University of Leicester

DEPARTMENT OF ECONOMICS

24/7

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Working Paper No. 11/51
November 2011

24/7*

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November 11, 2011

Abstract

This paper studies entry in a market where firms compete in shopping hours and prices. I show that an incumbent firm is able to choose its opening hours strategically to deter entry of a new firm. The potential effects of entry deterrence on social welfare depends on the degree of product differentiation. Entry deterrence increases social welfare when product differentiation is low, while it reduces social welfare when product differentiation is high. In terms of policy, the result of this model suggests that shopping hours deregulation is not always welfare enhancing.

JEL Numbers: D21; L51; L22.
Keywords: Entry; Product Differentiation; Shopping Hours.

*I would like to thank my supervisors Gianni De Fraja and Piercarlo Zanchettin for their continuous support and encouragement. I would also like to thank participants at the JEI 2011 (Valencia), EARIE Conference 2011 (Stockholm), NIE Doctoral Students Colloquium 2011 (Nottingham) and the SMYE 2011 (Groningen). All possible errors are mine.

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1 Introduction

Shopping hours regulation has been widely debated in many European countries.[1] In the past decades, countries such Sweden and the U.K. extended their opening hours in the retail industry. Others countries, e.g. Austria, Denmark, Finland and Norway, are more sceptical and maintain restriction on shopping hours. The main concern of shopping hours liberalization is how deregulation may affect the structure on the competitiveness of the retail industry. After deregulation, a firm with cost advantage might decide to open longer hours in order to attract additional consumers from those firms without such a cost advantage and, thus, affecting the composition of different firms' size in equilibrium.[2] On the demand side, since consumers might prefer to go shopping at different times, when opening hours are liberalized retailers can attract additional demand by extending their opening time and charge higher prices because some consumers are willing to pay for time flexibility. On the other hand, deregulation of shopping hours might also affect the incentives of entry into the market and how incumbent firms may respond to a threat of entry. This possible effect of shopping hours deregulation has not been studied, and this paper studies whether firms have incentive to use opening hours as a strategic variable when there is a threat of entry into the market. I show here that there may be entry deterrence strategies in a market where firms compete in shopping hours and prices. I also study the welfare implications of this behaviour and so the potential impact of shopping hours deregulation.

There is evidence from theoretical and empirical literature that changes in opening hours influence the structure of the retail industry. Early studies focused on the effect of longer opening hours on the demand faced by stores of different sizes and, thus, on their prices levels (Morrison and Newman, 1983; Tanguay et al., 1995). They show that shopping hours liberalization may cause a redistribution of sales from small to large stores, and higher prices at large stores. Most recently, studies have considered opening hours as a strategic variable among a fixed number of retailers and focused on how prices respond to shopping hours deregulation (Inderst and Irmen, 2005), and the relationship between equilibrium business hour configuration and flexibility of consumers to advance or postpone their shopping (Shy and Stenbacka, 2008). They found that retailers with longer opening hours charge higher prices because opening hour differentiation softens price competition. Wenzel (2011) extend their analysis to the case of firms' efficiency asymmetries. He shows that an independent retailer may choose full-time opening hours while the retail chain chooses part-time opening hours and gain from shopping hours deregulation when efficiency difference in favour of the

[1]See for example Morrison and Newman (1983), Ingene (1986), Kay and Morris (1987), Ferris (1990). Tanguay et al. (1995) study extending shopping hours in Canada.

[2]This may be the concern between the differences of large retail chains and smaller retailers.

retail chain is small. Conversely, an independent retailer may choose part-time opening hours while the retail chain chooses full-time opening hours and lose from shopping hours deregulation when efficiency difference in favour of the retail chain is large. Finally, Shy and Stenbacka (2008) consider the effect of shopping hours deregulation on the social welfare. They suggest there is no justification for restrictions on shopping hours. Similarly, Wenzel (2011) finds that shopping hours deregulation increases total welfare and consumer surplus.

My framework builds on Inderst and Irmen (2005) and Shy and Stenbacka (2008), but focuses on the possibility of entry in the market, in particular how an incumbent firm may respond to an entry threat. I consider a model of oligopolistic competition with product differentiation in two dimensions; space and time. I adapt the Hotelling (1929) model of spatial differentiation to study a market where firms compete for consumers with different preferences in their shopping hours. The interaction between the incumbent and a potential entrant is analysed in a three-stage game. In the first stage, the incumbent chooses its opening hours. In the second stage, after observing the decision taken by the incumbent, the potential entrant decides whether to enter or not the market and, in the entry case, chooses its opening time. Finally, in stage three and if entry occurs, incumbent and entrant compete in prices. The structure of the game is intended to capture the idea that the incumbent's decision about its opening hours can affect entry and the industry structure equilibrium.

The main result is that entry deterrence is possible: for some parameter values in the model, the incumbent uses opening hours to deter entry. This result has important implications in terms of social welfare. In fact, social welfare can be greater under entry deterrence when product differentiation is low. On the other hand, social welfare can be lower under entry deterrence when product differentiation is high. There are two possible driving forces for this result that depends on the industry equilibrium configuration comparison. First, when comparing entry deterrence with an equilibrium where both incumbent and entrant compete in the market and both firms open longer hours, industry profits is the main driving force. This is because with low product differentiation industry profits with entry deterrence are relative high, while industry profits are low with intense price competition. Second, when comparing entry deterrence with an equilibrium where both incumbent and entrant compete in the market but none of them open longer hours, consumer surplus is the main driving force. This is because with low product differentiation the positive effect on consumers' welfare of the longer opening hours chosen by the incumbent to pre-empt entry can dominate the negative effect of higher prices with entry deterrence. In terms of policy and contrary to the previous findings, the result in this setting suggests that shopping hours deregulation is not always welfare enhancing.

This paper contributes to two types of literature. The first contribution is to oligopolistic competition in multi-dimensional product differentiation, concerning location in space and time dimension.[3] Inderst and Irmen (2005), and Shy and Stenbacka (2008) analyse a two-stage game of duopolistic competition with a fixed number of firms, so they do not consider the possibility of entry into the market. The second contribution is to the literature of entry deterrence. I show that entry deterrence is possible in markets where firms compete in shopping hours and prices: an incumbent firm can use opening hours as a strategic commitment. This result closely relates to the pre-emption strategies studied by Bonanno (1987), Eaton and Lipsey (1977) and Schmalansee (1978), where pre-emption occurs through strategic brand proliferation: incumbents expand their products lines to leave no profitable niche to entrants. Longer opening hours can also be interpreted as product differentiation in the time dimension which creates barriers to entry in the sense of Bain (1956).

The remaining of the paper is organized as follow. Section 2 describes the model. Section 3 characterises the price equilibrium of the game. Section 4 analyses the equilibrium shopping hours and entry into the market. Section 5 studies the potential effects of an entry deterrence strategy in terms of social welfare. Section 6 discuss the possible effects of shopping hours deregulation. Finally, section 7 concludes. All proofs are relegated to the Appendix.

2 The Model

I use a model of duopolistic competition with product differentiation in two dimensions: space and time. For this purpose, I adapt the standard Hotelling (1929) model of spatial differentiation adding the time dimension. The location of firms is exogenous. This is to make the model tractable, and as the focus of the analysis is on entry into a market with shopping hours and price competition, is not a strong restriction.

2.1 Consumers

Consumers are differentiated along two dimensions: i) distance to stores' location, and ii) preferred shopping time. Along the first dimension, consumers are uniformly distributed along a unit line, $l \in [0, 1]$, and each of them buys, at a transportation cost of $\alpha > 0$ per

[3] The most well known models of two-dimensional product differentiation are from Economides (1989), Neven and Thisse (1990), and Tabuchi (1994). As pointed out by Inderst and Irmen (2005), the two-dimensional product differentiation we study has two major novelties. First, we consider a uniform distribution of consumer preferences with respect to shopping hours, capturing the empirical fact that most consumers prefer to go shopping during the day. Second, we consider a firm's product variant characterized by a point in the geographical space and an interval in the time space.

unit of distance, one unit of product. Along the time dimension consumers are of two types. The first type prefers to go shopping during the day, D, and the other prefers to go shopping during the night, N.[4] For example, when consumers go shopping to a grocery store, some of them may prefer to shop during day because they can find more variety of fresh products (vegetables, fruits, fish, etc.), while others shop during the night because of working hours restrictions. There is a unit mass of consumers of two types, which are distributed uniformly on the unit line: a mass λ of consumers of the first type and a mass $1-\lambda$ of consumers of the second type, for $\lambda \in \left(\frac{1}{2}, 1\right]$. This means that the number of consumers who prefer to shop during the day is greater than those who prefer to shop at night.

Each consumer can be represented in this model by a pair of coordinates (l, t), where l corresponds to the horizontal differentiation characteristic in the unit line, and $t \in \{D, N\}$ corresponds to the time preference of shopping hours. The utility function $U : [0,1] \times \{D, N\} \longrightarrow \mathbb{R}$ of a consumer with preference (l, t), derived from buying the product at a price p_i charged by a firm located at l_i with shopping hours t_i, is given by

$$U_{l,t}(l_i, t_i) = V - p_i - \alpha\,|l - l_i| - \beta(t, t_i) \tag{1}$$

where

$$\beta(t, t_i) = \begin{cases} \beta & \text{if } t \neq t_i, \\ 0 & \text{if } t = t_i. \end{cases}$$

$V \in \mathbb{R}_+$ is the consumers' basic utility derived from the consumption of the product and $p_i \in \mathbb{R}_+$ is the price a consumer pays to firm i for the product. I assume that V is sufficiently high such that the market is covered. The term $\alpha\,|l - l_i|$ is the transportation cost for the consumer, located at l, of shopping from firm i, located at l_i. The parameter $\beta(t, t_i)$ is associated with the time dimension and represents the distance between the consumer's preferred shopping time, t, and firm i's opening time, t_i. If $t \neq t_i$, the consumer has a cost β because the store is not open at the time she would like to buy the product, and if $t = t_i$ there is no cost in time because the consumer can buy the product at her preferred time. If the consumer does not buy the product she gets zero utility, $\bar{U}(t, l) = 0$.

[4] Inderst and Irmen (2005) and Shy and Stenbacka (2008) used the circular model of Salop (1979) to represent shopping time differentiation. In those models each point in the unit circle represents an ideal shopping time for a continuum of potential shoppers. My model simplifies consumers' ideal time to a discrete preference for shopping time; day and night.

2.2 Firms

I consider an oligopolistic sector with two firms, $i = I, E$, selling a product. The incumbent I is already in the market, while E is a potential entrant that can enter the market by paying a sunk cost $F > 0$.

The incumbent is located at 0 and the entrant may only locate at 1.[5] Along the time dimension, incumbent's decision of opening hours is a discrete choice between three options: i) open only during the day, D; ii) open only during the night, N; and iii) open all day, A, where $A = D \cup N$. Entrant decides entering or stay out, and if entrant enters the market, chooses between D, N or A. Notice that in the standard Hotelling model the two firms sell a product that is identical in all respects except one characteristic, which is the location where it is sold. In the present framework, firms sell a product which is different in the location where it is sold (spatial dimension) and also in the time when it is sold (time dimension).

For simplicity and without loss of generality, marginal costs are constant and normalised to zero. Firms incur in a fixed operating cost which depends on the opening time. I assume operating costs during the day or during the night are the same, $k > 0$, while the cost of operating all day is μk, for $\mu \in (1, 2]$. This means that firms can have increasing returns to scale in opening time.

The interaction between the incumbent and a potential entrant is analysed in a three-stage game. At *stage 1*, incumbent chooses its opening hours $t_I \in \{D, N, A\}$. Then, at *stage 2* and after knowing t_I, entrant chooses $t_E \in \{Out, D, N, A\}$. I assume, following Inderst and Irmen (2005) and Shy and Stenbacka (2008), that retailers commit to their opening hours choices in the long run. Finally, at *stage 3* and after having observed incumbent and entrant decisions, firms compete in prices, that is firm i chooses p_i, $i = I, E$, if the entrant has entered the market, otherwise the incumbent behaves as an unconstrained monopolist when there is no entry threat. I assume firms cannot price discriminate. I look for the subgame perfect Nash equilibrium ($SPNE$) in pure strategies.

3 Equilibrium Prices

In this section I analyse the equilibrium of the last stage of the game. In order to facilitate the exposition, I first study the case where the incumbent does not face an effective entry threat due to prohibitive entry costs, that is entry is never profitable for the entrant. In this situation the incumbent chooses prices as an unconstrained monopolist (Section 3.1). Then

[5] This assumption allows to focus the analysis on firms' decision of opening hours and, thus, the effect in the industry equilibrium.

I discuss the case where the entrant enters the market and both firms compete in prices (Section 3.2).

3.1 Incumbent without threat of entry

Assume the entry cost is sufficiently high that entry is never profitable for E. In this situation the incumbent behaves as an unconstrained monopolist. Suppose it charges p_I, then the demand function is given by

$$D_I(p_I) = \lambda\, min\{l_D; 1\} + (1-\lambda)\, min\{l_N; 1\} \quad (2)$$

where l_D and l_N are the consumers with day and night preference who are indifferent between shopping at I or not. The location of the indifferent consumer depends on the opening time chosen by the incumbent. When $t_I = D$, the marginal consumer for the first type is $l_D = \frac{V-p_I}{\alpha}$, and the marginal consumer for the second type is $l_N = \frac{V-p_I-\beta}{\alpha}$. When $t_I = N$, the marginal consumer for the first and second type are $l_D = \frac{V-p_I-\beta}{\alpha}$ and $l_N = \frac{V-p_I}{\alpha}$, respectively. When $t_I = A$, the marginal consumer is $l_D = l_N = \frac{V-p_I}{\alpha}$.[6]

The incumbents' payoff is $\Pi_I(p_I) = p_I D_I(p_I) - K$, where $K = k, \mu k$. So, given opening hours' decision, t_I, equilibrium prices is given by $p_I^* \in \arg\max_{p_I} \Pi_I(p_I)$. The prices and incumbents' payoff are showed in Table 1.

Since $\lambda \in (\frac{1}{2}, 1]$, prices are higher when $t_I = D$ than when $t_I = N$. Moreover, given that opening only during the day and opening only during the night have the same operating cost, incumbents' profits are higher when its store is open only during the day than when the store is open only during the night. When the incumbent opens all day prices are higher than the prices charged when the incumbent opens only during the day, meaning that consumers have to pay more for longer shopping hours. Intuitively, when the incumbent opens all day is able to charge a higher price because consumers with night shopping hours preferences want to benefit from the advantage of having zero disutility in the time dimension. Finally, the incumbent earns more profits opening only during the day than opening all day if $\frac{1}{2} + \frac{2\alpha}{\beta(1-\lambda)} \frac{(\mu-1)k}{\beta(1-\lambda)} > \frac{V}{\beta(1-\lambda)}$, and the equilibrium price is $p_I^* = \frac{V-\beta(1-\lambda)}{2}$. The opposite result emerges when the inequality reverse and the equilibrium price in this case is $p_I^* = \frac{V}{2}$. The second term in the LHS of the inequality can be interpreted as the transportation cost multiplied by the additional operation cost of opening all day, and the RHS is consumers' valuation from the consumption of the product, all in terms of the disutility in time for the night preference consumers.

[6] The marginal consumers are derived from (1).

3.2 Duopoly

Let us now suppose that the sunk cost is not so high to prevent the entrant from making any profit. Therefore, if E enters the market, the incumbent and the entrant compete in prices, given opening hours choices (t_I, t_E). Denote p_I and p_E the prices charged by the incumbent and entrant, respectively. Firms' demand functions are given by

$$\begin{aligned} D_I(p_I, p_E) &= \lambda\, min\{l_D; 1\} + (1-\lambda)\, min\{l_N; 1\} \qquad (3) \\ D_E(p_E, p_I) &= \lambda\, min\{(1-l_D); 1\} + (1-\lambda)\, min\{(1-l_N); 1\} \qquad (4) \end{aligned}$$

The marginal consumers are derived from (1) considering that a consumer is indifferent between shopping at I or E when $V - p_I - \alpha\,|l - l_I| - \beta(t, t_I) = V - p_E - \alpha\,|l - l_E| - \beta(t, t_E)$.

When $t_I = t_E$ both firms are open or closed at the consumers' preferred shopping time. Then, the location of the indifferent consumer is $l_D = l_N = \frac{1}{2} + \frac{p_E - p_I}{2\alpha}$. In this case, only the transportation costs and prices affect the consumers' decision of where to go shopping. Thus, when firms are open at the same time and given that consumers' time preference is uniformly distributed in the space, only spatial differentiation matters. In other words, with parallel opening hours there is no product differentiation with respect to time and there is product differentiation as in the standard Hotelling model.

When $t_I \neq t_E$ one firm is opened and the other firm is closed at a given period of time. In this case the indifferent consumers are different; $l_D \neq l_N$. The location of the indifferent consumer with day shopping time preference, l_D, is derived from $V - p_I - \alpha\,|l - l_I| - \beta(t_D, t_I) = V - p_E - \alpha\,|l - l_E| - \beta(t_D, t_E)$. The location of the indifferent consumer with night shopping time preference, l_N, is derived from $V - p_I - \alpha\,|l - l_I| - \beta(t_N, t_I) = V - p_E - \alpha\,|l - l_E| - \beta(t_N, t_E)$. For example, when the incumbent opens only during the day while the entrant opens only during the night, the marginal consumers are $l_D = \frac{1}{2} + \frac{p_E - p_I + \beta}{2\alpha}$ and $l_N = \frac{1}{2} + \frac{p_E - p_I - \beta}{2\alpha}$.

Notice that when $t_I \neq t_E$, β represents an advantage for the open firm because faces a higher demand while it is a disadvantage for the closed firm (faces a lower demand). This is because some consumers may want to minimize their disutility in time when purchasing at the opened firm, at the expense of a higher transportation cost. Therefore, given asymmetric time decisions, choices in prices, transportation costs and disutility in time all affect consumers' decision on where to go shopping.

Firm i's payoff is $\Pi_i(p_i, p_j) = p_i D_i(p_i, p_j) - K$, $i = I, E$ and $K = k, \mu k$. So, given opening hours decisions (t_i, t_j), equilibrium prices is given by $p_i^* \in \arg \underset{p_i}{max}\, \Pi_i(p_i, p_j^*)$. The prices and firm i's payoff are showed in Table 1.

Table 1: Equilibrium Prices and Profits

t_I	t_E	p_I^*	p_E^*	Π_I^*	Π_E^*
D	N	$\alpha - \frac{\beta(1-2\lambda)}{3}$	$\alpha + \frac{\beta(1-2\lambda)}{3}$	$\frac{(3\alpha-\beta(1-2\lambda))^2}{18\alpha} - k$	$\frac{(3\alpha+\beta(1-2\lambda))^2}{18\alpha} - k - F$
D	D	α	α	$\frac{\alpha}{2} - k$	$\frac{\alpha}{2} - k - F$
D	A	$\alpha - \frac{\beta(1-\lambda)}{3}$	$\alpha + \frac{\beta(1-\lambda)}{3}$	$\frac{(3\alpha-\beta(1-\lambda))^2}{18\alpha} - k$	$\frac{(3\alpha+\beta(1-\lambda))^2}{18\alpha} - \mu k - F$
D	Out	$\frac{V-\beta(1-\lambda)}{2}$	-	$\frac{(V-\beta(1-\lambda))^2}{4\alpha} - k$	0
N	N	α	α	$\frac{\alpha}{2} - k$	$\frac{\alpha}{2} - k - F$
N	D	$\alpha + \frac{\beta(1-2\lambda)}{3}$	$\alpha - \frac{\beta(1-2\lambda)}{3}$	$\frac{(3\alpha+\beta(1-2\lambda))^2}{18\alpha} - k$	$\frac{(3\alpha-\beta(1-2\lambda))^2}{18\alpha} - k - F$
N	A	$\alpha - \frac{\beta\lambda}{3}$	$\alpha + \frac{\beta\lambda}{3}$	$\frac{(3\alpha-\beta\lambda)^2}{18\alpha} - k$	$\frac{(3\alpha+\beta\lambda)^2}{18\alpha} - \mu k - F$
A	D	$\alpha + \frac{\beta\lambda}{3}$	$\alpha - \frac{\beta\lambda}{3}$	$\frac{(3\alpha+\beta\lambda)^2}{18\alpha} - \mu k$	$\frac{(3\alpha-\beta\lambda)^2}{18\alpha} - k - F$
A	N	$\alpha + \frac{\beta(1-\lambda)}{3}$	$\alpha - \frac{\beta(1-\lambda)}{3}$	$\frac{(3\alpha+\beta(1-\lambda))^2}{18\alpha} - \mu k$	$\frac{(3\alpha-\beta(1-\lambda))^2}{18\alpha} - k - F$
A	A	α	α	$\frac{\alpha}{2} - \mu k$	$\frac{\alpha}{2} - \mu k - F$
A	Out	$\frac{V}{2}$	-	$\frac{V^2}{4\alpha} - \mu k$	0

Notice that without any additional structure there are eleven equilibrium prices.

4 Equilibrium Shopping Hours and Entry

In this section I analyse opening hours decisions and entry into the market. Given the equilibrium prices, there are eleven possible outcomes. Notice that those equilibrium where the incumbent opens only during the night are not relevant for the purpose of this paper. First, in order to show that entry deterrence is feasible we need to discuss the case where the incumbent wants to choose longer opening hours only when it faces an entry threat. So, the incumbent chooses to open all day due to entry threat, otherwise the incumbent would choose to open only during the day. Second, in order to study the potential impact of shopping hours deregulation a benchmark case is needed: a situation with shopping hours restriction where firms are allow to open only during the day. Thus, in order to focus the analysis in those equilibria which are interesting for the purpose of this paper, Lemma 1 states that the incumbent will not choose to open only during the night.

Lemma 1. *Suppose* $6\alpha > \beta(1-\lambda)$ *and* $\lambda > \frac{2}{3}$*. Then,* $\Pi_I(D, t_E) > \Pi_I(N, t_E)$*.*

Lemma 1 means that, when transportation costs are relatively high in terms of the disutility for those consumers who prefer to go shopping during the night, $6\alpha > \beta(1-\lambda)$, and the number of consumers who prefer to buy during the day is relative high, $\lambda > \frac{2}{3}$, the incumbent will not choose to open only during the night. In other words, when the product differentiation in the space dimension is greater than the product differentiation in the time dimension and the *day demand* is higher than the *night demand*, then opening only during the night is not profitable for the incumbent.

4.1 Entrants' optimal choice

Let discuss E's optimal choice according to the opening times the incumbent might choose. The conditions in Lemma 1 imply that when the incumbent chooses opening only during the day, the entrant does not choose to open only during the night because $\Pi_E(D,D) > \Pi_E(N,D)$. Also, the conditions in Lemma 1 imply that when the incumbent chooses longer opening hours the entrant does not choose to open only during the night because $\Pi_E(N,A) < \Pi_E(D,A)$. Hence, opening only during the night is not an entrants' optimal response when the incumbent opens only during the day or opens all day. Intuitively, opening only during the night is not profitable because the product differentiation in the space dimension is greater than the product differentiation in the time dimension and the *day demand* is higher than the *night demand*. Thus, according to Lemma 1 the entrant chooses to enter the market (or not) and its opening time according to Lemma 2.

Lemma 2. *When Lemma 1 holds, entrants' optimal choice is as follows.*
When $t_I = D$*:*

(a) If $(\mu-1)k > \frac{\beta(1-\lambda)}{3} + \frac{\beta^2(1-\lambda)^2}{18\alpha}$*, then* $t_E^* = D$*;*

(b) If $(\mu-1)k < \frac{\beta(1-\lambda)}{3} + \frac{\beta^2(1-\lambda)^2}{18\alpha}$*, then* $t_E^* = A$*;*

(c) If $\Pi_E(D,D) < 0$ *and* $\Pi_E(A,D) < 0$*, then* $t_E^* = Out$*;*

(d) If $\Pi_E(D,D) > 0$ *and* $\Pi_E(A,D) > 0$*,* $t_E^* = D$*.*

When $t_I = A$*:*

(e) If $6\alpha > \beta$ *and* $(\mu-1)k > \frac{\beta\lambda}{3} - \frac{\beta^2\lambda^2}{18\alpha}$*, then* $t_E^* = D$*;*

(f) If $(\mu-1)k < \frac{\beta\lambda}{3} - \frac{\beta^2\lambda^2}{18\alpha}$ *and* $(\mu-1)k < \frac{\beta(1-\lambda)}{3} - \frac{\beta^2(1-\lambda)^2}{18\alpha}$*, then* $t_E^* = A$*;*

(g) If $\Pi_E(D,A) < 0$ *and* $\Pi_E(A,A) < 0$*, then* $t_E^* = Out$*.*

Lemma 2 *(a)* state that when the additional cost of operating all day is relatively high the entrant chooses to open only during the day. If the additional cost of operating all day is low it is more convenient to choose longer opening hours (Lemma 2 *(b)*). Lemma 2 *(c)* means that the entrant does not enter the market when the incumbent opens only during the day because entry cost is so high. Lemma 2 *(d)* means that the entrant wants to enter the market because, by doing so, it makes profits. Lemma 2 *(e)* and *(f)* state whether the entrant chooses opening only during the day or all day according to the additional cost of operating all day, respectively. Finally, Lemma 2 *(g)* means that the entrant cannot make profits in the market when the incumbent chooses longer opening hours. This happens when the entry cost is so high that the entrant stays out of the market. Notice that the entrant chooses $t_E = Out$ independent of the incumbents' shopping hours decision because the entry cost F is so high that the entrant cannot make any profits in the market (Lemma 2 *(c)* and *(g)*). The entrant can also choose $t_E = Out$ only if the incumbent chooses longer opening hours. Indeed, the entrant may wants to enter the market when the incumbent opens only during the day (it makes profits) but chooses to stay out of the market when the incumbent opens all day. This distinction is important for the purpose of this paper and it is necessary for the analysis at the first stage.

4.2 Incumbents' optimal choice

Let us now analyse the incumbents' optimal choice. Notice that Lemma 1 implies that neither the incumbent nor the entrant choose to open only during the night. Thus, the possibles SPNE are characterise in the following result.

Proposition 1. *The industry equilibria is as follows.*

When Lemma 1 holds:

(a) If $(\mu-1)k > \frac{\beta(1-\lambda)}{3} + \frac{\beta^2(1-\lambda)^2}{18\alpha}$, $(3\alpha-\beta\lambda)^2 < 18\alpha(k+F)$, $\frac{V^2}{4\alpha} > (\mu-1)k + \frac{\alpha}{2}$, *and* $2V\beta(1-\lambda) < 4\alpha(\mu-1)k + \beta^2(1-\lambda)^2$, *then* (t_I^*, t_E^*)*=(A, Out);*

(b) If $(\mu-1)k > \frac{\beta(1-\lambda)}{3} + \frac{\beta^2(1-\lambda)^2}{18\alpha}$, *then* (t_I^*, t_E^*)*=(D, D);*

(c) If $(\mu-1)k < -\frac{\beta(1-\lambda)}{3} + \frac{\beta^2(1-\lambda)^2}{18\alpha}$, *then* (t_I^*, t_E^*)*=(A, A).*

When F *is so high that* $\Pi_E(t_E, t_I) \leq 0$*:*

(d) If $\frac{1}{2} + \frac{2\alpha}{\beta(1-\lambda)}\frac{(\mu-1)k}{\beta(1-\lambda)} > \frac{V}{\beta(1-\lambda)}$, *then* (t_I^*, t_E^*)*=(D, Out);*

(e) If $\frac{1}{2} + \frac{2\alpha}{\beta(1-\lambda)}\frac{(\mu-1)k}{\beta(1-\lambda)} < \frac{V}{\beta(1-\lambda)}$, *then* (t_I^*, t_E^*)*=(A, Out);*

Next, I discuss in more detail each of the possible SPNE described in Proposition 1.

4.2.1 Entry Deterrence

Entry deterrence occurs when an incumbent firm, facing entry threat, uses a strategic variable to discourage potential entrants from entering into the market, otherwise the incumbent would not choose to use such strategic variable.[7]

Proposition 1 *(a)* is the main result of the paper and shows that an entry deterrence strategy (ED) is possible in a market with shopping hours and price competition. In this equilibrium I chooses $t_I = A$ at the first stage (otherwise $t_I \neq A$) in order to induce E to choose $t_E = Out$ at the second stage (otherwise $t_E \neq Out$). In this case the incumbent modifies its behaviour to thwart entry. In this framework, the strategic variable is opening hours. When the incumbent chooses longer opening hours to deter entry, this choice bind the incumbent to a particular output path (strategic commitment). This occurs because once the incumbent chooses longer opening hours is engaged in a commitment; incurs on additional costs (e.g. hire new personal or pay the personal for extra hours worked, negotiate a new insurance contract to cover night hours, etc.), and this commercial policy must be sustainable in the future. Intuitively, choosing longer opening hours to deter entry credibly commits the incumbent to keep on opening longer hours because if the incumbent decide to retract its decision, by opening only during the day, the incumbent would have a cost: renegotiate labour and insurance contracts, invest in advertising the new opening time.

Let discuss the equilibrium conditions for entry deterrence. The first inequality in Proposition 1 *(a)* implies that when the entrant wants to enter opening only during the day, it is not optimal for the incumbent to open all day. The second inequality means it is not profitable for E to open only during the day when I chooses to open all day, so the best choice for E is to stay out of the market when I opens all day. The third inequality means that *entry deterrence is desirable* by the incumbent because $\Pi_I(A, Out) > \Pi_I(D, D)$. The last inequality means that *entry deterrence is caused by the threat of entry*, otherwise the incumbent would choose to open only during the day when there is no entry threat because $\Pi_I(A, Out) < \Pi_I(D, Out)$.[8]

4.2.2 Other possible SPNE

Proposition 1 *(b)* consider *accommodation* equilibria. There is accommodation when the incumbent finds it is more profitable to let the entrant enter the market than to impose

[7]Following the literature of entry deterrence (see for example Neven, 1989), the key insight is that an incumbents' action cannot be easily undone (or incumbents' choice must be irreversible).

[8]This is the distinction pointed by Salop (1979) between natural (or innocent) barriers to entry and strategic barriers to entry. With the latter, the incumbent has to act strategically in order to protect the market.

costly barriers to entry.[9] In this type of equilibria, there is an entry threat and the incumbent accommodates to E's entry because $\Pi_I(t_I, t_E) > \Pi_I(t_I, Out)$, for $t_I \neq A$. The key element of accommodation strategy is that the incumbent will not choose to open longer hours when a potential firm wants to enter the market. According to this definition both firms compete in the market opening only during the day.

Proposition 1 *(c)* states the equilibrium where both firms compete in the market opening longer hours; (t_I^*, t_E^*)=(A, A). Intuitively, when the additional operating cost of opening all day is not sufficiently high, the incumbent and entrant will compete opening longer hours.

4.2.3 Incumbent without threat of entry

Proposition 1 *(d)* and *(e)* state the industry equilibrium when the incumbent faces no threat of entry: E chooses $t_E = Out$ at the second stage no matter what I has decided at the first stage because $\Pi_E(Out, t_i) > \Pi_E(t_E, t_i)$, for $t_E \neq Out$. This situation is possible when the sunk cost F is sufficiently high, then, in Bain's terminology (Bain, 1956), entry is blockaded. In this case, the incumbent chooses opening hours according to $t_I^* \in \arg\max_{t_I} \Pi_I(t_I, Out)$. Since $\Pi_I(D, Out) > \Pi_I(N, Out)$, as discussed in Section 3.1, the relevant choice is between opening only during the day or all day. Intuitively, the industry equilibrium condition in Proposition 1 *(d)* means that the transportation costs and additional operation costs of opening all day are greater than the consumers' product valuation, all express in terms of the disutility in time for the night preference consumers. Therefore, when the incumbent behaves as an unconstrained monopolist it is more profitable to open only during the day. Finally, proposition 1 *(e)* means that it is optimal to open all day when the transportation costs and the increasing returns to scale of opening longer hours are relatively low.

4.2.4 Numerical example

The industry equilibrium describe in proposition 1 depends on several parameters, so it cannot be easily visualized. For a better illustration that entry deterrence is possible, Figure 1 show the region where entry deterrence (ED) is feasible, the accommodation equilibrium (t_I^*,t_E^*)=(D, D) and the equilibriums when there is no entry threat (t_I^*,t_E^*)={(D, Out),(A, Out)}. Figure 1 (a) and (b) consider $V = 1$ and $V = \frac{3}{2}$, respectively, and the following parameters values: $\lambda = \frac{3}{4}$, $\mu = \frac{3}{2}$, $k = \frac{1}{4}$ and $F = \frac{1}{2}$ for (t_I^*,t_E^*)=(D, D). These figures show the interaction between the space product differentiation (α) and the time product differentiation (β) for each of those equilibriums. Given the space product differentiation where ED is feasible, the incumbent can sustain an entry deterrence strategy when those consumers who prefer to

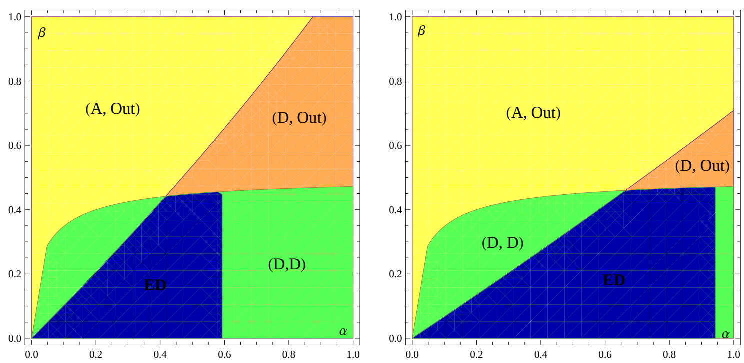
[9] This definition is according to Bains' terminology (Bain, 1956).

shop during the night have a higher disutility in time. Or, given time product differentiation where ED is viable, the incumbent can sustain an entry deterrence strategy when price competition is softened (increase in the transportation cost).

Figure 2 show the interaction between the operating cost k and the time product differentiation (β) for those equilibriums. I consider the following parameters values: $V = 1$, $\lambda = \frac{3}{4}$, $\alpha = \frac{1}{2}$, $F = \frac{1}{2}$ for (t_I^*,t_E^*)=(D, D), and $\mu = \frac{5}{4}$ in Figure 2 (a) and $\mu = 2$ in Figure 2 (b). Given the time product differentiation, it is more likely that the incumbent can sustain an entry deterrence strategy for relatively low k. In addition, when there are no increasing returns to scale in opening time ($\mu = 2$) the accommodation equilibrium (t_I^*,t_E^*)=(D, D) is more likely.

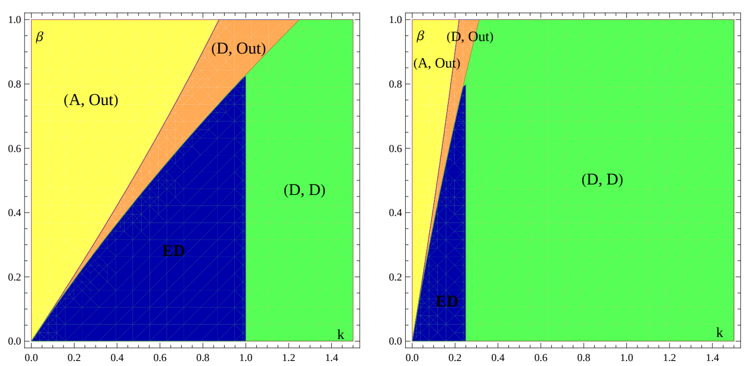
Finally, the region of pre-emption result has the same dimension of the parameter space. Indeed, it is possible to identify the region of ED in the space of the parameters. Therefore, the result showing that entry deterrence is feasible for that parametric specification is robust.

Figure 1: Equilibrium example I



(a) $V = 1$ (b) $V = \frac{3}{2}$

Figure 2: Equilibrium example II



(a) $\mu = \frac{5}{4}$ (b) $\mu = 2$

5 Welfare implications

In this section I discuss the potential effects of an entry deterrence strategy on social welfare (W), that is consumer surplus (CS) and industry profits. From last section we know there are five possible industry equilibriums and for the purpose of this section I compare the social welfare under entry deterrence with three possible equilibriums: (t_I^*,t_E^*)={(D,Out),(D,D),(A,A)}.[10] The first one is when the incumbent does not face an entry threat, so let call this unconstrained monopoly, M. The second is the accommodation type and let call it AC. The last equilibrium is where both firms compete opening longer hours; call this case AA. Table 2 shows the comparison between those equilibriums and the entry deterrence.[11]

Table 2: Social Welfare

Equilibrium	Consumer Surplus	Social Welfare
ED	$\frac{V-\alpha}{2}$	$\frac{V-\alpha}{2}+\frac{V^2}{4\alpha}-\mu k$
M	$\frac{V-\alpha-\beta(1-\lambda)}{2}$	$\frac{V-\alpha-\beta(1-\lambda)}{2}+\frac{(V-\beta(1-\lambda))^2}{4\alpha}-k$
AC	$\frac{V-5\alpha}{4-\beta(1-\lambda)}$	$\frac{V-\alpha}{4-\beta(1-\lambda)}-2k-F$
AA	$V-\frac{5}{4}\alpha$	$V-\frac{\alpha}{4}-2\mu k-F$

Let start comparing ED with M. Notice that price under ED is higher than with M (see Table 1), but the consumer surplus with ED is higher than with M ($CS^{ED} > CS^{M}$). This is because the incumbent opens for longer under ED, thus consumers have no disutility in time. So, when comparing ED with M, there are two opposed effects on consumer surplus: on the one hand, consumers have to pay higher prices under ED and, on the other hand, consumers suffer no disutility in time with ED. The social welfare with ED is higher than with M if $1+\frac{2V-\beta(1-\lambda)}{2\alpha} > \frac{2(\mu-1)k}{\beta(1-\lambda)}$. Thus, when the additional operating cost of longer opening hours, in terms of the disutility for the night preference consumers, is not sufficiently high, the society is better off under an entry deterrence equilibrium than under an unconstrained monopolist opening only during the day. The opposite result emerges when the additional operating cost of longer opening hours is sufficiently high. Then, society is worse off with ED.

When comparing ED with AC, the consumer surplus is higher with ED than with AC if $\frac{V}{2} < \beta(1-\lambda)+\frac{3}{4}\alpha$, and social welfare is higher with ED than with AC if $\frac{V^2}{4\alpha}+\beta(1-\lambda)+(2-\mu)k+F > \frac{2V+\alpha}{4}$. The opposite results come out when the inequalities reverse.

[10]The outcomes of an entry deterrence strategy and unconstrained monopolist opening all day are equivalent, so it is not necessary to compare these equilibriums.

[11]Details of the consumer surplus expressions in each case are in Appendix B.

When comparing ED with AA, consumer surplus is higher with ED than with AA if $V < \frac{3}{2}\alpha$, and social welfare is higher with ED than with AA if $V + \frac{1}{2}\alpha < \frac{V^2}{2\alpha} + 4\mu k + 2F$. The opposite results emerge when the inequalities reverse.

In order to determine which equilibrium is welfare enhancing when the incumbent faces an entry threat, two opposite effects may arise. On the one hand, when comparing ED with AC and AA, there is less competition due to a lower number of firms in the market under entry deterrence. This is a standard negative effect of entry deterrence because in a more concentrated market consumers pay a higher price.[12] On the other hand, the time dimension in this model adds a new effect when ED is compared with AC. As long as consumers have different shopping time preferences, longer opening hours due to entry deterrence can be viewed as an increased product valuation for consumers with night shopping hours preferences. Thus, these consumers do not have disutility in time because the incumbent opens all day, so this is a positive effect on welfare. Notice that the later effect does not appear when comparing ED with AA because consumers with night shopping hours preference have no disutility in time neither under ED nor in AA.

From the above analysis it is clear that which equilibrium is welfare enhancing depends on the parameters of the model. In particular, whether ED boosts welfare depending on the degree of product differentiation, which is measured by the transportation cost α. The following result states when social welfare can be enhanced (or not) by an entry deterrence strategy.

Proposition 2. *When product differentiation is (not) sufficiently high, an entry deterrence strategy is welfare (enhancing) reducing.*

Proposition 2 shows that entry deterrence boosts social welfare for low degree of product differentiation. Let consider first ED versus AA. Intuitively, when both firms are in the market and product differentiation is low, competition is tough. This increases consumer surplus but reduces industry profits in AA. Under entry deterrence and lower product differentiation consumer surplus and industry profits increase. Therefore, for a low degree of product differentiation, industry profits is the main driving force for a higher social welfare with ED than with AA. Now consider ED versus M and AC. For a lower degree of product differentiation profits are higher both in ED and M, while industry profit is lower in AC. On the other hand, under ED, contrary to M and AC, consumers suffer no disutility in time. Hence, the positive effect on consumers' welfare of the longer opening hours chosen by the incumbent to pre-empt entry dominates the negative effect of higher prices under ED. Therefore, for a low

[12]In this model, the prices are $p_I^{ED} = \frac{V}{2}$ and $p_I^{AC} = p_E^{AC} = p_I^{AA} = p_E^{AA} = \alpha$. Thus, the negative effect of less competition on prices requires that $\frac{V}{2} > \alpha$.

degree of product differentiation, the positive effect of longer opening hours on consumers' surplus is the main driving force for a higher social welfare with ED than with M and AC.

In order to better illustrate this result, I present a numerical example comparing consumer surplus and social welfare in terms of the degree of product differentiation. Assume the following set of parameters: $V = 1$, $\lambda = \frac{3}{4}$, $\mu = \frac{3}{2}$, $k = \frac{1}{4}$, $\beta = \frac{1}{2}$ and $F = \frac{1}{2}$ for AC. The results are shown in Figures 3 and 4. Notice that consumer surplus with ED is higher than with M, and consumer surplus with AC is greater than with ED only when α is relative small. For the range of transportation cost where ED is feasible (consistent with the result showed in Figure 1), consumers are better off with AA. Finally, social welfare is greater with ED than with M, AC and AA for low α.

Figure 3: Consumer Surplus

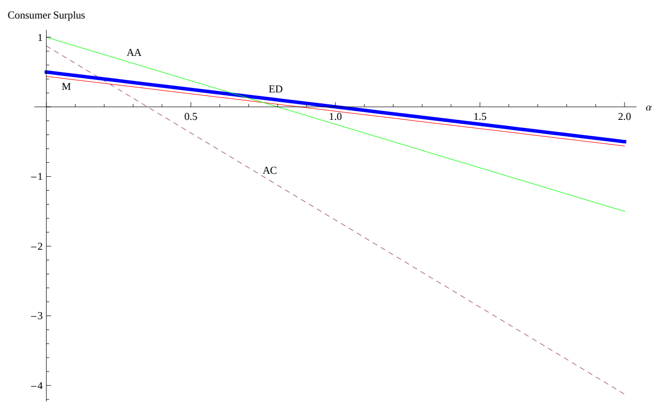
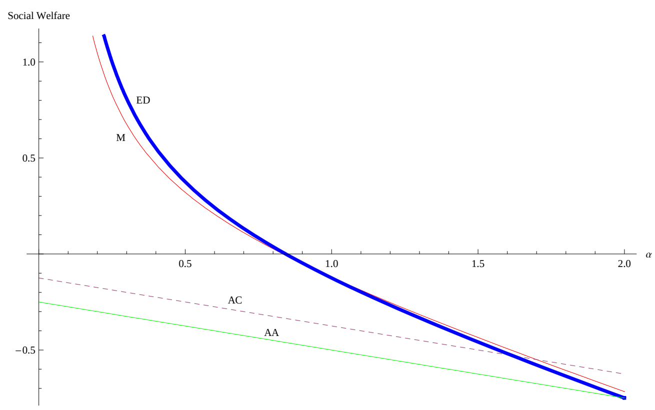


Figure 4: Social Welfare



6 Impact of shopping hours deregulation

This section examines the potential effects of shopping hours deregulation on the industry equilibrium and social welfare. So far I showed that without restrictions in firms' opening time the possible SPNE are: entry deterrence, accommodation equilibrium $(t_I^*, t_E^*)=(D, D)$, longer opening hours $(t_I^*, t_E^*)=(A, A)$ and the equilibriums when there is no entry threat $(t_I^*, t_E^*)=\{(D, Out), (A, Out)\}$.

The procedure to analyse the potential effects of shopping hours deregulation in this framework is the following. Let suppose that under opening time regulation firms are allowed to open only during the day, that is opening during the night time is prohibited. Also, assume that with regulation only the incumbent is active in the market. When shopping hours are deregulated, retail firms are free to open at night as well. Hence, the set of relevant SPNE are: entry deterrence, longer opening hours $(t_I^*, t_E^*)=(A, A)$ and the no entry threat equilibrium $(t_I^*, t_E^*)=(D, Out)$. The benchmark for the purpose of this section is described by the equilibrium $(t_I^*, t_E^*)=(D, Out)$ in Proposition 1 *(d)*. So, in order to analyse the effect of shopping hours deregulation we need to compare the situation with regulation, that is $(t_I^*, t_E^*)=(D, Out)$, with each of the two possible outcomes: ED and $(t_I^*, t_E^*)=(A, A)$.

I first compare the benchmark with ED. If deregulation leads to ED, then, as I discussed in the previous section, consumers pay a higher price but consumer surplus is higher with ED because consumers have no disutility in time. The social welfare with ED is higher than the welfare with a regulated market if $1 + \frac{2V-\beta(1-\lambda)}{2\alpha} > \frac{2(\mu-1)k}{\beta(1-\lambda)}$, otherwise welfare is higher with regulation. The opposite result emerges when operating cost of longer opening hours is sufficiently high.

Now, I compare the benchmark with the equilibrium where both firms open all day. If deregulation leads to both firms choosing longer opening hours, the consumer surplus under deregulation is higher than with regulation if $V + \beta(1-\lambda) > \alpha$, otherwise consumer surplus is higher under the regulated situation. The social welfare is higher with deregulation than with regulation if $V + \alpha + \beta(1-\lambda) + 2(1-2\mu)k > \frac{(V-\beta(1-\lambda))^2}{2\alpha} + 2F$, otherwise welfare is higher under the regulated situation.

Summing up, in this setting shopping hours deregulation is not necessary welfare enhancing. In particular, when the additional operating cost of longer opening hours (in terms of the disutility for the night preference consumers) is relatively high, society is better off with a regulated market. This implication is different from previous findings. Shy and Stenbacka (2008) suggest there is no justification for restrictions on shopping hours. Similarly, Wenzel (2011) shows that when there is efficiency differences between a retail chain and an independent retailer, shopping hours deregulation increases total welfare and consumer surplus.

7 Concluding remarks

This article explores whether an incumbent firm can use opening hours strategically to deter entry into the market. I also discuss the welfare implications of this behaviour and so the potential impact of shopping hours deregulation. I consider a model of oligopolistic competition with product differentiation in two dimensions; space and time. The interaction between an incumbent and a potential entrant is analysed in a three-stage competition with respect to shopping hours and prices.

The main result is that entry deterrence is possible in these types of markets: for some parameter values in the model, an incumbent firm is able to choose its opening hours to deter entry. The implications in terms of social welfare are important because entry deterrence is not necessary welfare reducing. In fact, entry deterrence can be welfare enhancing when product differentiation is low. On the contrary, social welfare decreases if product differentiation is high, then entry deterrence has a negative effect. In terms of policy, the results of this framework suggest, contrary to previous findings, that shopping hours deregulation is not always welfare enhancing.

This paper contributes to the literature of oligopolistic competition in multi-dimensional product differentiation and the public debate on shopping hours deregulation. This framework allows to analyse how changes in the regulation of shopping hours affect the incentives of incumbent firms to use opening hours as a strategic variable when there is an entry threat. Also, this paper contributes to the literature of entry deterrence. I show that entry deterrence is possible in markets in which firms compete in shopping hours and prices: an incumbent firm can use opening hours as a strategic commitment. This result closely relates to the pre-emption strategies studied by Bonanno (1987), Eaton and Lipsey (1977) and Schmalansee (1978), where pre-emption occurs through strategic brand proliferation. Longer opening hours can also be interpreted as product differentiation in the time dimension which creates barriers to entry in Bain's (1956) sense.

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A Proofs

Lemma 1.

Proof. By contradiction. Suppose $\Pi_I(D, t_E) < \Pi_I(N, t_E)$.

Define an entrant strategy when I chooses in $t_I \in \{D, N\}$ as:

Swamp: E chooses $t_E = A$ if $t_I \in \{D, N\}$.

Match: E chooses $t_E = D$ if $t_I = D$; and E chooses $t_E = N$ if $t_I = N$.

Evade: E chooses $t_E = N$ if $t_I = D$; and E chooses $t_E = D$ if $t_I = N$.

Suppose E chooses Swamp. Then, $\Pi_I(D, A) > \Pi_I(N, A)$. A contradiction.

Suppose E chooses Match. Then, $\Pi_I(D, D) \geqslant \Pi_I(N, A)$. A contradiction.

Suppose E chooses Evade. Then, $\Pi_I(D, N) > \Pi_I(N, D)$. A contradiction.

Suppose E chooses $t_E = D$ if I chooses $t_I = D$; and E chooses $t_E = A$ if I chooses $t_I = N$. Then, $\Pi_I(D, D) \gtreqless \Pi_I(N, A)$ if $6\alpha \gtreqless \beta\lambda$. So, if $6\alpha > \beta\lambda$, then $\Pi_I(D, D) > \Pi_I(N, A)$. A contradiction.

Suppose E chooses $t_E = N$ if I chooses $t_I = D$; and E chooses $t_E = A$ if I chooses $t_I = N$. Then, $\Pi_I(D, N) > \Pi_I(N, A)$. A contradiction.

Suppose E chooses $t_E = A$ if I chooses $t_I = D$; and E chooses $t_E = D$ if I chooses $t_I = N$. Then, $\Pi_I(D, A) \gtreqless \Pi_I(N, D)$ if $\lambda \gtreqless \frac{2}{3}$. So, if $\lambda > \frac{2}{3}$, then $\Pi_I(D, A) > \Pi_I(N, D)$. A contradiction.

Suppose E chooses $t_E = A$ if I chooses $t_I = D$; and E chooses $t_E = N$ if I chooses $t_I = N$. Then, $\Pi_I(D, A) \gtreqless \Pi_I(N, N)$ if $6\alpha \gtreqless \beta(1-\lambda)$. So, if $6\alpha > \beta(1-\lambda)$, then $\Pi_I(D, A) > \Pi_I(N, N)$. A contradiction.

Finally, notice that if $6\alpha > \beta(1-\lambda)$ hold, also $6\alpha > \beta\lambda$ hold. Then, if $6\alpha > \beta(1-\lambda)$ and $\lambda > \frac{2}{3}$, it is not true that $\Pi_I(D, t_E) < \Pi_I(N, t_E)$. Hence, it must be the case that $\Pi_I(D, t_E) > \Pi_I(N, t_E)$ when $6\alpha > \beta(1-\lambda)$ and $\lambda > \frac{2}{3}$. □

Proposition 1.

Proof. From Lemma 1 we know that $t_I = N$ is not a best response to $t_E \in \{D, N, A, Out\}$.

For part *(a)*: the proof follows directly from the discussion in Section 4.2.1.

For part *(b)*: Suppose I chooses $t_I = D$ at the first stage. Then, $t_E = N$ is not a best response if $\Pi_E(D, D) > \Pi_E(N, D)$. For this, it must be the case that $0 > \beta(1 - 2\lambda)$. Since $\lambda \in (\frac{2}{3}, 1]$, then $\Pi_E(D, D) > \Pi_E(N, D)$.

Next, $t_E = A$ is not a best response if $\Pi_E(D, D) > \Pi_E(A, D)$. For this, it must be the case that $\frac{6\alpha}{\beta(1-\lambda)} \frac{(\mu-1)k}{\beta(1-\lambda)} > \frac{2\alpha}{\beta(1-\lambda)} + \frac{1}{3}$ (*).

Suppose I chooses $t_I = A$ at the first stage. Then, $t_E = N$ is not a best response if $\Pi_E(D, A) > \Pi_E(N, A)$. Since $\lambda \in (\frac{2}{3}, 1]$, that is the case.

Next, $t_E = A$ is not a best response if $\Pi_E(D, A) > \Pi_E(A, A)$. For this, it must be the case that $\frac{6\alpha}{\beta(1-\lambda)} \frac{(\mu-1)k}{\beta(1-\lambda)} > \frac{2\alpha}{\beta(1-\lambda)} - \frac{1}{3}$ (**). Since (*) implies (**), then $\Pi_E(D, A) > \Pi_E(A, A)$. Hence, $t_E = D$ is the best response at the second stage.

At the first stage, I knows that (*) implies that E will choose $t_E = D$ for $t_I \in \{A, D\}$. But (*) also implies that $\Pi_I(D, D) > \Pi_I(A, D)$. Hence, the best response of I is to choose $t_I = D$.

For part *(c)*: the proof is analogous to that of part *(b)*.

To prove the second part of the proposition, assume F is so high that $\Pi_E(t_E, t_I) \leq 0$. Thus, E chooses $t_E = Out$ at the second stage regardless what I chooses at the first stage. Therefore, there is no entry threat and the incumbent chooses t_I such that $\Pi_I^*(t_I, Out)$, for $t_I \in \{D, N, A\}$, is the maximum payoff.

Since $\Pi_I(D, Out) > \Pi_I(N, Out)$, then $t_I = N$ is not the best response when $t_E = Out$.

For the first part *(d)*, $t_I = D$ is the best response when $t_E = Out$ if $\Pi_I(D, Out) > \Pi_I(A, Out)$. And $\Pi_I(D, Out) > \Pi_I(A, Out) \iff \frac{1}{2} + \frac{2\alpha}{\beta(1-\lambda)} \frac{(\mu-1)k}{\beta(1-\lambda)} > \frac{V}{\beta(1-\lambda)}$.

For the second part *(e)*, $t_I = A$ is the best response when $t_E = Out$ if $\Pi_I(D, Out) < \Pi_I(A, Out)$. And $\Pi_I(D, Out) < \Pi_I(A, Out) \iff \frac{1}{2} + \frac{2\alpha}{\beta(1-\lambda)} \frac{(\mu-1)k}{\beta(1-\lambda)} < \frac{V}{\beta(1-\lambda)}$. □

Proposition 2.

Proof. Let W^n, $n \in \{ED, M, AC, AA\}$, be the social welfare under the equilibriums ED, M, AC and AA, respectively:

$$W^{ED} = \frac{V-\alpha}{2} + \frac{V^2}{4\alpha} - \mu k.$$

$W^M = \frac{V-\alpha-\beta(1-\lambda)}{2} + \frac{(V-\beta(1-\lambda))^2}{4\alpha} - k,$

$W^{AC} = \frac{V-\alpha}{4-\beta(1-\lambda)} - 2k - F,$

$W^{AA} = V - \frac{\alpha}{4} - 2\mu k - F.$

Now, define: $g_1 = W^{ED} - W^M$, $g_2 = W^{ED} - W^{AC}$ and $g_3 = W^{ED} - W^{AA}$. In order to show that an entry deterrence strategy increases the social welfare when the degree of product differentiation, measured by the parameter α, is lower it must be showed that g_1, g_2, and g_3 are decreasing functions of α. Taking partial derivatives with respect to α:

$\frac{\partial g_1}{\partial \alpha} = \frac{-V^2+V-\beta(1-\lambda)^2}{4\alpha^2},$

$\frac{\partial g_2}{\partial \alpha} = -\frac{1}{2} + \frac{1}{4-\beta(1-\lambda)} - \frac{V^2}{4\alpha^2},$

$\frac{\partial g_3}{\partial \alpha} = -\frac{1}{4} - \frac{V^2}{4\alpha^2}.$

Finally,

$\frac{\partial g_1}{\partial \alpha} \lesseqgtr 0$ if $V \lesseqgtr V^2 + \beta(1-\lambda)^2,$

$\frac{\partial g_2}{\partial \alpha} \lesseqgtr 0$ if $\frac{1}{4-\beta(1-\lambda)} \lesseqgtr \frac{V^2}{4\alpha^2} + \frac{1}{2},$

$\frac{\partial g_3}{\partial \alpha} < 0.$

□

B Consumer Surplus

The consumer surplus in each possible equilibria is given by:

Unconstrained monopolist: $CS^M = \int_0^1 (V - p_I - \alpha(y - l_I))\, dy - \beta(1-\lambda).$

Accommodation AC: $CS^{AC} = \int_0^{\tilde{l}} (V - p_I - \alpha(y - l_I)\, dy + \int_{\tilde{l}}^1 (V - p_E - \alpha(l_E - y))\, dy - \beta(1-\lambda).$

Entry deterrence: $CS^{ED} = \int_0^1 (V - p_I - \alpha(y - l_I))\, dy.$

Longer opening hours AA: $CS^{AA} = \int_0^{\tilde{l}} (V - p_I - \alpha(y - l_I))\, dy + \int_{\tilde{l}}^1 (V - p_E - \alpha(l_E - y))\, dy.$