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Grade Inflation, Students' Social Background and String-Pulling

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Grade Inflation, Students' Social Background and String-Pulling*
[Preliminary]

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Abstract

In this paper I analyse grade inflation when students differ both in ability and social background. I consider a signalling game where a school may inflate the grade of low-ability students and a company decides whether ot hiring or not the students, and observes their grades. In the one-shot (repeated) version of the game, the company is aware (unaware) of the school strategy and the distribution of ability. The results suggest that a school can inflate grades in order to smooth down class differences in the job market. The results in the repeated game can explain how string-pulling can emerge as a job hiring strategy in the presence of grade inflation and school low reputation.

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1 Introduction

Grade inflation, i.e., the awarding of higher grades than students deserve, makes it more difficult to identify the truly excellent students, as more students come to obtain the highest possible grade. In many educational systems grade inflation has risen over the years and now it is strong in many countries.[1]

In this paper we analyse grade inflation by assuming that students differ both in ability and social background. The idea is to examine whether grade inflation may in fact have some positive effect in soothing class differences. I assume that students with advantaged social backgrounds are more likely to have high ability. Given the same distribution of innate ability within population with differing social backgrounds, an advantaged environment can help developing non-cognitive skills via parental and peer pressure so that, on average, the overall ability results higher for students with an advantaged background. The assumption is in line with past research documenting that family and environmental factors are major predictors of the individuals' ability (Cunha *et al.*, 2006, Carneiro and Heckman, 2003, Joshi and McCulloc, 2000).

I consider a signalling game between a school and a company. The school prepares students for a final exam and may inflate the grade of some low-

[1] In the United States, the evidence of the raise in grade inflation has been documented by Rojstaczer and Healy (2011). They collected historical data on letter grades awarded by more than 200 four-year colleges and universities. Their results confirm the drastic raisein the share of A grades awarded over the years. In Canada, the 52.6 % of high school graduates applying to Ontario universities in 1995 had an A average, then this had risen 61% in 2004. In 1995, 9.4 percent of high school graduates reported an A+ average, risen to a high of 14.9% in the 2003; the average grade of university applicants was 80% in 1997, and this percentage has steadily increased each year since (Allahar, A. and Côté, 2007). In the United Kingdom, graduates who graduate with First-Class Honours rose from 7.7% of total graduates in year 1996/97 to 14% in year 2008/09. For graduates with a upper-second honour, it rose from 41.1% of total graduates in year 1996/97 to 48% in year 2008/09 (Higher Education Statistic Agency). In Italy, the analysis "Stella" over graduates for years 2004 and 2005 reports one third of graduates achieved the highest grade (Modica, 2008).

ability students. I examine two alternative school types: an "employment-maximising" school, willing the highest number of students to obtain a job, and an "equalitarian" school, which values equity in the employment across advantaged and disadvantaged students. On the other side, the company wants to hire only high-ability students and observes the exam grade as a signal of ability.

In the one-shot version of the game, I assume that the company is aware of the distribution of ability in the students' population and the school strategy. In this case, the results suggest that a school can inflate grades to smooth down class differences in the job market. This may happen when more advantaged than disadvantaged students receive grade inflation. The result hinges upon the fact that more grade inflation to advantaged students may offset the higher probability of advantaged students of having high ability. The result is stronger the higher the school concern for disadvantaged students.

In the infinitely repeated version of the game, the company has no information on students' distribution of ability and school strategy at the beginning of each period, but acquires it at the end of it. The school can play the one-shot strategy or deviate from it by full inflating grades, since this maximises the single-period employment in the game with imperfect information. The company realises the behaviour of the school at the end of each period, and can punish possible deviations by hiring through string-pulling, such as appointing relatives, friends' relatives, references and so on.

The introduction of many periods and imperfect information makes weaker the beneficial effect of grade inflation compared to the one-shot case. Given the poor information of the company, the school has an incentive to deviate from the equilibria where grade inflation has a positive effect on disadvantaged students, as long as it is not so concerned on its reputation. Moreover, resorting to hire through string-pulling as a further negative effect on class differences, since an individual with advantaged background more likely has helpful "acquaintances" to obtain a job, a better networking and so forth. This result explains the facts occurring in some countries, such as Italy, where grade inflation is extremely high, every degree has the same legal value irrespective of the appointing school or university, so that there is no such concern on the institution reputation, and hence the best way of obtaining a job is through string-pulling.[2]

The economic literature only recently took interest on grade inflation, with few but noteworthy contributions. Yang and Yip (2003) present a model where universities have an incentive to inflate grades and they mutually reinforce each other's practice, determining a free-rider effect in grade inflation. In their equilibrium, universities with higher reputation inflate more. Bernhardt and Popov (2010) develop a similar model where they identify the increase over time in the quantity of good jobs as a driving force of grade inflation. They also extend their analysis by considering students with different social skills. Chan *et al.* (2007) develop a signalling model where firms observe the students' grade but ignore their ability and the proportion of talented ones in the population of students. This gives rise to an incentive to help some low ability students by giving them good grades. Indeed the labour market cannot fully distinguish whether this is due to a high grading standard or whether the school has a large proportion of talented students, and this in turn weakens the signal of good ones. Compared to these contributions, I cast aside the competition between universities, and I focus on the differences in students' social background and to the effects on string-pulling.

An analysis of grade inflation and the impact on the job market with student with different social background has been carried on by Schwager (2008) in a matching model. In this paper, students are matched with firms which offer different kinds of job, according to the grade and the expected ability. Regardless of social background, it is possible that mediocre students

[2] See Modica (2008) for some empirical evidence of grade inflation in Italy. For the legal value of an Italian degree, see the "D.M. 509/99, art. 4 comma 3 ". For some evidence of string-pulling in Italy, see Floris (2007).

receive a high grade caused by grade inflation. Also, the high-ability students from advantaged backgrounds may benefit from grade inflation since this shields them from the competition on the part of able and disadvantaged students. Compared to this analysis, I share the same assumptions on the distributions of ability with differing social backgrounds, but in my model disadvantaged students may benefit from the presence of inflation grade.

In a setup which is similar to the present one, Tampieri (2011) examines how students' social background affects the teaching effort and the students' opportunities in the job market. Like in the one-shot version of this model, the interaction between one school and one employer takes place, and students are no players. Nonetheless, the focus of the model is on the changes in the teaching effort according to the students' social background and how this influences the company behaviour, so that teaching effort is endogenously determined in equilibrium.

The remainder of the paper is organised as follows. The model is presented in Section 2. Section 3 examines the one-shot game with perfect information. Section 4 considers the repeated game with imperfect information. Section 5 concludes.

2 The model

I study an economy with a number of students, with mass normalised to one. Students attend all the same school and afterwards apply for a job in the same company. Students can have high (H) or low (L) ability and an advantaged (a) or disadvantaged (d) social background. Social background is public information, and can be seen as a one-dimensional measure of family environment, income, neighbourhood, ethnic origins and so forth. I denote as $\eta \in [0, 1]$ the proportion of advantaged students, and $p_a, p_d \in [0, 1]$ as the probability that an advantaged or disadvantaged student has high-ability, respectively, where $p_a > p_d$.

For simplicity I abstract from the students' effort and from schools or companies competition, and I focus on the interplay between one school and one company.

2.1 School

The school prepares students for a final exam, with equal teaching effort irrespective of the student type, and learns the students' ability through their tests and assessments results.[3] The possible exam outcomes are a high (A) or a low (B) grade. I assume that the student's probability of obtaining an A grade is 1 if she has high ability, while it is zero if the student has low ability.

Nonetheless, the school can influence job opportunities by inflating the grades of some low-ability students. In particular, a low-ability student receiving grade inflation has a probability $g \in (0, 1)$ of obtaining a high grade. This assumption can be interpreted as follows. The school is in reality composed by an institution manager and group of lecturers teaching different subjects. The final grades A and B just summarise the overall results in each subject. The probability g exogenously represents the proportion of teachers providing grade inflation. The school orientation about grade inflation, i.e., whether to allow some lecturers to do it or not depends on the institution manager, who in fact represents the school decision along the model, while the lecturers' choice is taken as given.[4]

The school benefit depends on the specific objective that it pursue. In what follows, I will consider two cases, that is a school whose objective is (i) to maximise total employment (*employment-maximising school*) and (ii) to soothe class differences by favouring disadvantaged students$\times$ (*equalitarian*

[3] The label "school" could be replaced by the label "university" without altering the ongoing analysis.

[4] An alternative interpretation is the case of European schooling systems, which feature national testings at the end of school/university and thus limiting the inflation grade power of an educational institution.

school).

2.2 Company

The company decides whether or not to hire a student and offers a single job type. I define *job capacity* as the maximum number of students that may be hired and I denoted it as $\Xi \in [0, 1]$. For simplicity, I assume that Ξ is exogenously related to the company's production potential, i.e., its size and technology, and it is completely independent by the company's hiring decision. Hence, neither the interaction between school and the company nor the students' type may influence Ξ.

Also, I rule out uncertainty in the company market and I assume that the students' ability determines the company's profit entirely. In particular, each high and low-ability student yields a profit of $\nu > 0$ and -1, respectively. The choice of ν and -1 is to simplify the algebra: other normalisations would complicate the analysis without changing any result. Assuming that a low-ability student gives negative profits can be interpreted in many ways: low-ability employees may have a marginal productivity which is lower than salary cost. In addition, the company may want to lay off a low-ability employee but this action still comes at a cost, e.g. industrial disputes, wasted training costs and time, and so on. Thus the company's payoff is given by

$$\Pi^C = \nu\omega(H) - \omega(L),$$

where $\omega(H)$ and $\omega(L)$ are the number of hired students with high or low ability, respectively.

The company observes the grade that a student obtains in a final exam as a signal of ability. The fact that the company can observe the social background of a student seems plausible: in the real world, a personnel manager can easily tell the job candidate social background through some information such as ethnic origins, name, address, language style, manners, clothing, and so on.

2.3 One-shot game

The interaction in the one-shot game can be described as follows. Nature draws the student types. Then, each student attends school and the school chooses whether to inflate the grades of some low-ability students in the final exam. Finally, each student applies for a job, and the company decides whether to hire her.

The equilibrium concept is the perfect Bayesian equilibrium, which is a combination of school and company strategies and beliefs where both agents maximise their payoff. After observing a grade, the company has a belief, consistent with Bayes' rule, about the student type, conditional on all the information he has: the student's grade, the distribution of ability according to the student's social background and the school strategy. For each grade, the company must maximise its expected profit, given its belief and the school strategy. In turn the school's strategy must maximise its expected payoff, given the company's strategy. Finally, job capacity requires that the number of hired students is at most Ξ.

I begin by introducing the notations of the school and company's actions:

- $x_a, x_d, \in [0, 1]$ are the probabilities that the school inflates the grade of an advantaged and disadvantaged low-ability student, respectively;
- $z_{Aa}, z_{Ba}, z_{Ad}, z_{Bd} \in [0, 1]$ are the probabilities that the company hires an advantaged student with a high or low grade and a disadvantaged student with a high or low grade, respectively.

I am now in a position to define the company beliefs on the students' ability. I denote them as $\pi(k \mid g, p_i, x_{ki})$, where $k \in \{H, L\}$ represents the ability level.

Definition 1 *The company's beliefs on the students' ability which are consistent with the Bayes' rule are*

$$\pi(H \mid g, p_i, x_i) = \frac{p_i}{p_i + g x_i (1 - p_i)},$$

and

$$\pi(L \mid g, p_i, x_i) = \frac{g x_i (1 - p_i)}{p_i + g x_i (1 - p_i)}.$$

Then I make the following assumption.

Assumption 1 $\Xi < \eta(p_a + g(1 - p_a)) + (1 - \eta)(p_d + g(1 - p_d)))$.

Assumption 1 requires job capacity to be lower than the highest possible number of high-grade students. This is necessary in order to focus the attention on those equilibria where social background can be used as an informative tool in the company's decisions. When Assumption 1 does not hold, the company may hire all the high-grade students irrespective of their social background, hence this would not affect their job opportunities. Nonetheless, there is a large empirical evidence showing that the students' social background does influence their job opportunities, so that I will focus on this case.[5]

3 Static results

3.1 Employment-maximising school

In this section I examine the static game with employment-maximising school. In this case, the school obtains a benefit $b > 0$ for every hired student, so

[5]For some empirical evidence, Glyn and Salverda (2000) and Berthoud and Blekesaune (2006) show that a disadvantaged social background negatively affects the chance of finding a job in OECD countries and the UK, respectively.

that its payoff is given by

$$\Pi^S = b(\omega(H) + \omega(L)).$$

The equilibria are summarised in the following proposition.

Proposition 1 *Let Assumption 1 hold and the school be employment-maximising.*

Case 1. Large proportion of high-ability students in both populations. *If*

$$p_a \geq p_d \geq \frac{g}{(\nu + g)},$$

then the school and company strategy is: $x_a = x_d = 1$ *and*

$$z_{Aa} = 1, z_{Ad} = \frac{\Xi - \eta\left(p_a + g\left(1 - p_a\right)\right)}{\left(1 - \eta\right)\left(p_d + g\left(1 - p_d\right)\right)}, z_{Ba} = z_{Bd} = 0,$$

respectively.

Case 2. Large proportion of high-ability students in the advantaged populations. *If*

$$p_a \geq \frac{g}{(\nu + g)} \geq p_d,$$

then the school and company strategy is:

$$x_a = 1, x_d = \frac{p_d \nu}{\left(1 - p_d\right) g},$$

and

$$z_{Aa} = 1, z_{Ad} = \frac{\Xi - \eta\left(p_a + g\left(1 - p_a\right)\right)}{\left(1 - \eta\right)\left(p_d + x_d g\left(1 - p_d\right)\right)}, z_{Ba} = z_{Bd} = 0,$$

respectively.

Case 3. Small proportion of high-ability students in both populations. *If*

$$\frac{g}{(\nu + g)} \geq p_a \geq p_d,$$

then the school and company strategy is:

$$x_a = x_d = \frac{p_d \nu}{(1 - p_d)\, g},$$

and

$$z_{Aa} = z_{Ad} = \frac{\Xi}{[\eta(p_a + x_a g\,(1 - p_a)) + (1 - \eta)\,(p_d + x_d g\,(1 - p_d))]}, z_{Ba} = z_{Bd} = 0,$$

respectively.

Proof. See Appendix. ■

In Case 1, the school provides every student with grade inflation, since there is a large amount of high ability students in the population of both social backgrounds. The company obtains a positive expected profit by high-grade students from both advantaged and disadvantaged backgrounds. Thus, his optimal strategy is to hire all of them, but Assumption 1 prevents this possibility, and force the company to choose between these two types. He will hire all the advantaged students, since they give a higher expected profit, and the disadvantaged students will be hired only for the remainder of job capacity.

In Case 2, the company prefers not to hire all the disadvantaged and high-grade students, while with low grade inflation it is indifferent between hiring a student with advantaged or disadvantaged background. In Case 2 and 3, the school provides grade inflation to a lower number of disadvantaged and low-ability students, so as to increase the chance that a disadvantaged and high-grade student has high ability. It is worth noting that the company never hires a low-grade student, since a low-grade student has low ability with probability one.

Figure 1. Employment-maximising school

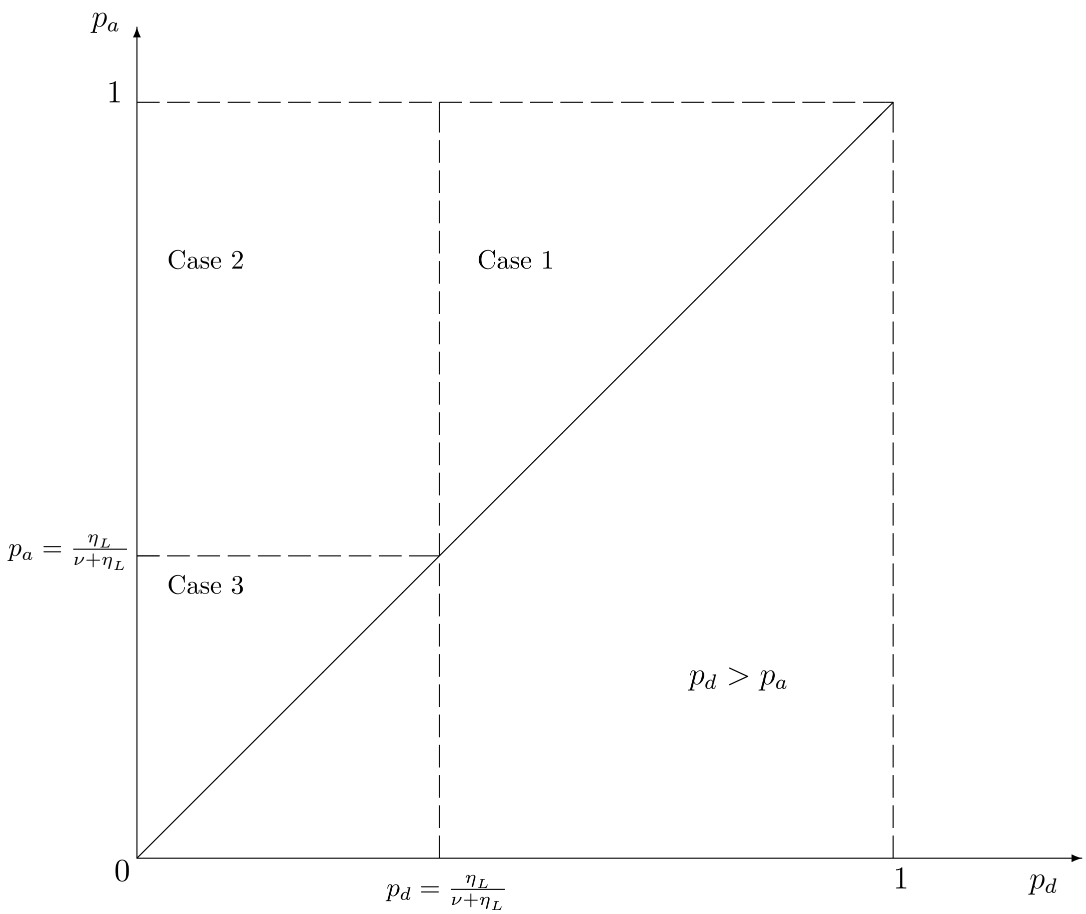

Figure 1 illustrates Proposition 1 for a given value of g, where $p_a > p_d$ holds above the upward-sloping 45 degrees line. This is the key assumption and makes the company obtain a higher expected payoff by hiring advantaged students, given the same grade inflation. However this may not happen if the school inflates grades more for advantaged students. Indeed this would raise the expected quality of the disadvantaged and high-grade students. In Case 1, the amount of students obtaining a job is maximised when the school gives grade inflation to all of them, since the company thinks that a high grade student very likely has high ability, irrespective of her social background.

In Case 2 and 3, the school maximises the amount of hired disadvantaged students by providing more grade inflation to advantaged students. More specifically, with middle grade inflation the company still prefers advantaged rather than disadvantaged and high-grade students. In Case 3, disadvantaged students have the same job opportunities of the advantaged ones. The reason of this result is the following. Since the school inflates less the grades of disadvantaged rather than advantaged students, the grade inflation effect is stronger for the latter. This result differs from other results on grade inflation (Schwager, 2008), where the job opportunities of high-ability and disadvantaged students are penalised by the grade inflation of low-ability and advantaged students.

Proposition 1 shows that, in the presence of an employment-maximising school, grade inflation has ambiguous effects on the job opportunities of disadvantaged students according to which equilibrium holds. This depends on the population distribution of ability and the size of grade inflation g, i.e., on how many low-ability students receive a high grade with grade inflation. This is summarised in the following corollary.

Corollary 1 *An increase in the size of grade inflation diminishes the employment opportunities and the provision of grade inflation.*

Proof. For all $x_d, x_a \in [0,1]$, differentiation of

$$\frac{\Xi - \eta(p_a + g(1-p_a))}{(1-\eta)(p_d + x_d g(1-p_d))} \text{ and } \frac{\Xi}{\eta(p_a + x_a g(1-p_a)) + (1-\eta)(p_d + x_d g(1-p_d))}$$

with respect to g yields

$$\frac{\partial}{\partial g}\frac{\Xi - \eta(p_a + g(1-p_a))}{(1-\eta)(p_d + x_d g(1-p_d))} =$$

$$\frac{\eta}{\eta - 1}\frac{1-p_a}{p_d + x_d g(1-p_d)} - \frac{(1-p_d)(\Xi - \eta(p_a + g(1-p_a)))}{(1-\eta)(p_d - x_d g(1-p_d))^2} < 0,$$

and

$$\frac{\partial}{\partial g}\frac{\Xi}{\eta(p_a + x_a g\,(1-p_a)) + (1-\eta)\,(p_d + x_d g\,(1-p_d))} =$$

$$\frac{\Xi\,(x_d\,(1-\eta)\,(p_d-1) - \eta x_a\,(1-p_a))}{(\eta\,(p_a - g x_a\,(p_a-1)) - (\eta-1)\,(p_d - g x_d\,(p_d-1)))^2} < 0,$$

respectively. Differentiation of

$$\frac{p_d}{(1-p_d)}\frac{\nu}{g}$$

with respect to g yields

$$\frac{\partial}{\partial g}\frac{p_d}{(1-p_d)}\frac{\nu}{g} = -\frac{\nu p_d}{g^2\,(1-p_d)} < 0.$$

■

An raise in the size of grade inflation makes the number of high-grade students increase. Thus their probability of being hired decreases and in turn probability of obtaining grade inflation decreases.

3.2 Equalitarian school

In this paragraph I consider the case where the school specifically aims to improve the conditions of disadvantaged students. This can be the situation where the school is managed by a social planner with the aim to soothe differences in the job market outcomes by students differing in social background. I assume that the school gives a higher weight to the hiring of a disadvantaged student, hence its payoff is given by

$$\Pi^S = b_a\eta(\omega(H) + \omega(L)) + b_d\,(1-\eta)\,(\omega(H) + \omega(L)),$$

where $b_d > b_a$.

The result can be summarised in the following proposition.

Proposition 2 *Let Assumption 1 hold and the school be equalitarian.*

Case 1. Large proportion of high ability students in the advantaged population. *If*

$$p_a \geq \frac{g}{(\nu + g)},$$

then the school and company strategy are:

$$x_a = 1;\ x_d = \frac{p_d\,(1 - p_a)}{p_a\,(1 - p_d)},$$

and

$$z_{Aa} = \frac{\Xi - (1 - \eta)\,(p_d + x_d g\,(1 - p_d))}{\eta(p_a + g\,(1 - p_a))}, z_{Ad} = 1, z_{Ba} = z_{Bd} = 0,$$

respectively.

Case 2. Small proportion of high ability students in both populations. *If*

$$\frac{g}{(\nu + g)} \geq p_a \geq p_d,$$

then the school and company strategy are:

$$x_a = \frac{p_a \nu}{(1 - p_a)\,g},\ x_d = \frac{p_d \nu}{(1 - p_d)\,g},$$

and

$$z_{Aa} = \frac{\Xi - (1 - \eta)\,(p_d + x_d g\,(1 - p_d))}{\eta(p_a + x_a g\,(1 - p_a))}, z_{Ad} = 1, z_{Ba} = z_{Bd} = 0,$$

respectively.

Proof. See Appendix. ■

Proposition 2 shows that a school caring of the job opportunities of the disadvantaged students can use grade inflation as a tool to achieve this goal.

4 Repeated game and string-pulling

In this section I consider a setting where the game is infinitely repeated and the company does not know the school strategy at the beginning of each period, but only learns it through the true abilities of the students once they have been hired, which can be inferred by profits. Also, I assume that the company ignores the ability and social background distributions p_i and η before seeing its own profit but obtains it afterwards. Finally, I assume that the ability distribution and the number of advantaged or disadvantaged students change over time, so as the company cannot infer it based on the previous generations. In what follows, I will consider the case with the employment-maximising school only. The reason is that, with equalitarian school, the company has a unique strategy along the entire parameter space (g, p_a, p_d), so that the school has no incentives to deviate from the one-shot game strategy.

At the first period, the company decides its hiring policy according to an exogenous belief on the school strategy. The school chooses it among the ones presented in Proposition 1. The school may start with a strategy that is "correct" according to the parameter values (g, p_a, p_d) and thus replicating the correspondent one-shot equilibrium. After checking its own profit and thus obtaining perfect information, the company will know that the school played the one-shot strategy and will trust it in the next period.

However, the one-shot equilibrium strategy not necessarily is the one that gives the school highest expected payoff, and the school may deviate by increasing it but losing the trust of the company. In particular, the school has 3 possible strategies to choose according to Proposition 1. Between them, it always prefers to fully inflate, in order to obtain the maximum payoff. The reason is simple. Given the assumptions on the company information and any possible company belief, the employment (and in turn the school payoff) is maximised in the single period through a full inflating strategy. As a consequence, if the parameter values (g, p_a, p_d) are such that the one-shot equilibrium is Case 1, then the school never deviates from that case, and may have an incentive to do it if the one-shot equilibrium is with Case 2 or 3.

If the school deviates (Case 2 and 3), then in the following period the company hires from out of the school through string-pulling (from now on, SP), that is informal channels, accepting friends pressure for hiring some relatives, and so forth. Let us assume that the company payoff obtained by using SP is lower than the one obtained by the one-shot equilibrium but higher than the case where the school deviates to full inflation:

$$\Pi^C \left(\text{one-shot eq.}\right) > \Pi^C \left(\text{SP}\right) > \Pi^C \left(\text{dev.}\right).$$

This assumption is reasonable as SP cannot be an objective and systematic selection tool. Also, I assume that the company can cover its hiring necessities through SP for a proportion $q \in (0, 1]$ only.

When the company hires through SP, then the school obtains zero benefits. For simplicity I assume that the company will adopt a grim-trigger strategy *à la* Friedman (1971) in the sense that, if the school plays the one-shot strategy, the company will trust it in the following period, otherwise the company will hire from SP forever for a proportion q and from the school for the other part, and the school will play equilibrium 1 forever. In this contest, the discount factor δ can be interpreted as a measure of the importance that the school gives to its reputation. The more the school cares of it, the higher δ will be. The condition for the stability of full collusion under grim trigger strategies is:

$$\frac{\Pi^S \left(\text{one-shot eq.}\right)}{1-\delta} \geq \Pi^S \left(\text{dev.}\right) + \frac{\delta}{1-\delta} \left(1-q\right) \Pi^S \left(\text{dev.}\right), \tag{1}$$

that is met by all

$$\delta \geq \delta^* = \frac{\Pi^S \left(\text{dev}\right) - \Pi^S \left(\text{one-shot eq.}\right)}{q \Pi^S \left(\text{dev}\right)}.$$

It is straightforward to notice that the threshold level δ^* ensuring that the school plays the one-shot strategy decreases the higher the chance for the company to hire from SP, since the company's outside option is stronger. The ongoing discussion can be summarised in:

Proposition 3 *Let Assumptions 1 and 2 hold and the company plays a grim-trigger strategy in the infinitely repeated game:*

(i) for $p_a \geq p_d \geq g/(\nu+g)$ then the equilibrium in the repeated game with imperfect information has the same solution of the one-shot game;

(ii) for all $g/(\nu+g) \geq p_d$, then the equilibrium in the repeated game with imperfect information has the same solution of the one-shot game for:

$$\delta \geq \delta^* = \frac{\Pi^S\,(cheats) - \Pi^S\,(school\ correct)}{q\Pi^S\,(cheats)},$$

otherwise the school deviates and the company hires through SP for a proportion q for all the periods onward.

Proposition 3 shows that the school has an incentive to fully inflate over time. This result may explain the situation in those countries where grade inflation is strong and companies largely hire through SP, such as Italy. Note that the deviation equilibrium is more likely to occur in the case where the school has no concern on her reputation.[6] In an educational system where the degree obtained in each institution has the same legal value, a school/university has no strong concerns about its reputation.[7]

Moreover, by having in mind the differences in social background, grade inflation loses its beneficial features on soothing job opportunities. Since a

[6] An alternative scenario is the case with many schools/universities, where grade inflation can emerge as a free-rider problem (see Yang and Yip, 2003), since every school has incentive in inflating (every school is doing it!), so that the relative importance attributed to each school does not change but the absolute importance of the school system (δ) becomes low.

[7] By imagining a setting with competition between schools then δ becomes a "collective" measure of reputation (see Tirole, 1996).

student with an advantaged background has more linking and connections compared to an individual with disadvantaged background, it is more likely that a student hired with SP has advantaged background (see Caliendo *et al.*, 2009, for some recent evidence on that).

5 Concluding remarks

In this paper I analysed whether grade inflation may in fact have some positive effect in soothing class differences, and at the same time if can explain the emerging of job string pulling. In the one-shot version of the game, the school can inflate grades to smooth down class differences in the job market by inflating the grades of more advantaged rather than disadvantaged students.

In the infinitely repeated version of the game the beneficial effect of grade inflation is weaker with respect to the one-shot case. Given the poor information of the company, the school has an incentive to deviate from the equilibria where grade inflation has a positive effect on disadvantaged students, as long as it is not so concerned on its reputation. Moreover, resorting to hire through string-pulling as a further negative effect on class differences, since an individual with advantaged background more likely has helpful "acquaintances" to obtain a job, a better networking and so forth. This result explains the facts occurring in some countries where there is strong grade inflation, school and universities have no such concern on their reputation and thus the best way of obtaining a job is through string-pulling. Further analysis should examine the effects on welfare on string-pulling, the resulting inefficiencies and some policy consideration in order to exacerbate it.

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Appendix

Proof of Proposition 1

Along the proof, we leave aside the analysis about low-grade students. Indeed these students are never hired since they have low ability with probability 1.

Case 1

Company. The company strategy is

$$z_{Aa} = 1; z_{Ba} = 0, z_{Ad} = \frac{\Xi - \eta(p_a + g(1-p_a))}{(1-\eta)(p_d + g(1-p_d))}; z_{Bd} = 0.$$

The company's beliefs for advantaged students are

$$\pi(H \mid A, a) = \frac{p_a}{p_a + g(1-p_a)}$$

and

$$\pi(L \mid A, a) = \frac{g(1-p_a)}{p_a + g(1-p_a)},$$

if the student has a high grade. Thus the expected profit for hiring an advantaged and high-grade student is: [8]

$$\Pi^{C}\left(E,A,a\right)=\frac{p_{a}}{p_{a}+g\left(1-p_{a}\right)}\nu-\frac{g\left(1-p_{a}\right)}{p_{a}+g\left(1-p_{a}\right)}.$$

This must be

$$\frac{p_{a}}{p_{a}+g\left(1-p_{a}\right)}\nu-\frac{\eta\left(1-p_{a}\right)}{p_{a}+g\left(1-p_{a}\right)}\geq 0,$$

and, after few passages, $p_{a}\geq g/\left(\nu+g\right)$.

The company's beliefs for disadvantaged students are

$$\pi\left(H\mid A,d\right)=\frac{p_{d}}{p_{d}+g\left(1-p_{d}\right)}$$

and

$$\pi\left(L\mid A,d\right)=\frac{g\left(1-p_{d}\right)}{p_{d}+g\left(1-p_{d}\right)},$$

The expected profit for hiring one disadvantaged and high-grade student is

$$\Pi^{C}\left(E,A,d\right)=\frac{p_{d}}{p_{d}+g\left(1-p_{d}\right)}\nu-\frac{g\left(1-p_{d}\right)}{p_{d}+g\left(1-p_{d}\right)}.$$

This must be

$$\frac{p_{d}}{p_{d}+g\left(1-p_{d}\right)}\nu-\frac{g\left(1-p_{d}\right)}{p_{d}+g\left(1-p_{d}\right)}\geq 0,$$

and, after few passages,

$$p_{d}\geq\frac{g}{\left(\nu+g\right)}.$$

Then the company needs to compare the expected profit obtained by high grade students with different social background: this is $\Pi^{C}\left(E,A,a\right)>\Pi^{C}\left(E,A,d\right)$, as $p_{a}>p_{d}$. As a consequence,the company admits all the advantaged and

[8]The first letter in parenthesis of the company expected profit indicates the action performed by the employer, where E indicates "to hire" and N "to not". the second letter specifies the student's grade, where A and B stands for high and low grade, respectively. Finally, a and d indicates the student's social background.

high-grade students and the disadvantaged ones only for the remainder of job capacity.

School. The school strategy is $x_a = 1; x_d = 1$.[9] The expected payoff of this strategy is:

$$\Pi^S = \Xi b,$$

which is the maximum payoff that the school can acquire, so that this is the dominant strategy.

Finally, the job capacity constraint requires:

$$\eta(p_a + g\,(1 - p_a)) + z_{Ad}\,(1 - \eta)\,(p_d + g\,(1 - p_d)) \leq \Xi,$$

so that

$$z_{Ad} = \frac{\Xi - \eta(p_a + g\,(1 - p_a))}{(1 - \eta)\,(p_d + g\,(1 - p_d))}$$

Case 2

Since

$$p_a \geq \frac{g}{(\nu + g)},$$

the company and school strategy for advantaged students does not change compared to the previous case. So the proof will focus on the strategies for disadvantaged students.

Company. The company strategy is

$$z_{Aa} = 1; z_{Ba} = 0, z_{Ad} = \frac{\Xi - \eta(p_a + g\,(1 - p_a))}{x_d\,(1 - \eta)\,(p_d\,(1 + \nu))}; z_{Bd} = 0.$$

Since

$$p_d < \frac{g}{(\nu + g)},$$

[9] The first letters in the parenthesis of the school expected payoff indicates the action performed by the school, where G and NG stands for "to give" and "not to give grade inflation". The second letter specifies the student's social background.

if the school inflates the grades of all disadvantaged and low-ability students ($x_d = 1$), then the company would not hire any of them. Hence the school needs to provide less inflation grade. The company's beliefs for disadvantaged students are

$$\pi(H \mid A, d) = \frac{p_d}{p_d + g x_d (1 - p_d)}$$

and

$$\pi(L \mid A, d) = \frac{g x_d (1 - p_d)}{p_d + g x_d (1 - p_d)},$$

if the student has a high grade. Thus the expected profit for hiring a disadvantaged student is

$$\Pi^C(E, A, d) = \frac{p_d}{p_d + g x_d (1 - p_d)} \nu - \frac{g x_d (1 - p_d)}{p_d + g x_d (1 - p_d)}.$$

The company would hire all the disadvantaged students only if this is higher or equal to zero, and that is the case if

$$x_d = \frac{p_d \nu}{(1 - p_d) g},$$

To be a probability, then

$$\frac{p_a \nu}{(1 - p_a) g} < 1$$

by which $p_d < g / (\nu + g)$. This confirms the parameters space where this equilibrium lies. Like in the previous equilibrium, the company needs to compare the expected profit obtained by high grade students with different social background, obtaining the same result as before in favour of the advantaged students: $\Pi^C(E, A, a) > \Pi^C(E, A, d)$.

School. The school expected payoff is:

$$\Pi^S = b \left[\eta(p_a + g(1 - p_a)) + z_{Ad}(1 - \eta)(p_d + x_d g(1 - p_d)) \right].$$

By decreasing x_d, the Π^S decreases, while by increasing it to more than

$\frac{p_a \nu}{(1-p_a) g}$, then the company would not hire disadvantaged students, given the distribution of ability in the disadvantaged population. Thus this is the school best reply.

Finally, the job capacity constraint requires:

$$\eta(p_a + g(1-p_a)) + z_{Ad}(1-\eta)(p_d + x_d g(1-p_d)) \leq \Xi,$$

so that

$$z_{Ad} = \frac{\Xi - \eta(p_a + g(1-p_a))}{(1-\eta)(p_d + x_d g(1-p_d))}$$

Case 3

Since

$$p_d < \frac{g}{(\nu + g)},$$

the company and school strategy for disadvantaged students does not change compared to the previous case. So the proof will focus on the strategies for advantaged students.

Company. The company strategy is

$$z_{Aa} = 1; z_{Ba} = 0, z_{Ad} = \frac{\Xi - \eta(p_a + g(1-p_a))}{(1-\eta)(p_d(1+\nu))}; z_{Bd} = 0.$$

Since

$$p_a < \frac{g}{(\nu + g)},$$

if the school inflates the grades of all advantaged and low-ability students ($x_a = 1$), then the company would not hire any of them. Hence the school needs to provide less inflation grade. The company's beliefs for advantaged students are

$$\pi(H \mid A, a) = \frac{p_a}{p_a + g x_a (1-p_a)}$$

and

$$\pi(L \mid A, a) = \frac{gx_a(1-p_a)}{p_a + gx_a(1-p_a)},$$

if the student has a high grade. Thus the expected profit for hiring a disadvantaged and high-grade student is

$$\Pi^C(E, A, a) = \frac{p_a}{p_a + gx_a(1-p_a)}\nu - \frac{gx_a(1-p_a)}{p_a + gx_a(1-p_a)}.$$

The company would hire all the disadvantaged students only if this is higher than zero, and that is the case if

$$x_a = \frac{p_a\nu}{(1-p_a)g},$$

To be a probability, then

$$\frac{p_a\nu}{(1-p_a)g} < 1$$

by which $p_a < g/(\nu+g)$. This confirms the parameters space where this equilibrium lies. Like in the previous equilibrium, the company needs to compare the expected profit obtained by high grade students with different social background, obtaining the same result as before in favour of the advantaged students: $\Pi^C(E, A, a) > \Pi^C(E, A, d)$.

School. The school expected payoff is:

$$\Pi^S = b\left[z_{Aa}\eta(p_a + x_a g(1-p_a)) + z_{Ad}(1-\eta)(p_d + x_d g(1-p_d))\right],$$

By decreasing x_a or x_d, Π^S would decrease, while by increasing them to a more than $\frac{p_a\nu}{(1-p_a)g}$ and $\frac{p_a\nu}{(1-p_a)g}$, respectively, would lead the company not to hire anyone, since in both populations of students the proportion of low ability students is too high.

Finally, the job capacity constraint requires:

$$z_{Aa}\eta(p_a + x_a g(1-p_a)) + z_{Ad}(1-\eta)(p_d + x_d g(1-p_d)) \leq \Xi,$$

since the company strategy is $z_{Aa} = z_{Ad}$,

$$z_{Aa} = z_{Ad} = \frac{\Xi}{[\eta(p_a + x_a g(1 - p_a)) + (1 - \eta)(p_d + x_d g(1 - p_d))]}$$

Proof of Proposition 2

Case 1

Company. The company strategy is

$$z_{Aa} = \frac{\Xi - (1 - \eta)(p_d + g(1 - p_d))}{\eta(p_a + g(1 - p_a))}; z_{Ba} = 0, z_{Ad} = 1; z_{Bd} = 0.$$

The company's beliefs for advantaged students are

$$\pi(H \mid A, a) = \frac{p_a}{p_a + g(1 - p_a)}$$

and

$$\pi(L \mid A, a) = \frac{g(1 - p_a)}{p_a + g(1 - p_a)},$$

if the student has a high grade. Thus the expected profit for hiring an advantaged and high-grade student is:

$$\Pi^C(E, A, a) = \frac{p_a}{p_a + g(1 - p_a)}\nu - \frac{g(1 - p_a)}{p_a + g(1 - p_a)}.$$

This must be

$$\frac{p_a}{p_a + g(1 - p_a)}\nu - \frac{\eta(1 - p_a)}{p_a + g(1 - p_a)} \geq 0,$$

and, after few passages, $p_a \geq g/(\nu + g)$.

The company's beliefs for disadvantaged students are

$$\pi(H \mid A, d) = \frac{p_d}{p_d + x_d g(1 - p_d)}$$

and

$$\pi(L \mid A, d) = \frac{x_d g (1 - p_d)}{p_d + x_d g (1 - p_d)},$$

The expected profit for hiring one disadvantaged and high-grade student is

$$\Pi^C(E, A, d) = \frac{p_d}{p_d + x_d g (1 - p_d)} \nu - \frac{x_d g (1 - p_d)}{p_d + x_d g (1 - p_d)}.$$

This must be

$$\frac{p_d}{p_d + x_d g (1 - p_d)} \nu - \frac{x_d g (1 - p_d)}{p_d + x_d g (1 - p_d)} \geq 0,$$

and sufficient condition is:[10]

$$p_d \geq \frac{g}{(\nu + g)}.$$

Then the company needs to compare the expected profit obtained by high grade students with different social background. This needs to be $\Pi^C(E, A, d) > \Pi^C(E, A, a)$:

$$\frac{p_d \nu - x_d g (1 - p_d)}{p_d + x_d g (1 - p_d)} \geq \frac{p_a \nu - g (1 - p_a)}{p_a + g (1 - p_a)}$$

$$(p_d \nu - x_d g (1 - p_d)) (p_a + g (1 - p_a)) \geq (p_a \nu - g (1 - p_a)) (p_d + x_d g (1 - p_d))$$

so that the school strategy is:

$$x_a = 1, x_d = \frac{p_d (1 - p_a)}{p_a (1 - p_d)}.$$

In order to be a probability, it is necessary that $\frac{p_d (1 - p_a)}{p_a (1 - p_d)} < 1$, leading to $\frac{p_d}{(1 - p_d)} < \frac{p_a}{(1 - p_a)}$, which respects the assumption $p_a > p_d$. In other words, the school inflates less the disadvantaged students according to the distribution of ability in both populations.

[10] The condition is $p_d \geq \frac{x_d g}{(\nu + x_d g)} < \frac{g}{(\nu + g)}$.

School. The school expected payoff is:

$$\Pi^S = b_a \eta \left(p_a + g\left(1 - p_a\right)\right) + b_d \left(1 - \eta\right) \left(p_d + x_d g \left(1 - p_d\right)\right),$$

which is the maximum payoff that the school can acquire, so that this is the dominant strategy.

Finally, the job capacity constraint requires:

$$z_{Aa} \eta (p_a + g\left(1 - p_a\right)) + \left(1 - \eta\right) \left(p_d + x_d g \left(1 - p_d\right)\right) \leq \Xi,$$

so that

$$z_{Aa} = \frac{\Xi - \left(1 - \eta\right) \left(p_d + x_d g \left(1 - p_d\right)\right)}{\eta (p_a + g\left(1 - p_a\right))}.$$

In the case where

$$p_d < \frac{g}{\nu + g},$$

the school needs to provide less inflation grade to disadvantaged students than the previous case, otherwise the company would not hire any of them. The expected profit for hiring a disadvantaged student is

$$\Pi^C \left(E, A, d\right) = \frac{p_d}{p_d + g x_d \left(1 - p_d\right)} \nu - \frac{g x_d \left(1 - p_d\right)}{p_d + g x_d \left(1 - p_d\right)}.$$

The company would hire all the disadvantaged students only if this is higher or equal to zero, and that is the case if

$$x_d \leq \frac{p_d \nu}{\left(1 - p_d\right) g},$$

Like in the previous equilibrium, the company needs to compare the expected profit obtained by high grade students with different social background.

Then the company needs to compare the expected profit obtained by high grade students with different social background: this is $\Pi^C \left(E, A, d\right) >$

$\Pi^C(E, A, a)$:
$$\frac{p_d\nu - x_d g(1-p_d)}{p_d + x_d g(1-p_d)} \geq \frac{p_a\nu - g(1-p_a)}{p_a + g(1-p_a)}$$
$$(p_d\nu - x_d g(1-p_d))(p_a + g(1-p_a)) \geq (p_a\nu - g(1-p_a))(p_d + x_d g(1-p_d))$$

so that the school strategy for disadvantaged students is:

$$x_d = \frac{p_d(1-p_a)}{p_a(1-p_d)}.$$

Thus it is necessary that:

$$\frac{p_d(1-p_a)}{p_a(1-p_d)} \leq \frac{p_d\nu}{(1-p_d)g},$$

and after few passages, the condition is:

$$p_a \geq \frac{g}{\nu + g}.$$

School. The school expected payoff is:

$$\Pi^S = b_d(1-\eta)(p_d + x_d g(1-p_d)) + b_a\eta(p_a + g(1-p_a)).$$

By decreasing x_d, the Π^S decreases, while by increasing it to more than $\frac{p_a\nu}{(1-p_a)g}$, then the company would not hire disadvantaged students, given the distribution of ability in the disadvantaged population. Thus this is the school best reply.

Finally, the job capacity constraint requires:

$$z_{Aa}\eta(p_a + g(1-p_a)) + b[(1-\eta)(p_d + x_d g(1-p_d))] \leq \Xi,$$

so that
$$z_{Aa} = \frac{\Xi - b(1-\eta)(p_d + x_d g(1-p_d))}{\eta(p_a + g(1-p_a))}$$

Case 2

Company. The company strategy is

$$z_{Aa} = 1; z_{Ba} = 0, z_{Ad} = \frac{\Xi - \eta(p_a + g(1 - p_a))}{(1-\eta)(p_d(1+\nu))}; z_{Bd} = 0.$$

Since

$$p_a < \frac{g}{(\nu + g)},$$

if the school inflates the grades of all advantaged and low-ability students ($x_a = 1$), then the company would not hire any of them. Hence the school needs to provide less inflation grade. The expected profit for hiring an advantaged and disadvantaged student are

$$\Pi^C(E, A, a) = \frac{p_a}{p_a + gx_a(1-p_a)}\nu - \frac{gx_a(1-p_a)}{p_a + gx_a(1-p_a)},$$

and

$$\Pi^C(E, A, d) = \frac{p_d}{p_d + gx_d(1-p_d)}\nu - \frac{gx_d(1-p_d)}{p_d + gx_d(1-p_d)},$$

respectively. The two conditions are $\Pi^C(E, A, a) \geq 0$ and $\Pi^C(E, A, d) \geq \Pi^C(E, A, a)$. by solving the system

$$\begin{cases} \dfrac{p_a\nu - x_a g(1-p_a)}{p_a + x_a g(1-p_a)} \geq 0 \\ \dfrac{p_d\nu - x_d g(1-p_d)}{p_d + x_d g(1-p_d)} \geq \dfrac{p_a\nu - x_a g(1-p_a)}{p_a + x_a g(1-p_a)} \end{cases}$$

w.r.t. x_a and x_d we obtain

$$x_a = \frac{p_a\nu}{(1-p_a)g}, \; x_d = \frac{p_d\nu}{(1-p_d)g}.$$

School. The school expected payoff is:

$$\Pi^S = b_a z_{Aa}\eta(p_a + x_a g(1-p_a)) + b_d z_{Ad}(1-\eta)(p_d + x_d g(1-p_d)),$$

By decreasing x_a or x_d, Π^S would decrease, while by increasing them to a more than $\frac{p_a \nu}{(1-p_a)g}$ and $\frac{p_d \nu}{(1-p_d)g}$, respectively, would lead the company not to hire anyone, since in both populations of students the proportion of low ability students is too high.

Finally, the job capacity constraint requires:

$$z_{Aa}\eta(p_a + x_a g\,(1 - p_a)) + (1 - \eta)\,(p_d + x_d g\,(1 - p_d)) \leq \Xi,$$

so that

$$z_{Aa} = \frac{\Xi - (1 - \eta)\,(p_d + x_d g\,(1 - p_d))}{\eta(p_a + x_a g\,(1 - p_a))}. \blacksquare$$

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