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A Nonstructural Analysis"

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The Macroeconomy and the Yield Curve:
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Abstract: We estimate a model with latent factors that summarize the yield curve (namely, level, slope, and curvature) as well as observable macroeconomic variables (real activity, inflation, and the stance of monetary policy). Our goal is to provide a characterization of the dynamic interactions between the macroeconomy and the yield curve. We find strong evidence of the effects of macro variables on future movements in the yield curve and much weaker evidence for a reverse influence. We also relate our results to a traditional macroeconomic approach based on the expectations hypothesis.

Key Words: Yield curve, term structure, interest rates, macroeconomic fundamentals, factor model, state-space model

JEL Codes: G1, E4, C5

1. Introduction

Macroeconomists, financial economists, and market participants all have attempted to build good models of the yield curve, yet the resulting models are very different in form and fit. In part, these differences reflect the particular modeling demands of various researchers and their different motives for modeling the yield curve (e.g., interest rate forecasting or simulation, bond or option pricing, or market surveillance). Still, an unusually large gap is apparent between the yield curve models developed by macroeconomists, which focus on the role of expectations of inflation and future real economic activity in the determination of yields, and the models employed by financial economists, which eschew any explicit role for such determinants. This paper takes a step toward bridging this gap by formulating and estimating a yield curve model that integrates macroeconomic and financial factors.

Many other recent papers have also modeled the yield curve, and they can be usefully categorized by the extent and nature of the linkages permitted between financial and macroeconomic variables. First, there are many yield curve models that ignore any macroeconomic linkage. Foremost among these are the very popular factor models that dominate the finance literature–especially those that impose a no-arbitrage restriction. For example, Knez, Litterman, and Scheinkman (1994), Duffie and Kan (1996), and Dai and Singleton (2000) all consider models in which a handful of unobserved factors explain the entire set of yields. These factors often have interpretations in terms of level, slope, and curvature, but they are not linked explicitly to the macroeconomy.

Our analysis also uses a latent factor model of the yield curve; however, we explicitly incorporate macroeconomic factors into the model as well. In this regard, our work is more closely related to Ang and Piazzesi (2003), Hördahl, Tristani, and Vestin (2002), and Wu (2002), who explicitly incorporate macro determinants into multi-factor yield curve models. However, those papers only consider a unidirectional macro linkage because output and inflation are assumed to be determined independently of the shape of the yield curve, but not vice versa. This same assumption is made in the vector autoregression (VAR) analysis of Evans and Marshall (1998, 2001) where neither contemporaneous nor lagged values of the bond yields enter the equations driving the economy. In contrast to this assumption of a one-way macro-to-yields link, the opposite view is taken in another large literature typified by Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998), which assumes a yields-to-macro link and focuses only on the unidirectional predictive power of the yield curve for the economy. The two assumptions of these literatures–one-way yields-to-macro or macro-to-yields links–are testable hypotheses that are special cases of our model and are examined below. Indeed, we are particularly interested in analyzing the potential bidirectional feedback from the yield curve to the economy and back again. Some of the work closest to our own allows a feedback from an implicit inflation target derived from the yield curve to help determine the dynamics of the macroeconomy, such as Kozicki and Tinsley

(2001), Dewachter and Lyrio (2002), and Rudebusch and Wu (2003). In our analysis, we allow for a more complete set of interactions in a general nonstructural framework.

Our basic framework for the yield curve is a latent factor model, although not the usual no-arbitrage factor representation typically used in the finance literature. Such no-arbitrage factor models often appear to fit the cross-section of yields at a particular point in time, but they do less well in describing the dynamics of the yield curve over time (e.g., Duffee, 2002; Brousseau, 2002). Such a dynamic fit is crucial to our goal of relating the evolution of the yield curve over time to movements in macroeconomic variables. To capture yield curve dynamics, we use a three-factor term structure model based on the classic contribution of Nelson and Siegel (1987), interpreted as a model of level, slope, and curvature, as in Diebold and Li (2002). This model has the substantial flexibility required to match the changing shape of the yield curve, yet it is parsimonious and easy to estimate. We do not explicitly enforce the no-arbitrage restriction. However, to the extent that it is satisfied in the data – as is likely for many but not necessarily all of the U.S. Treasury bill and bond obligations that we study – it will also likely be approximately satisfied in our estimates, as our model is quite flexible.

In Section 2, we describe and estimate a basic "yields-only" version of our yield curve model – that is, a model without macroeconomic variables included. To estimate this model, we introduce a unified state space modeling approach that lets us simultaneously fit the yield curve at each point in time and estimate the underlying dynamics of the factors. This one-step approach improves upon the two-step estimation procedure of Diebold and Li (2002) and provides a unified framework to examine the yield curve and the macroeconomy.

In section 3, we incorporate macroeconomic variables into our framework. To complement the nonstructural nature of our yield curve representation, we also use a simple nonstructural VAR representation of the macroeconomy. The focus of our examination is the nature of the linkages between the factors driving the yield curve and macroeconomic fundamentals.

In section 4, we relate our framework to the expectations hypothesis, which has been studied intensively in macroeconomics. Our framework is different, but closely related, as we make clear.

We offer concluding remarks in section 5.

2. A Yield Curve Model Without Macro Factors

In this section, we introduce a factor model of the yield curve without macroeconomic variables, which is useful for at least two reasons. First, methodologically, it proves to be a convenient vehicle for introducing the state space framework that we use throughout the paper. Second, and substantively, the estimated "yields-only" model serves as a useful benchmark to which we compare our full model that

incorporates macroeconomic variables.

2.1 A Factor Model Representation

        The factor model approach expresses a potentially large set of yields of various maturities as a function of just a small set of unobserved factors. Denote the set of yields as $y(\tau)$, where τ denotes maturity. Among practitioners and especially central banks,[1] a very popular representation of cross-section of yields at any point in time is the Nelson and Siegel (1987) curve:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}-e^{-\lambda\tau}\right),$$ (1)

where β_1, β_2, β_3 **and** λ are parameters.[2] Moreover, as shown by Diebold and Li (2002), the Nelson-Siegel representation can be interpreted in a dynamic fashion as a latent factor model in which β_1, β_2, and β_3 are time-varying level, slope, and curvature factors.[3] Then, the terms that multiply these factors can be interpreted as factor loadings. Thus, we write

$$y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}-e^{-\lambda\tau}\right),$$ (2)

where L_t, S_t, and C_t are the time-varying β_1, β_2, and β_3. We illustrate this interpretation with our empirical estimates below.

        If the dynamics of L_t, S_t, and C_t follow a vector autoregressive process of first order, then the model immediately forms a state-space system.[4] The transition equation, which governs the dynamics of the state vector, is

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[1] As described in documentation from the Bank for International Settlements (1999), many central banks have adopted the Nelson-Siegel yield curve (or some slight variant) for fitting bond yields.

[2] Our Nelson-Siegel yield curve equation (1) corresponds to equation (2) of Nelson Siegel (1987). Their notation differs from ours in a potentially confusing way: they use m for maturity and $1/\tau$ for the constant λ.

[3] More precisely, Diebold and Li show that β_2 and hence S_t actually correspond to the *negative* of slope as traditionally defined ("long minus short yields"). For ease of discussion, we prefer simply to call β_2 and hence S_t "slope," so we define slope as "short minus long."

[4] As is well-known, ARMA state vector dynamics of any order may be readily accommodated in state space form. We maintain the VAR(1) assumption only for transparency and parsimony.

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix} , \tag{3}$$

$t = 1, \ldots, T$. The measurement equation, which relates a set of N yields to the three unobservable factors, is

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \dfrac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \dfrac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \dfrac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} & \dfrac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ \vdots & \vdots & \vdots \\ 1 & \dfrac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} & \dfrac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{pmatrix} , \tag{4}$$

$t = 1, \ldots, T$. In an obvious vector/matrix notation, we write the state space system as

$$(f_t - \mu) = A (f_{t-1} - \mu) + \eta_t \tag{5}$$

$$y_t = \Lambda f_t + \varepsilon_t. \tag{6}$$

For linear least squares optimality of the Kalman filter, we require that the white noise transition and measurement disturbances be orthogonal to each other and to the initial state:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] \tag{7}$$

$$E(f_0 \eta_t') = 0 \tag{8}$$

$$E(f_0 \varepsilon_t') = 0 . \tag{9}$$

In much of our analysis, we assume that the Q and H matrices are diagonal. The assumption of a diagonal

H matrix, which implies that the deviations of yields of various maturities from the yield curve are uncorrelated, is quite standard. For example, in estimating the no-arbitrage term structure models, i.i.d. "measurement error" is added to the observed yields. This assumption is also required for computational tractability given the large number of observed yields used. However, the assumption of a diagonal *Q* matrix, which implies that shocks to the term structure factors are uncorrelated, is less common and is examined below.

2.2 Yields-Only Model Estimation

We examine U.S. Treasury yields with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months. Our data are derived from bid/ask average price quotes, from January 1972 through December 2000 using the well-known Fama-Bliss (1987) approach, as described in Diebold and Li (2002). The yields are measured as of the beginning of each month; this timing convention is immaterial for the yields-only model but will be more important when macro variables are introduced.

As discussed above, the yields-only model forms a state-space system, with a VAR(1) transition equation summarizing the dynamics of the vector of the latent state variables, and a linear measurement equation relating the observed yields to the state vector. Several parameters must be estimated. The (3x3) transition matrix *A* contains 9 free parameters, the (3x1) mean state vector $\boldsymbol{\mu}$ contains 3 free parameters, and the measurement matrix $\boldsymbol{\Lambda}$ contains 1 free parameter, $\boldsymbol{\lambda}$. Moreover, the transition and disturbance covariance matrix *Q* contains 3 free parameters (one disturbance variance for each of the three latent level, slope and curvature factors), and the measurement disturbance covariance matrix *H* contains 17 free parameters (one disturbance variance for each of the 17 yields). All told, then, 33 parameters must be estimated by numerical optimization – a very challenging, but not insurmountable, numerical task.

For a given parameter configuration, we use the Kalman filter to compute optimal yield predictions and the corresponding prediction errors, after which we proceed to evaluate the Gaussian likelihood function of the yields-only model using the prediction-error decomposition of the likelihood. We initialize the Kalman filter using the unconditional mean (zero) and unconditional covariance matrix of the state vector.[5] We maximize the likelihood by iterating the Marquart and Berndt-Hall-Hall-Hausman algorithms, using numerical derivatives, optimal stepsize, and a convergence criterion of 10^{-6} for the change in the norm of the parameter vector from one iteration to the next. We impose non-negativity on all estimated variances by estimating log variances; then we convert to variances by

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[5] For details of Kalman filtering and related issues such as initialization of the filter, see Harvey (1981) or Durbin and Koopman (2001).

exponentiating and compute asymptotic standard errors using the delta method. We obtain startup parameter values as follows: we use the Diebold-Li two-step methods to obtain the initial transition equation matrix, we initialize all variances at 1.0, and we initialize λ at the value given in Diebold and Li (2002).

In Table 1 we present estimation results for the yields-only model. The estimate of the A matrix indicates highly persistent own dynamics of L_t, S_t, and C_t, with estimated own-lag coefficients of .99, .94 and .83, respectively. Cross-factor dynamics appear unimportant, with the exception of a minor but statistically significant effect of S_{t-1} on L_t. The estimates also indicate that persistence decreases (as measured by the diagonal elements of A), and transition shock volatility increases (as measured by the diagonal elements of Q), as we move from L_t to S_t to C_t. The remaining estimates appear sensible; the mean level is approximately 8 percent, the mean slope is approximately -1.5 percent, and the mean curvature is insignificantly different from 0.[6] Finally, the estimated λ of .077 implies that the loading on the curvature factor is maximized at a maturity of 23.3 months.

The yields-only model fits the yield curve remarkably well. The first column of Table 2 contains estimated measurement error standard deviations (i.e., the square roots of the estimated diagonal elements of H), expressed in basis points, for each of the 17 maturities that we consider.[7] At the crucial middle range of maturities from 6 to 60 months, the average pricing error is just 10 basis points or so. The average pricing error increases at very short and very long maturities but nevertheless remains quite small.

As an additional check of model adequacy, we also tried a four-factor extension,

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \beta_{4t}\left(\frac{1-e^{-2\lambda\tau}}{2\lambda\tau}\right), \tag{10}$$

and a five-factor extension,

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{\tau}{2}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{4t}\left(\frac{1-e^{-\lambda\tau}}{\lambda^2\tau} - \frac{e^{-\lambda\tau}}{\lambda}\right) + \beta_{5t}\left(\frac{1-e^{-2\lambda\tau}}{2\lambda\tau}\right), \tag{11}$$

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[6] Recall that we define slope as short minus long, so that a negative mean slope means that yields tend to increase as maturity lengthens.

[7] That is, the table contains square roots of our estimates of the diagonal elements of the H matrix, times 10,000 (multiplication by 100 to express in percent, followed by multiplication by 100 again to express in basis points).

as in Björk and Christensen (1999). The extended models, however, provide negligible improvement in model fit.[8] This lack of improved fit is clear from a comparison of the first column of Table 2 to the third and fourth columns, in which we report measurement error standard deviations for the 4- and 5-factor models. For example, in pricing bonds with maturities ranging from 9 months to 84 months, the measurement error standard deviations for the 4- and 5-factor models are only about one or two basis points smaller than those of the 3-factor model.

We use the Kalman smoother to obtain optimal extractions of the latent level, slope and curvature factors. We display them in various ways. In Figure 1, we plot the three estimated factors together for comparative assessment, and in Figures 2, 3, and 4 we plot the respective factors in isolation of each other, but together with various empirical proxies and potentially related macroeconomic variables.[9]

In Figure 1, we see that the level factor displays very high persistence and is of course positive, in the neighborhood of 8 percent. In contrast, the slope and curvature are less persistent and assume both positive and negative values. The unconditional variances of the slope and curvature factors are roughly equal but are composed differently: slope has higher persistence and lower shock variance, whereas curvature has lower persistence and higher shock variance. Interestingly, slope and curvature appear highly related at business cycle frequencies; perhaps they are better interpreted as one factor ("slorvature"?). The simple correlation is 0.25, but the correlation at business cycle frequencies is 0.5 and at very high frequencies is -0.07.[10]

In Figure 2 we show the estimated level and two closely-linked comparison series: a common empirical proxy for level (namely, an average of short-, medium- and long-term yields, $(y_t(3) + y_t(24) + y_t(120))/3$), and a measure of inflation (the 12-month percent change in the price deflator (P_t) for personal consumption expenditures, namely, $100*(P_t - P_{t-12})/P_{t-12}$). The high correlation between $\hat{L}_t$ and $(y_t(3) + y_t(24) + y_t(120))/3$, which is .80, lends credibility to our interpretation of L_t as a level factor. The correlation between $\hat{L}_t$ and actual inflation, which is .43, is consistent with a link between the level of the yield curve and inflationary expectations, as suggested by the Fisher equation. This link is a common theme in the recent macro-finance literature, including Kozicki and Tinsley (2001), Dewachter and Lyrio (2002), Hördahl, Tristani, and Vestin (2002), and Rudebusch and Wu (2003).

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[8] These results are consistent with Dahlquist and Svensson (1994) who compare the Nelson and Siegel model with a more complex functional form and find no improvement to using the latter.

[9] In each case, the latent factors based on full-sample parameter estimates.

[10] More precisely, the correlation between Hodrick-Prescott "trends" in β_2 and β_3 is 0.55, and the correlation between the deviations from "trends" in β_2 and β_3 is -0.07.

In Figure 3 we show the estimated slope and two comparison series, to which the slope factor closely linked: a standard empirical slope proxy ($y_t(3) - y_t(120)$), and an indicator of macroeconomic activity (demeaned capacity utilization). The high correlation between $\hat{S}_t$ and $y_t(3) - y_t(120)$ (.98) lends credibility to our interpretation of S_t as a slope factor. The correlation between $\hat{S}_t$ and capacity utilization (.39) suggests that yield curve slope, like yield curve level, is intimately connected to the cyclical dynamics of the economy.

Finally, in Figure 4 we show the estimated curvature together with a standard empirical curvature proxy, $2y_t(24) - y_t(3) - y_t(120)$, to which C_t is closely linked with a correlation of .96, which again lends credibility to our interpretation of C_t as a curvature factor. Unfortunately, as shown in our combined yields-macro model in the next section, we know of no reliable macroeconomic links to C_t.

3. A Yield Curve Model With Macro Factors

Given the ability of the level, slope, and curvature factors to provide a good representation of the yield curve, it is of interest to relate them to macroeconomic variables. This can be done readily in an expanded version of the above state-space framework. In the next five subsections, we analyze the dynamic interactions between the macroeconomy and the yield curve and assess their importance.

3.1 The Yields-Macro Model: Specification and Estimation

We wish to characterize the relationships among $\hat{L}_t$, $\hat{S}_t$, $\hat{C}_t$ and the macroeconomy. Our measures of the economy include three key variables: manufacturing capacity utilization (CU_t), the federal funds rate (FFR_t), and annual price inflation ($INFL_t$).[11] These three variables represent, respectively, the level of real economic activity relative to potential, the monetary policy instrument, and the inflation rate, which are widely considered to be the minimum set of fundamentals needed to capture basic macroeconomic dynamics.[12]

A straightforward extension of the yields-only model adds the three macroeconomic variables to the set of state variables and replace equations (5)-(7) with

$$(f_t - \mu) = A (f_{t-1} - \mu) + \eta_t \tag{5$'$}$$

$$y_t = \Lambda f_t + \varepsilon_t \tag{6$'$}$$

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[11] The variable *INFL* is the 12-month percent change in the price deflator for personal consumption expenditures, and *FFR* is the monthly average funds rate.

[12] See, for example, Rudebusch and Svensson (1999) and Kozicki and Tinsley (2001)

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] , \qquad (7')$$

where $f_t' = (L_t, S_t, C_t, CU_t, FFR_t, INFL_t)$ and the dimensions of A, μ, Λ, η_t and Q are increased as appropriate. This system forms our "yields-macro" model, to which we will compare our earlier "yields-only" model.[13] Our baseline yields-macro model continues to assume the independence of the structural errors, so the expanded (6x6) transition shock covariance matrix Q remains diagonal. This will be important given our focus on the nature of the interactions between macroeconomic and financial variables. Unlike the usual types of identification strategies adopted in macroeconomic models–notably the common Cholesky factorization which assumes a contemporaneous recursive structure–the baseline yields-macro model is completely neutral with respect to the ordering of the variables. However, we will examine the robustness of our results to an alternative identification strategy.

In Table 3 we display the estimates of the parameters of the yields-macro model, which contains the crucial macro and term structure interactions. Individually, many of the off-diagonal elements appear insignificant; however, as we discuss below, key blocks of coefficients appear jointly significant. In particular, there is evidence of a block linkage from the macroeconomy to the yield curve, but not conversely.

The time series of estimates of the level, slope, and curvature factors in the yields-macro model are very similar to those obtained in the yields-only model. Thus, as shown in the second column of Table 2, the measurement error volatilities associated with the yields-macro model are essentially identical to those of the yields-only model , with volatilities of approximately 10 basis points for the 6- to 96-month yields.

3.2 Macroeconomic and Yield Curve Impulse Response Functions

We examine the dynamics of the complete yields-macro system by via impulse response functions, which we show in Figure 5, along with ninety percent confidence intervals. We will consider four groups of impulse responses in turn: macro responses to macro shocks, macro responses to yield curve shocks, yield curve responses to macro shocks, and the own response of the yield curve.

The responses of the macro variables to macro shocks match the typical impulse responses

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[13] We have maintained a first-order VAR structure for simplicity and tractability; however, based on some limited exploration of second-order models, it appears that our results are robust to this assumption as well.

produced in small estimated macro models of the kind commonly used in monetary policy analysis (e.g., Rudebusch and Svensson, 1999). The macro variables all show significant persistence.[14] In addition, an increase in the funds rate depresses capacity utilization over the next few years, similar to the aggregate demand response in Rudebusch and Svensson (1999). The funds rate, in turn, rises with capacity utilization and–with only marginal significance–with inflation in a fashion broadly consistent with an estimated Federal Reserve monetary policy reaction function. Finally, inflation exhibits a clear aggregate supply response to increased capacity utilization and, over time, declines in response to a funds rate increase.[15]

The yield curve components add some interesting elements to the macro responses. The macro variables have negligible responses to shocks in the slope or curvature factors; however, an increase in the level factor does raise capacity utilization, the funds rate, and inflation. Recall from Figure 2 the close connection between inflation and the level factor. The macro responses exhibited in Figure 5 are consistent with the above interpretation of the level factor as the bond market's perception of long-run inflation. Under this interpretation, an increase in the level factor–that is, an increase in future perceived inflation–lowers the ex ante real interest rate when measured as $i_t - L_t$, which is followed by a near-term economic boom.[16] However, during our sample the Fed has apparently accommodated only a small portion of the expected rise in inflation. The nominal funds rate rises significantly in response to the level shock, damping utilization, and limiting the rise in inflation to only about a quarter of the initial shock to the level.

Now consider the response of the yield curve to the macro variables. While the curvature factor shows very little response, the slope factor responds directly to positive shocks in all three macro variables. For example, an increase in the funds rate almost immediately pushes up the slope factor so the yield curve is less positively sloped (or more negatively sloped). Positive shocks to utilization, and to a lesser extent inflation, also induce similar though more delayed movements in the tilt. These reactions are

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[14] The interpretation of the persistence of FFR—the policy rate manipulated by the Fed—is open to some debate. However, Rudebusch (2002) argues that it does not indicate "interest rate smoothing" or "monetary policy inertia"; instead, it reflects serially correlated unobserved factors to which the Fed responds.

[15] There is a marginally significant initial upward response of inflation to the funds rate – a so-called "price puzzle" – which is typical in small VARs of this kind.

[16] Another common measure of the real rate, an ex post $i_t - INFL_t$, does not appear to be appropriate, as a positive shock to inflation does not boost economic activity (see Rudebusch and Svensson 1999).

consistent with a monetary policy response that raises the short end of the term structure in response to positive output and inflation surprises. However, shocks to the macro variables also affect the level of the term structure. In particular, surprises to actual inflation appear to give a long-run boost to the level factor. Such a reaction is consistent with long-inflation expectations not being firmly anchored, so a surprise increase in inflation (or even in real activity) feeds through to a expectations of higher future inflation, which raises the level factor.[17] A positive shock to the funds rate is also followed by a small temporary jump in the level factor. In principle, a surprise increase in the monetary policy rate could have two quite different effects on inflation expectations. On the one hand, if the central bank has a large degree of credibility and transparency, then a tightening could indicate a lower inflation target and a likely lowering of the level factor. Alternatively, a surprise tightening could indicate that the central back is worried about overheating and inflationary pressures in the economy–news that would boost future inflation expectations and the level factor. Evidently, over our sample, the later effect has dominated.

Finally, consider the block of own-dynamics of the term structure factors. The three factors exhibit significant persistence. In addition, a shock to the slope factor, which raises the short end of the yield curve relative to the long end, is associated with an increase in the overall level of rates. If such a slope movement is associated with a surprise monetary tightening, then a drop in the future inflation level might appear warranted. Conversely, a surprise increase in the level factor, which we interpret as higher inflation expectations, is associated with loosening of policy as measured by the slope factor and a lowering of the short end of the term structure relative to the long end.[18]

3.3 Macroeconomic and Yield Curve Variance Decompositions

Variance decompositions provide a popular metric for analyzing macro and yield curve interactions. Table 4 provides variance decompositions of the 1-month, 12-month, and 60-month yields at forecast horizons of 1, 12, and 60 months. Decompositions are provided for both the yields-only and the yields-macro models. At a 1-month horizon, very little of the variation in rates is driven by the macro factors (14, 8, and 3 percent for the 1-month, 12-month, and 60-month yield, respectively). This suggests a large amount of short-term idiosyncratic variation in the yield curve that is unrelated to macroeconomic fundamentals. However, at longer horizons, the macro factors quickly become more influential, and at a

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[17] Gurkaynak, Sack, and Swanson (2003) and Rudebusch and Wu (2003) discuss such a mechanism.

[18] Overall then, in important respects, this analysis improves on the usual monetary VAR, which contains a flawed specification of monetary policy (Rudebusch, 1998). In particular, the use of level, slope, and the funds rate allows a much more subtle and flexible description of policy.

60-month horizon, they account for about half (46 percent to 62 percent) of the variation in rates. This contribution is similar to the results in Ang and Piazzesi's (2003) macro model.

Table 5 examines the variance decompositions for the macroeconomic variables based on the joint yields-macro model and a "macro-only" model, which is a simple first-order VAR for CU, FFR, INFL. In both models, the macro variables dominate the term structure factors in accounting for the variation in the macro variables.

Taken together, the variance decompositions suggest that the effects of the yield curve on the macro variables are much less important than the effects of the macro variables on the term structure. To correctly interpret this result, it is important to note that an interest rate—the federal funds rate—is also included among the macro variables. That is, we are asking what would the yield curve add to a standard small macro model, such as the Rudebusch-Svensson (1999) model. We are not arguing that interest rates do not matter, but that, for our specification and sample, the funds rate is perhaps, to a rough approximation, a sufficient statistic for interest rate effects in macro dynamics, which is a conclusion consistent with Ang, Piazzesi, and Wei (2003).

3.4 Formal Tests of Macro and Yield Curve Interactions

As noted above, the coefficient matrix A shown in Table 3 is crucial for assessing interactions between macroeconomic variables and the term structure, particularly under our assumption of diagonal Q. Partition the 6x6 A matrix into four 3x3 blocks, as

$$A = \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}. \tag{12}$$

We are interested in a variety of cases, in particular $A_3 = 0$ (block exogeneity of the term structure relative to the macro variables), $A_2 = 0$ (block exogeneity of the macro variables relative to the term structure), and $A_2 = A_3 = 0$ (block exogeneity of each for the other).

We report results of both likelihood ratio and Wald tests in Table 6. Both tests overwhelmingly reject the hypothesis that $A_2 = 0$; that is, there is strong evidence that macro variables do influence the term structure. The test statistics, which are distributed as $\chi^2(9)$ under the null hypothesis, have values greater than 100. In contrast, there is noticeably less evidence against the hypothesis of $A_3 = 0$; that is, there is less evidence of yield curve effects on macro variables. The likelihood ratio test statistic falls to less than 25, although one would still reject the null at conventional levels, and the Wald statistic falls to a mere 12.5, which is insignificant at conventional levels.

3.5 Robustness to an Alternative Identification Strategy

Our results above are based on the assumption of no contemporaneous correlation between the

shocks—that is, we assume that the matrices Q and H are diagonal. As noted above, the assumption of a diagonal H matrix is quite standard; however, the assumption of a diagonal Q matrix is much less common. Assumption of diagonal Q is convenient because it allows us to sidestep issues of structural identification and highlight macro-finance linkages without being tied to a particular contemporaneous causal ordering or other identification scheme, which have been so contentious in the so-called identified or structural VAR literature. However, it is useful to examine whether our results are robust to the assumption of such a causal ordering. Therefore, we estimate a version of the macro-yields model that allows for a non-diagonal Q matrix. The estimated Q matrix is provided in Table 7. Several of the off-diagonal covariances appear significant individually; in particular, the slope and curvature factors appear positively correlated with the federal funds rate. In addition, a Wald test clearly rejects the joint hypothesis that all covariances are zero. Still, the A matrix associated with this model is essentially no different from the one shown in Table 3 for the diagonal Q version.

Producing impulse responses from the non-diagonal Q version of the yields-macro model requires an identification of the covariances shown in Table 7. Following common practice, we do this by assuming a particular recursive causal ordering of the variables; namely, we order the variables L_t, S_t, C_t, CU_t, $INFL_t$, FFR_t. We order the term structure factors prior to the macro variables because they are dated at the beginning of each month.[19] In Figure 6 we show both point and interval estimates of the impulse responses of this non-diagonal Q version of the yields-macro model, along with the earlier-reported point estimates of the impulse responses from the diagonal Q version for comparison. With only one or two exceptions out of 36 impulse response functions, the differences are negligible. The same is of course also true for the variance decompositions, which we show for the non-diagonal Q model in Table 8, and which are very close to those reported earlier for the diagonal Q version.

4. Examining The Expectations Hypothesis

It is useful to contrast our representation of the yield curve with others that have appeared in the literature. Here we relate our yield curve modeling approach to the traditional macroeconomic approach based on the expectations hypothesis.

The expectations hypothesis of the term structure states that movements in long rates are due to movements in expected future short rates. Any term or risk premia are assumed to be constant through time. In terms of our notation above, which pertains to the pure discount bond yields in our data set, the theoretical bond yield consistent with the expectations hypothesis, can be defined as

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[19] We obtained similar results by using end-of-period yields data and ordering the variables as CU_t, $INFL_t$, FFR_t, L_t, S_t, C_t.

$$y_t(m)^{EH} \equiv (1/m) \sum_{i=0}^{m-1} E_t \, y_{t+i}(1) + c \, , \tag{13}$$

where c is a term premium—that is, some predictable excess return on the —period bond over the *1*-period bond—that may vary with the maturity *m* but assumed to be constant through time.

The expectations hypothesis has long been a key building block in macroeconomics both in casual inference and formal modeling (for example, Fuhrer and Moore 1995 or Rudebusch 1995). However, severe failures in the expectations hypothesis have been documented at least since Macaulay (1938). Campbell and Shiller (1991) and Fuhrer (1996) provide recent evidence on the failure of the expectations hypothesis, and we use their methodology to examine the expectations hypothesis in the context of our model. Specifically, we compare the theoretical bond yields, $y_t(m)^{EH}$, which are constructed according to equation (13) and assume that the expectations hypothesis is true, with the actual bond yields $y_t(m)$. The theoretical bond yields at each point in time are computed from the projected future path of the 1-month rate and equation (13). (The 1-month forward rates are obtained by iterating forward the estimated yields-macro model using equation (5') and the measurement equation (6') for $y_t(1)$.) These theoretical EH yields are shown along with the actual yields in Figure 7 for six particular maturities (*m* = 3, 12, 24, 36, 60, 120). As in Fuhrer (1996), over the whole sample, there are large deviations between the actual yields and the ones consistent with the expectations hypothesis—especially for the yields of longer maturity.[20]

Still, the actual and theoretical rates do appear to be generally moving in the same direction most of the time. Over the whole sample, the correlation between the actual 10-year and 1-month yield spread, $y_t(120) - y_t(1)$, and the EH theoretical spread, $y_t(120)^{EH} - y_t(1)$, is .60. Furthermore during certain periods, the actual and theoretical rates move very closely together. This partial success of the expectations hypothesis is consistent with the views of Fuhrer (1996) and Kozicki and Tinsley (2001), who argue that much of the apparent failure of the expectations hypothesis reflects the assumption of a constant Fed reaction function—and in particular a constant inflation target—over the entire sample. Indeed, the expectations hypothesis fits much better over the second half of the sample, when inflation expectations are often considered to better anchored.

5. Summary and Conclusions

We have specified and estimated a yield curve model that incorporates both yield factors (level, slope, and curvature) and macroeconomic factors (real activity, inflation, and the stance of monetary

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[20] Not surprisingly, a formal statistical test along the lines of Krippner (2002) rejects the restrictions placed by the expectations hypothesis on the yields-macro model.

policy). The model's convenient state-space representation facilitates estimation, the extraction of latent yield-curve factors, and testing of hypotheses regarding dynamic interactions between the macroeconomy and the yield curve. Interestingly, we find strong evidence of macroeconomic effects on the future yield curve and less-strong evidence of yield curve effects on future macroeconomic activity. Hence, although bi-directional causality is likely present, effects in the tradition of Ang and Piazzesi (2003) seem relatively more important than those in the tradition of Estrella and Hardouvelis (1991), Estrella and Mishkin (1998), and Stock and Watson (2000). Finally, we relate our yield curve modeling approach both to a traditional macroeconomic approach based on the expectations hypothesis. The results indicate that the expectations hypothesis may hold reasonably well during certain periods, but that it does not hold across the entire sample.

Appendix

Calculation of Impulse Response Functions and Variance Decompositions

In this appendix, we describe the computation of the impulse response functions (IRFs) and variance decompositions (VDs) for our models. All models in the paper can be written in VAR(1) form,

$$y_t = \mu + Ay_{t-1} + \varepsilon_t, \tag{A1}$$

where y_t is an $m \times 1$ vector of endogenous variables, μ is the constant vector and A is the $m \times m$ transition matrix. The residuals ε_t follow

$$\varepsilon_t \sim N(0, \Omega), \tag{A2}$$

where Ω is a (potentially non-diagonal) variance-covariance matrix. In order to find the IRFs and VDs, we must write the VAR(1) in moving average (MA) form. Letting I_m denote the $m \times m$ identity matrix, the unconditional mean of y is $c = (I_m - A)^{-1} \mu$, and we can write the system as

$$(y_t - c) = \mu + A(y_{t-1} - c) + \varepsilon_t. \tag{A3}$$

Assuming that A satisfies the conditions for covariance stationarity, we can write the MA representation of the VAR as

$$(y_t - c) = \psi_0 \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \dots = \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i}, \tag{A4}$$

where $\psi_0 = I_m$ and $\psi_i = A^i$ for $i = 1, 2, \dots$

A.1. Impulse-Response Functions

We define the IRF of the system as the responses of the endogenous variables to one unit shocks in the residuals. One would often see responses to a "one standard deviation" shock instead of the "one unit" shock that we use. Because all the variables we use in our analysis are in percentage terms we find it more instructive to report results from one percentage point shocks to the residuals.

In most applied work involving VARs and IRFs, the variance-covariance matrix of the residuals are not diagonal and one would use a method to transform the residuals such as the Cholesky

decomposition. As a result, the residuals of the new system do not have a clear meaning because they are linear combinations of the original residuals. In our main model, because $\boldsymbol{\Omega}$ is already diagonal by assumption we don't need to transform our system and therefore we can interpret the IRFs as being responses to the original shocks. We summarize the procedure in the next subsection. Since we also consider the extension of our main model to a general $\boldsymbol{\Omega}$, in the following subsection we show how IRFs can be computed in that framework.

A.1.1 Diagonal $\boldsymbol{\Omega}$

The response of y_t^i to a one unit shock to ε_{t-k}^j is

$$\frac{\partial y_t^i}{\partial \varepsilon_{t-k}^j} = (\boldsymbol{\psi}_k)_{ij}, \tag{A5}$$

where $(\boldsymbol{\psi}_k)_{ij}$ is the *(i,j)* element of the corresponding matrix.

To compute the asymptotic standard errors of the IRFs, we follow Lütkepohl (1990). Let $\boldsymbol{\alpha} = vec(\boldsymbol{A})$ and $\boldsymbol{\sigma} = vec(\boldsymbol{\Omega})$. Suppose that

$$\sqrt{T} \begin{bmatrix} \hat{\alpha} - \alpha \\ \hat{\sigma} - \sigma \end{bmatrix} \to N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_\alpha & 0 \\ 0 & \Sigma_\sigma \end{pmatrix} \right]. \tag{A6}$$

Then the asymptotic distribution of the IRFs can be derived using

$$\sqrt{T} \, vec(\hat{\boldsymbol{\psi}}_k - \boldsymbol{\psi}) \to N\left(0, G_k \Sigma_\alpha G_k'\right), \tag{A7}$$

where

$$G_k = \frac{\partial vec(\boldsymbol{\psi}_k)}{\partial \alpha'} = \sum_{i=0}^{k-1} (A')^{k-1-i} \otimes \boldsymbol{\psi}_i. \tag{A8}$$

A.1.2 Non-Diagonal $\boldsymbol{\Omega}$

As mentioned above, when $\boldsymbol{\Omega}$ is non-diagonal, we cannot compute the IRFs using the original residuals, $\boldsymbol{\varepsilon}_t$. We use Cholesky decomposition to obtain a lower-triangular matrix P that satisfies $\boldsymbol{\Omega} = PP'$ to defining $v_t = P^{-1}\boldsymbol{\varepsilon}_t$. While in the case where $\boldsymbol{\Omega}$ is non-diagonal the ordering of the variables did not matter, the matrix P and therefore the IRFs are sensitive to the choice of the ordering of the variables. The new residuals now have a diagonal variance covariance matrix, $E\left(v_t v_t'\right) = P^{-1}\boldsymbol{\Omega}P^{-1'} = P^{-1}PP'P^{-1'} = I_m$. The response of y_t to a one unit shock to v_{t-k}^j is

$$\frac{\partial y_t}{\partial v_{t-k}^j} = \frac{(\psi_k)p_j}{\sqrt{\Omega_{jj}}}, \tag{A9}$$

where p_j is the jth column of P and Ω_{jj} is the (j,j) element of the original variance covariance matrix.

To compute the asymptotic standard errors of the IRFs, assume (A6) holds where now $\sigma = vech(\Omega)$ and *vech* is an operator which stack only the elements of a matrix on or below the diagonal. We denote Θ_k the matrix where the (i,j) element shows the response of the ith variable in period t to a one standard deviation shock in the jth variable in period t-k, that is, the jth column of Θ_k is given by $(\psi_k)p_j$. Then the asymptotic distribution of Θ_k can be derived using

$$\sqrt{T} \, vec(\hat{\Theta}_k - \Theta) \rightarrow N(0, C_k \Sigma_\alpha C_k' + \bar{C}_k \Sigma_\sigma \bar{C}_k'), \tag{A10}$$

where

$$C_0 = 0, \quad C_k = (P' \otimes I_m) G_k, \quad k=1,2,... \tag{A11}$$

$$\bar{C}_k = (I_m \otimes \psi_k) H, \quad k=0,1,2... \tag{A12}$$

$$H = \frac{\partial vec(P)}{\partial \sigma'} = L_m' \{L_m(I_{m^2} + K_{mm})(P \otimes I_m) L_m'\}^{-1}. \tag{A13}$$

and L_m Is a matrix such that $vech(F) = L_k vec(F)$ and K_{mm} is a matrix such that $K_{mm} vec(G) = vec(G')$ and G_k is as defined in (A8).

A.2. Variance Decompositions

The contribution of the jth variable to the Mean Squared Error (MSE) of the s-period ahead forecast, under the assumption that Ω is diagonal is given by

$$MSE^j(s) = var(\varepsilon_t^j) \left[I_j I_j' + \psi_1 I_j I_j' \psi_1' + ... + \psi_{s-1} I_j I_j' \psi_{s-1}' \right], \tag{A14}$$

while for the case where Ω is non-diagonal it is given by

$$MSE^j(s) = \left[p_j p_j' + \psi_1 p_j p_j' \psi_1' + ... + \psi_{s-1} p_j p_j' \psi_{s-1}' \right], \tag{A15}$$

The MSE is given by

$$MSE(s) = \sum_{j=1}^{k} MSE^j(s), \tag{A16}$$

where both *MSE* and *MSE^j* are *m×m* matrices. We define the VD as the MSE of the ith variable due to the jth variable at horizon s. It is given by

$$VDC_i^j(s) = \frac{MSE_{ii}^j(s)}{MSE_{ii}(s)}, \tag{A17}$$

where *MSE$_{ii}$* and *MSE$_{ii}^j(s)$* denote the *(i,i)* elements of the respective matrices.

  To compute the VD of the yields, we combine the result above with the measurement equation for the yields,

$$m_\tau = \beta_1 + \alpha(\tau)\beta_2 + \gamma(\tau)\beta_3, \tag{A18}$$

where $\alpha(\tau)$ and $\gamma(\tau)$ follow from equation (1). The contribution of the jth variable to the *MSE* of the s-period ahead forecast of the yield at maturity τ is given by

$$M\tilde{S}E^j(s,\tau) = MSE_{11}^j(s) + \alpha(\tau)^2 MSE_{22}^j(s) + \gamma(\tau)^2 MSE_{33}^j(s) \tag{A19}$$
$$+ 2\alpha(\tau)MSE_{12}^j(s) + 2\gamma(\tau)MSE_{13}^j(s) + 2\alpha(\tau)\gamma(\tau)MSE_{23}^j(s),$$

where we assume that the factors β_1, β_2 and β_3 are the first three elements of the vector y. Similarly, the MSE of the s-period ahead forecast of the yield at maturity τ is given by

$$M\tilde{S}E(s,\tau) = \sum_{j=1}^{k} M\tilde{S}E^j(s,\tau), \tag{A20}$$

and the VD of the yield with maturity τ is the MSE of the yield due to the jth variable at horizon s:

$$V\tilde{D}C_i^j(s,\tau) = \frac{M\tilde{S}E_{ii}^j(s,\tau)}{M\tilde{S}E_{ii}(s,\tau)}. \tag{A21}$$

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Table 1
Yields-Only Model Parameter Estimates

	L_{t-1}	S_{t-1}	C_{t-1}	μ	Q_{ii}
L_t	**0.99**	**0.03**	-0.02	**8.04**	**0.10**
	(0.01)	(0.01)	(0.01)	(1.77)	(0.01)
S_t	-0.03	**0.94**	0.04	**-1.44**	**0.38**
	(0.02)	(0.02)	(0.02)	(0.62)	(0.02)
C_t	0.03	0.02	**0.83**	-0.40	**0.84**
	(0.03)	(0.02)	(0.03)	(0.52)	(0.07)

Notes: Each row presents coefficients from the transition equation for the respective state variable. Bold entries denote parameter estimates significant at the 5 percent level. Standard errors appear in parentheses.

Table 2
Standard Deviations of Measurement Errors of Yields

Maturity	Yields-Only	Yields-Macro	4-Factor	5-Factor
3	26.82	26.87	20.91	21.21
6	7.44	7.46	5.73	3.81
9	9.02	8.88	9.63	9.62
12	10.47	10.46	10.85	10.57
15	9.92	9.94	9.01	8.06
18	8.64	8.58	7.55	6.79
21	7.86	7.81	7.25	6.94
24	7.23	7.15	7.37	7.64
30	7.25	7.25	6.68	6.79
36	7.90	7.88	7.13	6.42
48	10.31	10.26	10.09	8.60
60	9.27	9.25	8.66	6.86
72	10.03	10.07	9.83	9.22
84	11.13	11.16	11.36	11.51
96	10.60	10.56	10.27	9.65
108	15.06	15.05	14.95	10.83
120	17.26	17.26	18.74	15.39

Notes: We report the standard deviations of the measurement errors, expressed in basis points, for yields of various maturities measured in months.

Table 3
Yields-Macro Model Parameter Estimates

	L_{t-1}	S_{t-1}	C_{t-1}	CU_{t-1}	FFR_{t-1}	$INFL_{t-1}$	μ	Q_{ii}
L_t	**0.89**	**-0.07**	-0.02	-0.01	**0.08**	0.02	**7.60**	**0.09**
	(0.03)	(0.04)	(0.02)	(0.01)	(0.03)	(0.02)	(1.13)	(0.01)
S_t	**-0.41**	**0.50**	0.03	**0.03**	**0.36**	0.03	**-1.48**	**0.30**
	(0.07)	(0.07)	(0.04)	(0.01)	(0.06)	(0.03)	(0.35)	(0.03)
C_t	0.17	0.15	**0.84**	0.01	-0.12	0.00	-0.43	**0.84**
	(0.12)	(0.12)	(0.04)	(0.02)	(0.10)	(0.04)	(0.61)	(0.08)
CU_t	0.09	0.00	0.00	**1.00**	-0.08	-0.02	**80.72**	**0.37**
	(0.08)	(0.08)	(0.03)	(0.01)	(0.07)	(0.02)	(1.17)	(0.03)
FFR_t	0.02	-0.06	0.02	**0.05**	**0.97**	0.05	**6.61**	**0.42**
	(0.08)	(0.09)	(0.03)	(0.02)	(0.06)	(0.05)	(1.30)	(0.03)
$INFL_t$	-0.04	-0.03	-0.01	**0.03**	0.04	**0.99**	**4.28**	**0.05**
	(0.03)	(0.03)	(0.01)	(0.00)	(0.02)	(0.01)	(1.43)	(0.00)

Notes: Each row presents coefficients from the transition equation for the respective state variable. Bold entries denote parameter estimates significant at the 5 percent level. Standard errors appear in parentheses.

Table 4
Variance Decompositions of Yields

	Horizon	L	S	C	CU	FFR	INFL
			1-Month Yield				
Yields-Only Model	1	0.22	0.78	0.00	--	--	--
	12	0.21	0.77	0.02	--	--	--
	60	0.20	0.77	0.03	--	--	--
Yields-Macro Model	1	0.23	0.62	0.00	0.00	0.14	0.00
	12	0.11	0.16	0.00	0.09	0.63	0.01
	60	0.23	0.08	0.07	0.35		0.03
			12-Month Yield				
Yields-Only Model	1	0.32	0.53	0.16	--	--	--
	12	0.33	0.60	0.08	--	--	--
	60	0.31	0.64	0.05	--	--	--
Yields-Macro Model	1	0.33	0.37	0.22	0.00	0.08	0.00
	12	0.23	0.11	0.10	0.07	0.49	0.01
	60	0.29	0.07	0.11	0.31	0.18	0.04
			60-Month Yield				
Yields-Only Model	1	0.67	0.13	0.20	--	--	--
	12	0.69	0.26	0.05	--	--	--
	60	0.55	0.43	0.02	--	--	--
Yields-Macro Model	1	0.67	0.06	0.23	0.00	0.03	0.00
	12	0.54	0.05	0.08	0.01	0.31	0.01
	60	0.37	0.07	0.11	0.25	0.15	0.06

Notes: Each entry gives the proportion of the forecast variance (at the specified forecast horizon) for a 1-, 12- or 60-month yield that is explained by the particular state variable.

Table 5
Variance Decompositions of Macroeconomic Variables

	Horizon	*L*	*S*	*C*	*CU*	*FFR*	*INFL*
				CU			
Macro-Only Model	1	--	--	--	1.00	0.00	0.00
	12	--	--	--	0.89	0.10	0.01
	60	--	--	--	0.61	0.27	0.12
Yields-Macro Model	1	0.00	0.00	0.00	1.00	0.00	0.00
	12	0.03	0.00	0.00	0.78	0.17	0.00
	60	0.05	0.01	0.02	0.55		0.08
				FFR			
Macro-Only Model	1	--	--	--	0.00	1.00	0.00
	12	--	--	--	0.04	0.95	0.01
	60	--	--	--	0.34	0.64	0.02
Yields-Macro Model	1	0.00	0.00	0.00	0.00	1.00	0.00
	12	0.04	0.01	0.00	0.10	0.83	0.01
	60	0.20	0.04	0.07	0.34	0.33	0.02
				INFL			
Macro-Only Model	1	--	--	--	0.00	0.00	1.00
	12	--	--	--	0.21	0.00	0.78
	60	--	--	--	0.53	0.26	0.21
Yields-Macro Model	1	0.00	0.00	0.00	0.00	0.01	0.98
	12	0.00	0.03	0.04	0.20	0.03	0.70
	60	0.03	0.02	0.08	0.48	0.19	0.20

Notes: Each entry gives the proportion of the forecast variance (at the specified forecast horizon) for capacity utilization, the funds rate or inflation that is explained by the particular state variable.

Table 6
Tests of Transition Matrix Restrictions

Statistic	$A_2 = 0$	$A_3 = 0$	$A_2 = 0$ and $A_3 = 0$
LR	101.94	24.85	123.81
	(0.00)	(0.00)	(0.00)
Wald	115.34	12.50	249.23
	(0.00)	(0.19)	(0.00)

Notes: p-values appear in parentheses.

Table 7
Estimated Q Matrix of Yields-Macro Model

	L_t	S_t	C_t	CU_t	FFR_t	$INFL_t$
L_t	**0.09**	-0.02	0.05	**0.04**	0.03	**0.01**
	(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.00)
S_t		**0.30**	0.01	**0.08**	**0.22**	-0.01
		(0.03)	(0.04)	(0.02)	(0.03)	(0.01)
C_t			**0.81**	0.04	**0.17**	0.00
			(0.09)	(0.04)	(0.04)	(0.01)
CU_t				**0.37**	**0.15**	**0.02**
				(0.03)	(0.04)	(0.01)
FFR_t					**0.45**	0.00
					(0.03)	(0.01)
$INFL_t$						**0.05**
						(0.00)

Notes: Bold entries denote parameter estimates significant at the 5 percent
level. Standard errors appear in parentheses.

Table 8

Variance Decompositions for Yields-Macro Model with Non-Diagonal Q

Horizon	L	S	C	CU	FFR	INFL
			1-Month Yield			
1	0.15	0.75	0.02	0.01	0.07	0.00
12	0.19	0.34	0.04	0.15	0.28	0.01
60	0.34	0.13	0.08	0.31	0.12	0.02
			12-Month Yield			
1	0.30	0.44	0.22	0.00	0.04	0.00
12	0.33	0.22	0.13	0.11		0.01
60	0.41	0.08	0.12	0.28	0.09	0.03
			60-Month Yield			
1	0.68	0.09	0.21	0.00	0.02	0.00
12	0.66	0.06	0.08	0.04	0.15	0.01
60	0.52	0.02	0.10	0.24	0.08	0.04
			CU			
1	0.05	0.06	0.00	0.89	0.00	0.00
12	0.10	0.03	0.02	0.73	0.12	0.01
60	0.12	0.05	0.05	0.50	0.18	0.10
			FFR			
1	0.03	0.37	0.06	0.04	0.50	0.00
12	0.16	0.23	0.04	0.17	0.39	0.01
60	0.32	0.10	0.09	0.31	0.16	0.02
			INFL			
1	0.02	0.00	0.00	0.04	0.00	0.94
12	0.04	0.00	0.03	0.29	0.01	0.63
60	0.09	0.05	0.11	0.44	0.11	0.20

Notes: Each entry gives the proportion of the forecast variance (at the specified forecast horizon) for a 1-, 12- or 60-month yield or for capacity utilization, the funds rate, or inflation that is explained by the particular state variable.

Figure 1
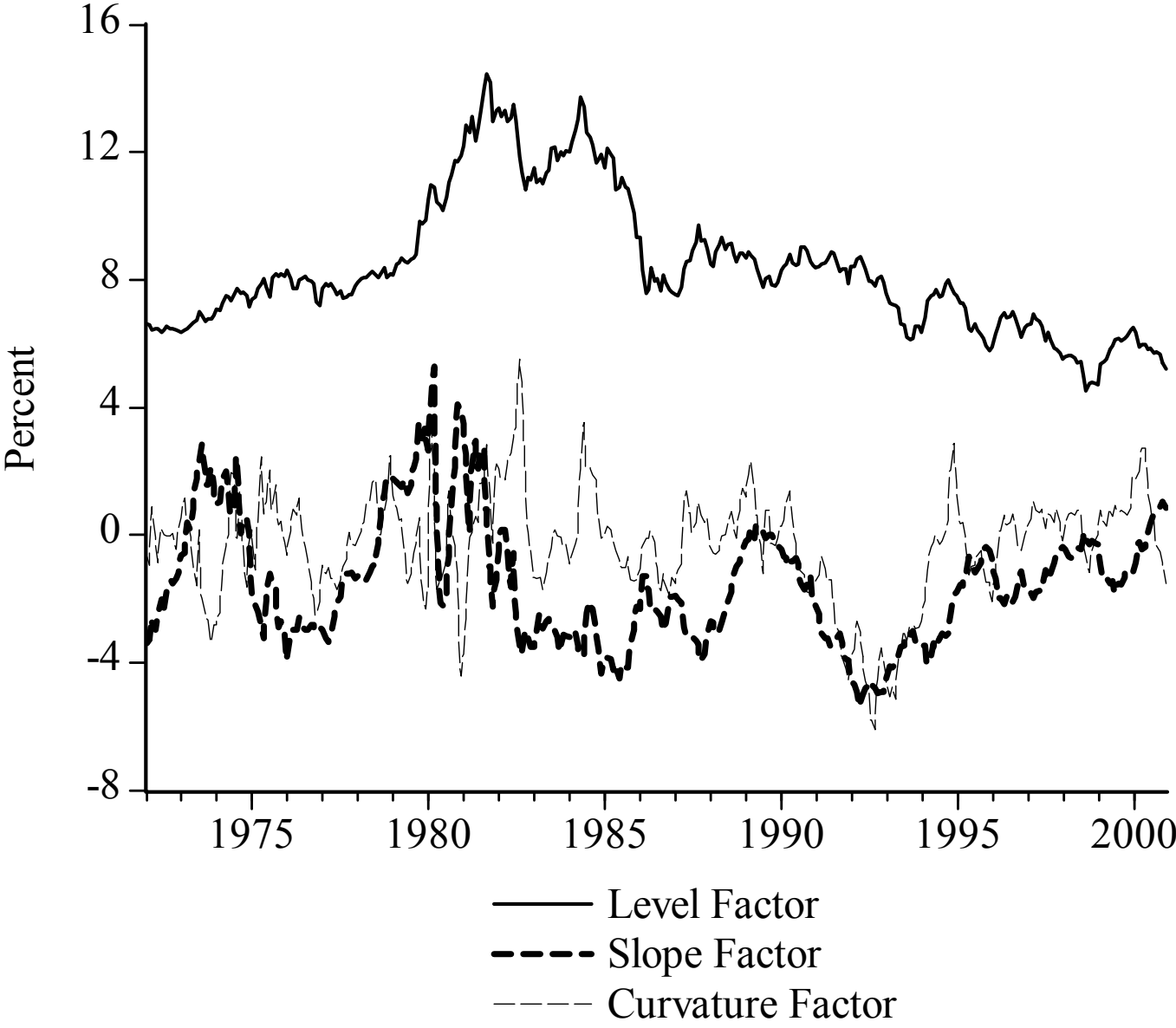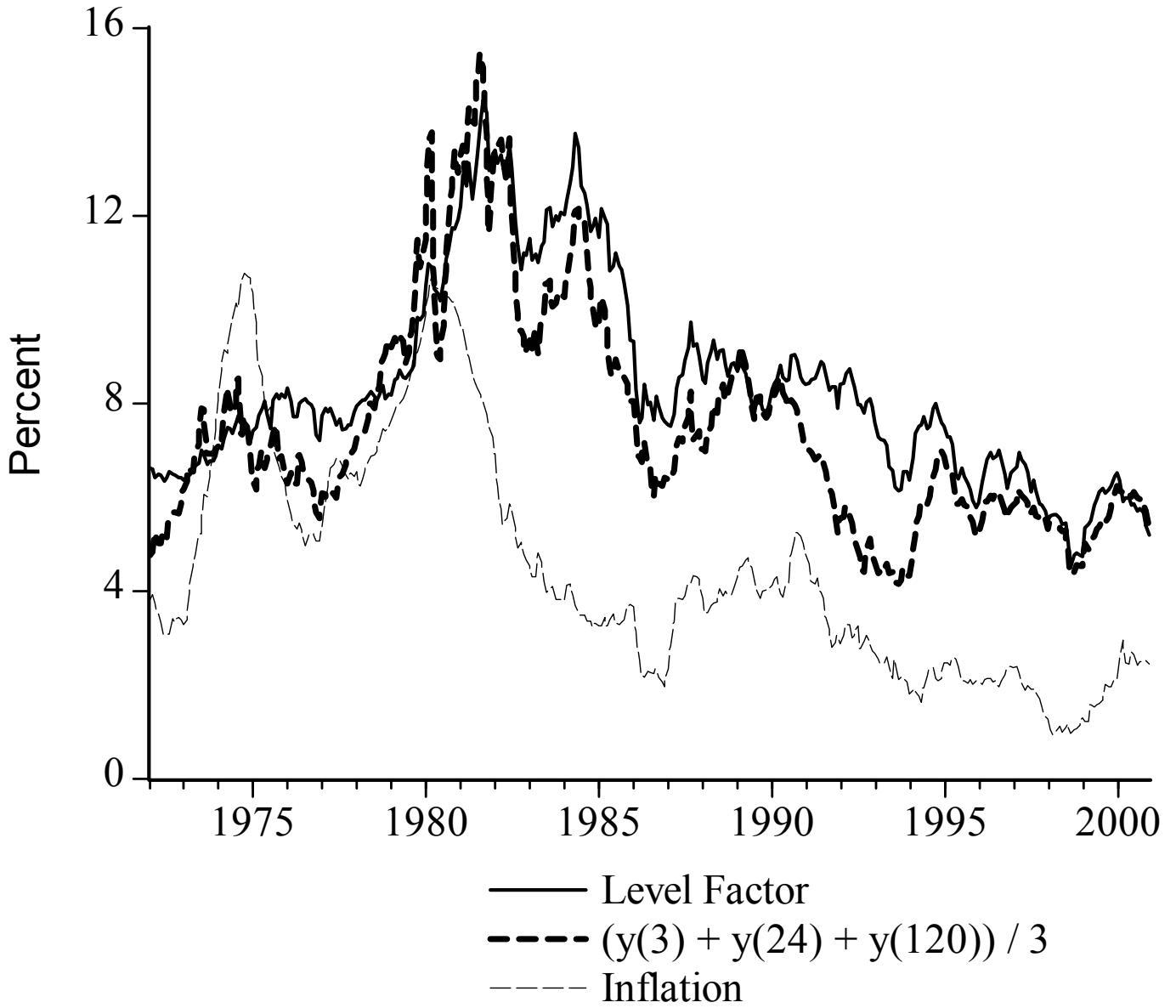**Estimates of Level, Slope, and Curvature in Yields-Only Model**

Figure 2
Yields-Only Model Level Factor and Empirical Counterparts

Figure 3
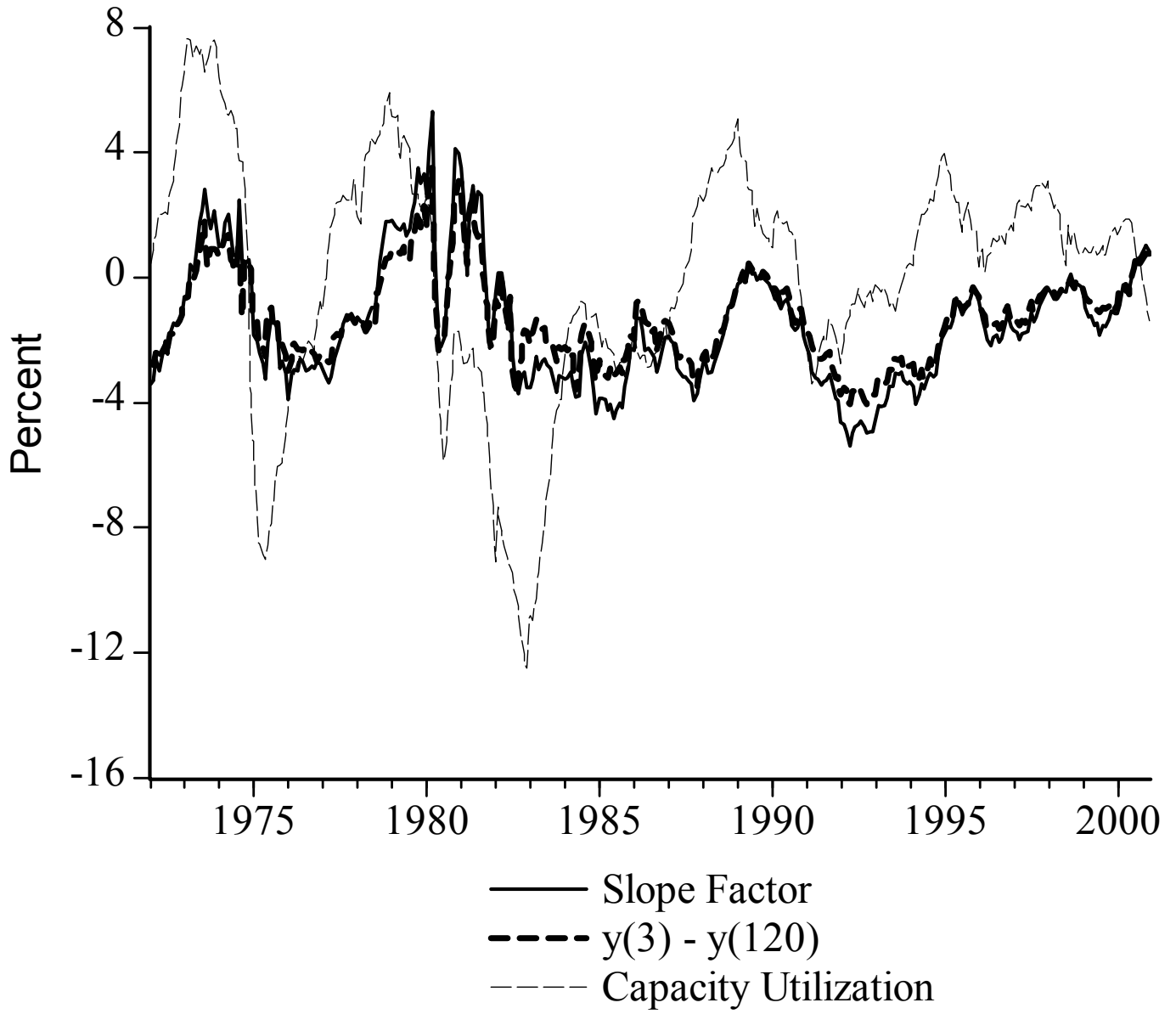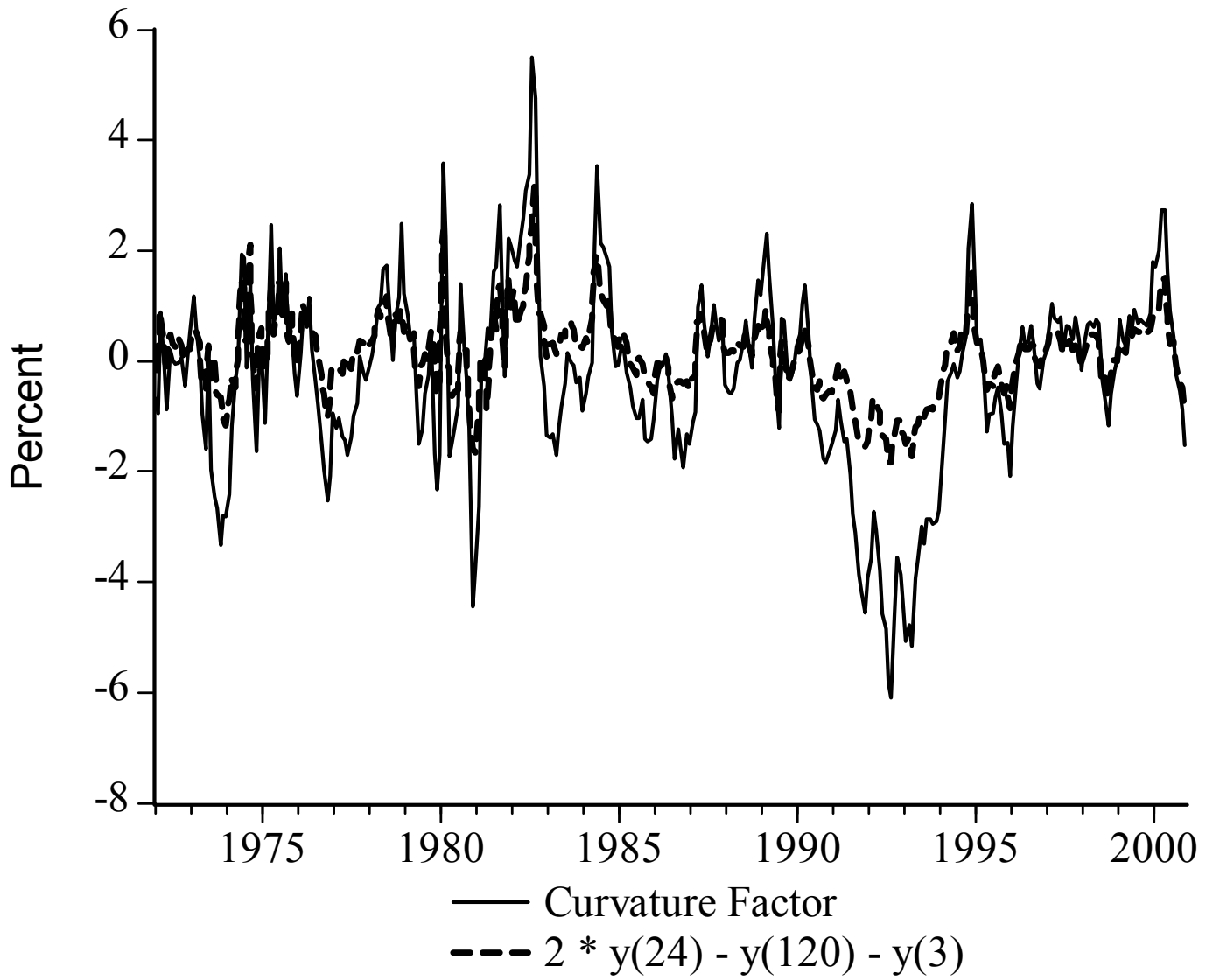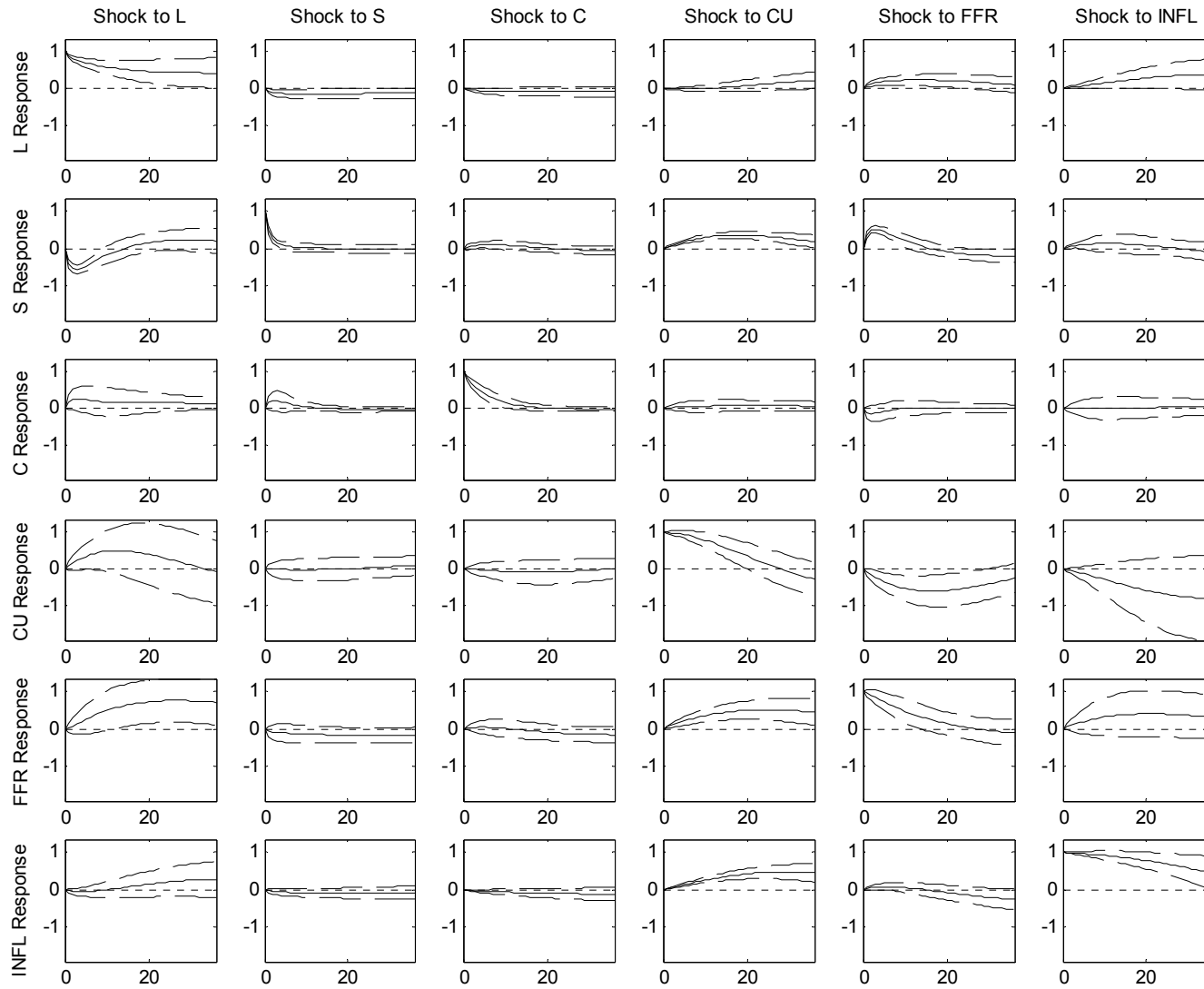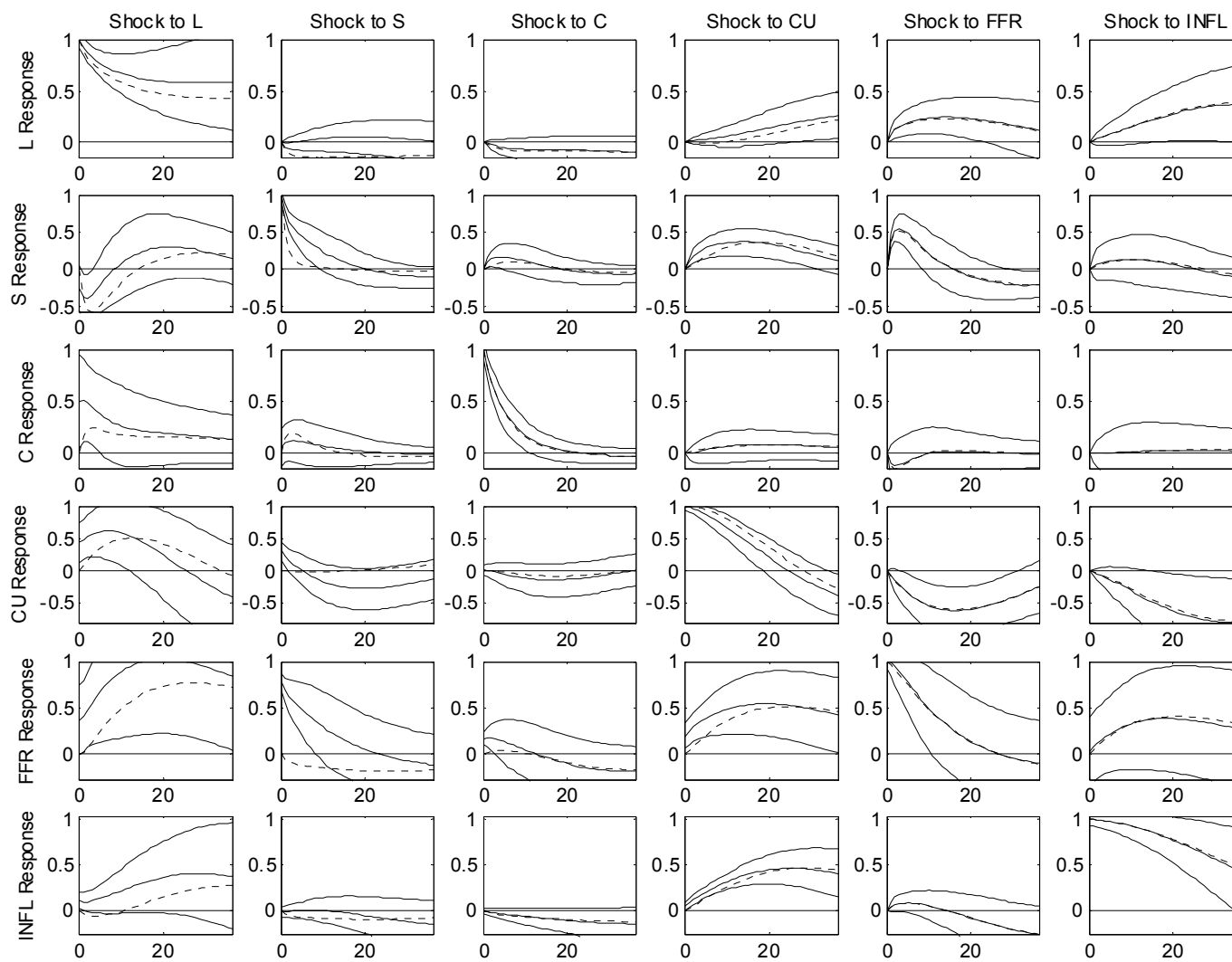**Yields-Only Model Slope Factor and Empirical Counterparts**

Figure 4
Yields-Only Model Curvature Factor and Empirical Counterpart
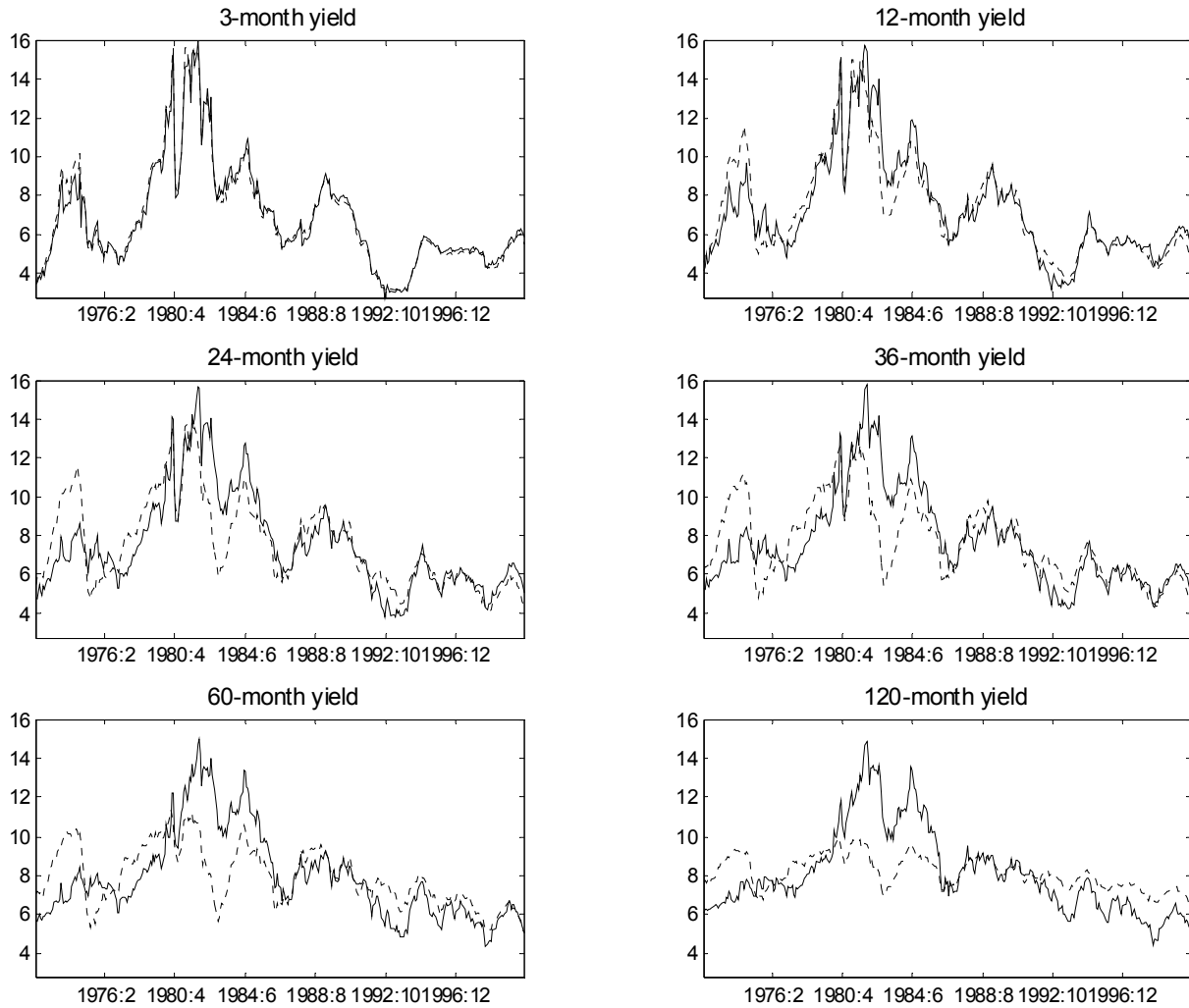
Figure 5
Impulse Responses of the Yields-Macro Model

Figure 6
Impulse Responses of Yields-Macro Model
Diagonal Q Point Model (Dashed), Non-Diagonal Q Model with Confidence Interval Estimates (Solid)

Figure 7
Actual Yields and Yields Implied by the Expectations Hypothesis

Notes : The actual yields are shown as solid lines. The yields implied by the expectations hypothesis are shown as dotted lines.