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COORDINATION FAILURES, PHILANTHROPY, AND PUBLIC POLICY

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Coordination Failures, Philanthropy, and Public Policy*

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Abstract

We focus on an "equilibrium analysis" of coordination problems in giving that lead to multiple equilibria; the notion of strategic complements and substitutes turns out to be useful in this regard. Some societies can get stuck at a low level of giving while others might, by accident or policy, be able to coordinate on a higher level of giving. Ceteris-paribus, this furnishes one plausible reason for heterogeneity in philanthropy. We give conditions under which tax exemptions to private giving can have perverse effects by reducing equilibrium private giving. Direct government grants to charity, possibly temporary, can enable an economy stuck at an equilibrium with a low level of giving to attain an equilibrium with a high level of giving. Therefore, direct government grants can crowd-in private giving to charity. The paper contributes to the economics of philanthropy as well as to an understanding of the role of public policy in the face of private coordination failures.

Keywords: multiple equilibria, crowding-in, strategic substitutes and complements, tax deductibility of charitable contributions, direct grants, charitable redistribution, voluntary contributions to public goods.

JEL Classification: D6, H2, H4.

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1. Introduction

Philanthropic activity is associated with several well known stylized facts; we will highlight three important ones.

1. Private philanthropic activity is important and varies significantly across countries. Figures for the United States, where philanthropic activity has been particularly well documented, are suggestive of the magnitudes. In 1995, as a percentage of income, giving has varied between 1.5 - 2.1 percent, 68.5 percent of all households gave to charity with the average gift being $1081 and those with the lowest incomes give over 4 percent of income to charity. There is significant cross country heterogeneity. As a percentage of GDP, over the period 1995-2000, such activity was in excess of 4 percent for the Netherlands and Sweden; between 3 and 4 percent for Norway and Tanzania; between 2 to 3 percent for France, United Kingdom, USA, Israel and Spain; and less than 0.5 percent for India, Pakistan, Brazil, Mexico and Poland[1].

2. Private individuals are by no means the only contributors to charity. Charities receive substantial direct grants from the government, the corporate sector, charitable trusts and foundations, and (in the West) from lottery money. Governments, operating normally through direct grants, are typically the single most important contributors to charities. On average, in the developed countries, charities receive close to half their total budget directly as grants from the government, while the average for developing countries is about 21.6 percent[2].

3. Contributions to charity are tax deductible in many cases. However, the deduction is not generally at the full marginal rate of tax and is typically capped at some upper rate[3]. Irrespective of the rate, the tax deduction acts as a reduction in the price of giving to charity.

In light of these stylized facts we broadly ask the following questions. First, why is there individual heterogeneity in the magnitude of giving, say, across groups, regions or countries? Second, what is the role of direct grants to charities made by the government?

[1]These figures are from Salamon et. al. (2004). The figures do not include donations to religious congregations which are a substatial component for countries such as the USA.

[2]These figures are from the Johns Hopkins Comparative Nonprofit Sector Project. The data can be accessed from their website http://www.jhu.edu/~cnp/. Non-governmental organizations are also an important source of contributors. In the UK, for instance, charitable trusts and foundations give grants worth £700 million a year while the National Lottery, another major contributor, had, by September 2001, awarded more than 45,000 grants worth a total of more than £2 billion through its Community Fund.

[3]In the USA, deductions are allowed at a rate of up to 50 percent of taxable income for donations to public charity but at a rate of 30 percent when donations are to a private foundation. In Canada the rate of deductions allowed increases from 17 percent to 29 percent depending on the amount donated.

Third, what is the precise role of tax deductions (or subsidies) to giving? All three of these questions have been asked before and there is by now an extensive literature on these issues[4]. However, the literature has typically focussed on a *unique equilibrium* in giving. This, we believe, masks several features of the problem when there are *multiple equilibria* in giving arising from the uncoordinated efforts of a very large number of givers in an economy.

Before we outline our approach and the main results, we situate our paper in the context of some of the related theoretical literature.

1.1. Related Literature

Rigorous theoretical work in the area began with Becker (1974), which was extended by Bergstrom, Blume and Varian (1986). These models stress the following important feature of the problem. Charitable giving has the nature of a public good. Givers care only about the sum total of giving by all givers; such givers are said to be motivated by a feeling of *altruism* or *benevolence*. Thus, only the total donations received by the charity enter an individual's utility function and, so, giving by one individual is a perfect substitute by another. This gives rise to a free rider problem. Any individual giver is indifferent to the source of a gift received by the charity, hence, would prefer someone else to make the donation. The model has some unattractive features, however. If public grants are financed by taxes on givers, they completely crowd out private giving, leaving the total supply of the public good unchanged[5], which does not seem to be supported by the empirical evidence. Furthermore, when the number of givers is large, even distortionary taxes become approximately neutral and redistribution of incomes between givers is neutral; see, for instance, Andreoni (2004).

An alternative approach was developed by Cornes and Sandler (1984) and Andreoni (1989, 1990)[6]. This approach introduces a direct utility, or *warm glow*, from one's own giving, but retains *altruism*[7]. Warm glow ensures that one's own giving is not a perfect substitute for the giving of others, hence, creating an additional incentive to give, which mitigates the free rider problem. It also obviously implies that government grants to charities do not crowd out private donations completely because the two are imperfect

[4] Andreoni (2004) provides a fairly exhaustive survey of the issues.

[5] At a corner solution in which no one contributes, and so no crowding-out can take place, then there is a potential efficiency enhancing role for the government in giving direct grants to the charity financed by taxes on givers.

[6] A variant of this approach was orginally suggested by Becker (1974) in a footnote.

[7] The literature does not precisely pin down the source of *warm glow* to one specific factor. Andreoni (2004) explains it thus: "humans are moral- they enjoy doing what is right. They are emotional, empathic and sympathetic- they enjoy gratitude and recognition, they enjoy making someone else happy, and they feel relieved from guilt when they become a giver. Put more simply and more generally, people may experience a *warm glow* from giving."

substitutes from the point of view of givers. There is strong support for warm glow in experimental data; see for instance, Andreoni(1993), Palfrey and Prisby (1997).

The theoretical literature has also explored the implications of tax deductions for charitable giving; see for instance, Feldstein (1980), Boadway and Keen (1993), Diamond (2003). In general, subsidies in this literature increase private giving by individuals, while direct government grants, because they rely on distortionary taxation, are less attractive.

Our paper is most closely related to Andreoni (1998) and complements his analysis. In Andreoni (1998), multiple equilibria arise if and only if there are non-convexities in the production technology of the public good in the following sense. Aggregate donations must exceed a certain threshold for production of the public good to be possible[8]. It can be shown that there are two Nash equilibria. One in which none contribute so the public good is not produced, and another in which at least one individual contributes and a positive level of the public good is produced. Our paper differs from this approach in the following ways. First, and most fundamentally, we show that multiple equilibria can arise even when there are no non-convexities in production. Furthermore, in our model, aggregate contributions are positive in the lower, as well as the higher, equilibrium. Second, we perform a welfare analysis in a general equilibrium model with a government budget constraint. Hence, the public policy variables are determined "optimally". Furthermore, the "socially optimal" public policy depends on which of the multiple equilibria is achieved. Third, in Andreoni (1998), public policy can help move an economy stuck at a low equilibrium in giving to a high equilibrium. However, this requires that the government have the ability to levy taxes based on individual characteristics. This might be difficult, even illegal, in practice. We need, however, the much weaker restriction that the government be able to observe and verify the income of individuals. Fourth, our focus is on the aggregate giving of all individuals in an economy rather than on issues of the startup of a specific charity.

1.2. Multiple Equilibria

The existing literature, with the notable exception of Andreoni (1998), focusses on a unique equilibrium. But it is entirely plausible that coordination problems among diverse and numerous givers give rise to multiple equilibria. For instance, if you conjecture that the aggregate gift of others to a charity will be high, so that the charity is 'large', then you might also be induced to contribute a high amount. This could occur, for instance, if you get a greater warm glow from contributing to larger, presumably more prominent, charities[9]. In this case, private giving and the size of the charity are *strategic complements*.

[8]Such non-convexities seem more important when a charity or a cause is to be set up rather than in the continued operation of the charity.

[9]Larger charities might also be a signal of greater importance of the cause for which the charity seeks funding, or reputation of the charity in channeling funds efficiently and honestly etc. Hence, it is plausible

Analogously, if your conjecture is a lower aggregate amount, then you might also contribute a lower amount, because, on accounts of strategic complements, your warm glow is now smaller. Hence, there could be several, self fulfilling, rational expectations equilibria. In some, giving is *high*, while in others, giving is *low*[10].

The high equilibrium could, of course, be welfare improving. However, coordination problems might prevent private individuals from attaining it. Resultingly, the economy might get stuck in the low equilibrium. Public policy could then play an important role in engineering a move from the low to the high equilibrium. We focus on this feature of the problem.

The possibility of multiple equilibria creates the potential for a richer set of results.

1.3. Some Results and Intuition

1. Multiple equilibria are suggestive of a natural explanation for cross country (or group or regional) heterogeneity in giving. Among identical societies some can get stuck at a low level of giving, while others might find themselves in an equilibrium with a high level of giving. The existing models, with a unique equilibrium, would have to rely on heterogeneity in tastes or preferences to explain this phenomenon.

2. We show that if private individual giving and aggregate giving are strategic complements, then subsidizing private individual giving can have the perverse effect of reducing aggregate giving. In this case, we show how public policy in the form of direct government grants to charity can engineer an escape from an equilibrium with a low level of giving to a better equilibrium with a high level of giving. Therefore, direct government grants can *crowd-in* rather than *crowd-out* private giving to charity. This obviously contrasts with the classical analysis of subsidies to charities, in the context of a unique equilibrium in giving, which is unfavorable to direct government grants; see, for instance, Feldstein (1980). In light of our analysis, then, (potentially temporary) government grants could be recommended on normative criteria.
   Some intuition behind our second set of results is as follows. Consider a situation of multiple equilibria in aggregate giving, that has two equilibria, *low* and *high*. We show, in plausible cases, that at the *low* equilibrium subsidies can actually *reduce* aggregate giving. In such a case, suppose that the government gives a (possibly temporary) direct grant to charity, financed by taxation, at least equal to the aggregate giving at the *low* equilibrium. The sum of public and private contributions now exceeds the level attained at the *low* equilibrium, which rules out the *low* equilibrium

that consumers might feel more assured in giving to such charities.

[10]We also show, formally, that when private giving and the size of the charity are *strategic substitutes* then the equilibrium is unique.

as a candidate. With only two possible equilibria, high and low, this must push the economy to attain the *high* equilibrium, by default, because the low equilibrium is ruled out. Furthermore, it might be the case that at the *high* equilibrium, subsidies are effective. In that case, replacing the direct government grant by private contributions induced through subsidies is welfare improving because grants provide no warm glow, while private giving does. Hence, temporary direct government grants allow public policy to solve the private coordination problem.

3. Finally, we give conditions that specify the optimal mix of public contributions (through direct government grants) and private contributions to charity. We establish sufficient conditions for giving to charity to be funded entirely by private individual donations, possibly subsidized from taxation, and there is no public contribution to charity. For other parameter values, however, subsidies can have perverse effects and direct government grants can improve welfare.

Throughout, our focus is on *equilibrium analysis.* We provide no rigorous analysis of the time path between two equilibria. Further comments on this issue can be found in Section 7 below.

1.4. Schematic Outline

The schematic outline of the paper is as follows. Section 2 describes the general theoretical model. Section 3 derives the equilibrium of the model and its comparative static results. Section 4 examines multiple equilibria in aggregate giving. Section 5 performs a welfare analysis and characterizes the normatively optimal public policy. Section 6 provides two illustrative examples of the general theoretical model applied respectively to (1) contributions to charity that support income redistribution, and (2) voluntary gifts to public good provision, in particular, when the public good takes the form of some essential public infrastructure[11]. In Section 7 we collect, briefly, some of our thoughts on dynamic analysis of the game. Section 8 explores the implications of strategic giving and illustrates the robustness of our results. Finally Section 9 concludes. All proofs are collected in the appendix.

2. Model

There are three main types of players in the economy, (1) *consumers*, (2) a *fiscal authority* (which we will simply refer to as the *Government*), and (3) *charities*. There are

[11]This paper can also be viewed as a model of voluntary contributions to public goods, as the second example shows. However, for expositional considerations, throughout, we will couch the model in terms of giving to charity.

n consumers indexed by $i = 1, 2, ..., n$. Consumer i has an exogenously fixed income of $m^i \geq 0$ each period. Define the vector of individual incomes $\mathbf{m} = (m^1, m^2, ..., m^n)$ and the aggregate of all incomes, M, as

$$M = \sum\nolimits_{i=1}^{n} m^i. \tag{2.1}$$

Assume that $M > 0$, i.e., at least some consumers have positive income.

2.1. Fiscal Instruments

The government exercises the following three types of fiscal instruments.

1. An income tax on individual income, m^i, at the rate t, $0 \leq t < 1$.
2. A subsidy to "private" giving to charity at the rate s, $0 \leq s < 1$.
3. Direct "public" contribution to charity given by $D \geq 0$.

2.2. Consumers

The utility function of consumer i is given by

$$u^i \left(c^i, g^i, G\right) \tag{2.2}$$

The consumer derives utility from private consumption expenditure, $c^i \geq 0$, from 'own' giving to charity (*warm glow*), $g^i \geq 0$, and from the aggregate level of giving to charity (*altruism*), $G \geq 0$. We will use subscripts to denote partial derivatives. The assumptions on preferences are that u^i is C^2 and $u^i_1 > 0$, $u^i_2 \geq 0$, $u^i_3 \geq 0$ with $u^i_2 > 0$ for some i.

The budget constraint of consumer i is given by

$$c^i + (1 - s)\, g^i \leq (1 - t)\, m^i \tag{2.3}$$

The RHS of (2.3) is the (net of tax) income of the individual, while the LHS gives the two sources of expenditure, private consumption expenditure and (net of subsidy) private giving to charities.

Furthermore, we assume that each g^i is a small fraction of G, so that each consumer takes the aggregate G as given[12]. Thus, in making her decision to allocate after-tax income

[12]Our motivation for doing so is that, in most situations of interest, the individual contribution is a negligible fraction of the total budget of a charity. This deviates from the standard literature, which is usually based on a Nash equilibrium in giving to charity. Such an approach complicates the analysis (even with identical consumers), without changing the qualitative conclusions, or significantly altering the quantitative results. In Section 8 below, we show this to be the case by performing simulation analysis to compare the two cases of strategic and non-strategic giving. Our approach can, of course, be made completely rigorous by adopting an appropriate measure-theoretic formulation with a continuum of consumers. Again, we have found this to considerably complicate our paper, without adding anything to either the conclusions or the literature on the measure-theoretic approach to economics.

between private consumption and giving to charity, the consumer takes as given $\mathbf{m}$, s, t, G and maximizes u^i given in (2.2) subject to the budget constraint (2.3). Since $u_1^i > 0$, the budget constraint (2.3) holds with equality. Hence, we can use it to eliminate c^i from (2.2). Letting $U^i\left(g^i, G; s, t\right)$ be the result, we have

$$U^i\left(g^i, G; s, t\right) = u^i\left(\left(1-t\right)m^i - \left(1-s\right)g^i, g^i, G\right) \tag{2.4}$$

Notice that for notational convenience we have suppressed reference to $\mathbf{m}$ because throughout our analysis we keep it fixed. The consumer's problem can be restated as

$$\text{Maximize}_{\langle g^i | s,t,G\rangle}\ U^i\left(g^i, G; s, t\right) \tag{2.5}$$

Following Bulow, Geanakoplos and Klemperer (1985), g^i, G are strategic complements (strategic substitutes), if and only if

$$\frac{\partial^2 U^i}{\partial g^i \partial G} \geq 0 \quad (\leq 0) \tag{2.6}$$

From (2.4) and (2.6), g^i, G are strategic complements (strategic substitutes), if and only if

$$u_{23}^i - \left(1-s\right)u_{13}^i \geq 0 \quad (\leq 0) \tag{2.7}$$

where subscript i, j refers to the cross partial derivative between the i^{th} and j^{th} arguments.

2.3. Government

We shall assume that the proceeds of the income tax, tM, are spent towards tax relief on donations to charity, $s\sum_{i=1}^{n} g^i$, and on aggregate direct grants from the Government to the charities, $D \geq 0$. Therefore, the (balanced) Government budget constraint is

$$tM = D + s\sum_{i=1}^{n} g^i \tag{2.8}$$

The Government chooses its instruments s, t, D so as to maximize a social welfare function

$$U = U\left(u^1, u^2, ..., u^n\right) \tag{2.9}$$

which we assume to be a strictly increasing function of the individual utility functions, $u^1, u^2, ..., u^n$, of the consumers[13].

[13]Note that the resulting social optimum is constrained by the available set of instruments $\{s, t, D\}$. In particular, an even better social optimum may be available if subsidies and taxes $\left\{s^i, t^i\right\}$ could be varied across individuals. We assume that this is either not desirable or not possible (although it is straightforward to extend our analysis to cope with the more general case).

2.4. Charities

In order to focus on the simultaneous determinants of private giving and the influence of public policy, we assume that charities are passive players in the game[14]. They merely collect all donations from private consumers, $\sum_{i=1}^{n} g^i$, and from the government, D. The production function for charities is a simple linear function that converts the sum of all giving $\sum_{i=1}^{n} g^i + D$ to some aggregate output $G \geq 0$, so

$$G = D + \sum_{i=1}^{n} g^i \tag{2.10}$$

3. Equilibrium Giving and Public Policy

Consider a consumer who enjoys warm glow, i.e., u_2^i is not identically zero, and has strictly positive income, $m^i > 0$. We assume that, for such a consumer, the problem in (2.5) has the unique interior solution $g^i(s,t,G)$. In other words, and using the budget constraint (2.3),

$$0 < g^i(s,t,G) < \frac{1-t}{1-s} m^i \tag{3.1}$$

It follows that for $m^i > 0$, $g^i(s,t,G)$ satisfies the first order condition

$$(1-s)\, u_1^i = u_2^i \ , \quad i = 1, 2, ..., n \tag{3.2}$$

where subscript j denotes partial derivative with respect to the j^{th} argument. We assume that the second order condition holds, hence,

$$(1-s)^2 u_{11}^i - 2(1-s)\, u_{12}^i + u_{22}^i < 0 \tag{3.3}$$

If a consumer does not enjoy any warm glow from giving, i.e., $u_2^i \equiv 0$, then,

$$g^i \equiv 0 \tag{3.4}$$

In Example 1 of Section 6.1 we consider the possibility that some, but not all, consumers enjoy warm glow from giving. Hence, for the former set of consumers who enjoy warm glow, the equilibrium (strictly positive) level of private giving is the solution to (3.2). For the latter set of consumers $g^i \equiv 0$.

Since u^i is C^2, it follows that g^i is C^1. Differentiating (3.2) implicitly w.r.t. G we get that for consumers who enjoy warm glow:

$$\frac{\partial g^i}{\partial G} = \frac{u_{23}^i - (1-s)\, u_{13}^i}{-(1-s)^2 u_{11}^i + 2(1-s)\, u_{12}^i - u_{22}^i} \tag{3.5}$$

[14]Hence, while we feel that in actual practice charities could be strategic players, using means such as bundling, marketing etc. to attract additional donations, we abstract here from these issues.

For consumers who do not enjoy warm glow, (3.4) gives

$$\frac{\partial g^i}{\partial G} = 0 \tag{3.6}$$

From (2.7), (3.3), (3.5), (3.6) it follows that g^i, G are strategic complements (strategic substitutes), if and only if

$$\frac{\partial g^i}{\partial G} \geq 0 \quad (\leq 0) \tag{3.7}$$

Example 1 in Section 6.1, below, considers an economy in which public policy and private charitable giving redistribute income towards the poor. The strategic complementarity between g^i, G arises because the warm glow that a consumer receives from an extra unit of giving is larger if the charity is larger (and so perhaps the charity is more prominent).

Example 2 in Section 6.2, below, considers an economy in which voluntary giving finances a public infrastructure good that is essential to enjoy private consumption. The strategic complementarity between g^i, G arises because a higher G results in a higher utility from a given level of consumption expenditure but a reduced marginal utility of private consumption. In response to an increase in G, a utility maximizing consumer will then spend less on consumption and give more to charity, thus, $\frac{\partial g^i}{\partial G} \geq 0$.

3.1. The Aggregate Desire to Give to Charity

Given *actual* aggregate donation to charity, G, the aggregate of all *desired* donations, $D+\sum_{i=1}^{n} g^i$, may be different. Therefore, we introduce a new function, F, which represents the aggregate of all desires to give to charity

$$F = D + \sum\nolimits_{i=1}^{n} g^i (s, t, G) \tag{3.8}$$

We substitute from (2.8) into (3.8) to get

$$F (s, t, G) = tM + (1 - s) \sum\nolimits_{i=1}^{n} g^i (s, t, G) \tag{3.9}$$

From (3.1), (3.9) it follows that

$$0 < F (s, t, G) < M \tag{3.10}$$

Hence, w.l.g., we may view $F (s, t, .)$ as a mapping from $[0, M]$ to $[0, M]$:

$$F (s, t, .) : [0, M] \to [0, M] \tag{3.11}$$

From (3.9)

$$\frac{\partial F}{\partial G} = F_G = (1 - s) \sum\nolimits_{i=1}^{n} \frac{\partial g^i}{\partial G} \tag{3.12}$$

From (3.7), (3.12) it follows that if g^i, G are strategic complements (strategic substitutes), then

$$F_G \geq 0 \quad (\leq 0) \tag{3.13}$$

We summarize the results on strategic complements (substitutes) in Lemma 1 below.

Lemma 1 *: (a) The following are equivalent.*
(i) g^i, G *are strategic complements (strategic substitutes).*
(ii) $u^i_{23} - (1-s)\, u^i_{13} \geq 0 \;\; (\leq 0)$.
(iii) $\frac{\partial g^i}{\partial G} \geq 0 \;\; (\leq 0)$.
(b) If g^i, G *are strategic complements (strategic substitutes), then* $F_G \geq 0 \;\; (\leq 0)$**.**

3.2. Equilibria

The economy is in equilibrium if, and only if, the aggregate of all desires to donate to charity, F, equals the aggregate of all donations, G, i.e.

$$G(s,t) = F(s,t,G(s,t)) \tag{3.14}$$

Proposition 1 *: An equilibrium exists. At equilibrium,* $0 < G(s,t) < M$*. If* g^i, G *are strategic substitutes then the equilibrium is unique.*

3.3. Equilibrium Analysis

We wish to investigate how aggregate equilibrium giving to charity, $G(s,t)$, depends on the policy instruments, s, t. Differentiating (3.14) implicitly, then rearranging, gives

$$G_s(s,t) = \frac{F_s}{1 - F_G} \tag{3.15}$$

$$G_t(s,t) = \frac{F_t}{1 - F_G} \tag{3.16}$$

$$G_{tt}(s,t) = \frac{(F_{tt} + F_{tG}G_t)(1 - F_G) + F_t(F_{tG} + F_{GG}G_t)}{(1 - F_G)^2} \tag{3.17}$$

Proposition 2 is obvious from (3.13), (3.15), (3.16), but we state is formally for future reference.

Proposition 2 *: Let* $j = s$ *or* t*. Assume* $F_j > 0$*. If* $F_G < 1$*, then* $G_j(s,t) > 0$*. However, if* $F_G > 1$*, then* $G_j(s,t) < 0$*. In particular, if* g^i, G *are strategic substitutes then* $G_j(s,t) > 0$*.*

The common sense expectation is that $G_s(s,t) > 0$, i.e., subsidies to private giving induce additional private giving. However, the possibility of $F_G > 1$ gives rise to a perverse comparative static effect, $G_s(s,t) < 0$, namely, that subsidies reduce private aggregate giving. The intuition is that if $F_G > 1$ we have an excessive desire to contribute to charity. However, a low level is needed to dampen the excessive desire to contribute. The result is a low level of contribution in equilibrium. The various cases arising from Proposition 2 are illustrated below.

A. Common-Sense Case: $F_s > 0$, $F_G < 1$.

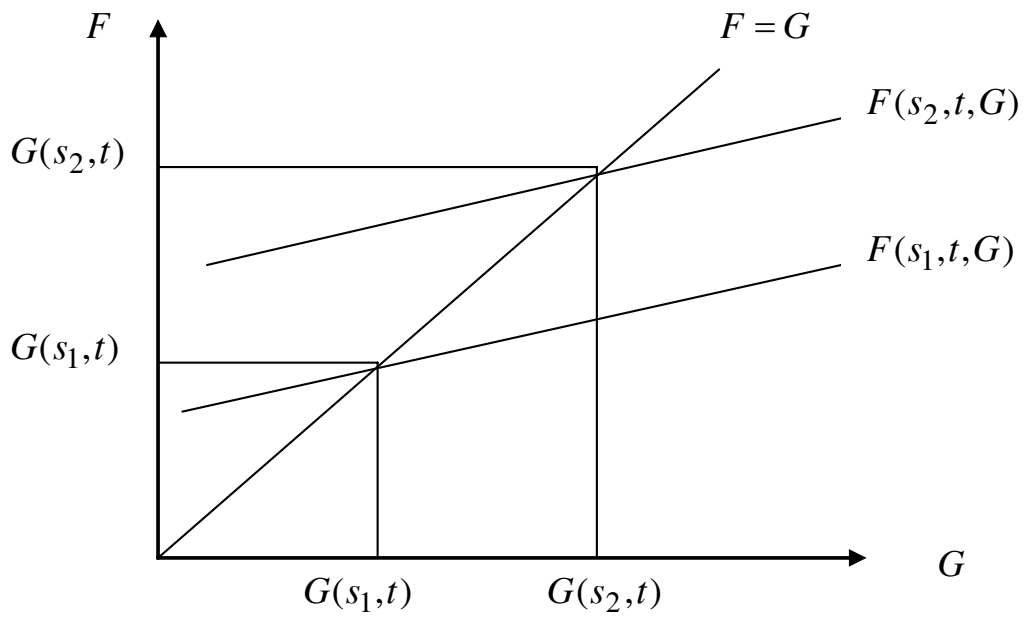

Figure 3.1: **The Case** $F_s > 0$ **and** $0 < F_G < 1$, $s_1 < s_2$.

We illustrate the case $F_s > 0$ and $0 < F_G < 1$ in Figure 3.1. As stated in Proposition 2, it is obvious that the optimal level of aggregate giving is increasing in the subsidy to individual giving i.e. $G_s > 0$. The same result also holds true when $F_s > 0$ and $F_G \leq 0$. In particular, if g^i, G are strategic substitutes, and so $F_G \leq 0$, aggregate donations nevertheless increase in equilibrium when the price of giving is reduced.

B. The Perverse Case: $F_s > 0$, $F_G > 1$.

In Case B, illustrated in Figure 3.2, equilibrium aggregate contributions are decreasing as the price of giving becomes more attractive (larger s). The intuition is that when

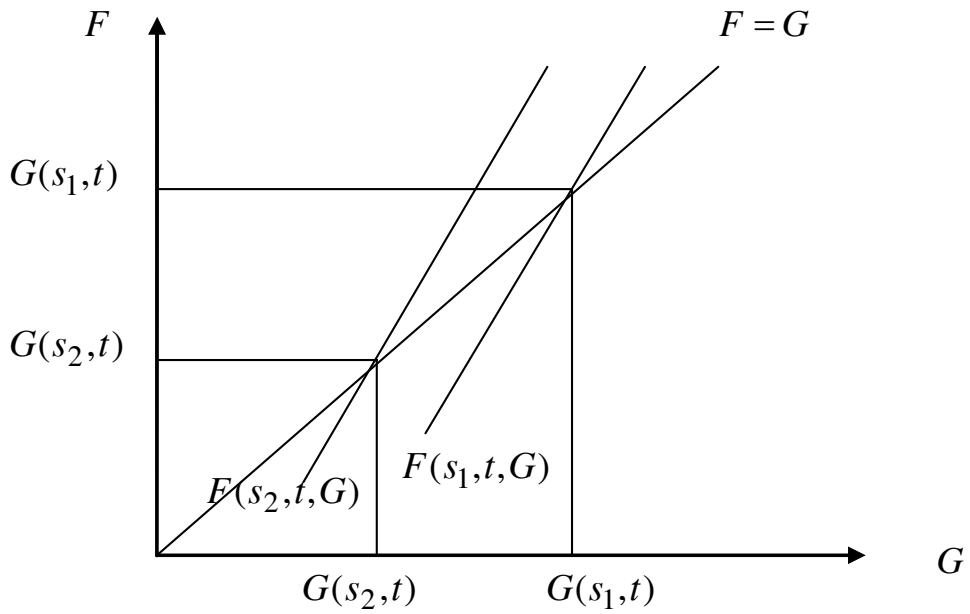

Figure 3.2: **The Case** $F_s > 0$, $F_G > 1$, $s_1 < s_2$.

$F_G > 1$ and $F_s > 0$, each consumer knows that a decrease in the price of giving will induce additional private desire to give ($F_s > 0$). However, because $F_G > 1$, consumers over-react to expectations of an increase in G. Thus, G needs to fall in order to ensure equilibrium in the market for charity[15].

This case provides a possible explanation of the surprising effects of the tax reforms of the 1980's on charitable giving in the US; see, for instance, Clotfelter (1990) for the evidence. The tax reforms increased the price of giving by reducing the tax preference for charitable donations. Contrary to the predictions of most economists, charitable contributions continued to rise in the following years. However, this empirical result is consistent with our analysis when $F_G > 1$. Similar observations would apply to the equilibria of Section 4 where $F_G > 1$.

[15]To aid intuition, imagine an upward sloping demand curve that cuts the supply curve from below. Following an increase in income, we get the perverse result that price (and quantity) will fall. In demand theory this case requires atypical assumptions such as giffen goods. By contrast, in the charity context such cases arise naturally; strategic complementarity being a necessary condition.

4. Multiple Equilibria

In this section we assume that $F_s > 0$, $s_1 < s_2$, $F_G > 0$ (i.e. g^i, G are strategic complements; see Lemma 1)[16]. However, we distinguish between two main cases here, $F_{GG} < 0$ and $F_{GG} > 0$.

A. $F_s > 0,\ F_G > 0, F_{GG} < 0.$

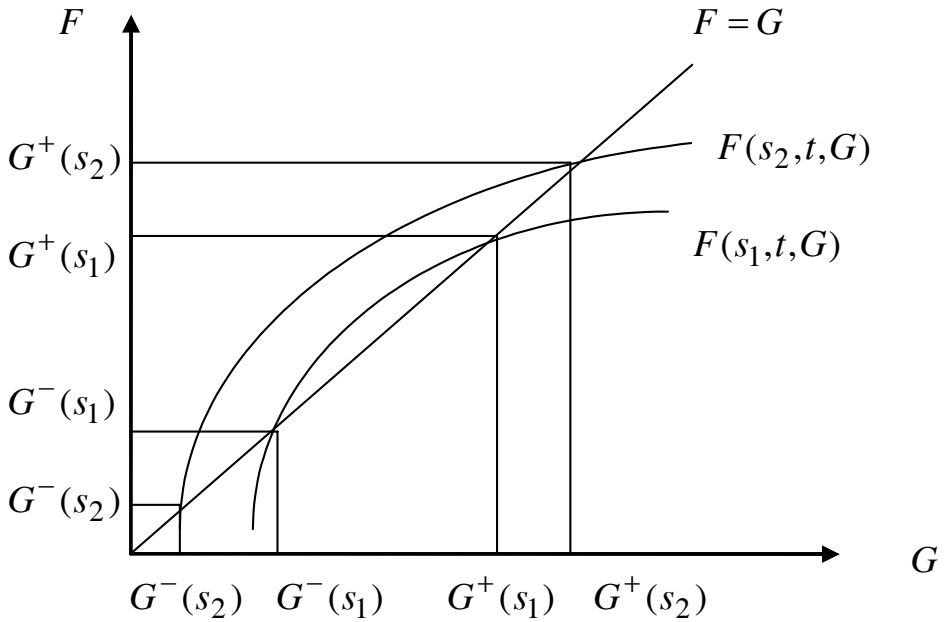

Figure 4.1: **The Case** $F_s > 0,\ F_G > 0, F_{GG} < 0,\ s_1 < s_2.$

This case is illustrated in Figure 4.1. For each level of s, we have two equilibria, one *low* (superscripted with a $-$ sign) and the other *high* (superscripted with a $+$ sign); we have omitted the argument t for greater visual clarity. Starting at the high equilibrium, aggregate donation to charity can be increased by increasing tax exemption for charitable contribution, from s_1 to s_2. However, at the low equilibrium, increasing tax allowances reduces aggregate donations even further.

A better policy would be a one-off lumpsum grant from the Government, D, directly to the charity. We will illustrate, in Section 6, that this moves the economy permanently

[16]In a different setting, in Cooper and John (1998), individual team members choose their effort levels, strategically, given the effort levels of other team members. They show that, in a symmetric Nash equilibrium, strategic complementarity in effort levels among the team members is a necessary condition for multiple equilibria.

from the low equilibrium to the high equilibrium. Once the economy reaches the high equilibrium, the socially optimal public policy can be implemented. For the moment, the following heuristic explanation might suffice.

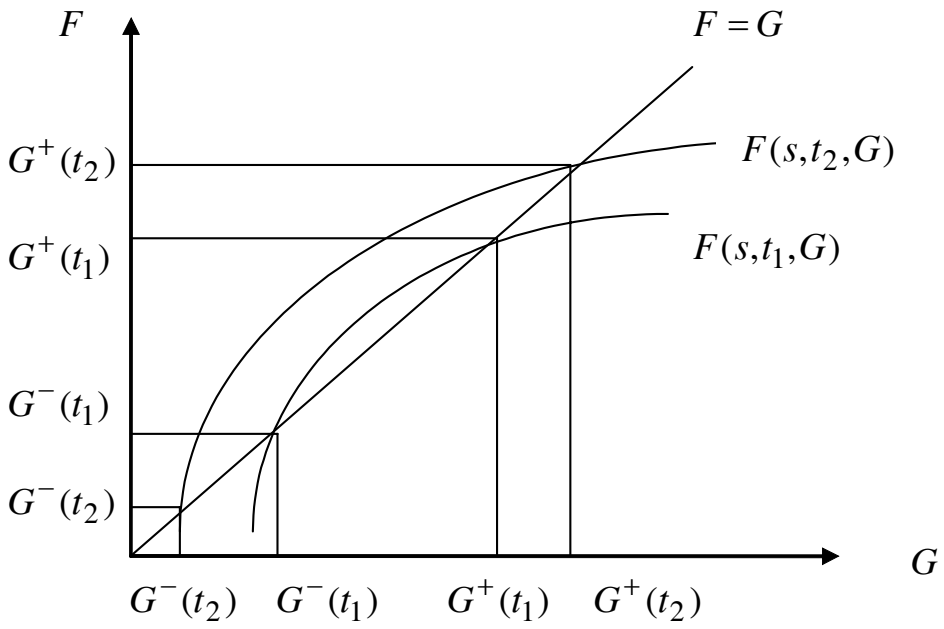

Figure 4.2: **Multiple equilibria when** $F_t > 0$, $F_G > 0, F_{GG} < 0$, $t_1 < t_2$.

Suppose that the situation is as depicted in Figure 4.2. We start with $t = t_1$ and the *low* equilibrium given by $G^-(t_1)$. Suppose that it is desirable to move the economy to a *high* equilibrium. In the absence of any direct grants from the government, it might be the case that subsidies to private giving, financed by taxation, are counterproductive. This would happen, for instance, if $G_t^- < 0$, $G_s^- < 0$; as is the case in Example 1 of Section 6. It would seem, then, that no government intervention is effective.

Consider, however, an alternative policy in which the Government makes a direct grant to the charity equal to (or greater than) $G^-(t_1)$, financed by an income tax levied at the rate $t = t_2 > t_1$. This implies that the aggregate desire to give to charity is $D + \sum_{i=1}^{n} g^i > G^-(t_1)$; recall our original assumption that $g^i > 0$ for at least one i. Hence, the equilibrium aggregate giving, G, must be greater than $G^-(t_1)$. Since the economy is now on the $F(s, t_2, G)$ locus and $G > G^-(t_1) > G^-(t_2)$ the sole candidate for equilibrium is the *high* equilibrium, $G^+(t_2) > G^+(t_1)$. Once G^+ is established, each individual will make her private giving decision conditional on G^+, thereby raising her own giving to a higher level such that equilibrium beliefs about G^+ become self-fulfilling.

So what should (optimal) public policy look like when the high equilibrium, G^+, is established? It would be welfare improving if the direct public grant, D, can be withdrawn and replaced by an equivalent contribution by the private sector. This is because an equivalent amount contributed by the private sector is welfare improving on account of the warm glow that individuals receive from giving. But why might public policy be able to induce additional private giving at the new equilibrium, G^+, when it could not do so at the old equilibrium, G^-? The reason is that the signs of G_t^+, G_s^+ could be different from those of G_t^-, G_s^-. In particular, it might be the case that $G_t^+ > 0$, $G_s^+ > 0$ (while $G_t^- < 0$, $G_s^- < 0$); as is the case in Example 1 in Section 6.1. In this case, the perverse effects of public policy that arise in the low equilibrium are reversed in the high equilibrium. Hence, once the high equilibrium is established the common-sense comparative statics obtain, so public policy can then be used to induce additional private giving that is consistent with replacing the direct government grant completely. The latter can then be withdrawn. Hence, temporary direct government grants can be welfare improving[17].

B. $F_s > 0,\ F_G > 0, F_{GG} > 0.$

Suppose, as in Figure 4.3, that the economy is at the lowest equilibrium, $G^-(s_1)$. Here, an increase in tax exemptions, from s_1 to s_2, would move the economy to the next, higher, equilibrium, $G^-(s_2)$; but at an increased tax rate.

A much better policy could be a grant from the government directly to the charity. This would move the economy to the high equilibrium. The situation is depicted in Figure 4.4. The intuition is identical to the one for Case A, hence, we will only sketch the outline of the argument.

Start with the *low* equilibrium, $G^-(t_1)$. In this case, subsidies to private giving, financed by taxation, are effective in raising the equilibrium from $G^-(t_1)$ to $G^-(t_2) > G^-(t_1)$ (see Figure 4.4). However, as pointed out above, this entails a permanently higher tax rate. Suppose now that the government gives a direct grant to charity, D, equal to at least $G^-(t_1)$, and finances the grant by raising taxes at the rate $t_2 > t_1 \geq 0$. Therefore, the aggregate desire to give to charity is $D + \sum_{i=1}^{n} g^i > G^-(t_1)$. Because $t = t_2$ we are now on the $F(s, t_2, G)$ locus, hence, the only possible candidate for equilibrium is the *high* equilibrium, $G^+(t_2)$. Once the high equilibrium is established then the Government can withdraw its direct grant and reduce the tax rate to its original level.

[17]We do not consider direct grants by non-governmental institutions, such as the national lottery in the UK or private charitable trusts and organizations. However, such grants can be accomodated in our framework. These will, in principle, perform a role that is similar to direct government grants except that non-governmental organisations cannot levy taxes.

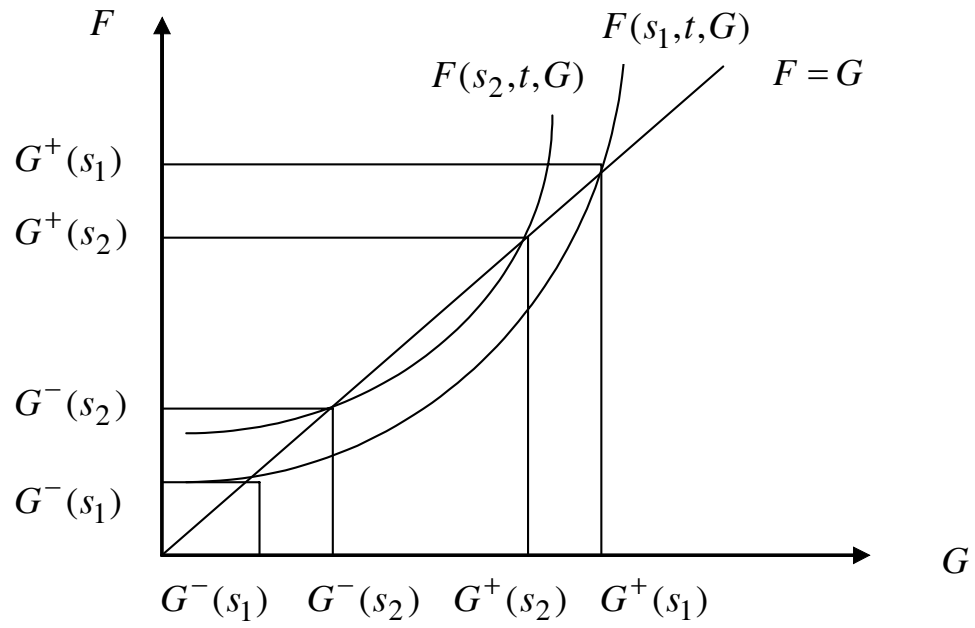

Figure 4.3: **The Case** $F_s > 0,\ F_G > 0, F_{GG} > 0,\ s_1 < s_2$.

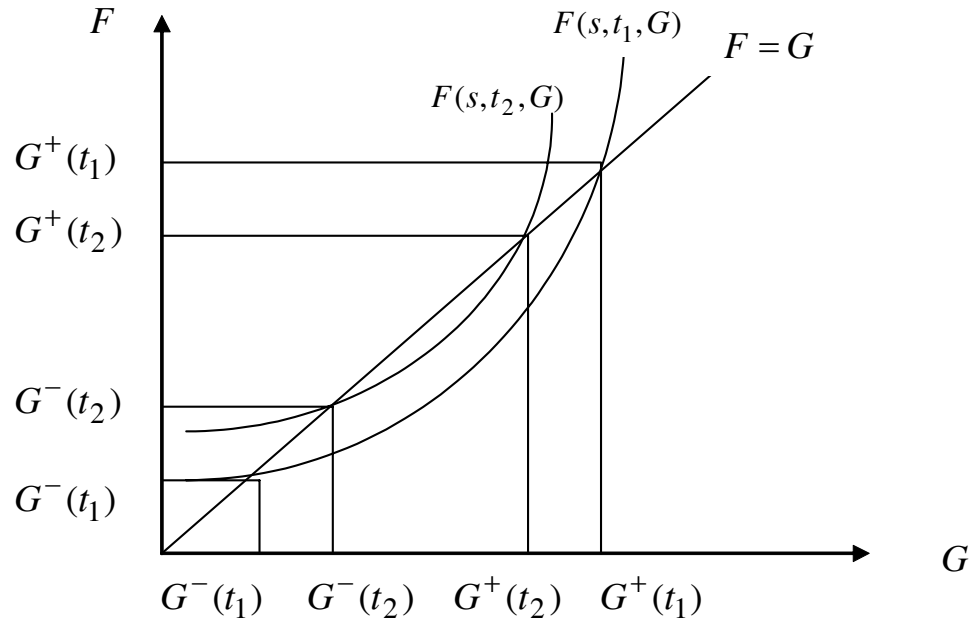

Figure 4.4: **Multiple equilibria when** $F_t > 0,\ F_G > 0, F_{GG} > 0,\ t_1 < t_2$.

5. Welfare Analysis

Substituting $g^i(s,t,G(s,t))$ and $G(s,t)$ in the utility function (2.4) gives the consumer's indirect utility function

$$v^i(s,t) = u^i\left[(1-t)m^i - (1-s)g^i(s,t,G(s,t)), g^i(s,t,G(s,t)), G(s,t)\right] \tag{5.1}$$

Differentiating (5.1), implicitly, using (3.4) and the first order conditions (3.2) (or appealing to the envelope theorem) gives

$$v_s^i = u_1^i g^i + u_3^i G_s \tag{5.2}$$

$$v_t^i = -u_1^i m^i + u_3^i G_t \tag{5.3}$$

where G_s, G_t are given by (3.15), (3.16) respectively.

Substituting the indirect utility function from (5.1) into (2.9) we get the government's indirect utility function

$$V(s,t) = U\left(v^1(s,t), v^2(s,t), ..., v^n(s,t)\right) \tag{5.4}$$

Differentiating (5.4) using the chain rule, and (5.2), (5.3), we get

$$V_s = \sum\nolimits_{i=1}^{n} U_i u_1^i g^i + G_s \sum\nolimits_{i=1}^{n} U_i u_3^i \tag{5.5}$$

$$V_t = -\sum\nolimits_{i=1}^{n} U_i u_1^i m^i + G_t \sum\nolimits_{i=1}^{n} U_i u_3^i \tag{5.6}$$

Propositions 3, 4 and 5 below derive the optimal mix between private and public giving to charity in different cases. We shall use these propositions extensively in Section 6; in particular, these propositions will be crucial to determine the optimal public policy at different equilibria.

Proposition 3 *: If $G_t \leq 0$ then $s = t = 0$. Thus, all giving to charity is private giving and there are no direct Government grants to charity. Conversely, if at a social optimum $t > 0$ then $G_t > 0$.*

Proposition 3 implies that in an optimum characterized by $G_t \leq 0$, no government intervention is needed, *warm glow* and/ or *altruism* suffice to maximize social welfare.

Proposition 4 *: If at an optimum $G_s \geq 0$ then s attains its maximum possible value and the government makes no direct contributions to charity i.e. $D = 0$. Conversely, if a social optimum involves positive government donations to charity i.e. $D > 0$ then, necessarily, $G_s < 0$.*

Proposition 4 shows that if subsidies are effective then no direct government grant is needed. This is because when *private* donations replace an identical amount of *public* donations welfare improves on account of the warm glow received by private givers.

Proposition 5 *: If at a social optimum $F_s \geq 0$ and $F_G < 1$, or if $F_s \leq 0$ and $F_G > 1$, then all contributions to charity come from individual private donations. No direct government grant is involved.*

6. Examples

In this section we present two illustrative examples of the general theoretical model. In the first example, charitable contributions provide income to consumers who, otherwise, have no income. Hence, private contributions supplement the public, redistributive, activities of the Government. In the second example, charitable contributions finance public good provision. Formally, because contributions to the public good are also voluntary, the basic framework of our paper applies to the second example as well.

6.1. Example 1: Redistribution and contribution to charity

We consider an economy where some consumers have no income. Their consumption expenditure is financed entirely by either charitable donations made by other 'caring' consumers with positive income and/ or by direct grants, financed through taxation, made by the Government.

Let there be n consumers in total. Of these, p consumers, $0 < p < n$, have positive income ($m^i > 0$); they are indexed by $i = 1, 2, ..., p$. The other $n - p$ consumers have no income ($m^i = 0$); they are indexed by $i = p + 1,\ p + 2, ..., n$. All incomes are costlessly verifiable by the Government.

The aggregate of all donations to charity (private and public), G, is divided equally among the consumers with no income; so each recipient gets $\frac{G}{n-p}$.

Of the p consumers with positive incomes, k, $0 < k \leq p$, care about the plight of those with no income. Each of these *caring* consumers has the utility function

$$u^i = \ln c^i + a^i g^i G; \quad a^i > 0; i = 1, ..., k \tag{6.1}$$

where

$$\frac{1}{a^i G} < \frac{1-t}{1-s} m^i; \quad i = 1, ..., k \tag{6.2}$$

The other $p - k$ consumers have positive income but do not care about those with no income. The utility function of the latter two groups of consumers (the non-caring with positive income and those with no income) is given by

$$u^i = \ln c^i; \quad i = k + 1, k + 2, ..., p, p + 1, ..., n \tag{6.3}$$

From Lemma 1, (6.1), (6.3) it follows that g^i, G are strategic complements.

Let m be the aggregate income of the caring consumers. Then, for $k < p$

$$m = \sum_{i=1}^{k} m^i < \sum_{i=1}^{p} m^i = \sum_{i=1}^{n} m^i = M \tag{6.4}$$

and, for $k = p$,

$$m = M \tag{6.5}$$

Also, let

$$A = \sum_{i=1}^{k} \frac{1}{a^i} > 0 \tag{6.6}$$

Proposition 6 *: The only economically interesting cases occur when*

$$[m + t\,(M - m)]^2 > 4(1 - s)A$$

in which case, we have two distinct real positive equilibria

$$0 < G^- < G^+$$

These are given by

$$G^{\pm} = \frac{1}{2}\left[m + t\,(M - m) \pm \sqrt{[m + t\,(M - m)]^2 - 4(1 - s)A}\right].$$

The resulting equilibria are as in Figure 4.2. In the figure, t_1, t_2 are two feasible tax rates such that $t_1 < t_2$.

Suppose an economy is at the low equilibrium, G^-, and also suppose that it is socially desirable to move it to the high equilibrium, G^+. How can this be done? From (10.17) and (10.18) in the appendix: $G_t^- \leq 0$. Hence, from Proposition 3, it would appear that the best policy is no intervention i.e. $s = t = 0$, leaving the economy at the low equilibrium, G^-. However, an alternative policy is possible, as we shall now describe; see Figures 6.1, 6.2.

Set the tax rate t as follows.

$$t = G^-(0)/M \tag{6.7}$$

Since $G^-(0)$ is an equilibrium, it must be affordable. Hence, $0 < G^-(0) < M$. It follows that $0 < t < 1$ and, hence, feasible. Set

$$s = 0 \tag{6.8}$$

$$D = tM = G^-(0) \tag{6.9}$$

Thus, the Government gives a direct grant equal to G^- $(0)(D > G^-(0)$ will also work) financed from an income tax. Since $g^i > 0$ for some i (i.e. $i = 1, 2, ..., k$) it follows, from

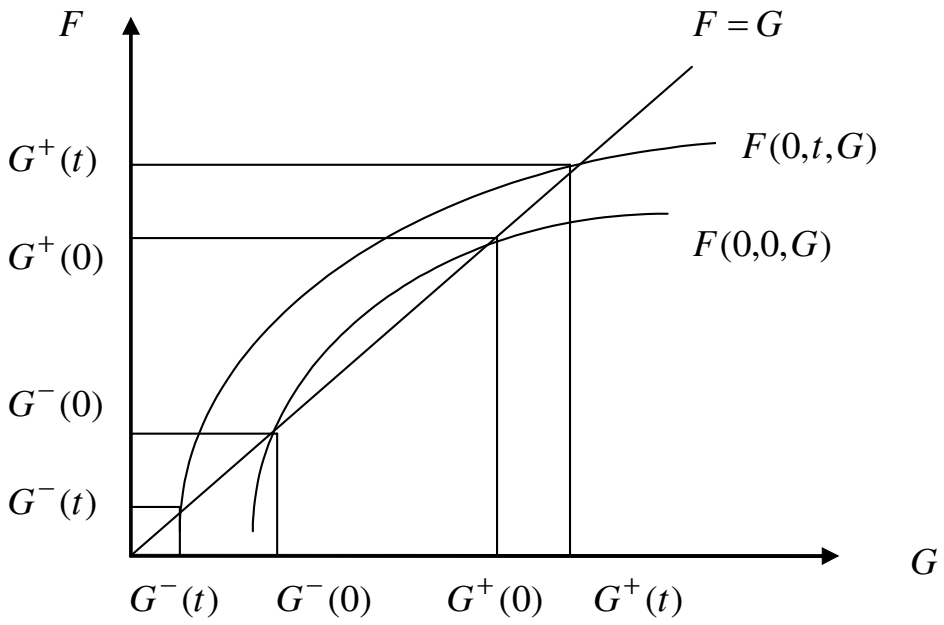

Figure 6.1: Multiple equilibria when $F_t > 0$, $F_G > 0$, $F_{GG} < 0$.

(2.10), that $G > D = G^-(0)$. Hence, because we are on the $F(0, t, G)$ locus (see Figure 6.1) and the equilibrium aggregate donation $G > G^-(0) > G^-(t)$, the only possible candidate for equilibrium is

$$G = G^+(t). \tag{6.10}$$

Once the economy is at the good equilibrium, G^+, s,t can be adjusted to their socially optimum values. We now address this issue.

6.1.1. Socially optimal public policy at the new equilibrium

At the low equilibrium, G^-, private individuals could not be induced to make additional contributions because of the perverse comparative static effects, $G_s^- < 0$, $G_t^- < 0$. These comparative static effects can be reversed at the high equilibrium. We have two cases to consider.

1. If $k < p$, so that not all consumers with positive income contribute to charity, then, from (6.4), $m < M$. Hence, from (10.16), (10.17) in the appendix $G_s^+ > 0$, $G_t^+ > 0$ i.e. the comparative static effects are reversed at the high equilibrium; see Figure 6.2. Depending on parameter values, the optimal tax rate may be positive, in which case it can be found by setting $V_t = 0$ in (5.6). From (10.8), (10.15) in the appendix

and Proposition 5 it follows that $D = 0$. Thus, once the economy has moved from G^- to G^+, the direct grant from the Government to charity should be phased out. In the new, socially optimal, equilibrium all the revenue is used to subsidize private donations to charity. The intuition is that direct Government grants do not generate warm glow, which private giving does. Hence, if the direct grant could be withdrawn but private individuals induced to contribute an equivalent amount (and so derive warm glow) then society's welfare is increased.

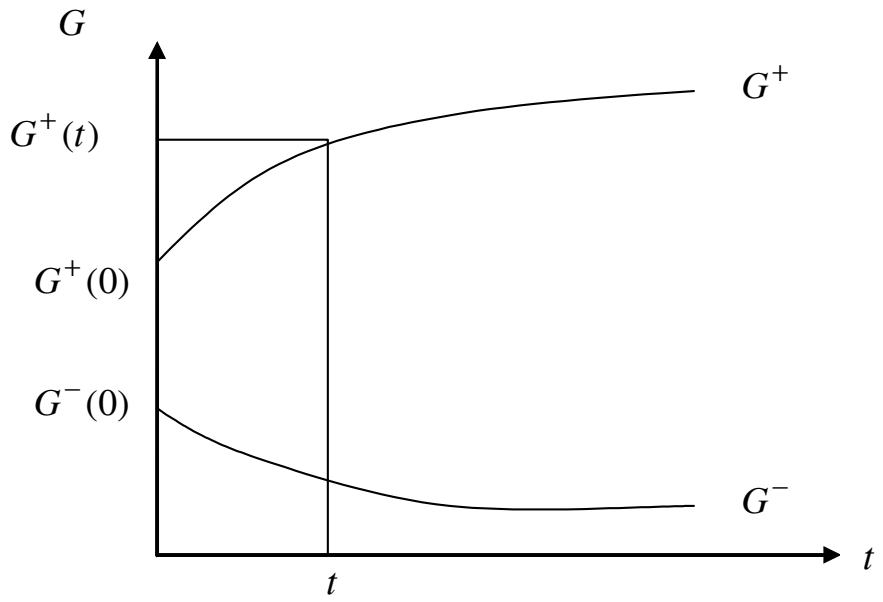

Figure 6.2: **The response of equilibrium G to the tax rate.**

2. If $k = p$ (so that all consumers with positive income contribute to charity) then, from (6.5), $m = M$. Hence, from (10.18), $G_t^+ = 0$. It follows, from Proposition 3, that $s = t = 0$. Thus, once the (one-off) grant to charity (financed by an income tax) has shifted the economy from the bad equilibrium, G^-, to the good equilibrium, G^+, no further government intervention is needed.

6.1.2. A numerical illustration

Consider an economy with $n = 1900$ consumers. Of these, 1000 do not have an income of their own. They are supported by charity. The other $p = 900$ consumers each has a positive income $m^i = 1$; $i = 1, 2, , ..., 900$. Of these $k = 450$ get a *warm glow* from giving to charity. Specifically, $a^i = 0.01$ for $i = 1, 2, , ..., 450$. The others do not derive warm glow

from giving to charity, i.e., $a^i = 0$ for $i = 451, 452, , ..., 1900$. We thus have

$$m = \sum\nolimits_{i=1}^{450} m^i = 450$$

$$M = \sum\nolimits_{i=1}^{900} m^i = \sum\nolimits_{i=1}^{1900} m^i = 900$$

$$A = \sum\nolimits_{i=1}^{450} \frac{1}{a^i} = \frac{450}{0.01} = 45000$$

Initially, assume $s = t = 0$. Then $G^- = 150$, $G^+ = 300$. Suppose the locus passing through G^- is socially inferior to that passing through G^+. How can government policy shift the economy onto the better locus?

Since $G_s^- < 0$, $G_t^- < 0$, the best policy would appear to be no intervention: $s = t = 0$. However, there is an alternative. The government sets $s = 0$, $t = 1/6$. This raises a total tax revenue equal to $tM = 900/6 = 150$. Since $G_t^- < 0$, it follows that $G^- < 150$ at $t = 1/6$. The government makes a direct grant $D = 150$ to the charity. Since $g^i > 0$, $i = 1, 2, , ..., 450$, we must have $G = D + \sum\nolimits_{i=1}^{450} g^i > D = 150 > G^-$. Hence, $G = G^+$. Once the economy is on the G^+ locus, s, t can be given their optimal values.

6.2. Example 2: Voluntary contributions to a public good

Individuals often voluntarily contribute to, and directly use, several kinds of public goods such as health services and education[18]. Suppose that the utility function of consumer i, $i = 1, 2, ..., n$ is given by

$$u^i = u^i\left(c^i, g^i, G\right) = \left(1 - a^i\right) \ln\left(c^i - \frac{b^i}{G}\right) + a^i \ln g^i \tag{6.11}$$

where

$$0 < a^i < 1;\ b^i > 0 \tag{6.12}$$

$$\frac{b^i}{G} < (1 - t)m^i \tag{6.13}$$

Condition (6.13) guarantees that consumer i has enough disposable income, $(1 - t)m^i$, to sustain a level of private consumption expenditure, c^i, greater than $\frac{b^i}{G}$ and also a positive level of donation to charity, g^i. It is straightforward to check that $u_1^i > 0$, $u_2^i > 0$, $u_3^i > 0$.

This example can be given the following interpretation. Private (voluntary) contributions to public goods, $\sum\nolimits_{i=1}^{n} g^i$, plus public contribution, D, financed from taxation, provide the necessary infrastructure for private consumption. Higher utility is derived, from a given level of private consumption expenditure, c^i, the higher is aggregate expenditure on infrastructure, $G = D + \sum\nolimits_{i=1}^{n} g^i$.

[18] For the US, education, health and human services account for the greatest proportion of private giving after religion; see Table 3 in Andreoni (2004).

From Lemma 1, (6.11) it follows that g^i, G are strategic complements. Let

$$B = \sum\nolimits_{i=1}^{n} a^i m^i + t \sum\nolimits_{i=1}^{n} \left(1 - a^i\right) m^i \tag{6.14}$$

$$C = \sum\nolimits_{i=1}^{n} a^i b^i \tag{6.15}$$

Proposition 7 *: The only economically interesting cases occur when*

$$B^2 > 4C$$

in which case, we have two distinct real positive equilibria

$$0 < G^- < G^+$$

These are given by

$$G^{\pm} = \frac{1}{2}\left[B \pm \sqrt{B^2 - 4C}\right].$$

Hence, the economy has two equilibria. One characterized by low voluntary contributions to the public good causing low aggregate spending on the public good infrastructure, G^-. From (6.11), we see that, to achieve any specific utility level, high private consumption expenditure is needed. From the budget constraint, (2.3), we see that less income can be contributed to the public good, perpetuating the low expenditure on infrastructure.

The other equilibrium is characterized by high contributions to the public good, causing high aggregate expenditure on infrastructure, G^+. In turn, this implies that relatively less private consumption expenditure is needed to reach any specific utility level. Hence, relatively more income is left over to donate to charity, perpetuating high expenditure on infrastructure.

The resulting equilibrium is as in Figure 4.2. In the figure, t_1, t_2 are two feasible tax rates such that $t_1 < t_2$.

Suppose an economy is at the low contribution to the public good/ poor infrastructure equilibrium G^-; and suppose it is socially desirable to move the economy to the high contribution/ good infrastructure equilibrium G^+. How can this transition be achieved? By (10.28), $G_s = 0$ so subsidies are ineffective.

From (10.35) $G_t^- < 0$, so, from Proposition 3, it would appear that, at the low equilibrium G^-, the best feasible policy is no intervention: $s = t = 0$, leaving the economy at the low equilibrium G^-.

However, an alternative policy is possible, as we shall now describe; see Figure 6.1, 6.2.

Set a tax rate t, given by

$$t = \frac{G^-(0)}{M} \tag{6.16}$$

Since $G^{-}(0)$, being an equilibrium, must be affordable, i.e., $0 < G^{-}(0) < M$, it follows that $0 < t < 1$ and, hence, feasible. Set

$$s = 0 \tag{6.17}$$

$$D = tM = G^{-}(0) \tag{6.18}$$

i.e. the government gives a direct grant to public good provision equal, at least, to $G^{-}(0)$, and financed from an income tax. Since $g^{i} > 0$ it follows, from (2.10), that $G > D = G^{-}(0)$. Since we are now on the $F(0, t, G)$ locus (see Figure 6.1) and the equilibrium aggregate donation $G > G^{-}(0) > G^{-}(t)$, hence, the only possible equilibrium is

$$G = G^{+}(t) \tag{6.19}$$

Once the economy is on the high contribution/ good infrastructure equilibrium, s and t can be adjusted to their socially optimal values. But what are the socially optimal values? We now address this question.

6.2.1. Socially optimal public policy at the new equilibrium

In view of (10.33), (10.35) and (10.37) in the appendix, the graphs of $G^{\pm}$ are as in Figure 6.2. Once the economy has moved to the new equilibrium, G^{+}, the direct grant from the government to the public good can be phased out. The reason is that since Government grants and individual giving are imperfect substitutes, hence, a drawback of grants is that *warm glow* does not accrue to individuals. Thus, welfare can be improved by withdrawing the grant and getting individuals to contribute instead.

In the low equilibrium, G^{-}, we have seen above that the optimal policy solution is $s = t = 0$. However, at the high equilibrium, $G_{t}^{+} > 0$ (see (10.35) or Figure 6.2) and so the comparative static results are reversed. The optimal tax rate, which can be found from (5.6), balances the loss in private consumption against the gain arising from the additional amount of the public good. Also, from (5.5), $V_{s} > 0$, hence, it is welfare improving to provide additional subsidies. Thus, all tax revenues are used to finance subsidies on charitable donations. In the socially optimal solution at the high equilibrium, G^{+}, therefore, $s > 0$, $t > 0$ while $D = 0$ (see Proposition 4).

6.2.2. Numerical Illustration

As a numerical illustration, consider an economy of $n = 50$ identical consumers each with income $m^{i} = 1$. Choose $a^{i} = 0.1$ and $b^{i} = 0.8$. Suppose that initially, $s = t = 0$. Then (6.14), (6.15), (10.31) give

$$G^{-} = 1 \tag{6.20}$$

$$G^+ = 4 \tag{6.21}$$

and the feasibility condition (6.13) is satisfied.

Suppose that the economy is, initially, at the low equilibrium $G^- = 1$. If the move to the high equilibrium is considered desirable, then, from (6.16) we get that the required tax rate, t, is

$$t = \frac{G^-}{M} = \frac{1}{50} = 0.02 \tag{6.22}$$

and the feasibility condition (6.13) still holds at this tax rate. The government uses its entire tax revenue $tM = 1 = G^-$ to make a contribution $D = G^- = 1$ to the public good. Since $g^i > 0$, $G = D + \sum_{i=1}^{n} g^i > 1 = G^-$. Hence, the only candidate for equilibrium is $G = G^+$. Once the economy is on the high locus G^+, the policy parameters s, t can be adjusted to their socially optimal values. This will involve the phasing out of the direct grant. Once the final position of the, new, socially optimal, equilibrium is established, the government uses all tax revenues to subsidize voluntary contributions to the public good. The direct grant, here, is only a temporary measure to shift the economy from the low, G^-, locus to the high, G^+, locus.

7. Dynamics

In this paper we have concentrated exclusively on equilibrium analysis. For certain purposes, a rigorous analysis of the time path of adjustment may be required. This, however, will involve specifying a precise list of the information sets of each of the players, the updating rules, and a learning process[19]. This, however, lies beyond the scope of the current paper.

The reader of this paper might have wondered about the stability properties of the equilibria. To study the stability of equilibria, an adjustment process needs to be specified. A popular class of adjustment processes in economics is given by the following *partial adjustment scheme*. Let G^* be an equilibrium, $G(k)$ the value of G in period k and λ a constant that satisfies $0 \leq \lambda \leq 1$. Then the adjustment process is given by

$$G(k+1) = (1-\lambda)\, G^* + \lambda F\left(s, t, G(k)\right) \tag{7.1}$$

Stability of G^* is guaranteed if $|\lambda F_G\left(s, t, G^*\right)| \leq \beta < 1$, which is ensured by choosing a sufficiently small λ. In examples 1 and 2 in Section 6, and in the special case of (7.1) with $\lambda = 1$, G^- is unstable while G^+ is stable. In general, however, and without

[19] At the moment there is a lack of consenus in the profession about the appropriate model of learning and there is 'too large' a variety of models to choose from. These include reinforcement learning, learning through fictitious play, learning direction theory, Bayesian learning, imitation learning, experience weighted attraction learning, etc. For a survey of the experimental evidence on these theories, see Camerer (2003)

precise specification, there is no presumption of stability/ instability associated with these equilibria.

8. Strategic Giving

We have assumed in this paper that when any individual i makes her decision to give to charity, g^i, she takes as given aggregate contributions, G. Our view is that, typically, the contribution of each individual is negligible relative to the total budget of the charity. In making their decisions individuals do not imagine that their gifts will make an appreciable difference to the charity's total budget. This, however, is in contrast to the standard literature which makes the converse assumption i.e. in making her decision, g^i, the individual takes as given $G^{-i} = G - g^i$, the contributions of all others, and behaves strategically with respect to all others.

One would expect that, as the number of individuals increases, the results of the two approaches should converge. Here, we show this to be the case for our two examples in Section 6[20].

8.1. Nash Equilibria for Example 1

We first derive the symmetric Nash equilibria (SNE) for Example 1 in Section 6. Suppose that all *caring* individuals with positive income are identical. Using the same parameter values as in the numerical example in subsection 6.1.2 we have $a^i = a = 0.01$, $m^i = m = 1$, $i = 1, 2, ..., 450$. For simplicity, we report the case $s = t = D = 0$. Other cases are similar.

The optimization problem of the i^{th} such individual is

$$\underset{\langle g^i | G^{-i} \rangle}{Maximize} \quad u^i = \ln c^i + 0.01 g^i \left(g^i + G^{-i} \right) G \tag{8.1}$$

subject to the individual budget constraint (2.3). We have replaced total aggregate giving, G, by $g^i + G^{-i}$. In making her decision, the consumer now takes as given the contribution of all others, G^{-i}. The first order condition to the maximization problem in (8.1) is

$$\frac{1}{1 - g^i} = 0.01(G + g^i) \tag{8.2}$$

In a SNE, $g^i = kG$ where $k = 450$. Substituting $g^i = 450G$ in (8.2) we get the following quadratic equation in G

$$4.51G^2 - 2029.5G + 202500 = 0 \tag{8.3}$$

Solving for the two Nash equilibria G^{N-}, G^{N+} we get

$$G^{N-} = 149.33 \tag{8.4}$$

[20] For a more rigourous treatment of the more general case, see al-Nowaihi and Dhami (2005).

$$G^{N+} = 300.66 \tag{8.5}$$

For non-strategic giving, the two equilibria, given in subsection 6.1.2, were $G^{-} = 150$, $G^{+} = 300$. Comparing to (8.4), (8.5) we find that the equilibria are virtually identical, as claimed earlier, although the number of givers, 450, is relatively small.

8.2. Nash Equilibria for Example 2

We show here, analytically, the effect on the equilibrium magnitudes as $n \to \infty$ as well as simulation exercises for smaller values of n. To enable the derivation of analytical results, rewrite (6.11) as

$$u^i\left(c^i, g^i, G\right) = \left(1 - a^i\right) \ln\left(c^i - \frac{b^i/n}{G/n}\right) + a^i \ln g^i \tag{8.6}$$

Define the constant $d^i = b^i/n$ and rewrite (8.6) as[21]

$$u^i\left(c^i, g^i, G\right) = \left(1 - a^i\right) \ln\left(c^i - \frac{d^i}{(G/n)}\right) + a^i \ln g^i \equiv w^i\left(c^i, g^i, \frac{G}{n}\right) \tag{8.7}$$

Substituting out c^i in (8.7) using the budget constraint (2.3) we get

$$w^i\left((1-t)\, m^i - (1-s)\, g^i, g^i, \frac{G}{n}\right) = \left(1 - a^i\right) \ln\left((1-t)\, m^i - (1-s)\, g^i - \frac{d^i}{(G/n)}\right) + a^i \ln g^i \tag{8.8}$$

8.2.1. General Case

Under non-strategic giving, each consumer chooses her contribution, given the per capita aggregate contribution, G/n. Therefore, the problem of the i^{th} consumer is to maximize (8.8) given G. The first order condition is

$$-(1-s)w_1^i + w_2^i = 0 \tag{8.9}$$

Under strategic giving, on the other hand, each consumer chooses her contribution, given the contribution of all others, G^{-i}. Therefore, the problem of the consumer is to maximize (8.8) given G^{-i}. The first order condition is

$$-(1-s)w_1^i + w_2^i + \frac{1}{n} w_3^i = 0 \tag{8.10}$$

[21] Note that d^i is now a parameter of the model. This reformulation is necessary because as n increases, G increases and so effectively the parameter b^i in the original formulation falls relative to G. In other words, such a replication of the economy alters the underlying model, which is not admissible. The reformulation in terms of the parameter d^i, however, is not subject to this problem.

Comparing (8.9), (8.10) we see that, because w_3^i is bounded, as $n \to \infty$, the first order conditions for strategic and non-strategic giving coincide. Note that this result does not depend on any particular function form, hence, it is completely general.

It remains to show, as we did in subsection 8.1, that even for smaller n, the equilibria of the two models are reasonably close. To keep the exposition simple we focus on symmetric equilibria below. Set $d^i = d$, $a^i = a$ and $m^i = m$ for all i. Furthermore, to simplify the exposition we report the case $s = t = D = 0$.

8.2.2. Symmetric Nash Equilibria

Using (8.10), the first order condition in the case of a SNE is

$$\frac{1-a}{m-g-\frac{nd}{G}}\left[\frac{nd}{G^2}-1\right]+\frac{a}{g}=0 \tag{8.11}$$

In a SNE, $g = \frac{G}{n}$. Substituting $G = ng$ in (8.11) we get the following quadratic equation for individual donation, g

$$g^2 - amg + d\left[a - \frac{1-a}{n}\right] = 0 \tag{8.12}$$

Solving (8.12) we get the two solutions for individual giving as

$$g^{N\pm} = \frac{1}{2}\left[ma \pm \sqrt{m^2a^2 - 4d\left(a - \frac{1-a}{n}\right)}\right] \tag{8.13}$$

where superscript 'N' denotes the Nash outcome

8.2.3. Symmetric Non-Strategic Equilibria

In this case the consumer takes as given aggregate giving. Using (8.9), the first order condition is

$$\frac{(1-a)}{m-g-\frac{nd}{G}} = \frac{a}{g} \tag{8.14}$$

In a symmetric equilibrium $g = G/n$. Substituting $G = ng$ in (8.14) we get a quadratic equation in g

$$g^2 - amg + ad = 0 \tag{8.15}$$

which has two solutions g^- and g^+ given by

$$g^{\pm} = \frac{1}{2}\left[am \pm \sqrt{a^2m^2 - 4ad}\right] \tag{8.16}$$

Clearly, from (8.13), (8.16), $g^{N+} \to g^+$ and $g^{N-} \to g^-$, as $n \to \infty$.

8.2.4. Simulations

In this subsection we address, numerically, the question of whether the results under strategic and non-strategic giving converge as the number of givers increases? From (8.16), the equilibrium level of individual giving in the non-strategic equilibrium is independent of the number of givers, n. Substituting $m = 1$, $a = 0.1$, $b = 0.8$, $d = \frac{b}{50} = 0.016$ in (8.16), it can be checked that for any n

$$g^- = 0.2 \tag{8.17}$$

$$g^+ = 0.8 \tag{8.18}$$

Below, we report the simulation results for the optimal individual giving in the strategic case, $g^{N\pm}$, as n increases, using $m = 1$, $a = 0.1$, $d = 0.016$.

TABLE-I : Multiple Nash Equilibria as the Number of Givers Varies

n	100	500	1000	$10,000$
g^{N-}	0.0177	0.0195	0.0198	0.02
g^{N+}	0.0823	0.0805	0.0802	0.08

It is evident that even for relatively small values of n, g^{N-} and g^{N+} converge rapidly to g^- and g^+. This seems to accord with the actual size of a typical individual donation relative to the budget of a charity. Furthermore, it allows one to conduct comparative static results that would be difficult in the case of strategic giving, even if restricted to the case of a symmetric Nash equilibrium.

9. Conclusions

Uncoordinated individual giving leads to the possibility that there could be multiple equilibria in private giving. Furthermore, these equilibria can be Pareto ranked. An interesting question then is the following. Given that a society is stuck at a *low* equilibrium, can policy help it to attain a *high* equilibrium?

Coordination problems and the resulting multiple equilibria are endemic in economics, however, we typically lack a means and/ or a method of choosing among the possible equilibria. In the context of private philanthropic activity, we show that direct grants to charities, made by the government, can enable an economy to achieve a *high* equilibrium.

We also examine the optimality of alternative mix of private and public contributions to charity. We show, for some parameter values, that additional incentives to giving, such as those arising from the tax deductibility of private contributions, can actually worsen matters by reducing the aggregate of private contributions in equilibrium[22]. For

[22]See Figure 3.2, G^- in Figure 4.1 and G^+ in Figure 4.3.

other parameter values, however, and we provide sufficient conditions[23], giving to charity should be entirely funded by private individual contributions, possibly subsidized through taxation.

Throughout we focus on equilibrium analysis. The issues involved with the dynamics of time paths involve fundamental questions about the precise learning mechanisms to be used. Although progress on learning mechanisms is being made and in due course such mechanisms may enrich our model, currently such issues lie beyond the scope of this paper.

While we view the principal contribution of this paper as providing new insights into the literature on the economics of philanthropy it also enriches our understanding of multiple equilibria in general. In particular, the role of public policy in "multiple equilibria engineering", namely, policy induced jumps from a bad to a good equilibria, is not very well understood. We show precisely how, in the context of voluntary giving and an equilibrium analysis, such engineering might take place.

10. Appendix

Proof of Proposition 1: Let

$$H(s,t,G) = G - F(s,t,G) \tag{10.1}$$

We know from (3.10) that $0 < F(s,t,G) < M$, hence,

$$H(s,t,0) = -F(s,t,0) < 0$$

$$H(s,t,M) = M - F(s,t,M) > 0.$$

Since $H(s,t,G)$ is continuous, it follows that $H(s,t,G) = 0$ for some $G^* \in (0,M)$ i.e.

$$G^* = F(s,t,G^*)$$

Using (10.1)

$$\frac{\partial H}{\partial G} = 1 - \frac{\partial F}{\partial G} \tag{10.2}$$

From Lemma 1, when g^i, G are strategic substitutes, $\frac{\partial F}{\partial G} \leq 0$, hence, $\frac{\partial H}{\partial G} > 0$ for all possible values of G. Thus, there is a unique equilibrium value of G. On the other hand, from Lemma 1, strategic complementarities ensure that $\frac{\partial F}{\partial G} \geq 0$ (but not $\frac{\partial F}{\partial G} > 1$), hence, $H(.)$ is not monotonic in G so a unique equilibrium is not guaranteed.■

Proof of Proposition 3: Let (s,t) maximize social welfare (5.4). We have assumed that $m^i \geq 0$, with some $m^i > 0$, $u_1^i > 0$, $u_3^i \geq 0$ and $U_i > 0$. If $G_t \leq 0$ then, from (5.6), $V_t < 0$

[23]See Proposition 5.

and it follows that necessarily, $s = t = 0$. The last statement in the proposition is simply the contrapositive of the first.■

Proof of Proposition 4: Let (s, t) maximize social welfare (5.4). We have assumed that $g^i \geq 0$, with some $g^i > 0$, $u_1^i > 0$, $u_3^i \geq 0$ and $U_i > 0$. If $G_s \geq 0$ then the first order condition (5.5) implies that $V_s > 0$ and so s attains its maximum possible value. Recall from subsection 2.1 that the maximum value of s is bounded away from unity. The consequence of $s = 1$ is that, from (2.3), the price of giving is zero and so any individual with $u_2^i > 0$ would like to give an infinite amount to charity. Since individual private giving can be increased substantially by decreasing its price, it is best to channel all giving privately because of the additional benefit arising to each individual from warm glow and, therefore, $D = 0$. The last statement in the proposition is simply the contrapositive of the first.■

Proof of Proposition 5: Proposition 5 is an immediate consequence of (3.15) and Proposition 4. ■

Proof of Proposition 6: Maximizing the utility (6.1) subject to the budget constraint (2.3) gives

$$g^i(s, t, G) = \frac{1-t}{1-s} m^i - \frac{1}{a^i G}; \quad i = 1, ..., k \tag{10.3}$$

$$c^i(s, t, G) = \frac{1-s}{a^i G}; \quad i = 1, ..., k \tag{10.4}$$

From (6.2) and (10.3) we see that $g^i(s, t, G) > 0$ and from (10.4) $c^i(s, t, G) > 0$. On the other hand, from (6.3) we get that

$$g^i \equiv 0; \quad i = k+1, k+2, ..., n \tag{10.5}$$

From (3.9), (6.4) - (6.6), (10.3), the aggregate desire to give to charity is

$$F(s, t, G) = m + t(M - m) - \frac{1-s}{G} A \tag{10.6}$$

hence,

$$F_t = M - m \geq 0. \tag{10.7}$$

The inequality in (10.7) is strict for $m < M$, i.e., $k < p$.

$$F_s = \frac{A}{G} > 0 \tag{10.8}$$

$$F_G = (1-s)\frac{A}{G^2} > 0 \tag{10.9}$$

$$F_{GG} = -2(1-s)\frac{A}{G^3} < 0 \tag{10.10}$$

From (3.14) and (10.6), an equilibrium G, must satisfy the quadratic equation

$$G^2 - [m + t\,(M - m)]\,G + (1 - s)A = 0 \tag{10.11}$$

The quadratic equation in (10.11) has the solutions

$$G^{\pm} = \frac{1}{2}\left[m + t\,(M - m) \pm \sqrt{[m + t\,(M - m)]^2 - 4(1 - s)A}\right] \tag{10.12}$$

For real roots we need $[m + t\,(M - m)]^2 \geq 4(1-s)A$. If $[m + t\,(M - m)]^2 = 4(1-s)A$ then, from (10.9), $F_G = 1$. From (3.15), (3.16) it would follow that G_s, G_t are undefined. Hence, the only economically interesting cases occur when,

$$[m + t\,(M - m)]^2 > 4(1 - s)A \tag{10.13}$$

in which case we have two distinct real positive equilibria

$$0 < G^- < G^+. \tag{10.14}$$

Using the fact that for real numbers a, b, $a > b$: $\sqrt{a - b} > \sqrt{a} - \sqrt{b}$, as well as (10.9) and (10.12) - (10.14), we get

$$G = G^+ \Rightarrow F_G < 1; \quad G = G^- \Rightarrow F_G > 1 \tag{10.15}$$

From (3.15), (3.16), (10.7), (10.8), (10.15) we get

$$G_s^+ > 0, \quad G_s^- < 0 \tag{10.16}$$

$$G_t^+ > 0, \quad G_t^- < 0 \text{ (for } m < M, \text{ i.e., } k < p) \tag{10.17}$$

$$G_t^{\pm} = 0 \text{ (for } m = M, \text{ i.e., } k = p) \tag{10.18}$$

From (3.17), (10.7)

$$G_{tt}\,(s, t) = \frac{F_t F_{GG} G_t}{(1 - F_G)^2}$$

Since $F_{GG} < 0$, $F_t > 0$, $(1 - F_G)^2 > 0$, we get that the sign of G_{tt} is the reverse of that of G_t. From (10.17), (10.18) we then get

$$G_{tt}^+ < 0 \text{ and } G_{tt}^- > 0 \quad (m < M, \text{ i.e., } k < p) \tag{10.19}$$

$$G_{tt}^+ = 0 \quad (m = M, \text{ i.e., } k = p) \tag{10.20}$$

This completes the proof.■

Proof of Proposition 7: Applying the first order condition (3.2) to the utility function (6.11), and using the budget constraint (2.3), gives

$$g^i(s,t,G) = \frac{a^i}{1-s}\left[(1-t)\,m^i - \frac{b^i}{G}\right] \tag{10.21}$$

$$c^i(s,t,G) = \left(1-a^i\right)\left[(1-t)\,m^i - \frac{b^i}{G}\right] + \frac{b^i}{G} \tag{10.22}$$

From (6.12), (6.13) and (10.21), (10.22), we see that $g^i(s,t,G) > 0$ and $c^i(s,t,G) > \frac{b^i}{G}$. Furthermore, it is straightforward to verify that the second order conditions also hold. Hence, given s,t,G; $g^i(s,t,G)$, $c^i(s,t,G)$ maximize utility (6.11) subject to the budget constraint (2.3), and are unique.

Substituting from (10.21) into (3.9) we get the aggregate desire to give to charity, $F(s,t,G)$:

$$F(s,t,G) = \sum\nolimits_{i=1}^{n} a^i m^i + t\sum\nolimits_{i=1}^{n}\left(1-a^i\right)m^i - \frac{1}{G}\sum\nolimits_{i=1}^{n} a^i b^i \tag{10.23}$$

From (10.23) we get:

$$F_s = 0 \tag{10.24}$$

$$F_t = \sum\nolimits_{i=1}^{n}\left(1-a^i\right)m^i > 0 \tag{10.25}$$

$$F_G = \frac{1}{G^2}\sum\nolimits_{i=1}^{n} a^i b^i > 0 \tag{10.26}$$

$$F_{GG} = -\frac{2}{G^3}\sum\nolimits_{i=1}^{n} a^i b^i < 0 \tag{10.27}$$

From (3.15), (10.24), we get

$$G_s = \frac{F_s}{1-F_G} = 0 \tag{10.28}$$

From (10.24) and Proposition 5, it follows that, at a social optimum, $D = 0$, i.e. no direct grant from the government to the charity is involved. Giving to charity is entirely funded by private donations, which are subsidized from taxation if $s > 0$, $t > 0$.

To make further progress, we need to determine the equilibrium values of G. From (3.9), (3.14), (10.23), the equilibrium values of G are the solutions to the equation

$$G = \sum\nolimits_{i=1}^{n} a^i m^i + t\sum\nolimits_{i=1}^{n}\left(1-a^i\right)m^i - \frac{1}{G}\sum\nolimits_{i=1}^{n} a^i b^i. \tag{10.29}$$

Substituting (6.14), (6.15) in (10.29) we get

$$G^2 - BG + C = 0 \tag{10.30}$$

with solutions

$$G^{\pm} = \frac{1}{2}\left[B \pm \sqrt{B^2 - 4C}\right] \tag{10.31}$$

If $B^2 < 4C$, then no equilibrium exists. If $B^2 = 4C$ then a unique equilibrium exists, and is $G = \frac{B}{2} = \sqrt{C}$. But then, from (10.26), (6.15), $F_G = 1$. In this case, neither G_s nor G_t are defined, see (3.15), (3.16). Hence, the only interesting case is when

$$B^2 > 4C \tag{10.32}$$

In this case, (10.30) has two distinct real positive roots:

$$0 < G^- < G^+ \tag{10.33}$$

Using the fact that for real numbers $a > b > 0$: $\sqrt{a-b} > \sqrt{a} - \sqrt{b}$, as well as (10.26), (6.15) and (10.31) - (10.33), we get

$$G = G^+ \Rightarrow F_G < 1; \quad G = G^- \Rightarrow F_G > 1 \tag{10.34}$$

From (3.16), (10.25) and (10.34) we get

$$G_t^+ > 0, \quad G_t^- < 0 \tag{10.35}$$

From (3.17) and (10.25) we get

$$G_{tt} = \frac{F_{GG} F_t G_t}{\left(1 - F_G\right)^2} \tag{10.36}$$

Since $F_{GG} < 0$, $F_t > 0$, $\left(1 - F_G\right)^2 > 0$, we get that the sign of G_{tt} is the reverse of that of G_t. From (10.35) , we then get

$$G_{tt}^+ < 0, \quad G_{tt}^- > 0 \tag{10.37}$$

This completes the proof.■

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