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GROUP FORMATION AND GOVERNANCE

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Group formation and governance*

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Abstract

This paper studies the impact of the governance of a group, whether be it unanimity, simple majority or qualified majority, on its (endogenously derived) size, composition, and inclination to change the status quo. Somewhat surprisingly, we show that not only unanimity might favor the formation of larger groups than majority, but also a change of status quo.

Keywords: groups, endogenous formation, economies of scale, loss of control, governance, unanimity, majority.

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1 Introduction

Does unanimity favor the formation of large and conservative groups? Does majority favor the formation of small and pro-active groups? These two questions echo the general theme of this paper: the role of groups' governance on their sizes, compositions and inclinations to change status quo. Many human activities are naturally organized in groups: World Trade Organization, North Atlantic Treaty Organization, European Monetary Union, law groups, fisheries, industry cartels, just to name a few. Arguably, the main rationale for individuals to form groups is to benefit from *efficiency gains* such as economies of scale, exchanges of information, transfer of knowledge, specialization. Another essential, almost tautological, feature of a group is that each group member *imperfectly* controls the decision the group takes. And the governance of a group precisely determines the extent to which each member influences the decisions the group takes. As an example, the International Monetary Fund (IMF) uses a weighted voting scheme and requires a majority of 85 % of votes to adopt major decisions. With a weight of over 17 %, the governance of the IMF effectively grants a veto power to the United States of America, on the one hand. On the other, only 15 % is required to veto a proposal by the United States (See Leech (2002).) The IMF's governance thus determines the extent to which the United States can pass or block proposals.

A group might therefore take decisions that some of its members would not have taken on their own. In this paper, we interpret the difference in payoffs resulting from the decisions a group takes and the ones an individual would have taken were he pivotal as a *control cost* i.e., a cost associated with the partial loss of control over the group decisions. The governance of a group determines the magnitude of this control cost. The aim of this paper is then two-fold. First, it aims at analyzing the formation of a group as the trade-off between economies of scale and the control cost. Second, it analyzes how the governance of a group affects its size, composition and propensity to change a status quo.

To highlight the importance of the above trade-off, let us consider several examples. The first series of examples concerns the formation of international organizations. For instance, benefits from joining the World Trade Organization (WTO) include access to markets without discrimination, increased specialization and more coordinated trade policies. Decisions WTO takes are governed by qualified majority rules.[1] Another example is the European Council. When taking decisions on particularly sensitive areas such as asylum, taxation and the common foreign and security policy, the Council must be in unanimous agreement. Being an European member is, however, beneficial as it implies economies of scale and more coordinated policies. Similar considerations apply to the IMF or the European Monetary Union. Second, in industrial organization, research ventures and cartels are examples of groups that benefit from economies of scale. For instance, d'Aspremont and Jacquemin (1988) and Kamien and Zang (1993) study the formation of cooperative research ventures where firms benefit from cost-reduction. Similarly, Nocke (1999) studies the formation of cartels when firms face capacity constraints. Firms in a cartel benefit from increased capacity. In all these examples, the decisions a cartel takes e.g., which R&D projects to finance, are often compromises resulting from lengthly negotiations, and are likely to differ from the decision a single firm might take on its own. All these examples illustrate the ubiquity of our trade-off. In fact, it is hard to imagine situations, where it does not apply.

We propose a simple model to study the interplay between the governance of a group and its (endogenously derived) size, composition, and likelihood to change the status quo. In the model, individuals can either participate in a group or stand alone, and we assume that individuals have *private* valuations over two alternatives x and y; y being the status quo. Benefits to participate in a group are modeled as *cost reduction*: the more individuals in the group, the lower the cost per individual of taking action x or y is. Historically, groups have adopted a large variety of governances ranging from unanimity, qualified majority to consensus and many more (see Felsenthal and Machover (1998)). In this paper, we assume that the governance takes the form of a voting system, a practice adopted by many international organizations and boards of shareholders. More precisely, we assume that a quota of $\omega(n) \leq n$ votes is required to change the status quo in a group of n individuals. For instance, unanimity corresponds to $\omega(n) = n$ and simple majority to $\omega(n) = (n+1)/2$ if n is odd, and $n/2$ if n is even.

To get some intuitions on our results, assume that it is costless to maintain

[1] The WTO continues GATTs tradition of making decisions not by voting but by consensus. Where consensus is not possible, the WTO agreement allows for voting. The WTO Agreement envisages several specific situations involving voting, which are governed either by the unanimity rule, or a two-thirds majority rule or a three-quarters majority rule.

the status quo, and that the cost of changing it is equally shared among members of the group. On the one hand, consider an individual who prefers the alternative x over the status quo y. If he participates in a group of n individuals *and* alternative x is voted, then he is better off because of the economies of scale. However, if the group maintains the status quo, he is worse off. On the other hand, suppose that the same individual prefers instead y over x, unless he shares the cost with at least n^* other individuals. The risk for him is now to join a group with less than n^* individuals *and* x being voted. Consequently, upon deciding whether to join a group, an individual has to trade-off the potential cost reduction with the potential risk that his less preferred alternative is chosen.

The governance of a group is therefore at the heart of the above trade-off. It is indeed instrumental in determining the likelihood that the less preferred alternative of an individual is chosen and, therefore, influences the composition and size of the group at an equilibrium. For instance, with unanimity, individuals who prefer the status quo on their own are weakly better off by participating in a group: they can always veto the adoption of x if the group is not large enough to make a change of status quo attractive. Assume all these individuals decide to join the group. Should the individuals preferring a change of status quo also join the group? Cost sharing argues in favor of joining the group, while unanimity makes it harder to change the status quo and argues against joining the group. However, the more individuals in the group, the lower the cost of changing the status quo per individual, and the more likely the status quo changes. And, as the group gets larger, changing the status quo becomes more attractive even for individuals who, on their own, prefer to maintain the status quo. In other words, by joining the group individuals endogenously increase the likelihood that the group changes the status quo. Consequently, unanimity might well lead all or almost all individuals to form the group. To contrast with, majority does not endow individuals with veto power. It is now risky not only for individuals preferring a change of status quo on their own, but also for individuals preferring the status quo on their own to form the group. It follows majority favors the formation of smaller groups than unanimity. Somewhat surprisingly, unanimity might also favors a change of status quo. Intuitively, if larger groups form under unanimity and economies of scale are strong enough, then changing the status quo becomes the best course of action for most individuals in a group. By favoring the formation of smaller groups, majority fails to capitalize on the strong economies of scale. This suggests that unanimity, often blamed for the European inertia of the last two decades, was only a scapegoat: most likely the true culprit is the lack of synergies among European countries. I believe this observation applies also to the popular belief that deciding by consensus is responsible of the relative inertia of international organizations such as the IMF or UN. Furthermore, despites its simplicity, the model is sufficiently rich to account for a wide variety of sizes and compositions of groups as the outcome of a simple but fundamental trade-off between economies of scale and control cost.

Related literature. This paper is part of the abundant literature on coalition formation games. One stream of this literature uses reduced-form models to study the formation of coalitions. This approach is very useful when the objective is not the detailed analysis of the emergence of agreements, but the analysis of their stability. For instance, d'Aspremont et al. (1983) have proposed a simple game with an open membership rule to analyze the external and internal stability of groups. This important contribution opened the way for numerous applications to industrial organization (see Bloch (2003)) and environmental economics (e.g., Barrett (1994)). (See also Hart and Kurz (1983)). Another stream uses extensive-form games which allow one to describe in great details aspects of the bargaining leading to the formation of coalitions. See, among others, Bloch (1996), Genicot and Ray (2003) or Ray and Vohra (1999, 2001). The present paper follows the approach of d'Aspremont et al., in that the group formation game is modeled as an open membership game with, however, incomplete information. The focus of the paper is on stability and governance. More closely related is the literature on the formation of clubs and the provision of local public goods (e.g., Casella (1992), Jehiel and Scotchmer (1997, 2001)). In this literature, as in the present paper, an individual trades off the benefit to participate in a group (sharing the cost of providing a public good) with the "risk" that the group provides a sub-optimal level of the public good from the individual perspective. The present paper differs from this literature in two important respects, however. First, the group formation game is explicitly modeled and analyzed. Second, and more importantly, the main focus of the paper is the interplay between the internal mode of governance of a group, and its size, composition, and propensity to change the status quo. To the best of my knowledge, this has not been the focus of the aforementioned literature.[2] Lastly, the literature on optimal voting schemes e.g., Barbera and Jackson (2004), Maggi and Morelli (2006) or Messmer and Polborn (2004), addresses the complementary issue of group optimal voting schemes. This literature differs from the present paper in that the present paper takes the voting scheme as given and endogenizes the group, while the former literature endogenizes the voting scheme and takes the group as given. A notable exception, however, is Maggi and Morelli (2006). These authors consider a dynamic model where several countries repeatedly vote on various issues, but vote outcomes are not enforceable. They show that the optimal self-enforcing voting scheme is majority if the discount factor is high enough (i.e., punishments are credible) and unanimity, otherwise. The intuition is clear: if the discount factor is not high enough, even the worst punishment (continuation payoff) cannot incentive countries in disagreement with the change of status quo to abide by it. In that case, the status quo has to be maintained, unless all countries agree to change it i..e, the voting rule is unanimity. They then consider the optimal group size and voting scheme at each period, and show that larger groups are associated with higher discount factors. Unanimity is therefore associated with smaller groups. On the surface, this result seems to contrast with our own result. However, both models are hardly comparable as their Maggi and Morelli's is dynamic and mostly concerned with self-enforcing voting, while we consider the stability of (endogenously formed) groups with enforceable voting.

The paper is organized as follows. Section 2 presents the model. The equilibrium analysis is exposed in Sections 3 and 4, while Section 5 contains the main results of the paper on governance and groups. Section 6 presents some extensions. Proofs are collected in the Appendix.

2 A model of group formation

We consider a model with costly actions and N individuals. Individuals can form a group to benefit from cost reduction. However, the decisions the group takes might differ from the decisions an individual would have taken had he be pivotal: this is an implicit cost to join a group. The group governance

[2] See also the literature on the formation of political parties e.g., Besley and Coate (1997), Levy (2004), Osborne and Tourky (2005) for more applications of coalition formation games.

partly determines this implicit cost and, consequently, its size, composition and inclination to change the status quo (to be defined later).

Formally, individuals not participating in the group and the group have to decide, each, whether to maintain the "status quo" (action y) or to change it (action x). For simplicity, we normalize the payoff of the status quo to zero. Taking action x yields a benefit to individual i of $\theta_i b_x$ with $b_x > 0$. Natural interpretations of our model include: adopting a new standard or technology, choosing whether to finance a R&D project and, more broadly, political or economic decisions. The parameter $\theta_i \in [0,1]$ is individual i's *private* valuation (type) of the benefit of taking action x. We assume that it is common knowledge that the $(\theta_i)_{i=1,\ldots,N}$ are the realizations of the random variables $(\tilde{\theta}_i)_{i=1,\ldots,N}$ independently and identically distributed (*i.i.d.*) with distribution μ. Unless indicated otherwise, μ is assumed to be absolutely continuous with respect to the Lebesgue measure.

Furthermore, changing the status quo is costly. We can think of this cost as an administrative cost, the cost to gather and process information, the cost to implement the new technology, etc. The cost to take action x is $c_x(n)$ per individual in a group of n members. We assume that $c_x(\cdot)$ is *non-increasing* in n, and $c_x(1) := c_x$.[3] For instance, if the cost to take action x is fixed, the group might equally share it among its n members, in which case $c_x(n) = c_x/n$. Thus, if an individual is member of a group composed of n individuals and the group takes action x, his payoff is $\theta_i b_x - c_x(n)$, higher than the payoff he gets if he takes action x on his own. By joining a group, an individual benefits from economies of scale (cost reduction).

An important assumption of the model is that the choice of x or y by an individual or the group does not affect the payoff of others. This assumption helps to focus on the interaction between the formation and stability of groups and governance and considerably simplifies the analysis. Moreover, it is natural in some environments. For instance, the choice of x by an individual (a country) or a group might be the provision of new public goods such as hospitals or schools, which cannot be used by other individuals (countries). In that case, only the citizens of country i benefit from the provision of the public good. In some other environments, however, the decision of an individual does impact on the payoff of others e.g., the adoption of trade

[3] All our results go through if we assume $b_x > b_y$, $c_x(n) > c_y(n)$ for each n, and $c_x(n) - c_y(n)$ decreasing in n, with obvious notations.

tariffs or technological standards (e.g., HD DVD vs. Blu Ray discs). In the presence of externalities, the model needs to be appropriately modified.[4]

Governance. A central feature of the model is the governance of a group. Historically, groups have adopted a large variety of governances ranging from voting to consensus without vote (NATO) and many more. Voting, however, is the most common form of governances. We therefore consider voting as the modes of governance in this paper. More specifically, we assume that a quota of $\omega(n)$ votes is required to adopt decision x i.e., to change the status quo, in a group of n individuals. For instance, if $\omega(n) = n$ for any n, a group changes the status quo only if all its members unanimously agree to do so, while if $\omega(n) = (n + 1)/2$ if n is odd and $\omega(n) = 1 + n/2$ if n is even, a simple majority is required to change the status quo. We can already note that since there are only two alternatives x and y, *sincere voting* is weakly dominant regardless of the type of an individual. We focus on equilibria featuring sincere voting in the rest of the paper.

Forming a group. To focus on the interaction between modes of governance, composition and group sizes, we consider a (very) simple two-stage game. In the first stage, all individuals *simultaneously* decide either to participate in a *unique* group, or to stand-alone (open membership game). In the second stage, the members of the group vote for an action to be taken by the group. The stand-alone individuals also choose between x and y. While our model abstracts from interesting aspects of group formation e.g., dynamic formation, entry and exit, multiple groups, it incorporates most of the ingredients needed to meaningfully study the interaction between the stability of a group and its mode of governance. Indeed, we can note that, in equilibrium, the group formed is externally and internally stable i.e., no type of a stand-alone individual has an incentive to join the group (external stability) and no type of a group member has an incentive to leave the group (internal stability). Moreover, Section 6 presents some extensions of the model, e.g., multiple groups or entry and exit, and it is argued that most of the qualitative results obtained in this (voluntarily) simple model remain valid in more general models.

[4]For instance, we can write $\theta_i b_x + B_x(m)$ for the benefit to take action x when individual i's private valuation is θ_i and m individuals take action x: $B_x(m)$ represents the externalities. With such modifications, the equilibrium characterization of Section 4 remains valid.

3 Cost reduction versus loss of control

Notation: Hereafter, $]\underline{a}, \overline{a}[$ denotes the open interval with endpoints $\underline{a}$ and $\overline{a}$ while $(\underline{a}, \overline{a})$ denotes the point in $\mathbb{R}^2$ with coordinates $\underline{a}$ and $\overline{a}$.

In the next two sections, we analyze the group formation game for a given mode of governance. Section 5 will study how equilibria vary as the mode of governance changes. For simplicity, we focus on *symmetric* perfect Bayesian equilibria. We now consider the problem an individual faces in taking his decision whether to participate in a group or to stand-alone.

Suppose that individual i participates in a group of n individuals. If individual i is pivotal (i.e., if he expects exactly $\omega(n) - 1$ members of the group, other than him, to vote for x), his payoff is $\max(\theta_i b_x - c_x(n), 0)$ since by voting x the group takes decision x, and individual i's payoff is $\theta_i b_x - c_x(n)$, while it is 0 if he votes y. Whether individual i, whenever pivotal, takes action x or y depends on his private valuation and the number of individuals participating in the group. If individual i is not pivotal, his vote does not influence the decision of the group, and his payoff is $\theta_i b_x - c_x(n)$ if more than $\omega(n)$ members of the group other than himself vote for x, and 0 otherwise.

Let $s : [0, 1] \to \{0, 1\}$, $\theta_i \mapsto s(\theta_i)$, be a symmetric equilibrium function, where "0" is interpreted as "stand alone" and "1" as "participate," and define

$$\theta^n := \begin{cases} 0 & \text{if} \quad c_x(n) \leq 0, \\ \frac{c_x(n)}{b_x} & \text{if} \quad b_x > c_x(n) > 0, \\ 1 & \text{if} \quad c_x(n) \geq b_x. \end{cases} \tag{1}$$

For $b_x > c_x(n) > 0$, θ^n is the type of an individual that would be indifferent between action x and y in a group of n individuals. Note that θ^n is decreasing in the number n of group members and increasing in the cost c_x of action x. Any member j of a group composed of n individuals votes for x if and only if $\theta_j \geq \theta^n$. Therefore, the probability $\beta(n, s)$ that individual j votes for x, conditional on participating in a group of n individuals and strategy s, is

$$\beta(n, s) := \Pr\left(\theta_j \geq \theta^n \mid \theta_j \in \left\{\theta'_j \in [0, 1] : s(\theta'_j) = 1\right\}\right). \tag{2}$$

Note that β depends on the group size n and its composition i.e., the set of types that join the group, $\{\theta'_j \in [0, 1] : s(\theta'_j) = 1\}$. It follows that the probability that exactly m out of $n - 1$ individuals, other than individual i, vote for x follows a binomial density with parameters $(\beta(n, s), n - 1)$. We denote $\alpha_{n-1}(m, s)$ the probability that exactly m individuals, other than individual i, vote for x in a group of n. In particular, the probability that individual i is pivotal in a group of n individuals is

$$\alpha_{n-1}(\omega(n)-1, s) = \beta(n, s)^{\omega(n)-1}(1-\beta(n, s))^{n-\omega(n)}\binom{n-1}{\omega(n)-1}.$$

It is important to stress that the probability to be pivotal depends not only on the mode of governance (through $\omega(n)$), but also on the size of the group and its composition (through $\beta(n, s)$).

The probability that any individual $j \neq i$ joins the group in a symmetric equilibrium is $\mu(\{\theta_j' \in [0,1] : s(\theta_j') = 1\})$ and since types are *i.i.d.*, the probability that exactly $(n-1)$ individuals other than i join the group is

$$\begin{aligned}\varphi(n-1, s) := \big[\mu(\{\theta_j' \in [0,1] : s(\theta_j') = 1\})\big]^{n-1} \\ \big[1-\mu(\{\theta_j' \in [0,1] : s(\theta_j') = 1\})\big]^{N-n}\binom{N-1}{n-1},\end{aligned} \tag{3}$$

a binomial density with parameters $(\mu(\{\theta_j' \in [0,1] : s(\theta_j') = 1\}), N-1)$.

It follows that the expected payoff of individual i of type θ_i to join the group is:

$$\begin{aligned}\mathcal{E}^1(\theta_i, s) :=& \\ \sum\nolimits_{n=1}^{N} \varphi(n-1, s)\alpha_{n-1}(\omega(n)-1, s)\max(0, \theta_i b_x - c_x(n))& \\ +\sum\nolimits_{n=1}^{N} \varphi(n-1, s)\left(\left(\sum\nolimits_{m=\omega(n)}^{n-1} \alpha_{n-1}(m, s)\right)(\theta_i b_x - c_x(n))\right).&\end{aligned} \tag{4}$$

Alternatively, if individual i of type θ_i stands alone, his expected payoff is

$$\mathcal{E}^0(\theta_i) := \max(0, \theta_i b_x - c_x(1)). \tag{5}$$

Note that the expected payoff to participate in a group depends on the equilibrium strategy s. Thus, to characterize the equilibria, we should find a function s^* such that $s^*(p_i) = 1$ if and only if $\mathcal{E}^1(\theta_i, s^*) \geq \mathcal{E}^0(\theta_i)$, and $s^*(\theta_i) = 0$ if and only if $\mathcal{E}^1(\theta_i, s^*) \leq \mathcal{E}^0(\theta_i)$. Despite the simplicity of our model, this task will turn out to be a difficult one.

Observe that if the quota $\omega(n)$ in a group of n individuals decreases, the expected payoff $\mathcal{E}^1(\theta_i, s)$ of types $\theta_i \geq \theta^n$ increases, while the expected payoff $\mathcal{E}^1(\theta_i, s)$ of types $\theta_i < \theta^n$ decreases. There is no monotone relationship between the quota and the expected payoff to join the group.[5]

[5] This will prevent the use of monotone comparative statics technique.

The trade-off between cost reduction and the control cost is not immediately apparent from equations (4) and (5). The next equation highlights this trade-off by writing the difference in payoffs between participating in a group and standing alone:

$$\begin{gathered}\mathcal{E}^1(\theta_i, s) - \mathcal{E}^0(\theta_i) = \\ \sum\nolimits_{n=1}^{N} \varphi(n-1, s) \left[\max(0, \theta_i b_x - c_x(n)) - \max(0, \theta_i b_x - c_x(1)\right] \\ + \sum\nolimits_{n=1}^{N} \varphi(n-1, s) \big(\sum\nolimits_{m=\omega(n)}^{n-1} \alpha_{n-1}(m, s)\big) \left[(\theta_i b_x - c_x(n)) - \max(0, \theta_i b_x - c_x(n))\right] \\ + \sum\nolimits_{n=1}^{N} \varphi(n-1, s) \big(\sum\nolimits_{m=0}^{\omega(n)-2} \alpha_{n-1}(m, s)\big) \left[0 - \max(0, \theta_i b_x - c_x(n))\right].\end{gathered} \tag{6}$$

In equation (6), the second line captures the economies of scale in participating in a group, and is positive. Ceteris paribus, the more individuals are in the group, the higher the gains for individual i to participate in a group. The third and fourth lines capture the cost associated with the loss of control over the decision the group takes and their sum is negative. Conditional on participating in a group of n individuals and not being pivotal, individual i expects the group to take action x with probability $\sum\nolimits_{m=\omega(n)}^{n-1} \alpha_{n-1}(m, s)$ and action y with probability $\sum\nolimits_{m=0}^{\omega(n)-2} \alpha_{n-1}(m, s)$. Moreover, his payoff is $(\theta_i b_x - c_x(n))$ if action x is taken and 0, otherwise. Were individual i pivotal, his expected payoff would be $\max(0, \theta_i b_x - c_x(n))$. It follows that individual i's implicit cost to participate in the group is indeed given by the sum of the third and fourth lines in equation (6). It is worth noting that conditional on being in a group of n individuals, the cost of losing control is increasing in the quota $\omega(n)$ if $\theta_i > \theta^n$, and decreasing in the quota if $\theta_i < \theta^n$. Indeed, if individual i's type θ_i is greater than θ^n, he prefers action x to be chosen, but a larger quota makes it harder to change the status quo, hence to adopt action x.

The group governance is thus instrumental in determining the cost of (partly) losing control over the decision the group takes.

4 Equilibrium analysis

As a preliminary observation, note that a symmetric Bayesian equilibrium of the group formation game exists. Intuitively, if each type of each individual conjectures that every type of the other individuals will not participate in the group, then each type is indifferent between standing alone and participating, hence standing-alone is a best reply.[6] Thus, there always exists trivial equilibria in which any type of any individual stands alone. Moreover, observe that if $c_x(N) \geq b_x$, then *any function* $s : [0,1] \to \{0,1\}$ *is an equilibrium function.* Indeed, if the cost $c_x(N)$ of taking x in a group of N individuals (the grand group) offsets the maximal gain b_x to be made, then action y is a strictly dominant action regardless of an individual's type, and thus each type of each individual is indifferent between standing alone and participating in the group.[7] Moreover, the payoff to each individual is zero in any of those equilibria. However, if $c_x(N) < b_x$, it might exist others equilibria. The existence and characterization of such non-trivial equilibria is our next task.

We first start with an important result about the equilibrium functions s, that is, equilibrium functions are the indicator of some intervals.

Proposition 1 *All symmetric equilibrium functions* $s : [0,1] \to \{0,1\}$ *are the indicator of some intervals* $]\underline{\theta}, \overline{\theta}[$ *or* $[\underline{\theta}, \overline{\theta}]$ *.*

Proposition 1 states that any equilibrium has a *double cutoff nature*: for all types $\theta_i \in [0,1]$ such that $\theta_i \leq \underline{\theta}$ and $\theta_i \geq \overline{\theta}$, an individual stands alone.[8] Individuals with "similar" types form the group. The intuition behind this result is simple. The higher θ_i, the higher individual i's payoffs to participate in a group and to stand-alone are. However, the difference of expected payoffs $\mathcal{E}^1(\cdot, s) - \mathcal{E}^0(\cdot)$ is increasing for $\theta_i < \theta^1$ and decreasing for $\theta_i \geq \theta^1$. Thus, if we find a "low" type $\underline{\theta}$ and a "high" type $\overline{\theta}$ such that these two types are indifferent between participating in the group and standing alone, then every type in-between participates. This result drastically simplifies our problem: we will only need to focus on the change of $\underline{\theta}$ and $\overline{\theta}$ as $\omega(\cdot)$ varies to analyze the impact of group governances on the (expected) size and composition of

[6] Formally, consider the strategy $s^*(\theta_i) = 0$ for all $\theta_i \in [0,1]$. It follows that $\mathcal{E}^0(\theta_i) = \mathcal{E}^1(\theta_i, s^*)$ for all types θ_i, hence it is a best reply for all types of each individual to stand on their own.

[7] If we assume that, whenever indifferent between standing alone and participating in a group, an individual stands alone, then there exists a unique equilibrium in which any type of any individual stands alone.

[8] For the beliefs $\theta_i = \overline{\theta}$ or $\theta_i = \underline{\theta}$, an individual is indifferent between participating in the group and standing alone, hence standing alone is a best-reply. In the sequel, we assume, for simplicity, that *whenever indifferent, an individual stands alone.*

a group. Note that this result is reminiscent of the literature on local public goods, which also find that groups consist of "connected" types.

Before proceeding, two observations are worth making. First, individuals with extremely low valuations (weakly) prefer to stand alone. More precisely, unless the mode of governance is unanimity, participating in the group is a weakly dominated strategy for every types θ_i of an individual with $\theta_i < \theta^N$. To see this, note that for those types, action x is *strictly dominated* by action y regardless of whether they stand alone or participate in a group of any size. Thus, the mere possibility that the group takes action x implies that they prefer to stand on their own: they have nothing to gain from participating in a group. Hence, it follows that $\underline{\theta} \geq \theta^N$. However, if the governance is unanimity, each of these types can veto the adoption of x; participating in the group is then undominated. With unanimity, there might therefore exist equilibria with $\underline{\theta} < \theta^N$. To see this, let us consider a simple example. Suppose that there are two individuals $N = 2$, μ is the uniform distribution on $(0, 1)$, $b_x = 1/2$, $c_x(1) = 3/10$, and $c_x(2) = 1/4$. We have that $\theta^1 = 3/5$ and $\theta^2 = 1/2$, and we can then show that the indicator function of $[0, 5/8]$ is an equilibrium.

The second observation is that not only individuals who would take action x standing on their own, but also individuals who would take action y standing on their own, join the group. Formally, we have $\underline{\theta} < \theta^1 \leq \overline{\theta}$. (A complete proof is found in Appendix.) For instance, it is easy to see that any types of an individual between θ^2 and θ^1 join the group. For those types, the payoff to stand-alone is zero, while their payoff to be in a group of two individuals or more is strictly positive. However, for individuals with types between θ^3 and θ^2, matters are more complicate as there is the risk to be in a group of only two individuals and action x being taken (action x has negative payoff for those types). Similarly, individuals with types above θ^1 might join the group if the likelihood of action y being chosen is sufficiently small.[9] Again, the likelihood of an action to be taken depends on the governance.

We can now continue the equilibrium characterization. To be an equilibrium, the two thresholds $\underline{\theta}$ and $\overline{\theta}$ have to balance two opposite forces. First, if individuals with valuations above $\overline{\theta}$ were to join the group, the likelihood to change the status quo increases, hence causing individuals with valuations close to $\underline{\theta}$ to leave the group. Second, if individuals with valuations below

[9]Note that with unanimity, joining the group is weakly dominant for all types $\theta_i < \theta^1$.

$\underline{\theta}$ were to join the group, the likelihood to change the status quo decreases, hence causing individuals with valuations close to $\overline{\theta}$ to leave the group if the increased economies of scale do not offset the increased "control cost." In equilibrium, these two forces have to be *exactly* balanced.

From Proposition 1, knowing the open interval $]\underline{\theta}, \overline{\theta}[$ is isomorphic to knowing the strategy s, and, thus, we "substitute" s by $\underline{\theta}, \overline{\theta}$ in Equations (2)-(4). For instance, the probability that any individual participates in the group is $\mu(]\underline{\theta}, \overline{\theta}[)$ since $\{\theta_i \in [0,1] : s(\theta_i) = 1\} =]\underline{\theta}, \overline{\theta}[$ in a symmetric equilibrium. Quite naturally, we characterize a non-trivial equilibrium as the zero of a map and show that such a zero exists. Define the map $\Gamma : \Sigma := \left\{(\underline{\theta}, \overline{\theta}) \in [0,1] \times [0,1] : \overline{\theta} \geq \underline{\theta}\right\} \to \mathbb{R}^2$, with

$$\Gamma(\underline{\theta}, \overline{\theta}) = \left(\begin{array}{c} \Gamma^1(\underline{\theta}, \overline{\theta}) \\ \Gamma^2(\underline{\theta}, \overline{\theta}) \end{array} \right) := \left(\begin{array}{c} \mathcal{E}^1(\underline{\theta}, \underline{\theta}, \overline{\theta}) - \mathcal{E}^0(\underline{\theta}) \\ \mathcal{E}^1(\overline{\theta}, \underline{\theta}, \overline{\theta}) - \mathcal{E}^0\left(\overline{\theta}\right) \end{array} \right). \tag{7}$$

Note that the map Γ is a continuous function of $\underline{\theta}$ and $\overline{\theta}$. An equilibrium $(\underline{\theta}, \overline{\theta})$ is the solution of $(\underline{\theta}, 1 - \overline{\theta}) \cdot \Gamma(\underline{\theta}, \overline{\theta}) \geq 0$, with $\Gamma(\underline{\theta}, \overline{\theta}) = 0$ if $(\underline{\theta}, \overline{\theta}) \neq (0,1)$. As already mentioned, the set $\left\{(\underline{\theta}, \overline{\theta}) : \underline{\theta} = \overline{\theta}\right\}$ is contained in $\Gamma^{-1}(0) := \left\{(\underline{\theta}, \overline{\theta}) : \Gamma(\underline{\theta}, \overline{\theta}) = 0\right\}$.[10] A non-trivial equilibrium $(\underline{\theta}, \overline{\theta})$ is then a zero of Γ, which does not belong to the set $\left\{(\underline{\theta}, \overline{\theta}) : \underline{\theta} = \overline{\theta}\right\}$. In a non-trivial equilibrium, the probability to participate in the group is strictly positive.

Theorem 1 *If $c_x(N) < b_x$, there exists a non-trivial equilibrium.*

Thus, if there are potential gains to form a group, an equilibrium exists in which some types of individuals form a group. Several additional remarks are worth making. First, if $\theta^2 = 0$, *the grand group is the unique non-trivial equilibrium.* Intuitively, if $\theta^2 = 0$, that is if $c_x(2) = 0$ or b_x is infinitely large, every types of any individual in a group of two individuals or more agree that the best action is x. Since there is no disagreement over the best decision to take in a group, the grand group forms. Moreover, participating in a group is a weakly dominant strategy. Second, if in two non-trivial equilibria, the probability to participate in the group is the same, then these two equilibria are identical. Proposition 2 formally states this result.

Proposition 2 *If in two non-trivial equilibria $(\underline{\theta}, \overline{\theta})$ and $(\underline{\theta}', \overline{\theta}')$, the probability to participate in the group is the same, i.e., $\mu(]\underline{\theta}, \overline{\theta}[) = \mu(]\underline{\theta}', \overline{\theta}'[)$, then $(\underline{\theta}, \overline{\theta}) = (\underline{\theta}', \overline{\theta}')$.*

[10]This is equivalent to $s(\theta_i) = 0$ for all $\theta_i \in [0,1]$.

In the previous discussion, we have shown that the group formation game possesses trivial equilibria and, at least, one non-trivial equilibrium.[11] This multiplicity of equilibria should not be too disturbing: it rather nicely mirrors the fascinating variety of forms that groups exhibit in real-life. In the sequel, we assume that individuals coordinate on a most comprehensive equilibrium in order to compare the size and composition of the group as the mode of governance varies.

Definition 1 *An equilibrium* $(\underline{\theta}^*, \overline{\theta}^*)$ *is said to be a most comprehensive equilibrium if there does not exist another equilibrium* $(\underline{\theta}, \overline{\theta})$ *such that* $\mu(]\underline{\theta}, \overline{\theta}[) > \mu(]\underline{\theta}^*, \overline{\theta}^*[)$.

Thus, in a most comprehensive equilibrium, the probability to participate in the group is maximal. From Proposition 2, there exists a unique most comprehensive equilibrium.

A desirable property of a selected equilibrium is efficiency. For games of complete information, the concept of efficiency is clearly defined. However, for games of incomplete information, as ours, the concept of efficiency becomes more difficult to apprehend. In this paper, we use the concepts of *interim* efficiency (see Hölmstrom and Myerson (1983)).[12] If every individual prefers a given equilibrium over an alternative equilibrium when he knows his type, whatever his type might be, then the given equilibrium *interim* dominates the alternative one. And we say that an equilibrium is *interim* efficient if there exists no other equilibrium that *interim* dominates it. Thus, *interim* efficiency is the appropriate concept of efficiency for games of incomplete information in which the individuals already know their types when the play of the game begins.

For any mode of governance but unanimity, the most comprehensive equilibrium is efficient. To see this, consider the most comprehensive equilibrium. For any alternative equilibrium, there exists a set of types of positive measure participating in the group in the most comprehensive equilibrium and standing-alone in the alternative equilibrium; and these types of an individ-

[11] In fact, the argument used to prove the existence of at least one non-trivial equilibrium guarantees than there exists an odd number of non-trivial equilibria. Moreover, they are locally unique.

[12] Hölmstrom and Myerson make the distinction between classical efficiency and incentive-compatible efficiency. In the paper, we refer to their concept of classical efficiency.

ual obtain a higher expected payoff in the most comprehensive equilibrium. Therefore, no alternative equilibrium can interim dominate the most comprehensive equilibrium, hence the most comprehensive equilibrium is interim efficient. With unanimity, however, the most comprehensive equilibrium might not be efficient (see the numerical example in Section 5). Yet, the most comprehensive equilibrium has another interesting property: it minimizes the total expected cost under mild conditions.

Proposition 3 *Assume that the cost function satisfies:* $\lim_{n\to+\infty} c_x(n) = 0$, $nc_x(n)$ *is increasing in* n, *and* $\lim_{n\to+\infty} nc_x(n) < +\infty$. *There exists an integer* $\widehat{N}$ *such that for* $N > \widehat{N}$, *the most comprehensive equilibrium minimizes the total expected cost.*

Note that if the cost c_x of taking action x is equally shared among the group members, i.e., $c_x(n) = c_x/n$, then the assumptions of Proposition 3 are satisfied. To fix idea, suppose (for the time being) that all individuals have chosen action x and there are n individuals in the group. The total cost is $(N-n)c_x + nc_x(n)$, a decreasing function of n. The more individuals are in the group, the lower the total cost is. This is the main idea behind Proposition 3. However, matters are more complex since even though it is less costly for the group to take action x, the group might choose the costly action x more often. To get intuition for this, compare the total expected cost $Nc_x(1)\mu([\theta^1, 1])$ if all individuals stand alone and the total expected cost

$$Nc_x(N)\left(\sum_{m=\omega(N)}^{N} \mu([\theta^N, 1])^m (1-\mu([\theta^N, 1]))^{N-m}\binom{N}{m}\right),$$

if all individuals participate in the group. We indeed have cost reduction $c_x(N) < c_x$, but the group might take action x with a higher probability. Hence, an extremely large group might not be socially efficient. The conditions stated in Proposition 3 guarantees that the largest group is socially desirable, however.

5 Composition, size and governances

The aim of this section is to compare the composition, (expected) size and likelihood to change the status quo of the group at the most comprehensive equilibrium under two important modes of governance: unanimity $\omega(n) = n$ and qualified majority $\lfloor n/2 \rfloor + 1 \leq \omega(n) < n$, for all n. In particular, we want to confront the widespread belief that unanimity favors the formation of larger but less pro-active groups than majority with our theoretical predictions.

Formally, we define the likelihood to change the status quo as the probability that a group of at least two individuals forms and takes decision x i.e.,

$$\sum_{n=2}^{N} \varphi(n,s) \left(\sum_{m=\omega(n)}^{n} \alpha_n(m,s) \right).$$

We first present a simple numerical example.

5.1 A numerical example

Let N=3 and assume that μ is the uniform distribution on $[0,1]$, $b_x = 1$, $c_x(1) = 0.50$, $c_x(2) = 0.25$ and $c_x(3) = 0.16$. If an individual of type θ_i stands alone, his payoff is $\mathcal{E}^0(\theta_i) = \max(0, \theta_i - 0.50)$. We now compute the (expected) payoff to join the group under unanimity and simple majority.

Unanimity. With unanimity, the group adopts decision x if only if all its members unanimously agree to do so, that is, $\omega(n) = n$ for $n \in \{1,2,3\}$. From Eq. (4), the (expected) payoff of an individual of type θ_i to participate in the group is (after simplifications) :

$$\begin{aligned} \mathcal{E}^1_{una}(\theta_i, \underline{\theta}, \overline{\theta}) = (1 - (\overline{\theta} - \underline{\theta}))^2 \max(0, \theta_i - 0.50) + \\ 2(1 - (\overline{\theta} - \underline{\theta}))(\overline{\theta} - \min(\max(\underline{\theta}, 0.25), \overline{\theta})) \max(0, \theta_i - 0.25) + \\ (\overline{\theta} - \min(\max(\underline{\theta}, 0.16), \overline{\theta}))^2 \max(0, \theta_i - 0.16). \end{aligned}$$

Majority. With three individuals, majority differs from unanimity only in the event that all the individuals participate in the group. Moreover, an individual is pivotal in a group of three individuals only in the event that exactly one of the two other individuals vote for x. The (expected) payoff of an individual of type θ_i to participate in a group is therefore:

$$\begin{aligned} \mathcal{E}^1_{maj}(\theta_i, \underline{\theta}, \overline{\theta}) = (1 - (\overline{\theta} - \underline{\theta}))^2 \max(0, \theta_i - 0.50) + \\ 2(1 - (\overline{\theta} - \underline{\theta}))(\overline{\theta} - \min(\max(\underline{\theta}, 0.25), \overline{\theta})) \max(0, \theta_i - 0.25) + \\ 2(\overline{\theta} - \min(\max(\underline{\theta}, 0.16), \overline{\theta}))(\max(\min(\overline{\theta}, 0.16), \underline{\theta}) - \underline{\theta}) \max(0, \theta_i - 0.16) + \\ (\overline{\theta} - \min(\max(\underline{\theta}, 0.16), \overline{\theta}))^2 (\theta_i - 0.16). \end{aligned}$$

What is the most comprehensive equilibrium under unanimity and majority? The most comprehensive equilibrium is $(0, 1)$ under unanimity while it is $(0.16, 1)$ under majority.[13] Intuitively, under both governances, the benefit to be made for cost sharing largely offsets the "control cost." Indeed, in a group of two individuals, the cost c_x is halved and almost halved again in a group of three individuals. Moreover, the probability to change the status quo is about 0.60 ($\approx 0.84^3$) under unanimity and 0.80 ($\approx 0.84^3 + 3 \times 0.16 \times 0.65^2$) under majority. This example confirms the widespread belief that unanimity favors the formation of larger groups and majority the formation of more pro-active groups.

However, assume instead that $c_x(1) = 0.28$ i.e., there are less economies of scale in joining a group. (Preferences are more "homogeneous" too.) Under unanimity, $(0, 1)$ is not an equilibrium anymore. If individuals with very low valuations join the group, it is optimal for individuals with very high valuations to not join the group. For individuals with high valuations, the "control cost" is now much larger than the gain to be made from cost sharing. The most comprehensive equilibrium under unanimity is $(0.11, 1)$.[14] Intuitively, if a group of two or three individuals forms, the "control cost" is relatively small as only individuals with valuations in $(0.11, 0.16)$ (respectively $(0.11, 0.25)$) would veto a change of status quo in a group of three (respectively two). Since the likelihood of forming a group of two or three individuals and the status quo being maintained is relatively low, individuals with very high valuations have an incentive to join the group (because of cost sharing).[15] The probability to change the status quo is 0.73.

With majority, the most comprehensive equilibrium is $(0.16, 0.55)$. From the payoff to join a group under majority, it is clear that $\underline{\theta} = 0.16$ as it is the unique non-trivial solution to $\mathcal{E}^1_{maj}(\underline{\theta}, \underline{\theta}, \overline{\theta}) = 0$. And the largest solution of $\mathcal{E}^1_{maj}(\overline{\theta}, 0.16, \overline{\theta}) = \overline{\theta} - 0.28$ is 0.55. Since it is weakly dominant for all individuals with valuations between $(0.16, 0.28)$ to join the group, individuals

[13] Note that if $(1 - \theta_3)^2(1 - c_x(3)) \geq (1 - c_x(1))$, then $(0, 1)$ is the most comprehensive equilibrium under unanimity.

[14] The indicator of $(0.16, 1)$ is also an equilibrium and Pareto dominates the most comprehensive equilibrium, hence proving that the most comprehensive equilibrium might not be interim-efficient under unanimity.

[15] Note that it is a weakly dominant strategy for individuals with valuations in $(0.16, 0.28)$ to join the group.

with very high valuations prefer to stay on their own. Those individuals require larger (expected) economies of scale to offset the "control cost" and, consequently, more individuals in the group. However, with majority, it is weakly dominant for individuals with valuations smaller than 0.16 to stand on their own. As with the IMF weighted scheme discussed in the introduction, majority makes it easier to change the status quo, but also to maintain it. Moreover, the probability to change the status quo is only 0.22! Therefore, not only unanimity might favor the formation of larger groups, but also of more pro-active groups. The next section provides more general results.

5.2 Unanimity vs. Majority

To start with, let us consider the unanimity rule. The payoff to individual i of type θ_i is:

$$\mathcal{E}^1_{una}(\theta_i, s) = \sum_{n=1}^{N} \varphi(n-1, s)\alpha_{n-1}(n-1, s)\max(0, \theta_i b_x - c_x(n)). \quad (8)$$

From Eq. (8), we deduce that for all types $\theta_i \in]\theta^N, \theta^1]$ of individual i, it is weakly dominant to participate in the group, while types in $[0, \theta^N]$ are indifferent. Indeed, with unanimity, each individual has the power to veto a change of status quo and, therefore, individuals with types below θ^1 (weakly) prefer to join the group. However, types above θ^1 prefer the alternative x, and joining a group entails the risk to be vetoed. Therefore, some might join, some might not.

Before presenting general results on governances and groups, let us consider the simple case in which no individual on their own finds it profitable to change the status quo i.e., $c_x > b_x$ ($\theta^1 = 1$). We already know from the above arguments that $\underline{\theta}_{una} \leq \theta^N$ in any non-trivial equilibrium with unanimity, while $\underline{\theta}_{maj} \geq \theta^N$ in any equilibrium with a qualified majority. Since $\theta^1 = 1$, it follows that $[0, 1]$ is the most comprehensive equilibrium with unanimity. With qualified majority, the most comprehensive equilibrium is $[\underline{\theta}_{maj}, 1]$ with $\underline{\theta}_{maj} \geq \theta^N$. We therefore have that the expected size of the group is larger under unanimity than majority, a prediction that confirms our intuition. Furthermore, we have the following result about the likelihood to change the status quo.

Proposition 4 *If $\mu([\theta^N, 1])^N \geq (1 - \mu([0, \theta^N]))^{N-1}((N-1)\mu([\theta^2, 1]) + 1)$ and $c_x > b_x$, unanimity not only maximizes the expected size of the group, but also the probability to change the status quo.*

The intuition behind Proposition 4 is as follows. We already know that if $c_x \geq b_x$ (i.e., $\theta^1 = 1$), unanimity favors the formation of larger groups than majority. In fact, with unanimity, the grand group forms and the probability to change the status quo is $\mu([\theta^N, 1])^N$. Moreover, with majority, any "group" of size 1 maintains the status quo since $\theta^1 = 1$, and this occurs with probability $N\mu([\underline{\theta}_{maj}, 1])^1\mu([0, \underline{\theta}_{maj}))^{N-1}$. In any most comprehensive equilibrium $(\underline{\theta}_{maj}, 1)$ with majority, this probability is bounded from below by $N\mu([\theta^2, 1])^1\mu([0, \theta^N))^{N-1}$ since $\theta^N \leq \underline{\theta}_{maj} < \theta^2$ in equilibrium. It then follows that the probability to change the status quo is bounded from above by the right-hand side of the inequality presented in Proposition 4 (see the proof for more details.) This sufficient condition is not easily satisfied, however. That is not to say that majority "generically" favors the formation of more pro-active groups. The upper bound derived in Proposition 4 assumes that any group of two or more individuals changes the status quo with probability one and, consequently, is not the tightest possible. Moreover, it is worth pointing out that if there are strong economies of scale in forming large groups, i.e., $\lim_{n\to\infty} c_x(n) = 0$, then majority always favor the formation of more pro-active groups for N large enough.

We now turn to the general case in which some individuals find it profitable to change the status quo even standing on their own i.e., $c_x < b_x$ ($\theta^1 < 1$). The next proposition presents a condition under which unanimity again favors the formation of larger groups than majority.

Proposition 5 *If* $n \ln(\mu([\theta^{n+1}, \theta^n])) - \ln(\mu([0, \theta^{n+1}])) \geq 0$ *for any* $n \geq 1$*, then unanimity favors the formation of larger groups than majority.*

Before giving the intuition behind Proposition 5, let us first interpret the joint condition on the indifference thresholds $(\theta^n)_{n=1,\dots,N}$ and the distribution of types μ. Suppose that μ is the uniform distribution, the condition then states that the total cost of changing the status quo is decreasing in the size of the group i.e., $nc_x(n) \geq (n+1)c_x(n+1)$. Equivalently, the cost per individual to change the status quo is rapidly decreasing as the size of the group increases.[16] More generally, the condition implies that economies of scale are growing as the size of the group increases; the rate of growth being determined by the distribution μ. In turn, this rapid growth of economies of scale implies that the expected gain to join the group offsets the risk that

[16] In particular, it implies that $\lim_{n\to+\infty} c_x(n) = 0$.

individuals in the group do not unanimously agree to change the status quo. It follows that the most comprehensive equilibrium with unanimity is the indicator of $[0, 1]$ and, therefore, unanimity favors the formation of larger groups than majority. This is the main intuition behind Proposition 5. The next proposition complements Proposition 5: it states that unanimity leads to the formation of more pro-active groups only if it favors the formation of larger groups than majority.

Proposition 6 *If the expected size of the group with unanimity is smaller than the expected size of the group with majority, then majority favors the change of status quo.*

The intuition is again simple. If the expected size of the group is smaller with unanimity than with majority, it means that individuals forming the group under unanimity have lower valuations than those forming the group under majority.[17] Therefore, individuals forming the group under unanimity are less likely to change the status quo. This is a selection effect. Together with Proposition 5, this suggests that unanimity induces more pro-active groups than majority only if economies of scale are rapidly growing in the size of the group.

To sum up, we have seen that not only unanimity might favor the formation of larger groups than majority, but also the formation of more pro-active groups. Large economies of scale are necessary. For otherwise, majority favors the formation of more pro-active groups, although they might be of smaller sizes. Finally, we might wonder whether there is a monotone relationship between the governance of a group and its size, composition, and inclination to change the status quo. While we have not been able to prove more general results (and this despites the simplicity of the model), numerical examples suggest that this is not always the case.[18]

Assume that there are ten players ($N = 10$), μ is the uniform distribution and $b_x = 11$. Table 1 gives the most comprehensive equilibrium thresholds $\underline{\theta}$ and $\overline{\theta}$ along with the probability to change the status quo and the expected group size when $c_x(n) = 10 - (n - 1)$ as a function of the mode of governance. For instance, "majority + 1" corresponds to $\omega(n) = \min(\lceil n/2 \rceil + 1, n)$. For these parameter values, we can note that the higher the quota is, the larger

[17] Remember $\underline{\theta}_{una} \leq \underline{\theta}_{maj}$ in any equilibrium.

[18] Matlab codes are available upon request.

the group is as well as the smaller the probability to change to status quo is. These numerical results agree well with our intuition. Ceteris paribus, the higher the quota is, the lower the likelihood to change the status quo is and, consequently, the lower the incentive for individuals with high valuations to join the group. However, with these parameter values, there are strong economies of scale to join the group ($c_x(1) = 10$ while $c_x(10) = 1$, ten times smaller!). In equilibrium, this latter effect dominates and all types of an individual with $\theta_i > \theta^1$ join the group. It follows that more types of an individual with $\theta_i < \theta^1$ find it profitable to join the group if the mode of governance is "closer" to unanimity.

ω	$\underline{\theta}$	$\overline{\theta}$	prob. to change status quo	group size
unanimity	0.0	1	0.3885	10
majority	0.3579	1	0.9003	6.42
majority +1	0.2725	1	0.8317	7.28
majority −1	0.4413	1	0.9582	5.59
una $n \leq 5$, maj $n > 5$	0.3048	1	0.836	6.95
maj $n \leq 5$, una $n > 5$	0.0911	1	0.5684	9.08

Table 1: $c_x(n) = 10 - (n - 1)$.

Furthermore, we can note that unanimity for small groups ($n \leq 5$) and majority for larger groups ($n > 6$) favors the formation of more pro-active groups, while majority for small groups ($n \leq 5$) and unanimity for larger groups ($n > 6$) favors the formation of larger groups. Thus, following Maggi and Morelli (2006), if the "optimal" mode of governance of larger groups is majority, then stable groups are more likely to be pro-active.

Table 2 below shows, however, that there might not always exist a monotone relationship between mode of governance, group size, and likelihood to change the status quo. For instance, "unanimity −1" favors the formation of a more pro-active group than "majority +1" even though the quota required to change the status quo with "unanimity −1" is higher.

5.3 Efficiency

A natural issue is the characterization of the mode(s) of governance that maximize the total (ex-ante) welfare. Again, despite the simplicity of our

ω	$\underline{\theta}$	$\overline{\theta}$	prob. to change status quo	group size
unanimity	0.0	0.9094	0.0004	9.01
majority	0.5759	1	0.812	4.24
majority +1	0.5455	1	0.6542	4.54
unanimity −1	0.561	1	0.7385	4.39

Table 2: $c_x(n) = 10 - (n-1)$ if $n \leq 5$, $c_x(n) = 6$ if $n \geq 6$.

model, the complexity of the equilibrium characterization does not make it possible to satisfactorily address this issue.[19] However, we can address a more modest issue. Assume that $\theta^N = 0$ and, consequently, the unique efficient outcome is for the grand group to form. Does some modes of governance lead to the formation of the grand group, while others do not? Clearly, regardless of the mode of governance, the most comprehensive equilibrium is for each type of each individual to join the group (i.e., the indicator function of [0,1]). To discriminate among modes of governance, we focus on the *optimal* modes of governance for which forming the grand group is the *unique* profile of strategies that survives iterated deletion of weakly dominated strategies. Our next proposition states that unanimity is the unique optimal mode of governance under a mild assumption.

Proposition 7 *Let $\theta^{N-1} > 0$. If the following condition holds*

$$\mu(c_x(2)/b_x, c_x(1)/b_x)^N (c_x(1) - c_x(2)) - \sum_{n=1}^{N} \mu(0, c_x(2)/b_x)^n \mu(c_x(2)/b_x, c_x(1)/b_x)^{N-n} \binom{N}{n} (b_x - c_x(1)) \geq 0, \tag{9}$$

then unanimity is optimal, while all other modes of governance are not.

The intuition for Proposition 7 is simple. With unanimity, it is clearly weakly dominated for individuals with types lower than θ^1 to stand on their own. Let us delete all strategies such that $s(\theta_i) = 0$ for all $\theta_i \in [0, \theta_1]$. Turning to individuals with types higher than θ^1, the smallest expected gain to join the

[19]There are several technical problems: 1) the payoff to participate to the group has points of non-differentiability, 2) even if we can apply the implicit function theorem, we cannot sign the derivatives e.g., $d\overline{\theta}/d\omega(n)$ cannot be signed, and 3) monotone comparative statics a la Milgrom and Roberts (1994) does not work either since $\mathcal{E}^1$ is not monotone in $\omega(n)$.

group is given by the first line of Equation (9), while the largest loss to join the group is given by the second line of Equation (9). Under the condition stated in Proposition 7, it follows that it is weakly dominated for individuals with types higher than θ^1 to stand on their own. With a mode of governance other than unanimity, standing on their own is weakly dominated for any type θ_i^* in $[0, \theta^{N-1})$ only if all other types join the group, i.e., if all strategies but $s(\theta_i) = 1$ for all $\theta_i \in [0,1] \setminus \{\theta_i^*\}$ have been deleted. For such a type, joining the group gives a positive payoff only if the grand group is formed. However, at best, we can only delete all strategies such that $s(\theta_i) = 0$ for all $\theta_i \in [\theta^{N-1}, 1]$. Note that this result does not depend on the order of deletion of weakly dominated strategies.

Thus, if the objective of a social planner is to (fully) implement the unique efficient outcome, i.e., forming the grand group, as the unique outcome of the process of iterated deletion of weakly dominated strategies, then unanimity has a strong appeal.

6 Extensions

In this section, we propose some extensions of the model and discuss the robustness of our results.

Complete information. An important assumption of the model is that the valuations $(\theta_i)_{i=1,\ldots,N}$ are private information of each individual. This assumption is crucial for the mode of governance to matter. To see this, assume that types are commonly known and define $\mathcal{C}^* := \{i : \theta_i \geq \theta^{n^*}\}$ with $n^* = \max\{n \in \{1, \ldots, N\} : |\{i : \theta_i \geq \theta^n\}| \geq n\}$ as the largest group whose all members agree to change the status quo. We can then show that the strategy profile $s_i = 1$ for all $i \in \mathcal{C}^*$, and $s_i = 0$ otherwise, is the most comprehensive Nash equilibrium of our game.[20] Moreover, this is regardless of the mode of governance.

[20]To see this, observe that $|\{i : \theta_i \geq \theta^{n^*}\}| = n^*$. If not, we have $|\{i : \theta_i \geq \theta^{n^*}\}| = m > n^*$ implying that $|\{i : \theta_i \geq \theta^m\}| \geq m$ since $\theta^m \leq \theta^{n^*}$, a contradiction with the definition of n^*. Individuals in $\mathcal{C}^*$ have clearly no incentives to deviate. As for individuals not in $\mathcal{C}^*$, suppose that one of them deviates. The size of the group is then $n^* + 1$, and the deviation is profitable to player i only if $\theta_i > \theta^{n^*+1}$, which is impossible by definition of n^*. The proof that this is the most comprehensive equilibrium is easily made by contradiction and available upon request.

Entry and exit. An implicit assumption of the model is that members of the group cannot exit the group after either observing how many individuals join the group or the vote outcome. This assumption is reasonable if there is a sufficiently high cost to exit the group. However, our qualitative results are not altered if individuals can exit the group. Indeed, note that if individuals can exit the group after their initial decision to enter the group, then joining the group at the initial stage is weakly dominant. An individual can always exit the group later and gets his stand-alone payoff. It follows that if exit can only take place after the initial decision to enter the group (i.e., after observing how many individuals have decided to join the group), then all equilibria are equivalent to the ones analyzed in this paper.

If, however, individuals can exit the group after the vote, the equilibria are different but the same trade-off and qualitative results remain. To see this, note that conditional on y being chosen, all individuals with types above θ^1 exit the group while the other types stay. Conditional on x being chosen, we clearly have that all types above θ^1 stay and all types below θ^N exit. It follows that there exists a threshold θ^* such that all individuals with types above θ^* stay in the group (by monotonicity of the payoff in θ_i). Moreover, if $\mu(0, \theta^N) = 0$, then the most comprehensive equilibrium consists of all individuals forming the group and voting for x, regardless of the mode of governance.[21] However, if $\mu([0, \theta^N]) > 0$, then the most comprehensive equilibrium under unanimity is the indicator of $[0, \theta^1]$, while it is the indicator of $[\theta^*, 1]$ under majority. Majority thus favors a change of status quo. And a sufficient condition for unanimity to favor larger groups than majority is $\mu([0, \theta^N]) > 1/2$. This suggests that the qualitative results of this paper are robust to the possibility of exit from the group. A full-fledged analysis of entry and exit is, nonetheless, left for future research.

Many choices. Another important assumption of the model is that the group has to take a unique decision. Instead, suppose that the group has to take T decisions, sequentially. A more complicate trade-off emerges, but the main intuitions are the same. On the one hand, an individual still benefits from economies of scale by participating in the group. On the other hand, he still faces the risk that the group adopts a sequence of decisions that

[21]To see this, consider a strategy profile such that $s(\theta_i) = 1$ for all $\theta_i \in [0, 1]$ i.e., all individuals join the group regardless of their types. Since $\mu([0, \theta^N]) = 0$, all individuals in the group unanimously agree to change the status quo and, consequently, no individual has an incentive to exit the group.

differs from the sequence of decisions the individual would have taken were he pivotal. Alternatively, suppose that after each vote, each member of the group has the option to freely exit the group and each stand-alone individual has the option to freely join the group. The group formation game is then the finite repetition of the (constituent) game analyzed in the present paper. And following the idea found in the literature on repeated games, we can use equilibria of our game to construct equilibrium strategies of this new repeated game.[22]

Multiple groups. As alluded in the introduction, the literature on jurisdictions and the local provision of public goods is closely related to the present work. Following this literature (e.g., Jehiel and Scotchmer (2001)), we define a (symmetric) *free mobility* equilibrium as a finite partition $\{C_k\}_{k=1}^K$ of the space of valuations $[0,1]$ such that the two following conditions hold: 1) for all $\theta_i \in C_k$, $\mathcal{E}(\theta_i, C_k) \geq \mathcal{E}(\theta_i, C_{k'})$ for all k', and 2) $\mathcal{E}(\theta_i, C_k) \geq \max(0, \theta_i b_x - c_x)$.[23] In the definition, the first condition states that the expected payoff of individual i of valuation $\theta_i \in C_k$ is better off joining the group C_k than any other group $C_{k'}$. The second condition simply states that an individual is not compelled to participate in a group, and should get at least his stand-alone payoff $\mathcal{E}^0(\theta_i)$. Note that the definition allows for the existence of several groups. What would be a free mobility equilibrium? First, since payoff functions satisfy a single crossing property, it is immediate to see that groups must be intervals. Second, assume that the mode of governance is not unanimity and $N > 2$. Suppose that there exists a group C_k such that $C_k \cap [0, \theta^N] \neq \emptyset$ and $C_k \not\subseteq [0, \theta^N)$. Clearly, condition 2) of the definition is violated for any $\theta_i \in C_k \cap [0, \theta^N)$. For those types, changing the status quo is strictly dominated regardless of the size of the group and, therefore, the mere possibility that the group C_k changes the status quo (i.e., takes action

[22] Assuming that valuations are independently drawn at each period t.

[23] The expected payoff $\mathcal{E}(\theta_i, C_k)$ is given by:

$$\sum_{n=1}^{N} \mu(\theta_i \in C_k)^{n-1} \mu(\theta_i \notin C_k)^{N-n-1} \binom{N-1}{n-1}$$

$$\left(\sum_{m=\omega(n)}^{n} \mu(\theta_i \geq \theta^n | \theta_i \in C_k)^m \mu(\theta_i < \theta^n | \theta_i \in C_k)^{n-1-m} \binom{n-1}{m} (\theta_i b_x - c_x(n)) + \right.$$

$$\left. \mu(\theta_i \geq \theta^n | \theta_i \in C_k)^{\omega(n)-1} \mu(\theta_i < \theta^n | \theta_i \in C_k)^{n-\omega(n)} \binom{n-1}{\omega(n)} \max(0, \theta_i b_x - c_x(n)) \right).$$

x) implies that their expected payoff is strictly negative in the group C_k. It follows that $[0, \theta^N)$ has to be a group, say C_1.[24] Next, consider the group $C_2 = [\theta^N, \theta^*)$. By continuity of the payoff function, we have that for all valuations in C_2 sufficiently close to θ^N, their expected payoff is strictly negative, which again contradicts condition 2) of the definition. Therefore, no free mobility equilibrium exists. In other words, it is impossible to organize individuals in groups such that all individuals receive their stand-alone payoffs. However, if there are only two individuals i.e., $N = 2$, then $\{[0, \theta^2), [\theta^2, 1]\}$ is a free mobility equilibrium: the first group does not change the status quo while the second does. For $N = 2$, this equilibrium is the *unique* non-trivial equilibrium of the group formation game analyzed in this paper. Lastly, with unanimity, it is easy to see that there exists a $\theta^* \in (\theta^N, \theta^1)$ such that $\{[0, \theta^*), [\theta^*, 1]\}$ is a free mobility equilibrium with the group composed of the individuals with the higher valuations being more likely to change the status quo. This last equilibrium differs from the one analyzed in the paper.

Finally, suppose that individuals can endogenously form several groups or stand alone i.e., the strategy of an individual is a map from $[0, 1]$ to $\{0, 1, \ldots, K\}$ where "0" is interpreted as "stand alone", "k" as "participate in group k." It is immediate to see that the equilibria analyzed in the present paper survive. Indeed, it is a coordination game, and if each individual conjectures that his opponents are using the equilibrium strategy found in this paper i.e., $s(\theta_i) = 0$ if $\theta_i \notin [\underline{\theta}, \overline{\theta}]$ and $s(\theta_i) = k$ if $\theta_i \in [\underline{\theta}, \overline{\theta}]$, then it is a best reply to follow strategy s. Henceforth, most of our results remain valid in this more general model allowing for multiple groups. However, there might exist other equilibria. This is left for future research.

7 Appendix

Proof of Proposition 1

Remember that

$$\mathcal{E}^1(\theta_i, s) :=$$
$$\sum_{n=1}^{N} \varphi(n-1, s)[\alpha_{n-1}(\omega(n) - 1, s) \max(0, \theta_i b_x - c_x(n))$$
$$+ (\sum_{m=\omega(n)}^{n-1} \alpha_{n-1}(m, s))(\theta_i b_x - c_x(n))]$$

[24] Or the union of several groups.

is strictly increasing in θ_i regardless of s, and thus strictly quasi-concave. Define $T := \{\theta_i \in [0,1] : \theta_i < \theta^1\}$, as the set of types that choose action x in a group of one or more individuals, and denote T^c the complement of T in $[0,1]$. In the sequel, we write $\mathcal{E}^1(\theta_i, \cdot)$ for " $\mathcal{E}^1(\theta_i, s)$ for any strategy function s".

Consider $(\theta_i, \theta_i') \in [0,1] \times [0,1]$ such that $\mathcal{E}^1(\theta_i, \cdot) \geq \max(0, \theta_i b_x - c_x)$ $\mathcal{E}^1(\theta_i', \cdot) \geq \max(0, \theta_i' b_x - c_x)$, and any $a \in (0,1)$. We shall show that

$$\mathcal{E}^1(a\theta_i + (1-a)\theta_i', \cdot) > \max(0, (a\theta_i + (1-a)\theta_i') b_x - c_x). \tag{10}$$

First, if $(\theta_i, \theta_i') \in T \times T$, Eq. (10) is trivially satisfied since $\mathcal{E}^1$ is strictly quasi-concave in θ_i. Second, if $\theta_i \in T$, $\theta_i' \in T^c$, and $a\theta_i + (1-a)\theta_i' \in T$, we shall show that

$$\mathcal{E}^1(a\theta_i + (1-a)\theta_i', \cdot) > 0.$$

One again, this is trivially true by the strict quasi-concavity of $\mathcal{E}^1$. Third, if $\theta_i \in T$, $\theta_i' \in T^c$, and $a\theta_i + (1-a)\theta_i' \in T^c$, we shall show that

$$\mathcal{E}^1(a\theta_i + (1-a)\theta_i', \cdot) > (a\theta_i + (1-a)\theta_i') b_x - c_x. \tag{11}$$

To prove this last statement, we first need a Lemma.

Lemma 1 *For all $\theta_i \in T^c$, $\mathcal{E}^1(\theta_i, \cdot) - (\theta_i b_x - c_x)$ is decreasing in θ_i.*

Proof First, observe that for all $\theta_i \in T^c$,

$$\mathcal{E}^1(\theta_i, \cdot) =$$
$$\sum_{n=1}^{N} \varphi(n-1, \cdot) \Big(\sum_{m=\omega(n)-1}^{n-1} \alpha(m, \cdot))(\theta_i b_x - c_x(n)).$$

Its slope λ is thus a point in the set Λ with

$$\Lambda := co \left\{ b_x, \ldots, (\textstyle\sum_{m=\omega(N)-1}^{N-1} \alpha(m, \cdot)) b_x \right\},$$

the convex hull of $\{b_x, \ldots, (\sum_{m=\omega(N)-1}^{N-1} \alpha(m, \cdot)) b_x\}$. We then have

$$\lambda^* := \arg\sup_{\lambda \in \Lambda} \lambda = b_x.$$

Finally, the slope of $\theta_i b_x - c_x$ is b_x, and thus $\mathcal{E}^1(\theta_i, \cdot) - (\theta_i b_x - c_x)$ is decreasing in θ_i. $\square$

By Lemma 1, it thus follows that (11) holds. Similarly, we can show that if $(\theta_i, \theta_i') \in T^c \times T^c$, and $a\theta_i + (1-a)\,\theta_i' \in T^c$, (11) holds. This completes the proof. $\square$

Binomial formula. In this section, we give a result about binomial sums for increasing finite sequences $\{a_n\}_{n=1}^N$. i.e., sequences with $a_1 \le a_2 \le ... \le a_N$. This result is used in a subsequent proof. Consider

$$f(p) = \sum_{n=0}^{N} a_n \binom{N}{n} p^n (1-p)^{N-n}.$$

We want to show that $f(p)$ is increasing in p. Differentiating with respect to p, we have

$$\begin{aligned} f'(p) &= \sum_{n=0}^{N} a_n \binom{N}{n} \left[np^{n-1} (1-p)^{N-n} - (N-n)p^n (1-p)^{N-n-1} \right] \\ &= \sum_{n=0}^{N} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np) \\ &= \sum_{n<Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np) \\ &+ \sum_{n\ge Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np). \end{aligned}$$

For $n < Np$, we have $a_n \le a_{[Np]}$, and since $n - Np < 0$ for such n, it follows that $a_n (n - Np) \ge a_{[Np]} (n - Np)$. Thus, the first summation satisfies

$$\sum_{n<Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np) \ge a_{[Np]} \sum_{n<Np} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np).$$

Similarly, for the second summation it holds that

$$\sum_{n\ge Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np) \ge a_{[Np]} \sum_{n\ge Np} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np),$$

because $a_n \ge a_{[Np]}$ and $n - Np \ge 0$. Combining the two inequalities yields

$$\begin{aligned} f'(p) &\ge a_{[Np]} \sum_{n=0}^{N} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np) \\ &= a_{[Np]} \sum_{n=0}^{N} n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} - Np \sum_{n=1}^{N} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} \\ &= a_{[Np]} (Np - Np) = 0, \end{aligned}$$

which is the desired result. Note that if there is at least one strict inequality between the a_n's, a strict inequality for $f'(p)$ will follow. Moreover, if we consider a decreasing sequence i.e., $a_1 \geq a_2 \geq ... \geq a_N$, the reverse inequality holds.

Proof of Theorem 1

To prove the existence of at least one non-trivial equilibrium, we rely on arguments from Index Theory. Note that we do not use usual fixed point arguments since we cannot guarantee that the domain of $\Gamma(\underline{\theta}, \overline{\theta}) - (\underline{\theta}, \overline{\theta})$ is Σ. Remember that if $N = 2$, there is a non-trivial equilibrium with $(\underline{\theta}, \overline{\theta}) = (\theta^2, 1)$. From now, assume $N \geq 3$.

First, observe that a non-trivial equilibrium necessarily satisfies $(\underline{\theta}, \overline{\theta}) \in T \times T^c \subset \Sigma$ (T^c being the complement of T in $[0, 1]$), with

$$T := \left\{ \theta_i \in [0, 1] : \theta_i < \theta^1 \right\},$$

the set of types that choose action y whenever they stand alone. The proof proceeds by contradiction. First, suppose that $(\underline{\theta}, \overline{\theta}) \in T \times T$, then we have $\mathcal{E}^1(\underline{\theta}, \underline{\theta}, \overline{\theta}) = 0$ from the definition of T and an equilibrium. Since $\mathcal{E}^1$ is increasing in θ_i (see (4)), we then have $\mathcal{E}^1(\overline{\theta}, \underline{\theta}, \overline{\theta}) > 0$, a contradiction. Second, suppose that $(\underline{\theta}, \overline{\theta}) \in T^c \times T^c$, then we have $\mathcal{E}^1(\overline{\theta}, \underline{\theta}, \overline{\theta}) - \overline{\theta} b_x - c_x = 0$ from the definition of T^c and an equilibrium. As already mentioned, $\mathcal{E}^1(\cdot, \underline{\theta}, \overline{\theta}) - \mathcal{E}^0(\cdot)$ is decreasing in θ_i for $\theta_i \in T^C$ (see Lemma 1), hence $\mathcal{E}^1(\underline{\theta}, \underline{\theta}, \overline{\theta}) - \underline{\theta} b_x - c_x > 0$, again a contradiction. Finally, if $(\underline{\theta}, \overline{\theta}) = (0, 1)$, it is trivially true. Therefore, at a non-trivial equilibrium, we have $\underline{\theta} < \theta^1 \leq \overline{\theta}$. This implies that $\beta(n, s) \neq 0$ in any non-trivial equilibrium.

Second, we have $\theta^N \leq \underline{\theta}$ at a non-trivial (undominated) equilibrium if the mode of governance is not the unanimity. Note that since $c_x(N) < b_x$, we have $\theta^N > 0$. By contradiction, suppose that $\theta^N > \underline{\theta}$ at a non-trivial equilibrium, hence all types $\theta_i \in]\underline{\theta}, \theta^N[$ participate in the group. However, for all types $\theta_i \in]\underline{\theta}, \theta^N[$, we have $\mathcal{E}^1(\theta_i, \underline{\theta}, \overline{\theta}) < 0 = \mathcal{E}^0(\theta_i)$ independently of $\overline{\theta}$ since for these types, action x is strictly dominated by y (i.e., $\theta_i b_x < c_x(N)$). Hence $\underline{\theta} \in [\theta^N, \theta^1[$. (In other words, $\mathcal{E}^1(\theta_i, s) < 0$ for any $\theta_i \leq \theta^N$ at any non-trivial equilibrium s with $N \geq 3$.) Similarly, it is easy to see that, independently of $\underline{\theta} \in T$, we have $\overline{\theta} \neq \theta^1$. It follows that a non-trivial equilibrium point $(\underline{\theta}, \overline{\theta})$ necessarily belongs to $[\theta^N, \theta^1[\times]\theta^1, 1]$, an open subset of Σ.

Third, if the mode of governance is unanimity, we might have a non-trivial equilibrium with $\underline{\theta} < \theta^N$ since types $\theta_i \in [\underline{\theta}, \theta^N]$ can veto decision x with probability 1. In other words, $\mathcal{E}^1(\theta_i, \underline{\theta}, \overline{\theta}) = 0 = \mathcal{E}^0(\theta_i)$ for those types.

The last step in proving the existence of a non-trivial equilibrium consists in proving the existence of a zero of Γ. To do so, we construct a mapping (homotopy) $h : [\theta^N, \theta^1] \times [\theta^1, 1] \to \mathbb{R}^2$ that admits a unique zero in the interior of its domain and that has the same degree than Γ, hence Γ admits a zero.[25]

The mapping $(\underline{\theta}, \overline{\theta}) \mapsto h(\underline{\theta}, \overline{\theta})$ is given by:

$$h(\underline{\theta}, \overline{\theta}) = \begin{pmatrix} h_1(\underline{\theta}, \overline{\theta}) \\ h_2(\underline{\theta}, \overline{\theta}) \end{pmatrix} = \begin{pmatrix} \frac{\theta^N + \theta^1}{2} + \underline{\theta} \\ \frac{\theta^1 + 1}{2} - \overline{\theta} \end{pmatrix}.$$

Note that the determinant of the Jacobian matrix of h is -1, hence is of full rank, and the index of h is $+1$. It follows that h has a zero. Moreover, we have the following boundary conditions for h. $\lim_{\underline{\theta} \to \theta^N} h_1(\underline{\theta}, \overline{\theta}) < 0$, $\lim_{\underline{\theta} \to \theta^1} h_1(\underline{\theta}, \overline{\theta}) > 0$, $\lim_{\overline{\theta} \to \theta^1} h_2(\underline{\theta}, \overline{\theta}) > 0$, and $\lim_{\overline{\theta} \to 1} h_2(\underline{\theta}, \overline{\theta}) < 0$. As for Γ, from the above observations, we have the following boundary conditions. $\lim_{\underline{\theta} \to \theta^N} \Gamma_1(\underline{\theta}, \overline{\theta}) \leq 0$, $\lim_{\underline{\theta} \to \theta^1} \Gamma_1(\underline{\theta}, \overline{\theta}) \geq 0$, $\lim_{\overline{\theta} \to \theta^1} \Gamma_2(\underline{\theta}, \overline{\theta}) \geq 0$.

In a technical appendix available upon request, I prove the following:

Corollary A *Let $f : \operatorname{int}[0,1]^n \to \mathbb{R}^n$ be a continuous mapping. If for any $x = (x_1, \ldots, x_i, \ldots, x_n) \in [0,1]^n$ such that $x_i = 0$, $f_i(x) \leq 0$, for any $x = (x_1, \ldots, x_i, \ldots, x_n) \in [0,1]^n$ such that $x_i = 1$, $f_i(x) \geq 0$, then f has a zero in the interior of $[0,1]^n$.*

We can then apply Corollary A to prove the existence of a zero of Γ. More precisely, if $\lim_{\overline{\theta} \to 1} \Gamma_2(\underline{\theta}, \overline{\theta}) \leq 0$, then the existence follows directly from Theorem A. If $\lim_{\overline{\theta} \to 1} \Gamma_2(\underline{\theta}, \overline{\theta}) \geq 0$, we have that $\overline{\theta} = 1$, and the proof follows then by the Intermediate Value Theorem. □

Proof of Proposition 2

Consider two non-trivial equilibria, $(\underline{\theta}, \overline{\theta})$ and $(\underline{\theta}', \overline{\theta}')$, such that the two equilibria have the expected size of the group i.e., $\mu(]\underline{\theta}, \overline{\theta}[) = q = \mu(]\underline{\theta}', \overline{\theta}'[)$. We have to show that $(\underline{\theta}', \overline{\theta}') = (\underline{\theta}, \overline{\theta})$. First, suppose that the mode of governance is unanimity i.e., $\omega(n) = n$ for $n \in N$, and without loss of generality assume $\underline{\theta} < \underline{\theta}' < \overline{\theta} < \overline{\theta}'$. For all $\theta_i \in [0,1]$, a simple computation gives:

$$\mathcal{E}^1_{una}(\theta_i, \underline{\theta}, \overline{\theta}) - \mathcal{E}^1_{una}(\theta_i, \underline{\theta}', \underline{\theta}') =$$
$$\sum\nolimits_{n=1}^{N} \varphi(n-1, \underline{\theta}, \overline{\theta}))[\alpha(n-1, \underline{\theta}, \overline{\theta}) - \alpha(n-1, \underline{\theta}', \overline{\theta}')] \max(0, \theta_i b_x - c_x(n)),$$

[25]Loosely speaking, the degree of a function at a 0 with respect to a bounded, open set counts the solution in that set in a particular way. Two functions have the same degree at 0 if they do no point into opposites directions at the boundary. See Mass-Colell (1985).

since $\varphi(n-1,\underline{\theta},\overline{\theta}) = \varphi(n-1,\underline{\theta}',\overline{\theta}')$ for all $n \in \{0,\ldots,N-1\}$. Moreover, we have $\mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[) \leq \mu(]\max(\underline{\theta}',\theta^n),\overline{\theta}'[)$ with at least one n for which the inequality is strict, hence $\alpha(n-1,\underline{\theta},\overline{\theta}) \leq \alpha(n-1,\underline{\theta}',\overline{\theta}')$ with at least one n for which the inequality is strict. It follows that $0 = \mathcal{E}^1(\underline{\theta},\underline{\theta},\overline{\theta}) < \mathcal{E}^1(\underline{\theta},\underline{\theta}',\overline{\theta}')$ implying that $\underline{\theta}' \leq \underline{\theta}$ for $(\underline{\theta}',\overline{\theta}')$ to be an equilibrium (i.e., $\mathcal{E}^1(\underline{\theta}',\underline{\theta}',\overline{\theta}') = 0$), hence $]\underline{\theta}',\overline{\theta}'[\supset]\underline{\theta},\overline{\theta}[$, contradicting $\mu(]\underline{\theta},\overline{\theta}[) = \mu(]\underline{\theta}',\overline{\theta}'[)$.

For general modes of governance $\omega(\cdot)$, the same arguments apply noticing that $\alpha(\cdot,\underline{\theta}',\overline{\theta}')$ first-order stochastically dominates $\alpha(\cdot,\underline{\theta},\overline{\theta})$. □

Proof of Proposition 3

For any non-trivial equilibrium $(\underline{\theta},\overline{\theta})$, conditionally on n individuals participating in a group, the total expected cost is

$$(N-n)\mu[\overline{\theta},1]c_x + \sum_{m=\omega(n)}^{n} \mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[)^m(1-\mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[))^{n-m}\binom{n}{m}nc_x(n),$$

that is, the probability that $(N-n)$ individuals standing alone choose action x (remember that $\overline{\theta} \geq \theta^1 > \underline{\theta}$ in a non-trivial equilibrium) and the probability that the group chooses action x in a group of n individuals. Moreover, the probability that exactly n individuals participate in the group is

$$\varphi\left(n,\underline{\theta},\overline{\theta}\right) = \left[\mu(]\underline{\theta},\overline{\theta}[)\right]^n \left[1-\mu(]\underline{\theta},\overline{\theta}[)\right]^{N-n}\binom{N}{n}.$$

Hence, the total expected cost is given by

$$Nc_x(1-\mu(]\underline{\theta},\overline{\theta}[))\mu\left([\overline{\theta},1]\right) + \sum_{n=0}^{N}\varphi(n,\underline{\theta},\overline{\theta})\sum_{m=\omega(n)}^{n}\mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[)^m(1-\mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[))^{n-m}\binom{n}{m}nc_x(n). \tag{12}$$

Now consider two non-trivial equilibria $(\underline{\theta}^*,\overline{\theta}^*)$ and $(\underline{\theta},\overline{\theta})$ such that $\mu(]\underline{\theta}^*,\overline{\theta}^*[) > \mu(]\underline{\theta},\overline{\theta}[)$. The first term in Equation (12) is clearly smaller for the equilibrium $(\underline{\theta}^*,\overline{\theta}^*)$ than $(\underline{\theta},\overline{\theta})$. As for the second term, the complexity of the finite binomial sum of terms, which also depends on $\underline{\theta}$ and $\overline{\theta}$, does not make it possible to sign its variation. Nonetheless, it follows from the assumptions in the text that it is bounded. As N gets larger, the variation in the first term dominates the variation in the second term, and thus we can conclude that for two equilibria $(\underline{\theta}^*,\overline{\theta}^*)$ and $(\underline{\theta},\overline{\theta})$ such that $\mu(]\underline{\theta}^*,\overline{\theta}^*[) > \mu(]\underline{\theta},\overline{\theta}[)$, a larger group is socially desirable. □

Proof of Proposition 4

The probability to change the status quo is $(\mu([\theta^N, 1]))^N$ with unanimity at the most comprehensive equilibrium $(0, 1)$.

Let $(\underline{\theta}, 1)$ be the most comprehensive equilibrium with majority. Clearly, we have $\theta^N \leq \underline{\theta} < \theta^2$. The probability P that the group change the status quo with majority is

$$P = \sum_{n=1}^{n=N} \mu([\underline{\theta}, 1])^n \mu([0, \underline{\theta}))^{N-n} \binom{N}{n}$$
$$\left(\sum_{m=\omega(n)}^{n} \left(\frac{\mu([\max(\underline{\theta}, \theta^n), 1])}{\mu([\underline{\theta}, 1])} \right)^m \left(\frac{\mu([0, \max(\underline{\theta}, \theta^n)))}{\mu([\underline{\theta}, 1])} \right)^{n-m} \binom{n}{m} \right). \tag{13}$$

Since $\max(\underline{\theta}, \theta^1) = \theta^1 = 1$, the probability to change the status quo is nil if a group of a single individual is formed, which occurs with probability $N\mu([\underline{\theta}, 1])^1 \mu([0, \underline{\theta}))^{N-1}$. It follows that

$$\begin{aligned} P \quad & < 1 - \mu([0, \underline{\theta}))^N - N\mu([\underline{\theta}, 1])^1 \mu([0, \underline{\theta}))^{N-1} \\ & = 1 - \mu([0, \underline{\theta}))^{N-1}((N-1)\mu([\underline{\theta}, 1]) + 1) \\ & \leq 1 - \mu([0, \theta^N))^{N-1}((N-1)\mu([\theta^2, 1]) + 1), \end{aligned}$$

which gives the condition in the text.

$\square$

Proof of Proposition 5

Assume that the indicator function of $[\underline{\theta}, \overline{\theta}]$ with $\overline{\theta} < 1$ is the most comprehensive equilibrium under unanimity. (Remember that $\underline{\theta} \leq \theta^N$.) We want to show that there exists a $\theta^* > \overline{\theta}$ such that the indicator function of $[\underline{\theta}, \theta^*]$ is also an equilibrium function under the assumption stated in Proposition 5, thus contradicting the assumption that $[\underline{\theta}, \overline{\theta}]$ is the most comprehensive equilibrium.

First, let us show that the sequence

$$\left(\left(\frac{\mu(\theta^n, \theta^*)}{\mu(\underline{\theta}, \theta^*)} \right)^n \right)_{n \in N}$$

is increasing in n. The sequence is increasing if for any $n \geq 1$, we have

$$\left(\frac{\mu(\theta^n, \theta^*)}{\mu(\underline{\theta}, \theta^*)} \right)^n \leq \left(\frac{\mu(\theta^{n+1}, \theta^*)}{\mu(\underline{\theta}, \theta^*)} \right)^{n+1}.$$

Taking Neperien logarithmic on both sides, we have

$$n(\ln(\mu(\theta^n,\theta^*)) - \ln(\mu(\underline{\theta},\theta^*))) \leq (n+1)(\ln(\mu(\theta^{n+1},\theta^*)) - \ln(\mu(\underline{\theta},\theta^*))).$$

This is equivalent to

$$\begin{aligned} n\ln(\mu(\theta^n,\theta^*)) &\leq (n+1)\ln(\mu(\theta^{n+1},\theta^*)) - \ln(\mu(\underline{\theta},\theta^*)) \\ &= (n+1)(\ln(\mu(\theta^{n+1},\theta^n)) + \ln(\mu(\theta^n,\theta^*))) - \ln(\mu(\underline{\theta},\theta^*)). \end{aligned}$$

It follows that a sufficient condition for the sequence to be increasing is

$$n\ln(\mu([\theta^{n+1},\theta^n])) - \ln(\mu([0,\theta^{n+1}])) \geq 0,$$

the condition stated in Proposition 5.

Second, the difference in payoffs $\mathcal{E}^1(\theta_i,\underline{\theta},\theta^*) - \mathcal{E}^1(\theta_i,\underline{\theta},\overline{\theta})$ is:

$$\begin{aligned} &\sum(\varphi(n-1,\underline{\theta},\theta^*) - \varphi(n-1,\underline{\theta},\overline{\theta}))\left(\frac{\mu(\theta^n,\theta^*)}{\mu(\underline{\theta},\theta^*)}\right)^n \max(0,\theta_i b_x - c_x(n)) + \\ &\sum\varphi(n-1,\underline{\theta},\overline{\theta})\left(\left(\frac{\mu(\theta^n,\theta^*)}{\mu(\underline{\theta},\theta^*)}\right)^n - \left(\frac{\mu(\theta^n,\overline{\theta})}{\mu(\underline{\theta},\overline{\theta})}\right)^n\right)\max(0,\theta_i b_x - c_x(n)) \end{aligned}$$

Since the sequence $\left(\left(\frac{\mu(\theta^n,\theta^*)}{\mu(\underline{\theta},\theta^*)}\right)^n \max(0,\theta_i b_x - c_x(n))\right)_{n\in N}$ is increasing, it follows from the Binomial formula (see above) that the first line is positive. It is also easy to check that the second line is positive. Hence, $\mathcal{E}^1(\theta_i,\underline{\theta},\theta^*) \geq \mathcal{E}^1(\theta_i,\underline{\theta},\overline{\theta})$ for all θ_i. Moreover, it is strictly positive for all $\theta_i > \theta^1$. It follows then from the intermediate value theorem that there exists a $\theta^* > \overline{\theta}$ such the indicator function of $[\underline{\theta},\theta^*]$ is an equilibrium. By repeating this argument, we have that at the most comprehensive equilibrium for unanimity, $\overline{\theta} = 1$. □

Proof of Proposition 6 Let the indicator function of $[\underline{\theta},\overline{\theta}]$ and $[\underline{\theta}^*,\overline{\theta}^*]$ be, respectively, the equilibrium under unanimity and majority. Observe that since the expected size of the group under unanimity is smaller than under majority, we have $\underline{\theta} \leq \underline{\theta}^* \leq \overline{\theta} \leq \overline{\theta}^*$. It follows that

$$\mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[) \leq \mu(]\max(\underline{\theta}^*,\theta^n),\overline{\theta}^*[), \tag{14}$$

with a strict inequality if $\mu(]\underline{\theta},\overline{\theta}[) < \mu]\underline{\theta}^*,\overline{\theta}^*[$. The probability to change the status quo with unanimity is written (after simplifications) as

$$(1-\mu(]\underline{\theta},\overline{\theta}[))^N \sum_{n=0}^{N}\left(\frac{\mu(]\max(\underline{\theta},\theta^n),\overline{\theta}[)}{1-\mu(]\underline{\theta},\overline{\theta}[)}\right)^n \binom{N}{n},$$

while the probability to change the status quo with majority is written as

$$(1-\mu(]\underline{\theta}^*,\overline{\theta}^*[))^N$$

$$\sum_{n=0}^{N}\binom{N}{n}\sum_{m=\omega(n)}^{n}\left(\frac{\mu(]\max(\underline{\theta}^*,\theta^n),\overline{\theta}^*[)}{1-\mu(]\underline{\theta}^*,\overline{\theta}^*[)}\right)^m\left(\frac{\mu(]\underline{\theta}^*,\overline{\theta}^*[)-\mu(]\max(\underline{\theta}^*,\theta^n),\overline{\theta}^*[)}{1-\mu(]\underline{\theta}^*,\overline{\theta}^*[)}\right)^{n-m}\binom{n}{m}.$$

Clearly, if $\mu(]\underline{\theta},\overline{\theta}[)=\mu]\underline{\theta}^*,\overline{\theta}^*[$, Proposition 6 follows. To complete the proof, observe that the expected probability to change the status quo is decreasing in $\mu(]\underline{\theta},\overline{\theta}[)$. □

Proof of Proposition 7 The proof follows directly from the discussion in the text. □

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