitable giving is a source of *warm glow* but direct government grants are not (see the discussion in Remark 3). Hence, once the economy is established on the G^{+} locus in Figure 5.1, it would be welfare improving to replace government grants by an equivalent amount of private giving, if possible. Propositions 6, 7, 8 in Section 6, below, characterize the optimal policy when we are on the G^+ locus.

6. Welfare analysis

We now ask what should be the optimal policy parameters s, t of the government when its objective is to maximize the social welfare function (2.9). This also allows us to directly answer the question that we posed in subsection 5.2.2 above: What should be the optimal mix of policy parameters once the high equilibrium gets established and (using the stability of beliefs) the economy moves along the G^+ locus.

Substituting $g_i(s, t, G^*(s, t))$ and $G^*(s, t)$ (where G^* lies either on the G^- or the G^+ locus) in the individual utility function (2.12) gives the consumer's indirect utility function

$$v^i(s, t) = u^i[(1-t)m_i + \tau_i - (1-s)g_i(s, t, G^*(s, t)), g_i(s, t, G^*(s, t)), G^*(s, t)]. \quad (6.1)$$

Differentiating (6.1) implicitly using Lemma 1(b), or using the envelope theorem, gives

$$v_s^i = u_1^i g_i + u_3^i G_s^*, \quad (6.2)$$

$$v_t^i = -u_1^i m_i + u_3^i G_t^*, \quad (6.3)$$

where, the partial derivatives G_s^*, G_t^* are given by Proposition 3(a),(b), respectively.

Substituting the indirect utility function from (6.1) into (2.9) we get the government's indirect (social) utility function

$$V(s, t) = U\left(v^1(s, t), v^2(s, t), ..., v^n(s, t)\right). \quad (6.4)$$

Differentiating (6.4), using the chain rule, and (6.2), (6.3), we get

$$V_s = \sum\nolimits_{i=1}^n U_i u_1^i g_i + G_s^* \sum\nolimits_{i=1}^n U_i u_3^i, \quad (6.5)$$

$$V_t = -\sum\nolimits_{i=1}^n U_i u_1^i m_i + G_t^* \sum\nolimits_{i=1}^n U_i u_3^i, \quad (6.6)$$

where subscripts on the utility functions, U, u denote appropriate partial derivatives.

Propositions 6, 7 and 8, below, derive the optimal mix between private and public giving to charity in different cases. These propositions are used extensively in Section 7 and are crucial in determining the optimal public policy at different equilibria.

Proposition 6, below, implies that in a social optimum where $G_t^* \leq 0$ no government intervention is needed, *warm glow* and/or *altruism* suffice to maximize social welfare.

Proposition 6 : *Let (s^*, t^*) be a social optimum where $G_t^* \leq 0$ and some $g_i^* > 0$, then $s^* = t^* = D^* = 0$. Thus all giving to charity is private giving.*

We now show, in Proposition 7, that if subsidies are effective then no direct government grant is needed. This is because when *private* donations replace an identical amount of *public* donations, welfare improves on account of the warm glow received by private givers.

Proposition 7 *: If (s^*, t^*) is a social optimum where $G_s^* \geq 0$ and some $g_i^* > 0$, then $D^* = 0$.*

Proposition 8 *: Let G^* be a social optimum with some $g_i^* > 0$. Suppose (i) $F_s \geq 0$ and $F_G < 1$ at G^* or (ii) $F_s \leq 0$ and $F_G > 1$ at G^*. Then all charitable contributions come from individual private donations and $D^* = 0$.*

The intuition behind Proposition 8 can be seen from the results in Propositions 4, 5. In Proposition 8(i), for instance, the comparative statics are normal (and not perverse), hence, private giving can be encouraged through the use of subsidies. Since private giving leads to warm glow, and an improvement in welfare, it is optimal to generate all charitable giving through private giving, rather than by direct government grants. A similar intuition explains Proposition 8(ii).

We can now use these propositions to revisit the question that we posed in subsection 5.2.2, above. Namely, the question of the optimal policy once the economy gets established on the G^+ locus. Once established on the locus G^+ comparative statics are normal, i.e., $G_s^+ > 0$. From Proposition 7 we know that, in this case, s attains its maximum value and the government makes no direct contributions to charity, i.e., $D = 0$. All tax revenues are then used to finance subsidies to private giving. The reason is that private giving confers warm glow on individuals and, hence, raises welfare relative to the same level of giving being undertaken directly by the government. Hence, it is optimal to set $D = 0$.

7. Competitive equilibrium outcomes for Examples 1 and 2 (Section 3)

We now apply the general theory developed so far to Examples 1 and 2 (see Section 3).

7.1. Example 1: Charitable contributions as public redistribution

Consider the setup of Example 1 (subsection 3.1) that we now take as given. Let m be the aggregate income of the caring consumers (see Figure 3.1). Then

$$m = \sum_{i=1}^{k} m_i \leq \sum_{i=1}^{p} m_i = \sum_{i=1}^{n} m_i = M. \tag{7.1}$$

Let

$$A = \sum_{i=1}^{k} \frac{1}{a_i} > 0. \tag{7.2}$$

Proposition 9 summarizes the main results.

Proposition 9 : *(a) (Multiple equilibria) The only economically interesting cases occur when* $[m + t(M - m)]^2 > 4(1 - s)A$*. In this case, we have two distinct, real, positive, equilibria* $0 < G^-(s,t) < G^+(s,t)$*. These are given by*

$$G^{\pm}(s,t) = \frac{1}{2}\left[m + t(M - m) \pm \sqrt{[m + t(M - m)]^2 - 4(1 - s)A}\right],$$

and, correspondingly,

$$g_i^{\pm}\left(s,t,G^{\pm}(s,t)\right) = \frac{1-t}{1-s}m_i - \frac{1}{a_i G^{\pm}(s,t)},\ i = 1, ..., k,$$

$$g_i^{\pm} = 0,\ i = k+1, k+2, ..., n.$$

(b) (Increasing and concave desire to contribute) For all G*, the aggregate desire to give,* F*, (i) responds positively to subsides i.e.,* $F_s > 0$ *and, (ii) it is increasing and concave i.e.,* $F_G > 0$*,* $F_{GG} < 0$*. So we have the case depicted in Figures 5.2-5.4. Furthermore,*

$$G = G^+ \Rightarrow F_G < 1;\ G = G^- \Rightarrow F_G > 1.$$

(c) (Perverse and normal comparative statics) The comparative statics with respect to the subsidy are perverse at the low equilibrium and normal at the high equilibrium, i.e., $G_s^- < 0$*,* $G_s^+ > 0$*. For* $m < M$ *(and so* $k < p$*) the same holds for the comparative static result with respect to the tax rate, i.e.,* $G_t^- < 0$*,* $G_t^+ > 0$*. For* $k = p$*,* $G_t^{\pm} = 0$*.*

The equilibria are as in Figures 5.2-5.4. Suppose that an economy is at the low equilibrium, G^-, and also suppose that it is socially desirable to move it to the high equilibrium, G^+. How can this be done? The situation is identical to the one presented in Section 5.2, and so we follow the same solution method.

From Proposition 9(b), due to perverse comparative statics at the low equilibrium, $G_t^- \leq 0$. Hence, from Proposition 6, it would appear that the best policy is no intervention i.e., $s = t = 0$, leaving the economy at the low equilibrium, G^-. However, an alternative policy is possible, as we shall now describe; see Figure 7.1.

Set the tax rate t as $t = G^-(0,0)/M$. Since $G^-(0,0)$ is an equilibrium, and there is an interior solution to consumption (from (2.4)), so $0 < G^-(0,0) < M$. It follows that $0 < t < 1$ and, hence, it is feasible. Set s, D as follows: $s = 0$ and $D = tM = G^-(0,0)$.

Thus, the government gives a direct grant equal to $G^-(0,0)$ financed with an income tax (any grant $D > G^-(0,0)$ will also work). Since $g_i > 0$ for some i ($i = 1, 2, ..., k$) it follows, from (2.11), that $G > D = G^-(0,0)$. Hence, because we are now on the $F(0,t,G)$ locus (see Figure 7.1) and the equilibrium aggregate donation $G > G^-(0,0) > G^-(0,t)$, the only possible candidate for equilibrium is

$$G = G^+(0,t). \tag{7.3}$$

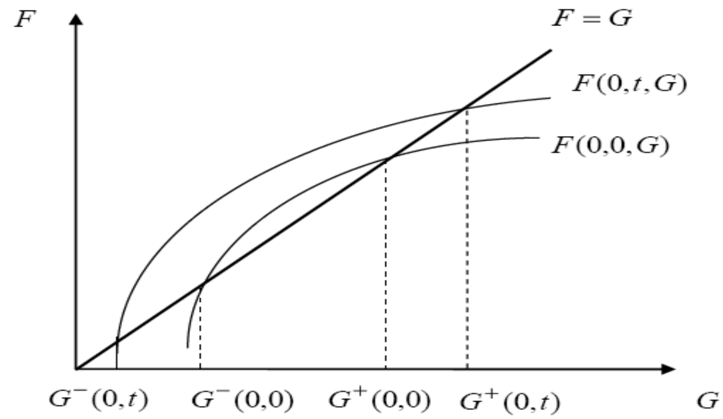


Figure 7.1: Multiple equilibria when $F_t > 0$, $F_G > 0$, $F_{GG} < 0$.

Once the economy is allowed to establish itself at the high equilibrium, $G^{+}(0,t)$, the stability of beliefs (see subsection 5.1 above) ensures that the economy is on the G^{+} locus. The fiscal parameters, s,t can then be adjusted to their socially optimum values, assuming that the economy is on the G^{+} locus, using the results of Section 6. We now address this issue.

7.1.1. Socially optimal public policy at the new equilibrium

At the low equilibrium, $G^{-}(0,0)$, private individuals cannot be induced to make additional contributions because of the *perverse* comparative statics, $G_s^{-}(0,0) < 0$, $G_t^{-}(0,0) < 0$. In contrast, the comparative statics at the high equilibrium, $G^{+}(s,t)$, for any values of s,t are *normal*. Suppose that we allow the economy to establish itself at the G^{+} locus. We now illustrate the insights of subsection 5.2 for the concrete case of Example 1 (subsection 3.1). Consider two cases.

1. If $k < p$, so that not all consumers with positive income are caring and so do not contribute to charity, then, $m < M$. From Proposition 9(c), $G_s^{+} > 0$, $G_t^{+} > 0$, i.e., the comparative static effects are normal at the high equilibrium. Depending on the parameter values, the optimal tax rate may be positive, in which case it can be found by setting $V_t = 0$ in (6.6). From Proposition 9(b), at the high equilibrium G^{+}, $F_s > 0$, and $F_G < 1$, so, Proposition 8 implies that $D = 0$. Thus, once the economy has moved from the G^{-} locus to the G^{+} locus, the direct grant from the government to the charity is phased out. In the new, socially optimal, equilibrium, all contributions to charity are exclusively private (because only private contributions are associated with warm glow) and all income tax revenue is used to subsidize private donations to charity.

2. If $k = p$, so that all consumers with positive income contribute to charity, then $m = M$. Hence, from Proposition 9(c), $G_t^+ = 0$. It follows, from Proposition 6, that once the economy has established itself on the G^+ locus, $s = t = 0$. Thus, once the (one-off) direct government grant to charity (financed by an income tax) has shifted the economy from the bad equilibrium, G^-, to the good equilibrium, G^+, no further government intervention is needed and the entire charitable contributions come from voluntary private contributions. So at the new optimum, equilibrium is described by $G^+(0,0) = \sum_{i=1}^n g_i(0,0,G^+(0,0))$.

7.1.2. A numerical illustration of Example 1

We now illustrate the above results by a numerical simulation of Example 1. Specifically, let $n = 1900$, $p = 900$, $k = 450$, $a_i = 0.01$, $i = 1,2,...,450$, $a_i = 0$, $i = 451,...,1900$, $m_i = 1$ (so that $g_i^{\pm}$ can be interpreted as fraction of income given to charity), $i = 1,2,...,900$, $m_i = 0$, $i = 901,...,1900$, $\tau_i = 0$, $i = 1,2,...,900$, $\sum_{i=901}^{1900} \tau_i = G$. Thus,

$$m = \sum_{i=1}^{450} m_i = 450,\ M = \sum_{i=1}^{1900} m_i = \sum_{i=1}^{900} m_i = 900,\ A = \sum_{i=1}^{450} \frac{1}{a_i} = 45000. \quad (7.4)$$

Initially, assume that $s = t = 0$. Then, from (7.4) and Proposition 9(a), $G^-(0,0) = 150$, $G^+(0,0) = 300$; and, correspondingly, $g_i^- = \frac{1}{3}$, $g_i^+ = \frac{2}{3}$, $i = 1,...,450$, $g_i^{\pm} = 0$, $i = 451,...,1900$.

Suppose that the locus passing through G^- is considered to be socially inferior to that passing through G^+. How can government policy shift the economy onto the better locus? Since there are perverse comparative statics at the low equilibrium, i.e., $G_s^- < 0$, $G_t^- < 0$, the best policy would appear to be no intervention: $s = t = 0$. However, there is an alternative. The government sets $s = 0$, $t = 1/6$. This raises a total tax revenue equal to $tM = 900/6 = 150$. Since $G_t^- < 0$, it follows that $G^-(0,1/6) < 150$ at $t = 1/6$. The government makes a direct grant $D = G^-(0,0) = 150$ to the charity. Since $g_i > 0$, $i = 1,2,...,450$, we must have $G(0,1/6) = D + \sum_{i=1}^{450} g_i(0,1/6,G) > D = 150 = G^-(0,0) > G^-(0,1/6)$. Hence, the only possible equilibrium is $G = G^+(0,1/6)$. Once the economy is on the G^+ locus and the new equilibrium gets established (see subsection 5.1 above) s, t can be given their optimal values as shown in Propositions 6, 7, 8 in Section 6.

7.2. Example 2: Voluntary contributions to a public good

We now consider Example 2 (subsection 3.2). In this example, G has the interpretation of public infrastructure such as health and education. Define the constants B, C as:

$$B = \sum_{i=1}^n a_i m_i + t \sum_{i=1}^n (1 - a_i)\, m_i,\ C = \sum_{i=1}^n a_i b_i. \quad (7.5)$$

The main results for this example are listed in Proposition 10, below.

Proposition 10 : *(a) (Multiple equilibria) The only economically interesting cases occur when* $B^2 > 4C$*. In this case, we have two distinct real positive equilibria* $0 < G^-(s,t) < G^+(s,t)$*. These are given by*

$$G^{\pm}(s,t) = \frac{1}{2}\left(B \pm \sqrt{B^2 - 4C}\right), \tag{7.6}$$

and, correspondingly,

$$g_i^{\pm}(s,t,G^{\pm}(s,t)) = \frac{a_i}{1-s}\left[(1-t)\,m_i - \frac{b_i}{G^{\pm}(s,t)}\right],\ i = 1,2,...n.$$

(b) (Increasing and concave desire to contribute) The aggregate desire to give, F*, (i) responds positively to taxes, i.e.,* $F_t > 0$*, (ii) is unresponsive to subsidies, i.e.,* $F_s = 0$*, and, (iii) it is increasing and concave, i.e.,* $F_G > 0$*,* $F_{GG} < 0$ *(as in Figures 5.2-5.4). Furthermore,*

$$G = G^+ \Rightarrow F_G < 1;\ G = G^- \Rightarrow F_G > 1.$$

(c) (Neutral comparative statics) Comparative statics are neutral, i.e., $G_s^{\pm} = 0$*. Thus, subsidies are ineffective in influencing aggregate giving. Furthermore,* $G_t^- < 0$*,* $G_t^+ > 0$*.*

From Proposition 10, we know that the economy has two equilibria, as in Figure 7.1.

1. The low equilibrium is characterized by low voluntary contributions to the public good, causing low aggregate spending on the public good infrastructure, G^-. From (3.4), to achieve any specific utility level, high private consumption expenditure is needed. From the budget constraint, (2.7), we see that, as a consequence, less income can be contributed to the public good, which is a strategic complement, thus, perpetuating the low expenditure on infrastructure.

2. The high equilibrium is characterized by high contributions to the public good, causing high aggregate expenditure on infrastructure, G^+. In turn, this implies that relatively less private consumption expenditure is needed to reach any specific utility level. Hence, relatively more income is left over to donate to charity, perpetuating high expenditure on infrastructure.

Suppose that the economy is at the low equilibrium, G^-, and that it is socially desirable to move the economy to the high equilibrium, G^+. How can this transition be achieved? From Proposition 10(c), we know that incentives in the form of a subsidy will not work because $G_s^* = 0$. From Proposition 10(c), $G_t^- < 0$, so, from Proposition 6, it would appear that, at the low equilibrium G^-, the best feasible policy is no intervention: $s = t = 0$, leaving the economy at the low equilibrium G^-. However, an alternative policy is possible, as we shall now describe; see Figure 7.1. Set a tax rate t, given by

$$t = \frac{G^-(0,0)}{M}. \tag{7.7}$$

Since $G^-(0,0)$ is an equilibrium, $0 < G^-(0,0) < M$. Hence, it follows, from Proposition 2(a), that $0 < t < 1$, which is a feasible tax rate. Set $s = 0$ and $D = tM = G^-(0,0)$, i.e., the government gives a direct grant to public good provision equal to $G^-(0,0)$ that is financed from an income tax. Since $g_i > 0$ it follows, from (2.11), that $G > D = G^-(0,0)$. Since we are now on the $F(0,t,G)$ locus (see Figure 7.1) and the equilibrium aggregate donation $G > G^-(0,0) > G^-(0,t)$, the only possible equilibrium is $G = G^+(0,t)$. Once the high equilibrium is established, the stability of beliefs ensures that the economy is on the G^+ locus. The fiscal parameters s, t can then be adjusted to their socially optimal values as suggested in Section 6. We now turn to this issue.

7.2.1. Socially optimal public policy at the new equilibrium

From Proposition 10(b) the relevant graph is as in Figure 7.1. Once the economy has moved to the new equilibrium, G^+, the direct grant from the government, $D = G^-(0,0)$, towards the public good can be phased out. It is welfare improving to do so because of the normal comparative statics at the high equilibrium, and the fact that private giving confers warm glow, while an equivalent amount of direct grants does not.

In the low equilibrium, G^-, we have seen above that the optimal policy solution is $s = t = 0$. However, from Proposition 10(c), we know that at the high equilibrium, $G_t^+ > 0$, and so, the comparative static results are reversed from the low equilibrium. The optimal tax rate, which can be found from (6.6), balances the loss in private consumption against the gain arising from the additional amount of the public good. Also, from (6.5), $V_s > 0$, hence, it is welfare improving to provide additional subsidies. Thus, all tax revenues are used to finance subsidies on charitable donations. In the socially optimal solution at the high equilibrium, G^+, therefore, $s > 0$, $t > 0$ while $D = 0$ (see Proposition 7).

7.2.2. A numerical illustration of Example 2

As a numerical illustration of Example 2, consider an economy of $n = 50$ identical consumers, each with income $m_i = 1$ (so that $g_i^{\pm}$ can be interpreted as fraction of income given to charity). Choose $a_i = 0.1$ and $b_i = 0.8$ (the relevant utility function in given in (3.4)). Suppose that initially, $s = t = 0$. Then (7.5), (7.6) give $G^-(0,0) = 1$ and $G^+(0,0) = 4$, and the feasibility condition (3.5) is satisfied (see Figure 7.1). Correspondingly, $g_i^- = 0.02$, $g_i^+ = 0.08$, $i = 1, 2, ..., 50$.

Suppose that the economy is, initially, at the low equilibrium $G^{-}(0,0)=1$. If the move to a high equilibrium is considered desirable, then, from (7.7) we get that the required tax rate, t, is

$$t=\frac{G^{-}(0,0)}{M}=\frac{1}{50}=0.02, \tag{7.8}$$

and the feasibility condition (3.5) still holds at this tax rate. The government uses its entire tax revenue $tM=1=G^{-}(0,0)$ to make a contribution $D=G^{-}(0,0)=1$ to the public good. Since $g_i^*>0$, $G^*(0,0.02)=D+\sum_{i=1}^{n}g_i^*>1=G^{-}(0,0)$. In terms of Figure 7.1, the economy is on the $F(0,0.02,G)$ locus. Hence, the only candidate for equilibrium is $G=G^{+}(0,0.02)$. Once the economy is on the high equilibrium, $G^{+}(0,0.02)$, the policy parameters s,t can be adjusted to their socially optimal values. This will involve the phasing out of the direct grant. Once the final position of the, new, socially optimal, equilibrium is established, the government uses all tax revenues to subsidize voluntary contributions to the public good. The direct grant, here, is only a temporary measure to shift the economy from the low equilibrium locus G^{-} to the high equilibrium locus G^{+}.

8. Dynamics

In this paper we have concentrated exclusively on equilibrium analysis. For certain purposes, a rigorous analysis of the time path of adjustment may be required. This, however, will involve specifying a precise list of the information sets of each of the players, the updating rules, and a learning process[32]. This, however, lies beyond the scope of the current paper.

The reader of this paper might have wondered about the stability properties of the equilibria. To study the stability of equilibria, an adjustment process needs to be specified.

A popular adjustment process in economics is the partial adjustment scheme. Let G^* be an equilibrium, $G(x)$ the value of G at time, x, and λ a positive constant. Then the partial adjustment scheme is given by

$$\dot{G}(x)=\lambda\left[G^*-G(x)\right], \tag{8.1}$$

from which we can see that G^* is a *stable* equilibrium. Although a plausibility argument can be given for (8.1), it cannot be derived from the behavior of the decision makers in our model. The reason is that there are no dynamic constraints in our model. So if all

[32] At the moment there is a lack of consenus in the profession about the appropriate model of learning and there is 'too large' a variety of models to choose from. These include reinforcement learning, learning through fictitious play, learning direction theory, Bayesian learning, imitation learning, experience weighted attraction learning, etc. For a survey of the experimental evidence on these theories, see Camerer (2003)

agents believe that the resulting equilibrium will be G^*, then they will jump straight to G^*. It is not clear why they should follow (8.1).

Another popular adjustment process is

$$\dot{G}(x) = \mu \left[F(s, t, G(x)) - G(x) \right]. \tag{8.2}$$

For $\mu > 0$, (8.2) says that aggregate giving moves in the direction of the aggregate desire to give; which appears plausible. It is similar to the tatonnement process in general competitive equilibrium and the Cournot adjustment process in oligopoly theory. Both have been extensively studied in the past but are no more regarded as true dynamic processes.

For the moment, let us take (8.2) seriously. Suppose $\mu > 0$. If $F_G < 1$ at an equilibrium, G^*, then G^* is stable. This is compatible with either normal comparative statics (if $F_s > 0$) or perverse comparative statics (if $F_s < 0$). If $F_G > 1$, then G^* is unstable; and this is also compatible with normal comparative statics (if $F_s < 0$) or perverse comparative statics (if $F_s > 0$). The exact reverse happens if $\mu < 0$. Therefore, we cannot assume that an equilibrium with perverse comparative statics is unstable and, therefore, uninteresting.

Moreover, it can be argued that the correct stability notion is *saddle path stability*. This requires that the number of stable eigenvalues of a dynamic system (calculated at equilibrium) be equal to the number of predetermined variables and the number of unstable eigenvalues be equal to the number of jump variables. Our system (8.2) is one dimensional (though non-linear). We have no predetermined variables (our model lacks dynamic constraints). The one variable, G, of (8.2) is a jump variable (our model allows agents to choose any g_i and, hence, any G). Thus saddle path stability (which is arguably the correct notion of stability here) actually requires that (8.2) be unstable (as in simple macro models of exchange rate determination).

To summarize, the only natural dynamics for our model is the very simple one where agents jump to whatever equilibrium is in line with their expectations.

9. Strategic giving[33]

So far we have considered only competitive equilibria in giving in the sense that each giver, i, takes aggregate giving, G, as given. By contrast, in a static Nash noncooperative equilibrium each giver, i, takes as given the contributions of all others, $G_{-i} = G - g_i$. If g_i is small relative to G, one would expect that it would not make much difference whether giver i took G_{-i} as given or G as given. In subsection 9.1 we will show that this is indeed the case in the sense that, as the number of individuals increases, a Nash equilibrium in giving

[33] We are grateful for the comments of a referee in rewriting this section.

converges to a competitive equilibrium. Subsection 9.2 (9.3) shows that, for Example 1 (respectively 2), a Nash equilibrium outcome is within 1% of a competitive equilibrium if the number of givers is 200 (respectively 1600) or more.

9.1. Convergence of a Nash equilibrium in giving to a competitive equilibrium

We have to distinguish between the case where G increases due to an increase in g_i, $i = 1, 2, ..., n$, for *fixed* n and the case where G increases because of an *increase* in n. Thus, we rewrite the utility (2.2) of giver i as

$$u^i(c_i, g_i, G) = w^i\left(c_i, g_i, \frac{G}{n}\right). \tag{9.1}$$

In (9.1), we see that an increase in G, for fixed n, increases u^i (if $u_3^i > 0$) as before. However, an N fold replication of the economy will result in an increase in the number of givers from n to Nn and also an increase in total giving from G to NG. From (9.1) we see that this leaves w^i (and, hence, also u^i) unchanged, which is what we want.[34]

Substituting from the individual budget constraint (2.7) into (9.1) gives

$$U^i(g_i, G; s, t) = w^i\left((1-t)m_i + \tau_i - (1-s)g_i, g_i, \frac{G}{n}\right), \tag{9.2}$$

which is equivalent to (2.12). Differentiating (9.2) w.r.t. g_i *taking* G *as given*, yields

$$\frac{\partial U^i}{\partial g_i}(g_i, G; s, t) = -(1-s)w_1^i + w_2^i \text{ (}G\text{ given: Competitive case).} \tag{9.3}$$

Now substitute $g_i + G_{-i}$ for G in (9.2) to get

$$U^i(g_i, g_i + G_{-i}; s, t) = w^i\left((1-t)m_i + \tau_i - (1-s)g_i, g_i, \frac{g_i + G_{-i}}{n}\right). \tag{9.4}$$

Differentiating (9.4) w.r.t. g_i *taking* G_{-i} *as given*, yields

$$\frac{\partial U^i}{\partial g_i}(g_i, g_i + G_{-i}; s, t) = -(1-s)w_1^i + w_2^i + \frac{1}{n}w_3^i \text{ (}G_{-i}\text{ given: Nash case).} \tag{9.5}$$

Since w_3^i is a continuous function on the compact set $[0, F_{\max}]$ (Recall Definition 2), it follows that w_3^i is bounded. Hence, as $n \to \infty$, (9.3) goes over to (9.5). Hence, as $n \to \infty$, the set of Nash equilibria determined by the first order conditions derived from (9.5) goes over to the set of competitive equilibria determined by the first order conditions derived from (9.3). Hence, we have established the following proposition.

Proposition 11 : *A Nash equilibrium in giving converges to a competitive equilibrium in giving as the number of givers, n, goes to infinity.*

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[34] For Example 1 we have $u^i = \ln c_i + a_i g_i G = \ln c_i + \alpha_i g_i \frac{G}{n}$, where $\alpha_i = na$. We hold α_i fixed as we take the limit $n \to \infty$.

For Example 2 we have $u^i = (1-a_i)\ln\left(c_i - \frac{b_i}{G}\right) + a_i \ln g_i = (1-a_i)\ln\left(c_i - \frac{\beta_i}{G/n}\right) + a_i \ln g_i$, where $\beta_i = \frac{b_i}{n}$. We hold β_i fixed as we take the limit $n \to \infty$.

9.2. Nash equilibria for Example 1 (public redistribution)

Proposition 12 : *(a) For the numerical values given in subsection 7.1.2, above, there are two Nash equilibria. The following table compares the competitive outcomes (third column) with the Nash outcomes (fourth column) at the two equilibria.*

		Competitive	*Nash*
Aggregate giving	G^-	150	149.33
Aggregate giving	G^+	300	300.66
Individual giving	g_i^-, $i = 1, 2, ..., 450$	$\frac{1}{3}$	0.332
Individual giving	g_i^+, $i = 1, 2, ..., 450$	$\frac{2}{3}$	0.668
Individual giving	$g_i^\pm = 0$, $i = 451, ..., 1900$	0	0

(b) For k givers, $\frac{1/3 - g_i^-}{1/3} < r \Leftrightarrow k > \frac{2-r-r^2}{r+r^2}$ *and* $\frac{g_i^+ - 2/3}{2/3} < r \Leftrightarrow k > \frac{1-r-2r^2}{r+2r^2}$*. In particular, if $k > 197$, then the individual (and aggregate) Nash giving is within 1% of the competitive value.*

In Proposition 12(a) the competitive and Nash solutions are determined for the numerical values of the parameters in subsection 7.1.2. Replication of the economy does not affect the competitive equilibrium values that were derived in subsection 7.1.2 but it affects the Nash equilibrium values $g_i^\pm$. In part Proposition 12(b) we compare the percentage difference between the competitive outcome and $g_i^\pm$ for any arbitrary $r > 0$. The inequality $k > 197$ corresponds to the value $r = 0.01$.

9.3. Nash equilibria for Example 2 (public goods)

Proposition 13 : *(a) For the numerical values given in subsection 7.2.2, above, there are two Nash equilibria. Table-I compares the competitive outcomes (third column) with the Nash outcomes (fourth column) at the two equilibria.*

TABLE-I: Competitive and Nash equilibria for 50 givers

		Competitive	*Nash*
Aggregate giving	G^-	1	0.7766
Aggregate giving	G^+	4	4.2234
Individual giving	g_i^-, $i = 1, 2, ..., 50$	0.0200	0.0155
Individual giving	g_i^+, $i = 1, 2, ..., 50$	0.0800	0.0845

(b) For n givers, $\frac{0.02 - g_i^-}{0.02} < r \Leftrightarrow n > \frac{144}{9r+4r^2}$ *and* $\frac{g_i^+ - 0.08}{0.08} < r \Leftrightarrow n > \frac{9}{3r+16r^2}$*. In particular, if $n > 1592$, then the individual (and aggregate) Nash giving is within 1% of the competitive value.*

(c) For the same parameter values, Table-II reports the Nash levels of individual giving, $g_i^\pm$, as the number of individual givers, n, increases from 100 to 10,000.

TABLE-II: Convergence of Nash giving as the number of givers increases

n	100	500	1000	$10,000$
g_i^-	0.0177	0.0195	0.0198	0.0200
g_i^+	0.0823	0.0805	0.0802	0.0800

10. Conclusions

Private philanthropic activity is much studied in economics and is of immense economic importance. The empirical evidence suggests that the vast bulk of giving activity is undertaken by very large numbers of diverse, dispersed and small givers. For such givers, the competitive equilibrium model in economics would seem to be a natural fit. However, the existing literature has typically (but not always) described the activity of 'giving' within the ambit of a *symmetric Nash equilibrium* in strategic giving. Furthermore, the literature has, by and large, restricted itself to a *unique equilibrium* in giving. We relax both these features of the existing literature. Our framework is not restricted to the economics of charity. As our examples show, our results also have an equally important bearing on many public goods contexts, particularly those that involve private provision of public goods.[35]

Coordination problems and multiple equilibria are the norm in many situations of economic interest, but they are often ruled out by assumption. It is not surprising that uncoordinated private giving might lead to the possibility of multiple equilibria. We show that a necessary condition for multiple equilibria is our proposed condition of *aggregate strategic complementarity* between own-giving and aggregate-giving to a charity. Strategic complementarity at the level of each giver is sufficient but not necessary for aggregate strategic complementarity.

Once one allows for multiple equilibria that can potentially be ranked according to some social criteria, the following question arises naturally. If society is stuck at a *low* equilibrium, characterized by low levels of giving, can public policy help it to attain a *high* equilibrium? This is not a trivial question because our understanding of engineering moves between alternative equilibria in economics is not very well developed. In the context of private philanthropic activity (and public good contexts), we show that temporary direct government grants to charities allow for such engineering of moves between equilibria.[36] The significance of this result, to our mind, goes beyond a charity or a public goods context.

We also perform a welfare analysis and examine the optimality of alternative mixes of private and public contributions to charity. We show, for some parameter values, that additional incentives to giving can reduce the aggregate of private contributions in equilib-

[35] One can also consider applications of this framework to the provision of public goods in a federation.

[36] This result, in our model, also has the potential to explain the efficacy of *seed money*, *leadership contributions*, and direct grants by large donors, e.g., the National Lottery (as in the UK).

rium (*perverse comparative statics*). In such cases, it might be best to finance charitable giving, if required, by direct government grants financed through taxation. For other parameter values, however, giving to charity responds well to incentives (*normal comparative statics*). In this case, charitable giving should be entirely funded by private individual contributions, possibly subsidized through taxation. This is welfare improving because private giving leads to warm glow, while direct government grants do not.

The *aggregate desire to give function* played a key role in this paper. It may also be useful in other contexts.

Throughout we focus on equilibrium analysis. The dynamics of time paths from one equilibrium to another involve fundamental questions about the precise learning mechanisms to be used. Although progress on learning mechanisms is being made and in due course such mechanisms may enrich our model, currently such issues lie beyond the scope of this paper.

11. Appendix: Proofs

Proof of Proposition 1: (a) U^i is a continuous, strictly concave function (see (2.13)) defined on a compact interval (see (2.14)). Hence, a maximum exists and it is unique. (b) Denoting the optimal values by superscripted stars, from the restriction (2.4) we know that $c_i^* > \underline{c}_i$ and so $g_i^* < \frac{1}{1-s}\left[(1-t)\,m_i + \tau_i - \underline{c}_i\right]$. (c) If, in addition, (2.6) holds, then $g_i^* > 0$. ■

Proof of Lemma 1: If $g_i^* > 0$, then, in the light of Proposition 1b, g_i^* is an interior maximum and part (a) follows. Part (b) follows from (2.12). Appealing to the implicit function theorem, or differentiating the identity, $(1-s)\,u_1^i = u_2^i$, establishes parts (c), (d) and (e). ■

Proof of Lemma 2: Follows from (2.12) and Definition 1. ■

Proof of Lemma 3: (a) First, suppose that $g_i^* > 0$. The result then follows from Lemma 2 and Lemma 1(c).

(bi) Suppose that $\frac{\partial g_i^*}{\partial G} \geq 0$ (≤ 0) holds for all $g_i^* \geq 0$. Then, a fortiori, it holds for all $g_i^* > 0$. Hence, by (a), g_i and G are strategic complements (substitutes). (bii) Since u^i is C^2, g_i^* is C^1, i.e., $\frac{\partial g_i^*}{\partial G}$ is continuous. Now, suppose that g_i and G are strategic complements (substitutes). Then, by (a), $\frac{\partial g_i^*}{\partial G} \geq 0$ (≤ 0) holds for all $g_i^* > 0$. By the continuity of $\frac{\partial g_i^*}{\partial G}$, it follows that $\frac{\partial g_i^*}{\partial G} \geq 0$ (≤ 0) holds for $g_i^* \geq 0$. ■

Proof of Lemma 4: Follows from (4.7) and Lemma 3. ■

Proof of Proposition 2: In the text. ■

Proof of Lemma 5: From Definitions 2 and 5 we get $G^*(s,t) = tM + (1-s)\sum_{i=1}^{n} g_i^*(s,t,G^*(s,t))$. Implicitly differentiate this identity and rearrange to get $\sum_{i=1}^{n}\left(\frac{\partial g_i^*}{\partial s} + \frac{\partial g_i^*}{\partial G}G_s^*\right) = \frac{1}{1-s}\left(G_s^* + \sum_{i=1}^{n} g_i^*\right)$. ■

Proof of Proposition 3: Let G^* be an equilibrium at which $F_G \neq 1$. Then, from Proposition 2c, G^* is isolated. By Definition 5, $G^* = F(s,t,G^*)$. Differentiating this implicitly, and rearranging, gives the required results. ■

Proof of Corollary 1: Immediate from Proposition 3 and Definition 7. ■

Proof of Propositions 4 and 5: Obvious from Proposition 3(a), (b), respectively, and Lemma 4. ■

Proof of Lemma 6: From the government budget constraint, (4.2), evaluated at equilibrium, we get $D(s,t,G^*(s,t)) = tM - s\sum_{i=1}^{n} g_i^*(s,t,G^*(s,t))$. Rewrite this with D and t as the independent instruments, to get $D = tM - s(D,t)\sum_{i=1}^{n} g_i^*(s(D,t),t,G^*(s(D,t),t))$. Differentiate implicitly with respect to D and rearrange to get $\frac{\partial s}{\partial D} = -1/\sum_{i=1}^{n}\left[g_i^* + s\left(\frac{\partial g_i^*}{\partial s} + \frac{\partial g_i^*}{\partial G}\frac{\partial G^*}{\partial s}\right)\right]$. Use Lemma 5 to get $\frac{\partial s}{\partial D} = -1/\sum_{i=1}^{n}\left[g_i^* + \frac{s}{1-s}\left(G_s^* + \sum_{j=1}^{n} g_j^*\right)\right]$. ■

Proof of Proposition 6: Let (s^*, t^*) maximize social welfare (6.4). By assumption, $u_1^i > 0$, $u_3^i \geq 0$, $U_i > 0$ and $m_i \geq 0$ with some $m_i > 0$. If $G_t^* \leq 0$ then, from (6.6), it follows that $V_t < 0$. Hence, necessarily, $t^* = 0$. Since $D^* \geq 0$, $s^* \geq 0$ and some $g_i^* > 0$ it follows from the government budget constraint (2.8) that $s^* = D^* = 0$. ■

Proof of Proposition 7: Let (s^*, t^*) maximize social welfare (6.4). By assumption, $u_1^i > 0$, $u_3^i \geq 0$, $U_i > 0$ and $g_i^* \geq 0$ with some $g_i^* > 0$. Hence, if $G_s^* \geq 0$ then (6.5) implies that $V_s > 0$. Viewing D and t as the independent government instruments we get, $\frac{\partial V(s(D,t),t)}{\partial D} = V_s(s(D,t),t)\frac{\partial s(D,t)}{\partial D}$. From this and Lemma6 we get, $\frac{\partial V}{\partial D} = -V_s/\sum_{i=1}^{n}\left[g_i^* + \frac{s}{1-s}\left(G_s^* + \sum_{i=1}^{n} g_i^*\right)\right]$. Since $V_s > 0$, $G_s^* \geq 0$, $g_i^* \geq 0$ with some $g_i^* > 0$, it follows that $\frac{\partial V}{\partial D} < 0$. Suppose $D^* > 0$. Since $\frac{\partial V}{\partial D} < 0$, it follows that V can be increased further by reducing D, which contradicts the assumption that (s^*, t^*) maximizes social welfare. Hence, $D^* = 0$. ■

Proof of Proposition 8: From Proposition 3(a), $G_s^*(s,t) = \frac{F_s}{1-F_G}$. So when $F_s \geq 0$ and $F_G < 1$ at G^*, or if $F_s \leq 0$ and $F_G > 1$ at G^*, we get $G_s^*(s,t) > 0$. Proposition 7 then implies that s attains its maximum value and direct government grants are zero. ■

Proof of Proposition 9: Maximizing (3.1) subject to (2.7) gives

$$g_i^*(s,t,G) = \frac{1-t}{1-s}m_i - \frac{1}{a_i G},\ i = 1,...,k, \tag{11.1}$$

$$c_i^*(s,t,G) = \frac{1-s}{a_i G},\ i = 1,...,k. \tag{11.2}$$

From (3.2) and (11.1) we see that $g_i^*(s,t,G) > 0$ and from (11.2) $c_i^*(s,t,G) > 0$. On the other hand, from (3.3) we get that

$$g_i^* = 0,\ i = k+1, k+2, ..., n. \tag{11.3}$$

From (4.3), (7.1) - (7.2), (11.1), the aggregate desire to give to charity is

$$F(s,t,G) = m + t(M-m) - \frac{1-s}{G}A, \tag{11.4}$$

$$\Rightarrow F_t = M - m \geq 0. \tag{11.5}$$

The inequality in (11.5) is strict for $m < M$, i.e., $k < p$. Direct differentiation of (11.4) proves part (b) of the proposition, i.e., $F_s > 0$, $F_G > 0$, $F_{GG} < 0$.

$$F_s = \frac{A}{G} > 0,\ F_G = (1-s)\frac{A}{G^2} > 0,\ F_{GG} = -2(1-s)\frac{A}{G^3} < 0. \tag{11.6}$$

From Definition 5 and (11.4), an equilibrium G, must satisfy the quadratic equation

$$G^2 - [m + t(M-m)]G + (1-s)A = 0. \tag{11.7}$$

The quadratic equation in (11.7) has the solutions

$$G^{\pm}(s,t) = \frac{1}{2}\left[m + t(M-m) \pm \sqrt{[m + t(M-m)]^2 - 4(1-s)A}\right]. \tag{11.8}$$

For real roots we need $[m + t(M-m)]^2 \geq 4(1-s)A$. If $[m + t(M-m)]^2 = 4(1-s)A$ then, from (11.6), $F_G = 1$. From Proposition 3(a), (b) it would follow that G_s^*, G_t^* are undefined. Hence, the only economically interesting cases occur when,

$$[m + t(M-m)]^2 > 4(1-s)A, \tag{11.9}$$

in which case we have two distinct real positive equilibria

$$0 < G^-(s,t) < G^+(s,t), \tag{11.10}$$

and, correspondingly,

$$g_i^{\pm}\left(s,t,G^{\pm}(s,t)\right) = \frac{1-t}{1-s}m_i - \frac{1}{a_i G^{\pm}(s,t)},\ i = 1, ..., k, \tag{11.11}$$

$$g_i^{\pm} = 0,\ i = k+1, k+2, ..., n. \tag{11.12}$$

Using the fact that for real numbers a, b, $a > b$: $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$, as well as (11.6) and (11.8) - (11.10), we get

$$G = G^+ \Rightarrow F_G < 1,\ G = G^- \Rightarrow F_G > 1. \tag{11.13}$$

From Proposition 3(a), (b), (11.5), (11.6), (11.13) we get

$$G_s^+ > 0,\ G_s^- < 0, \tag{11.14}$$

$$G_t^+ > 0,\ G_t^- < 0 \text{ (for } m < M\text{, i.e., } k < p\text{)}, \tag{11.15}$$

$$G_t^{\pm} = 0 \text{ (for } m = M\text{, i.e., } k = p\text{)}. \tag{11.16}$$

Proof of Proposition 10: Applying Lemma 1(b) to the utility function (3.4), and using the budget constraint (2.7), gives

$$g_i^*(s,t,G) = \frac{a_i}{1-s}\left[(1-t)\,m_i - \frac{b_i}{G}\right], \tag{11.17}$$

$$c_i^*(s,t,G) = (1-a_i)\left[(1-t)\,m_i - \frac{b_i}{G}\right] + \frac{b_i}{G}. \tag{11.18}$$

From (3.5) and (11.17), (11.18), we see that $g_i^*(s,t,G) > 0$ and $c_i^*(s,t,G) > \frac{b_i}{G}$. Furthermore, it is straightforward to verify that the second order conditions also hold. Hence, given s,t,G; $g_i^*(s,t,G)$, $c_i^*(s,t,G)$ maximize utility (3.4) subject to the budget constraint (2.7), and are unique.

Substituting from (11.17) into (4.3) the aggregate desire to give, $F(s,t,G)$ is:

$$F(s,t,G) = \sum\nolimits_{i=1}^{n} a_i m_i + t\sum\nolimits_{i=1}^{n} (1-a_i)\,m_i - \frac{1}{G}\sum\nolimits_{i=1}^{n} a_i b_i. \tag{11.19}$$

From (11.19) we get:

$$F_s = 0,\ F_t = \sum\nolimits_{i=1}^{n} (1-a_i)\,m_i > 0; F_G = \frac{1}{G^2}\sum\nolimits_{i=1}^{n} a_i b_i > 0; F_{GG} = -\frac{2}{G^3}\sum\nolimits_{i=1}^{n} a_i b_i < 0. \tag{11.20}$$

From (11.20) and Proposition 3(a), we get

$$G_s^* = \frac{F_s}{1-F_G} = 0. \tag{11.21}$$

From (11.20) and Proposition 8, it follows that, at a social optimum, $D = 0$, i.e., no direct grant from the government to the charity is involved. Giving to charity is entirely funded by private donations, which are subsidized from taxation if $s > 0$, $t > 0$.

To make further progress, we need to determine the equilibrium values of G. From (4.3), (11.19) and Definition 5, the equilibrium values of G are the solutions to the equation

$$G = \sum\nolimits_{i=1}^{n} a_i m_i + t\sum\nolimits_{i=1}^{n} (1-a_i)\,m_i - \frac{1}{G}\sum\nolimits_{i=1}^{n} a_i b_i. \tag{11.22}$$

Substituting (7.5) in (11.22) we get

$$G^2 - BG + C = 0, \tag{11.23}$$

with solutions

$$G^{\pm}(s,t) = \frac{1}{2}\left[B \pm \sqrt{B^2 - 4C}\right]. \tag{11.24}$$

If $B^2 < 4C$, then no equilibrium exists. If $B^2 = 4C$ then a unique equilibrium exists, and is $G = \frac{B}{2} = \sqrt{C}$. But then, from (11.20), (7.5), $F_G = 1$. In this case, neither G_s^* nor G_t^* are defined, see Proposition 3(a), (b). Hence, the only interesting case is when $B^2 > 4C$.

In this case, (11.23) has two distinct real positive roots:

$$0 < G^{-}(s,t) < G^{+}(s,t), \tag{11.25}$$

and, correspondingly,

$$g_i^{\pm}(s,t,G^{\pm}(s,t)) = \frac{a_i}{1-s}\left[(1-t)\,m_i - \frac{b_i}{G^{\pm}(s,t)}\right], \; i = 1,2,...n. \tag{11.26}$$

Using the fact that for real numbers $a > b > 0$: $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$, as well as (11.20), (7.5) and (11.24) - (11.25), we get

$$G = G^{+} \Rightarrow F_G < 1, \; G = G^{-} \Rightarrow F_G > 1. \tag{11.27}$$

From (11.20), (11.27) and Proposition 3(b), we get $G_t^{+} > 0$, $G_t^{-} < 0$. ■

Proof of Proposition 12: Rewrite (3.1) as

$$u^i(c_i, g_i, G) = \ln c_i + \alpha_i g_i\left(\frac{G}{k}\right), \; i = 1,2,...,k, \tag{11.28}$$

where $\alpha_i = ka_i$. For the reasons explained in subsection 9.1, above, we shall take the limit, $k \to \infty$, keeping α_i fixed. We use the same parameter values as in subsection 7.1.2. Thus $a_i = 0.01$, $\alpha_i = 450a_i = 4.5$, $m_i = 1$ (so that $g_i^{\pm}$ can be interpreted as the fraction of income given to charity), $\tau_i = 0$, $s = t = 0$. Hence, the giver's budget constraint is $c_i = 1 - g_i$. Substituting these, along with $G = g_i + G_{-i}$, into (11.28) gives

$$u^i(1-g_i, g_i, g_i + G_{-i}) = \ln(1-g_i) + 4.5g_i\left(\frac{g_i + G_{-i}}{k}\right), \; i = 1,2,...,k. \tag{11.29}$$

Maximizing (11.29) with respect to g_i, given G_{-i}, gives

$$g_i^{\pm} = \frac{3(1-G) \pm \sqrt{9(1+G)^2 - 8k}}{6}, \; i = 1,2,...,k, \tag{11.30}$$

from which we see that $g_i = g_j$, $i,j = 1,2,...,k$. Substituting $G = kg_i$ in (11.30) gives $9(1+k)\,g_i^2 - 9(1+k)\,g_i + 2k = 0$. Solving gives

$$g_i^{\pm} = \frac{1}{2} \pm \frac{1}{6}\sqrt{9 - \frac{8}{1+1/k}}, \; i = 1,2,...,k. \tag{11.31}$$

From (11.31) it follows that $\frac{\frac{1}{3} - g_i^{-}}{\frac{1}{3}} < r \Leftrightarrow k > \frac{2-r-r^2}{r+r^2}$ and $\frac{g_i^{+} - \frac{2}{3}}{\frac{2}{3}} < r \Leftrightarrow k > \frac{1-r-2r^2}{r+2r^2}$. From these, and (11.31), the required results follow. ■

Proof of Proposition 13: Rewrite (3.4) as

$$u^i(c_i, g_i, G) = (1-a_i)\ln\left(c_i - \frac{\beta_i}{G/n}\right) + a_i \ln g_i, \tag{11.32}$$

where $\beta_i = b_i/n$. For the reasons explained in subsection 9.1, above, we shall take the limit, $n \to \infty$, keeping β_i fixed. We use the same parameter values as in subsection 7.2.2. Thus $a_i = 0.1$, $\beta_i = 0.8/50 = 0.016$, $m_i = 1$ (so that $g_i^{\pm}$ can be interpreted as the fraction of income given to charity), $\tau_i = 0$, $s = t = 0$. Hence, the giver's budget constraint is $c_i = 1 - g_i$. Substituting these, along with $G = g_i + G_{-i}$, into (11.32) gives

$$u^i (1 - g_i, g_i, g_i + G_{-i}) = 0.9 \ln \left(1 - g_i - \frac{0.016n}{g_i + G_{-i}}\right) + 0.1 \ln g_i. \tag{11.33}$$

Maximizing (11.33) with respect to g_i, given G_{-i}, gives

$$g_i = \frac{G^2 - 0.016nG}{10G^2 - 0.144n}. \tag{11.34}$$

From (11.34) we see that all g_i are equal. Substituting $G = ng_i$ in (11.34) gives $g_i^2 - 0.1g_i + 0.0016 - 0.0144n^{-1} = 0$. Solving gives

$$g_i^{\pm} = 0.05 \pm 0.03\sqrt{1 + 16n^{-1}}, \tag{11.35}$$

which leads to the figures in Tables I and II.

Also from (11.35) it follows that $\frac{0.02 - g_i^-}{0.02} < r \Leftrightarrow n > \frac{144}{9r+4r^2}$ and $\frac{g_i^+ - 0.08}{0.08} < r \Leftrightarrow n > \frac{9}{3r+16r^2}$. From these, the required results follow. ■

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