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INSURANCE AND PROBABILITY WEIGHTING FUNCTIONS

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Insurance and Probability Weighting Functions*

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Abstract

Evidence shows that (i) people overweight low probabilities and underweight high probabilities, but (ii) ignore events of extremely low probability and treat extremely high probability events as certain. Decision models, such as rank dependent utility (RDU) and cumulative prospect theory (CP), use *probability weighting functions.* Existing probability weighting functions incorporate (i) but not (ii). Our contribution is threefold. First, we show that this would lead people, even in the presence of fixed costs and actuarially unfair premiums, to insure fully against losses of sufficiently low probability. This is contrary to the evidence. Second, we introduce a new class of probability weighting functions, which we call *higher order Prelec probability weighting functions*, that incorporate (i) and (ii). Third, we show that if RDU or CP are combined with our new probability weighting function, then a decision maker will not buy insurance against a loss of sufficiently low probability; in agreement with the evidence. We also show that our weighting function solves the St. Petersburg paradox that reemerges under RDU and CP.

Keywords: Decision making under risk; Prelec's probability weighting function; Higher order Prelec probability weighting functions; Behavioral economics; Rank dependent utility theory; Prospect theory; Insurance; St. Petersburg paradox.

JEL Classification: C60(General: Mathematical methods and programming), D81(Criteria for decision making under risk and uncertainty).

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1. Introduction

The explanation of insurance is, rightly, regarded as one of the triumphs of Expected Utility Theory (EU). For example, it is a standard theorem of EU that people will insure fully if, and only if, they face actuarially fair premiums. Since insurance firms have to at least cover their costs, market premiums have to be above the actuarially fair ones. Thus EU provides a completely rational explanation of the widely observed phenomenon of under-insurance. This has the policy implication that if full-insurance is deemed necessary (because of strong externalities for example), then it has to be encouraged through subsidy or stipulated by law.

However, problems remain. The following are three well known examples. First, it is difficult for EU to explain the fact that many people simultaneously gamble and insure; see, for example, Peel, Cain and Law (2005). The gambling and insurance industries are too large and important for such behavior to be dismissed as quirky. Second, EU predicts that a risk averse decision maker will always buy some positive level of insurance, even when premiums are unfair. What is observed is that many people do not buy any insurance, even when available. Indeed for several types of risk, the government has to legislate the mandatory purchase of insurance. Third, when faced with an actuarially unfair premium, EU predicts that a decision maker, who is indifferent between full-insurance and not insuring, would strictly prefer probabilistic insurance to either. However, the experimental evidence is the reverse; see Kahneman and Tversky (1979, pp269-271).

These and other anomalies have motivated a number of alternatives to EU. The most important of these are *rank dependent utility theory* (RDU), see Quiggin (1982, 1993), and *cumulative prospect theory* (CP), see Tversky and Kahneman (1992). Both RDU and CP use probability weighting functions to overweight low probabilities and underweight high probabilities. However, the standard probability weighting functions infinitely overweight infinitesimal probabilities, in the sense that the ratio between the weight and the probability goes to *infinity*, as the probability goes to zero. Our first contribution is to show that this would lead people, when faced with an expected loss, to insure fully against that loss if it is of sufficiently low probability. This result holds even when premiums are actuarially unfair and there are fixed costs of insurance, provided a participation constraint is satisfied. Simulations suggest that this participation constraint is quite mild. This behavior is contrary to evidence, as we shall now see.

Two particularly striking examples are given in the seminal work of Kunreuther et al. (1978). These are the unpopularity of flood and earthquake insurance, despite heavy government subsidy to overcome transaction costs, reduce premiums to below their actuarially fair rates, to provide reinsurance for firms and provide relevant information. This not only contradicts the prediction of EU[1], it also contradicts the predictions of RDU and CP when standard probability weighting functions are used.

While there is considerable evidence that people overweight low probabilities and underweight high probabilities, it is also a common observation that they ignore events of extremely low probability and treat extremely high probability events as certain. We could follow Kahneman and Tversky (1979) and rely on an initial editing phase, where the decision maker chooses which improbable events to treat as impossible and which probable events to treat as certain. No doubt such editing does occur. However, as yet, there is no general theory of the editing phase. Instead, in our second contribution, we propose a new class of probability weighting functions that combine the editing phase with the probability weighting phase. While our proposed functions overweight low probabilities and underweight high probabilities, they also have the feature that the ratio between the weight and the probability goes to *zero* as the probability goes to zero. We call these *higher order Prelec probability weighting functions* because they are generalizations of Prelec's (1998) probability weighting function.

Our third contribution is to show that when RDU or CP is combined with any one of these new probability weighting functions, then a decision maker will not buy insurance against an expected loss of sufficiently low probability; in agreement with the evidence. To quote from Kunreuther et al. (1978, p248) "This brings us to the key finding of our study. The principal reason for a failure of the market is that most individuals do not use insurance as a means of transferring risk from themselves to others. This behavior is caused by people's refusal to worry about losses whose probability is below some threshold."[2]

Blavatskyy (2004) and Rieger and Wang (2006) show that the St. Petersburg paradox reemerges under CP, even with a strictly concave value function. Rieger and Wang (2006) derive a new probability weighting function that solves this paradox. We show that the higher order Prelec probability weighting functions also resolve this paradox.

The paper is structured as follows. In Section 2, we define the concepts of *infinite-overweighting* and *zero-underweighting*. In Section 3 we derive some properties of the Prelec weighting function. In Section 4, we present our proposed weighting function and derive its properties. Sections 5 and 6 derive the implications of the probability weighting functions for insurance when an individual uses, respectively, RDU and CP. Section 7

[1]To quote from Kunreuther et al. (1978, p240) "... a substantial number of those who have sufficient information for making decisions on the basis of the expected utility model frequently behave in a manner inconsistent with what would be predicted by the theory." Other studies that reach a similar conclusion, reviewed by Kunreuther et al. (1978, section 1.4), cover the decisions to wear seat belts, to obtain breast examinations, to stop smoking, to purchase subsidized crime insurance and to purchase flight insurance. The last of these, however, shows that people purchase too much flight insurance, compared to the prediction of EU.

[2]Kunreuther et al. (1978) is a major study, involving samples of thousands, survey data, econometric analysis and experimental evidence. All three methodologies give this same conclusion.

briefly considers the St. Petersburg paradox. Section 8 gives the conclusions.

2. Weighting of probabilities

In this section, we introduce the concepts of *infinite-overweighting* and *zero-underweighting* of *infinitesimal probabilities*. These will be crucial for the rest of the paper.

Definition 1 *: By a probability weighting function we mean a strictly increasing function* $w : [0,1] \to [0,1], w(0) = 0, w(1) = 1.$

Definition 2 *:We say that the probability weighting function, w, infinitely-overweights infinitesimal probabilities, if (a)* $\lim_{p\to 0} \frac{w(p)}{p} = \infty$ *and (b)* $\lim_{p\to 1} \frac{1-w(p)}{1-p} = \infty.$

Definition 3 *: We say that the probability weighting function, w, zero-underweights infinitesimal probabilities, if (a)* $\lim_{p\to 0} \frac{w(p)}{p} = 0$ *and (b)* $\lim_{p\to 1} \frac{1-w(p)}{1-p} = 0.$[3]

3. Prelec's probability weighting function

The Prelec (1998) probability weighting function has the attraction that it is parsimonious and is consistent with much of the available empirical evidence[4]. Therefore, we chose it as our starting point.

Definition 4 *: (Prelec, 1998). By the Prelec function we mean the probability weighting function* $w : [0,1] \to [0,1]$ *given by*

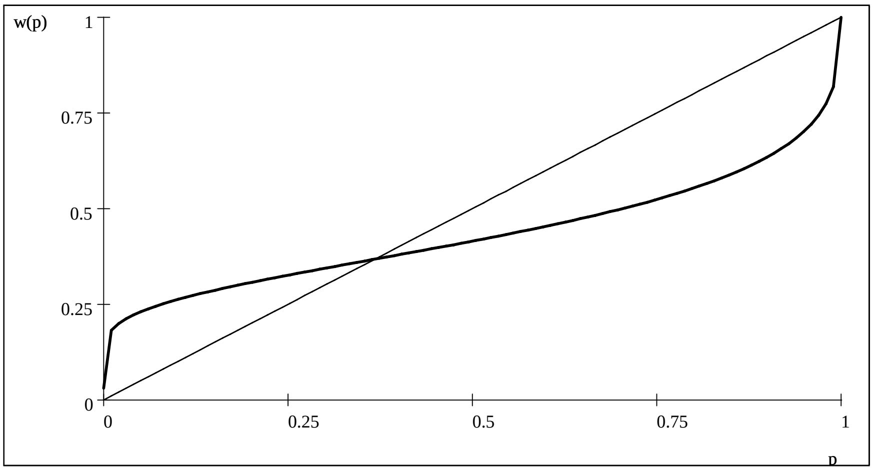
$$w(0) = 0 \tag{3.1}$$

$$w(p) = e^{-\beta(-\ln p)^{\alpha}};\ 0 < p \leq 1,\ 0 < \alpha < 1,\ \beta > 0 \tag{3.2}$$

We produce below, a graph of the Prelec function for $\alpha = 0.35$, $\beta = 1$.

[3]We could introduce the further concepts, *finite-overweighting* : $\lim_{p\to 0} \frac{w(p)}{p}, \lim_{p\to 1} \frac{1-w(p)}{1-p} \in (1,\infty)$ and *finite-underweighting* : $\lim_{p\to 0} \frac{w(p)}{p}, \lim_{p\to 1} \frac{1-w(p)}{1-p} \in (0,1)$. But we will not use these in this paper.

[4]Prelec (1998) gives a derivation based on 'compound invariance', Luce (2001) gives a derivation based on 'reduction invariance' and al-Nowaihi and Dhami (2005) give a derivation based on 'power invariance'. Since the Prelec function satisfies all three, 'compound invariance', 'reduction invariance' and 'power invariance' are all equivalent.



A Prelec function

Proposition 1 *: (Prelec, 1998, p505). For Prelec's function (Definition 2):* $\lim\limits_{p\to 0}\frac{w(p)}{p}=\infty$ *and* $\lim\limits_{p\to 1}\frac{1-w(p)}{1-p}=\infty$*, i.e.,* w*, infinitely-overweights infinitesimal probabilities.*[5]

Proof: For $0<p<1$, (3.2) gives $\ln\frac{w(p)}{p}=\ln w(p)-\ln p=-\beta(-\ln p)^{\alpha}-\ln p=(-\ln p)^{\alpha}\left((-\ln p)^{1-\alpha}-\beta\right)$ and since $0<\alpha<1$, we get $\lim\limits_{p\to 0}\ln\frac{w(p)}{p}=\infty$. Hence, $\lim\limits_{p\to 0}\frac{w(p)}{p}=\infty$. This proves the first part. To prove the second part, note that, as $p\to 1$, $1-w(p)\to 0$ and $1-p\to 0$. Hence, we evaluate $\lim\limits_{p\to 1}\frac{1-w(p)}{1-p}$ using L'Hopital's rule. This gives $\lim\limits_{p\to 1}\frac{1-w(p)}{1-p}=\lim\limits_{p\to 1}\frac{d(1-w(p))}{dp}/\lim\limits_{p\to 1}\frac{d(1-p)}{dp}=\lim\limits_{p\to 1}\frac{dw(p)}{dp}=\lim\limits_{p\to 1}\frac{\alpha\beta w(p)}{p(-\ln p)^{1-\alpha}}=\infty$.■

According to Prelec (1998, p505), these infinite limits capture the qualitative change as we move from certainty to probability and from impossibility to improbability. On the other hand, they contradict the observed behavior that people ignore events of very low probability and treat very high probability events as certain. In sections 5 and 6, below, we show that this leads to people fully insuring against all losses of sufficiently low probability, even with actuarially unfair premiums and fixed costs to insurance. This is

[5]Tversky and Kahneman (1992) propose the following probability weighting function, where the lower bound on γ comes from Rieger and Wang (2006),

$$w(p)=\frac{p^{\gamma}}{\left[p^{\gamma}+(1-p)^{\gamma}\right]^{\frac{1}{\gamma}}},0.5\leq\gamma<1$$

Clearly, this function also has the property $\lim\limits_{p\to 0}\frac{w(p)}{p}=\infty$. It can be shown that other probability weighting functions that have been proposed, for example, Gonzalez and Wu (1999) and Lattimore, Baker and Witte (1992), also have this feature. A notable exception is the probability weighting function of Rieger and Wang (2006).

contrary to observation. See, for example, Kunreuther et al. (1978). Following Kahneman and Tversky (1979), we could rely on an initial editing phase, where the decision maker chooses which improbable events to treat as impossible and which probable events to treat as certain. While we are persuaded by this choice heuristic, as yet, there is no general theory of the editing phase. In the next section, we propose a class of probability weighting functions that combine the editing phase with the probability weighting phase.

4. Higher order Prelec probability weighting functions

Lemma 1 : *(Prelec, 1998, p507, footnote). Prelec's function (Definition 2) can be written as*

$$w(0) = 0, w(1) = 1 \tag{4.1}$$

$$-\ln(-\ln w) = (-\ln\beta) + \alpha(-\ln(-\ln p)), 0 < p < 1 \tag{4.2}$$

Lemma 1 motivates the following development. Assume that $-\ln(-\ln w)$ can be expanded as a power series in $-\ln(-\ln p)$, i.e.,

$$-\ln(-\ln w) = \sum_{k=0}^{\infty} a_k (-\ln(-\ln p))^k, 0 < p < 1 \tag{4.3}$$

Lemma 2 : *If* $\sum_{k=0}^{\infty} a_k (-\ln(-\ln p))^k$ *is convergent, then (4.3) defines a function* $w : (0,1) \to (0,1)$.

Proof: The result follows from the fact that (4.3) is equivalent to

$$w(p) = \exp\left(-\exp\left(-\sum_{k=0}^{\infty} a_k (-\ln(-\ln p))^k\right)\right), 0 < p < 1 \tag{4.4}$$

Definition 5 : *By a higher order Prelec probability weighting function, we mean a probability weighting function (Definition 1) given by (4.1) and (4.4).*

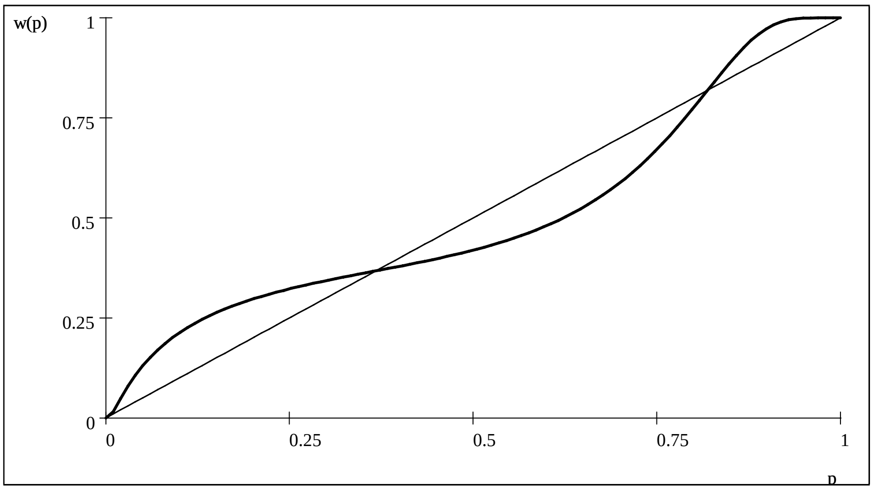
Note that a Prelec function (Definition 4) is a higher order Prelec probability weighting function. Hence, the class of functions defined by Definition 5 is not empty. In what follows, we will show that it is a rich class with interesting members.

Suppose that $a_n \neq 0$ but $a_k = 0$, for all $k > n$. Hence, (4.3) becomes

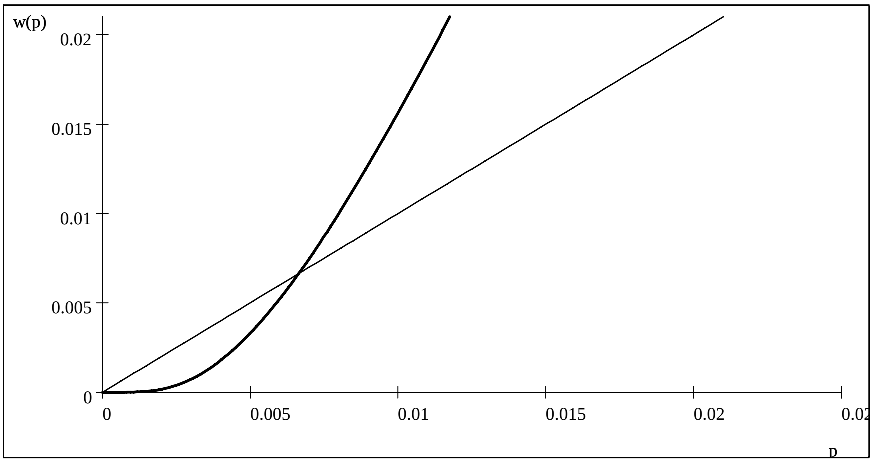
$$-\ln(-\ln w) = \sum_{k=0}^{n} a_k (-\ln(-\ln p))^k, 0 < p < 1, a_n \neq 0 \tag{4.5}$$

Definition 6 : *Let w be a probability weighting function that satisfies (4.1) and (4.5). We call w a Prelec probability weighting function of order n.*

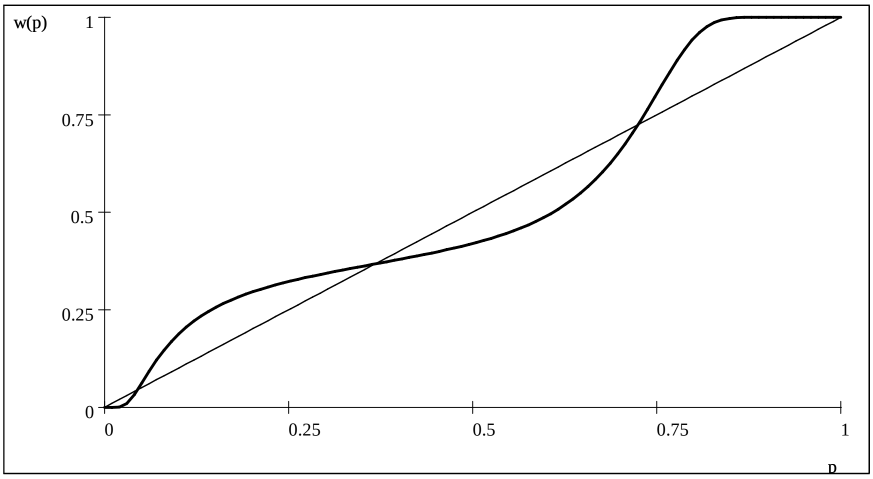
Clearly, a Prelec function is a Prelec probability weighting function of order 1. We produce some graphs below. First we plot a third order Prelec function (i) over the entire probability range and (ii) for low probabilities; we use the following parameter values: $a_1 = 0.35$, $a_3 = 0.25$, $a_0 = a_2 = 0$. Then we plot a fifth order Prelec function using the values: $a_1 = 0.35$, $a_3 = 0.25$, $a_5 = 0.20$, $a_0 = a_2 = a_4 = 0$.



A third order Prelec function



The third order Prelec function for low probabilities



A fifth order Prelec function

The following lemma states, without proof, a few useful mathematical facts.

Lemma 3 *: Let $y(x) = -\ln(-\ln x), x \in (0,1)$. Then*
(i) $y : (0,1) \overset{onto}{\rightarrow} (-\infty, \infty)$
(ii) y is a strictly increasing function of x.
(iii) $y(e^{-1}) = 0$,
(iv) $y(x) < 0 \Leftrightarrow x \in (0, e^{-1})$,
(v) $y(x) > 0 \Leftrightarrow x \in (e^{-1}, 1)$.
(vi) $y(x) \rightarrow -\infty \Leftrightarrow x \rightarrow 0$.
(vii) $y(x) \rightarrow \infty \Leftrightarrow x \rightarrow 1$.

Lemma 4 *: Let $w(p)$ be given by (4.5). Then, for p near 0 and p near 1, the behavior of $w(p)$ is dominated by the behavior of the leading term, $a_n(-\ln(-\ln p))^n$. Specifically, $-\ln(-\ln w) \rightarrow a_n(-\ln(-\ln p))^n$, as $p \rightarrow 0$ or as $p \rightarrow 1$.*

Proof: (4.5) can be written as follows

$$-\ln(-\ln w) = a_n(-\ln(-\ln p))^n \left[1 + \sum_{k=0}^{n-1} \frac{a_k}{a_n} (-\ln(-\ln p))^{-(n-k)}\right], 0 < p < 1, a_n \neq 0 \tag{4.6}$$

Let $p \rightarrow 1$. By Lemma 3 (vii), $-\ln(-\ln p) \rightarrow \infty$. Hence, $(-\ln(-\ln p))^{-(n-k)} \rightarrow 0$ for $k < n$. It follows, from (4.6), that $-\ln(-\ln w) \rightarrow a_n(-\ln(-\ln p))^n$. Let $p \rightarrow 0$. By Lemma 3 (vi) $-\ln(-\ln p) \rightarrow -\infty$. Hence, $(-\ln(-\ln p))^{-(n-k)} \rightarrow 0$ for $k < n$. It follows, from (4.6), that $-\ln(-\ln w) \rightarrow a_n(-\ln(-\ln p))^n$. ■

Proposition 2 : *The following defines a Prelec probability weighting function of order $2n+1$:*

$$w(0) = 0, w(1) = 1 \tag{4.7}$$

$$-\ln(-\ln w) = a_0 + \sum_{k=0}^{n} a_{2k+1} (-\ln(-\ln p))^{2k+1}, 0 < p < 1, a_{2k+1} \geq 0, a_{2n+1} > 0 \tag{4.8}$$

Proof: Since $-\ln(-\ln p)$ is a strictly increasing function of p, $2k+1$ is odd, $a_{2k+1} \geq 0, a_{2n+1} > 0$, it follows that $\sum_{k=0}^{n} a_{2k+1} (-\ln(-\ln p))^{2k+1}$ is a strictly increasing function of p. Hence, from (4.8), $-\ln(-\ln w)$ is a strictly increasing function of p. Hence, w is a strictly increasing function of p. Therefore, (4.7), (4.8) define a Prelec probability weighting function of order $2n+1$. ■

Proposition 2 gives sufficient, but not necessary, conditions for (4.5) to represent a Prelec function of order n. On the other hand, Proposition 3 (a), below, gives necessary, but not sufficient, conditions.

Proposition 3 : *Let $w(p)$ be a Prelec function of order n (Definition 6). Then*
(a) n is odd and $a_n > 0$.
(b) $\lim_{p\to 0} w(p) = 0$ and $\lim_{p\to 1} w(p) = 1$, i.e., $w(p)$ is continuous at zero and at one.
(c) If $n > 1$, then $\lim_{p\to 0} \frac{w(p)}{p} = 0$ and $\lim_{p\to 1} \frac{1-w(p)}{1-p} = 0$, i.e., w, zero-underweights infinitesimal probabilities.

Proof: Since $a_n \neq 0$,we have either $a_n > 0$ or $a_n < 0$. Suppose $a_n < 0$. By Lemma 3 (ii), (v), $-\ln(-\ln p)$ is a positive and strictly increasing function of $p \in (e^{-1}, 1)$. Hence, $(-\ln(-\ln p))^n$ is also a strictly increasing function for $p \in (e^{-1}, 1)$. Thus, $a_n (-\ln(-\ln p))^n$ is a strictly decreasing function for $p \in (e^{-1}, 1)$. From Lemma 4, it then follows that $-\ln(-\ln w)$, and hence w, is a strictly decreasing function for p sufficiently close to 1. But this cannot be, because a probability weighting function is strictly increasing. Hence, $a_n > 0$. Suppose n is even. By Lemma 3 (ii), (iv), $-\ln(-\ln p)$ is a negative and strictly increasing function of $p \in (0, e^{-1})$. Then $(-\ln(-\ln p))^n$, and hence also $a_n (-\ln(-\ln p))^n$, is a strictly decreasing function for $p \in (0, e^{-1})$. From Lemma 4, it then follows that $-\ln(-\ln w)$, and hence w, is a strictly decreasing function for p sufficiently close to 0. But this cannot be, because a probability weighting function is strictly increasing. Hence, n is odd. This proves part (a).

From part (a), Lemma 3 (vi) and (vii), and Lemma 4 we get $p \to 1 \Rightarrow -\ln(-\ln w) \to a_n (-\ln(-\ln p))^n \to \infty \Rightarrow w \to 1$. We also get $p \to 0 \Rightarrow -\ln(-\ln w) \to a_n (-\ln(-\ln p))^n \to -\infty \Rightarrow w \to 0$. This proves part (b).

To prove the first part of (c), write (4.6) as

$$\ln w = -e^{-a_n(-\ln(-\ln p))^n \left[1+\sum_{k=0}^{n-1} \frac{a_k}{a_n}(-\ln(-\ln p))^{-(n-k)}\right]}, 0 < p < 1, a_n > 0, n > 1, n \text{ odd} \tag{4.9}$$

Since $\ln \frac{w}{p} = \ln w - \ln p = \ln w + e^{\ln(-\ln p)}$, (4.9) gives for $0 < p < 1$, $a_n > 0$, $n > 1$, n odd:

$$\ln \frac{w}{p} = e^{\ln(-\ln p)} \left[1 - e^{-\ln(-\ln p)\left[1 - a_n(-\ln(-\ln p))^{n-1}\left[1 + \sum_{k=0}^{n-1} \frac{a_k}{a_n}(-\ln(-\ln p))^{-(n-k)}\right]\right]} \right] \tag{4.10}$$

Let $p \to 0$. By Lemma 3 (vi) $-\ln(-\ln p) \to -\infty$. Hence, $(-\ln(-\ln p))^{-(n-k)} \to 0$ for $k < n$. Furthermore, since $n - 1$ is a positive even number, and $a_n > 0$, it follows that $a_n (-\ln(-\ln p))^{n-1} \to \infty$. In the light of these facts, (4.10) gives $\ln \frac{w}{p} \to -\infty$,as $p \to 0$. Hence, $\frac{w(p)}{p} \to 0$,as $p \to 0$. This proves the first part of (c). To prove the second part of (c), write (4.5) as

$$\ln w = -e^{-\sum_{k=0}^{n} a_k(-\ln(-\ln p))^k}, 0 < p < 1, a_n > 0, n > 1, n \text{ odd} \tag{4.11}$$

In the light of part (b), we use L'Hopital's rule to evaluate $\lim_{p\to 1} \frac{1-w(p)}{1-p}$. This gives $\lim_{p\to 1} \frac{1-w(p)}{1-p} = \lim_{p\to 1} \frac{d(1-w(p))}{dp} / \lim_{p\to 1} \frac{d(1-p)}{dp} = \lim_{p\to 1} \frac{dw(p)}{dp} = \lim_{p\to 1} w(p) \frac{d \ln w(p)}{dp} = \lim_{p\to 1} w(p) \lim_{p\to 1} \frac{d \ln w(p)}{dp} = \lim_{p\to 1} \frac{d \ln w(p)}{dp}$.

From this, the fact that $\ln p = -e^{-[-\ln(-\ln p)]}$, from (4.11), and since $n > 1$, we get

$$\lim_{p\to 1} \frac{1 - w(p)}{1 - p}$$

$$= n \lim_{p\to 1} \frac{a_n (-\ln(-\ln p))^{n-1} \left[1 + \sum_{k=1}^{n-1} \frac{k a_k}{n a_n} (-\ln(-\ln p))^{-(n-k)}\right]}{p \exp\left(a_n (-\ln(-\ln p))^n \left[1 - \frac{1}{a_n}(-\ln(-\ln p))^{-(n-1)} + \sum_{k=0}^{n-1} \frac{a_k}{a_n} (-\ln(-\ln p))^{-(n-k)}\right]\right)}$$

$$= n \lim_{p\to 1} \frac{a_n (-\ln(-\ln p))^{n-1}}{e^{a_n(-\ln(-\ln p))^n}} = n(a_n)^{\frac{1}{n}} \lim_{x\to\infty} \frac{x^n}{x e^{x^n}} = 0,$$

where $x = (a_n)^{\frac{1}{n}} (-\ln(-\ln p))$. ■

Note that it follows from Definition 6 and Proposition 3(b), that a Prelec function of order n is continuous.

Proposition 3(c) formalizes the exact sense in which improbable events are ignored and probable events are treated as certain. Of course, how probable or improbable, depends on the parameters a_k.

Comparing the graphs of the (1st order) Prelec function with the 3rd and 5rd order Prelec functions, we see that they are similar for probabilities in the middle range. However, the higher order functions allow improbable events to be ignored and probable events to be treated as certain. In principle, the order, n, and the parameters, a_k, can be chosen to fit the data.

We now give an example of an infinite order Prelec function. Taking $a_1 = a > 0$ and, for $k \geq 1$, $a_{2k} = 0$, $a_{2k+1} \geq 0$, we get that (4.1) and (4.3) define a strictly increasing function, provided the series is convergent. An easy way to guarantee convergence is to take $a_{2k+1} = \frac{a}{(2k+1)!}$. Then, for any $p \in (0,1)$, the series in (4.3) converges (absolutely and uniformly) to $a_0 + \frac{1}{2}a\left(e^{-\ln(-\ln p)} - e^{\ln(-\ln p)}\right) = a_0 + a\sinh\left(-\ln\left(-\ln p\right)\right)$.[6] To get the 'right shape', we take $a \in (0,1)$. For probabilities in the middle range, $\sinh\left(-\ln\left(-\ln p\right)\right) \simeq -\ln\left(-\ln p\right)$, hence this function is a good approximation to the (first order) Prelec function for such probabilities. Using arguments similar to those in the proof of Proposition 3, it is straightforward to prove:

Proposition 4 *: Let $w(p)$ be defined by:*

$$w(0) = 0, w(1) = 1 \tag{4.12}$$

$$-\ln\left(-\ln w\right) = a_0 + a\sinh\left(-\ln\left(-\ln p\right)\right), 0 < p < 1, 0 < a < 1 \tag{4.13}$$

Then

(a) $w : [0,1] \to [0,1]$ is continuous and strictly increasing; w is C^∞ on $(0,1)$.

(b) $\lim_{p\to 0}\frac{w(p)}{p} = 0$ and $\lim_{p\to 1}\frac{1-w(p)}{1-p} = 0$, i.e., w, zero-underweights infinitesimal probabilities.

Definition 7 *: By the hyperbolic Prelec function (HP), we mean the probability weighting function defined by (4.12) and (4.13).*

Note that (4.12) and (4.13) define a two-parameter family of functions. Hence a hyperbolic Prelec function is just as parsimonious as the (1st order) Prelec function (Definition 4). The following is a graph of (4.12) and (4.13) for $a_0 = 0$ and $a = \frac{1}{2}$:

In the next two sections we compare the behavior of the decision maker when she uses, respectively, a probability weighting function of order 1 (which is just the standard Prelec function) and of order greater than one. We show that her behavior differs significantly between the two cases.

5. Rank dependent utility theory and insurance

In this section, we model the behavior of an individual using *rank dependent utility theory* (RDU), which we may regard as a conservative extension of *expected utility theory* (EU) to

[6] The *hyperbolic sin* function is defined as

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right).$$

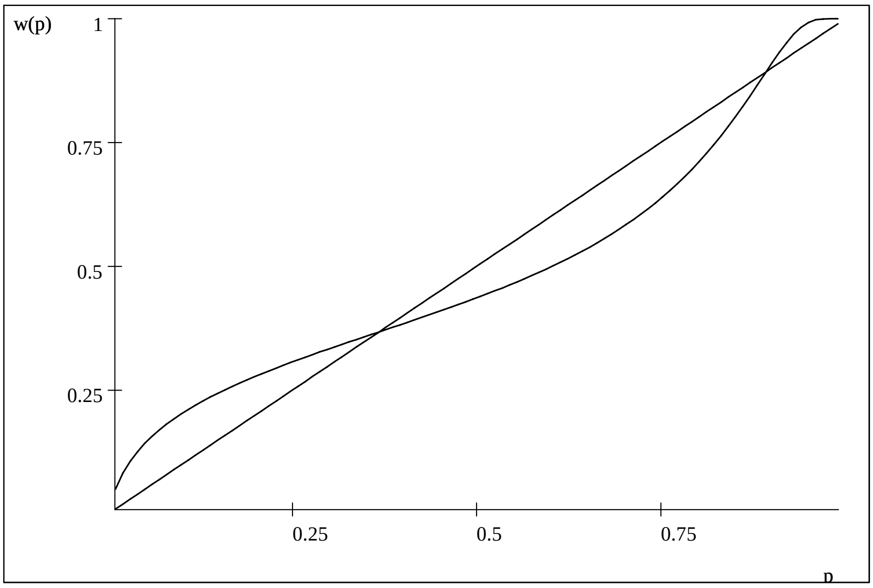


Figure 4.1: **A Hyperbolic Prelec Function**

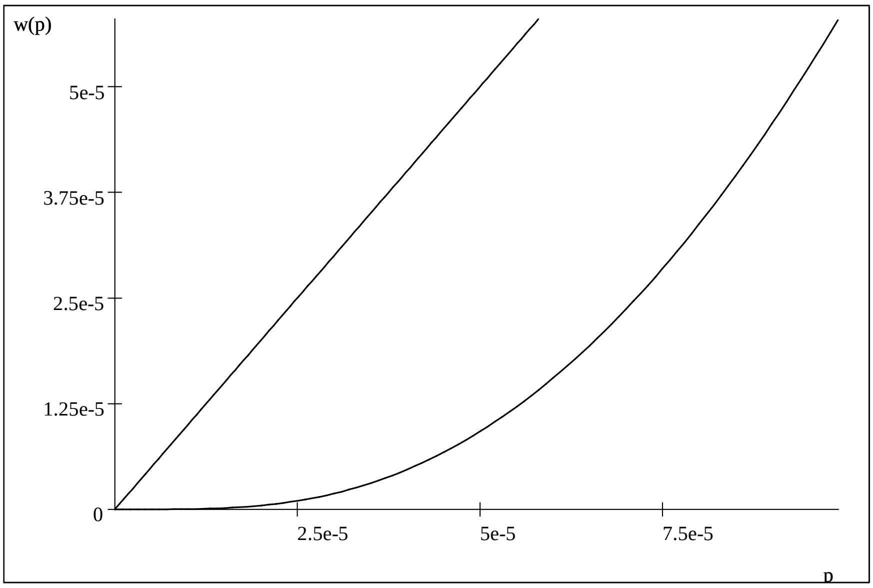


Figure 4.2: **A Hyperbolic Prelec Function For Low Probabilities**

the case where probabilities are transformed. In the next section, we consider *cumulative prospect theory* (CP), which is a more radical departure from expected utility theory. Consider a decision maker with initial wealth, W, probability weighting function, w, and utility function, u, where u is strictly concave, differentiable, $u' > 0$ and u' is bounded above[7] by (say) $\overline{u}'$. Assume that she faces the lottery: win x_1 with probability p_1 or x_2 with probability p_2, $x_1 \leq x_2$, $0 \leq p_i \leq 1$, $p_1 + p_2 = 1$ (if $x_i < 0$, then x_i is, in fact, a loss).

According to rank dependent utility theory, her expected utility will be $U = w(p_2) u(x_2) + [1 - w(p_2)] u(x_1)$. For $w(p) = p$, rank dependent utility theory reduces to standard expected utility theory. Note that the *higher* outcome, x_2, receives weight $w(p_2)$, while the *lower* outcome, x_1, receives weight $w(p_1 + p_2) - w(p_2) = w(1) - w(p_2) = 1 - w(p_2)$; see Quiggin (1982, 1993) for the details.

Suppose a decision maker can suffer the loss, $L > 0$, with probability p. She can buy coverage, C, at the cost $rC + f$, where $0 \leq C \leq L$, $0 < p < 1$, $0 < r < 1$, $f \geq 0$ and f is a fixed cost. We allow departures from the actuarially fair condition. We do so in a simple way by setting the insurance premium rate $r = (1 + \theta) p$. Thus, $\theta = 0$ corresponds to the actuarially fair condition, $\theta > 0$ to the actuarially unfair and $\theta < 0$ to the actuarially 'over-fair' condition. With probability $1 - p$, her wealth will become $W - rC - f$. With probability p, her wealth will become $W - rC - f - L + C \leq W - rC - f$. If she buys insurance, her expected utility under RDU will then be:

$$U_I(C) = w(1-p) u(W - rC - f) + [1 - w(1-p)] u(W - rC - f - L + C) \quad (5.1)$$

Since $U_I(C)$ is a continuous function on the non-empty compact interval $[0, L]$, an optimal level of coverage, C^*, exists.

For full insurance, $C = L$, (5.1) gives:

$$U_I(L) = u(W - rL - f) \quad (5.2)$$

On the other hand, if she does not buy insurance, her expected utility will be:

$$U_{NI} = w(1-p) u(W) + [1 - w(1-p)] u(W - L) \quad (5.3)$$

For the decision maker to buy insurance, the following *participation constraint* must be satisfied:

$$U_{NI} \leq U_I(C^*) \quad (5.4)$$

[7]The boundedness of u' is needed for part (b) of Proposition 5. This seems feasible on empirical grounds, since people do undertake activities with a non-zero probability of complete ruin, e.g., using the road, undertaking dangerous sports, etc. However, the boundedness of u' excludes such tractable utility functions as $\ln x$ and x^γ, $0 < \gamma < 1$. By contrast, the boundedness of u' is not a requirement in CP, as we shall see.

Proposition 5 : *Under RDU,*

(a) If a probability weighting function infinitely-overweights infinitesimal probabilities (Definition 2) then, for a given expected loss, the decision maker will insure fully for all sufficiently small probabilities.

(b) If a probability weighting function zero-underweights infinitesimal probabilities (Definition 3) then, for a given expected loss, a decision maker will not insure, for all sufficiently small probabilities.

Proof: Consider an expected loss

$$\overline{L} = pL \tag{5.5}$$

Differentiate (5.1) with respect to C to get

$$\begin{aligned} U_I'(C) &= -rw(1-p)\,u'(W - rC - f) && (5.6)\\ &\quad + (1-r)(1 - w(1-p))\,u'(W + (1-r)C - f - L) && (5.7) \end{aligned}$$

Since u is (strictly) concave, $u' > 0$ and $0 < r < 1$, it follows, from (5.6) that $U_I'(C)$ is a decreasing function of C. Hence,

$$U_I'(L) \leq U_I'(C) \leq U_I'(0) \text{ for all } C \in [0, L] \tag{5.8}$$

Replace r by $(1+\theta)p$ in (5.6), then divide both sides by p, to get

$$\begin{aligned} \frac{U_I'(C)}{p} &= -(1+\theta)\,w(1-p)\,u'(W - (1+\theta)pC - f)\\ &\quad + (1 - (1+\theta)p)\,\frac{1 - w(1-p)}{p}\,u'(W - (1+\theta)pC - f - L + C) \end{aligned} \tag{5.9}$$

For $C = 0$ and $C = L$, (5.9) gives (using (5.5)):

$$\frac{U_I'(0)}{p} = [1 - (1+\theta)p]\,\frac{1 - w(1-p)}{p}\,u'\left(W - f - \frac{\overline{L}}{p}\right) - (1+\theta)\,w(1-p)\,u'(W - f) \tag{5.10}$$

$$\frac{U_I'(L)}{p} = \left[\frac{1 - w(1-p)}{p} - (1+\theta)\right] u'\left(W - (1+\theta)\overline{L} - f\right) \tag{5.11}$$

Since $0 < (1+\theta)p < 1$, $0 < p < 1$, $0 < w(1-p) < 1$, $0 < u' < \overline{u}'$ we get, from (5.10),

$$\frac{U_I'(0)}{p} < \frac{1 - w(1-p)}{p}\,\overline{u}' - (1+\theta)\,w(1-p)\,u'(W - f) \tag{5.12}$$

Let

$$F(p) = \frac{1 - w(1-p)}{p} - (1+\theta) \tag{5.13}$$

From (5.11) and (5.13) we get,

$$\frac{U_I'(L)}{p} = F(p)\,u'\left(W - (1+\theta)\,\overline{L} - f\right) \tag{5.14}$$

From (5.14) we see that

$$U_I'(L) > 0 \Leftrightarrow F(p) > 0 \tag{5.15}$$

From (5.2), (5.3), (5.5), (5.13) and the facts that u is strictly increasing and strictly concave, simple algebra leads to

$$f < \overline{L}F(p) \Rightarrow U_{NI} < U_I(L) \tag{5.16}$$

Put

$$q = 1 - p \tag{5.17}$$

(a) Suppose $w(p)$ infinitely-overweights infinitesimal probabilities. Then, from (5.17) and Definition 2, $\lim_{p\to 0}\frac{1-w(1-p)}{p} = \lim_{q\to 1}\frac{1-w(q)}{1-q} = \infty$. Hence, from (5.13), for given expected loss, $\overline{L}$, we can find a $p_1 \in (0,1)$ such that, for all $p \in (0, p_1)$, we get $f < \overline{L}F(p)$. From (5.16) it follows that the participation constraint (5.4) is satisfied for all $p \in (0, p_1)$. From $f < \overline{L}F(p)$ we get that $F(p) > 0$ for all $p \in (0, p_1)$. From (5.15) it follows that $U_I'(L) > 0$ for all such p. From (5.8) it follows that $U_I'(C) > 0$ for all such p and, hence, the decision maker insures fully for all $p \in (0, p_1)$.

(b) Suppose $w(p)$ zero-underweights infinitesimal probabilities. Then, from (5.17) and Definition 3, $\lim_{p\to 0}\frac{1-w(1-p)}{p} = \lim_{q\to 1}\frac{1-w(q)}{1-q} = 0$. Hence, from (5.12), there exists $p_2 \in (0,1)$ such that for all $p \in (0, p_2)$, $U_I'(0) < 0$. Hence, from (5.8), $U_I'(C) < 0$ for all $C \in [0, L]$. Hence the optimal level of coverage is 0. ■

From Proposition 1 and Proposition 5(a), a decision maker using a Prelec probability weighting function (Definition 4), will fully insure against all losses of sufficiently small probability, provided the participation constraint (5.4) is satisfied. It is of interest to get a feel for how restrictive this participation constraint is. Example (1), below, suggests it is a weak restriction.

Example 1 : *The the first row of the following table gives losses from* 10 *(Dollars, say) to* 10, 000, 000*, with corresponding probabilities (row 2) ranging from* 0.1 *to* 0.000, 000, 1*; so that the expected loss in each case is* $\overline{L} = 1$*. In row 3 are the corresponding values of* $\frac{1-w(1-p)}{p}$ *for the Prelec function* $w(p) = e^{-(-\ln p)^{0.65}}$*, where the values* $\alpha = 0.65$ *and* $\beta = 1$ *are suggested by Prelec (1998).*

loss	10	100	1000	10,000	100,000	1,000,000	10,000,000
probability of loss	0.1	0.01	0.001	0.000,1	0.000,01	0.000,001	0.000,000,1
$\frac{1-w(1-p)}{p}$	2.0674	4.9039	11.161	25.088	56.219	125.88	281.83

From (5.16) we saw that the participation constrain (5.4) is satisfied if the fixed cost, f, is less than $\overline{L}F(p)$, where $F(p)$ is given by (5.13) and, in Example 1, $\overline{L} = 1$. Even for the high profit rate of 100% ($\theta = 1$), so that $F(p) = \frac{1-w(1-p)}{p} - 2$, we see, from the above table, that the upper bound on the fixed component of the cost of ensuring against an expected loss of one unit (e.g. one Dollar), so that the participation constraint is satisfied, is hardly restrictive for low probabilities. Thus, from Proposition 5(a), we see that using RDU in combination with the Prelec function of order 1, is likely to lead to misleading results, in that it would predict too much insurance.

On the other hand, from Propositions 3(c), 4(b) and 5(b), a decision maker using a Prelec probability weighting function of order $n > 1$ (Definitions 6 and 7) will not insure against any loss of sufficiently small probability, in agreement with observation.

6. Cumulative prospect theory and insurance

Several anomalies, among them the ones mentioned in the introduction, motivated the development of prospect theory (Kahneman and Tversky (1979), Tversky and Kahneman (1992)). In prospect theory, the carriers of utility are not levels of wealth, assets or goods, but differences between these and a reference point (*reference dependence*). The reference point is usually (but not necessarily) taken to be the status quo. The value function, as the utility function is called in prospect theory, is concave for gains but convex for losses (*declining sensitivity*). The disutility of a loss is greater than the utility of a gain of the same magnitude (*loss aversion*)[8]. Probabilities are transformed, so that small probabilities are overweighted but high probabilities are underweighted. A commonly used value function in prospect theory is

$$v(x) = x^{\gamma},\ 0 < \gamma < 1 \tag{6.1}$$

Consider a decision maker whose behavior is described by *cumulative prospect theory* (CP, Tversky and Kahneman, 1992). Let her initial wealth be W. Suppose she can suffer the loss, $L > 0$, with probability p. She can buy coverage, C, at the cost $rC + f$, where $0 \leq C \leq L$, $0 < p < 1$, $0 < r < 1$, $f \geq 0$ and f is a fixed cost. As in Section 5, $r = (1+\theta)p$. Take her current wealth, W, to be her reference point. With probability $1-p$, her wealth will become $W - rC - f$; which she codes as the loss $rC + f$, relative to her reference wealth, W. With probability p, her wealth will become $W - rC - f - L + C$; which she codes as the loss $L - C + rC + f \geq rC$, relative to her reference wealth W. Let $-v$ be her value function for the domain of *losses*, $v : [0,\infty) \to [0,\infty)$ where v is strictly

[8] Loss aversion is very important when the decision maker is in the domain of gains in one state of the world but in the domain of losses for another. However, loss aversion will not be important here because, as we shall see, the decision maker will always be in the domain of losses.

concave, $v(0) = 0$, v is differentiable on $(0, \infty)$ with $v' > 0$.[9] Her utility function under cumulative prospect theory will then be:

$$V_I(C) = -w(p)\, v(L - C + rC + f) - (1 - w(p))\, v(rC + f) \tag{6.2}$$

Since $V_I(C)$ is a continuous function on the non-empty compact interval $[0, L]$, an optimal level of coverage, C^*, exists.

For full insurance, $C = L$, (6.2) gives:

$$V_I(L) = -v(rL + f) \tag{6.3}$$

On the other hand, if she does not buy insurance, her expected utility will be (recall that $v(0) = 0$):

$$V_{NI} = -w(p)\, v(L) \tag{6.4}$$

For the decision maker to buy insurance, the following participation constraint (the analogue here of (5.4)) must be satisfied:

$$V_{NI} \leq V_I(C^*) \tag{6.5}$$

Proposition 6 : *Under CP,*

(a) A decision maker will insure fully against any loss, provided the participation constraint is satisfied.

(b) For Prelec's probability weighting function (Definition 4), for the value function (6.1) and for a given expected loss, the participation constraint (6.5) is satisfied for all sufficiently small probabilities.

(c) If a probability weighting function zero-underweights infinitesimal probabilities (Definition 3) then, for a given expected loss, a decision maker will not insure against any loss of sufficiently small probability.

Proof: (a) Since v is strictly concave, $-v$ is strictly convex. Hence, from (6.2), it follows that V_I is strictly convex. Since $0 \leq C \leq L$, it follows that $V_I(C)$ is maximized either at $C = 0$ or at $C = L$. Hence, if the participation constraint is satisfied, then the decision maker will fully insure against the loss.

(b) Consider the Prelec function (3.2) and the value function (6.1). Let

$$F(p) = \frac{e^{-\frac{\beta}{\gamma}(-\ln p)^{\alpha}}}{p} - (1 + \theta), \text{ (recall } 0 < \alpha < 1,\ \beta > 0,\ \gamma > 0) \tag{6.6}$$

Consider an expected loss

$$\overline{L} = pL \tag{6.7}$$

[9] These assumptions are satisfied by, for example, $v(x) = 1 - e^{-x}$ and $v(x) = x^{\gamma}$, $0 < \gamma < 1$. But they are not satisfied by $v(x) = \ln x$, since $\ln 0$ is not defined.

From (3.2), (6.1), (6.3), (6.4), (6.6) and (6.7), simple algebra leads to

$$f < \overline{L}F(p) \Rightarrow V_{NI} < V_I(L) \tag{6.8}$$

From (6.6) and Proposition 1, $\lim_{p \to 0} F(p) = \infty$. Hence, for given expected loss, $\overline{L}$, we get $f < \overline{L}F(p)$, for all sufficiently small p. From (6.8) it follows that the participation constraint is satisfied for all such small p.

(c) From (6.3) and (6.4) we get the following

$$\frac{V_I(L) - V_{NI}}{p} = v(L)\frac{w(p)}{p} - v\left((1+\theta)\overline{L} + f\right)\frac{1}{p} \tag{6.9}$$

$$\lim_{p \to 0} \frac{V_I(L) - V_{NI}}{p} = v(L)\lim_{p \to 0}\frac{w(p)}{p} - v\left((1+\theta)\overline{L} + f\right)\lim_{p \to 0}\frac{1}{p} \tag{6.10}$$

Suppose $w(p)$ zero-underweights infinitesimal probabilities. Then, from Definition 3, $\lim_{p \to 0} \frac{w(p)}{p} = 0$. Hence, the first term in (6.10) goes to 0 as p goes to 0. The second term in (6.10), however, goes to $-\infty$ as p goes to 0. Hence, there exists $p_2 \in (0,1)$ such that for all $p \in (0, p_2)$, $V_{NI} > V_I(L)$. ■

By Proposition 6(a), a decision maker will insure fully against any loss, provided the participation constraint (6.5) is satisfied. By Proposition 6(b), for Prelec's probability weighting function (Definition 4), for the value function (6.1) and for a given expected loss, the participation constraint (6.5) is satisfied for all sufficiently small probabilities. It is of interest to get a feel for how restrictive this participation constraint is. Example (2), below, suggests it is a weak restriction.

Example 2 : *The the first row of the following table gives losses from* 10 *(Dollars, say) to* 10,000,000, *with corresponding probabilities (row 2) ranging from* 0.1 *to* 0.000,000,1*; so that the expected loss in each case is* $\overline{L} = 1$. *In row 3 are the corresponding values of* $\frac{e^{-\frac{\beta}{\gamma}(-\ln p)^{\alpha}}}{p}$, *where the values* $\alpha = 0.65$ *and* $\beta = 1$ *are suggested by Prelec (1998) and* $\gamma = 0.88$ *is suggested by Tversky and Kahneman (1992).*

loss	10	100	1000	10,000	100,000	1,000,000	10,000,000
probability of loss	0.1	0.01	0.001	0.000,1	0.000,01	0.000,001	0.000,000,1
$\frac{e^{-\frac{1}{0.88}(-\ln p)^{0.65}}}{p}$	1.4169	4.6589	18.48	81.342	383.83	1906.3	9852.3

From (6.8) we saw that the participation constrain (6.5) is satisfied if the fixed cost, f, is less than $\overline{L}F(p)$, where $F(p)$ is given by (6.6) and, in Example 2, $\overline{L} = 1$. Even for the high profit rate of 100% ($\theta = 1$), so that $F(p) = \frac{e^{-\frac{\beta}{\gamma}(-\ln p)^{\alpha}}}{p} - 2$, we see, from the above table, that the upper bound on the fixed component of the cost of ensuring against an expected loss of one unit (e.g. one Dollar), so that the participation constraint is satisfied, is hardly restrictive for low probabilities. Thus, from Proposition 6(a) and (b), we see that using CP in combination with the Prelec function of order 1, is likely to lead to misleading results, in that it would predict too much insurance.

On the other hand, from Propositions 3(c),4(b) and 6(c), a decision maker using a Prelec probability weighting function of order $n > 1$ (Definition 6 and 7) will not insure against any loss of sufficiently small probability, in agreement with observation. However, by Proposition 6(a), CP predicts that, if a decision maker decides to insure, she will insure fully, even with a fixed cost of entry and an actuarially unfair premium.

7. St. Petersburg paradox

The St. Petersburg paradox occupies an important place in the history of economic thought, as it motivated von Neumann and Morgenstern (1947) to introduce expected utility into economics. Expected utility has remained ever since the main tool for analyzing decision making under risk. A simple version of the paradox runs as follows. If the first realization of 'heads' in a sequence of random throws of a fair coin occurs on the n-th throw, then the game ends and the recipient receives a payoff of 2^n monetary units. This game has an infinite expected payoff, yet experimental evidence suggests that subjects will pay only a modest finite sum to play this game. Bernoulli (1738) suggested that a decision maker maximized the expected utility of a lottery rather than the expected monetary value. Blavatskyy (2004) and Rieger and Wang (2006)[10] have shown that this paradox reemerges under CP. They prove that, even with a strictly concave value function, the Bernoulli lottery will have an infinite expected utility. Rieger and Wang (2006) go on to show that the probability weighting function[11]:

$$w(p) = p + \frac{3(1-b)}{1-a+a^2}\left[ap - (1+a)p^2 + p^3\right],\ a \in \left(\frac{2}{9}, 1\right), b \in (0,1) \tag{7.1}$$

solves the paradox by generating a finite expected utility under CP.[12]

By direct calculation, or by applying Theorem 1 of Rieger and Wang (2006), it is straightforward to show that any of the generalized Prelec probability weighting functions (of order $n > 1$), will also generate a finite expected utility for the St. Petersburg paradox. Thus, the generalized Prelec functions solve both the insurance paradox and the St. Petersburg paradox.

[10] We cannot do justice to Rieger and Wang (2006) in this brief section. The reader is urged to read their paper.

[11] Rieger and Wang state, incorrectly, that $a \in (0,1)$. For sufficiently low a and $p \simeq \frac{1}{3}$, $w(p)$, as given by (7.1), is *decreasing* in p. The lower bound of $\frac{2}{9}$ on a is sufficient, but not necessary, for $w(p)$ to be strictly increasing.

[12] From (7.1): $\lim_{p\to 0} \frac{w(p)}{p} \in (1,\infty)$ and $\lim_{p\to 1} \frac{1-w(p)}{1-p} \in (1,\infty)$. Hence, these functions finitely-overweight infinitesimal probabilities, in the sense of footnote 3. Hence, unlike the higher order Prelec functions, they do not capture the empirical fact that people ignore extremely low probabilities and code extremely likely events as certain.

8. Conclusion

Decision models that rely on non-linear transformations of probabilities, for instance, rank dependent utility (RDU) and cumulative prospect theory (CP) require using a probability weighting function. However, the standard probability weighting functions infinitely overweight infinitesimal probabilities, in the sense that the ratio between the weight and the probability goes to infinity, as the probability goes to zero. In actual practice, individuals code very small probabilities as zero and very large ones as one. Given that many important decisions under uncertainty involve small probabilities, we show that the 'infinite overweighting of small probabilities' feature of existing probability weighting functions leads to predictions that contradict observed behavior.

Thus, individuals should insure fully even for ridiculously low probability natural hazards. This is at odds with the evidence. Indeed, governments often have to legislate compulsory insurance of several kinds and mortgage lenders have mandatory building and contents insurance requirements as a pre-condition to their lending.

Kahneman and Tversky (1979) proposed a two-step heuristic procedure to deal with these sorts of problems. In the first step, events associated with probabilities coded as zero are ignored. In the second step, a probability weighting function is used to make a choice from the surviving alternatives. We propose a class of probability weighting functions- *the higher order Prelec probability weighting functions*- that allow us to combine the two-step heuristic choice process of Kahneman and Tversky into one.

Furthermore, while our proposed functions overweight low probabilities and underweight high probabilities, they also have the feature that the ratio between the weight and the probability goes to zero as the probability goes to zero. This enables us to show that when RDU or CP is combined with any one of these new probability weighting functions, then a decision maker would not buy insurance against an expected loss of sufficiently low probability; in agreement with the evidence.

One attractive feature of the Prelec (1998) probability weighting function is that it has an axiomatic derivation. An interesting question, that lies beyond the scope of this paper, is what behavioral assumptions lead to the higher order Prelec probability weighting functions? This could be a fruitful line of inquiry. Another topic for future research could be to try to fit the proposed weighting function to data and estimate its parameters. This could potentially reveal important information about individual choice under uncertainty.

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