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"Global Yield Curve Dynamics and Interactions:
A Dynamic Nelson-Siegel Approach"

by

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Global Yield Curve Dynamics and Interactions: A Dynamic Nelson-Siegel Approach

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Abstract: The popular Nelson-Siegel (1987) yield curve is routinely fit to cross sections of intra-country bond yields, and Diebold and Li (2006) have recently proposed a dynamized version. In this paper we extend Diebold-Li to a global context, modeling a potentially large *set* of country yield curves in a framework that allows for both global and country-specific factors. In an empirical analysis of term structures of government bond yields for the Germany, Japan, the U.K. and the U.S., we find that global yield factors do indeed exist and are economically important, generally explaining significant fractions of country yield curve dynamics, with interesting differences across countries.

Key Words: Term Structure, Interest Rate, Dynamic Factor Model, Global Yield, World Yield, Bond Market

JEL Codes: G1, E4, C5

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1. Introduction

The yield curve is of great interest both to academics and market practitioners. Hence yield curve modeling has generated a huge literature spanning many decades, particularly as regards the term structure of government bond yields. Much of that literature is unified by the assumption that the yield curve is driven by a number of latent factors (e.g., Litterman and Scheinkman, 1991; Balduzzi, Das, Foresi and Sundaram, 1996; Bliss, 1997a, 1997b; Dai and Singleton, 2000). Moreover, in many cases the latent yield factors may be interpreted as level, slope and curvature (e.g., Andersen and Lund, 1997; Diebold and Li, 2006). The vast majority of the literature studies a single country's yield curve in isolation and relates domestic yields to domestic yield factors, and more recently, to domestic macroeconomic factors (e.g., Ang and Piazzesi, 2003; Diebold, Rudebusch and Aruoba, 2006).

Little is known, however, about whether common *global* yield factors are operative, and more generally, about the nature of dynamic cross-country bond yield interactions. One might naturally conjecture the existence of global bond yield factors, as factor structure is routinely present in financial markets, in which case understanding global yield factors is surely crucial for understanding the global bond market. Numerous questions arise. Do global yield factors indeed exist? If so, what are their dynamic properties? How do country yield factors load on the global factors, and what are the implications for cross-country yield curve interactions? How much of country yield factor variation is explained by global factors, and how much by country-specific factors, and does the split vary across countries in an interpretable way? Has the importance of global yield factors varied over time, perhaps, for example, increasing in recent years as global financial markets have become more integrated?

In this paper we begin to address such questions in the context of a powerful yet tractable yield curve modeling framework. Building on the classic work of Nelson and Siegel (1987) as dynamized by Diebold and Li (2006), we construct a hierarchical dynamic factor model for sets of country yield curves, in which country yields may depend on country factors, and country factors may depend on global factors. Using government bond yields for the U.S., Germany, Japan, and the U.K., we estimate the model and extract the global yield curve factors. We then decompose the variation in country yields and yield factors into the parts dues global and idiosyncratic components. Finally, we also explore the evolution (or lack thereof) of global yield curve dynamics in recent decades.

Our generalized Nelson-Siegel approach is related to, but distinct from, existing work that tends to focus on spreads between domestic bond yields and a "world rate" (e.g., Al Awad and Goodwin, 1998), implicit one-factor analyses based on the international CAPM (e.g., Solnik, 1974, 2004; Thomas and Wickens, 1993), multi-factor analyses of long bond spreads (e.g., Dungey, Martin and Pagan, 2000), and affine equilibrium analyses (e.g., Brennan and Xia, 2006). Instead we work in a rich environment where each country yield curve is driven by country factors, which in turn are driven both by global and country-specific factors. Hence we achieve an approximate global bond market parallel to the global real-side work of Lumsdaine and Prasad (2003), Gregory and Head (1999) and Kose, Otrok and Whiteman (2003).

We proceed as follows. In section 2 we describe our basic global bond yield modeling framework, and in section 3 we discuss our bond yield data for four countries. In section 4 we provide full-sample estimates and variance decompositions for the global yield curve model, and in section 5 we provide sub-sample results. We conclude in section 6.

2. Modeling Framework

Diebold and Li (2006), Diebold, Rudebusch and Aruoba (2006) and Diebold Piazzesi and Rudebusch (2005) show that, in a U.S. closed-economy environment, a generalized Nelson-Siegel model accurately approximates yield curve dynamics and provides good forecasts. Here we extend that framework to a multi-country environment, allowing for both global and country-specific factors.

Single-Country

The Diebold-Li factorization of the Nelson-Siegel yield curve for a single country (at a particular and arbitrary point in time) is

$$y_i(\tau) = l_i + s_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau}\right) + c_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau}-e^{-\lambda_i\tau}\right) + v_i(\tau), \qquad (1)$$

where $y_i(\tau)$ denotes the continuously-compounded zero-coupon nominal yield on a τ-month bond, l_i, s_i, c_i and λ_i are parameters, and $v_i(\tau)$ is a disturbance with standard deviation $\sigma_i(\tau)$. Following Diebold and Li, we dynamize the model by allowing the parameters to vary over time,

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau}\right) + c_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau}-e^{-\lambda_{it}\tau}\right) + v_{it}(\tau), \qquad (2)$$

and we interpret l_{it}, s_{it}, and c_{it} as *latent factors*. In particular, as shown by Diebold and Li, they are level, slope and curvature factors, respectively, because their factor loadings are a constant, a decreasing function of τ, and a concave function of τ. (Hence the notation *l*, *s* and *c*.) As the yield factors vary over time, this generalized Nelson-Siegel model can generate a variety of time-varying yield curve shapes.

Henceforth we will work with a simplified version of the yield curve (3). First, we will assume constancy of the λ_{it} parameters over countries and time. Following Diebold and Li (2006), there is little loss of generality from doing so, because λ primarily determines the maturity at which the curvature loading is maximized. Second, because the curvature factor is normally estimated with low precision due to missing data at very short and/or very long maturities in most of the countries used in our study, and because curvature lacks clear links to macroeconomic fundamentals as shown in Diebold, Rudebusch and Aruoba (2006), we focus on the model with level and slope factors only. Hence we write

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + v_{it}(\tau). \tag{3}$$

Note that (3) is effectively the measurement equation of a state space system with state vector $(l_{it}, s_{it})'$, as emphasized by Diebold, Rudebusch and Aruoba (2006). Hence the generalized Nelson-Siegel model does not need to be cast in state space form – it is *already* in state space form. Subsequently we will discuss the details of specific parameterizations of that state space form, but for now we simply note its immediate existence.

Multi-Country

We now move to an N-country framework. We allow global yields to be depend on global yield factors,

$$Y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + V_t(\tau), \tag{4}$$

where the $Y_t(\tau)$ are global yields and L_t and S_t are global yield factors. We endow the global yield factors with simple autoregressive dynamics,

$$\begin{pmatrix} L_t \\ S_t \end{pmatrix} = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \end{pmatrix} + \begin{pmatrix} U_t^l \\ U_t^s \end{pmatrix}, \tag{5}$$

where the U_t^n are disturbances such that $EU_t^n U_{t'}^{n'} = (\sigma^n)^2$ if $t=t'$ and $n=n'$, and 0 otherwise, $n = l, s$.

Each country's yield curve remains characterized by (3), but we now allow the country common factors, l_{it} and s_{it}, to load on the global factors L_t and S_t, as well as country idiosyncratic factors:

$$l_{it} = \alpha_i^l + \beta_i^l L_t + \varepsilon_{it}^l \tag{6a}$$

$$s_{it} = \alpha_i^s + \beta_i^s S_t + \varepsilon_{it}^s, \tag{6b}$$

where $\{\alpha_i^l,\ \alpha_i^s\}$ are constant terms, $\{\beta_i^l,\ \beta_i^s\}$ are loadings on global factors, and $\{\varepsilon_{it}^l,\ \varepsilon_{it}^s\}$ are country idiosyncratic factors, $i = 1, ..., N$. Because we include constant terms in (6), we assume with no loss of generality that the country idiosyncratic factors have zero mean. In addition, we make two sets of identifying assumptions. First, because the magnitudes of global factors and factor loadings are not separately identified, we assume that innovations to global factors have unit standard deviation, that is, $\sigma^n = 1$, $n = l, s$.[1] Second, to identify the signs of factors and factor loadings, we assume that the U.S. loadings on the global factors are positive; that is, we assume that $\beta_{us}^n > 0$, $n = l, s$.

As with the global factors, we allow the county idiosyncratic factors to have first-order autoregressive dynamics,

$$\begin{pmatrix} \varepsilon_{it}^l \\ \varepsilon_{it}^s \end{pmatrix} = \begin{pmatrix} \varphi_{i,11} & \varphi_{i,12} \\ \varphi_{i,21} & \varphi_{i,22} \end{pmatrix} \begin{pmatrix} \varepsilon_{i,t-1}^l \\ \varepsilon_{i,t-1}^s \end{pmatrix} + \begin{pmatrix} u_{it}^l \\ u_{it}^s \end{pmatrix}, \tag{7}$$

where the u_{it}^n are disturbances such that $Eu_{it}^n u_{i't'}^{n'} = (\sigma_i^n)^2$ if $i = i'$, $t = t'$ and $n = n'$, and 0 otherwise, $n = l, s$. We also assume the shocks to the global factors U_t^n and the shocks to the country-specific factors $u_{it}^{n'}$ are orthogonal: $EU_t^n u_{i,t-s}^{n'} = 0$, for all n, n', i , and s.

Many variations, extensions and specializations of this basic model are of course possible. For example, a useful specialization to facilitate tractable estimation would restrict the dynamic matrices in (5) and (7) to be diagonal. (We shall do this.) As another example, an interesting extension would include not only global factors, but also regional factors, in which case country factors could depend on regional factors, which in turn could depend on global factors. We shall not consider such extensions in this paper; instead, we now implement empirically our basic model sketched thus far.

3. Data Construction, Data Description, and Preliminary Analysis

In this section, prior to fitting the full global yield model, we discuss and describe the data. We perform several preliminary analyses that provide background, motivation and a foundation for the subsequent analysis.

Data Construction

Our data, generously supplied by Michael Brennan and Yihong Xia for 1985.09-2002.05 and extended by us to 2005.08, consist of government bond prices, coupon rates, and coupon structures, as

[1] This follows Sargent and Sims (1977) and Stock and Watson (1989).

well as issue and redemption dates, in local currency terms for the U.S., Germany, Japan, and the U.K.

We calculate zero-coupon bond yields using the unsmoothed Fama-Bliss (1987) approach.[2] We measure the bond yields on the second day of each month. We also apply several data filters designed to enhance data quality and focus attention on maturities with good liquidity. First, we exclude floating rate bonds, callable bonds and bonds extended beyond the original redemption date. Second, we exclude outlying bond prices less than 50 or greater than 130 because their price premium/discounts are too high and imply thin trading, and we exclude yields that differ greatly from yields at nearby maturities. Finally, we use only bonds with maturity greater than one month and less than fifteen years, because other bonds are not actively traded. To simplify our subsequent estimation, using linear interpolation we pool the bond yields into constant maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months, where a month is defined as 30.4375 days.

Data Description

In Figure 1 we show the government bond yield curves across countries and time. It is apparent that each yield curve displays substantial level movements. Cross-country comparison of the yield curves, moreover, reveals clear commonality in level movements. Yield curve slopes vary less, although they do of course vary, and Figure 1 suggests that they may also display some cross-country commonality in movements.

In Table 1 we report summary statistics for bond yields at representative maturities. Japanese yields are lowest on average, typically around two or three percent. All yield curves are upward-sloping, and yield volatility decreases with maturity. In addition, all yields are highly persistent for all countries, with average first-order autocorrelation greater than 0.95.

Preliminary Analysis

Our ultimate goal is to estimate the global yield model (4)-(7), extract the global yield factors, and so forth. To facilitate that goal, we first conduct a preliminary estimation of the Nelson-Siegel factors separately for each country. That is, we estimate the level and slope factors, $\{l_{it}, s_{it}\}$, $t = 1, ..., T$ and $i = 1, ..., N$, via a series of ordinary least squares regressions for each country, as in Diebold and Li (2006).

In Table 2 we present descriptive statistics for the estimated factors, movement in which

[2] Our zero-coupon bond yields are highly correlated with those obtained by Brennan and Xia (2006), who use a cubic spline and maturities of 3, 6, 12, 24, 36, 60, 84, 96, 108 and 120 months.

potentially reflects both global and country-specific influences.[3] The mean level factor is lowest for Japan, and the mean absolute slope factor is greatest for the U.S.[4] The comparatively steep average U.S. yield curve slope may reflect comparatively optimistic average U.S. growth expectations during our sample period. The factor autocorrelations reveal that all factors display persistent dynamics, with the level more persistent than the slope.

Of central interest is the possible existence of *commonality* in country level and/or slope factor dynamics, as predicted by our global factor model. To investigate this, in Figure 2a we plot superimposed estimated level factors for all the countries, and in Figure 2b we plot the superimposed estimated slope factors. For both sets of factors, there is clear visual evidence of commonality in factor dynamics.

Finally, as an alternative and complementary preliminary assessment of commonality of movements in country yield curves, we conduct a principal components analysis of the estimated level and slope factors. The results, reported in Table 3, suggest the existence of global factors. Specifically, the first principal component for levels explains more than ninety percent of level variation, and the first principal component for slopes explains roughly fifty percent of slope variation. We interpret this as suggesting the existence of one dominant global level factor, and one important (if not completely dominant) global slope factor. Armed with these preliminary and suggestive results, we now proceed to formal econometric estimation of our model, and extraction of the associated global factors.

4. Global Model Estimation

In this section, we estimate the global yield curve factor model, exploiting its state-space structure for both parameter estimation and factor extraction.

4a. State Space Representation

The global yield curve model has a natural state-space representation. The measurement equation is:

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[3] In section 4 we will explicitly decompose the country factors into global and country-specific components.

[4] It is interesting to note that the U.K. has the smallest ratio of mean to variance for both estimated factors. This is consistent with the U.K. yield data plotted in the lower right panel of Figure 1, which appears noticeably noisier than that for the other three countries. We are unsure as to whether the comparatively noisy U.K. data accurately reflect U.K. bond market conditions during our sample period, or whether they are simply of comparatively poor quality.

$$\begin{pmatrix} y_{1t}(\tau_1) \\ y_{1t}(\tau_2) \\ \dots \\ y_{Nt}(\tau_J) \end{pmatrix} = A \begin{pmatrix} \alpha_1^l \\ \alpha_1^s \\ \dots \\ \alpha_N^s \end{pmatrix} + B \begin{pmatrix} L_t \\ S_t \end{pmatrix} + A \begin{pmatrix} \varepsilon_{1t}^l \\ \varepsilon_{1t}^s \\ \dots \\ \varepsilon_{Nt}^s \end{pmatrix} + \begin{pmatrix} v_{1t}(\tau_1) \\ v_{1t}(\tau_2) \\ \dots \\ v_{Nt}(\tau_J) \end{pmatrix} \tag{8}$$

where

$$A = \begin{pmatrix} 1 & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}\right) & 0 & \dots & 0 \\ 1 & \left(\frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2}\right) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & \left(\frac{1-e^{-\lambda\tau_J}}{\lambda\tau_J}\right) \end{pmatrix} \tag{9}$$

$$B = \begin{pmatrix} \beta_1^l & \beta_1^s\left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}\right) \\ \beta_1^l & \beta_1^s\left(\frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2}\right) \\ \dots & \dots \\ \beta_N^l & \beta_N^s\left(\frac{1-e^{-\lambda\tau_J}}{\lambda\tau_J}\right) \end{pmatrix} \tag{10}$$

The transition equations are the union of (5) and (7).

Note that our approach fortunately does not require that we observe global yields or global yield factors. Global yields $Y_t(\tau)$ do not appear at all in the state space representation, the measurement equation of which instead relates observed *country* yields to the latent global yield factors L_t and S_t, which appear in the state vector. Once the model is estimated, L_t and S_t can be immediately extracted via the Kalman smoother. Hence we now turn to estimation.

4b. Model Estimation

Under a normality assumption for the measurement and transition shocks, Gaussian maximum likelihood estimates are readily obtained in principle via application of the Kalman filter to the model in state space form, as in the single-country framework of Diebold, Rudebusch and Aruoba (2006). In practice, however, maximum likelihood is particularly difficult to implement in multi-country environments, because of the large number of parameters to be estimated. Hence we maintain the normality assumption but take a different, Bayesian, approach, using Markov Chain Monte Carlo (MCMC) methods to perform a posterior analysis of the model, in the tradition of recent Bayesian estimation of large-scale dynamic factor models of real economic activity (e.g., Kim and Nelson, 1998; Kose, Otrok and Whiteman, 2003; Bernanke, Boivin and Eliasz, 2005).

We use a convenient multi-step estimation method in the tradition of Diebold and Li (2006), but which still exploits the state space structure emphasized by Diebold, Rudebusch and Aruoba (2006). In the first step, we estimate the model (3) separately for each country to obtain the series of level and slope factors, l_{it} and s_{it}. In the second step, we estimate a dynamic latent factor model composed of the country factor decomposition equations (6), the global factor dynamics (5), and the country idiosyncratic factor dynamics (7).

Motivated by the results of single-country analyses, which indicate little cross-factor dynamic interaction, we assume that the VARs given by equations (5) and (7) have diagonal autoregressive coefficient matrices. This drastically simplifies the second-step estimation, because it implies that we can estimate the model factor-by-factor, first estimating the model relating the four country level factors to the global level factor, and then estimating the model relating the four country slope factors to the global slope factor. For each of the two state-space models there are seventeen parameters to estimate: one autoregressive coefficient for the global factor, four intercepts, four loadings on the global factor, four autoregressive coefficients for the idiosyncratic factors, and four standard deviations for the innovations to the idiosyncratic factors.

Our state space models, whether for levels or slopes, are simply statements that certain "data" Z_t (the set of country level or slope factors estimated in the first step), conditional on a set of parameters φ and a latent variable F_t (the global level or slope factor), have a certain normal distribution. We use MCMC methods – effectively just Gibbs sampling – to produce draws from the joint posterior distribution $p(\varphi, F|Z)$ by iterating on the conditionals $p(\varphi|F,Z)$ and $p(F|\varphi,Z)$.[5] At generic iteration i,

[5] We use $\underline{N}(0,1)$ priors for all factor loadings and serial correlation coefficients, $N(\bar{Z}_i,1)$ for all intercepts where $\bar{Z}_i$ is sample mean of i^{th} data series, and inverse gamma priors for all variances. Given the bounded likelihood and proper priors we use, the joint posterior is well-behaved, and the empirical

we first draw from $p(\varphi|F^{(i-1)},Z)$ by exploiting the fact that, conditional on F, the state space measurement equations (6) are simply regressions with autoregressive errors, so draws from the conditional posterior $p(\varphi|F^{(i-1)},Z)$ are readily obtained via the Chib-Greenberg (1994) algorithm. After drawing from $p(\varphi|F^{(i-1)},Z)$, we then draw from $p(F|\varphi^{(i)},Z)$, which is analogous to a standard signal extraction problem, except that we seek not just to know the posterior mean but rather the entire posterior distribution. The solution is provided by the multi-move Gibbs sampling algorithm of Carter and Kohn (1994), which lets us draw entire realizations of the factor F, governed by $p(F|\varphi^{(i)},Z)$.[6]

4c. Estimated Parameters and Factors

We present the estimation results in Table 4. Consider first the results for the country level factors, all of which load positively on the global level factor, which is highly serially correlated. The global level factor loadings in the country level factor equations are estimated with high precision, with posterior means much greater than posterior standard deviations. The country-specific level factors are also generally highly persistent. Relative to the middle-of-the-road results for the U.S. and U.K., the German level loading on the global level factor is larger, and the persistence of the German-specific level factor much smaller, implying that the dynamics of the German yield level match closely those of the global level. Conversely, and again relative to the U.S. and U.K., the Japanese level loading on the global level factor is smaller, and the persistence of the Japan-specific level factor larger, implying that the Japanese yield level is comparatively divorced from the global level. The Japanese results are particularly sensible given the very low level of Japanese yields, relative to those elsewhere, in the second half of our sample.

Now consider the results for the country slope factors. In parallel with the earlier results for the level factors, all slope factors load positively on the global slope factor, which is very highly serially correlated, albeit slightly less so than the global level factor. The country-specific slope factors are also generally highly persistent. A number of novel results are apparent as well. The German slope factor effectively does not load on the global slope factor; instead, its movements appear completely idiosyncratic, just the opposite of the behavior of the German level factor. The U.K. slope results also differ from those for the U.K. level: U.K. slope loads entirely on global slope with little persistence in its

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distribution of draws from the conditional posteriors converges to the joint marginal posterior as the number of iterations goes to infinity. We use chains of length 40,000, discarding the first 20,000 draws as a burn-in, and then using the remaining 20,000 to sample from the marginal posteriors. See the Appendix for details.

[6] For an insightful exposition of the Carter-Kohn algorithm, and creative application to dynamic factor models, see Kim and Nelson (1999).

country-specific slope factor. Like its yield level, the Japanese slope loading on the global slope factor is small (and insignificant), and the persistence of the Japan-specific slope factor large, implying that the Japanese yield slope is also comparatively divorced from the global slope.

In Figure 3 we plot the posterior means of the global level and slope factors extracted, along with two posterior standard deviation bands. The narrow bands indicate that the factors are estimated with high precision. In Figure 4 we plot the global level and slope factors together with the earlier-discussed first principal components of country levels and country slopes. The global factors and first principal components are highly correlated, which is reassuring. It is important to note, however, that although related, the extracted global factors and first principal components are not at all identical.

4d. Links to the Global Macroeconomy

Several studies, for example Ang and Piazzesi (2003) and Diebold, Rudebusch and Aruoba (2006), show that latent country yield factors are linked to, and interact dynamically with, domestic macroeconomic factors. The key factors, moreover, are inflation and real activity, which are linked to the yield curve level and slope, respectively.

In parallel fashion, our extracted global level and slope factors reflect the major developments in global inflation and real activity during the past twenty years. The temporal decline in the global level factor reflects the reduction of inflation in the industrialized countries; the correlation between our extracted global level factor and average G-7 inflation during our sample is 0.75.[7] Similarly, movements in the global slope factor reflect the global business cycle, with the global slope factor peaking just before the two global recessions of the early 1990s and early 2000s.[8] The correlation between our extracted global slope factor and average G-7 GDP annual growth during our sample is 0.27.[9]

All told, the picture that emerges is one of hierarchical linkage: Country yields load off country factors, which in turn load off global factors. This is similar, for example, to standard results in other markets such as equities, in which excess returns on country stocks are driven by country factors, which are in turn driven by global factors, leading to an "international CAPM" (e.g., Solnik, 1974, 2004; Thomas and Wickens, 1993). Moreover, this hierarchical factor structure of global asset markets mirrors

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[7] We use inflation data from the IMF's *International Financial Statistics*.

[8] Our slope factor tracks the negative of yield curve slope, as shown in Diebold and Li (2006), so large values correspond to a flat or inverted yield curve. Hence it seems that flat or inverted global yield curves tend to lead global recessions.

[9] We use G-7 quarterly GDP data from the IMF's *International Financial Statistics*, and we correlate it with our extracted global slope factor aggregated to quarterly frequency.

that of macroeconomic fundamentals: Country real activity is typically driven by one real factor (e.g., Stock and Watson, 1989), but country factors load on global factors, consistent with an "international business cycle" (e.g., Kose, Otrok and Whiteman, 2003).

4e. Variance Decompositions

We conduct two sets of variance decompositions. First, we decompose the variation in country level and slope factors into parts driven by global yield variation and country-specific variation. Second, we directly decompose the variation in country yields as opposed to country factors, determining for any given country yield the fraction of its variance due to variation in underlying global factors, whether level or slope.

The global and country-specific factors extracted from a finite sample of data may be correlated even if they are truly uncorrelated in population. Hence we orthogonalize the extracted factors by regressing the country factor on the extracted global factor and updating the global factor loading and country-specific factor variance accordingly. Then from (6) we have:

$$var(l_{it}) = (\beta_i^l)^2 var(L_t) + var(\varepsilon_{it}^l) \quad (11a)$$

$$var(s_{it}) = (\beta_i^s)^2 var(S_t) + var(\varepsilon_{it}^s), \quad (11b)$$

for $i = 1, ..., N$. This splits the variation in each country factor into a part driven by the corresponding global factor and a part driven by the corresponding country-specific factor.

We present the results in Table 5, reporting the posterior median as well as the fifth and ninety-fifth posterior percentiles for each variance share.[10] Variation in the global level factor L explains a large fraction of the variation in country level factors, for all countries, typically in the range of sixty to eighty percent. Indeed variation in L explains almost *all* variation in the German level factor, which is natural because, as discussed earlier, the German level factor loads heavily on L and the German-specific level factor is simply white noise with a small variance.

The country-factor variance decomposition results for slopes are more diverse. Variation in the global slope factor accounts for small percentage of the variation in the U.S. and German slope factors, indicating comparative independence of the U.S. and German business cycles. In contrast, variation in

[10] We present explicit posterior percentiles, rather than posterior means and standard deviations as elsewhere in the paper, because the posterior distributions of the variance shares are quite highly skewed, whereas the posterior distributions elsewhere in the paper are not.

the global slope factor accounts for almost *all* of the variation in the U.K. slope factor, consistent with the earlier-discussed fact that the U.K. slope factor loads almost entirely on the global slope factor.

Now we provide variance decompositions for yields, as opposed to yield factors. We regress Nelson-Siegel fitted yields on our extracted global level and slope factors jointly as implied by equations (3) and (6), and we calculate the variance of the bond yield explained jointly by the global level and slope factors in that regression. In Figure 5 we show the variance decompositions; more precisely, we show the posterior median of the fraction of variation coming from global factors for bonds yields at four representative maturities of 3, 12, 60 and 120 months. The shaded area is the range from the fifth percentile to the ninety-fifth percentile of the posterior distribution.

There are three major results. First, and strikingly, for all countries and maturities, variation in the global factor is responsible for a large share of variation in yields. The global share is never less than a third, typically more than half, and often much more than half. Hence global yield factors are important drivers of country bond yields. Second, the global share of bond yield variation is smallest for the U.S. across all maturities, consistent with relative independence of the U.S. market. Third, the global share tends to increase with maturity, which effectively means that the importance of the global level factor tends to increase with maturity (naturally, since long yields load little on slope), emphasizing the crucial importance of inflation expectations in pricing long bonds.

5. Sub-Sample Analysis

In this section we assess the evidence on the stability (or lack thereof) of the dynamics linking the four countries' yield curves. As mentioned earlier, for example, one might naturally conjecture the enhanced importance of global yield factors in recent decades, due to enhanced global bond market integration.

We split our sample into two equal-length sub-samples, 1985:9-1995:8 and 1995:9-2005:8. In Table 6 we report sub-sample summary statistics for the country level and slope factors. The mean levels are lower in the second sub-sample, reflecting the global disinflation of the past twenty years. In contrast, the mean slopes vary less across the sub-samples, reflecting comparatively stable cyclical patterns in real activity.[11]

In Table 7 we report log likelihoods for the full-sample models and sub-sample models, as well as

[11] Germany is an exception. Its mean slope is positive in the first sub-sample and negative in the second sub-sample.

the corresponding log posterior odds. The log posterior odds (assuming flat priors) is simply the difference between the sum of the sub-sample log likelihoods and the full-sample log likelihood. The evidence strongly suggests instability for both the country level and slope systems.

In an attempt to uncover the sources of instability, we report sub-sample estimation results in Table 8 and sub-sample variance decompositions in Table 9. The results display interesting nuances and are certainly more involved than, for example, a simple uniform increase in importance of the global factor in the second sub-sample.

Consider first the level factors. For three of the four countries, the importance of the global level factor for country yield levels either increases or remains roughly unchanged across the sub-samples; the exception is Germany. The comparative importance of the German country-specific factor may perhaps be linked to the successful emergence of the Eurozone in the second-subsample, so that in recent years Germany loads less on the global level factor than do Japan, the U.K. and the U.S.

The evolving role of the global slope factor is more involved. First, the importance of the global slope factor for country yield slopes decreases across the sub-samples, except for Germany. Second, Japan's slope factor has a negative loading on the global slope factor in the second sub-sample, reflecting the unique characteristics of the Japanese economy post-1995. Similarly, global slope factor variation accounts for only a very small fraction of Japanese slope factor variation in the second sub-sample.

In Figure 6 we report the sub-sample variance decompositions of yields, as opposed to factors. For both subsamples, the global shares of bond yield variation are economically important. Just as the global factors impact the country factors differently across the two sub-samples, so too do they impact the dynamics of yields differently. Overall the evidence suggests that global factors play a larger and important role in the second sub-sample, but the details are again richly nuanced. In the first sub-sample, the contribution of global factor greatly differs across countries, with the lowest share of 20% for the U.S., which again reflects the independence of the U.S. bond market in the first sub-sample. Moreover, the importance of global factors decreases with maturity for all countries except Germany. For the second sub-sample, the global factors account for more than 50% of variation in bond yields for all the bonds considered except for the 3 and 12 months bond issued by Japan.[12] The importance of global factors largely increases with maturity with the exception of U.K. Moreover, the global shares for the countries in our sample have more homogeneity in the second sub-sample, in particular for bonds with maturity longer than five years. This result, together with the increasing role of global yield factors,

[12] The exception of Japan's 3- and 12-month bond yields reflects the distinct short-term monetary policy in Japan due to its low inflation after 1995.

reflects the greater degree of globalization of the world economy as well as better integration of bond markets.

6. Summary and Concluding Remarks

We have extended the yield curve model of Nelson-Siegel (1987) and Diebold-Li (2006) to a global setting, proposing a hierarchical model in which country yield level and slope factors may depend on global level and slope factors as well as country-specific factors. Using a monthly dataset of government bond yields for Germany, Japan, the U.S. and the U.K. from 1985:9 to 2005:8, we extracted global factors and country-specific factors for both the full sample and the 1985:9-1995:8 and 1995:9-2005:8 sub-samples. The results indicate strongly that global yield level and slope factors do indeed exist and are economically important, accounting for a significant fraction of variation in country bond yields. Moreover, the global yield factors appear linked to global macroeconomic fundamentals (inflation and real activity) and appear more important in the second sub-sample.

We look forward to future work producing one-step estimates in an environment with many countries, richer country-factor and global-factor dynamics, richer interactions with macroeconomic fundamentals, and time-varying yield volatility. Such extensions remain challenging, however, because of the prohibitive dimensionality of the estimation problem. Presently we are estimating 240+17=257 parameters in each of our separate global level and slope models, composed of the 17 parameters discussed earlier, plus the 240 parameters corresponding to the values of the state vector at 240 different times. Incorporating the generalizations mentioned above could easily quadruple the number of parameters, necessitating the use of much longer Markov Chains.

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Table 1: Descriptive Statistics for Bond Yields

U.S.							
Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	4.76	2.07	0.87	9.32	0.95	0.68	0.17
12	5.19	2.10	1.03	9.72	0.98	0.71	0.20
60	6.02	1.90	1.74	10.05	0.96	0.66	0.40
120	6.73	1.59	3.35	10.83	0.91	0.46	0.32

Germany							
Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	3.38	1.61	1.98	7.98	-	-	-
12	5.08	2.06	1.07	9.59	0.98	0.77	0.26
60	5.56	1.79	2.20	9.32	0.98	0.77	0.42
120	5.57	1.17	3.18	8.42	0.96	0.52	0.17

Japan							
Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	2.14	2.55	0.01	8.35	-	-	-
12	2.36	2.44	0.01	8.41	0.99	0.83	0.57
60	2.95	2.15	0.17	8.10	0.98	0.82	0.59
120	2.79	1.62	0.16	6.56	0.83	0.59	0.42

U.K.							
Maturity (Months)	Mean	Standard Deviation	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	6.92	3.00	3.29	14.77	-	-	-
12	7.04	3.02	2.67	14.23	0.94	0.75	0.46
60	7.31	2.65	2.30	13.47	0.98	0.80	0.56
120	7.12	2.44	3.64	13.34	0.98	0.80	0.67

Notes to table: All yield data are monthly, 1985.09-2005.08. $\hat{\rho}(\tau)$ denotes the sample autocorrelation at displacement τ. Because of missing data, we do not compute $\rho(\tau)$ for 3-month bonds for Germany, Japan, and UK. See text for details.

Table 2: Descriptive Statistics for Estimated Country Level and Slope Factors

U.S.							
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
$\hat{l}_{it}$	6.87	1.72	2.89	11.34	0.94	0.59	0.48
$\hat{s}_{it}$	-2.39	1.61	-5.90	0.72	0.92	0.37	-0.13

Germany							
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
$\hat{l}_{it}$	6.20	1.58	2.57	9.39	0.97	0.68	0.42
$\hat{s}_{it}$	-1.63	2.10	-5.87	4.51	0.96	0.44	-0.23

Japan							
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
$\hat{l}_{it}$	3.48	1.94	0.41	7.61	0.98	0.82	0.63
$\hat{s}_{it}$	-1.63	1.35	-4.32	1.72	0.95	0.63	0.22

U.K.							
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
$\hat{l}_{it}$	7.23	2.38	2.96	12.36	0.97	0.82	0.65
$\hat{s}_{it}$	-0.34	2.18	-5.65	5.56	0.90	0.36	0.02

Notes to table: All yield data are monthly, 1985.09-2005.08. $\hat{\rho}(\tau)$ denotes the sample autocorrelation at displacement τ. See text for details.

Table 3: Principal Components Analysis for Estimated Country Level and Slope Factors

Level Factors, $\hat{l}_{it}$, $i = US, GM, JP, UK, \quad t = 1, ..., T$

	PC 1	PC 2	PC 3	PC 4
Eigenvalue	3.64	0.27	0.10	0.05
Variance Prop.	0.91	0.05	0.03	0.01
Cumulative Prop.	0.91	0.96	0.99	1.00

Slope Factors, $\hat{s}_{it}$, $i = US, GM, JP, UK, \quad t = 1, ..., T$

	PC 1	PC 2	PC 3	PC 4
Eigenvalue	1.99	1.01	0.69	0.30
Variance Prop.	0.50	0.25	0.17	0.08
Cumulative Prop.	0.50	0.75	0.92	1.00

Notes to table: All yield data are monthly, 1985.09-2005.08. For each set of estimated country level factors and slope factors, we report the eigenvalues, variance proportions and cumulative variance proportions associated with the four principal components.

Table 4: Estimates of The Global Yield Curve Model Parameters

Global Level Factor

$$L_t = \underset{(0.01)}{0.99} L_{t-1} + U_t^l$$

Global Slope Factor

$$S_t = \underset{(0.02)}{0.94} S_{t-1} + U_t^s$$

Country Level Factors

$$l_{US,t} = \underset{(0.30)}{6.77} + \underset{(0.03)}{0.18} L_t + \varepsilon_{US,t}^l \qquad \varepsilon_{US,t}^l = \underset{(0.03)}{0.89}\varepsilon_{US,t-1}^l + \underset{(0.02)}{0.43} v_{US,t}^l$$

$$l_{GM,t} = \underset{(0.19)}{6.29} + \underset{(0.01)}{0.26} L_t + \varepsilon_{GM,t}^l \qquad \varepsilon_{GM,t}^l = \underset{(0.10)}{0.07}\varepsilon_{GM,t-1}^l + \underset{(0.02)}{0.12} v_{GM,t}^l$$

$$l_{JP,t} = \underset{(0.68)}{3.16} + \underset{(0.02)}{0.11} L_t + \varepsilon_{JP,t}^l \qquad \varepsilon_{JP,t}^l = \underset{(0.01)}{0.98}\varepsilon_{JP,t-1}^l + \underset{(0.01)}{0.28} v_{JP,t}^l$$

$$l_{UK,t} = \underset{(0.58)}{7.17} + \underset{(0.03)}{0.22} L_t + \varepsilon_{UK,t}^l \qquad \varepsilon_{UK,t}^l = \underset{(0.02)}{0.95}\varepsilon_{UK,t-1}^l + \underset{(0.02)}{0.39} v_{UK,t}^l$$

Country Slope Factors

$$s_{US,t} = \underset{(0.48)}{-2.24} + \underset{(0.05)}{0.14} S_t + \varepsilon_{US,t}^s \qquad \varepsilon_{US,t}^s = \underset{(0.03)}{0.92}\varepsilon_{US,t-1}^s + \underset{(0.03)}{0.58} v_{US,t}^s$$

$$s_{GM,t} = \underset{(0.72)}{-1.53} + \underset{(0.05)}{0.06} S_t + \varepsilon_{GM,t}^s \qquad \varepsilon_{GM,t}^s = \underset{(0.02)}{0.96}\varepsilon_{GM,t-1}^s + \underset{(0.03)}{0.60} v_{GM,t}^s$$

$$s_{JP,t} = \underset{(0.53)}{-1.70} + \underset{(0.03)}{0.03} S_t + \varepsilon_{JP,t}^s \qquad \varepsilon_{JP,t}^s = \underset{(0.02)}{0.95}\varepsilon_{JP,t-1}^s + \underset{(0.02)}{0.41} v_{JP,t}^s$$

$$s_{UK,t} = \underset{(0.42)}{-0.48} + \underset{(0.07)}{0.77} S_t + \varepsilon_{UK,t}^s \qquad \varepsilon_{UK,t}^s = \underset{(0.09)}{0.03}\varepsilon_{UK,t-1}^s + \underset{(0.09)}{0.42} v_{UK,t}^s$$

Notes to table: We report Bayesian estimates of the global yield curve model (4)-(7), obtained using monthly yields 1985.09-2005.08. We show posterior means, with posterior standard deviations in parentheses. We define $v_{i,t} \equiv u_{i,t}/\sigma_i$, so that $u_{i,t} \equiv \sigma_i v_{i,t}$. See text for details.

Table 5: Variance Decompositions of Country Level and Slope Factors

Level Factors

		U.S.	Germany	Japan	U.K.
	5th Percentile	63.04	99.04	79.80	80.39
Global	Median	63.88	99.40	80.51	81.03
	95th Percentile	64.91	99.67	81.40	81.80
	5th Percentile	36.96	0.96	20.20	19.61
Country	Median	36.12	0.60	19.49	18.97
	95th Percentile	35.09	0.33	18.60	18.20

Slope Factors

		U.S.	Germany	Japan	U.K.
	5th Percentile	16.45	14.39	34.52	93.39
Global	Median	18.03	15.93	36.48	95.93
	95th Percentile	19.95	17.79	39.02	98.17
	5th Percentile	83.55	85.61	65.48	6.61
Country	Median	81.97	84.07	63.52	4.07
	95th Percentile	80.05	82.21	60.98	1.83

Notes to table: For each country, we decompose country level and slope factor variation into parts coming from global factor variation and country-specific factor variation. We estimate the underlying model using monthly yield data, 1985.09-2005.08. We show posterior medians together with posterior fifth and ninety-fifth percentiles. See text for details.

Table 6a: First Sub-Sample Descriptive Statistics for Estimated Country Level and Slope Factors, 1985.09-1995.08

U.S.

Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	8.18	1.10	5.64	11.34	0.87	0.15	0.09	-3.22**
$\hat{s}_{it}$	-2.62	1.69	-2.87	0.60	0.94	0.39	-0.34	-1.98

Germany

Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	7.37	0.92	5.64	9.39	0.95	-0.36	-0.27	-1.74
$\hat{s}_{it}$	1.31	1.97	-5.36	2.97	0.94	0.51	-0.03	-1.42

Japan

Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	5.21	0.91	3.04	7.61	0.90	0.33	-0.30	-2.25
$\hat{s}_{it}$	-1.11	1.51	-3.89	1.72	0.93	0.48	-0.16	-0.90

U.K.

Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
$\hat{l}_{it}$	9.32	1.10	6.01	12.36	0.89	0.38	-0.10	-2.39
$\hat{s}_{it}$	-0.14	2.57	-5.65	5.56	0.91	0.41	-0.14	-2.20

Notes to table: $\hat{\rho}(\tau)$ denotes the sample autocorrelation at displacement τ, and ADF denotes the augmented Dickey-Fuller statistic with augmentation lag length selected using the Akaike information criterion. Single and double asterisks denote statistical significance at the ten and five percent levels, respectively. See text for details.

Table 6b: Second Sub-Sample Descriptive Statistics for Estimated Country Level and Slope Factors, 1995.09-2005.08

U.S.								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
l_{it}	5.56	1.12	2.89	7.35	0.90	0.34	0.05	-2.31
s_{it}	-2.16	1.49	-5.90	0.72	0.90	0.32	0.16	-1.97

Germany								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
l_{it}	5.03	1.19	2.57	7.36	0.95	0.28	-0.17	-1.28
s_{it}	-1.95	2.18	-5.87	4.51	0.96	0.29	-0.36	-1.52

Japan								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
l_{it}	2.50	1.01	0.68	4.77	0.93	0.41	0.15	-1.92
s_{it}	-2.15	0.92	-4.32	-0.54	0.91	0.35	0.11	-1.91

U.K.								
Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$	ADF
l_{it}	5.17	1.16	2.96	7.84	0.93	0.44	0.00	-2.47
s_{it}	-0.54	1.49	-4.08	3.49	0.86	0.09	-0.04	-2.87*

Notes to table: $\hat{\rho}(\tau)$ denotes the sample autocorrelation at displacement τ, and ADF denotes the augmented Dickey-Fuller statistic with augmentation lag length selected using the Akaike information criterion. Single and double asterisks denote statistical significance at the ten and five percent levels, respectively. See text for details.

Table 7: Log Posterior Odds Associated With Structural Stability

	Level Factors		Slope Factors
		Log Likelihoods	
Full-sample: 1985:9 - 2005:8	-320.48		-878.07
Sub-sample: 1985:9 - 1995:8	-195.65		-458.98
Sub-sample: 1995:9 - 2005:8	-99.51		-406.23
		Log Posterior Odds	
	25.32		12.86

Notes to table: We show Gaussian log likelihoods and posterior odds associated with structural stability analysis of the model (4)-(7). The log posterior odds value (assuming flat priors) is simply the difference between the sum of the sub-sample log likelihoods and the full-sample log likelihood.

Table 8a: First Sub-Sample Estimates of Global Yield Curve Model Parameters, 1985.09-1995.08

Global Level Factor

$$L_t = \underset{(0.03)}{0.95} L_{t-1} + U_t^l$$

Global Slope Factor

$$S_t = \underset{(0.03)}{0.94} S_{t-1} + U_t^s$$

Country Level Factors

$$l_{US,t} = \underset{(0.35)}{7.99} + \underset{(0.05)}{0.14} L_t + \varepsilon^l_{US,t} \qquad \varepsilon^l_{US,t} = \underset{(0.04)}{0.87} \varepsilon^l_{US,t-1} + \underset{(0.03)}{0.43} v^l_{US,t}$$

$$l_{GM,t} = \underset{(0.22)}{7.56} + \underset{(0.02)}{0.29} L_t + \varepsilon^l_{GM,t} \qquad \varepsilon^l_{GM,t} = \underset{(0.13)}{0.05} \varepsilon^l_{GM,t-1} + \underset{(0.03)}{0.13} v^l_{GM,t}$$

$$l_{JP,t} = \underset{(0.41)}{5.10} + \underset{(0.04)}{0.19} L_t + \varepsilon^l_{JP,t} \qquad \varepsilon^l_{JP,t} = \underset{(0.05)}{0.90} \varepsilon^l_{JP,t-1} + \underset{(0.02)}{0.31} v^l_{JP,t}$$

$$l_{UK,t} = \underset{(0.40)}{9.39} + \underset{(0.05)}{0.24} L_t + \varepsilon^l_{UK,t} \qquad \varepsilon^l_{UK,t} = \underset{(0.05)}{0.86} \varepsilon^l_{UK,t-1} + \underset{(0.03)}{0.46} v^l_{UK,t}$$

Country Slope Factors

$$s_{US,t} = \underset{(0.68)}{-2.38} + \underset{(0.06)}{0.18} S_t + \varepsilon^s_{US,t} \qquad \varepsilon^s_{US,t} = \underset{(0.03)}{0.94} \varepsilon^s_{US,t-1} + \underset{(0.04)}{0.53} v^s_{US,t}$$

$$s_{GM,t} = \underset{(0.77)}{-1.28} + \underset{(0.07)}{0.04} S_t + \varepsilon^s_{GM,t} \qquad \varepsilon^s_{GM,t} = \underset{(0.03)}{0.95} \varepsilon^s_{GM,t-1} + \underset{(0.04)}{0.61} v^s_{GM,t}$$

$$s_{JP,t} = \underset{(0.70)}{-1.40} + \underset{(0.06)}{0.08} S_t + \varepsilon^s_{JP,t} \qquad \varepsilon^s_{JP,t} = \underset{(0.03)}{0.94} \varepsilon^s_{JP,t-1} + \underset{(0.03)}{0.49} v^s_{JP,t}$$

$$s_{UK,t} = \underset{(0.47)}{-0.59} + \underset{(0.11)}{0.94} S_t + \varepsilon^s_{UK,t} \qquad \varepsilon^s_{UK,t} = \underset{(0.14)}{0.03} \varepsilon^s_{UK,t-1} + \underset{(0.14)}{0.41} v^s_{UK,t}$$

Notes to table: We report Bayesian estimates of the global yield curve model (4)-(7), obtained using monthly yields 1985.09-1995.08. We show posterior means, with posterior standard deviations in parentheses. We define $\nu_{i,t} \equiv u_{i,t}/\sigma_i$, so that $u_{i,t} \equiv \sigma_i \nu_{i,t}$. See text for details.

Table 8b: Second Sub-Sample Estimates of Global Yield Curve Model Parameters, 1995.09-2005.08

Global Level Factor	Global Slope Factor
$L_t = \underset{(0.02)}{0.96} L_{t-1} + U_t^l$	$S_t = \underset{(0.04)}{0.91} S_{t-1} + U_t^s$

Country Level Factors

$$l_{US,t} = \underset{(0.22)}{5.59} + \underset{(0.04)}{0.29} L_t + \varepsilon^l_{US,t} \qquad \varepsilon^l_{US,t} = \underset{(0.19)}{0.15}\varepsilon^l_{US,t-1} + \underset{(0.05)}{0.28} v^l_{US,t}$$

$$l_{GM,t} = \underset{(0.55)}{4.56} + \underset{(0.03)}{0.12} L_t + \varepsilon^l_{GM,t} \qquad \varepsilon^l_{GM,t} = \underset{(0.02)}{0.95}\varepsilon^l_{GM,t-1} + \underset{(0.02)}{0.25} v^l_{GM,t}$$

$$l_{JP,t} = \underset{(0.49)}{1.43} + \underset{(0.03)}{0.05} L_t + \varepsilon^l_{JP,t} \qquad \varepsilon^l_{JP,t} = \underset{(0.03)}{0.94}\varepsilon^l_{JP,t-1} + \underset{(0.02)}{0.22} v^l_{JP,t}$$

$$l_{UK,t} = \underset{(0.31)}{4.99} + \underset{(0.04)}{0.17} L_t + \varepsilon^l_{UK,t} \qquad \varepsilon^l_{UK,t} = \underset{(0.16)}{0.85}\varepsilon^l_{UK,t-1} + \underset{(0.03)}{0.31} v^l_{UK,t}$$

Country Slope Factors

$$s_{US,t} = \underset{(0.58)}{-2.18} + \underset{(0.06)}{0.09} S_t + \varepsilon^s_{US,t} \qquad \varepsilon^s_{US,t} = \underset{(0.04)}{0.90}\varepsilon^s_{US,t-1} + \underset{(0.04)}{0.65} v^s_{US,t}$$

$$s_{GM,t} = \underset{(0.78)}{-1.83} + \underset{(0.06)}{0.11} S_t + \varepsilon^s_{GM,t} \qquad \varepsilon^s_{GM,t} = \underset{(0.02)}{0.95}\varepsilon^s_{GM,t-1} + \underset{(0.04)}{0.58} v^s_{GM,t}$$

$$s_{JP,t} = \underset{(0.52)}{-1.86} - \underset{(0.04)}{0.04} S_t + \varepsilon^s_{JP,t} \qquad \varepsilon^s_{JP,t} = \underset{(0.03)}{0.93}\varepsilon^s_{JP,t-1} + \underset{(0.02)}{0.33} v^s_{JP,t}$$

$$s_{UK,t} = \underset{(0.46)}{-0.75} + \underset{(0.09)}{0.67} S_t + \varepsilon^s_{UK,t} \qquad \varepsilon^s_{UK,t} = \underset{(0.13)}{0.05}\varepsilon^s_{UK,t-1} + \underset{(0.11)}{0.39} v^s_{UK,t}$$

Notes to table: We report Bayesian estimates of the global yield curve model (4)-(7), obtained using monthly yields 1995.09-2005.08. We show posterior means, with posterior standard deviations in parentheses. We define $v_{i,t} \equiv u_{i,t}/\sigma_i$, so that $u_{i,t} \equiv \sigma_i v_{i,t}$. See text for details.

Table 9a: First Sub-Sample Variance Decompositions of Country Level and Slope Factors, 1985:09-1995:08

Level Factors

		U.S.	Germany	Japan	U.K.
	5th Percentile	12.12	96.26	54.88	41.41
Global	Median	13.69	97.97	57.09	43.39
	95th Percentile	16.12	99.23	60.05	45.86
	5th Percentile	87.88	3.74	45.12	58.59
Country	Median	86.31	2.03	42.91	56.61
	95th Percentile	83.88	0.77	39.95	54.14

Slope Factors

		U.S.	Germany	Japan	U.K.
	5th Percentile	21.28	5.34	61.69	93.66
Global	Median	23.50	6.72	64.01	96.95
	95th Percentile	26.21	8.17	67.62	99.26
	5th Percentile	78.72	94.66	38.31	6.34
Country	Median	76.50	93.28	35.99	3.05
	95th Percentile	73.79	91.83	32.38	0.74

Notes to table: For each country, we decompose country level and slope factor variation into parts coming from global factor variation and country-specific factor variation. We estimate the underlying model using monthly yield data, 1985.09-1995.08. We show posterior medians together with posterior fifth and ninety-fifth percentiles. See text for details.

Table 9b: Second Sub-Sample Variance Decompositions of Country Level and Slope Factors, 1995:09-2005:8

Level Factors

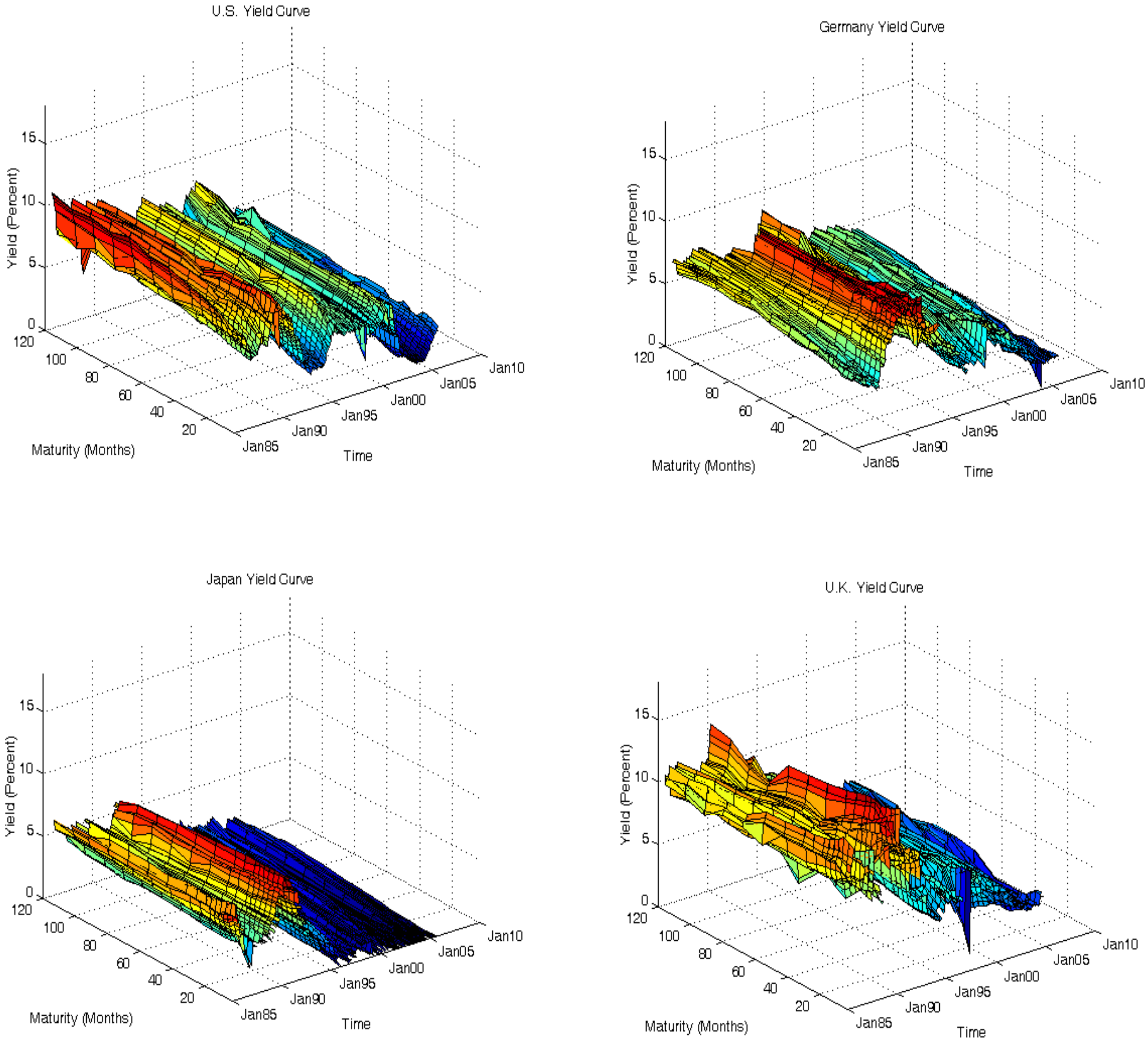
		U.S.	Germany	Japan	U.K.
	5th Percentile	88.76	43.29	51.12	52.50
Global	Median	93.49	47.63	55.54	56.82
	95th Percentile	96.04	55.06	61.21	63.07
	5th Percentile	11.24	56.71	48.88	47.50
Country	Median	6.51	52.37	44.46	43.18
	95th Percentile	3.96	44.94	38.79	36.93

Slope Factors

		U.S.	Germany	Japan	U.K.
	5th Percentile	13.08	32.48	0.99	87.97
Global	Median	15.41	35.99	1.93	93.25
	95th Percentile	18.90	41.17	3.36	97.52
	5th Percentile	86.92	67.52	99.01	12.03
Country	Median	84.59	64.01	98.07	6.75
	95th Percentile	81.10	58.83	96.64	2.48

Notes to table: For each country, we decompose country level and slope factor variation into parts coming from global factor variation and country-specific factor variation. We estimate the underlying model using monthly yield data, 1995.09-2005.08. We show posterior medians together with posterior fifth and ninety-fifth percentiles. See text for details.

Figure 1
Yield Curves Across Countries and Time



Notes to figure: All yield data are monthly, 1985.09 through 2005.08.

Figure 2a
Estimated Country Level Factors

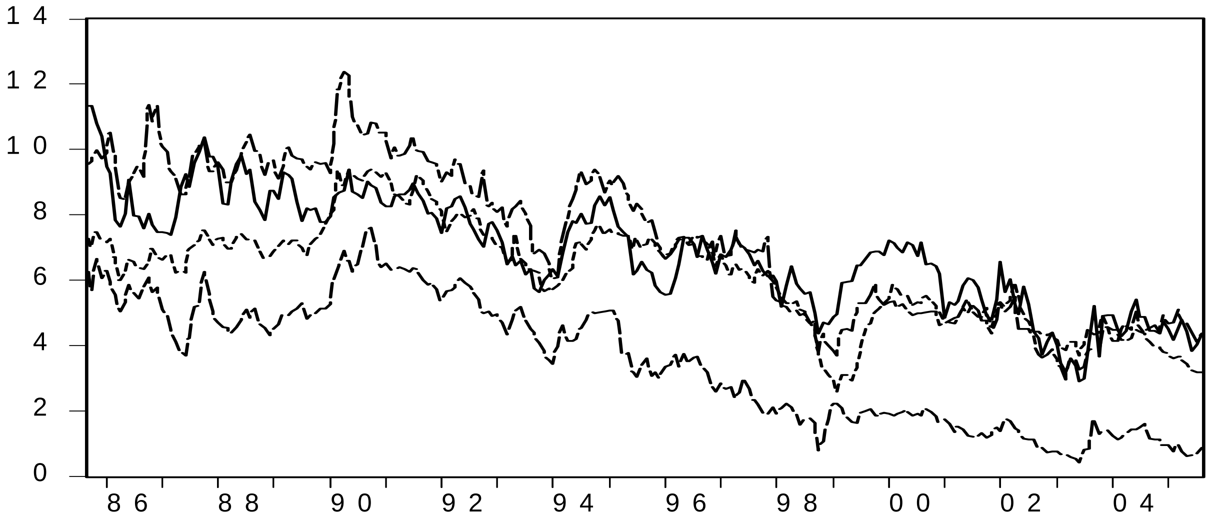
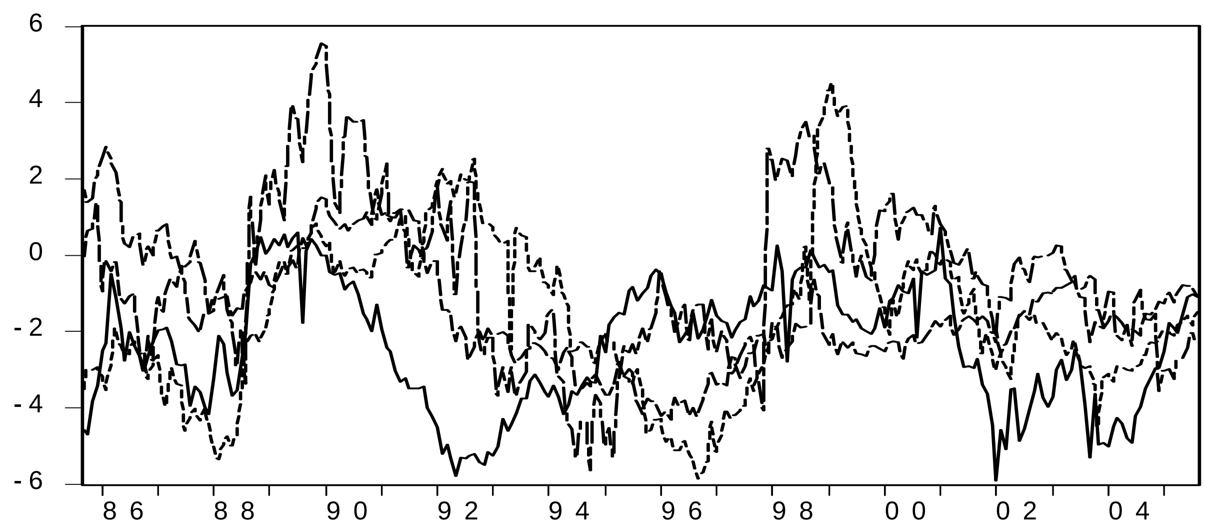


Figure 2b:
Estimated Country Slope Factors



Notes to figure: We show estimates of the country level and slope factors, obtained via step one of the Diebold-Li (2006) two-step procedure, for each of four countries, 1985.09 through 2005.08. Panel a shows levels, and panel b shows slopes. See text for details.

Figure 3a
Extracted Global Level Factor
Posterior Mean and Two Posterior Standard Deviation Band

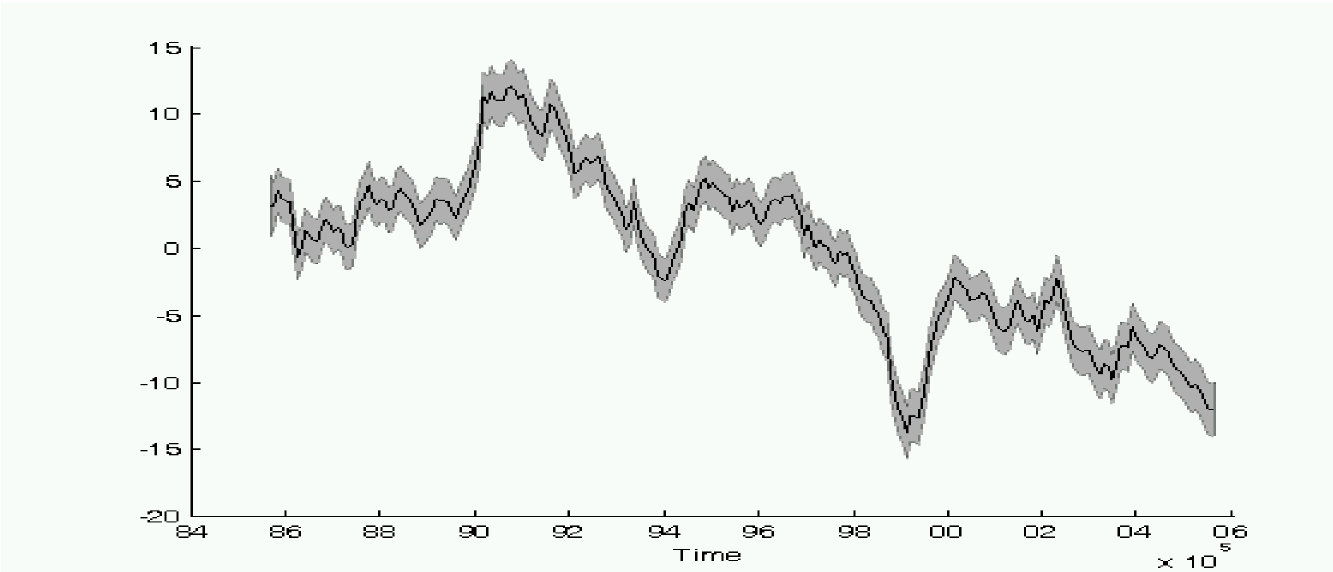
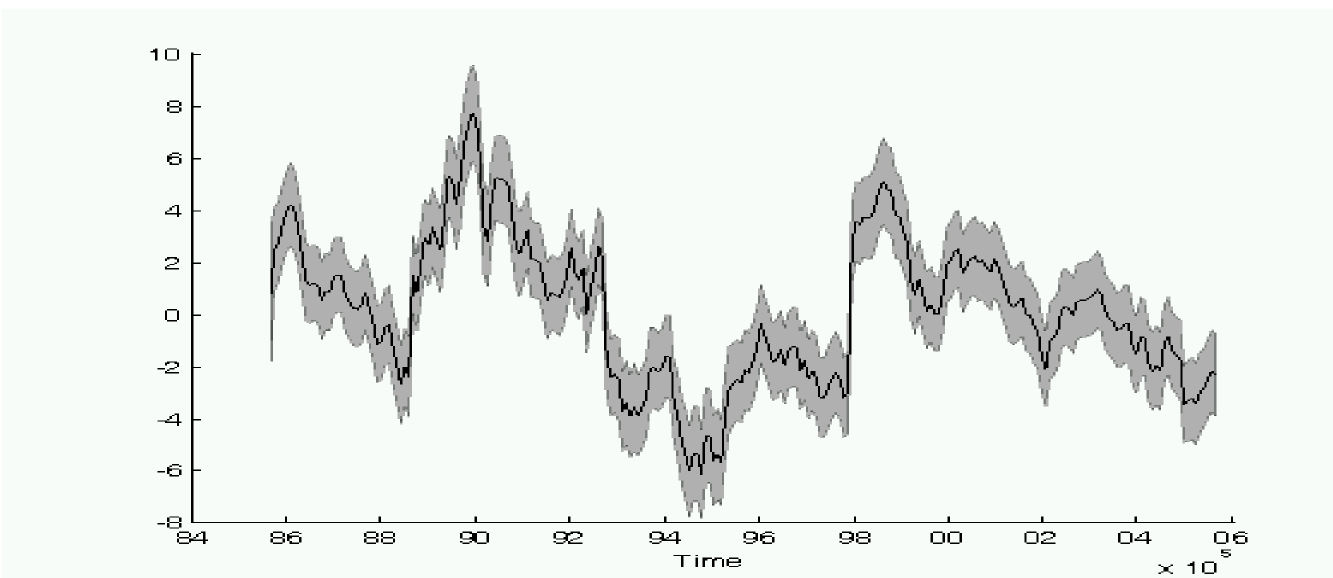


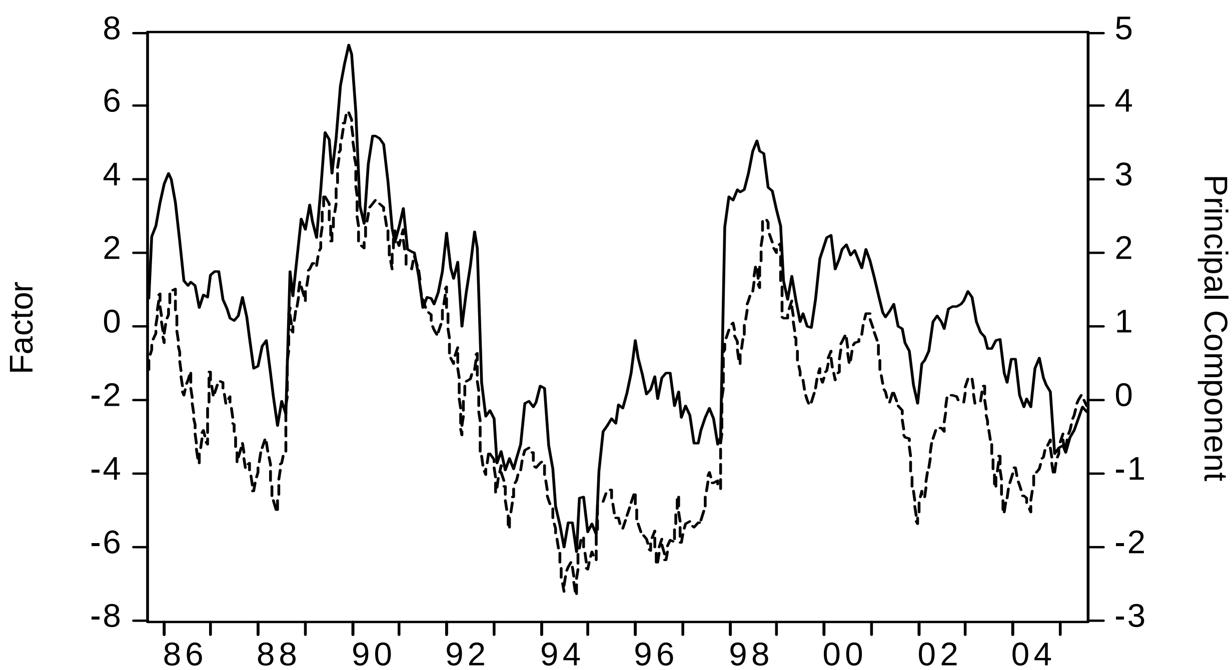
Figure 3b
Extracted Global Slope Factor
Posterior Mean and Two Posterior Standard Deviation Band



Notes to figure: We obtain posterior draws for the time series of global level and slope factors via the Carter-Kohn (1994) algorithm. We plot the posterior mean and two standard deviation band, 1985.09 through 2005.08. See text for details.

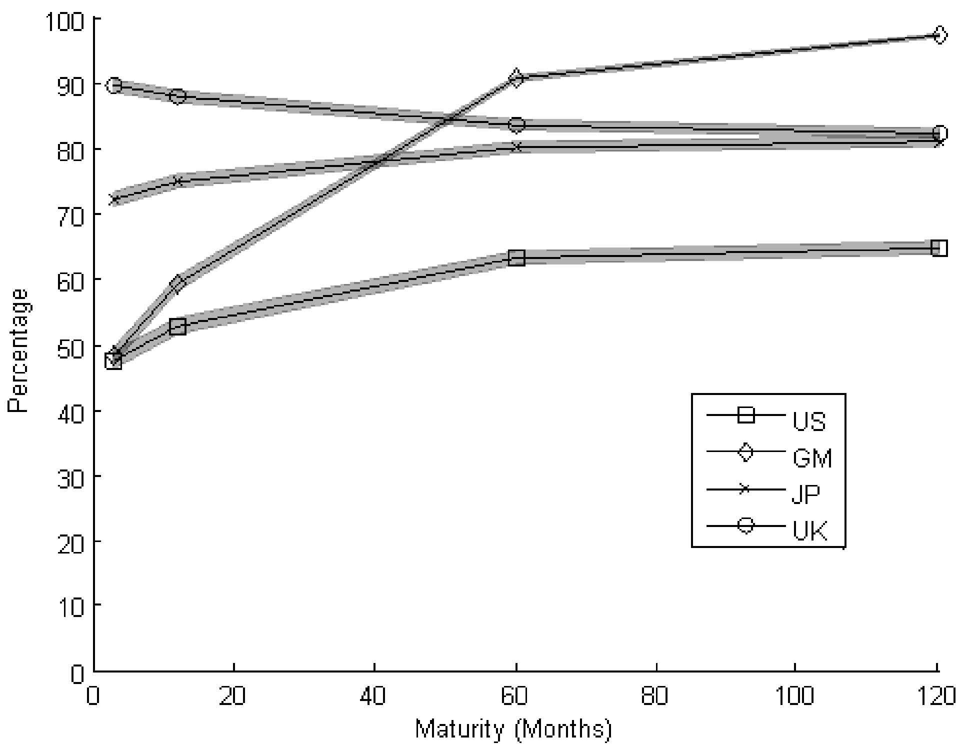
Figure 4a
Posterior Mean of Global Level Factor vs. First Principal Component of Country Yield Levels

Figure 4b
Posterior Mean of Global Slope Factor vs. First Principal Component of Country Yield Slopes



Notes to figure: We show the posterior mean as a solid line and the first principal component as a dashed line, 1985.09 through 2005.08.

Figure 5
Variance Decomposition of Country Yields



Notes to figure: For each country and four maturities, we decompose country yield variation into parts coming from global factor variation and country-specific factor variation. We estimate the underlying model using monthly yield data, 1985:09-2005:08. We show posterior medians together with posterior fifth and ninety-fifth percentile band of the fractions of variation coming from global factor. See text for details.

Figure 6a
Variance Decomposition of Country Yields, 1985:09-1995:08

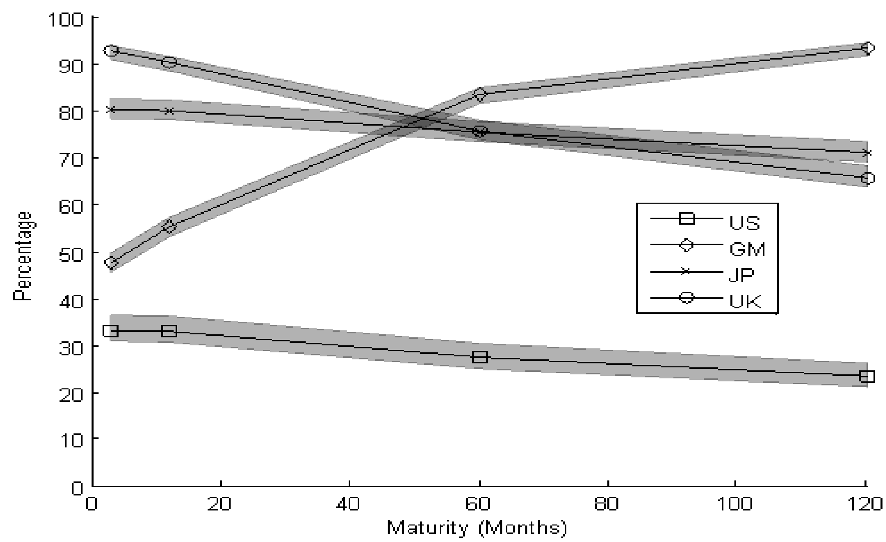
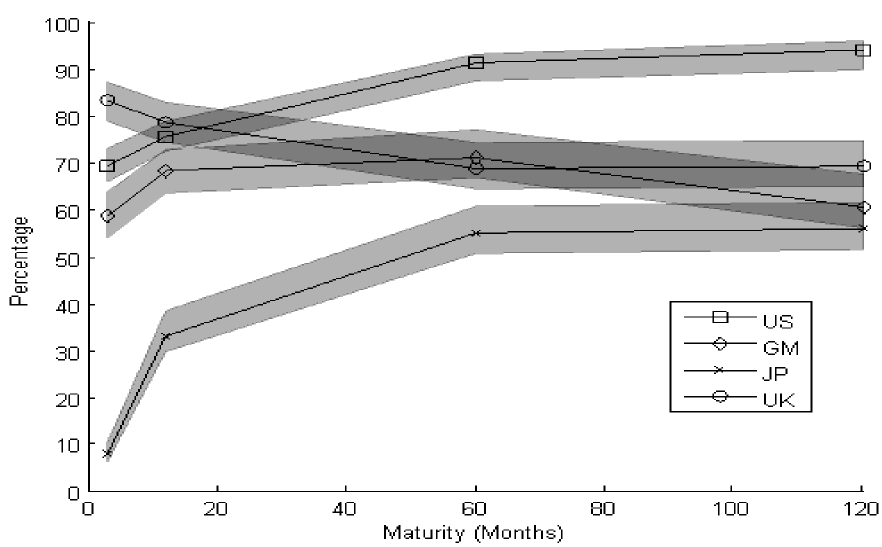


Figure 6b
Variance Decomposition of Country Yields, 1995:09-2005:08



Notes to figure: For each country and four maturities, we decompose country yield variation into parts coming from global factor variation and country-specific factor variation. We estimate the underlying model using monthly yield data. We show posterior medians together with posterior fifth and ninety-fifth percentile band of the fractions of variation coming from global factor. See text for details.

Appendix: Details of MCMC Estimation

We estimate the dynamic generalized Nelson-Siegel model in equation (3), (5), (6) and (7) using a two-step procedure. Let φ denote the set of parameters $(\alpha_i^n,\ \beta_i^n,\ \varphi_i,\ \sigma_i^n,\ \Phi)$, $i = 1, ..., K$, where $n = l, s$. Denote by F the corresponding set of global factors $\{L_t, S_t\}$.

In the first step, we estimate model (3) using OLS to obtain level and slope factors, l_{it} and s_{it}, separately for each country. We treat the estimated country level and slope factors as observed data Z_t in the second step estimation. In the second step, we view the dynamic factor model in equation (5)- (7) as a Gaussian probability density for the data $\{l_{it}, s_{it}\}$, $i = 1, ..., K$, or Z_t, conditional on a set of parameters φ and a set of latent global factors Z_t. Under the assumption of diagonal autoregressive coefficient matrices, we estimate the model factor-by-factor.

Consider, for example, the level factor. The density is $p(Z|\varphi,F)$ where Z denotes the $KT\times 1$ vector of country level factor data and F denotes the $T\times 1$ vector of global level factor data. In addition, conditional on the parameters there is a specification of a Gaussian probability density $p(F|\varphi)$ for F itself. Given a prior distribution for φ, $p(\varphi)$, the joint posterior distribution for the parameters and the latent variables is given by the product of the likelihood and prior, $p(\varphi,F|Z) = p(Z|\varphi,F)\,p(F|\varphi)\,p(\varphi)$.

Given the conjugate prior for φ, we first obtain the two conditional densities $p(\varphi|F,Z)$ and $p(F|\varphi,Z)$ via the Chib-Greenberg (1994) algorithm. Then we generate an artificial sample (φ^j, F^j) for $j = 1, ..., J$ and get the joint posterior distribution $p(\varphi, F|Z)$. More precisely, we proceed as follows:

<u>(1) Conditional distribution of the parameters given the factor, $p(\varphi|F,Z)$.</u>

We first generate the conditional distribution of measurement equation parameters given the factor. Conditional on the data and draws of the latent factor F^0, the state space measurement equations (6) are a set of independent regressions with autoregressive errors given in equation (7). To begin, define $a_i = (\alpha_i^l, \beta_i^l)'$, $l_i = (l_{i1}, ..., l_{it})'$ and $F = (L_1, ..., L_T)'$, for $i = 1, ..., K$. We use the usual conjugate prior densities, $a_i \sim N_2(\tilde{a}_i, \tilde{A}_i)$, $\varphi_i \sim N_1(\tilde{\varphi}_i, \tilde{V}_i)\, I[s(\varphi)]$, $(\sigma_i^l)^2 \sim IG(\tilde{\nu}_i/2, \tilde{\delta}_i/2)$, where IG is the inverted gamma distribution and $I[s(\varphi)]$ is the indicator function for stationarity. We use standard normals for (a_i, φ_i) and improper inverted gammas for all variances; experiments with tighter and looser priors for both the factor loadings and the autoregressive parameters did not produce qualitatively important changes in the results.

We first generate a_i, conditional on φ_i, $(\sigma_i^l)^2$, F, and l_i, $i = 1, 2, ..., K$. By multiplying both sides of equation (6a) by $1 - \varphi_{i,11} L$ and using equation (7) we have $l_{it}^* = [(1-\varphi_{i,11})\quad F_t^*]' a_i + u_{it}^l$, where $l_{it}^* = l_{it} - \varphi_{i,11}\, l_{i,t-1}$, and $F_t^* = L_t - \varphi_{i,11} L_{t-1}$. In matrix notation this is

$$l_i^* = X^* a_i + u_t^l, \tag{A1}$$

where $X^* = [(1-\varphi_{i,11})\vec{1}\quad F^*]$ and $\vec{1}$ is a vector of ones. This is a standard regression inference problem with known variance. The posterior is then $a_i|\,\varphi_i,\ (\sigma_i^l)^2,\ F,\ l_i \sim N(\bar{a}_i, \bar{A}_i)$, where $\bar{a}_i = [\tilde{A}_i^{-1} + (\sigma_i^l)^{-2} X^{*\prime} X^*]^{-1} [\tilde{A}_i^{-1}\tilde{a}_i + (\sigma_i^l)^{-2} X^{*\prime} l_i^*]$ and $\bar{A}_i = [\tilde{A}_i^{-1} + (\sigma_i^l)^{-2} X^{*\prime} X^*]^{-1}$.

We also need to generate φ_i, conditional on a_i, $(\sigma_i^l)^2$, F, and l_i, $i = 1, 2, ..., K$. We focus on equation (7), which is a standard AR(1) regression with known variance. From equation (6a) we can generate $\varepsilon_{it}^{l} = l_{it} - [1 \;\; L_t]' a_i$. In matrix notation equation (7) is

$$e_i^{\,l} = E_i^{\,l} + u_i^{\,l}, \tag{A2}$$

where $E_l^{\,i}$ is a vector of lagged country idiosyncratic factors e_{it}^{l}. The posterior is then $\varphi_i | \, a_i, \, (\sigma_i^l)^2, \, F, \, l_i \sim N(\bar{\varphi}_i, \bar{V}_i) \, I[s(\varphi)]$, where $\bar{\varphi}_i = [\tilde{V}_i^{-1} + (\sigma_i^l)^{-2} E_i' E_i]^{-1} [\tilde{V}_i^{-1} \tilde{\varphi}_i + (\sigma_i^l)^{-2} E_i' e_i]$, $\bar{V}_i = [\tilde{V}_i^{-1} + (\sigma_i^l)^{-2} E_i' E_i]^{-1}$.

Finally, we need to generate $(\sigma_i^l)^2$, conditional on a_i, φ_i, F, and l_i, $i = 1, 2, ..., K$. Given a_i and φ_i, we can use either equation (A1) or (A2) to derive the posterior distribution of $(\sigma_i^l)^2$, which is $(\sigma_i^l)^2 | \, a_i, \, \varphi_i, \, F, \, l_i \sim IG(\bar{\nu}_i/2, \bar{\delta}_i/2)$, where $\bar{\nu}_i = (\tilde{\nu}_i + T - 1)$, $\bar{\delta}_i = \tilde{\delta}_i + (l_i^* - X^* a_i)'(l_i^* - X^* a_i)$.

Next, we generate the conditional distribution of the autoregressive parameters Φ_{11} in the global level factor transition equation, given the factor F. Conditioning on the factor, the factor dynamics equation (5) is independent of the rest of the model. In matrix notation, the global level factors follow $F = X\Phi_{11} + U^l$, where X is a vector of lagged global level factors L_t.

We assume a normal prior for Φ_{11}, given by $N_1(\tilde{\Phi}_{11}, \tilde{A}) \; I[s(\Phi_{11})]$, where $I[s(\Phi_{11})]$ is an indicator function for stationarity. The posterior for Φ_{11} is then $\Phi_{11} | \, F \sim N(\bar{\Phi}_{11}, \bar{A}) \, I[s(\Phi_{11})]$, where $\bar{\Phi}_{11} = (\tilde{A}^{-1} + X'X)^{-1} (\tilde{A}^{-1} \tilde{\Phi}_{11} + X' F_l)$, $\bar{A} = (\tilde{A}^{-1} + X'X)^{-1}$.

<u>(2) Conditional distribution of the dynamic factor given the parameters, $p(F|\varphi, Z)$</u>

We next need to generate a random draw F^1 from the conditional density $p(F|\varphi, Z)$. Based on a result of Carter and Kohn (1994), Kim and Nelson (1999) show that draws from the conditional distribution of the latent factors F can be obtained as follows. First, put the model in a standard state-space form. Multiplying both sides of equation (6a) by $1 - \varphi_{i,11} L$ and using equation (7) we have the measurement equation:

$$\begin{bmatrix} l_{1t}^* \\ \cdot \\ \cdot \\ l_{Kt}^* \end{bmatrix} = \begin{bmatrix} \alpha_1^l (1 - \varphi_{1,11}) \\ \cdot \\ \cdot \\ \alpha_K^l (1 - \varphi_{K,11}) \end{bmatrix} + \begin{bmatrix} \beta_1^l & -\beta_1^l \varphi_{1,11} \\ \cdot & \cdot \\ \cdot & \cdot \\ \beta_K^l & -\beta_K^l \varphi_{K,11} \end{bmatrix} \begin{bmatrix} L_t \\ L_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t}^{\,l} \\ \cdot \\ \cdot \\ u_{Kt}^{\,l} \end{bmatrix} \quad ,$$

or $l_t^* = b + H F_t^L + e_t$, with

$$E(e_t e_t') = R = \begin{bmatrix} (\sigma_1^l)^2 & 0 & 0 & . & 0 \\ 0 & (\sigma_2^l)^2 & 0 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & (\sigma_K^l)^2 \end{bmatrix} ,$$

where $l_{it}^* = l_{it} - \varphi_{i,11} l_{i,t-1}$, $i = 1, 2, ..., K$. Similarly , the transition equation is:

$$\begin{bmatrix} L_t \\ L_{t-1} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} L_{t-1} \\ L_{t-2} \end{bmatrix} + \begin{bmatrix} U_t^l \\ 0 \end{bmatrix} ,$$

or $F_t^L = G F_{t-1}^L + w_t$, with

$$E(w_t w_t') = Q = \begin{bmatrix} (\sigma_l)^2 & 0 \\ 0 & 0 \end{bmatrix} .$$

The covariance matrix of the shocks to the transition equation, Q, is not positive-definite (Kim and Nelson (1999), pp. 194-196), but the first $J \times J$, $J=1$ block of the Q matrix, denoted by Q^*, is positive definite. Denote the first J rows of F_{t+1}^L by F_{t+1}^{L*}, and the first J rows of G by G^*.

We then run the Kalman filter forward to obtain estimates $F_{T|T}^L$ of the factors in period T, and their covariance matrix $P_{T|T}$ based on all available sample information. Then, for $t = T-1,...,1$, we proceed backward to generate draws $F_t^L | F_{t+1}^{L*}$ from $F_t | F_{t+1}^{L*}, Z, \varphi \sim N(F_{t|t, F_{t+1}^{L*}}, P_{t|t, F_{t+1}^{L*}})$, where $F_{t|t, F_{t+1}^{L*}}^L = F_{t|t}^L + P_{t|t} G^{*\prime} [G^* P_{t|t} G^{*\prime} + Q^*]^{-1} [F_{t+1}^{L*} - G^* F_{t|t}^L]$, $P_{t|t, F_{t+1}^{L*}} = P_{t|t} - P_{t|t} G^{*\prime} [G^* P_{t|t} G^{*\prime} + Q^*]^{-1} G^* P_{t|t}$, and $F_{t|t}^L$ and $P_{t|t}$ are updated estimates of the factor F_t^L and its covariance in the Kalman filter. We then use the first J rows of $F_t^L | F_{t+1}^{L*}$ as our generated global level factors F_t.

(3) Joint posterior distribution of the parameters and the factors

We repeat steps 2 and 3, drawing $\varphi^j \sim p(\varphi | F^{j-1}, Z)$ and $F^j \sim p(F | \varphi^{j-1}, Z)$ at each step. The empirical distribution of draws from the conditional posterior densities converges to the joint marginal posterior distribution as the number of iterations goes to infinity. We use chains of length 40,000 in the simulation. We discard the first 20,000 draws and use the remaining 20,000 to sample from the marginal posteriors. For chains of length 40,000 or greater, the results were the same. Sampling from the known conditional posterior densities of factors and parameters is equivalent to sampling from their unknown joint posterior distribution. We can then draw inferences about the parameters and factors from the posterior distribution.