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Global Sourcing of a Complex Good  
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**DISCUSSION  
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# Global Sourcing of a Complex Good\*

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## Abstract

We analyze a firm that produces a final good from multiple intermediates that can each be sourced domestically or from a low-wage country. The model explicitly incorporates that sourcing decisions of intermediates are interdependent. Equilibrium predictions depend crucially on a key modeling assumption—the nature of the trade friction that foreign production has to overcome. If production abroad involves a fixed cost, offshoring one intermediate unambiguously facilitates offshoring of other intermediates. However, if production abroad involves incomplete contracts, offshoring one intermediate almost always makes it more difficult to offshore others. We illustrate that the pattern in prices at which successive automotive parts are imported into the U.S. accords better with the predictions of the incomplete contracting model, except for a few countries with the best governance indicators.

JEL codes: L23, D23, F12

Keywords: Outsourcing, property rights, FDI.

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# 1 Introduction

International trade in manufactured products has been growing rapidly for decades and intermediate inputs make up an increasing share of it. Hummels, Ishii, and Yi (2001) illustrate that the value share of imported intermediates in exports has increased by more than 30% from 1970 to 1990. Antràs (2003) further highlights that almost half of all trade in manufactures takes place within firm boundaries. Both patterns suggest an important role for international fragmentation of production.

Some worry that this process is accelerating as new producers are rapidly being integrated in global production networks and even production of advanced products switches to low-cost destinations. Because demand for different intermediates is interrelated through final product demand, their sourcing decisions are naturally connected. The particular question we pose is whether past decisions to offshore production of fast-maturing parts leads firms to advance or delay offshoring of subsequent parts.

We first study theoretically how the offshoring decisions of different intermediates influence each other in a model that generates a product life-cycle. Production switches from high-wage to low-wage locations as goods mature and inputs are standardized. Antràs and Rossi-Hansberg (2009) have argued that theoretical insights depend crucially on proper modeling of the organizational aspects of production. We demonstrate that two alternative ways of introducing the cost of producing in the low-wage location—using incomplete contracts or fixed costs—lead to opposing predictions. We then investigate which modeling assumption leads to predictions most in line with the empirical patterns for U.S. imports of automotive parts.

The contribution of our study is threefold. First, we distinguish explicitly between final goods and intermediates and show how this leads to interdependent sourcing decisions. Our approach is closest to the incomplete contracting model in Antràs (2005) that generates an endogenous production life-cycle.<sup>1</sup> Formal contract enforcement tends to be weaker in less developed countries, making it more difficult to specify supplier investment and remuneration contractually (Nunn 2007). The cost advantage of low-wage (South) countries is balanced by a production inefficiency as the contracting friction leads to underinvestment in specific inputs. Only when products have matured sufficiently does it become profitable to produce them in a low-wage location. When we add intermediate products to this model, the offshoring decisions become interdependent because substituting between interme-

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<sup>1</sup>The property rights model has been a popular framework to study offshoring. Most existing models assume that decisions are made for each product independently (Antràs and Helpman, 2004; Feenstra and Hanson, 2005; Grossman and Helpman, 2005). The model in Spencer and Qiu (2001) features multiple inputs, but outsourcing decisions are still made for each one individually.

diates is harder than substituting between final products.<sup>2</sup>

A few studies have looked at such dependency, relying on mechanisms that are arguably less relevant in the context of cost-driven offshoring. The need to disclose sensitive business information or coordinate production decisions introduces a direct complementarity between outsourcing decisions of different intermediates in Novak and Stern (2009). The externality is indirect in the model of Acemoglu, Antràs, and Helpman (2007) where aggregate output is increasing in the total number of intermediates, which is interpreted as the level of technology. In the models of Bartel, Lach, and Sicherman (2005) and Lileeva and Van Biesebroeck (2011) the interaction is between sourcing decisions of different single-product firms.

The second contribution is to show that the standard neoclassical framework, where offshoring is driven by technology or cost considerations, can generate the same endogenous production life-cycle as the incomplete contracting model. If technology differed across countries, a per-unit offshoring cost would be enough to generate an interesting model, as in Grossman and Rossi-Hansberg (2008). We assume all countries use the same technology, but introduce a fixed cost for each intermediate that is produced in the low-wage location, as in Helpman, Melitz, and Yeaple (2004) or Nocke and Yeaple (2008). We again find that a product is only offshored after it has matured sufficiently, when variable cost savings outweigh the additional fixed cost.

Introducing intermediates in this alternative framework leads to starkly different predictions. In a model where South production involves a fixed cost, offshoring is accelerating. Intermediates that mature later, move to the low-cost production location at an earlier stage of maturity, i.e. when they still require relatively more high-skill inputs. In contrast, in a model where South production involves incomplete contracts, the reverse pattern applies in most cases. Offshoring becomes gradually more difficult. Intermediates that mature later, need to reach a higher level of maturity before they are profitably produced in the low-cost location.

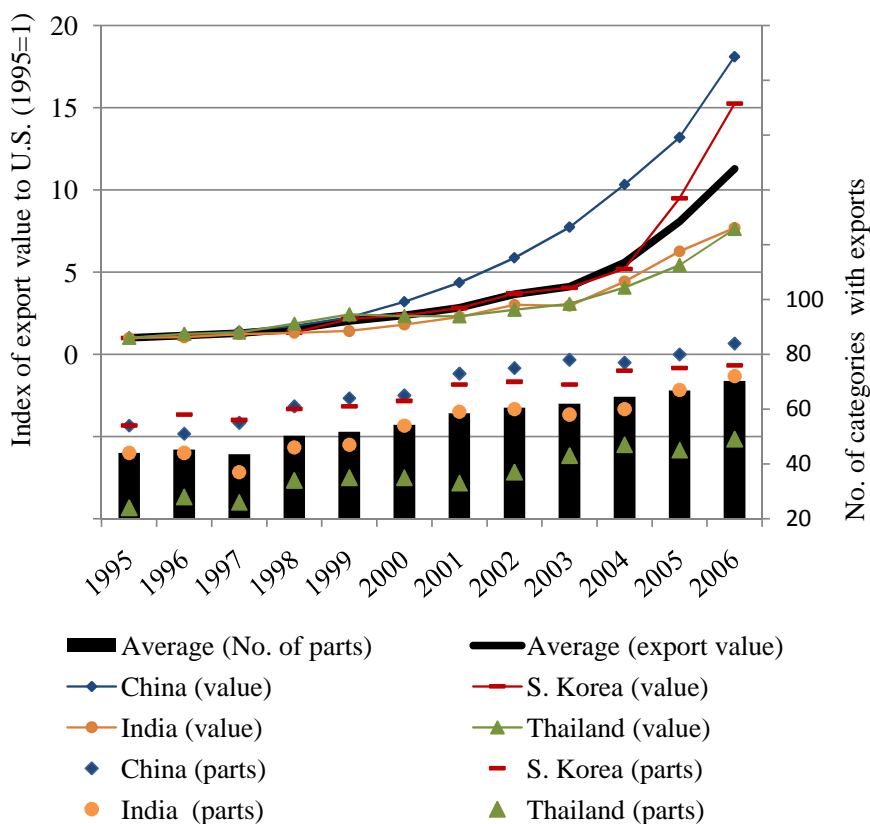
Our third contribution is to use these models to shed light on the following patterns in the trade data that are shared by many industries and countries. The top panel in Figure 1 illustrates that U.S. imports of automotive components from four Asian countries with emerging auto sectors has risen almost exponentially between 1995 and 2006. The bottom panel illustrates that this growth in value has coincided with a large increase in the number of different components imported into the United States.<sup>3</sup>

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<sup>2</sup>Recent papers by Antràs and Chor (2010) and Schwarz and Suedekum (2010) also focus on supply chain choices, but use models with constant cost-savings from offshoring.

<sup>3</sup>Products are defined at the H.S. 10-digit level. Details on the data and the sample are provided in Section 4 where we test between two predictions of the two models we develop first.

Figure 1: U.S. imports of automotive parts from countries with emerging auto sectors



The attractiveness of foreign sourcing is clearly rising over time, even at an increasing rate. Rising productivity and falling unit costs in foreign sectors are partial explanations, but unlikely to account for the entire increase. In our model, the decision to import one component from abroad changes the profitability of importing other components as well. We naturally expect that more sophisticated components will only be imported at a later time, but it is an open question whether they should mature less or more than earlier products before sourcing from a low-wage location becomes profitable. In the automotive sector it is notoriously difficult to substitute between suppliers (Clark and Fujimoto 1991) and the interdependencies in the model are likely to be important.

To test between the opposing predictions derived from the two modeling assumptions, we need information on the order in which parts mature and on the maturity reached by each part when sourcing switches. A ranking of parts that applies to all countries is recovered using the method in Feenstra and Rose (2000). As a proxy for the maturity level of each imported part, we calculate the difference between its import price and the corresponding import prices from a few advanced countries. We only use this ratio in the first year the United States imports a particular component

from a country to proxy for the maturity threshold when sourcing switches. Time and country fixed effects are included in the analysis to control for wage differences. We find that the patterns in automotive components trade tend to be most supportive of the incomplete contracts assumption, especially for countries with poor governance according to the World Bank indicators.

The paper is organized along the lines of the three contributions. In Section 2 we add intermediates to the incomplete contracting model of Antràs (2005) and illustrate the interdependence of sourcing decisions. In Section 3 we show that a much simpler model with a fixed cost for foreign production also leads to a production life-cycle, but the interaction of the sourcing decisions with intermediates is starkly different. In Section 4 we compare the predictive power of both models using data on U.S. automotive parts imports. Section 5 concludes with a summary of findings and with a few broader implications.

## 2 An incomplete contracting model of outsourcing

### 2.1 Setup

We first study interdependent sourcing by adding a layer of intermediates into the model of Antràs (2005). A firm producing a final good can use a North or South supplier for each intermediate. Because institutions are less developed in South, production there suffers from contract incompleteness. Investments in specific inputs used to produce an intermediate in South are non-verifiable and cannot be contracted on. Inputs will be chosen noncooperatively and the South supplier and the firm bargain to split the relationship-specific quasi-rent. This introduces a trade friction, not present when the intermediate is produced in North where input levels are contractible.

We follow Antràs (2005) by assuming an iso-elastic demand for the final product:

$$R = \beta^{-\beta} y^\beta, \quad \beta \in (0, 1). \quad (1)$$

$R$  is the sales revenue and  $\beta > 0$  implies that final products are substitutes.<sup>4</sup> The novelty in our model is that the final good  $y$  is produced from several intermediates  $y_k$ , with  $k = 1, \dots, K$ . Rather than deciding the sourcing of the final good directly, the firm needs to make sourcing decisions for all intermediates.

Production is according to a constant elasticity of substitution technology with

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<sup>4</sup>A love-of-variety Dixit-Stiglitz utility function over varieties can rationalize this demand. We study only one variety and normalize the constant term to simplify some of the expressions below.

constant returns to scale:

$$y = \left( \sum_{k=1}^K \frac{1}{K} y_k^\alpha \right)^{1/\alpha}, \quad \alpha \leq \beta. \quad (2)$$

Intermediates are indexed by  $k$ .  $\alpha \leq \beta$  implies that it is more difficult for the firm to substitute between intermediates than it is for consumers to substitute between final products. Implicitly, this is what defines intermediates. In the short run it is difficult to change a product's design and substitute between its components in production. If  $\alpha < 0$  the intermediates are complements, while for  $\alpha > 0$  they are substitutes.<sup>5</sup> If  $\alpha = \beta$  the difference between intermediates and final product varieties vanishes and the sourcing pattern in our model becomes identical to that in Antràs (2005).

Each intermediate is itself produced with a constant returns to scale Cobb-Douglas production technology:

$$y_k = \left( \frac{x_{kl}}{z_k} \right)^{z_k} \left( \frac{x_{kh}}{1 - z_k} \right)^{1 - z_k}. \quad (3)$$

The high and low-skill inputs  $x_h$  and  $x_l$  are interpreted as specific investments.<sup>6</sup> The firm chooses  $x_h$  directly, with each unit produced using one unit of North labor. It can choose to source  $x_l$  from a North supplier, in which case it can specify the level directly and reward the North supplier for its production costs. If it chooses to source  $x_l$  from South, the weaker contract environment makes it impossible to specify contractually the supplier's investment and remuneration. The South supplier will choose  $x_l$  non-cooperatively and bargain with the firm over the ex-post division of the quasi-rents.

Wages in North are normalized to unity and South wages are strictly lower. The relative South wage is  $w \equiv w_S/w_N < 1$ . Sourcing an intermediate from South involves a trade-off between cheaper low-skill inputs and a production inefficiency due to noncooperative input choices. The low-skill input elasticity  $z_k$  will play a key role as the advantage of sourcing intermediate  $k$  from South increases with  $z_k$ .

The international trade literature has used several variations of the property rights model, see Helpman (2006) for an overview. In the model analyzed in Antràs (2005) there are two possible organizational forms in South: (i) an arms-length relationship between the firm and the South supplier with equal bargaining power and no secondary markets and (ii) an FDI structure where the South supplier is a

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<sup>5</sup>An extreme case is the Leontief production technology, the limiting case for  $\alpha \rightarrow -\infty$ . Intermediates that are close substitutes, e.g. red and blue paint, are aggregated in one intermediate.

<sup>6</sup>The  $k$  index is omitted for  $x_{kl}$  and  $x_{kh}$  wherever it is obvious.

subsidiary and the firm still has access to a fraction  $\delta$  of the low-skill input off the equilibrium path. Both forms can be nested in a more general bargaining model with a share  $m$  going to the supplier and the balance to the firm (Antràs and Helpman 2004). To focus on our main contribution, we only consider the case of  $m = 1/2$ , which is equivalent to the arms-length organization in Antràs (2005).<sup>7</sup>

In terms of terminology, each *final good*  $y$  is composed of  $K$  *intermediates*  $y_k$ , which are themselves produced with two *inputs*  $x_h$  and  $x_l$ . The model has three stages. First, the *firm* simultaneously selects *suppliers* in North or South for each intermediate. Second, for intermediates sourced in South the firm and supplier choose input levels simultaneously and noncooperatively. For intermediates sourced in North the firm chooses both input levels. Assembly can simply be thought of as one intermediate. Third, production takes place and the firm chooses a price to sell the final product in a monopolistically competitive market.

## 2.2 Firm behavior

The model is solved backwards as follows. In the third stage, past input choices determine the output of all intermediates and the final product. Given the assumptions of monopolistic competition and CES demand, the optimal price is a constant markup over marginal cost. In the second stage, input levels are chosen while taking the impact on outputs of all intermediates and the slope of final product demand into account. This choice is taken differently for intermediates sourced in North or South. In the first stage, the firm selects the sourcing configuration, i.e. one location for each intermediate, that maximizes its profits over all possible configurations.

### (a) North suppliers

For North suppliers contracts are complete and inputs  $x_h$  and  $x_l$  can be considered as chosen directly by the firm that produces the final product to maximize its profit. The first order condition for each input equalizes its marginal revenue product to the marginal cost:

$$\frac{\partial R}{\partial y} \cdot \frac{\partial y}{\partial y_k} \cdot \frac{\partial y_k}{\partial x_i} = w_N \equiv 1, \quad i = h, l.$$

Dividing the two first order conditions reveals that the firm will always produce at a point where

$$\frac{x_l}{z} = \frac{x_h}{1-z}, \quad (4)$$

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<sup>7</sup>The second organizational form amounts to  $m = (1 - \delta^\sigma)(1 - \eta)$ , with  $\eta$  the bargaining weight of the supplier and  $\sigma$  the elasticity of substitution across products. With  $\alpha = \beta$  it equals  $1/(1 - \beta)$ , but with  $\alpha < \beta$  it is a more complicated function of the  $\beta$  and  $\alpha$  parameters.



and the production cost is constant at  $c_k^N = 1$  ( $= AC_N = MC_N$ ). Equation (4) determines the input mix for the two inputs, while the absolute input levels are determined by equating the marginal revenue contribution of intermediate  $k$  to its marginal cost:

$$\frac{\partial R}{\partial y} \frac{\partial y}{\partial y_k} = c_k^N \equiv 1. \quad (5)$$

This condition will be taken into account when determining the optimal price of the final good, which pins down the optimal quantity and implicitly also the output of all intermediates.

(b) South suppliers

For South suppliers contracts are incomplete and the firm and its supplier will choose inputs noncooperatively. Payoffs are determined in an ex-post bargaining game over the surplus generated with the specific inputs with both parties assigned equal bargaining strength. For the supplier, we use the standard assumption that there are no secondary markets for its input, which puts its outside option to zero.

To facilitate the exposition, we assume that the firm has the option of using a generic intermediate  $k$ . It is never used in equilibrium, but if it were the sales revenue net of the cost of the generic intermediate  $R'$  would be independent of the specific inputs  $x_{kl}$  and  $x_{kh}$ . The difference between  $R$  and  $R'$  is positive and equals the quasi-rent from this one bilateral relationship. Introducing  $R'$  leaves the rent associated with other intermediates outside the bargaining with the supplier of  $k$ .<sup>8</sup>

The Nash bargaining solution is obtained if the firm and supplier choose inputs to maximize the following profits:

$$\begin{aligned} \text{Firm:} \quad & \max_{x_h} \frac{1}{2}(R - R') + R' - \sum_{l \neq k} c_j - w_N x_h \\ \text{Supplier:} \quad & \max_{x_l} \frac{1}{2}(R - R') - w_S x_l \end{aligned}$$

Noncooperative choices of  $x_h$  and  $x_l$  lead to the following first order conditions:

$$\begin{aligned} \frac{1}{2} \frac{\partial R}{\partial y} \frac{\partial y}{\partial y_k} \frac{\partial y_k}{\partial x_h} &= w_N \\ \frac{1}{2} \frac{\partial R}{\partial y} \frac{\partial y}{\partial y_k} \frac{\partial y_k}{\partial x_l} &= w_S. \end{aligned}$$

Dividing the two illustrates that the optimal input mix still equates the relative

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<sup>8</sup>Note that introducing  $R'$  has no effect on the equilibrium. In the absence of generic intermediates, the ex-ante payment would simply adjust to reflect the greater bargaining strength of the supplier. Either way, the South supplier will earn its reservation wage in equilibrium.

productivities to the relative wage:

$$\frac{x_l^S/z}{x_h^N/(1-z)} = \frac{w^S}{w^N} \equiv w. \quad (6)$$

Optimal pricing and output is again determined by the demand function and the first order conditions for all  $k$ , which for a South-sourced intermediate equals

$$\frac{1}{2} \frac{\partial R}{\partial y} \frac{\partial y}{\partial y_k} = w_S^{z_k} w_N^{1-z_k} \equiv c_k^S. \quad (7)$$

With incomplete contracts, the firm and supplier choose input levels below first best as they bear the full cost but only receive one half of the proceeds. Underinvestment drives a wedge between the marginal revenue product of intermediate  $k$  and the (efficient) marginal production cost. With equal bargaining weight, only one half of the marginal revenue is taken into account and output will be inefficiently low. From the final good producer's point of view the relevant marginal cost of procuring input  $k$  in South is twice the production cost:

$$MC_k^S = 2c_k^S.$$

Only when  $z_k$  is sufficiently high will marginal procurement costs for intermediate  $k$  be lower in South.

An additional benefit of South sourcing is the lump sum transfer that the firm receives when it selects a South supplier. This payment accomplishes that in equilibrium South suppliers receive their reservation wage  $w_S$  for all low-skill inputs they provide and all ex-post rents accrue to the firm (Helpman, 2006). If the input mix is undistorted, which will be the case with equal bargaining strengths, the average cost of sourcing intermediate  $k$  from South will equal the production cost

$$AC_k^S = (x_h + wx_l)/y_k \equiv c_k^S = \frac{1}{2}MC_k^S.$$

The marginal cost exceeds the average by a factor of two and the difference contributes to profits.<sup>9</sup>

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<sup>9</sup>Unequal bargaining strength for the firm and the supplier, indicated by  $1-m$  and  $m$ , would lead to a number of differences: (i) The relative productivities are equalized to  $w(1-m)/m$  and the input-mix is distorted as the firm and supplier underinvest to different extents. (ii) Actual production costs differ from  $c_k^S = w^{z_k}$ . (iii)  $MC_k$  exceeds the efficient production cost  $c_k^S$  by a factor  $[(1-m)^{1-z_k} m^{z_k}]^{-1}$  (iv)  $AC_k$  is lower than  $MC_k$  by a factor  $[(1-m)(1-z_k) + mz_k]$ , which can be higher or lower than the efficient production cost  $c_k^S$ . (v) The implicit markup over the procurement cost depends on the maturity of South-sourced intermediate, rather than equalling  $(1/\beta)$  (see below). For  $m < 1/2$ , the equilibrium markup rises with maturity. The low bargaining weight of the supplier makes its underinvestment ever more costly and production increasingly inefficient as the part matures. The firm prefers to extract more profit per output rather than

(c) Firm (final good producer)

Conditional on the sourcing decision for each part, equation (4) or (6) determines the optimal input mix and equation (5) or (7) determines the absolute input levels. Moreover, the output level of intermediate  $k$  depends on the sourcing pattern of all other intermediates. Substituting the derivatives of the CES demand expression (1) and the production function (2) in the first order conditions we can write

$$y_k = \beta \left( K \cdot \widetilde{MC} \right)^{-\frac{1}{1-\beta}} \left( \frac{MC_k}{\widetilde{MC}} \right)^{-\frac{1}{1-\alpha}}. \quad (8)$$

$\widetilde{MC}$  averages the procurement cost of all intermediates as a CES price index:  $\widetilde{MC}^{-\frac{\alpha}{1-\alpha}} = \frac{1}{K} \sum_{l=1}^K MC_l^{-\frac{\alpha}{1-\alpha}}$ , where  $MC_N$  or  $MC_l^S$  as defined in the two previous sections is used depending on the sourcing of each intermediate  $l$ .

The optimal output of intermediate  $k$  is naturally declining in  $MC_k$ . With  $\alpha < \beta$  it is also declining in the marginal cost of other inputs (cross-cost effect). Even though an increase in  $MC_l$  leads to substitution towards intermediate  $k$  in the production of the final good, this effect is outweighed by the lower sales of the final product as the cost increase raises the final product price. The substitution effect away from the final good that contains both intermediates  $k$  and  $l$  dominates the substitution between  $k$  and  $l$ . Stronger substitution between intermediates ( $\alpha$  closer to  $\beta$ ) strengthens the own-cost effect, but diminishes the cross-cost effect.

Given the CES demand assumption and monopolistic competition in the final good market, optimal price setting for the final product will be  $p^* = MC/\beta$  and total variable profits are  $(1 - \beta)R$ . The marginal cost of the final product equals  $(\sum_k MC_k y_k)/y$ . Through  $MC_k$  it depends directly on the sourcing decisions of all intermediates (North or South) and there is an additional indirect effect through the relative importance of each input ( $y_k/y$ ). A specific feature of the incomplete contracting model is that the ex-ante payments from South suppliers also contribute to profits. They equal the difference between the marginal and average procurement cost for an intermediate sourced in South.

Combining both contributions, we obtain

$$\begin{aligned} \pi &= p y - \sum_k AC_k y_k \\ &= (1 - \beta)R + \sum_{s \in S} (MC_s - AC_s) y_s \end{aligned}$$

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increase output. The underproduction of  $y_k$  is proportional to the deviation of  $m^z(1-m)^{1-z}$  from unity, making it optimal eventually to give more bargaining power to the supplier. The option of first producing in South using FDI and switching to an arms-length sourcing relationship for more mature intermediates in Antràs (2005) accomplishes just this.

$$= \sum_{n \in N} (1 - \beta) MC_n y_n / \beta + \sum_{s \in S} (MC_s - \beta AC_s) y_s / \beta.$$

Substituting in the definitions for the average costs and the output level of intermediates, and using  $n_S$  to indicate the number of South-sourced intermediates gives optimal profits in the incomplete contracting case as

$$\pi = \left( (1 - \beta)(K - n_S) + (1 - \frac{\beta}{2}) \sum_{s \in S} MC_s^{-\frac{\alpha}{1-\alpha}} \right) \times (\widetilde{MC})^{\frac{\beta-\alpha}{\alpha(1-\beta)}}. \quad (9)$$

We can divide the first term by  $\widetilde{MC}^{-\frac{\alpha}{1-\alpha}}$  such that it is a weighted average of the two price-cost markups. The second expression then amounts to total revenue, which can be seen by substituting the definition of  $y_k$  in  $\sum_k MC_k y_k / \beta$ .

### 2.3 Equilibrium

Conditional on the maturities of all intermediates, a  $1 \times K$  vector of  $z_k$  values, we can evaluate the above profit expression for each possible sourcing configuration. The sourcing equilibrium chosen by the firm in the first stage of the model is simply the one yielding the highest profits out of the  $2^K$  possible configurations.

Among the different sourcing configurations with  $n_S$  intermediates in South, it is profit maximizing to source the  $n_S$  intermediates with the highest  $z$  indices in South.<sup>10</sup> The optimal sourcing configuration conditional on the vector of maturities can then simply be characterized by  $n_S^*$ , the number of intermediates sourced in South. It will satisfy the following condition:

$$\pi(n_S^*) \geq \pi(j) \quad \text{for all } j \in \{1, \dots, K\}.$$

$n_S^*$  can be found by evaluating profits only for the  $K$  relevant configurations, always sourcing the  $j$  intermediates with highest  $z$  values in South.

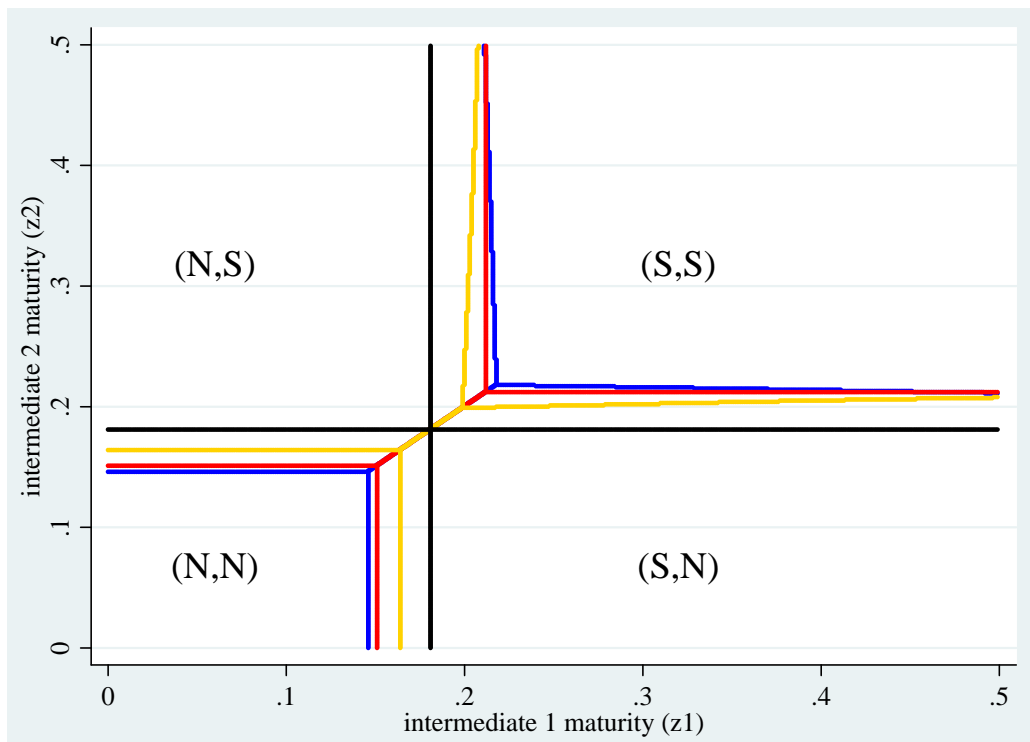
### 2.4 All possible equilibria in the two-intermediate case

Now that we know how to find the sourcing equilibrium for any vector of maturities, we want to characterize how the equilibrium configurations vary with maturity levels. In particular, we want to study how the sourcing decision of one intermediate depends on the sourcing of other intermediates. In the next section we demonstrate a few general properties. To provide intuition, we first illustrate graphically the

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<sup>10</sup>We show in Appendix A.2 that the derivative of profits with respect to the maturity  $z$  of an intermediate sourced in South is positive, while it does not vary with the maturity of parts sourced in North.

Figure 2: All sourcing equilibria in the two-intermediate case with incomplete contracts



Note: the lines partition the space into four areas with different optimal sourcing configurations for different values of  $\alpha$ . They refer, in order of the vertical segments, to  $\alpha = -8$  (blue),  $\alpha = 0.1$  (red),  $\alpha = 0.65$  (yellow), and  $\alpha = \beta = 0.8$  (black straight lines).

equilibria for all possible maturity vectors when there are only two intermediates.

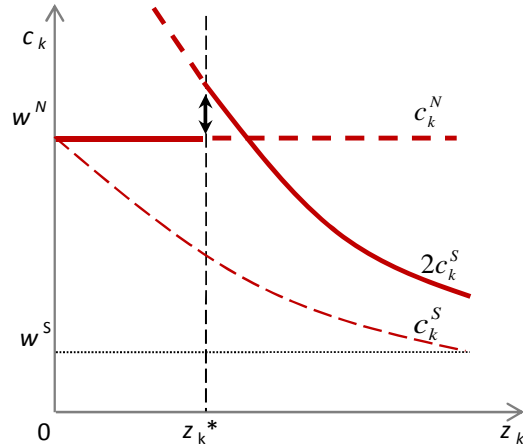
For each element in  $(z_1, z_2)$  space, we compare profits in the three possible sourcing configurations,  $n_S^* = 0, 1, \text{ or } 2$ , where the intermediate with highest  $z$  is sourced in South if  $n_S^* = 1$ . In Figure 2 the space is partitioned into four areas. When the two intermediates have similar maturities, they will most of the time be sourced in the same location. The low maturities at the bottom-left lead to (N,N) sourcing and the top-right area represents (S,S) equilibria. In the bottom-right and top-left areas only the intermediate with the highest  $z$  value is sourced in South and the other in North.

The colored borders between the areas indicate maturity thresholds ( $z_k^*$ ) where the firm is indifferent between different sourcing configurations. Lines with different colors are for different values of the  $\alpha$  parameter, the ease of substitution between intermediates.

In the extreme case of  $\alpha = \beta$ , the equilibrium is always the same as in Antràs (2005). When the sourcing of the first intermediate changes, there is no effect on the derived demand for the second intermediate. Substitution by the firm between

different intermediates of the same final product is exactly offset by substitution between different final products by consumers. As a result, optimal sourcing for each part is independent of the maturity or sourcing location of the other part.<sup>11</sup> The threshold maturity values that make the firm indifferent between sourcing a part in North or South are the black straight lines;  $z_1^*$  and  $z_2^*$  both equal the same constant.<sup>12</sup>

Figure 3: Maturity threshold and costs in the incomplete contracting case



When  $\alpha < \beta$  this independence is still present when the firm considers sourcing a first part in South. The maturity thresholds between the (N,N) area and the (N,S) or (S,N) areas do not depend on the maturity of the other part which is sourced in North. Note, however, that each vertical segment for part 1 is shifted to the left of the black line for the  $\alpha = \beta$  benchmark case. The first intermediate is offshored at a lower maturity level than would be the case for independent products. We explain this at some length because it provides an important insight into the mechanics of the model (affecting all sourcing decisions).

The thought experiment in Figure 2 is to raise  $z_1$  along a horizontal line holding  $z_2$  constant at a low level. When  $z_1$  crosses the  $z_1^*$  threshold, production of intermediate 1 switches from North to South and the procurement cost to the firm changes from  $MC_1^N = 1$  to  $MC_1^S = 2w^{z_1}$ . At the same time, the implicit markup on the intermediate rises from  $(1 - \beta)$  to  $(1 - \beta/2)$ . Figure 3 illustrates the evolution of costs as a function of a part's maturity and sourcing location. By definition, to the right

<sup>11</sup>The independence can be seen directly from the optimal output  $y_k$  in equation (8) which only depends on  $c_k$  in this case, not on  $\tilde{c}$ . Intermediates and final goods are now indistinguishable and the CES model leads to independent pricing in the final good market.

<sup>12</sup>In Antràs' model, South production by a subsidiary is a third sourcing option. It involves less underinvestment in the high-skill input and will be optimal for goods with intermediate maturity levels. Hence, there are two, constant maturity thresholds.

of the  $z^*$  threshold the advantages of South production dominate the production inefficiency.<sup>13</sup>

The firm can only be indifferent between sourcing an intermediate in North or South if the higher markup in South is compensated for by a higher procurement cost. Hence, it must be the case that  $MC^S(z^*) = 2c^S > c^N$  and the marginal procurement cost of the intermediate rises when sourcing switches from North to South. Immediately after a switch the firm will substitute away from the intermediate. However, although ex-post procurement costs are higher in South, total profitability including the ex-ante payment is higher. The greater difficulty of substituting away from South-sourced parts leads to a larger quasi-rent for these parts than in the case without interdependent demand. It encourages both the supplier and the firm to invest more in the production of South-sourced parts, making them already profitable at lower maturity levels.<sup>14</sup>

While  $\alpha < \beta$  facilitates offshoring of the first part, it makes it more difficult to offshore all parts. The entire (S,S) area lies in a subset of the top-right quadrant defined by the benchmark black lines. South production of the second part happens unambiguously at a more mature stage than it would for independent goods. There is a delay before sourcing of the second part switches to South as the maturity of the first part has to mature sufficiently for its marginal procurement cost in South to fall below its original marginal cost in North. This delaying effect is more pronounced for lower values of  $\alpha$ .

Because of this delay, the two areas discussed thus far, (N,N) and (S,S), do not touch. There is an intermediate range of  $z$  values where the intermediate with the highest value is produced in South and the second intermediate in North. The  $z^*$  threshold has an upward sloping section that coincides with the 45 degree line and separates the (N;S) and (S;N) areas.<sup>15</sup>

The section of the  $z_1^*$  threshold that separates the (N,S) and (S,S) areas at the top is a function of the maturity of the second part which is already produced in South and of the ease of substitution between the parts. For a given  $z_2$  it is more difficult to offshore part 1 the lower is  $\alpha$ . Better substitutes lowers the derived demand for part

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<sup>13</sup>The elasticity of the final good demand and the production function will influence the trade-off, as well as the maturity levels of other parts already produced in South.

<sup>14</sup>Another way to see this is through the relative impact on intermediates. The equilibrium price for the final product rises after a switch and consumers will substitute away from it. If  $\alpha = \beta$ , substitution by the firm and consumers balance out leaving demand for North-sourced intermediates unaffected. To find the maturity threshold, the firm only needs to equate the markup gain and cost increase for the affected part. If  $\alpha < \beta$ , the consumer's substitution dominates and the lower output for other intermediates raises the share of the switched part in profit. Because the profit margin is greater for South-sourced part, the lower  $\alpha$  parameter increases profits with South sourcing.

<sup>15</sup>The segment is found by equating  $\pi(S, N) = \pi(N, S)$  using the profit expression in equation (9) and solving for the  $z_1^*$  threshold. Because the two  $z$  values enter symmetrically on either side, the equation has a unique solution at  $z_1^*(z_2) = z_2$ .

1 which makes it easier to overcome its temporary increase in marginal procurement cost when it moves to South.

For a given  $\alpha$ , the sign of  $\partial z_1^*/\partial z_2$  depends on the sign of  $\alpha$ . If  $\alpha > 0$  and the two parts are substitutes (yellow line), the threshold slopes away from the black line. Higher maturity of part 2 makes it more difficult to offshore part 1. This is because the substitution away from part 1 is not sufficient to overcome the higher demand for part 1 induced by the lower cost of the final good when  $z_2$  is higher. In contrast, if  $\alpha < 0$  and intermediates are complements (blue line) offshoring part 1 becomes easier for a higher  $z_2$ . The cost share for the expensive part 1 is increasing in  $z_2$  and thus final demand and derived demand for part 1 is decreasing in  $z_2$ .

## 2.5 A dynamic interpretation

We derived the optimal sourcing configuration conditional on a vector of maturities and then we discussed how the equilibrium configuration varies over the  $(z_1, z_2)$  space and how it depends on the  $\alpha$  parameter. Now, we characterize how the sourcing equilibrium evolves when maturities evolve. In a dynamic interpretation, it is intuitive to view successive  $z$ -vectors as weakly-increasing and we derive a number of properties for the successive sourcing equilibria.

The first result holds all  $K - 1$  maturities constant and traces the sourcing equilibria when only  $z_k$  varies.

**Proposition 1** *Given the skill intensity of all other parts, there exists a  $z_k^*$  threshold such that part  $k$  should be produced in North if  $z_k < z_k^*$  and in South if  $z_k \geq z_k^*$ , with South production involving incomplete contracts.*

This result generalizes the equilibrium in Antràs (2005) to the case with interdependent intermediates. The key step in the proof is to show that  $\partial\pi(k \in S)/\partial z_k > 0$  while  $\partial\pi(k \in N)/\partial z_k = 0$ , holding the sourcing of other parts fixed. Hence, as  $z_k$  rises, profits with  $k$  in South will at some point dominate. The possibility of changing the sourcing of other parts can never overturn this result. Details are in the Appendix.

Note that the thresholds in Proposition 1 are functions of the maturities of other parts. It is possible that after sourcing of intermediate  $k$  has optimally moved to South at a particular  $z_k > z_k^*$  value, a sufficient increase in maturity of another part might change the optimal sourcing of intermediate  $k$  back to North. This can happen when another part passes  $k$  in terms of maturity and they switch sourcing locations. When parts are substitutes, it is even possible that an increase in the maturity of the most mature part forces optimal sourcing of another (less-mature)



South-sourced part back to North.<sup>16</sup>

A second result establishes that

**Corollary 2** *With incomplete contracts in South, it is never optimal to switch two or more parts to South simultaneously, even when they are of the same skill intensity.*

There does not exist any maturity vector such that the firm is indifferent between  $\pi(n_S^*)$  and  $\pi(n_S^* + 2)$ . If this were true, a small increase in one part's maturity could lead to an increase of the number of parts sourced in South by more than one. The proof is also in the Appendix.

Taken together, these results indicate that the succession of equilibria imply a product-life cycle for each intermediate: parts with low  $z$  levels are produced in North, while mature parts are offshored. As  $z_k$  increases, at some point it crosses a threshold and production of part  $k$  switches from North to South. Moreover, the optimal sourcing of other parts might change as a result. A North to South switch by part  $k$  cannot trigger North to South sourcing changes for other parts with constant maturity, but a sufficiently large increase in the maturity of a South-sourced part might induce a North to South switch for a complementary part. An increase in  $z_k$  might lead to a South to North sourcing change for other (constant) intermediates if  $k$  moves up in the part ordering or if parts are substitutes.

A third result establishes that the maturity thresholds for successive intermediates are increasing for parts that are substitutes, suggesting that the offshoring process is slowing down. For such a characterization to have a clear interpretation, intermediates must be unambiguously ranked by maturity. We assume that the high-skill input intensity of intermediates is increasing in the order  $k$ , i.e. at all times  $z_1 \geq z_2 \geq \dots \geq z_K$ , without any restriction on the differences between subsequent  $z$ 's. Lower ranked intermediates start maturing sooner and reach any specific  $z$  value before higher ranked intermediates. Higher ranked intermediates might catch up, but they can never surpass them.

We already established that intermediates will switch from North to South one by one in the order of the  $k$  ranking. The main question is then, how does the interdependence of sourcing decisions influence the maturity threshold at which parts are offshored?

We already discussed in the two-intermediate case of Figure 2 that the first part is offshored at a lower maturity threshold with interdependence, compared to the independent sourcing case. This generalizes to the case of  $K$  intermediates.

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<sup>16</sup>This can be seen in Figure 2 as the yellow threshold separating the (S,N) and (S,S) areas is upward-sloping. A sufficient increase in  $z_1$  can change the optimal sourcing location of part 2 from South back to North (holding  $z_2$  constant).

Additional parts produced in North do not affect the sourcing decision of the first part. The offshoring process thus gets started more easily if the trade friction takes the form of an incomplete contract in South countries. But once the process has started, how quickly does it spread to successive parts?

With only two intermediates, the second intermediate switches to South production at a higher level of maturity than intermediate 1. In the area below the 45-degree line, where the maturity ranking is constant,  $z_1^* < z_2^*$  in Figure 2. Comparing the thresholds for subsequent intermediates is more complicated if  $K > 2$  as they depend on the maturity levels of all parts already produced in South. It is even possible that intermediate  $k$  switches sourcing without a change in its own  $z_k$  index.

When intermediates are substitutes, we can still establish the following general result.

**Proposition 3** *If production in South involves incomplete contracts and  $z_k \geq z_{k+1}$  for all  $k$ , the sourcing thresholds that make the firm indifferent between North and South sourcing of intermediates  $k$  and  $k + 1$  satisfy  $z_k^* < z_{k+1}^*$  if  $\alpha > 0$ .*

The proof is in the Appendix and we also illustrate there why the result does not extend to the case of  $\alpha < 0$ .

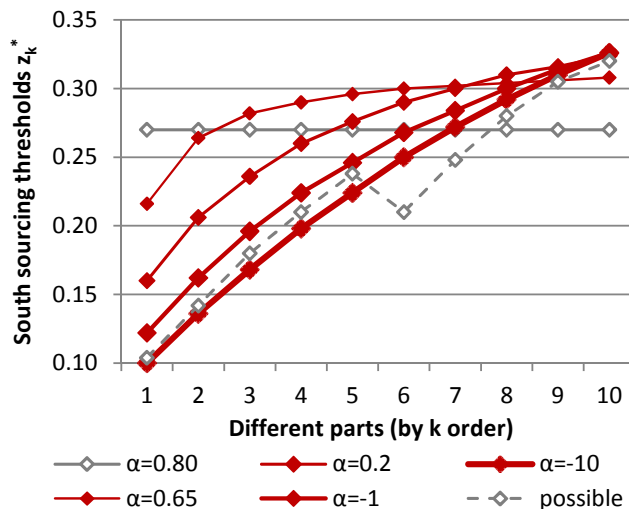
The intuition is as follows. After an initial dip in output when an additional part switches to South, output is generally increasing in the maturity of parts in South. The  $z_{k+1}^*$  threshold will be crossed at a higher output level than threshold  $z_k^*$  was. At this higher output level, switching sourcing of an additional part to South is more difficult because its marginal cost rises temporarily after the switch. It will thus require a higher maturity level until it is profitable.

If parts are substitutes this holds generally. If parts are complements, an alternative pattern is possible in extreme circumstances. If the maturity of South-sourced parts rises sufficiently between the offshoring of part  $k$  and  $k + 1$ , it can raise the total quasi-rent that part  $k + 1$  can hold-up so much that it is optimally offshored already at a maturity threshold lower than  $z_k^*$ . This can only happen if the firm is unable to substitute away from the more expensive parts and if the average maturity of inframarginal parts rises very much. It does make Proposition 3 only hold in general for  $\alpha > 0$ .

These effects can be quantitatively important. The  $z$  threshold to offshore the first part in Figure 2 was 0.145 (for  $\alpha = 0.1$ ), while the second part was never offshored before its maturity reached 0.22. The difference between the two thresholds is even larger for lower values of  $\alpha$  and for higher  $\alpha$  values the  $z_2^*$  threshold is increasing in the maturity of the South-sourced part 1.

Figure 4 illustrates the prediction of Proposition 3 when there are many parts. It

Figure 4: Sourcing thresholds in the incomplete contracting case



plots the optimal sourcing thresholds for successive parts for a particular assumption on the evolution of the  $z$  indices.<sup>17</sup> Each successive point corresponds to a maturity vector where the firm is indifferent between sourcing one additional intermediate in South. The maturity level of that marginal intermediate is indicated on the vertical axis. The lines connect the equilibria for different values of the  $\alpha$  parameter which governs the substitutability of parts. We used a sequential maturing of parts— $z_1$  goes from 0 to 1 before any other part starts maturing, followed by  $z_2$  going all the way from 0 to 1 before the remaining parts mature, etc.

The horizontal grey line is for  $\alpha = \beta = 0.8$ , in which case sourcing of all intermediates is decided independently and the thresholds are the same for all intermediates—cfr. the black straight lines in Figure 2. For lower  $\alpha$ 's, successive thresholds are always higher indicating a slow-down in outsourcing. Higher ranked parts need to mature further before they can be profitably produced in South. The pattern was the same for other modeling choices that we tried for the maturity progression, e.g. linear maturing with higher ranked intermediates starting with a constant advance and concave rate of maturing where parts are converging in maturity.

To verify that Proposition 3 does not extend to the case of  $\alpha < 0$ , we included a case with maturity bursts that extend to several parts, but not all. Because the thresholds are a declining function of  $\tilde{z}_S$ , the maturity of parts already produced in South, a large enough increase in  $\tilde{z}_S$  between parts  $k$  and  $k + 1$  can induce a drop in the maturity threshold, i.e. acceleration in offshoring. This possibility is indicated

<sup>17</sup>We used the following parameter values:  $K = 10$ ,  $\beta = 0.8$ ,  $w = 0.1$ .

with the dashed grey line in Figure 4.<sup>18</sup>

### 3 An alternative model of outsourcing: South production involves a fixed cost

#### 3.1 The model

The incomplete contracting approach in the previous section has three disadvantages. The model is relatively complicated. It leads to inconclusive predictions for the relative sourcing thresholds if parts are complements. When parts are substitutes, offshoring is predicted to require ever greater maturity, i.e. become more difficult, which seems counterintuitive in light of the observed patterns in Figure 1.

The lower wage in South provides an incentive to produce low-skilled inputs there. Without trade frictions all  $x_l$  would be produced in South if  $z > 0$ , making the problem uninteresting. Even a trade friction that is proportional to the variable production cost would lead to an uninteresting problem, as sourcing decisions of all intermediates would be independent events.<sup>19</sup> A simple alternative would be to introduce a fixed cost  $f$  for each part that is produced in South.

#### 3.2 Firm behavior

Conditional on the sourcing decisions of all intermediates much of the mechanics of the model are the same as before. The optimal input mix follows equation (4) for intermediates sourced in North and equation (6) for South parts, equating marginal products to the appropriate relative wage.

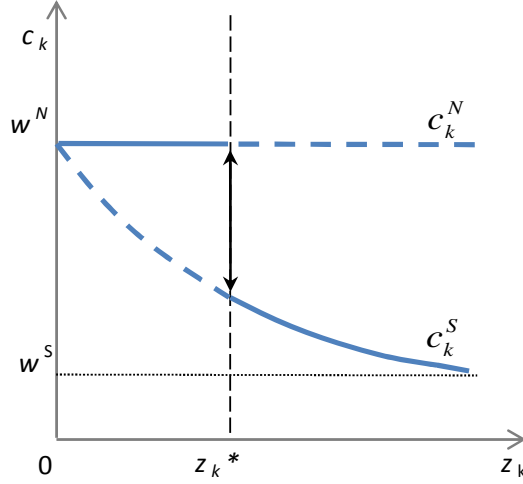
Equilibrium output is obtained by equating the marginal revenue product for each intermediate  $\partial R/\partial y_k$  to the relevant marginal production cost  $c_k^N$  or  $c_k^S$ . This is decided centrally by the North firm as it determines quantity and price for the final good, conditional on the sourcing pattern of all intermediates. For parts sourced in North the input levels are determined by equation (5), while for South parts the adjustment for the incomplete contracting friction disappears and the first order

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<sup>18</sup>This sequence is generated for  $\alpha = -10$  and assuming the following evolution for the  $z$  indices. The first five intermediates mature gradually together, which leads to the usual deceleration. They mature completely to  $z = 1$  before  $z_6 > 0$ . As a result, part 6 faces a much higher  $\tilde{z}_S$  than part 5 did, which induces a temporary acceleration, i.e.  $z_6^* < z_5^*$ . As this is only possible if the difference in  $\tilde{z}_S$  is sufficiently large, the parts following 6 will again show deceleration.

<sup>19</sup>Iceberg transportation costs or tariffs would lead to a landed marginal cost for intermediate  $k$  that is produced in South equal to  $c_k^S = w^{z_k}/\theta$  ( $0 < \theta < 1$ ), while the marginal cost of producing  $y_k$  in North is always unity. Any intermediate with  $z_k > \ln(\theta/w)$  would be produced in South irrespective of where other intermediates are produced. In general equilibrium, a non-tradables sector could fix the relative South wage rate and make the  $\ln(\theta/w)$  threshold a constant.

Figure 5: Maturity threshold and costs when South production entails fixed costs



condition becomes the same equation with the right hand side set to  $c_k^S = w^{z_k}$ .

The evolution of marginal costs in North and South and the maturity threshold where sourcing switches for a representative intermediate  $k$  are illustrated in Figure 5.

For a representative intermediate, Figure 5 illustrates that marginal production and thus also procurement costs are lower when the part is sourced in South.  $c_k^S$  is always below the constant marginal cost of North production, but only when the gap is sufficient to cover the fixed cost will production take place in South, i.e. when the average cost of production is lower in South. In addition to the fixed cost  $f$ , the  $z^*$  threshold is also a function of the optimal output levels under either sourcing mode  $y_k^N(z^*)$  and  $y_k^S(z^*)$ , which depend themselves on all parameters in the model, in particular the maturity levels of all parts already produced in South.

Pricing for the final good is unchanged and profits can be derived in the same way as before as

$$\pi(\tilde{c}, n_S) = (1 - \beta) (K\tilde{c})^{-\frac{\beta}{1-\beta}} - f \cdot n_S. \quad (10)$$

Profit depends on  $n_S$ , the number of intermediates produced in South, as South production raises fixed costs. It also depends on the average cost  $\tilde{c}$  which is defined similarly as the  $\widetilde{MC}$  aggregate before:  $\tilde{c}^{-\frac{\alpha}{1-\alpha}} = \frac{1}{K} \sum_{l=1}^K c_l^{-\frac{\alpha}{1-\alpha}}$ .

The profit equation nicely illustrates the different roles of the  $\beta$  and  $\alpha$  parameters. If all intermediates are produced in North,  $\tilde{c} = 1$  and the  $z$  parameters are irrelevant. Optimal quantities and price are determined solely by the number of intermediates and the substitutability of final products. This is because output in

the CES production function rises with  $K$  and because the equilibrium price-cost ratio equals  $1/\beta$ . When at least one intermediate is sourced from South, the average marginal production cost declines ( $\tilde{c} < 1$ ), equilibrium output is raised, and variable profits rise. The  $\tilde{c}$  average is declining in the  $z$  index of a South-sourced intermediate, but the  $\alpha$  parameter also plays a role now.

Implicitly, profit vary with  $\tilde{z}$ , which is defined as  $\tilde{c} = w^{\tilde{z}}$ , mirroring  $c_k^S = w^{z_k}$ . It is the appropriately weighted average maturity of parts and determines variable cost savings from South production. It differs from the simple unweighted average  $\bar{z}$  for two reasons. First, for components sourced in North the  $z$  value used in the  $\tilde{c}$  average is 0 because North production cost always remains at  $w_N = w^0 = 1$ . Second, the relative weight of intermediates depends on  $\alpha$ , which determines the substitution elasticity in production. If  $\alpha > 0$ , intermediates are substitutes and  $\tilde{z}$  will be higher than the unweighted average. Substitution towards South-sourced intermediates with high  $z$  raises their weight in the marginal cost of the final good and their  $z$ 's will receive a higher weight in the  $\tilde{z}$  aggregate as well. This effect is stronger if substitution between intermediates is easier (larger  $\alpha$ ). If  $\alpha < 0$  and intermediates are complements, the cost share of intermediates from North will be more than proportional and  $\tilde{z}$  will be lower than the unweighted average.

Comparing profit equation (10) to the one for the incomplete contracting model, in equation (9), reveals three differences.<sup>20</sup>

First, there are the explicit fixed costs associated with South production.

Second, the firm now considers the production costs  $c_s$  rather than the procurement costs  $MC_s$  for intermediates produced in South. Without rent sharing and inefficient (noncooperative) input choices, the costs considered by the firm to determine  $y^*$  are the lower production costs, rather than its own expenditures, i.e.  $w_N x_h$  and the fraction of the rent shared with the South supplier in the incomplete contracts case.

Third, in the current case the implicit markup on the cost of intermediates is a uniform  $(1 - \beta)$ . In the incomplete contracting place, this multiplier only applies to intermediates produced in North, with a higher markup  $(1 - \beta/2)$  applied to intermediates sourced in South. These come from the ex-ante payment for these intermediates that raises profits and show up as a higher equilibrium markup. Without a fraction of the profits received ex ante, the firm's profits are now more responsive

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<sup>20</sup>To facilitate comparison, we re-write equation (10) in the same form as (9):

$$\pi = g(K) \left( (1 - \beta)(K - n_S) + (1 - \beta) \sum_{s \in S} c_s^{-\frac{\alpha}{1-\alpha}} \right) \times (\tilde{c})^{\frac{\beta-\alpha}{\alpha(1-\beta)}} - f \cdot n_S.$$

to price increases and the equilibrium price markup will be lower.

### 3.3 Equilibrium

Conditional on the vector of maturities of all intermediates, the sourcing equilibrium is again found by comparing profits under all possible configurations. As in the incomplete contracting model, it is again optimal to offshore intermediates in the order of their maturity, indexed by  $k$ . The equilibrium can again be characterized by the number of intermediates sourced in South.

The first term in the profit equation (10), the variable profit, is monotonically decreasing in  $\tilde{c}$ , which is itself a decreasing function of  $n_S$ . Variable profits are thus increasing in  $n_S$ , which is intuitive. The second term in (10), the sum of fixed costs, increases linearly in  $n_S$ . In equilibrium, intermediates will be offshored until the variable cost savings are outweighed by the additional fixed cost. For any two sourcing configurations, there is an implicit threshold for the change in marginal cost  $\tilde{c}$  that makes the firm indifferent between the two configurations. The profit comparison boils down to

$$\tilde{c}(n_S)^{-\frac{\beta}{1-\beta}} - \tilde{c}(n'_S)^{-\frac{\beta}{1-\beta}} = \Delta n_S f \frac{K^{\frac{\beta}{1-\beta}}}{1-\beta}, \quad (11)$$

As before, we need to evaluate profits for all possible  $n_S \in \{1, \dots, K\}$  to find the optimum. We illustrate in the next section that it is important to also compare sourcing configurations that differ by more than one part, e.g.  $n_k$  and  $n_{k+2}$ .

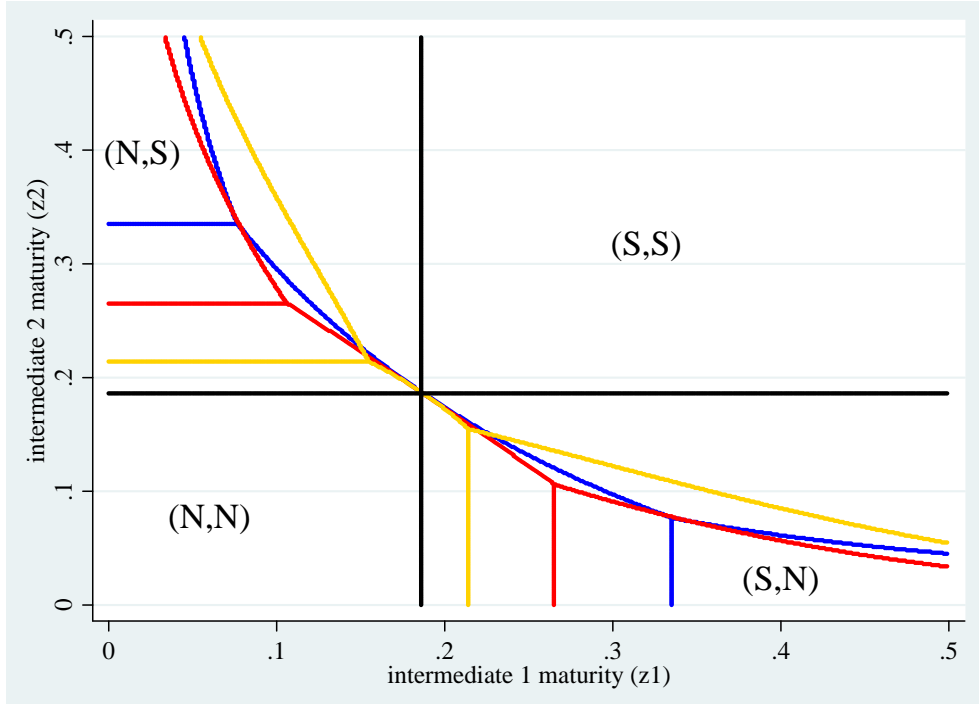
### 3.4 All possible equilibria in the two-intermediate case

For a final good that is made up of only two intermediates, we again illustrate graphically the sourcing equilibria for all points in  $(z_1, z_2)$  space. The two maturity thresholds  $z_1^*(z_2)$  and  $z_2^*(z_1)$  partition the space into four sourcing configurations which is illustrated for different values of  $\alpha$  in Figure 6.

The special case of independent intermediates,  $\alpha = \beta$ , is represented by the black straight lines. Both thresholds are independent of the maturity level of the other part. The contribution to profit of each intermediate is additive. Comparing alternative sourcing options for intermediate 1 in equation (11), the contribution of intermediate 2 cancels out on the left-hand side. In this case, the equilibrium from the much simpler model with fixed costs takes the exact same form as in the incomplete contracting model which was first analyzed by Antràs (2005).

If  $\alpha < \beta$ , the threshold  $z_1^*(z_2)$  at which the firm is indifferent to source intermediate 1 from North or South is a weakly decreasing function of the maturity level of the second intermediate. Equation (11) pins down the maturity threshold  $z_1^*$  for

Figure 6: All sourcing equilibria in the two-intermediate case with fixed costs



Note: The lines are the maturity thresholds that partition the space into four optimal sourcing configurations for different values of  $\alpha$ . In the order of the vertical segments, they refer to  $\alpha = \beta = 0.8$  (black straight lines),  $\alpha = 0.65$  (yellow),  $\alpha = 0.1$  (red),  $\alpha = -5$  (blue).

each value of  $z_2$  and the slope of this function depends on the sourcing location of intermediate 2, which can take three forms:

$$\text{Intermediate 2 in North} \quad [c_1^S(z_1^*)^{\tilde{\alpha}} + (c_2^N)^{\tilde{\alpha}}]^{\tilde{\beta}/\tilde{\alpha}} - [(c_1^N)^{\tilde{\alpha}} + (c_2^N)^{\tilde{\alpha}}]^{\tilde{\beta}/\tilde{\alpha}} = f \tilde{K} \quad (12)$$

$$\text{Intermediate 2 in South} \quad [c_1^S(z_1^*)^{\tilde{\alpha}} + (c_2^S(z_2))^{\tilde{\alpha}}]^{\tilde{\beta}/\tilde{\alpha}} - [(c_1^N)^{\tilde{\alpha}} + (c_2^S(z_2))^{\tilde{\alpha}}]^{\tilde{\beta}/\tilde{\alpha}} = f \tilde{K} \quad (13)$$

$$\text{Simultaneous switch} \quad [c_1^S(z_1^*)^{\tilde{\alpha}} + (c_2^S(z_2))^{\tilde{\alpha}}]^{\tilde{\beta}/\tilde{\alpha}} - [(c_1^N)^{\tilde{\alpha}} + (c_2^N)^{\tilde{\alpha}}]^{\tilde{\beta}/\tilde{\alpha}} = 2f \tilde{K} \quad (14)$$

The exponents are  $\tilde{\alpha} = -\alpha/(1-\alpha)$ ,  $\tilde{\beta} = -\beta/(1-\beta)$ , and  $\tilde{K} = 2^{-\tilde{\beta}}/(1-\beta)$  is a scaling term. Each of the equations (12)-(14) corresponds to a segment of the downward-sloping line that defines the  $z_1^*$  threshold in Figure 6. The three segments of the  $z_1^*$  threshold in Figure 2 for the incomplete contracting case can be defined similarly, but the algebra is more involved.

Equation (12) determines the border between the (N,N) and (S,N) areas. The maturity of intermediate 2 is irrelevant as its marginal cost is constant, leading to a vertical segment. The left-hand side increases monotonically in  $z_1$  as  $c_1^S$  falls from 1 to  $w$ . The equation has a unique solution for  $z_1^*$  if the maximum LHS at  $z_1 = 1$  exceeds  $\tilde{K}f$ .



In equation (13) the second intermediate is already produced in South and  $c_2^N$  is twice replaced with the lower cost  $c_2^S(z_2)$ . The threshold  $z_1^*$  has to be lower than in the previous case and sourcing for intermediate 1 will already be profitable in South at a smaller marginal cost advantage.<sup>21</sup> With intermediate 2 in South the total marginal cost of the final good is lower and equilibrium output higher. As long as  $\alpha < \beta$  substitution away from the relatively more expensive intermediate 1 will not be strong enough to overturn the higher derived demand for intermediate 1. At a higher production volume  $y_1^*$  it is easier to recover the fixed costs of South sourcing, which thus happens at lower  $z_1^*$ .

Moreover, the  $z_1^*$  threshold depends directly on  $z_2$ . Applying the implicit function theorem to equation (13) demonstrates that  $\partial z_1^*(z_2)/\partial z_2 < 0$ . The mechanism leading to a lower  $z_1^*$  if intermediate 2 is produced in South is reinforced with higher  $z_2$ . The border separating the (N,S) and (S,S) areas thus lays to the left of the vertical segment and is downward-sloping.

Finally, equation (14) determines the segment in the middle that separates the (N,N) and (S,S) areas. When the maturities of the intermediates are sufficiently similar, i.e. near the 45-degree line, it will be optimal to produce both intermediates in South at a maturity level where neither of the intermediates can alone be profitably produced in South. The output boost from switching one intermediate from North to South makes it profitable to make the same sourcing switch for the second component.

The reason is that variable profits are convexly increasing in  $\tilde{z}$ , while sourcing an additional intermediate in South only raises fixed costs by a constant amount. If a second intermediate is almost as mature as the first one, it is very well possible that  $\tilde{c}(2)^{\tilde{\beta}} - 1$  exceeds  $2f\tilde{K}$  before  $\tilde{c}(1)^{\tilde{\beta}} - 1$  exceeds  $f\tilde{K}$ .<sup>22</sup>

The lines in Figure 6 partition the space into four areas with a different optimal sourcing configuration. Differently colored lines correspond to different values of the  $\alpha$  parameter, which governs the substitutability of intermediates, holding constant the  $\beta$  parameter, which governs the substitutability of final products.

The maturity threshold to offshore the first component is higher if  $\alpha$  is low. Moving production of one intermediate to South has a smaller effect on profits if  $\alpha$

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<sup>21</sup>If  $\alpha > 0$ ,  $\tilde{\alpha} < 0$  and all  $(c^S)^{\tilde{\alpha}}$  terms are larger than one.  $\beta > \alpha$  further implies that  $\tilde{\beta}/\tilde{\alpha} > 1$  and the first term in equation (13) has to be lower than the corresponding term in (12) for the left hand sides of both equations to equalize. With  $\tilde{\alpha} < 0$  this happens if  $z_1^*$  is lower. If  $\alpha < 0$ , the  $(c^S)^{\tilde{\alpha}}$  terms are smaller than one, but the exponent  $\tilde{\beta}/\tilde{\alpha}$  turns negative. The first term in equation (13) now has to be greater than the corresponding term in (12), but this again means a lower  $z_1^*$ .

<sup>22</sup>In the general case with  $K$  intermediates, even more simultaneous switches are possible if  $\tilde{z}$  values for more parts are similar as well. In the incomplete contracting model, the equivalent of equation (14) does not have an interior solution, as demonstrated in Proposition 2. Instead, there is an ‘invert sourcing’ segment where the firm is indifferent between the sourcing configurations (S,N) and (N,S). This equation does not have a solution in the model with fixed costs.

is low because the firm cannot substitute as easily away from the expensive North-sourced intermediate as consumers can substitute between final goods. Output will not rise very much, making it harder to recover fixed costs and offshoring will only start at higher maturity levels.

The same difficulty to substitute away from the expensive intermediate raises its derived demand when another intermediate is already produced in South. This now has the opposite effect, making it easier to recover the fixed cost of offshoring an additional intermediate. This effect only goes so far, as for  $\alpha < 0$  intermediates are complements and the cost of the final good rises and its quantity falls introducing an opposing effect. The segment of the  $z_2^*$  threshold separating the (S,N) and (S,S) areas is lowest for  $\alpha$  close to zero. In any case, the second intermediate is offshored at a lower maturity level than the first, an acceleration in the offshoring process.

In sum, the difficulty of substituting between intermediates (low  $\alpha$ ) works as a complementarity. It increases the incentive to produce intermediates in the same place. The first effect is to delay offshoring of the first intermediate. The second effect is to accelerate offshoring of part 2 when part 1 is already produced in South. The third effect is to enlarge the segment of the threshold that separates the (N,N) and (S,S) areas.

### 3.5 A dynamic interpretation

We now adopt the same dynamic interpretation as before, i.e. we characterize how the sourcing configuration changes when maturities increase. First holding the maturities of all other intermediates constant, the maturing of one intermediate also leads to a production life-cycle in the model with fixed costs:

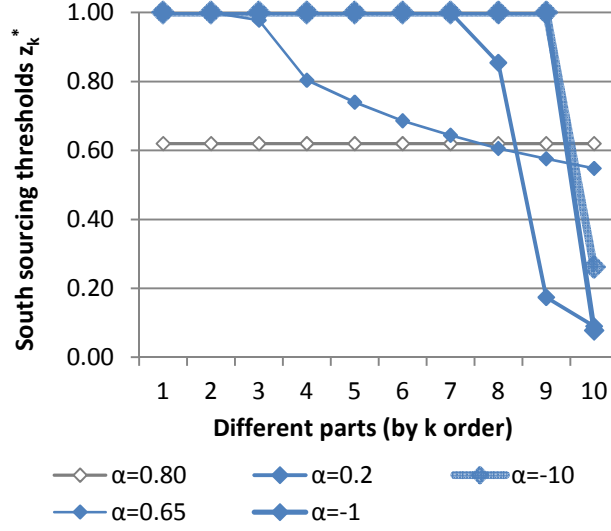
**Proposition 4** *Given the skill intensity of all other parts, there exists a  $z_k^*$  threshold such that part  $k$  should be produced in North if  $z_k < z_k^*$  and in South if  $z_k \geq z_k^*$ , with South production incurring a fixed cost.*

The equilibrium sourcing pattern satisfies the following property which was not the case for the incomplete contracting model:

**Corollary 5** *Once a part is sourced from South, no increase in the skill intensity of other parts will be able to switch optimal production of the part back to North.*

Proofs for both results are in the Appendix. Proposition 4 follows just like Proposition 1 from the slope of the profit derivative:  $\partial\pi(k \in S)/\partial z_k > 0$ . Corollary 5 follows from the key property that  $\partial^2\pi(k \in S)/\partial z_k \partial \tilde{c} < 0$ . Both derivatives hold conditional on the sourcing pattern of the other parts and we demonstrate that no change in their sourcing can overturn the result, while still be profitable. The last

Figure 7: Sourcing thresholds when South production entails fixed costs



derivative leads to the following comparative static result: the threshold for South sourcing of intermediate  $k$  depends negatively on the skill intensity of other parts, i.e.  $\partial z_k^*/\partial \tilde{z} \leq 0$ . If all elements of the full  $z$  vector are weakly higher, the equilibrium cannot have fewer intermediates produced in South.

For the third result we assume again that the ordering is preserved when intermediates mature. The pattern is the exact opposite from the incomplete contracting model and it holds for all possible values of the  $\alpha$  parameter.

**Proposition 6** *If production in South involves a fixed cost and  $z_k \geq z_{k+1}$  for all  $k$ , the sourcing thresholds that make the firm indifferent between North and South sourcing of intermediates  $k$  and  $k + 1$  satisfy  $z_k^* \geq z_{k+1}^*$ .*

With fixed costs, maturity thresholds for successive intermediates are unambiguously decreasing. When one intermediate is offshored, the marginal cost of the final product declines and its equilibrium output rises. Because  $\alpha < \beta$  the derived demand for each intermediate is certain to increase, which makes it easier to recover the fixed costs of South production for any intermediates still produced in North.

The interdependence of demand for different intermediates leads unambiguously to an acceleration in offshoring, i.e. later maturing intermediates make the switch at a lower level of maturity.<sup>23</sup> This effect can be quantitatively important. In the two-

<sup>23</sup>Note that there is no scope for forward looking behavior by the firm. Each decision is optimal given the exogenous evolution of the  $z$  indices. Variation in  $z^*$  thresholds for different parts is not the result of myopic behavior as the firm cannot influence any of the parameters that determine the thresholds.

intermediate case of Figure 6 with  $\alpha = 0.1$  (red line), the share of low-skill inputs in the first intermediate has to exceed 0.27 before it can be profitably sourced from South. The  $z_2^*$  threshold for the second part to switch sourcing as well immediately falls to 0.11 when intermediate one is produced in South. If  $z_1$  were to rise to 0.5, the  $z_2^*$  threshold would decline further to 0.04. Only one seventh of  $z_1^*$ .

The maturity threshold at which successive intermediates switch optimal production to South in the case of 10 intermediates and with the same sequential maturing as in Figure 4 is illustrated in Figure 7. In the benchmark case of  $\alpha = \beta = 0.8$  thresholds are constant, as before, but for lower  $\alpha$ 's they are weakly declining in the  $k$  ordering. This can be seen best for  $\alpha = 0.65$ . For even lower  $\alpha$ 's the simultaneous switching becomes very important, leading to  $z^* = 1$  for several parts, but still lower thresholds for later parts.

## 4 Empirical test

U.S. import statistics for automotive components in Figure 1 suggest a growing importance of imported parts over time. Not only are import volumes higher, countries gradually export a wider range of components to the United States, in line with the predictions of our theoretical model. Sourcing of intermediates from low-cost countries becomes more profitable once they have matured further. This effect would even operate if sourcing were decided independently for all intermediates.

When demands are interrelated, the two alternative ways we modeled trade friction produced starkly different predictions on the maturity levels at which successive intermediates are first offshored. When South production involves incomplete contracts, intermediates that mature sooner are first produced abroad at lower maturity levels than intermediates that mature later, barring some special cases. The reverse holds if South production involves fixed costs.

One could say that offshoring is slowing down in the first model, as the maturity levels at which intermediates are first offshored rise over time, while it is accelerating in the second model. These opposing predictions could be tested directly if we observed two pieces of information: the ordering of parts ( $1, \dots, K$ ) and the maturity levels of parts in the first year they are offshored ( $z_k^*$ ).

The information on U.S. automotive component imports that we use is from the NBER web site for the 1995-2001 period and from the Global Trade Atlas for 2002-2006.<sup>24</sup> The units of observation are import flows at the country-product-year level.

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<sup>24</sup>The first data source is described in Feenstra, Romalis, and Schott (2002) and available at <http://www.nber.org/data/>. The second source is maintained by a private company, Global Trade Information Service, with information at <http://www.gtis.com/>

The sample contains products at the 10-digit level of the Harmonized System (HS) classification within the HS870600, HS870701, HS870790, and HS8708 categories.<sup>25</sup> A total of 54 countries record positive exports to the United States for any of these automotive components over the 12 year sample period.

The parts ordering in the theory is intended to represent a particular feature of technology—the relative speed at which different intermediates mature—and it should apply to all countries. We used the method developed by Feenstra and Rose (2000) to construct a unique ranking of parts, using the country-specific ordering in which parts are first exports to the United States as inputs. The method aggregates over the order of initial exports for all countries, with a correction for products that are skipped by some countries, i.e. never exported to the United States.

We cannot derive a proxy for the maturity thresholds directly from the observed export flows. Because the rate of maturing over time is not necessarily constant, the predictions of an accelerating or decelerating offshoring process that we defined in terms of maturity thresholds does not necessarily translate into accelerating or decelerating import flows over time. Our solution is to recover a proxy for maturity from the unit value ratio, i.e. the import price.

We normalize the prices for all ‘South’ countries by the average price of U.S. imports from Japan, Germany, and the United Kingdom for the same product in the same year. These three countries have high and stable wages over the sample period and their import prices serve as a proxy for the unobserved North price of domestic produced parts. The gap between South and North prices serves as a proxy for the inverse of a part’s maturity.<sup>26</sup> If South sourcing only occurs at a very mature stage, this will show up as a low relative import price as the weight on the low-skilled input produced with low wages has become large.

Regression results of this proxy for maturity on the parts ordering are reported in Table 1. We only use the price gap for the first year that a country exports a part to the United States. To facilitate the interpretation we use the negative of the logarithm of relative prices as dependent variable in the regressions and we include country and year fixed effects to control for the general effect of wage differences. While it is a strong assumption to use the cardinality of the parts ranking as explanatory variable, it serves the illustrative purpose. We also include results where the parts index is interacted with the simple average of six governance measures compiled annually by the World Bank.<sup>27</sup>

A number of findings stand out. First, the overall relationship between the proxy

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<sup>25</sup>While sufficient for our illustrative purpose, it only represents approximately one half of all automotive component imports. The remainder is spread over sub-categories of automotive components within other broad product classes, e.g. HS840820 is “diesel engines for motor vehicles,” or

Table 1: Relationship between relative prices and the ordering of parts

	Dependent variable is $-\ln P_S/P_N$			
	(1)	(2)	(3)	(4)
part index (PI)	0.008*** (0.001)	0.010*** (0.002)		
PI * governance		-0.002* (0.001)		
PI * bad governance			0.014*** (0.002)	
PI * intermediate governance			0.007*** (0.002)	
PI * good governance			0.004* (0.002)	-0.005* (0.003)
No. of observations	3,186	3,186	3,186	203

Notes: A unit of observation is a 10-digit automotive part imported into the United States for the first time by a particular country.  $P_S$  is the unit value for that import flow and it is normalized by the average unit value of Japan, Germany, and the U.K. for the same part in that year. The part index ranks all components in the average order they are first exported to the United States, constructed as in Feenstra and Rose (2000), and it is the same for all exporting countries. Column (4) includes only Canada, France, and Mexico. \*\*\*, \*\*, \* indicate significance at the 1%, 5%, and 10% level.

for  $z_k^*$  and the parts ordering is positive, suggesting that offshoring is slowing down. This pattern cannot be generated by a model with only fixed costs as a trade friction to delay parts shifting to South. Both frictions might play a role, but the empirical pattern on the maturity thresholds suggests that incomplete contracts are especially important.

Second, the positive relationship between the ordering and the maturity is weakened by the quality of governance in the country. It suggests the incomplete contracting model is less dominant for countries with better governance. An alternative interpretation is that assemblers take advantage of better governance situations to increase the substitutability of their parts, i.e. be less rigid in their sourcing relationships, which raises the  $\alpha$  parameter in the production function and weakens linkage between parts.

Classifying countries into three groups based on the quality of governance leads to the same finding. The negative relationship between the parts ordering and maturity is strongest if governance is bad. Even for the subset of countries with the best governance, the correlation is still positive, but the coefficient becomes

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HS700721 is "Safety glass (laminated) for vehicles."

<sup>26</sup>In the theory this is direct because  $\ln AC^N/AC^S = -z \ln w$ .

<sup>27</sup>Available at <http://www.worldbank.org/wbi/governance>.

insignificant if the regression is run separately for each group and time dummies vary.

Finally, in the last column the regression is limited to three countries with excellent or good governance and with a well developed automotive sector: Canada, France, and Mexico. The parameter estimate turns negative here. For these three countries the empirical pattern is consistent with a model of complete contracts and only fixed costs delaying international sourcing of parts.

## 5 Conclusions

A model with only final goods can generate the intuitive predictions that goods need to reach a maturity threshold before they are offshored and that goods maturing more slowly are offshored later. The interaction of sourcing decisions for intermediates that are assembled into the same final good generates some more surprising predictions.

Foremost, for many predictions it matters crucially how costs associated with producing in a low-wage location are modeled. First, intermediates that mature at similar speeds are offshored jointly if South production requires a fixed cost, but never if contracts with South suppliers are incomplete. Second, in the fixed cost model offshoring starts later than would be the case for independent products, i.e. at a higher maturity level, but it starts earlier in the incomplete contracting model. Third, offshoring of successive intermediates is accelerating in the fixed cost model, i.e. happening at ever lower maturity levels, but unambiguously slowing down in the incomplete contracting model if intermediates are substitutes.

An illustrative empirical test of this last prediction using data on U.S. automotive parts imports suggests that incomplete contracts are a crucial ingredient to explain the observed patterns. Especially for countries with poor governance, we find that offshoring of parts is slowing down. Automotive components that were first imported in the United States only late in the sample period enter at significantly lower prices than components that entered earlier. We interpret this as having reached a higher level of maturity. Only for a few countries with good governance and mature automotive sectors is the pattern consistent with a model without incomplete contracts and only fixed costs.

Two broader implications of distinguishing between final goods and intermediates are worth highlighting. First, while we focus on the interrelation of sourcing decisions, the division of benefits from trade between the final good producer and individual suppliers also becomes interrelated in our model. It will depend on the substitutability of intermediates and on the rate at which individual parts mature,

both of which vary naturally across industries and our model can be used to study the implications. For example, much of the cost-saving on one intermediate might not benefit its particular supplier, see Dedrick, Kraemer, and Linden (2010) for an illustration using the Apple iPod.

Second, the dependency of the maturity threshold of one part on the maturity and sourcing location of other parts implies that the range of products that a country produces is not constant. If sourcing decisions were independent, the productivity-adjusted wage level would fully determine the maturity level for which a product could be profitably produced in each country. As products mature, their production would move from country to country in the order of their (productivity-adjusted) wage, always switching at the same maturity level. In contrast, our model predicts that at any given time countries will produce products over a range of maturity levels that overlap with that of other countries. The interrelation of demand for intermediates thus weakens the usual prediction of specialization according to comparative advantage.

In future work, we intend to derive a broader range of predictions from an extended model with multiple countries. If they can be ranked by unit cost the model would predict sequential sourcing chains. Combining information on wage and productivity differences, we could test predictions on offshoring order, speed, and interdependence. The acceleration or deceleration in offshoring could be evaluated for industries that differ in the substitutability of intermediates. Industries also differ in the importance of differentiated inputs, which according to Nunn (2007) is likely to correlate with the importance of incomplete contracts. A testable prediction of our model is then that offshoring is more likely to slow down over time in industries relying more on differentiated inputs.



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## Appendix I Proofs for the incomplete contracting model

### Proposition 1

We need to show that there exists one and only one cutoff for  $z_k$  that determines optimal sourcing for intermediate  $k$ , given the maturities  $z_L = (z_1, \dots, z_{k-1}, z_{k+1}, \dots, z_K)$  of all other intermediates.

We first assume that the production mode of all other parts is fixed and denoted by  $M_L(m_1, \dots, m_{k-1}, m_{k+1}, \dots, m_K)$  with  $m_j \in (N, S) \forall j$ . Conditional on  $z_L$ , profits  $\pi(m_k, M_L)$  are entirely determined by the production location of all parts. The profit difference  $\pi(S, M_L) - \pi(N, M_L)$  is (i) negative at  $z_k = 0$ , (ii) positive at  $z_k = 1$  for parameters that ensure South sourcing is viable, and (iii) monotonically increasing in  $z_k$ . This last feature follows from  $\partial\pi(N, M_L)/\partial z_k = 0$  and

$$\frac{\partial\pi(S, M_L)}{\partial z_k} = \left( \frac{-\alpha}{1-\alpha} \right) \frac{MC_k^{-\frac{\alpha}{1-\alpha}} \pi}{K - n_S + n_S \widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}}} \ln w \left[ \frac{\beta - \alpha}{\alpha(1-\beta)} + \frac{n_S(1 - \frac{\beta}{2}) \widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}}}{(K - n_S)(1-\beta) + n_S(1 - \frac{\beta}{2}) \widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}}} \right] > 0.$$

with  $\widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}} = 1/n_S \sum_{s \in S} MC_s^{-\frac{\alpha}{1-\alpha}}$ . There must be one and only one cutoff  $z_k^*(z_L, M_L)$  for part  $k$  such that profit maximization dictates: if  $z_k < z_k^*$ , part  $k$  is produced in North and in South if  $z_k \geq z_k^*$ .

We next consider that the production mode for other parts can vary. Profits with  $k$  produced in North satisfy  $\partial\pi(N, M_L)/\partial z_k = 0$  for each possible  $M_L$ . For a given  $z_L$ , one particular configuration, say  $M_{L^*}$ , will be optimal when  $k$  is produced in North, irrespective of  $z_k$ . This configuration also satisfies  $\partial\pi(S, M_{L^*})/\partial z_k > 0$ . Once  $z_k \geq z_k^*(z_L, M_{L^*})$  and profits with  $k$  in South exceed the highest possible profits with  $k$  in North, it is never profitable to produce  $k$  in North again if  $z_k$  does not fall. ■

### Corollary 2

According to Corollary 2, it is never optimal to switch more than one part from North to South at the same moment. We demonstrate this for part  $k$  and part  $k+1$ , when the skill intensity of all other parts is fixed. Use  $\pi(m_k, m_{k+1}, M_L)$  to denote the profit when part  $k$  and  $k+1$  are produced as  $m_k$  and  $m_{k+1}$ , and other parts according to  $M_L$ .

Assume it is optimal that both parts  $k$  and  $k+1$  switch from North to South simultaneously. In that case, the following equation implicitly defines the two cutoffs

$z_k^*$  and  $z_{k+1}^*$

$$\pi(N, N, M_L) = \pi(S, S, M'_L)$$

It allows for the possibility that  $M'_L$  differs from  $M_L$ . If simultaneous switching were indeed optimal, it implies that both profits above are higher than  $\pi(S, N, M_L)$  at  $z_k^*$ . Suppose  $\pi(N, N, M_L) = \pi(S, S, M'_L) = x\pi(S, N, M_L)$  with  $x \geq 1$ .

Note that we can write the profit equation for any sourcing configuration as

$$\begin{aligned} \pi(\bullet) &= K^{-\frac{\beta}{\alpha(1-\beta)}} \left( MC_k^{-\frac{\alpha}{1-\alpha}} + MC_{k+1}^{-\frac{\alpha}{1-\alpha}} + (K-2) \cdot \widetilde{MC}_L^{-\frac{\alpha}{1-\alpha}} \right)^{\frac{\beta-\alpha}{\alpha(1-\beta)}} \times \\ &\quad \left[ (1-\beta\tau_k)MC_k^{-\frac{\alpha}{1-\alpha}} + (1-\beta\tau_{k+1})MC_{k+1}^{-\frac{\alpha}{1-\alpha}} + (K-2)(1-\beta\tilde{\tau}_L)\widetilde{MC}_L^{-\frac{\alpha}{1-\alpha}} \right] \\ &= y(\bullet) \times [\text{mark-up}(\bullet)] \end{aligned}$$

always using the appropriate marginal costs and mark-ups for the chosen sourcing configuration  $m_k, m_{k+1}$ , and  $M_L$ :  $MC_j = 1$  and  $\tau_j = 1$  if  $m_j = N$  and  $MC_j = w^{z_j}$  and  $\tau_j = 1/2$  if  $m_j = S$ . The aggregates are then defined as  $(K-2)\widetilde{MC}_L^{-\frac{\alpha}{1-\alpha}} = \sum_{l \neq k, k+1} MC_l^{-\frac{\alpha}{1-\alpha}}$  and  $(K-2)(1-\beta\tilde{\tau}_L)\widetilde{MC}_L^{-\frac{\alpha}{1-\alpha}} = \sum_{l \neq k, k+1} (1-\beta\tau_l)MC_l^{-\frac{\alpha}{1-\alpha}}$ .

Given that the mark-up is a sum of  $K$  terms, we can decompose it linearly in the  $(N, S, M'_L)$  sourcing configuration as

$$\text{mark-up}(N, S, M'_L) = \text{mark-up}(N, N, M_L) + \text{mark-up}(S, S, M'_L) - \text{mark-up}(S, N, M_L).$$

Next, we substitute in the profit definition

$$\frac{\pi(N, S, M'_L)}{y(N, S, M'_L)} = \frac{\pi(N, N, M_L)}{y(N, N, M_L)} + \frac{\pi(S, S, M'_L)}{y(S, S, M'_L)} - \frac{\pi(S, N, M_L)}{y(S, N, M_L)},$$

and re-write it using the earlier equalities as we evaluate the expressions at  $z_k^*$  and  $z_{k+1}^*$

$$\frac{\pi(N, S, M'_L)}{y(N, S, M'_L)} = \frac{\pi(N, N, M_L)}{y(N, N, M_L)} + \frac{\pi(N, N, M_L)}{y(S, S, M'_L)} - \frac{\pi(N, N, M_L)}{xy(S, N, M_L)},$$

to find

$$\frac{\pi(N, S, M'_L)}{\pi(N, N, M_L)} = y(N, S, M'_L) \left( \frac{1}{y(N, N, M_L)} + \frac{1}{y(S, S, M'_L)} - \frac{1}{xy(S, N, M_L)} \right).$$

This expression is increasing in  $x$ . The concavity of the  $1/y(\bullet)$  function makes that

at  $x = 1$  we can sign the right-hand term:<sup>28</sup>

$$\frac{\pi(N, S, M'_L)}{\pi(N, N, M_L)} > 1.$$

These calculations indicate that at  $z_k^*$  and  $z_{k+1}^*$  the firm would make higher profits if only  $k$  were sourced in North and  $k + 1$  in South, rather than having both parts sourced in the same location. This contradicts the optimality of a simultaneous switch of both parts. ■

### Proposition 3

The assumption that  $z_{k+1} \leq z_k$  and the result in Corollary 2 that parts will not switch sourcing at the same time, guarantees that part  $k$  will already be produced in South when part  $z_{k+1}$  reaches its sourcing threshold. The zero profit condition that pins down  $z_{k+1}^*$  is the following

$$\begin{aligned} & \left( (K - k) + k \cdot \widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}} \right)^{\frac{\beta-\alpha}{\alpha(1-\beta)}} \left( (1 - \beta)(K - k) + \left(1 - \frac{\beta}{2}\right)k \cdot \widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}} \right) \\ &= \left( (K - k - 1) + MC_{k+1}^{*\frac{-\alpha}{1-\alpha}} + k \cdot \widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}} \right)^{\frac{\beta-\alpha}{\alpha(1-\beta)}} \\ & \quad \left( (1 - \beta)(K - k - 1) + \left(1 - \frac{\beta}{2}\right)(MC_{k+1}^{*\frac{-\alpha}{1-\alpha}} + k \cdot \widetilde{MC}_S^{-\frac{\alpha}{1-\alpha}}) \right) \end{aligned}$$

Applying the implicit function theorem to the above equation reveals that  $sign(\partial z_{k+1}^* / \partial \tilde{z}_S) = sign(\alpha)$ .

When  $\alpha > 0$ , the minimum  $z_{k+1}^*$  is achieved at the minimum  $\tilde{z}_S$ . By definition,  $\tilde{z}_S$  is the average  $z_j$  for all parts  $j = 1, \dots, k$  already sourced from South (with necessarily  $z_j \geq z_j^*$  and, recall,  $z_j \geq z_{k+1}$ ). Therefore, the lowest possible  $\tilde{z}_S$  will be  $z_{k+1}^*$  itself, which is achieved only if  $z_1 = \dots = z_k = z_{k+1}^*$ . It must be the case that  $z_k^* \leq z_{k+1}^*$ . ■

We illustrate why the proposition might not hold down when  $\alpha < 0$ . Now the minimum  $z_{k+1}^*$  is achieved at  $\tilde{z}_S = 1$ . Use  $\pi(m_k, m_{k+1}, \tilde{z}_S)$  to denote the profit function when parts  $k$  and  $k + 1$  are produced in  $m_k$  and  $m_{k+1}$  and the  $k - 1$  parts (out of  $K - 2$ ) that are produced in South are characterized by  $\tilde{z}_S$ . The lowest possible threshold for  $z_{k+1}^*$  is thus defined as  $\pi(S, N, 1) = \pi(S, S, 1)$ , with also  $z_k = 1$ ,

<sup>28</sup>This can be seen from the following inequality:

$$y(N, N, M_L)^{-1} + y(S, S, M'_L)^{-1} > y(N, S, M'_L)^{-1} + y(S, N, M_L)^{-1}$$

i.e.,

$$\begin{aligned}
\underline{[z_{k+1}^*]} &= \left( (K-k) + k(2w)^{-\frac{\alpha}{1-\alpha}} \right)^{\frac{\beta-\alpha}{\alpha(1-\beta)}} \times \left( (1-\beta)(K-k) + \left(1 - \frac{\beta}{2}\right)k(2w)^{-\frac{\alpha}{1-\alpha}} \right) \\
&= \left( (K-k-1) + MC_{k+1}^{*\frac{-\alpha}{1-\alpha}} + k(2w)^{-\frac{\alpha}{1-\alpha}} \right)^{\frac{\beta-\alpha}{\alpha(1-\beta)}} \times \\
&\quad \left( (1-\beta)(K-k-1) + \left(1 - \frac{\beta}{2}\right)(MC_{k+1}^{*\frac{-\alpha}{1-\alpha}} + k(2w)^{-\frac{\alpha}{1-\alpha}}) \right)
\end{aligned}$$

The highest possible threshold for  $z_k^*$  is achieved when  $\tilde{z}_S = z_k^*$  and defined by  $\pi(N, N, z_k^*) = \pi(S, N, z_k^*)$ , i.e.,

$$\begin{aligned}
\overline{[z_k^*]} &= \left( (K-k+1) + (k-1)\widetilde{MC}_k^{*\frac{-\alpha}{1-\alpha}} \right)^{\frac{\beta-\alpha}{\alpha(1-\beta)}} \times \\
&\quad \left( (1-\beta)(K-k+1) + \left(1 - \frac{\beta}{2}\right)(k-1)\widetilde{MC}_k^{*\frac{-\alpha}{1-\alpha}} \right) \\
&= \left( (K-k) + k\widetilde{MC}_k^{*\frac{-\alpha}{1-\alpha}} \right)^{\frac{\beta-\alpha}{\alpha(1-\beta)}} \left( (1-\beta)(K-k) + \left(1 - \frac{\beta}{2}\right)k\widetilde{MC}_k^{*\frac{-\alpha}{1-\alpha}} \right)
\end{aligned}$$

Note that  $\pi(S, N, 1)$ , the left hand side of  $\underline{[z_{k+1}^*]}$ , is always larger than  $\pi(S, N, z_k^*)$ , the right hand side of  $\overline{[z_k^*]}$ . Assume that  $x\pi(S, N, 1)|_{z_k=1} = \pi(S, N, z_k^*)|_{z_k=z_k^*} = \pi(N, N, z_k^*)|_{z_k=z_k^*}$  with  $0 < x \leq 1$ .

As in the previous case for  $\alpha > 0$ , we verify whether  $\underline{z_{k+1}^*} \geq \overline{z_k^*}$  as this implies directly that in all situations  $z_{k+1}^* \geq z_k^*$ . We do this by evaluating the equality that defines  $\underline{z_{k+1}^*}$  at  $z_{k+1} = \overline{z_k^*}$ . Because  $\pi(S, S, 1)$  is increasing in  $z_{k+1}$ , we know that if  $[\pi(S, N, 1) - \pi(S, S, 1)]_{z_{k+1}=\overline{z_k^*}} \geq 0$  the sourcing threshold for part  $k+1$  must be higher than this value, and thus  $z_{k+1}^* \geq \overline{z_k^*} \geq z_k^*$ .

We use the same approach as for the proof of Corollary 2 in Appendix A.3. Profits can be written as the product of output and a mark-up term. Because the latter is a sum over all parts, it can be re-written as the combination of the mark-ups in three different configurations:

$$\pi(S, S, 1)|_{z_{k+1}=\overline{z_k^*}} = y(S, S, 1) \left[ \frac{\pi(S, N, 1)}{y(S, N, 1)} + \frac{\pi(S, N, z_k^*)}{y(S, N, z_k^*)} - \frac{\pi(N, N, z_k^*)}{y(N, N, z_k^*)} \right]$$

Substituting in the above two equalities, we obtain

$$\pi(S, S, 1)|_{z_{k+1}=\overline{z_k^*}} = y(S, S, 1) \left[ \frac{\pi(S, N, 1)}{y(S, N, 1)} + \frac{x\pi(S, N, 1)}{y(S, N, z_k^*)} - \frac{x\pi(S, N, 1)}{y(N, N, z_k^*)} \right]$$

and thus

$$\frac{\pi(S, S, 1)}{\pi(S, N, 1)}|_{z_{k+1}=\overline{z_k^*}} = y(S, S, 1) \left[ \frac{1}{y(S, N, 1)} + \frac{x}{y(S, N, z_k^*)} - \frac{x}{y(N, N, z_k^*)} \right],$$

which is increasing in  $x$  because  $y(S, N, z_k^*) < y(N, N, z_k^*)$ .<sup>29</sup>

Similar to Appendix A.3, the right-hand side is positive when  $x = 1$ . Hence, for  $x \rightarrow 1$  it will be true that  $\pi(S, S, 1) - \pi(S, N, 1)|_{z_{k+1}=\bar{z}_k^*} > 1$  and the prediction Proposition 6 will still hold. Recall that  $x$  is defined as  $\frac{\pi(S, N, z_k^*)|_{z_k=z_k^*}}{\pi(S, N, 1)|_{z_k=1}}$ , which is the inverse of the profit gap when the maturities of all  $k$  South sourced parts increases from  $z_k^*$  to 1. If this difference is small, the proposition will still hold and  $z_{k+1}^* \geq z_k^*$ . On the contrary, if the profit difference is large, there might be cases where  $z_{k+1}^* < z_k^*$ . The probability of this happening is lower when  $z_k^*$  is already high.

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<sup>29</sup>Multiplying through the  $y(S, S, 1)$  factor, the first term on the right captures the lower output when more parts are sourced in South, while the next two terms capture the net increase in profit margin, which is increasing in  $x$ .

## Appendix II Proofs for the fixed cost model

### Proposition 4

The proof of Proposition 4 follows the same logic as that of Proposition 1. At  $z_k = 0$ , the fixed costs involved in South production guarantee that  $\pi(N, M_L) = \pi(S, M_L) + f > \pi(S, M_L)$  and intermediate  $k$  is produced in North. We limit attention to the case where at  $z_k = 1$  the profits satisfy  $\pi(N, M_L) \leq \pi(S, M_L) - f$ , i.e. South production is optimal at some maturity level.<sup>30</sup> Because  $\partial\pi(N, M_L)/\partial z_k = 0$  and

$$\frac{\partial\pi(S, M_L)}{\partial z_k} = -\beta K^{-\frac{\beta}{\alpha(1-\beta)}} \left[ c_k^{-\frac{\alpha}{1-\alpha}} + \sum_{j \in M_L} c_j^{-\frac{\alpha}{1-\alpha}} \right]^{\frac{\beta(1-\alpha)}{\alpha(1-\beta)}-1} c_k^{-\frac{1}{1-\alpha}} \cdot \frac{\partial c_k}{\partial z_k} > 0,$$

the difference  $\pi(N, M_L) - \pi(S, M_L)$  declines monotonically with  $z_k$ . There must be one and only one cutoff  $z_k^*(z_L, M_L)$  for part  $k$  such that profit maximization dictates: if  $z_k < z_k^*$ , part  $k$  is produced in North and in South if  $z_k \geq z_k^*$ .

Even when  $M_L$  is not fixed, a change in the sourcing of other parts can never make North sourcing of  $k$  profitable again if  $z_k$  exceeds the threshold that makes South production more profitable than the highest possible profit with part  $k$  in North. Hence, for any  $z_k < z_k^*$ , part  $k$  will be produced in North; while for any  $z_k \geq z_k^*$ , part  $k$  is produced in South. ■

### Corollary 5

To prove Corollary 5, we only need to show that once part  $k$  is produced in South even a change in  $z_L$ , possibly accompanied with a change in  $M_L$ , will not make North production of part  $k$  profitable again either. Due to the series of bilateral contracts, a change in  $M_L$  only enters the above profit functions through its effect on  $\tilde{c}_L^{-\frac{\alpha}{1-\alpha}}$  and  $n_L$ . Following the reasoning above, each possible  $M_L$  leads to a unique cutoff that governs optimal sourcing of component  $k$  and we denote it by  $z_k^*(z_L, M_L)$ .

Consider two alternative choices of  $M_L$ :  $M^A$  and  $M^B$ . Let's assume that  $\tilde{c}_L^A < \tilde{c}_L^B$ , which requires that choice  $A$  has more parts produced in South ( $n^A > n^B$ ), otherwise  $B$  would be dominated and never chosen.<sup>31</sup> Note that the slope of the marginal profit function ( $\partial\pi/\partial z_k$ ) is declining in  $\tilde{c}_L$ ,

$$\frac{\partial^2\pi(S, \cdot)}{\partial z_k \partial \tilde{c}_L} = \frac{\beta(\beta - \alpha)}{(1 - \beta)(1 - \alpha)} K^{-\frac{\beta}{\alpha(1-\beta)}} \left[ c_k^{-\frac{\alpha}{1-\alpha}} + (K-1)\tilde{c}_L^{-\frac{\alpha}{1-\alpha}} \right]^{\frac{\beta(1-\alpha)}{\alpha(1-\beta)}-2} c_k^{-\frac{1}{1-\alpha}} \tilde{c}_L^{-\frac{1}{1-\alpha}} \cdot \frac{\partial c_k}{\partial z_k} < 0,$$

<sup>30</sup>This is not a necessary condition because the possibility of changing  $M_L$  can trigger a boost in derived demand for intermediate  $k$ , which facilitates offshoring for given  $z_k$ .

<sup>31</sup>We use strict rather than weak inequalities, otherwise one of the plans would still weakly dominate the other.



which implies that  $\partial\pi(S, M^A)/\partial z_k > \partial\pi(S, M^B)/\partial z_k$ .

One possibility is that  $\pi(N, M^A) \geq \pi(N, M^B)$ , which implies  $\pi(S, M^A) \geq \pi(S, M^B)$  for  $z_k = 0$ . Given the steeper marginal profit function for  $A$ ,  $\pi(S, M^A) \geq \pi(S, M^B)$  for all possible  $z_k$ .  $A$  dominates  $B$  and the unique cutoff  $z_k^*(z_L, M^A)$  is the only one relevant to determine optimal sourcing for part  $k$  (all other parts are produced according to  $M^A$  for any value of  $z_k$ ).

The other possibility is that  $\pi(N, M^A) < \pi(N, M^B)$ , which implies  $\pi(S, M^A) < \pi(S, M^B)$  for  $z_k = 0$ . The steeper slope of  $\pi(S, M^A)$  against  $z_k$  leads to the following three cases:

1.  $\pi(S, M^A) < \pi(S, M^B)$  even for  $z_k = 1$ : then plan  $B$  dominates  $A$  and  $z_k^*(z_L, M^B)$  is the only relevant cutoff determining optimal sourcing of part  $k$ .
2.  $\pi(S, M^A)$  cuts  $\pi(S, M^B)$  at  $z^{**} \in (z_k^*(z_L, M^B), 1)$ : then  $z_k^*(z_L, M^B)$  is the cutoff for component  $k$ , and  $M^B$  is chosen when  $z_k$  is very small until  $z_k$  reaches  $z^{**}$ .
3.  $\pi(S, M^A)$  cuts  $\pi(S, M^B)$  at  $z^{***} \in (0, z_k^*(z_L, M^B))$ : then  $z^{***}$  is the relevant cutoff value for component  $k$ .  $M^B$  is chosen while  $z_k$  is relatively small and  $k$  is produced in North. When  $z_k$  reaches the cutoff point, not only component  $k$  is switched to South production, but also all other parts are now produced according to  $M^A$ , which sources more other components from South as well.

In each case, once the lower variable cost combination  $M^A$  is chosen,  $M^B$  cannot dominate  $M^A$  with further increase in  $z_k$ , which is Corollary 5. ■

## Proposition 6

Given the assumption that the component index order is fixed, it generally has to be true that if component  $k$  is offshored, any component with index  $l < k$  (and thus with lower skill intensity) should be offshored as well. Otherwise, the firm can raise profits by switching  $k$  to North and produce the lower index component in South instead. It would incur the same fixed costs, but lower variable cost.

There are two possibilities for part  $k+1$ . It will either be offshored first when part  $k$  is already produced in South, or both parts can switch sourcing simultaneously. In the latter case,  $z_{k+1}^* \leq z_k^*$  is directly satisfied by the definition of the index order.

We only need to show that the same inequality holds in the first case, when component  $k$  is already sourced from South when  $k+1$  first moves there. When  $z_k$  reaches the  $z_k^*$  threshold, components with index higher than  $k$  are still sourced

from North. The cutoff  $z_k^*$  is defined as

$$\pi(N, N, M_L) = \pi(S, N, M'_L). \quad (*)$$

The first arguments denote the production location of component  $k$ , the second arguments component  $k + 1$ , and the last arguments the production choice of all other components.

There are two possibilities for  $M'_L$ . First, if component  $k$  switches to South production when component  $k - 1$  is already produced there, then  $M'_L = M_L$ . Second, if component  $k$  is switched to South simultaneously with component  $k - 1$ ,  $M'_L$  is a combination with lower variable cost and higher fixed cost than  $M_L$ . Note that the definition also implies that  $\pi(N, N, M_L)|_{z_k=z_k^*} \geq \pi(N, N, M'_L)|_{z_k=z_k^*}$ , which will be used later.

The cutoff  $z_{k+1}^*$  can be defined similarly as

$$\pi(S, N, M'_L) = \pi(S, S, M''_L). \quad (**)$$

$M''_L$  equals  $M'_L$  if part  $k + 1$  switches sourcing alone, or it can be a lower variable cost higher fixed cost combination where some components with index higher than  $k + 1$  switch sourcing to South simultaneously.

Because the LHS of the  $z_{k+1}^*$  definition (\*\*) is constant in  $z_{k+1}$  and the RHS is increasing,  $z_{k+1}^* \leq z_k^*$  will be satisfied if  $\pi(S, N, M'_L)|_{z_{k+1}=z_k^*} \leq \pi(S, S, M''_L)|_{z_{k+1}=z_k^*}$ . A sufficient condition for this is

$$\pi(S, N, M'_L)|_{z_{k+1}=z_k^*} \leq \pi(S, S, M''_L)|_{z_{k+1}=z_k^*} \quad \text{SC}$$

We can show this condition must be satisfied as follows. Define  $c^*$  as  $c^S$  evaluated at  $z_k^*$ ,  $c_{k+}^S$  as  $c^S$  evaluated at  $z_k \geq z_k^*$ , and  $(\tilde{c}'_L)^{-\alpha'}$  as the average marginal cost for the  $(K - 2)$  parts in  $M'_L$  using the earlier aggregation scheme. Also define  $\alpha' = \alpha/(1 - \alpha)$ ,  $\beta' = \beta/(1 - \beta)$ . If  $\beta > \alpha > 0$ ,  $-\alpha' < 0$  such that  $(c_{k+}^S)^{-\alpha'} > (c^*)^{-\alpha'} > (c^N)^{-\alpha'}$ . Moreover,  $g(x) = x^{\beta'/\alpha'}$  will be a convex function and the following inequality will then hold:

$$\begin{aligned} & \left[ (c^N)^{-\alpha'} + (c^N)^{-\alpha'} + (K - 2)(\tilde{c}'_L)^{-\alpha'} \right]^{\frac{\beta'}{\alpha'}} + \left[ (c_{k+}^S)^{-\alpha'} + (c^*)^{-\alpha'} + (K - 2)(\tilde{c}'_L)^{-\alpha'} \right]^{\frac{\beta'}{\alpha'}} \\ & > \left[ (c^*)^{-\alpha'} + (c^N)^{-\alpha'} + (K - 2)(\tilde{c}'_L)^{-\alpha'} \right]^{\frac{\beta'}{\alpha'}} + \left[ (c_{k+}^S)^{-\alpha'} + (c^N)^{-\alpha'} + (K - 2)(\tilde{c}'_L)^{-\alpha'} \right]^{\frac{\beta'}{\alpha'}} \end{aligned}$$

If  $\alpha < 0$ , the inverse inequalities apply,  $(c_{k+}^S)^{-\alpha'} < (c^*)^{-\alpha'} < (c^N)^{-\alpha'}$ , but the  $g(x)$  function becomes concave leading to the same overall inequality.

Multiplying both sides by  $(1 - \beta)K^{-\beta'/\alpha}$  and adding  $2f + 2I'_L f$ , we can rewrite

the inequality as

$$\begin{aligned}
\pi(N, N, M'_L)|_{z_k=z_k^*} + \pi(S, S, M'_L)|_{z_{k+1}=z_k^*} &> \pi(S, N, M'_L)|_{z_k=z_k^*} + \pi(S, N, M'_L)|_{z_{k+1}=z_k^*} \\
\pi(N, N, M_L)|_{z_k=z_k^*} + \pi(S, S, M'_L)|_{z_{k+1}=z_k^*} &> \pi(S, N, M'_L)|_{z_k=z_k^*} + \pi(S, N, M'_L)|_{z_{k+1}=z_k^*} \\
\pi(S, S, M'_L)|_{z_{k+1}=z_k^*} &> \pi(S, N, M'_L)|_{z_{k+1}=z_k^*}
\end{aligned}$$

The substitution in the second line strengthens the inequality, as indicated with the definition of  $z_k^*$ . The definition of  $z_k^*$  itself allows the elimination of the first terms on both sides altogether. Hence we find that the sufficient condition holds with strict inequality.

In other words, when  $z_{k+1} = z_k^*$ , producing part  $k + 1$  in South generates higher profit than producing it at North. This must mean that the actual cutoff value  $z_{k+1}^* \leq z_k^*$ . ■

