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GROUP FORMATION AND GOVERNANCE

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Group formation and governance*

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Abstract

This paper studies the impact of the governance of a group, whether be it unanimity, simple majority or qualified majority, on its size, composition, and inclination to change the status quo. Somewhat surprisingly, we show that not only unanimity might favor the formation of larger groups than majority, but also a change of status quo. This paper therefore suggests that unanimity, often blamed for the European inertia of the last two decades, was only a scapegoat.

Keywords: groups, coalitions, alliances, endogenous formation, cost reduction, loss of control, governance, unanimity, majority.

JEL Classification Numbers: D7

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1 Introduction

Does unanimity favor the formation of large and conservative groups? Does majority favor the formation of small and pro-active groups? These two questions echo the general theme of this paper: the role of groups' governance on their sizes, compositions and inclinations to change status quo. Many human activities are naturally organized in groups, alliances, partnerships or coalitions: World Trade Organization, North Atlantic Treaty Organization, European Monetary Union, law groups, fisheries, marriages, industry cartels, just to name a few.[1] Arguably, the main rationale for individuals to form groups is to benefit from *efficiency gains* such as economies of scale, exchanges of information, transfer of knowledge, specialization. Another essential feature is that the decisions a group takes are often partly out of control of individuals composing the group or, in the hands of a few of its members: a board of shareholders, a hiring committee, etc. And the governance of a group precisely determines the extent to which members of the group influence the decisions the group takes. For instance, the International Monetary Fund (IMF) uses a weighted voting scheme to adopt decisions, and requires a majority of 85 % of votes to adopt major decisions. With a weight of over 17 %, the governance of the IMF effectively grants a veto power to the United States of America, on the one hand. On the other hand, only 15 % is required to veto a proposal by the United States (See Leech (2002).) As another example, a quorum of 16 votes out of 20 is required for the board of shareholders of *Le Monde* to appoint a director. (See *Le Monde*, 23/05/2007.)

Consequently, a group might take decisions that some of its members would not have taken on their own. In this paper, we interpret the difference in payoffs resulting from the decisions a group takes and the ones an individual would have taken on their own as a *cost associated with the loss of control over the group decisions*. And the governance of a group determines the magnitude of this implicit cost. The aim of this paper is two-fold. First, it aims at analyzing the formation of a group as the trade-off between efficiency gains and the cost associated with the partial loss of control over the decisions a group takes. Second, it analyzes how the governance of a group affects its size, composition and propensity to change a status quo.

[1] In this paper, the word "group" is used generically for groups, alliances, coalitions, organizations, etc.

To highlight the prevalence of this trade-off, let us consider several examples. The first series of examples concerns the formation of international organizations. For instance, benefits from joining the World Trade Organization (WTO) include access to markets without discrimination, increased specialization and more coordinated trade policies. Decisions WTO takes are governed by qualified majority rules.[2] Thus, the WTO might take a decision, say to maintain a trade tariff, that some of its members would have abolished. Another example is the European Council. When taking decisions on particularly sensitive areas such as asylum, taxation and the common foreign and security policy, the Council must be in unanimous agreement. Being an European member is, however, beneficial as it implies economies of scale and more coordinated policies. Similar considerations apply to the IMF or the European Monetary Union (Kohler (2002)).[3] Second, in industrial organization, cartels and research ventures are examples of groups that benefit from economies of scale. For instance, Nocke (1999) studies the formation of cartels when firms face capacity constraints. Firms in a cartel benefit from increased capacity. Similarly, d'Aspremont and Jacquemin (1988), Kamien and Zang (1993), study the formation of cooperative research ventures where firms benefit from cost-reduction.[4] In all these examples, the decisions a cartel takes e.g., which R&D projects to finance, are often compromises resulting from lengthly negotiations, and are likely to differ from the decision a single firm might take on its own. As a last example, individuals often invest in asset funds not only to economize on monitoring, legal, or screening costs but also to share risks. However, the investment decisions i.e., the portfolio allocations, the fund management takes is likely to differ from the decision an individual would take due to different attitudes toward risks, opinions or time horizons. (See Diamond (1984), Boyd and Prescott (1986) and Genicot and Ray (2003).)

We propose a simple model with costly actions to analyze the consequences in terms of size, composition, and likelihood to change the status quo of the above trade-off. In the model, individuals can either participate

[2]The WTO continues GATTs tradition of making decisions not by voting but by consensus. Where consensus is not possible, the WTO agreement allows for voting. The WTO Agreement envisages several specific situations involving voting, which are governed either by the unanimity rule, or a two-thirds majority rule or a three-quarters majority rule.

[3]Maggi and Rodríguez-Clare (2006) develop a model of self-enforcing voting that explains most of the modes of governance encountered in international organizations.

[4]For further examples, we refer the reader to Bloch (2003).

in a group or stand alone, and we assume that individuals have private valuations over two alternatives x and y, y being the status quo. Benefits to participate in a group are modelled as cost reduction: the more individuals in the group, the lower the cost per individual of taking action x or y is. Historically, groups have adopted a large variety of governances ranging from unanimity, qualified majority to consensus and many more (see Felsenthal and Machover (1998)). In this paper, we assume that the governance takes the form of a voting system, a practice adopted by many international organizations and boards of shareholders. More precisely, we assume that a quorum of $\omega(n) \leq n$ votes is required to change the status quo in a group of n individuals. For instance, unanimity corresponds to $\omega(n) = n$ and simple majority to $\omega(n) = (n+1)/2$ if n is odd, and $n/2$ if n is even.

To get some intuitions on the results, assume that it is costless to maintain the status quo, and that the cost of changing it is equally shared among members of the group. On the one hand, consider an individual who prefers the alternative x over the status quo y. If he participates in a group of n individuals *and* alternative x is voted, then he is better off because of the economies of scale. However, if the group maintains the status quo, he is worse off. On the other hand, suppose that the same individual rather prefers y over x unless he shares the cost with at least n^* other individuals. The risk for him is now to join a group with less than n^* individuals and x being voted. Consequently, upon deciding whether to join a group, an individual has to trade-off the potential cost reduction with the potential risk that his less preferred alternative is chosen. This trade-off implies, in turn, that individuals with "similar" valuations form the group; more extreme individuals stand on their own.

As alluded above, the governance of a group is clearly instrumental in determining the likelihood that the less preferred alternative of an individual is implemented and, therefore, influences the composition and size of the group. For instance, with unanimity, individuals who prefer the status quo on their own are weakly better off by participating in a group: they can always veto the adoption of x if the group is not large enough to make a change of status quo attractive. Furthermore, if those individuals are numerous enough, then even the individuals preferring a change of status quo join the group. Indeed, the strong economies of scale now offset the risk of the status quo to be maintained. And, as the group gets larger and larger, changing the status quo becomes more and more attractive. This suggests that unanimity, often blamed for the European inertia of the last two decades, is only a scapegoat: the true culprit is the lack of synergies among European countries.

Related literature. This paper is part of the abundant literature on coalition formation games. One part of this literature uses stregic-form games which are reduced form models, useful when the objective is not the analysis of the emergence of agreements but the analysis of their stability. The first ancestral exclusive membership game was proposed by Von Neumann and Morgenstern (1944) in their seminal book which marked the beginning of game theory (see also Hart and Kurz (1983)). At the same time, d'Aspremont et al. (1983) proposed a simpler game with an open-membership rule, which opened the way for numerous applications to industrial organization (see Bloch (2003)) and environmental economics (e.g., Barrett (1994)). A parallel literature uses extensive form games which allow one to describe, some aspects of the bargaining leading to the agreement. See for example Bloch (1996), or Ray and Vohra (1999, 2001). The present paper follows the approach of d'Aspremont et al., in that the group formation game is modelled as an open membership game with incomplete information, however. More closely related is the literature on the formation of clubs and the provision of local public goods (e.g., Casella (1992), Jehiel and Scotchmer (1997, 2001)). In this literature, as in the present paper, an individual trades off the benefit to participate in a group (sharing the cost of providing a public good) with the "risk" that the group provides a sub-optimal level of the public good from the individual perspective. The paper differs from this literature in two important respects, however. First, the group formation game is explicitly modelled and analyzed. Second, and more importantly, the main focus of the paper is the interplay between the internal mode of governance of a group, and its size, composition, and propensity to change the status quo. To the best of my knowledge, this has not been the focus of the aforementioned literature. Finally, we can cite the literature on the formation of political parties e.g., Besley and Coate (1997), Levy (2004), Osborne and Tourky (2005).

The paper is organized as follows. Section 2 presents the model. The equilibrium analysis is exposed in Section 3, while Section 4 contains the main results of the paper on governance and groups. Section 5 concludes. Proofs are collected in the Appendix.

2 A model of group formation

We consider a model with costly actions and N individuals. Individuals can form a group to benefit from cost reduction. However, the decision the group takes might differ from the decision any individual would have taken on their own: this is an implicit cost to join a group. The group governance partly determines this implicit cost and, consequently, its size, composition and inclination to change the status quo.

Formally, individuals not participating in the group and the group have to decide, each, whether to maintain the status quo (action y) or to change it (action x). For simplicity, we normalize the payoff of the status quo to zero. Taking action x yields a benefit to individual i of $\theta_i b_x$ with $b_x > 0$. Natural interpretations of our model include: adopting a new standard or technology, choosing whether to finance an investment in a financial asset or a R&D project, and more broadly any political or economic decision. The parameter $\theta_i \in [0, 1]$ is individual i's *private* valuation of the benefit of taking action x. We assume that it is common knowledge that the $(\theta_i)_{i=1,\ldots,N}$ are the realizations of the random variables $(\tilde{\theta}_i)_{i=1,\ldots,N}$ independently and identically distributed (*i.i.d.*) with distribution μ. Unless indicated otherwise, μ is assumed to be absolutely continuous with respect to the Lebesgue measure.

Furthermore, changing the status quo is costly. We can think of this cost as an administrative cost, the cost to gather and process information, the cost to implement the new technology, etc. The cost to take action x is $c_x(n)$ per individual in a group of n members. We assume that $c_x(\cdot)$ is *non-increasing* in n, and $c_x(1) = c_x$.[5] For instance, if the cost to take action x is fixed, the group might share it among its n members. Thus, if an individual is member of a group composed of n individuals and the group takes action x, his payoff is $\theta_i b_x - c_x(n)$, higher than the payoff he gets if he takes action x on their own. By joining a group, an individual benefits from economies of scale (cost reduction).

[5]All our results go through if we assume $b_x > b_y$, $c_x(n) > c_y(n)$ for each n, and $c_x(n) - c_y(n)$ decreasing in n, with obvious notations.

2.1 Governance

A central feature of the model is the governance of a group. Historically, groups have adopted a large variety of governances ranging from voting to consensus without vote (NATO) and many more. Voting, however, is the most common form of governances. We therefore consider voting as the modes of governance in this paper. More specifically, we assume that a quorum of $\omega(n)$ votes is required to adopt decision x i.e., to change the status quo, in a group of n individuals. For instance, if $\omega(n) = n$ for any n, a group changes the status quo only if all its members unanimously agree to do so, while if $\omega(n) = (n+1)/2$ if n is odd and $\omega(n) = n/2$ if n is even, a simple majority is required to change the status quo. We can already note that since there are only two alternatives x and y, *sincere voting* is weakly dominant regardless of the type of an individual. We focus on equilibria featuring sincere voting in the rest of the paper.

The governance of a group (social choice rule) selects the decision that a "qualified" majority of its members prefer. An alternative for the group would be to select the decision that is ex-post efficient, that is, to select x if $b_x(\sum_{i \in \text{Group}} \theta_i) \geq nc_x(n)$ and y, otherwise. Does it exist a mechanism that implements this rule? The short answer is no: an ex-post efficient, budget-balanced, incentive-compatible and individually rational mechanism does not exist in our framework (See Fudenberg and Tirole (1991)). Note that with this alternative, the implicit cost to join the group would have been nil.

2.2 Forming a group

To focus on the interaction between modes of governance, composition and group sizes, we consider a (very) simple two-stage game. In the first stage, all individuals *simultaneously* decide either to participate in a *unique* group, or to stand-alone (open membership game). In the second stage, the members of the group vote for an action to be taken by the group. The stand-alone individuals also choose between x and y. While our model abstracts from interesting aspects of group formation e.g., dynamic formation, entry and exit, multiple groups, it incorporates most of the ingredients to meaningfully study the interaction between group formation and modes of governance. We can also note that the group is externally and internally stable in equilibrium.[6]

3 A numerical example

This section is under revision. A new example will be provided soon.

Before analyzing the model, we present a simple numerical example with three individuals that illustrates some of our results. Assume that μ is the uniform distribution on $[0,1]$, $b_x = 1$, $c_x(1) = 0.38$, $c_x(2) = 0.25$ and $c_x(3) = 0.16$. If an individual stands alone, he takes either action x and gets a payoff of $\theta_i - 0.38$ or action y and gets a payoff of 0. His payoff to stand alone is therefore $\max(0, \theta_i - 0.38)$. What is the expected payoff of an individual if he decides to participate in a group? Let us assume that equilibrium strategies $s_i : [0,1] \to \{0,1\}$, where "0" stands for "stand alone" and "1" for "participate," are the indicator of some interval $[\underline{\theta}, \overline{\theta}]$.[7]

Unanimity. With unanimity, the group adopts decision x if only if all its members unanimously agree to do so, that is, $\omega(n) = n$ for $n \in \{1,2,3\}$. Moreover, individual i is *pivotal* in a group of n individuals in the event that $n-1$ individuals (conditional on being in the group) vote for x. That is, individual i is pivotal in a group of n with probability $\Pr(\theta_j b_x - c_x(n) \geq 0 | \theta_j \in [\underline{\theta}, \overline{\theta}])^{n-1}$. Lastly, the probability that $n-1$ individuals other than i participates in the group follows a binomial distribution with parameters $((\overline{\theta} - \underline{\theta}), N-1)$. Henceforth, the expected payoff of an individual of type θ_i to participate in the group with unanimity is given by:

$$\begin{aligned} \mathcal{E}^1_{una}(\theta_i, \underline{\theta}, \overline{\theta}) &= (1 - (\overline{\theta} - \underline{\theta}))^2 \max(0, \theta_i - 0.38) + \\ 2(\overline{\theta} - \underline{\theta})(1 - (\overline{\theta} - \underline{\theta})) & \left(\frac{\overline{\theta} - \min(\max(\underline{\theta}, 0.25), \overline{\theta})}{\overline{\theta} - \underline{\theta}} \right) \max(0, \theta_i - 0.25) + \\ (\overline{\theta} - \underline{\theta})^2 & \left(\frac{\overline{\theta} - \min(\max(\underline{\theta}, 0.16), \overline{\theta})}{\overline{\theta} - \underline{\theta}} \right)^2 \max(0, \theta_i - 0.16). \end{aligned}$$

We can first note that if $\overline{\theta} = \underline{\theta}$, an individual is indifferent between participating in a group or standing alone. It follows that standing alone is an equilibrium of this game. Besides these trivial equilibria, there exist other

[6] Formally, assume that any individual i can either locate at $\mathcal{G}$ or $\{i\}$ where $\mathcal{G}$ stands for the group. In equilibrium, no individual has an incentive to deviate to another location.

[7] We will show that this is indeed the case in any equilibrium.

equilibria. For instance, there is an equilibrium with $(\underline{\theta}, \overline{\theta}) = (0, 1)$. And the probability to change the status quo is around 0.60 (0.84^3).

Majority. With three individuals, majority differs from unanimity only in the event that all the three individuals participate in the group. Moreover, an individual is pivotal in a group of 3 in the event that exactly one of the two other individuals vote for x. The expected payoff of an individual of type θ_i to participate in a group with majority is therefore:

$$\begin{aligned}
\mathcal{E}^1_{maj}(\theta_i, \underline{\theta}, \overline{\theta}) = (1 - (\overline{\theta} - \underline{\theta}))^2 \max(0, \theta_i - 0.38) + \\
2(\overline{\theta} - \underline{\theta})(1 - (\overline{\theta} - \underline{\theta})) \left(\frac{\overline{\theta} - \min(\max(\underline{\theta}, 0.25), \overline{\theta})}{\overline{\theta} - \underline{\theta}} \right) \max(0, \theta_i - 0.25) + \\
2(\overline{\theta} - \underline{\theta})^2 \left(\frac{\overline{\theta} - \min(\max(\underline{\theta}, 0.16), \overline{\theta})}{\overline{\theta} - \underline{\theta}} \right) \left(\frac{\min(\max(\underline{\theta}, 0.16) - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \right) \max(0, \theta_i - 0.16) + \\
(\overline{\theta} - \underline{\theta})^2 \left(\frac{\overline{\theta} - \min(\max(\underline{\theta}, 0.16), \overline{\theta})}{\overline{\theta} - \underline{\theta}} \right)^2 (\theta_i - 0.16).
\end{aligned}$$

Note that $\mathcal{E}^1_{maj}(\theta_i, \underline{\theta}, \overline{\theta}) \leq \mathcal{E}^1_{una}(\theta_i, \underline{\theta}, \overline{\theta})$ for all $\theta_i < 0.16$ and $\mathcal{E}^1_{maj}(\theta_i, \underline{\theta}, \overline{\theta}) > \mathcal{E}^1_{una}(\theta_i, \underline{\theta}, \overline{\theta})$ for all $\theta_i > 0.38$. An equilibrium with majority is $(\underline{\theta}, \overline{\theta} = (0.16, 0.82)$ and the probability to change the status quo is around 0.88.

4 Cost reduction versus loss of control

Notation: Hereafter, $]\underline{a}, \overline{a}[$ denotes the open interval with endpoints $\underline{a}$ and $\overline{a}$ while $(\underline{a}, \overline{a})$ denotes the point in $\mathbb{R}^2$ with coordinates $\underline{a}$ and $\overline{a}$.

In the next two sections, we analyze the group formation game for a given governance. And Section 5 will study how equilibria vary as the mode of governance changes. For simplicity, we only consider *symmetric* perfect Bayesian equilibrium. We now consider the problem an individual faces in taking his decision whether to participate in a group or to stand-alone.

Suppose that individual i participates in a group of n individuals. If individual i is pivotal (i.e., if he expects exactly $\omega(n) - 1$ members of the group to vote for x), his payoff is $\max(\theta_i b_x - c_x(n), 0)$ since by voting x the group takes decision x, and individual i's payoff is then $\theta_i b_x - c_x(n)$, while it is 0 if he votes y. Hence, whether individual i, whenever pivotal, takes action x or y depends on his type (valuation) and the number of individuals participating in the group. If individual i is not pivotal, his vote does not influence the decision of the group, and his payoff is $\theta_i b_x - c_x(n)$ if more than $\omega(n)$ members of the group other than himself vote for x, and 0 otherwise.

Let $s : [0,1] \to \{0,1\},\ \theta_i \mapsto s(\theta_i)$ be a symmetric equilibrium function, where "0" is interpreted as "stands alone" and "1" as "participates," and define

$$\theta^n := \begin{cases} 0 & \text{if} \quad c_x(n) \leq 0, \\ \frac{c_x(n)}{b_x} & \text{if} \quad b_x > c_x(n) > 0, \\ 1 & \text{if} \quad c_x(n) \geq b_x. \end{cases} \tag{1}$$

For $b_x > c_x(n) > 0$, θ^n is the type of an individual that would be indifferent between action x and y in a group of n individuals. Note that θ^n is decreasing in the number n of group members and increasing in the cost c_x of action x. Any member of a group composed of n individuals votes for x if and only if $\theta_i \geq \theta^n$. Therefore, the probability $\beta(n,s)$ that individual j votes for x, conditional on participating in a group of n individuals is[8]

$$\beta(n,s) := \Pr\left(\theta_j \geq \theta^n \mid \theta_j \in \left\{\theta'_j \in [0,1] : s(\theta'_j) = 1\right\}\right). \tag{2}$$

It follows that the probability that exactly m out of $n-1$ individuals, other than i, vote for x follows a binomial density with parameters $(\beta(n,s), n-1)$. Note that $\beta(\cdot,\cdot)$ depends on the group size and its composition. We denote $\alpha_{n-1}(m,s)$ the probability that exactly m individuals, other than i, vote for x in a group of n. In particular, the probability that individual i is pivotal in a group of n individuals is

$$\alpha_{n-1}(\omega(n)-1,s) = \beta(n,s)^{\omega(n)-1}(1-\beta(n,s))^{n-\omega(n)}\binom{n-1}{\omega(n)-1}.$$

The probability to be pivotal therefore depends on the mode of governance $\omega(n)$, the size of the group n, and its composition i.e., the set of types that join the group. The probability that any individual $j \neq i$ joins the group in a symmetric equilibrium is $\mu(\{\theta_j \in [0,1] : s(\theta_j) = 1\})$ and since types are *i.i.d.*, the probability that exactly $(n-1)$ individuals other than i join the group is

$$\begin{aligned} \varphi(n-1,s) := {} & [\mu(\{\theta_j \in [0,1] : s(\theta_j) = 1\})]^{n-1} \\ & [1-\mu(\{\theta_j \in [0,1] : s(\theta_j) = 1\})]^{N-n}\binom{N-1}{n-1}, \end{aligned} \tag{3}$$

[8] If $\mu\{p \in [0,1] : s(p) = 1\} = 0$, then $\beta(\cdot,s) \equiv 0$.

a binomial density with parameters $(\mu(\{\theta_j \in [0,1] : s(\theta_j) = 1\}), N-1)$. The expected payoff of individual i of type θ_i to join the group is therefore:

$$\begin{aligned} &\mathcal{E}^1(\theta_i, s) := \\ &\sum\nolimits_{n=1}^{N} \varphi(n-1, s)[\alpha_{n-1}(\omega(n)-1, s)\max(0, \theta_i b_x - c_x(n)) \\ &+(\sum\nolimits_{m=\omega(n)}^{n-1} \alpha_{n-1}(m, s))(\theta_i b_x - c_x(n))]. \end{aligned} \tag{4}$$

Alternatively, if individual i of type θ_i stands alone, his expected payoff is

$$\mathcal{E}^0(\theta_i) := \max(0, \theta_i b_x - c_x(1)). \tag{5}$$

Note that the expected payoff to participate in a group is dependent on the equilibrium strategy s. Thus, to characterize the equilibria, we should find a function s^* such that $s^*(p_i) = 1$ if and only if $\mathcal{E}^1(\theta_i, s^*) \geq \mathcal{E}^0(\theta_i)$, and $s^*(\theta_i) = 0$ if and only if $\mathcal{E}^1(\theta_i, s^*) \leq \mathcal{E}^0(\theta_i)$. Despite the simplicity of our model, this task will turn out to be a difficult one.

The trade-off between cost reduction and the cost associated with the loss of control is not immediately apparent from equations (4) and (5). The next equation highlights this trade-off by writing the difference in payoffs between participating in a group and standing alone:

$$\begin{aligned} &\mathcal{E}^1(\theta_i, s) - \mathcal{E}^0(\theta_i) = \\ &\sum\nolimits_{n=1}^{N} \varphi(n-1, s)\left[\max(0, \theta_i b_x - c_x(n)) - \max(0, \theta_i b_x - c_x(1)\right] \\ &+\sum\nolimits_{n=1}^{N} \varphi(n-1, s)(\sum\nolimits_{m=\omega(n)}^{n-1} \alpha_{n-1}(m, s))\left[(\theta_i b_x - c_x(n)) - \max(0, \theta_i b_x - c_x(n))\right] \\ &+\sum\nolimits_{n=1}^{N} \varphi(n-1, s)(\sum\nolimits_{m=0}^{\omega(n)-2} \alpha_{n-1}(m, s))\left[0 - \max(0, \theta_i b_x - c_x(n))\right]. \end{aligned} \tag{6}$$

In Equation (6), the second line captures the economies of scale in participating in a group, and is positive. Ceteris paribus, the more individuals are in the group, the higher the gains for individual i to participate in a group. The third and fourth lines capture the cost associated with the loss of control over the decision the group takes and their sum is negative. Conditional on participating in a group of n individuals and not being pivotal, individual i expects the group to take action x with probability $\sum\nolimits_{m=\omega(n)}^{n-1} \alpha_{n-1}(m, s)$ and action y with probability $\sum\nolimits_{m=0}^{\omega(n)-2} \alpha_{n-1}(m, s)$. Moreover, his payoff is

$(\theta_i b_x - c_x(n))$ if action x is taken and 0, otherwise. Were individual i pivotal, his expected payoff would be $\max(0, \theta_i b_x - c_x(n))$. It follows that individual i's implicit cost to participate in the group is indeed given by the sum of the third and fourth lines in Equation (6).

The group governance is thus instrumental in determining the cost of (partly) losing control over the decision the group takes. Conditional on being in a group of n individuals, the cost of losing control is increasing in the quorum $\omega(n)$ if $\theta_i > \theta^n$, and decreasing in the quorum if $\theta_i < \theta^n$. Indeed, if individual i's type θ_i is greater than θ^n, he prefers action x to be chosen, but a larger quorum makes it harder to change the status quo, hence to adopt action x.

5 Equilibrium analysis

As a preliminary observation, note that a symmetric Bayesian equilibrium of the group formation game exists. Intuitively, if each type of each individual conjectures that every type of the other individuals will not participate in the group, then each type is indifferent between standing alone and participating, hence standing-alone is a best reply.[9] Thus, there always exists trivial equilibria in which any type of any individual stands alone. Moreover, observe that if $c_x(N) \geq b_x$, then *any function* $s : [0,1] \to \{0,1\}$ *is an equilibrium function*. Indeed, if the cost $c_x(N)$ of taking x in a group of N individuals (the grand group) offsets the gain b_x to be made, then action y is a strictly dominant action regardless of an individual's type, and thus each type of each individual is indifferent between standing alone and participating in the group.[10] Moreover, the payoff to each individual is zero in any of those equilibria. However, if $c_x(N) < b_x$, it might exist others equilibria. The existence of such non-trivial equilibria is our next task.

[9] Formally, consider the strategy $s^*(\theta_i) = 0$ for all $\theta_i \in [0,1]$. It follows that $\mathcal{E}^0(\theta_i) = \mathcal{E}^1(\theta_i, s^*)$ for all types θ_i, hence it is a best reply for all types of each individual to stand on their own.

[10] If we assume that, whenever indifferent between standing alone and participating in a group, an individual stands alone, then there exists a unique equilibrium in which any type of any individual stands alone.

5.1 Extreme types stand on their own

We first start with an important result about the equilibrium functions s, that is, equilibrium functions are the indicator of some intervals.

Proposition 1 *All symmetric equilibrium functions* $s : [0,1] \to \{0,1\}$ *are the indicator of some intervals* $]\underline{\theta}, \overline{\theta}[$ *or* $[\underline{\theta}, \overline{\theta}]$ *.*

Proposition 1 states that any equilibrium has a *double cutoff nature*: for all types $\theta_i \in [0,1]$ such that $\theta_i \leq \underline{\theta}$ and $\theta_i \geq \overline{\theta}$, an individual stands alone.[11] Thus, extreme types do not participate in the group; individuals with "similar" types form the group. The intuition behind this result is simple. The higher θ_i, the higher individual i's payoff to participate in a group and to stand-alone are. Furthermore, we can show that the difference of expected payoffs $\mathcal{E}^1(\cdot, s) - \mathcal{E}^0(\cdot)$ is increasing for $\theta_i < \theta^1$ and decreasing for $\theta_i \geq \theta^1$. Thus, if we find a "low" type $\underline{\theta}$ and a "high" type $\overline{\theta}$ such that these two types are indifferent between participating in the group and standing alone, then every type in-between participates. This result drastically simplifies our problem: we will only need to focus on the change of $\underline{\theta}$ and $\overline{\theta}$ as $\omega(\cdot)$ varies to analyze the impact of group governances on the (expected) size and composition of a group. Moreover, note that this result is reminiscent of the literature on local public goods, which also find that groups consist of "connected" types.

Before going further, two observations are worth doing. First, individuals with extremely low valuations (weakly) prefer to stand alone. More precisely, participating in the group is a weakly dominated strategy for every types of an individual with $\theta_i < \theta^N$, unless the mode of governance is unanimity. To see this, note that for those types, action x is *strictly dominated* by action y regardless of whether they stand alone or participate in a group of any size. Thus, the mere possibility that the group takes action x implies that they prefer to stand on their own: they have nothing to gain from participating in a group. However, if the governance is unanimity, each of these types can veto the adoption of x; participating in the group is then not dominated. Hence, it follows that $\underline{\theta} \geq \theta^N$, unless the mode of governance is unanimity. With unanimity, there might exist equilibria with $\underline{\theta} < \theta^N$, however. To see

[11] For the beliefs $\theta_i = \overline{\theta}$ or $\theta_i = \underline{\theta}$, an individual is indifferent between participating in the group and standing alone, hence standing alone is a best-reply. In the sequel, we assume, for simplicity, that *whenever indifferent, an individual stands alone.*

this, let us consider a simple example. Suppose that there are two individuals $N = 2$, μ is the uniform distribution on $[0, 1]$, $b_x = 1/2$, $c_x(1) = 3/10$, and $c_x(2) = 1/4$. We have that $\theta^1 = 3/5$ and $\theta^2 = 1/2$. We can then show that the indicator function of $[0, 5/8]$ is an equilibrium. Moreover, the (expected) group size is $5/4$ and the probability that the group changes the status quo is $1/40$. For the argument sake, suppose now that it suffices that one individual votes for x to change the status quo. We obtain that the indicator function of $]1/2, 1]$ is the unique non-trivial equilibrium function. Intuitively, since it is a weakly dominated strategy for types below $1/2$ (θ^2) to join the group, each individual knows that the types of his opponent that might join the group is above $1/2$, hence take action x in a group of size 2. It then follows that there is no cost associated with the loss of control over the group's decision, while there are gains to be made from cost sharing.[12] Moreover, the expected group size is 1 and the probability that the group changes the status quo is $3/10$.[13] This simple example suggests that unanimity favors the formation of large groups while majority favors the change of status quo. However, this example does not illustrate a general principle: unanimity might also favor a change of status quo. The intuition is simple. By favoring the formation of very large groups, unanimity maximizes the gains to be made from cost reduction. And, therefore, changing the status quo becomes more attractive.

The second observation is that not only individuals who would take action x standing on their own, but also individuals who would take action y standing on their own, join the group. Formally, we have $\underline{\theta} < \theta^1 \leq \overline{\theta}$. (A complete proof is found in Appendix.) For instance, it is easy to see that any types of an individual in between θ^2 and θ^1 join the group. For those types, the payoff to stand-alone is zero, while their payoff to be in a group of two individuals or more is strictly positive. However, for individuals with types in between θ^3 and θ^2, matters are more complicate as there is the risk to be in a group of only two individuals and action x being taken (action x has negative payoff for those types), unless the mode of governance is una-

[12] Note that the profile of strategy $s(\theta_i) = 1$ if $\theta_i \in]1/2, 1]$ and $s(\theta_i) = 0$, otherwise, is rationalizable.

[13] Assuming $c_x(2) = 1/10$, we have that the indicator of $[0, 1]$ is an equilibrium function with unanimity and of $[1/5, 1]$ with majority. The probability to change the status quo is $16/25$ with unanimity and $19/25$ with majority. The expected group size is 2 with unanimity and $8/5$ with majority.

nimity. Similarly, individuals with types above θ^1 might join the group if the likelihood of action y being chosen is sufficiently small. Again, the likelihood of an action to be taken depends on the governance.

We can now continue the equilibrium characterization. We first take advantage of Proposition 1 to rewrite the problem of determining s. From Proposition 1, it follows that knowing the open interval $]\underline{\theta}, \overline{\theta}[$ is isomorphic to knowing the strategy s, and, thus, we substitute s by $\underline{\theta}, \overline{\theta}$ in Equations (2)-(4). Moreover, we have that the probability that any individual participates in the group is $\mu(]\underline{\theta}, \overline{\theta}[)$ since $\{p \in [0,1] : s(p) = 1\} =]\underline{\theta}, \overline{\theta}[$ in a symmetric equilibrium. Hence, the probability that exactly $(n-1)$ individuals other than i participate in the group follows a binomial density with parameters $(\mu(]\underline{\theta}, \overline{\theta}[), N-1)$. Quite naturally, we now characterize a non-trivial equilibrium as the zero of a map, and show that such a zero exists. Define the map $\Gamma : \Sigma := \left\{(\underline{\theta}, \overline{\theta}) \in [0,1] \times [0,1] : \overline{\theta} \geq \underline{\theta}\right\} \to \mathbb{R}^2$, with

$$\Gamma(\underline{\theta}, \overline{\theta}) = \left(\begin{array}{c} \Gamma^1(\underline{\theta}, \overline{\theta}) \\ \Gamma^2(\underline{\theta}, \overline{\theta}) \end{array} \right) := \left(\begin{array}{c} \mathcal{E}^1(\underline{\theta}, \underline{\theta}, \overline{\theta}) - \mathcal{E}^0(\underline{\theta}) \\ \mathcal{E}^1(\overline{\theta}, \underline{\theta}, \overline{\theta}) - \mathcal{E}^0\left(\overline{\theta}\right) \end{array} \right). \tag{7}$$

Note that the map Γ is a continuous function of $\underline{\theta}$ and $\overline{\theta}$. An equilibrium $(\underline{\theta}, \overline{\theta})$ is the solution of $(\underline{\theta}, 1-\overline{\theta}) \cdot \Gamma(\underline{\theta}, \overline{\theta}) \geq 0$, with $\Gamma(\underline{\theta}, \overline{\theta}) = 0$ if $(\underline{\theta}, \overline{\theta}) \neq (0,1)$. As already mentioned, the set $\left\{(\underline{\theta}, \overline{\theta}) : \underline{\theta} = \overline{\theta}\right\}$ is contained in $\Gamma^{-1}(0) := \left\{(\underline{\theta}, \overline{\theta}) : \Gamma(\underline{\theta}, \overline{\theta}) = 0\right\}$.[14] Moreover, it is easy to show that these points are *critical* points, that is to say, the Jacobian of Γ evaluated in $\left\{(\underline{\theta}, \overline{\theta}) : \underline{\theta} = \overline{\theta}\right\}$ does not have full rank. A non-trivial equilibrium $(\underline{\theta}, \overline{\theta})$ is then a zero of Γ, which does not belong to the set $\left\{(\underline{\theta}, \overline{\theta}) : \underline{\theta} = \overline{\theta}\right\}$, hence, in a non-trivial equilibrium, the probability to participate in the group is strictly positive.

Theorem 1 *If $c_x(N) < b_x$, there exists a non-trivial equilibrium.*

Thus, if there are potential gains to form a group, an equilibrium exists in which some types of individuals form a group. Several additional remarks are worth doing.

First, if $\theta^2 = 0$, *the grand group is the unique non-trivial equilibrium.* Intuitively, if $\theta^2 = 0$, that is if $c_x(2) = 0$ or b_x infinitely large, every types of any individual in a group of two individuals or more agree that the best action is x. Since there is no disagreement over the best decision to take in a

[14] This is equivalent to $s(\theta_i) = 0$ for all $\theta_i \in [0,1]$.

group, the grand group forms. Moreover, participating in a group is a weakly dominant strategy. Second, if in two non-trivial equilibria, the probability to participate in the group is the same, then these two equilibria are identical.

Lemma 1 *If in two non-trivial equilibria $(\underline{\theta}, \overline{\theta})$ and $(\underline{\theta}', \overline{\theta}')$, the probability to participate in the group is the same, i.e., $\mu(]\underline{\theta}, \overline{\theta}[) = \mu(]\underline{\theta}', \overline{\theta}'[)$, then $(\underline{\theta}, \overline{\theta}) = (\underline{\theta}', \overline{\theta}')$.*

In the previous discussion, we have shown that the group formation game possesses trivial equilibria and, at least, one non-trivial equilibrium.[15] This multiplicity of equilibria should not be too disturbing: it rather nicely mirrors the fascinating variety of forms that groups exhibit in real-life. In the sequel, we nonetheless assume that individuals coordinate on a most comprehensive equilibrium, as defined below.

Definition 1 *An equilibrium $(\underline{\theta}^*, \overline{\theta}^*)$ is said to be a most comprehensive equilibrium if there does not exist another equilibrium $(\underline{\theta}, \overline{\theta})$, such that*

$$\mu(]\underline{\theta}, \overline{\theta}[) > \mu(]\underline{\theta}^*, \overline{\theta}^*[).$$

Thus, in a most comprehensive equilibrium, the probability to participate in the group is maximal. A desirable, if not essential, property of a selected equilibrium is efficiency. For games of complete information, the concept of efficiency is clearly defined. However, for games of incomplete information, as ours, the concept of efficiency becomes more difficult to apprehend. In this paper, we use the concepts of *interim* efficiency (see Hölmström and Myerson (1983)).[16] If every individual prefers a given equilibrium over an alternative equilibrium when he knows his type, whatever his type might be, then the given equilibrium *interim* dominates the alternative one. And we say that an equilibrium is *interim* efficient if there exists no other equilibrium that *interim* dominates it. Thus, *interim* efficiency is the appropriate concept of efficiency for games of incomplete information in which the individuals already know their type when the play of the game begins. We show that a most comprehensive equilibrium has some appealing properties.

[15] In fact, the argument used to prove the existence of at least one non-trivial equilibrium guarantees than there exists an odd number of non-trivial equilibria. Moreover, they are locally unique.

[16] Hölmström and Myerson make the distinction between classical efficiency and incentive-compatible efficiency. In the paper, we refer to their concept of classical efficiency.

Lemma 2 *There exists a unique most comprehensive equilibrium. Moreover, it is interim efficient.*

Uniqueness of the most comprehensive equilibrium follows from Lemma 1. As for efficiency, consider the most comprehensive equilibrium. For any alternative equilibrium, there exists a set of types of positive measure participating in the group in the most comprehensive equilibrium and standing-alone in the alternative equilibrium; and these types of an individual obtain a higher expected payoff in the most comprehensive equilibrium. Therefore, no alternative equilibrium can interim dominate the most comprehensive equilibrium, hence the most comprehensive equilibrium is interim efficient. Besides interim efficiency, the most comprehensive equilibrium has another interesting property: it minimizes the total expected cost under mild conditions. Thus, the most comprehensive equilibrium would be the one selected by a social planner, who aims at minimizing the total expected cost.

Proposition 2 *Assume that the cost function satisfies:* $\lim_{n\to+\infty} c_x(n) = 0$*,* $nc_x(n)$ *is increasing in* n*, and* $\lim_{n\to+\infty} nc_x(n) < +\infty$*. There exists an integer* $\widehat{N}$ *such that for* $N > \widehat{N}$*, the most comprehensive equilibrium minimizes the total expected cost.*

Note that if the cost c_x of taking action x is equally shared among the group members, i.e., $c_x(n) = c_x/n$, then the assumptions of Proposition 2 are satisfied. To fix idea, suppose (for the time being) that all individuals have chosen action x and there are n individuals in the group. The total cost is $(N-n)c_x + nc_x(n)$, a decreasing function of n. The more individuals are in the group, the lower the total cost is. This is the main idea behind Proposition 2. However, matters are more complex since the group might choose with a higher probability action x than stand-alone individuals. In other words, it is less costly for the group to take action x, but the group might take action x more often. To get intuition for this result, compare the total expected cost $\mu([\theta^1, 1])Nc_x(1)$ if all individuals stand alone and the total expected cost

$$\sum_{m=\omega(N)}^{N} \mu([\theta^N, 1])^m (1 - N\mu([\theta^N, 1]))^{N-m} \binom{N}{m} Nc_x(N),$$

if all individuals participate in the group. We indeed have cost reduction $c_x(N) < c_x$, but the group might take action x with a higher probability. Hence, an extremely large group might not be socially efficient. The conditions stated in Proposition 2 insures that the largest group is socially desirable, however.

6 Composition, size and governances

The aim of this section is to compare the composition, (expected) size and likelihood to change the status quo of the most comprehensive group under two important modes of governance: unanimity $\omega(n) = n$ and qualified majority $\lceil n/2 \rceil \leq \omega(n) < n$, for all n.

To start with, let us consider the unanimity rule. The payoff to individual i of type θ_i is:

$$\mathcal{E}^1_{una}(\theta_i, s) = \sum\nolimits_{n=1}^{N} \varphi(n-1, s)\alpha_{n-1}(n-1, s) \max(0, \theta_i b_x - c_x(n)). \qquad (8)$$

From Eq. (8), we deduce that for all types $\theta_i \in]\theta^N, \theta^1]$ of individual i, it is weakly dominant to participate in the group, while types in $[0, \theta^N]$ are indifferent. Indeed, with unanimity, each individual has the power to veto a change of status quo, and therefore individuals with types below θ^1 (weakly) prefer to join the group. However, types above θ^1 prefer the alternative x, and joining a group entails the risk to be vetoed. Therefore, some might join, some might not. For instance, consider the example with $N = 3$, uniform distribution of types, $b_x = 1/2$, $c_x(1) = 3/10$, $c_x(2) = 1/4$ and $c_x(3) = 1/5$, then the indicator function of $[0, 1]$ is the most comprehensive equilibrium. And it clearly maximizes the size of the group. This simple example illustrates an important aspect: unanimity favors the formation of larger groups than majority.

Before presenting general results on governances and groups, let us consider the simple case in which no individual on their own finds it profitable to change the status quo i.e., $c_x > b_x$ ($\theta^1 = 1$). We already know from the above arguments that $\underline{\theta}_{una} \leq \theta^N$ in any equilibrium with unanimity, while $\underline{\theta}_{maj} > \theta^N$ in any equilibrium with a qualified majority. Since $\theta^1 = 1$, it follows that $[0, 1]$ is the most comprehensive equilibrium with unanimity. With qualified majority, the most comprehensive equilibrium is $[\underline{\theta}_{maj}, 1]$ with $\underline{\theta}_{maj} > \theta^N$. We therefore have that the expected size of the group is larger under unanimity than majority, a prediction that confirms our intuition.

Furthermore, we have the following result about the likelihood to change the status quo.

Lemma 3 *If $b_x > c_x$ and $\lim_{n\to\infty} c_x(n) = 0$, there exists a $\overline{N}$ such that for $N \geq \overline{N}$, unanimity not only maximizes the expected size of the group, but also the probability to change the status quo.*

We already know that if $c_x > b_x$, unanimity favors the formation of larger groups than majority. Moreover, if there are strong economies of scale in forming large groups, unanimity induces more pro-active groups i.e., groups with a higher probability to change the status quo. The intuition behind this result is simple: whenever unanimity maximizes the expected size of the group, it also maximizes the economies of scale. As a consequence, more individuals are now willing to change the status quo. At the extreme, when economies of scale become extremely large, *all* individuals find it profitable to change the status quo and no one vetoes a change of status quo. By favoring the formation of smaller groups, majority fails to capitalize on these very large economies of scale.

We now turn to the general case in which some individuals find it profitable to change the status quo even standing on their own i.e., $\theta^1 < 1$. The next proposition presents a condition under which unanimity favors the formation of larger groups than majority.

Proposition 3 *If $n \ln(\mu([\theta^{n+1}, \theta^n])) - \ln(\mu([0, \theta^{n+1}])) \geq 0$ for any $n \geq 1$, then unanimity favors the formation of larger groups than majority.*

Before giving the intuition behind Proposition 3, let us first interpret the joint condition on the indifference thresholds $(\theta^n)_{n=1,\ldots,N}$ and the distribution of types μ. Suppose that μ is the uniform distribution, the condition then states that the total cost of changing the status quo is decreasing in the size of the group i.e., $nc_x(n) \geq (n+1)c_x(n+1)$. Equivalently, the cost per individual to change the status quo is rapidly decreasing as the size of the group increases.[17] More generally, the condition implies that economies of scale are growing as the size of the group increases; the rate of growth being determined by the distribution μ. In turn, this rapid growth of economies of scale implies that the expected gain to join the group offsets the risk that individuals in the group do not unanimously agree to change the status quo.

[17] In particular, it implies that $\lim_{n\to+\infty} c_x(n) = 0$.

It follows that the most comprehensive equilibrium with unanimity is the indicator of $[0, 1]$, and therefore unanimity favors the formation of larger groups than majority. This is the main intuition behind Proposition 3. The next proposition complements Proposition 3: it states that unanimity leads to the formation of more pro-active groups only if it favors the formation of larger groups than majority.

Proposition 4 *If the expected size of the group with unanimity is smaller than the expected size of the group with majority, then majority favors the change of status quo.*

The intuition is again simple. If the expected size of the group is smaller with unanimity than with majority, it means that individuals forming the group under unanimity have lower valuations than those forming the group under majority.[18] Therefore, individuals forming the group under unanimity are less likely to change the status quo. This is a selection effect. Together with Proposition 3, this suggests that unanimity induces more pro-active groups than majority only if economies of scale are rapidly growing in the size of the group.

To sum up, we have seen that not only unanimity might favor the formation of larger groups than majority, but also the formation of more pro-active groups. Large economies of scale are necessary. For otherwise, majority favors the formation of more pro-active groups, although they might be of smaller sizes (See example in Section 4). Finally, we might wonder whether there is a monotone relationship between the governance of a group and its size, composition, and inclination to change the status quo. Numerical examples show that this is not the case.[19]

7 Extensions

In this section, we propose some extensions of the model and discuss the robustness of our results.

Complete information. An important assumption of the model is that the valuations $(\theta_i)_{i=1,...,N}$ are private information of each individual. This assumption is crucial for the mode of governance to matter. To see this,

[18] Remember $\underline{\theta}_{una} \leq \underline{\theta}_{maj}$ in any equilibrium.

[19] Matlab codes are available upon request.

assume that types are commonly known and define $\mathcal{C}^* := \{i : \theta_i \geq \theta^{n^*}\}$ with $n^* = \arg\max_{n \in N}(|\{i : \theta_i \geq \theta^n\}| \geq n)$ as the largest group whose all members agree to change the status quo. We can then show that the strategy profile $s_i = 1$ for all $i \in \mathcal{C}^*$, and $s_i = 0$ otherwise, is the most comprehensive Nash equilibrium of our game.[20] Moreover, this is regardless of the mode of governance.

Entry and exit. An implicit assumption of the model is that members of the group cannot exit the group after either observing how many individuals join the group or the vote outcome. This assumption is reasonable if there is a sufficiently high cost to exit the group. However, our qualitative results are not altered if individuals can exit the group. Indeed, note that if individuals can exit the group after their initial decision to enter the group, then joining the group at the initial stage is weakly dominant. An individual can always exit the group later and gets his stand-alone payoff. It follows that if exit can only take place after the initial decision to enter the group (i.e., after observing how many individuals have decided to join the group), then all equilibria are equivalent to the ones analyzed in this paper. If, however, individuals can exit the group after the vote, the equilibria are different but the same trade-off and qualitative results remain. To see this, note that conditional on y being chosen, all individuals with types above θ^1 exit the group while the other types stay. Conditional on x being chosen, we clearly have that all types above θ^1 stay and all types below θ^N exit. It follows that there exists a threshold θ^* such that all individuals with types above θ^* stay in the group. Moreover, we can easily show that if $\mu(0, \theta^N) = 0$, then the most comprehensive equilibrium consists of all individuals forming the group and voting for x, regardless of the mode of governance. However, if $\mu(0, \theta^N) > 0$, then the most comprehensive equilibrium under unanimity is the indicator of $[0, \theta^1]$ while it is the indicator of $[\theta^*, 1]$ under majority. Majority thus favors a change of status quo. And a sufficient condition for unanimity to favor larger groups than majority is $\mu([0, \theta^N]) > 1/2$. This suggests that the qualitative results of this paper are robust to the possibility of exit from the

[20]To see this, observe that $|\{i : \theta_i \geq \theta^{n^*}\}| = n^*$. If not, we have $|\{i : \theta_i \geq \theta^{n^*}\}| = m > n^*$ implying that $|\{i : \theta_i \geq \theta^m\}| \geq m$, a contradiction with the definition of n^*. Individuals in $\mathcal{C}^*$ have clearly no incentives to deviate. As for individuals not in $\mathcal{C}^*$, suppose that one of them deviates. The size of the group is then $n^* + 1$, and the deviation is profitable to player i only if $\theta_i > \theta^{n^*+1}$, which is impossible by definition of n^*. The proof that is the most comprehensive equilibrium is available upon request.

group. A full-fledged analysis of entry and exit is, nonetheless, left for future research.

Many choices. Another important assumption of the model is that the group has to take a unique decision. Instead, suppose that the group has to take T decisions, sequentially. A more complicate trade-off emerges, but the main intuitions are the same. On the one hand, an individual still benefits from economies of scale by participating in the group. On the other hand, he still faces the risk that the group adopts a sequence of decisions that differs from the sequence of decisions the individual would take on his own. Alternatively, suppose that after each vote, each member of the group has the option to freely exit the group and each stand-alone individual has the option to freely join the group. The group formation game is then the finite repetition of the (constituent) game analyzed in the present paper. And following the idea found in the literature on repeated games, we can use equilibria of our game to construct equilibrium strategies of this new repeated game.[21]

Multiple groups. As alluded in the introduction, the literature on jurisdictions and the local provision of public goods is closely related to the present work. Following this literature (e.g., Jehiel and Scotchmer (2001)), we define a (symmetric) *free mobility* equilibrium as a finite partition $\{C_k\}_{k=1}^{K}$ of the space of valuations $[0,1]$ such that the two following conditions hold: 1) for all $\theta_i \in C_k$, $\mathcal{E}(\theta_i, C_k) \geq \mathcal{E}(\theta_i, C_{k'})$ for all k', and 2) $\mathcal{E}(\theta_i, C_k) \geq \max(0, \theta_i b_x - c_x)$.[22] In the definition, the first condition states that the expected payoff of an individual i of valuation $\theta_i \in C_k$ is better off joining the group C_k than any other group $C_{k'}$. The second condition simply states that an individual is not compelled to participate in a group, and should get at least his stand-alone payoff $\mathcal{E}^0(\theta_i)$. Note that the definition allows for

[21] Assuming that valuations are independently drawn at each period t.

[22] The expected payoff $\mathcal{E}(\theta_i, C_k)$ is given by:

$$\sum_{n=1}^{N} \mu(\theta_i \in C_k)^{n-1} \mu(\theta_i \notin C_k)^{N-n-1} \binom{N-1}{n-1}$$

$$\left(\sum_{m=\omega(n)}^{n} \mu(\theta_i \geq \theta^n | \theta_i \in C_k)^m \mu(\theta_i < \theta^n | \theta_i \in C_k)^{n-1-m} \binom{n-1}{m} (\theta_i b_x - c_x(n)) + \right.$$

$$\left. \mu(\theta_i \geq \theta^n | \theta_i \in C_k)^{\omega(n)-1} \mu(\theta_i < \theta^n | \theta_i \in C_k)^{n-\omega(n)} \binom{n-1}{\omega(n)} \max(0, \theta_i b_x - c_x(n)) \right).$$

the existence of several groups. What would be a free mobility equilibrium? First, since payoff functions satisfy a single crossing property, it is immediate to see that groups must be intervals. Second, assume that the mode of governance is not unanimity and $N > 2$. Suppose that there exists a group C_k such that $C_k \cap [0, \theta^N] \neq \emptyset$ and $C_k \not\subseteq [0, \theta^N)$. Clearly, condition 2) of the definition is violated for any $\theta_i \in C_k \cap [0, \theta^N)$. For those types, changing the status quo is strictly dominated regardless of the size of the group and, therefore, the mere possibility that the group C_k changes the status quo (i.e., takes action x) implies that their expected payoff is strictly negative in the group C_k. It follows that $[0, \theta^N)$ has to be a group, say C_1. Next, consider the group $C_2 = [\theta^N, \theta^*)$. By continuity of the payoff function, we have that for all valuations in C_2 sufficiently close to θ^N, their expected payoff is strictly negative, which again contradicts condition 2) of the definition. Therefore, no free mobility equilibrium exists. In other words, it is impossible to organize individuals in groups such that all individuals receive their stand-alone payoffs. However, if there are $N = 2$, $\{[0, \theta^2), [\theta^2, 1]\}$ is a free mobility equilibrium: the first group does not change the status quo while the second does. For $N = 2$, this equilibrium is the *unique* non-trivial equilibrium of the group formation game analyzed in this paper. Lastly, with unanimity, it is easy to see that there exists a $\theta^* \in (\theta^N, \theta^1)$ such that $\{[0, \theta^*), [\theta^*, 1]\}$ is a free mobility equilibrium with the group composed of the individuals with the higher valuations being more likely to change the status quo. This last equilibrium differs from the one analyzed in the paper.

Finally, suppose that individuals can endogenously form several groups or stand alone i.e., the strategy of an individual is a map from $[0, 1]$ to $\{0, 1, \ldots, K\}$ where "0" is interpreted as "stand alone", "k" as "participate in group k." It is immediate to see that the equilibria analyzed in the present paper survive. Indeed, it is a coordination game, and if each individual conjectures that his opponents are using the equilibrium strategy found in this paper i.e., $s(\theta_i) = 0$ if $\theta_i \notin [\underline{\theta}, \overline{\theta}]$ and $s(\theta_i) = k$ if $\theta_i \in [\underline{\theta}, \overline{\theta}]$, then it is a best reply to follow strategy s. Henceforth, most of our results remain valid in this more general model allowing for multiple groups. However, there might exist other equilibria. This is left for future research.

8 Appendix

Proof of Proposition 1

Remember that

$$\mathcal{E}^1(\theta_i, s) :=$$
$$\sum\nolimits_{n=1}^{N} \varphi(n-1,s)[\alpha_{n-1}(\omega(n)-1,s)\max(0,\theta_i b_x - c_x(n))$$
$$+(\sum\nolimits_{m=\omega(n)}^{n-1} \alpha_{n-1}(m,s))(\theta_i b_x - c_x(n))]$$

is strictly increasing in θ_i regardless of s, and thus strictly quasi-concave. Define $T := \{\theta_i \in [0,1] : \theta_i < \theta^1\}$, as the set of types that choose action x in a group of one or more individuals, and denote T^c the complement of T in $[0,1]$. In the sequel, we write $\mathcal{E}^1(\theta_i, \cdot)$ for " $\mathcal{E}^1(\theta_i, s)$ for any strategy function s".

Consider $(\theta_i, \theta_i') \in [0,1] \times [0,1]$ such that $\mathcal{E}^1(\theta_i, \cdot) \geq \max(0, \theta_i b_x - c_x)$ $\mathcal{E}^1(\theta_i', \cdot) \geq \max(0, \theta_i' b_x - c_x)$, and any $a \in (0,1)$. We shall show that

$$\mathcal{E}^1(a\theta_i + (1-a)\theta_i', \cdot) > \max(0, (a\theta_i + (1-a)\theta_i')b_x - c_x). \tag{9}$$

First, if $(\theta_i, \theta_i') \in T \times T$, Eq. (9) is trivially satisfied since $\mathcal{E}^1$ is strictly quasi-concave in θ_i. Second, if $\theta_i \in T$, $\theta_i' \in T^c$, and $a\theta_i + (1-a)\theta_i' \in T$, we shall show that

$$\mathcal{E}^1(a\theta_i + (1-a)\theta_i', \cdot) > 0.$$

One again, this is trivially true by the strict quasi-concavity of $\mathcal{E}^1$. Third, if $\theta_i \in T$, $\theta_i' \in T^c$, and $a\theta_i + (1-a)\theta_i' \in T^c$, we shall show that

$$\mathcal{E}^1(a\theta_i + (1-a)\theta_i', \cdot) > (a\theta_i + (1-a)\theta_i')b_x - c_x. \tag{10}$$

To prove this last statement, we first need a Lemma.

Lemma 4 *For all $\theta_i \in T^c$, $\mathcal{E}^1(\theta_i, \cdot) - (\theta_i \Delta_x - c_x)$ is decreasing in θ_i.*

Proof First, observe that for all $\theta_i \in T^c$,

$$\mathcal{E}^1(\theta_i, \cdot) =$$
$$\sum_{n=1}^{N} \varphi(n-1, \cdot)(\sum_{m=\omega(n)-1}^{n-1} \alpha(m, \cdot))(\theta_i b_x - c_x(n)).$$

Its slope λ is thus a point in the set Λ with

$$\Lambda := co\left\{b_x, \ldots, (\sum\nolimits_{m=\omega(N)-1}^{N-1} \alpha(m,\cdot))b_x\right\},$$

the convex hull of $\{b_x, \ldots, (\sum\nolimits_{m=\omega(N)-1}^{N-1} \alpha(m,\cdot))b_x\}$. We then have

$$\lambda^* := \underset{\lambda\in\Lambda}{\arg\sup}\lambda = b_x.$$

Finally, the slope of $\theta_i b_x - c_x$ is b_x, and thus $\mathcal{E}^1(\theta_i,\cdot) - (\theta_i b_x - c_x)$ is decreasing in θ_i. □

By Lemma 4, it thus follows that (10) holds. Similarly, we can show that if $(\theta_i, \theta_i') \in T^c \times T^c$, and $a\theta_i + (1-a)\theta_i' \in T^c$, (10) holds. This completes the proof. □

Binomial formula. In this section, we give a result about binomial sums for increasing finite sequences $\{a_n\}_{n=1}^N$. i.e., sequences with $a_1 \le a_2 \le ... \le a_N$. This result is used in a subsequent proof. Consider

$$f(p) = \sum_{n=0}^{N} a_n \binom{N}{n} p^n (1-p)^{N-n}.$$

We want to show that $f(p)$ is increasing in p. Differentiating with respect to p, we have

$$\begin{aligned} f'(p) &= \sum_{n=0}^{N} a_n \binom{N}{n} \left[np^{n-1}(1-p)^{N-n} - (N-n)p^n(1-p)^{N-n-1}\right] \\ &= \sum_{n=0}^{N} a_n \binom{N}{n} p^{n-1}(1-p)^{N-n-1}(n-Np) \\ &= \sum_{n<Np} a_n \binom{N}{n} p^{n-1}(1-p)^{N-n-1}(n-Np) \\ &+ \sum_{n\ge Np} a_n \binom{N}{n} p^{n-1}(1-p)^{N-n-1}(n-Np). \end{aligned}$$

For $n < Np$, we have $a_n \le a_{[Np]}$, and since $n - Np < 0$ for such n, it follows that $a_n(n-Np) \ge a_{[Np]}(n-Np)$. Thus, the first summation satisfies

$$\sum_{n<Np} a_n \binom{N}{n} p^{n-1}(1-p)^{N-n-1}(n-Np) \ge a_{[Np]} \sum_{n<Np} \binom{N}{n} p^{n-1}(1-p)^{N-n-1}(n-Np).$$

Similarly, for the second summation it holds that

$$\sum_{n\geq Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np) \geq a_{[Np]} \sum_{n\geq Np} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np),$$

because $a_n \geq a_{[Np]}$ and $n - Np \geq 0$. Combining the two inequalities yields

$$\begin{aligned} f'(p) &\geq a_{[Np]} \sum_{n=0}^{N} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np) \\ &= a_{[Np]} \sum_{n=0}^{N} n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} - Np \sum_{n=1}^{N} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} \\ &= a_{[Np]} (Np - Np) = 0, \end{aligned}$$

which is the desired result. Note that if there is at least one strict inequality between the a_n's, a strict inequality for $f'(p)$ will follow. Moreover, if we consider a decreasing sequence i.e., $a_1 \geq a_2 \geq ... \geq a_N$, the reverse inequality trivially holds.

Proof of Theorem 1

To prove the existence of at least one non-trivial equilibrium, we rely on arguments from Index Theory. Note that we do not use usual fixed point arguments since we cannot guarantee that the domain of $\Gamma(\underline{\theta}, \overline{\theta}) - (\underline{\theta}, \overline{\theta})$ is Σ. Remember that if $N = 2$, there is a non-trivial equilibrium with $(\underline{\theta}, \overline{\theta}) = (\theta^2, 1)$. From now, assume $N \geq 3$. (CHECK whether for unanimity something different must be said).

First, observe that a non-trivial equilibrium necessarily satisfies $(\underline{\theta}, \overline{\theta}) \in T \times T^c \subset \Sigma$ (T^c being the complement of T in $[0, 1]$), with

$$T := \left\{ \theta_i \in [0,1] : \theta_i < \theta^1 \right\},$$

the set of types that choose action y whenever they stand alone. The proof proceeds by contradiction. First, suppose that $(\underline{\theta}, \overline{\theta}) \in T \times T$, then we have $\mathcal{E}^1(\underline{\theta}, \underline{\theta}, \overline{\theta}) = 0$ from the definition of T and an equilibrium. Since $\mathcal{E}^1$ is increasing in θ_i (see (4)), we then have $\mathcal{E}^1(\overline{\theta}, \underline{\theta}, \overline{\theta}) > 0$, a contradiction. Second, suppose that $(\underline{\theta}, \overline{\theta}) \in T^c \times T^c$, then we have $\mathcal{E}^1(\overline{\theta}, \underline{\theta}, \overline{\theta}) - \overline{\theta} b_x - c_x = 0$ from the definition of T^c and an equilibrium. As already mentioned, $\mathcal{E}^1(\cdot, \underline{\theta}, \overline{\theta}) - \mathcal{E}^0(\cdot)$ is decreasing in θ_i for $\theta_i \in T^C$ (see Lemma 4), hence $\mathcal{E}^1(\underline{\theta}, \underline{\theta}, \overline{\theta}) - \underline{\theta} b_x - c_x > 0$, again a contradiction. Finally, if $(\underline{\theta}, \overline{\theta}) = (0, 1)$, it is trivially true. Therefore, at a non-trivial equilibrium, we have $\underline{\theta} < \theta^1 \leq \overline{\theta}$. This implies that $\beta(n, s) \neq 0$ in any non-trivial equilibrium.

Second, we have $\theta^N \leq \underline{\theta}$ at a non-trivial (undominated) equilibrium if the mode of governance is not the unanimity. Note that since $c_x(N) < b_x$, we have $\theta^N > 0$. By contradiction, suppose that $\theta^N > \underline{\theta}$ at a non-trivial equilibrium, hence all types $\theta_i \in]\underline{\theta}, \theta^N[$ participate in the group. However, for all types $\theta_i \in]\underline{\theta}, \theta^N[$, we have $\mathcal{E}^1(\theta_i, \underline{\theta}, \overline{\theta}) < 0 = \mathcal{E}^0(\theta_i)$ independently of $\overline{\theta}$ since for these types, action x is strictly dominated by y (i.e., $\theta_i b_x < c_x(N)$). Hence $\underline{\theta} \in [\theta^N, \theta^1[$. (In other words, $\mathcal{E}^1(\theta_i, s) < 0$ for any $\theta_i \leq p^N$ at any non-trivial equilibrium s with $N \geq 3$.) Similarly, it is easy to see that, independently of $\underline{\theta} \in T$, we have $\overline{\theta} \neq \theta^1$. It follows that a non-trivial equilibrium point $(\underline{\theta}, \overline{\theta})$ necessarily belongs to $[\theta^N, \theta^1[\times]\theta^1, 1]$, an open subset of Σ.

Third, if the mode of governance is unanimity, we might have a non-trivial equilibrium with $\underline{\theta} < \theta^N$ since types $\theta_i \in [\underline{\theta}, \theta^N]$ can veto decision x with probability 1. In other words, $\mathcal{E}^1(\theta_i, \underline{\theta}, \overline{\theta}) = 0 = \mathcal{E}^0(\theta_i)$ for those types.

The last step in proving the existence of a non-trivial equilibrium consists in proving the existence of a zero of Γ. To do so, we construct a mapping (homotopy) $h : [\theta^N, \theta^1] \times [\theta^1, 1] \to \mathbb{R}^2$ that admits a unique zero in the interior of its domain and that has the same degree than Γ, hence Γ admits a zero.[23]

The mapping $(\underline{\theta}, \overline{\theta}) \mapsto h(\underline{\theta}, \overline{\theta})$ is given by:

$$h(\underline{\theta}, \overline{\theta}) = \begin{pmatrix} h_1(\underline{\theta}, \overline{\theta}) \\ h_2(\underline{\theta}, \overline{\theta}) \end{pmatrix} = \begin{pmatrix} \frac{\theta^N + \theta^1}{2} + \underline{\theta} \\ \frac{\theta^1 + 1}{2} - \overline{\theta} \end{pmatrix}.$$

Note that the determinant of the Jacobian matrix of h is -1, hence is of full rank, and the index of h is $+1$. It follows that h has a zero. Moreover, we have the following boundary conditions for h. $\lim_{\underline{\theta} \to \theta^N} h_1(\underline{\theta}, \overline{\theta}) < 0$, $\lim_{\underline{\theta} \to \theta^1} h_1(\underline{\theta}, \overline{\theta}) > 0$, $\lim_{\overline{\theta} \to \theta^1} h_2(\underline{\theta}, \overline{\theta}) > 0$, and $\lim_{\overline{\theta} \to 1} h_2(\underline{\theta}, \overline{\theta}) < 0$. As for Γ, from the above observations, we have the following boundary conditions. $\lim_{\underline{\theta} \to \theta^N} \Gamma_1(\underline{\theta}, \overline{\theta}) \leq 0$, $\lim_{\underline{\theta} \to \theta^1} \Gamma_1(\underline{\theta}, \overline{\theta}) \geq 0$, $\lim_{\overline{\theta} \to \theta^1} \Gamma_2(\underline{\theta}, \overline{\theta}) \geq 0$.

In a technical appendix available on my webpage, I prove the following:

Corollary A *Let $f : \operatorname{int}[0,1]^n \to \mathbb{R}^n$ be a continuous mapping. If for any $x = (x_1, \ldots, x_i, \ldots, x_n) \in [0,1]^n$ such that $x_i = 0$, $f_i(x) \leq 0$, for any $x = (x_1, \ldots, x_i, \ldots, x_n) \in [0,1]^n$ such that $x_i = 1$, $f_i(x) \geq 0$, then f has a zero in the interior of $[0,1]^n$.*

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[23]Loosely speaking, the degree of a function at a 0 with respect to a bounded, open set counts the solution in that set in a particular way. Two functions have the same degree at 0 if they do no point into opposites directions at the boundary. See Mass-Colell (1985).

We can then apply Corollary A to prove the existence of a zero of Γ. More precisely, if $\lim_{\overline{\theta}\to 1}\Gamma_2(\underline{\theta},\overline{\theta}) \leq 0$, then the existence follows directly from Theorem A. If $\lim_{\overline{\theta}\to 1}\Gamma_2(\underline{\theta},\overline{\theta}) \geq 0$, we have that $\overline{\theta}=1$, and the proof follows then by the Intermediate Value Theorem. □

Proof of Lemma 1

Consider two non-trivial equilibria, $(\underline{\theta},\overline{\theta})$ and $(\underline{\theta}',\overline{\theta}')$, such that the two equilibria have the expected size of the group i.e., $\mu(]\underline{\theta},\overline{\theta}[) = q = \mu(]\underline{\theta}',\overline{\theta}'[)$. We have to show that $(\underline{\theta}',\overline{\theta}') = (\underline{\theta},\overline{\theta})$. First, suppose that the mode of governance is the unanimity i.e., $\omega(n) = n$ for $n \in N$, and without loss of generality assume $\underline{\theta} < \underline{\theta}' < \overline{\theta} < \overline{\theta}'$. For all $\theta_i \in [0,1]$, a simple computation gives:

$$\mathcal{E}^1_{una}(\theta_i,\underline{\theta},\overline{\theta}) - \mathcal{E}^1_{una}(\theta_i,\underline{\theta}',\underline{\theta}') =$$
$$\sum_{n=1}^{N} \varphi(n-1,\underline{\theta},\overline{\theta}))[\alpha(n-1,\underline{\theta},\overline{\theta}) - \alpha(n-1,\underline{\theta}',\overline{\theta}')]\max(0,\theta_i b_x - c_x(n)),$$

since $\varphi(n-1,\underline{\theta},\overline{\theta}) = \varphi(n-1,\underline{\theta}',\overline{\theta}')$ for all $n \in \{0,\ldots,N-1\}$. Moreover, we have $\mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[) \leq \mu(]\max(\underline{\theta}',\theta^n),\overline{\theta}'[)$ with at least one n for which the inequality is strict, hence $\alpha(n-1,\underline{\theta},\overline{\theta}) \leq \alpha(n-1,\underline{\theta}',\overline{\theta}')$ with at least one n for which the inequality is strict. It follows that $0 = \mathcal{E}^1(\underline{\theta},\underline{\theta},\overline{\theta}) < \mathcal{E}^1(\underline{\theta},\underline{\theta}',\overline{\theta}')$ implying that $\underline{\theta}' \leq \underline{\theta}$ for $(\underline{\theta}',\overline{\theta}')$ to be an equilibrium (i.e., $\mathcal{E}^1(\underline{\theta}',\underline{\theta}',\overline{\theta}') = 0$), hence $]\underline{\theta}',\overline{\theta}'[\supset]\underline{\theta},\overline{\theta}[$, contradicting $\mu(]\underline{\theta},\overline{\theta}[) = \mu(]\underline{\theta}',\overline{\theta}'[)$.

For general modes of governance $\omega(\cdot)$, the same arguments apply noticing that $\alpha(\cdot,\underline{\theta}',\overline{\theta}')$ first-order stochastically dominates $\alpha(\cdot,\underline{\theta},\overline{\theta})$. □

Proof of Lemma 3

The probability to change the status quo is $(\mu([\theta^N,1]))^N$ with unanimity. Since $\lim_{n\to\infty} c_x(n) = 0$, for any $\varepsilon > 0$, there exists a $\overline{N}$ such that $\theta^N \in (0,\varepsilon)$ for any $N > \overline{N}$. It implies that $\mu(\theta^N,1) \to 1$ as N goes to infinity. For otherwise, μ is not absolutely continuous with respect to the Lebesgue measure. It follows that $\lim_{N\to\infty}(\mu([\theta^N,1]))^N = 1$. The probability that the group change the status quo with majority is

$$\sum_{n=1}^{n=N} \varphi(n,\underline{\theta}_{maj},1) \sum_{m=\omega(n)}^{n} \alpha(m,\underline{\theta}_{maj},1) < 1, \tag{11}$$

which is bounded from above by 1 since $\underline{\theta}_{maj} > \theta^N$ for any N. This completes the proof. □

Proof of Proposition 2

For any non-trivial equilibrium $(\underline{\theta}, \overline{\theta})$, conditionally on n individuals participating in a group, the expected total cost is

$$(N-n)\mu[\overline{\theta}, 1]c_x + \sum_{m=\omega(n)}^{n} \mu(]\max(\underline{\theta}, \theta^n), \overline{\theta}[)^m (1-\mu(]\max(\underline{\theta}, \theta^n), \overline{\theta}[))^{n-m} \binom{n}{m} nc_x(n),$$

that is, the probability that $(N-n)$ individuals standing alone choose action x (remember that $\overline{\theta} \geq \theta^1 > \underline{\theta}$ in a non-trivial equilibrium) and the probability that the group chooses action x conditional on the mode of governance in a group of n individuals. Moreover, the probability that exactly n individuals participate in the group is

$$\varphi\left(n, \underline{\theta}, \overline{\theta}\right) = \left[\mu(]\underline{\theta}, \overline{\theta}[)\right]^n \left[1 - \mu(]\underline{\theta}, \overline{\theta}[)\right]^{N-n} \binom{N}{n}.$$

Hence, the total expected cost is given by

$$Nc_x(1 - \mu(]\underline{\theta}, \overline{\theta}[))\mu\left([\overline{\theta}, 1]\right) +$$
$$\sum_{n=0}^{N} \varphi(n, \underline{\theta}, \overline{\theta}) \sum_{m=\omega(n)}^{n} \mu(]\max(\underline{\theta}, \theta^n), \overline{\theta}[)^m (1 - \mu(]\max(\underline{\theta}, \theta^n), \overline{\theta}[))^{n-m} \binom{n}{m} nc_x(n). \tag{12}$$

Now consider two non-trivial equilibria $(\underline{\theta}^*, \overline{\theta}^*)$ and $(\underline{\theta}, \overline{\theta})$ such that $\mu(]\underline{\theta}^*, \overline{\theta}^*[) > \mu(]\underline{\theta}, \overline{\theta}[)$. We can easily show that the first term in Equation (12) is smaller for the equilibrium $(\underline{\theta}^*, \overline{\theta}^*)$ than $(\underline{\theta}, \overline{\theta})$. As for the second term, the complexity of the finite binomial sum of terms, which also depends on $\underline{\theta}$ and $\overline{\theta}$, does not make it possible to sign its variation. Nonetheless, it is bounded. As N gets larger, the variation in the first term dominates the variation in the second term, and thus we can conclude that for two equilibria $(\underline{\theta}^*, \overline{\theta}^*)$ and $(\underline{\theta}, \overline{\theta})$ such that $\mu(]\underline{\theta}^*, \overline{\theta}^*[) > \mu(]\underline{\theta}, \overline{\theta}[)$, a larger group is socially desirable. □

Proof of Proposition 3

Assume that $[\underline{\theta}, \overline{\theta}]$ is the most comprehensive equilibrium function for unanimity with $\overline{\theta} < 0$. (Remember that $\underline{\theta} \leq \theta^N$.) We want to show that there exists a $\theta^* > \overline{\theta}$ such that $[\underline{\theta}, \theta^*]$ is also an equilibrium function under the assumption stated in Proposition 3, thus contradicting the assumption that $[\underline{\theta}, \overline{\theta}]$ is the most comprehensive equilibrium. First, let us show that the sequence

$$\left(\left(\frac{\mu(\theta^n, \theta^*)}{\mu(\underline{\theta}, \theta^*)}\right)^n\right)_{n \in N}$$

is increasing in n. The sequence is increasing if for any $n \geq 1$, we have

$$\left(\frac{\mu(\theta^n, \theta^*)}{\mu(\underline{\theta}, \theta^*)}\right)^n \leq \left(\frac{\mu(\theta^{n+1}, \theta^*)}{\mu(\underline{\theta}, \theta^*)}\right)^{n+1}.$$

Taking Neperien logarithmic on both sides, we have

$$n(\ln(\mu(\theta^n, \theta^*)) - \ln(\mu(\underline{\theta}, \theta^*))) \leq (n+1)(\ln(\mu(\theta^{n+1}, \theta^*)) - \ln(\mu(\underline{\theta}, \theta^*))).$$

This is equivalent to

$$\begin{aligned} n \ln(\mu(\theta^n, \theta^*)) &\leq (n+1) \ln(\mu(\theta^{n+1}, \theta^*)) - \ln(\mu(\underline{\theta}, \theta^*)) \\ &= (n+1)(\ln(\mu(\theta^{n+1}, \theta^n)) + \ln(\mu(\theta^n, \theta^*))) - \ln(\mu(\underline{\theta}, \theta^*)). \end{aligned}$$

It follows that a sufficient condition for the sequence to be increasing is

$$n \ln(\mu([\theta^{n+1}, \theta^n])) - \ln(\mu([0, \theta^{n+1}])) \geq 0,$$

the condition stated in Proposition 3. Second, let us compute the difference in payoffs:

$$\begin{aligned} &\sum(\varphi(n-1, \underline{\theta}, \theta^*) - \varphi(n-1, \underline{\theta}, \overline{\theta})) \left(\frac{\mu(\theta^n, \theta^*)}{\mu(\underline{\theta}, \theta^*)}\right)^n \max(0, \theta_i b_x - c_x(n)) + \\ &\sum \varphi(n-1, \underline{\theta}, \overline{\theta}) \left(\left(\frac{\mu(\theta^n, \theta^*)}{\mu(\underline{\theta}, \theta^*)}\right)^n - \left(\frac{\mu(\theta^n, \overline{\theta})}{\mu(\underline{\theta}, \overline{\theta})}\right)^n\right) \max(0, \theta_i b_x - c_x(n)) \end{aligned}$$

Since the sequence $\left(\left(\frac{\mu(\theta^n, \theta^*)}{\mu(\underline{\theta}, \theta^*)}\right)^n \max(0, \theta_i b_x - c_x(n))\right)_{n \in N}$ is increasing, it follows from the Binomial formula that the first line is positive. It is also easy to check that the second line is positive. Moreover, it is strictly positive for all $\theta_i > \theta^1$. It follows then from the intermediate value theorem that there exists a $\theta^* > \overline{\theta}$ such $[\underline{\theta}, \theta^*]$ is an equilibrium. By repeating this argument, we have that at the most comprehensive equilibrium for unanimity, $\overline{\theta} = 1$. $\square$

Proof of Proposition 4 Let the indicator function of $[\underline{\theta}, \overline{\theta}]$ and $[\underline{\theta}^*, \overline{\theta}^*]$ be, respectively, the equilibrium under unanimity and majority. Observe that since the expected size of the group under unanimity is smaller than under majority, we have $\underline{\theta} \leq \underline{\theta}^* \leq \overline{\theta} \leq \overline{\theta}^*$. It follows that

$$\mu(]\max(\underline{\theta}, \theta^n), \overline{\theta}[) < \mu(]\max(\underline{\theta}^*, \theta^n), \overline{\theta}^*[). \tag{13}$$

The probability to change the status quo with unanimity is written (after simplifications) as

$$(1 - \mu(]\underline{\theta}, \overline{\theta}[))^N \sum_{n=0}^{N} \left(\frac{\mu(]\max(\underline{\theta}, \theta^n), \overline{\theta}[)}{1 - \mu(]\underline{\theta}, \overline{\theta}[)} \right)^n \binom{N}{n},$$

while the probability to change the status quo with majority is written as

$$(1 - \mu(]\underline{\theta}^*, \overline{\theta}^*[))^N$$

$$\sum_{n=0}^{N} \binom{N}{n} \sum_{m=\omega(n)}^{n} \left(\frac{\mu(]\max(\underline{\theta}^*, \theta^n), \overline{\theta}^*[)}{1 - \mu(]\underline{\theta}^*, \overline{\theta}^*[)} \right)^m \left(\frac{\mu(]\underline{\theta}^*, \overline{\theta}^*[) - \mu(]\max(\underline{\theta}^*, \theta^n), \overline{\theta}^*[)}{1 - \mu(]\underline{\theta}^*, \overline{\theta}^*[)} \right)^{n-m} \binom{n}{m}.$$

Clearly, if $\mu(]\underline{\theta}, \overline{\theta}[) = \mu]\underline{\theta}^*, \overline{\theta}^*[$, Proposition 4 follows. Next, if the expected size $\mu(]\underline{\theta}, \overline{\theta}[)$ is lower than $\mu]\underline{\theta}^*, \overline{\theta}^*[$, we can show that the expected probability to change the status quo is reduced, and this completes the proof. □

References

d'Aspremont, Claude and Jacquemin, Alexis, "Cooperative and Non-cooperative R&D in Duopoly with Spillovers," *American Economic Review*, 1988, 78(5), pp. 1133-37.

d'Aspremont, Claude and Jacquemin, Alexis, and Gabszewicz, Jean Jaskold, and Weymark, John. "On the Stability of Collusive Price Leadership," *Canadian Journal of Economics,* 1983, 16(1), pp. 17-25.

Barrett, S. "Self-enforcing international environmental agreements ," *Oxford Economic Papers*, 1994, 46, pp. 878-894.

Besley, Timothy and Coate, Stephen. "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, 1997, 108(1), pp. 85-114.

Bloch, Francis. "Group and Network Formation in Industrial Organization: A Survey," March 2003, forthcoming in Group Formation in Economics: Networks, Clubs and Coalitions (G. Demange and M. Wooders, eds.), Cambridge University Press.

Boyd John, and Prescott Edward. "Financial Intermediary-Coalitions," *Journal of Economic Theory*, 1986, 39, pp. 211-232.

Diamond, Douglas. "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 1984, 51, pp. 393-414.

Casella, Alexandra. "On Markets and Clubs: Economics and Political Integration of Regions with Unequal Productivity," *American Economic Review*, 1992, 82, pp. 115-121.

Farell, Joseph and Suzanne Scotchmer. "Partnerships," *Quarterly Journal of Economics,* 1988, 103, pp. 279298.

Felsenthal, Dan S. and Moshé Machover. *The measurement of voting power : theory and practice, problems and paradoxes*, Cheltenham : Edward Elgar, 1998.

Fudenberg, Drew and Jean Tirole. *Game Theory*, MIT Press, 1991.

Genicot, Garance and Debraj, Ray. "Group Formation in Risk-Sharing Arrangements," *Review of Economic Studies,* 2003, 70, pp. 87-113.

Hart, Sergiu and Kurz, Mordecai. "Endogenous Formation of Coalitions," **Econometrica**, 1983, 51, pp. 1047-1064.

Hölmström, Bengt and Myerson, Roger. "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica*, 1983, 51, pp. 1799-1819.

Jehiel, Philippe and Scotchmer, Suzanne. "Free Mobility and the Optimal Number of Jurisdictions," *Annales d'Economie et de Statistique,* 1997, 45, pp. 219–231.

Jehiel, Philippe and Scotchmer, Suzanne. "Constitutional Rules of Exclusion in Jurisdiction Formation," *Review of Economic Studies,* 2001, 68, pp. 393–413.

Kamien, Morton I and Zang, Israel. "Competing Research Joint Ventures," *Journal of Economics and Management Strategy,* 1993, 2(1), pp. 23-40.

Kohler, Marion. "Coalition formation in international monetary policy games", *Journal of International Economics,* 2002, 56, pp. 371-385.

Konishi, Hideo and Ray, Debraj. "Coalition Formation as a Dynamic Process," *Journal of Economic Theory*, 2003, 110, pp. 1-41.

Leech, Dennis. "Voting Power in the Governance of the International Monetary Fund," *Annals of Operations Research*, 2002, 109, pp. 375-397.

Levy, Gilat. "A Model of Political Parties," *Journal of Economic Theory,* 2004, 115(2), pp. 250-277.

Maggi, Giovanni and Morelli, Massimo. "Self-Enforcing Voting in International Organizations," *American Economic Review,* forthcoming, 2006.

Mas-Colell, Andreu. *The theory of general economic equilibrium : a differentiable approach*, New York : Cambridge University Press, 1985.

Nocke, Volker. "Cartel Stability under Capacity Constraints: The Traditional View Restored," mimeo, 2002.

Osborne, Martin J. and Tourky, Rabee. "Party formation in single-issue politics," Mimeo University of Toronto, 2005.

Ray, Debraj and Vohra, Rakish. "A Theory of Endogenous Coalition Structures", *Games and Econonomic Behavior*, 26, 1999, pp. 286-336.

Ray, Debraj and Vohra, Rakish. "Coalitional Power and Public Goods," *Journal of Political Economy*, 2001, 109, pp. 1355–1384.

Sandler, Todd and Hartley, Keith. "Economics of Alliances: The Lessons for Collective Action," *Journal of Economics Perspectives,* 2001, pp. 869-896.

Selten, Reinhard. "A Simple Model of Imperfect Competition When 4 Are Few and 6 Many," *International Journal of Game Theory,* 1973, 2, pp. 141-201.