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Robert Aumann's and Thomas Schelling's Contributions to Game Theory: Analyses of Conflict and Cooperation

1. Introduction

Wars and other conflicts are among the main sources of human misery. A minimum of cooperation is a prerequisite for a prosperous society. Life in an anarchic "state of nature" with its struggle of every man against every man is, in Thomas Hobbes' (1651) famous phrase, "solitary, poor, nasty, brutish, and short".

Social scientists have long attempted to understand the fundamental causes of conflict and cooperation. The advent of game theory in the middle of the twentieth century led to major new insights and enabled researchers to analyze the subject with mathematical rigor. The foundations of game theory were laid out in the classic book by John von Neumann and Oscar Morgenstern, *The Theory of Games and Economic Behavior*, published in 1944. The 1994 economics laureates John Harsanyi, John Nash and Reinhard Selten added solution concepts and insights that substantially enhanced the usefulness and predictive power of non-cooperative game theory. The most central solution concept is that of Nash equilibrium. A strategy combination (one strategy for each player) constitutes a Nash equilibrium if each player's strategy is optimal against the other players' strategies.[1] Harsanyi showed that this solution concept could be generalized to games of incomplete information (that is, where players do not know each others' preferences). Selten demonstrated that it could be refined for dynamic games and for games where players make mistakes with (infinitesimally) small probabilities. Nevertheless, the great intellectual achievements of these researchers would have been to little avail, had game-theoretic tools not been applied to address salient questions about society.

The work of two researchers, Robert J. Aumann and Thomas C. Schelling, was essential in developing non-cooperative game theory further and bringing it to bear

[1] A non-cooperative game in normal form consists of a list of players, a set of strategies available to each player, and a function that specifies the payoff consequences to all players of each strategy combination.

on major questions in the social sciences.[2] Approaching the subject from different angles—Aumann from mathematics and Schelling from economics—they both perceived that the game-theoretic perspective had the potential to reshape the analysis of human interaction. Perhaps most importantly, Schelling showed that many familiar social interactions could be viewed as non-cooperative games that involve both common and conflicting interests, and Aumann demonstrated that long-run social interaction could be comprehensively analyzed using formal non-cooperative game theory.

Although their writings on conflict and cooperation were well received when they appeared in the late 1950s, it took a long time before Aumann's and Schelling's visions came to be fully realized. The delay reflects both the originality of their contributions and the steepness of the subsequent steps. Eventually, and especially over the last twenty-five years, game theory has become a universally accepted tool and language in economics and in many areas of the other social sciences. Current economic analysis of conflict and cooperation builds almost uniformly on the foundations laid by Aumann and Schelling.

2. Schelling

Thomas Schelling's book *The Strategy of Conflict* (1960) launched his vision of game theory as a unifying framework for the social sciences. Turning attention away from zero-sum games, such as chess, where players have diametrically opposed interests, he emphasized the fact that almost all multi-person decision problems contain a mixture of conflicting and common interests, and that the interplay between the two concerns could be effectively analyzed by means of non-cooperative game theory. The stage had been set by Nash (1950a,1951), who had proven that there exist (Nash) equilibria in all games with finitely many pure strategies. Schelling took on the complementary task of deducing the equilibria for interesting classes of games and evaluating whether these games and their equilibria were instructive regarding actual economic and social interaction. He did this against the background of the world's first nuclear arms race and came to contribute greatly to our understanding of its implications.

[2] While cooperative game theory starts out from a set of potential binding agreements and players' preferences over them, non-cooperative game theory starts out from players' strategy sets and preferences over the associated outcomes.

2.1. Conflict, commitment and coordination. Schelling's earliest major contribution is his analysis of behavior in bilateral bargaining situations, first published as an article (Schelling, 1956) and later reprinted as Chapter 2 of Schelling (1960). Here, bargaining is interpreted broadly: besides explicit negotiations—for example between two countries or between a seller and a buyer—there is also "bargaining" when two trucks loaded with dynamite meet on a road wide enough for one, to cite one of Schelling's characteristically graphical examples.

Bargaining always entails some conflict of interest in that each party usually seeks an agreement that is as favorable as possible. Yet, any agreement is better for both parties than no agreement at all. Each player has to balance the quest for a large "share of the pie" against the concern for agreement. When Schelling wrote his article, economists' work on bargaining had typically taken a cooperative or normative approach, by asking questions such as: what is a fair outcome? An exception was Nash, who modeled bargaining both with a cooperative (Nash 1950b) and a non-cooperative (Nash, 1953) approach. While Nash's formulations allow elegant mathematical analyses by way of abstracting from many realistic bargaining tactics, Schelling examines the bargaining tactics a player can use in order to tilt the outcome in his or her favor — emphasizing in particular that it may be advantageous to worsen one's own options in order to elicit concessions from the opponent. It can be wise for a general to burn bridges behind his troops as a credible commitment towards the enemy not to retreat. Similarly, the owners of a firm may profitably appoint a manager with limited powers to negotiate, and a politician may gain from making public promises that would be embarrassing to break. Such tactics work if the commitment is irreversible or can only be undone at great cost, while commitments that are cheap to reverse will not elicit large concessions. However, if both parties make irreversible and incompatible commitments, harmful disagreement may follow.

Let us illustrate some of the key issues by means of a stylized and simple example. Suppose that two countries disagree over the right to a patch of territory.[3] Each country can choose to mobilize military force or refrain from doing so. If both mobilize there is a high probability of war, while the probability of a peaceful agreement about

[3] For more elaborate game-theoretic analyses of commitment in bargaining, see for example Crawford (1982), Muthoo (1996), and Güth, Ritzberger and van Damme (2004).

division of the territory is low. Let the expected payoff to each country be zero if both mobilize. If instead both countries refrain from mobilization, a peaceful agreement about division of the territory has a high probability, while the probability of war is small. In this case, each country obtains a positive expected payoff b. However, if only one country mobilizes, it can take complete control of the territory without war, and neither the other country nor any other party can force a military retreat by the occupant. The aggressor obtains payoff a while the loser's payoff is c, where $a > b > c > 0$, war thus being the worst outcome.[4] This simple "mobilization game" can be described by the following payoff bi-matrix, where one player (here country) chooses a row and the other simultaneously chooses a column, with the row player's payoff listed first in each entry:

	Mobilize	Refrain
Mobilize	$0, 0$	a, c
Refrain	c, a	b, b

TABLE 1

This game belongs to a class of games known as "Chicken," sometimes called "Hawk-Dove." Such games have three Nash equilibria: two pure and one mixed. The pure equilibria entail mobilization by exactly one country; if one country expects the other to mobilize, then it is optimal to refrain from mobilization. The mixed equilibrium entails randomized mobilization by each country and thus a positive probability of war.

The pure equilibria are plausible in situations where the two countries have some means to coordinate on either equilibrium. For example, a small perturbation of the game that would create even a tiny asymmetry in the payoffs, may be enough for both players to expect mobilization by the player who has the most to gain from it, thus rendering that equilibrium "salient" or "focal." According to Schelling, it is likely that humans are capable of such coordination in many situations, while a purely formal analysis is likely to be unable to capture the principles of salience or

[4]In some conflict situations, war is less undesirable than the humiliation that may be associated with lack of mobilization when the other country mobilizes. In such situations, $a > b > 0 > c$, and the game becomes a prisoners' dilemma, with mutual mobilization as the outcome, see section 3.1.

focality in the game in question: "One cannot, without empirical evidence, deduce what understandings can be perceived in a non-zero sum game of maneuver any more than one can prove, by purely formal deduction, that a particular joke is bound to be funny" is a famous quote by Schelling (1960, p.164). Instead, equilibrium selection is "an area where experimental psychology can contribute to game theory" (*ibid.* p.113).[5]

Absent any commonly understood coordination principle, the game's mixed equilibrium appears more plausible. Each country is then uncertain about the other's move, assigning some probability p to the event that the other country will mobilize. The Nash equilibrium probability of mobilization is $p = (a-b)/(a-b+c)$, rendering each country indifferent whether to mobilize.[6] It follows that, for plausible parameter values, the probability of war is decreasing in the loser's payoff c; the key to minimizing the risk of war is not only to contain the winner's gain but equally importantly to improve the loser's payoff.[7]

Mobilizing and threatening to mobilize are not equivalent. A formal analysis of deterrence is complicated and requires specifying a dynamic game with several stages, but with Schelling's intuition as a guide it is possible to proceed without detailed mathematics. The study of credible deterrence through so-called "second-strike" strategies takes up a major part of *The Strategy of Conflict*. Schelling emphasizes that investments in deterrence can become dangerous in case of false warnings as well as when misjudging the adversary's interests and intentions.

Suppose that Country 1 can pre-commit to mobilize if Country 2 mobilizes. More precisely: first Country 1 chooses whether to refrain from mobilization altogether or to commit to mobilize if and only if Country 2 mobilizes. Thereafter, Country 2 observes 1's move and decides whether or not to mobilize. If payoffs are as described in

[5]By now, there is a sizeable experimental literature on focal points in bargaining as well as in other games, much of it informed and inspired by Schelling. We return to the coordination problem below.

[6]Mobilizing yields expected payoff $(1-p)a$, while refraining yields $pc + (1-p)b$. Equating the two determines the equilibrium probability p.

[7]By statistical independence between the two players' randomization, the probability for war is $q = p^2\mu + (1-p)^2\nu$, where μ is the probability for war when both countries mobilize and $\nu < \mu$ the probability for war when no country mobilizes. It follows that q is increasing in p for all $p > \nu/(\mu+\nu)$. Hence q is decreasing in c if $c < (a-b)\mu/\nu$.

Table 1, the (subgame perfect) equilibrium outcome will be that Country 1 makes the mobilization commitment, and both countries refrain from mobilization. Indeed, it is sufficient that Country one commits to mobilize with a sufficiently high probability.[8] Such deterrence thus guarantees a peaceful outcome—a balance of terror.

Suppose, moreover, that Country 1 is uncertain whether Country 2 actually prefers war to the negotiated outcome. In the game-theoretic parlance (based on Harsanyi's work), Country 1 now has incomplete information about Country 2's payoffs. Should Country 1 still commit to mobilize if country Country 2 mobilizes? Schelling's analysis reveals that the optimal commitment strategy is then often to choose a probability of mobilization that is less than one. In other words, in the face of an enemy's military escalation, a country should threaten to let the situation "slip out of hand" rather than commit to certain retaliation, or in Schelling's words, make "threats that leave some things to chance." The reason is that a modest probability of war may be enough to deter the enemy's mobilization.[9]

Another virtue of uncertain retaliation threats is that credibility is easier to attain the smaller is the own expected retaliation cost. In fact, Schelling suggested that a good way to meet enemy aggression is to engage in "brinkmanship" – gradually stepping up the probability of open conflict. Since each step is small, credibility can be sustained by the anger and outrage that builds steadily against an unrelenting opponent, and since the opponent can reduce the probability of conflict by relenting, the probability of conflict is kept low. As Schelling observed, most children understand brinkmanship perfectly.

The above analysis implies that countries should keep the adversary guessing about their response to aggression, at the same time ensuring that forceful retaliation

[8]Suppose Country 1 can commit to any probability $\pi \in [0,1]$ of retaliation if Country 2 mobilizes. If Country 2's preferences are as in Table 1, deterrence requires that $b \geq (1-\pi)a$ or, equivalently, that $\pi \geq 1 - b/a = \pi^*$.

[9]Let θ be the probability that Country 1 attaches to the possibility that Country 2 prefers to mobilize regardless of the retaliation threat. For $\pi < \pi^*$, Country 2 will still mobilize for sure, so the payoff to Country 1 is then $(1-\pi)\,c$, a decreasing function of π. For $\pi \geq \pi^*$ its expected payoff is $\theta(1-\pi)c + (1-\theta)b$, again a decreasing function of π. Hence, deterrence (choosing $\pi = \pi^*$) is optimal for Country 1 if and only if $\theta(1-\pi^*)c + (1-\theta)b$ is at least as large as the payoff c from not retaliating ($\pi = 0$), or, equivalently, if and only if $\theta \leq (1-c/b)/(1-c/a)$.

is regarded as a real option. Two other insights are also quite immediate. First, deterrence only works when retaliatory weapons can be shielded in case of an enemy attack; war prevention thus requires invulnerable basing of weapons—such as missile silos—rather than protection of population centers. Second, instability is dangerous. The balance of terror is maintained only as long as retaliation is sufficiently probable and harsh compared to the gains from occupation. War can be ignited by changes in preferences as well as in technology, and successful attempts at disarmament have to be balanced throughout.

Schelling's analysis of "credible commitments" demonstrated that some Nash equilibria are more plausible than others, inspiring Reinhard Selten's subgame perfection refinement of the Nash equilibrium concept.[10] Schelling's and Selten's work on strategic commitment initiated a lively economics literature. The analyses of strategic investment in oligopoly markets developed by, among others, Avinash Dixit and Schelling's student Michael Spence (a 2001 laureate) are leading examples of applied work on commitment that took off in the late 1970s (see, for example, Spence, 1977, and Dixit, 1980). Their analyses show that a firm operating in an imperfectly competitive market can increase its profits by changing its cost structure, even if its unit production cost increases as a result. For example, a firm can credibly commit to a high volume of output by investing in an expensive plant with low marginal costs. Even if average costs thereby go up, losses due to inefficient production can be outweighed by the gains generated by competitors' less aggressive behavior.

The literature on monetary policy institutions provides another example of the idea of strategic commitment at work. Here, the major point is that under certain circumstances, voters and politicians are better off delegating monetary policy to decision makers with other preferences than their own. Since firms and trade unions take the expected monetary policy into account when setting prices and wages, an independent central banker can be superior to an elected politician even if the politician

[10] A Nash equilibrium in an extensive-form game is *subgame perfect* (Selten, 1965) if it induces a Nash equilibrium in every subgame. Since Nash equilibrium only requires optimality on the path of play, Nash equilibria may well rely on "threats" or "promises" that will not subsequently materialize. Subgame perfection eliminates many such equilibria, and, in later work Selten (1975) developed a stronger refinement, "perfection."

at each point in time would act in accordance with the current public interest.[11]

Sometimes conflicts of interest may appear so strong as to be insoluble. The best strategy for an individual may result in the worst outcome for a group. The short-run gains from cheating on an agreement might by far outweigh the short-run losses. Schelling (1956) noted that "What makes many agreements enforceable is only the recognition of future opportunities for agreement that will be eliminated if mutual trust is not created and maintained, and whose value outweighs the momentary gain from cheating in the present instance." (op. cit. p. 301). Thus, if the parties take a long perspective and do in fact interact repeatedly, their common interests may be sufficiently strong to sustain cooperation. In fact, Schelling went further: "Even if the future will bring no recurrence, it may be possible to create the equivalence of continuity by dividing the bargaining issue into consecutive parts." That is, people can structure their relationships, by extending interaction over time, in such a way as to reduce the incentive to behave opportunistically at each point in time.

When Schelling first made these observations and conjectures, game theory had not advanced far enough to allow him to articulate them precisely, far less prove them. Gradually, however, the literature on repeated games and "Folk Theorems" (discussed below) demonstrated how present cooperation can be credibly sustained by the threat of conflict in similar situations in the future. As for Schelling's assertion that it is sometimes possible to sustain agreement by decomposing one large cooperative action into several small ones, it took the profession more than forty years to fully develop the formal argument. Lockwood and Thomas (2002) demonstrate in a two-player model that private provision of public goods can often be substantially higher if the parties can take turns contributing than if they can only make one round of contributions each.[12] By gradually increasing their contribution, implicitly threatening to stop the increase if the other does so, each party holds out a carrot to the other. However, fully efficient contribution levels are only attainable under strong additional assumptions, such as zero discounting (Gale, 2001) or non-smooth payoff

[11]Last year's economics prize was awarded to Finn Kydland and Edward Prescott in part for having identified and analyzed problems of commitment in economic policy-making. The macroeconomic literature on delegation is largely inspired by their work, see e.g. Rogoff (1985).

[12]Admati and Perry (1991) were the first to tackle the problem head on, but their analysis considered a fairly special environment and only yielded weak support for Schelling's conjecture.

functions (Marx and Matthews, 2000). These analyses can potentially explain why progress is necessarily gradual in many areas where cooperative actions are costly to reverse. Examples include military disarmament, environmental cooperation, and shrinkage of production capacity in a declining market.

Gradual cooperation occurs not only among humans. The biologist John Maynard Smith describes the mating behavior of the black hamlet, a hermaphrodite coral reef fish which carries both sperm and eggs simultaneously (Maynard Smith, 1982, pages 159-160). When the fish mates, it engages in several rounds of "egg trading" where it alternately lays eggs and fertilizes the eggs of its partner. The proposed explanation is that it is cheaper to produce sperm than eggs, so if all eggs were being laid at once, the fish playing the male role in this first round might not produce any eggs thereafter, preferring instead to play the male role again with another fish willing to produce eggs. By saving some eggs to be used as a reward for the other fish's eggs, each fish lowers the partner's incentive to defect. This is but one example from evolutionary biology where Schelling's analysis has relevance.

Schelling also studied a class of social interactions that involve little or no conflict of interest, so-called pure coordination games. These are games where all players prefer coordination on some joint course of action and no player cares about which coordinated course of action is taken. For example, it may not matter to a team of workers who carries out which task, as long as the team gets its job done. In this case, coordination may be easy if players can communicate with each other but appears difficult without communication. By experimenting with his students and colleagues, Schelling discovered that they were often able to coordinate rather well without communicating even in unfamiliar games that had an abundance of Nash equilibria. As an example, consider the game where two people are asked to select a positive integer each. If they choose the same integer both get an award, otherwise no award is given. In such a setting, the majority tends to select the number 1. This number is distinctive, since it is the smallest positive integer. Likewise, in many other settings, Schelling's experimental subjects were able to utilize contextual details, joint references, and empathy in order to identify "focal" equilibria.[13] It seems likely that

[13] Subsequent attempts to discover fundamental coordination principles include Mehta, Starmer and Sugden (1994a,b). Camerer (2003, Chapter 7), gives an overview of coordination experiments.

many social conventions and organizational arrangements have emerged because they facilitate coordination. Inspired by Schelling's analysis of coordination in common interest games, the philosopher David Lewis specified the compelling hypothesis that language itself has emerged as a convention (Lewis, 1969).

A final interesting class of social decision problems are interactions in which participants are mutually distrustful. For example, two generals may both agree that war is undesirable, and will hence prepare for peace as long as they both think that the other will do likewise. Yet, if one general suspects that the other is preparing for war, then his best response may be to prepare for war as well—when war is less undesirable than being occupied.[14] As Schelling (1966, page 261) notes, this idea had already been clearly formulated by Xenophon (in the fourth century B.C.). A more recent version of the argument is due to Wohlstetter (1959), who in turn inspired Schelling. The analysis was advanced by Schelling (1960, Chapter 9), who expressed it in game-theoretic terms and considered explicitly the role of uncertainty in triggering aggression. To illustrate the possibility that war is caused solely by mutual distrust, consider the following payoff bi-matrix (the first number in each entry being the payoff to the row player):

	War	Peace
War	$2, 2$	$3, 0$
Peace	$0, 3$	$4, 4$

TABLE 2

Each player has the choice between going to war and behaving peacefully. The two pure-strategy Nash equilibria are (War, War) and (Peace, Peace). If players are rational, carry out their plans perfectly, and have no uncertainty about the opponent's payoff, Schelling (1960, p.210) thought that peace would be the most plausible outcome of such a game (a position that is not shared by all game theorists). However, Schelling (1960, p.207) also contended that a small amount of nervousness about the opponent's intentions could be contagious enough to make the peaceful equilibrium crumble: "If I go downstairs to investigate a noise at night, with a gun in my hand,

For recent theoretical work, see Binmore and Samuelson (2005).

[14]This was not the case in the previous example, where war was the worst outome.

and find myself face to face with a burglar who has a gun in his hand, there is a danger of an outcome that neither of us desires. Even if he prefers just to leave quietly, and I wish him to, there is danger that he may *think* I want to shoot, and shoot first. Worse, there is danger that he may think that *I* think *he* wants to shoot." Schelling did attempt a formal analysis of this surprise attack dilemma, but since game theory at that time lacked a proper framework for studying games with incomplete information, it is fair to say that his modeling did less than full justice to his intuition.[15]

The Strategy of Conflict has had a lasting influence on the economics profession as well as on other social sciences. It has inspired, among other things, the detailed analysis of bargaining in historical crisis situations (see e.g. Snyder and Diesing, 1977). The book and its sequels *Strategy and Arms Control* (1961, coauthored with Morton Halperin) and *Arms and Influence* (1966), also had a profound impact on military theorists and practitioners in the cold war era, played a major role in establishing "strategic studies" as an academic field of study, and may well have contributed significantly to deterrence and disarmament among the superpowers.[16]

2.2. Other contributions. Over the forty-five years since the publication of *The Strategy of Conflict*, Thomas Schelling has continued to produce a series of novel and useful ideas. We briefly mention two of them here.

In a much cited article from 1971, Schelling analyzed how racially mixed societies and neighborhoods can suddenly become segregated as the proportion of inhabitants of one race gradually slides below a critical level. A modest preference for not forming part of a minority in one's neighborhood, but not necessarily favoring dominance of one's own race, can cause small microshocks to have drastic consequences at the macro level. Besides providing a convincing account of an important social policy problem, Schelling here offers an early analysis of "tipping"—the rapid movement from one equilibrium to another—in social situations involving a large number of in-

[15]Recently, Baliga and Sjöström (2004) have provided a formal analysis.

[16]The secrecy surrounding military issues makes it difficult to assess the exact impact of Schelling's work on the behavior of superpowers. However, a clue is that in 1993 Schelling won the National Academy of Sciences (U.S.) Award for Behavioral Research Relevant to the Prevention of Nuclear War.

dividuals. The tipping phenomenon is pursued in several different contexts in another of Schelling's influential books, *Micromotives and Macrobehavior* from 1978, and has been further analyzed by other social scientists.

The next seminal set of ideas is explored in a sequence of articles on self-command, notably Schelling (1980, 1983, 1984a, 1992).[17] Here, Schelling observes that we do many things that we wish we would rather not do, for example smoking and drinking too much or exercising and saving too little. He also explores the limits of self-management and the associated challenges for public policy. Interestingly, the importance of credible commitments is no smaller in this context of intrapersonal conflicts than in the interpersonal conflicts which occupied Schelling at the beginning of his career. Over the last decade, with the rise of behavioral economics, the issue of limited self-command has received widespread attention.[18] There are now many papers in leading economics journals on procrastination, under-saving, and unhealthy consumption.

In sum: the "errant economist" (as Schelling has called himself) turned out to be a pre-eminent pathfinder.

3. Aumann

Robert Aumann has played an essential role in shaping game theory. He has promoted a unified view of the very wide domain of strategic interactions, encompassing many apparently disparate disciplines, such as economics, political science, biology, philosophy, computer science and statistics. Instead of using different constructs to deal with various specific issues—such as deterrence, perfect competition, oligopoly, taxation and voting—Aumann has developed general methodologies and investigated where these lead in each specific application. His research is characterized by an unusual combination of breadth and depth. Some contributions contain involved analysis while others are technically simple but conceptually profound. His fundamental works have both clarified the internal logic of game-theoretic reasoning and expanded game theory's domain of applicability.

[17] The first two of these are reprinted as Chapters 3 and 4 in Schelling (1984b).

[18] For early formal analyses of such problems, see e.g. Strotz (1956) and Phelps and Pollak (1968).

3.1. Long-term cooperation. Among Aumann's many contributions, the study of long-term cooperation has arguably had the most profound impact on the social sciences. As pointed out above, a great deal of interaction is long-term in nature, sometimes of indefinite duration. Countries often have an opportunity to gain some advantage at their neighbors' expense. Competing firms may take daily or monthly production and pricing decisions, conditioned in part on their competitors' past behavior. Farmers may join together to manage some common resource, such as a pasture or water source, etc. It is therefore important to study recurrent interaction with a long horizon.

The difference between short-term and long-term interaction is perhaps most easily illustrated by the well-known prisoners' dilemma game. This is a two-person game, where each player has two pure strategies, to "cooperate" (C) or "defect" (D). The players choose their strategies simultaneously. Each player's dominant strategy is D—that is, D is an optimal strategy irrespective of the other's strategy—but both players gain if they both play C. When played once, the game thus admits only one Nash equilibrium: that both players "defect." However, the equilibrium outcome is worse for both players than the strategy pair where both "cooperate." An example is given by the following payoff bi-matrix, where, as before, the first number in each entry is the payoff to the row player and the second number the payoff to the column player.[19]

	C	D
C	$2, 2$	$0, 3$
D	$3, 0$	$1, 1$

TABLE 3

Suppose that the same two players meet every day, playing the prisoner' dilemma over and over again, seeking to maximize the average daily payoff stream over the infinite future. In this case, it can be shown that cooperation in every period is an equilibrium outcome. The reason is that players can now threaten to punish any

[19]The payoffs are assumed to be von Neumann-Morgenstern utilities truly capturing the motives of the players. If the payoffs are monetary instead, it is perfectly possible that a rational player would choose C out of of concern for the other player's income.

deviation from cooperative play today by refusing to cooperate in the future. That is, the short-term gain from defection today is more than outweighed by the reduction in future cooperation.

In fact, Aumann (1959) proved a much more general result, concerning any "supergame" G^* that consists of the infinite repetition of any given game G. Essentially, he showed that any average payoff that is feasible in the supergame and does not violate individual rationality (see below) in the "stage game" G can be sustained as a Nash equilibrium outcome in G^*. Moreover, he demonstrated that the result holds even if robustness is required with respect to joint deviations by coalitions of players.

Let us state the result more precisely. A *pure strategy* in G^* is a decision rule that assigns a pure strategy in G to each period and for every history of play up to that period. The set of pure strategies in G^* is thus infinite and contains very complex strategies. The main result of the paper specifies exactly the set of strong equilibrium payoffs of G^*.[20] A *strong equilibrium*, a solution concept due to Aumann (1959), is a strategy profile such that no group (subset, coalition) of players can, by changing its own strategies, obtain higher payoffs to all members of the group.[21] Nash equilibrium is thus the special case in which the deviating group always consists of exactly one player. Aumann showed that the set of strong equilibrium payoffs coincides with the so-called β-core of the game G that is being repeated. The β-core, a version of the core, essentially requires that no group of players can guarantee themselves higher payoffs—even if the others would "gang up" against them.

When Aumann's result is applied to deviating groups of size one, the result is a so-called Folk Theorem for repeated games. According to this theorem, the set of Nash equilibrium payoffs of an infinitely repeated game G^* coincides with the set of feasible and individually rational payoffs. A payoff vector—a list of payoffs, one for each player— is *feasible* if it is the convex combination of payoff vectors that can be obtained by means of pure strategies in G, and a payoff level is *individually rational* for a player if it is not less than the lowest payoff in G to which the other players can "force" the player down.[22] The gist of the argument is to provide strategies in

[20] Aumann defines payoffs in G^* by means of a certain limit of time averages of payoffs in G.

[21] Not all games have such equilibria.

[22] The set of individually rational payoffs can be defined as follows. For each (pure or mixed) strategy combination of the other players in G, let the player in question play a (pure or mixed)

G^* that constitute "threats" against deviations from strategies in G^* that implement the given payoff vector.

In the prisoners' dilemma considered here, the set of feasible and individually rational payoff pairs consists of all payoff pairs that can be obtained as convex combinations of the payoff pairs in Table 3 and where no payoff is below 1. To see this, first note that each player can guarantee himself a payoff of at least 1 by playing D. Second, the four pure-strategy pairs result in payoff pairs $(2,2)$, $(1,1)$, $(3,0)$ and $(0,3)$. The set of feasible payoff pairs is thus the polyhedron with these pairs as vertices. The shaded area in Figure 1 below is the intersection of these two sets. All these payoff pairs, and no others, can be obtained as time-average payoffs in Nash equilibrium of the infinitely repeated play of this game.

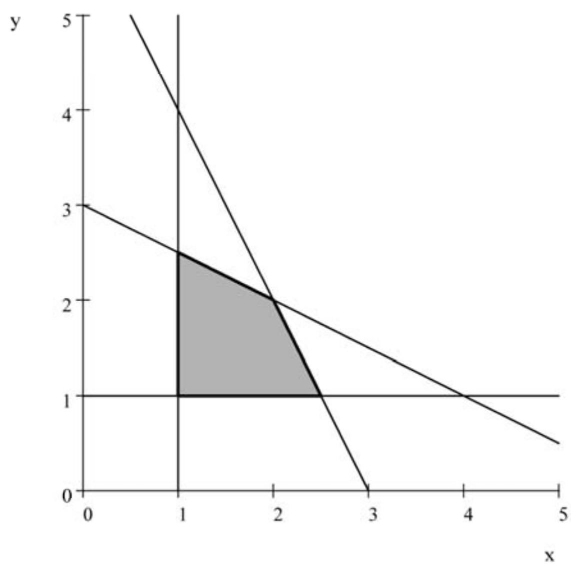

Figure 1: The set of feasible and individually rational payoff pairs in the prisoners' dilemma game in Table 3.

Applied to the game in Table 1, the Folk Theorem claims that all payoff pairs that are convex combinations of $(0,0)$, (a,c), (c,a) and (b,b), and where no payoff is

best reply. The minimal value among the resulting payoffs to the latter defines the lower bound on that player's individually rational payoffs.

below c, can be obtained as time-average payoffs in Nash equilibrium of the infinitely repeated play of that game. In particular, the "good" outcome (b, b) is sustainable—despite the fact that it is not an equilibrium of the game when played once. Deviations from prescribed play can be threatened by "minmaxing" the deviator, that is, the other player randomizes between the two pure strategies in such a way as to minimize the deviator's expected payoff when the latter plays his or her best reply against this "punishment." Such punishments can also sustain other outcomes as equilibria of the infinitely repeated games, for example alternating play of C and D according to some prescribed pattern. Applied to more complex games, such punishments can temporarily force players' payoffs below all Nash equilibrium payoff levels in the stage game G. For example, firms in repeated quantity (Cournot) competition can punish deviations from collusive behavior (such as implicit cartel agreements to restrain output) by temporarily "flooding" the market and thereby forcing profits down to zero.

In the 1950s, several game theorists had conjectured that rational players should be able to cooperate—for example play C in the above prisoners' dilemma—if the game would only continue long enough (see Section 5.5 in Luce and Raiffa, 1957). Its folklore flavor is the reason why the result came to be referred to as a "Folk Theorem." As indicated above, Schelling (1956) definitely believed the folk wisdom and deemed it to be empirically relevant. Still, it was Aumann's precise and general statement and proof that laid the foundation for subsequent analyses of repeated interactions. Later, Friedman (1971) established a useful, although partial result for repeated games: if players discount future payoffs to a sufficiently small extent, then outcomes with higher payoffs to all players than what they would receive in a pure-strategy Nash equilibrium of the underlying stage game G can be obtained as equilibria in the infinitely repeated game.

During the cold war, between 1965 and 1968, Robert Aumann, Michael Maschler and Richard Stearns collaborated on research on the dynamics of arms control negotiations. Their work became the foundation of the theory of repeated games with incomplete information, that is, repeated games in which all or some of the players do not know which stage game G is being played, see Aumann and Maschler (1966, 1967, 1968), Stearns (1967) and Aumann, Maschler and Stearns (1968). For example, a firm might not know a competitor's costs and a country might not know another country's arsenal of military weapons or the other country's ranking of alternative agreements. The extension introduces yet another strategic element: incentives to conceal or reveal private information to other players. How might a person, firm or country who has extra information utilize the advantage? How might an ignorant player infer information known to another player by observing that player's past actions? Should an informed player take advantage of the information for short-run gains, thereby risking to reveal his information to other players, or should he conceal the information in order to gain more in the future? Building on the work of John Harsanyi, Aumann, Maschler and Stearns brought game theory to bear on these subtle strategic issues. Their work is collected and commented upon in Aumann and Maschler (1995).

Aumann and Shapley (1976) and Rubinstein (1976, 1979) refined the analysis of repeated games with complete information by showing that all feasible and individually rational outcomes can also be sustained as *subgame-perfect* Nash equilibria. In the context of an infinitely repeated game, subgame perfection essentially requires that the players, in the wake of a unilateral deviation from the equilibrium path of play, have incentives to play according to the equilibrium. In particular, subgame perfection requires that no player will ever have an incentive to deviate from punishing a deviator, nor to deviate from punishing a player who deviates from punishing a player, etc. Many Nash equilibria are not subgame perfect, and it was by no means clear that such a seemingly stringent refinement would leave intact the entire set of Nash equilibrium payoffs of supergames. Indeed, as Aumann and Shapley showed, if players discount future payoffs, and strive to maximize the expected present value of their own payoff stream, then the set of subgame perfect equilibrium outcomes may be significantly smaller than the set of Nash equilibrium outcomes. For while the Nash equilibrium criterion does not depend on the "costs" of "punishing" deviators, the subgame perfection criterion does. However, their generalized Folk Theorem establishes that the distinction between subgame perfect and Nash equilibrium disappears if there is no discounting.

The theory of repeated games has flourished over the last forty years, and we now have a much deeper understanding of the conditions for cooperation in ongoing relationships. Following a characterization of optimal punishments by Abreu (1988), it became easier to find the set of sustainable equilibrium payoffs in repeated games. Fudenberg and Maskin (1986) established Folk Theorems for subgame perfect equilibrium in infinitely repeated games with discounting and an arbitrary (finite) number of players. Aumann and Sorin (1989) showed that players' bounded recall can shrink the set of equilibria to those that are socially efficient, and Abreu, Dutta and Smith (1994) essentially characterized the class of games for which the Folk Theorem claim holds under infinite repetition and discounting.

An example of subgame-perfect equilibrium in an infinitely repeated game with discounting is when n identical firms with no fixed costs and constant marginal cost c sell the same product and are engaged in dynamic price competition in a market. Each firm announces a price in each period and consumers buy only from the firm(s) with the lowest price, with their demand spread evenly over these firms. If this interaction took place only once, then the resulting market price would be the same as under perfect competition: $p = c$. However, when the interaction takes place over an indefinite future where profits are discounted at a constant rate, many other equilibrium outcomes are possible if the discounting is not too severe. For example, all firms may start out by setting the monopoly price $\hat{p} > p$ and continue doing so until a price deviation has been detected, from which period on all firms set the competitive price $p = c$. Such as strategy profile constitutes a subgame perfect equilibrium if $\delta \geq 1 - 1/n$, where $\delta \in (0, 1)$ is the discount factor—the factor by which future profits are discounted each period.[23] The more competitors there are, the harsher is thus the condition on the discount factor—and hence the harder it is to sustain collusion.

[23]To see this, let $\Pi(p)$ be the industry profit when all firms quote the same price p, and assume that this function is continuous and unimodal with maximum at $p = \hat{p}$. The strategy profile here described constitutes a subgame perfect equilibrium if and only if $\frac{1}{n}\Pi(\hat{p})/\,(1-\delta)$ is (weakly) exceeds $\Pi(p)$ for all $p < \hat{p}$. The first quantity is the present value of the firm's profit if it continues to set the collusive price $\hat{p}$, while $\Pi(p)$ is the present value of the profit to a firm if it undercuts the collusive price by posting a price $p < \hat{p}$ — such a firm will earn zero profit in all future periods because all firms will subsequently price at marginal cost. By continuity of the function Π, the required inequality holds if and only if $\delta > 1 - 1/n$. There is no incentive to deviate from punishment of a deviator, should a deviation occur, since all profits are zero as soon as any firm quotes $p = c$.

Other strands of the literature examine the possibilities of long-term cooperation when players are impatient and only have access to noisy signals about past behavior; prominent early contributions include Green and Porter (1984) and Abreu, Pearce and Stacchetti (1990). More recent related contributions concern long-lived players, as well as imperfect public and private monitoring.[24] There is also a literature on cooperation in *finitely* repeated games, that is, when the stage game G is repeated a finite number of times. For example, Benoit and Krishna (1985) established Folk-theorem-like results for repeated games with multiple Nash equilibria when the time horizon is finite but long, and Kreps, Milgrom, Roberts and Wilson (1982) showed that if a prisoners' dilemma is repeated sufficiently many times it takes only a small amount of incomplete information about payoffs to sustain cooperation most of the time, although conflict will break out in the last couple of rounds. Neyman (1999) showed that cooperation in a finitely repeated prisoners' dilemma is possible even under complete information if the time horizon is not commonly known (see below for a brief discussion of common knowledge in games). Another important contribution to the literature on repeated games is Axelrod (1984), whose experimental tournaments suggest that simple strategies such as "tit-for-tat" perform well in populations of boundedly rational players.

All these subsequent insights owe much to Aumann's innovative and fundamental research. When studying cooperation among agents with partly conflicting interests, whether these are firms in a capitalist marketplace—as in many of the first applications—or farmers sharing a common grassland or irrigation system—as in Ostrom (1990)—the theory of repeated games is now the benchmark paradigm.

The theory of repeated games helps to explain a wide range of empirical findings, notably why it is often harder to sustain cooperation when there are many players, when players interact infrequently, when there is a high probability that interaction will cease for exogenous reasons, when the time horizon is short, and when others' behavior is observed after a delay. Price wars, trade wars and other economic and social conflicts can often be ascribed to one or more of these factors. The repeated-games framework also sheds light on the existence and functioning of a variety of

[24]See Fudenberg and Levine (1994), Fudenberg, Levine and Maskin (1994), Kandori (2002), and Ely, Hörner and Olszewski (2005)

institutions, ranging from merchant guilds (Greif, Milgrom, and Weingast, 1994) and the World Trade Organization (Maggi, 1999) to the mafia (Dixit, 2003).

3.2. Other contributions. Aumann has made numerous important contributions to other aspects of game theory and its application to economics. Here, we only mention a few of them.

Players' knowledge about each others' strategy sets, information and preferences is of utmost importance for their choice of course of action in a game. Thus it is natural to ask: What epistemic assumptions imply equilibrium play by rational players? Game theorists were largely silent on this fundamental question, and economists carried out equilibrium analyses without worrying too much about it, until Aumann established the research agenda sometimes called interactive epistemology. In his paper "Agreeing to disagree" (1976), Aumann introduced to game theory the concept of "common knowledge," a concept first defined by Lewis (1969). An event is *common knowledge* among the players of a game if it is known by all players, if all players know that it is known by all players, if all players know that all players know that it is known by all players etc., ad infinitum. Roughly, Aumann proved that if two players have common knowledge about each other's probability assessments concerning some event, then these assessments must be identical. Aumann's counter-intuitive "agreement result" has had a considerable effect on the theoretical analysis of trade in financial markets, see e.g. Milgrom and Stokey (1982).

In the 1980s, Bernheim (1984) and Pearce (1984) showed that players' rationality and their common knowledge of the game and of each others' rationality does not, in general, lead to Nash equilibrium, not even in games with a unique Nash equilibrium. A decade later, Aumann and Brandenburger (1995) established tight sufficient epistemic conditions for Nash equilibrium play.

As mentioned above, Aumann defined the concept of strong equilibrium, which is a refinement of Nash equilibrium. In two papers, published in 1974 and 1987, he also defined another solution concept that is "coarser" than Nash equilibrium: *correlated equilibrium.* Unlike Nash equilibrium, correlated equilibrium permits players' strategies to be statistically dependent, and thus Nash equilibrium emerges as the special case of statistical independence. Such correlation is possible if players can condition their strategy choice on correlated random variables, such as distinct but related observations of the weather, a news event, or some other variable feature of their environment. In a correlated equilibrium, each player's conditioned choice is optimal, given the others' decision rules.

The set of correlated equilibrium outcomes of a complete-information game also provides the limits to cooperation when players can communicate freely, possibly through an impartial mediator, prior to choosing their strategies in the underlying game. When each player's observed random variable is a recommendation from an impartial mediator, a correlated equilibrium is a collection of recommendations such that no player can increase his or her expected payoff by a unilateral deviation from his or her recommendation. In the mobilization game discussed above (see Table 1), it can be shown that there are correlated equilibria in which war is avoided completely, while the negotiation payoff pair (b, b) is attained with positive probability. To see this, suppose that a mediator recommends exactly one of the countries to refrain from mobilization with equal probability π for each country, and recommends both to refrain from mobilization with the remaining probability, $1 - 2\pi$. If $\pi > 2b/(2b+a-c)$ each country will refrain from mobilization if and only if it receives this recommendation.[25] For a careful discussion of the link between the concept of correlated equilibrium and the role of communication in games, see Myerson (1991, Chapter 6).

Aumann (1987) showed that correlated equilibrium can be viewed as a natural extension of Bayesian decision theory to non-cooperative games. In this interpretation, rational players (according to the definition of rationality due to Savage, 1954) will play a correlated equilibrium if their rationality and their probabilistic priors are common knowledge.

Aumann also made noteworthy contributions to other areas of economics; one is his joint work on decision theory with Frank J. Anscombe (Anscombe and Aumann, 1963), another is his continuum model of perfect competition (Aumann 1964, 1966), and a third is his joint work with Mordecai Kurz and Abraham Neyman on applications of game theory to political economy (Aumann and Kurz, 1977, Aumann, Kurz

[25] If a country does not receive a recommendation, then it knows that the other country received a recommendation to refrain, in which case mobilization is optimal. If a country receives a recommendation, then the expected payoff of refraining from mobilization is $\pi c + (1 - 2\pi)\, b$ and this exceeds πa, the expected payoff of mobilization.

and Neyman, 1983 and 1987).

4. Recommended readings

The work of Thomas Schelling is accessible also to non-specialists and we recommend consulting his original publications. Aumann's writings are highly technical, but usually also contain easily accessible discussions. See Aumann (1981) for a survey of the repeated games literature up till then, and Aumann and Maschler (1995) for a discussion of early work on repeated games with incomplete information. For a readable and almost entirely non-technical introduction to game theory, see Dixit and Nalebuff (1991); this book discusses long-term cooperation in Chapter 4 and credible commitments in Chapter 6. For comprehensive books on game theory, see Dixit and Skeath (2004) for an introductory text and Fudenberg and Tirole (1991) and Myerson (1991) for advanced and technical expositions. Aumann's and Schelling's personal (if not necessarily current) views on game theory, may be found in Aumann (1985) and Schelling (1967). For more bibliographic and personal details about the two game theorists, see Zeckhauser's (1989) portrait of Schelling, and Hart's (2005) interview with Aumann.

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