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"A Gap for Me: Entrepreneurs and Entry"
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A Gap for Me: Entrepreneurs and Entry

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Abstract

We present a theory of entrepreneurial entry and exit decisions. Knowing their own managerial talent, entrepreneurs decide *which* market to enter, where markets differ in size. We obtain a striking sorting result: each entrant in a large market is more efficient than any entrepreneur in a smaller market since competition is endogenously more intense in larger markets. This result continues to hold when entrepreneurs can export their output to other markets, thereby incurring a unit transport cost or tariff. The sorting and price competition effects imply that the number of entrants (and hence product variety) may actually be smaller in larger markets. In the stochastic dynamic extension of the model, we show that the churning rate of entrepreneurs is higher in larger markets.

Keywords: entrepreneurship, entry, exit, firm turnover, industry dynamics.

JEL: L11, L13, M13.

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If I can make it there, I'll make it anywhere
It's up to you - New York, New York
Frank Sinatra (New York, New York)

1 Introduction

This paper presents a simple theory of entrepreneurial entry (and exit) decisions. The two main questions addressed in this paper are the following. First, what is the relationship between the size of a market and the talent of its entrepreneurs – when entrepreneurs can decide which market to enter? Second, what is the relationship between the size of a market and the turnover (or "churning") rate of entrepreneurs?

Our model of entrepreneurship is based on the idea that (potential) entrepreneurs differ in their managerial talent. Moreover, we assume that (young) entrepreneurs are "mobile" in that they can freely choose the market they want to enter. For instance, the entrepreneurs may be chefs or restaurateurs who can decide in which city to open a new restaurant, or innovative firms choosing in which country or state to locate their operations. An alternative interpretation is that instead of deciding which geographical market to enter, entrepreneurs must choose which industry to enter at a given location. This paper fills a gap in the industrial organization literature by tackling the important question of how a population of heterogeneous entrepreneurs will allocate itself across different markets. We find that this allocation will be biased, which has wide-reaching implications both for policy and for empirical work.

We address two main issues in this paper. First, and most importantly, we investigate the relationship between market size and the talent of entrepreneurs. This analysis bears on the large empirical literature on productivity differences across firms and markets; see Bartelsman and Doms (2000) for a survey. Following Sveikauskas (1975) and Henderson (1986), many empirical studies have confirmed that firms are more productive in larger cities or more densely populated regions. These productivity differences have typically been interpreted as evidence for agglomeration externalities. Our theory suggests that such agglomeration economies may be less important than previously thought – since we show that this type of findings could alternatively be explained by our theory, which relies purely on the self-selection of entrepreneurs. In fact, as Marshall (1890) observed in his *Principles of Economics*:

> The large towns and especially London absorb the very best blood from all the rest of England; the most enterprising, the most highly gifted, those with the highest physique and the strongest characters go there to find scope for their abilities.

Hence, productivity in larger towns may be higher not because larger markets make firms more productive but rather because the more capable entrepreneurs enter the larger markets.

The second main issue addressed in this paper is the relationship between market size and the turnover rate of entrepreneurs. Again, our analysis is motivated by a large body of empirical literature in industrial organization and labor economics analyzing the pattern of firm entry and exit, and gross job creation and destruction. Several interesting regularities have been identified (see, for instance, Caves (1998), Cabral (1997), and Davis and Haltiwanger (1999)). First, cross-industry differences in the level of firm turnover (or gross job reallocation) are large in magnitude and persistent over time. Second, the ranking of industries by the level of firm turnover is very similar from one country to another. Third, entry and exit rates are positively correlated across industries; that is, industries with high exit rates are likely to exhibit high entry rates as well. These regularities suggest that certain industry characteristics (such as the pattern of demand or technology) determine the turnover level. Most previous theoretical models of dynamic industry equilibrium (e.g., Jovanovic (1982), Lambson (1991), Hopenhayn (1992), Ericson and Pakes (1995), and Asplund and Nocke (2005)) have assumed that firms are identical when they decide whether or not to enter a market, and so cross-industry predictions result from comparative statics. As Asplund and Nocke (2005), we analyze the effect of market size on churning rates. In contrast to the existing literature, however, we analyze the dynamic industry equilibrium in a model with *multiple* markets, where heterogeneous entrepreneurs can self-select into markets.

We consider the entry (and exit) decisions of a pool of heterogeneous entrepreneurs. Knowing her own talent, each entrepreneur decides which market enter, where markets differ only in their size. What is the resulting relationship between the size of a market and the talent of its entrepreneurs? Existing models of competition seem to suggest that (almost) no restriction can be placed on the equilibrium pattern of talents. To take the simplest example, if firms behave as price-takers producing a single homogeneous product, then all firms would prefer to enter the market with the highest market price. Free entry then implies that the equilibrium price must be the same in all markets, in which case all entrepreneurs are indifferent between all markets, and very little can be said about the relationship between market size and the efficiency levels of firms.[1]

In this paper, we propose an alternative model where each entrepreneur has a unique "idea": the knowledge to produce a distinct product. We begin by positing that the entrepreneurial input at a given location is "essential" (or the entrepreneurial span of control exhibits strongly diminishing returns across markets), so that each entrepreneur enters at most one market. Post-entry competition is therefore imperfect, and the intensity of competition in each market is the result of entrepreneurial entry decisions. We assume that the quality of an entrepreneur's idea varies with her talent. We then obtain a striking sorting result. In the unique equilibrium, the most capable entrepreneurs all enter the largest market, somewhat less capable entrepreneurs enter the next largest market, and so on, with the least talented group entering the smallest market. That is, *the larger is the market, the more talented are its entrants*.

This sorting result follows from little-known properties of standard models of imperfect competition with heterogeneous firms, and may be explained as follows. Free (but costly) entry implies that the toughness of price competition depends on the costs and benefits of the firms serving that market. If the market price were the same across all markets, then all firms would prefer to enter the larger market because they can expect to make more sales at a given price. So any equilibrium clearly entails lower prices in larger markets, so that some firms will prefer to enter the smaller markets. Which firms remain in the larger markets? Comparing

[1] The same result would obtain in a Dixit-Stiglitz type model of monopolistic competition. There, each entrant in a given market faces the same residual demand curve of the form $D(p) = \psi p^{-\sigma}$, where σ is a parameter of the utility function, and ψ the endogenous demand level. Since each entrepreneur prefers to enter the market with the highest demand level ψ, free entry implies that, in equilibrium, each firm faces the same (residual) demand curve in all markets.

two markets of different size, an entrepreneur now faces the following trade off. In a larger market, she can expect to make greater sales (since there are more consumers) but her price-cost margins are narrower (since competition is endogenously more intense). However, a more efficient entrepreneur with a lower marginal costs will have a larger price-cost margin and thus will benefit relatively more from the increased sales a larger market allows. In equilibrium, more efficient entrepreneurs will therefore enter the larger market, while less efficient entrepreneurs will enter the smaller market. Perhaps surprisingly, the self-selection of entrepreneurs into markets implies that the number of firms, and hence product variety, may actually be greater in a *smaller* market than in a larger one.

The sorting result continues to hold when entrepreneurs have to incur (weakly) higher fixed costs in larger markets. Sorting by talent also obtains when entrepreneurs can "export" their output to other markets, thereby incurring a unit transport cost or tariff. Depending on the level of transport costs, no entrepreneur may enter the smallest market(s). In the limit as transport costs become small – for instance, due to trade liberalization – entrepreneurs enter *only* the largest market, which has potentially important implications for trade policies.

In a dynamic extension, we analyze the relationship between churning of entrepreneurs and market size. To generate endogenous churning, we assume that the quality of an entrepreneur's idea changes stochastically over time, for example because of shocks to consumers' tastes. Provided that entrepreneurial efficiencies do not change at too fast a rate, the stationary equilibrium again exhibits sorting of the most efficient entrants into the largest markets. Entry and exit will occur simultaneously into the same industry at the same location: entrepreneurs with good draws continue to survive in the market while entrepreneurs with sufficiently bad draws decide to leave it and are replaced by new entrants. Most importantly, *the equilibrium rate of firm turnover is higher in larger markets*, and so the expected life span of firms in such markets is shorter. Consequently, entrepreneurial firms tend to be younger in larger markets. This is consistent with the empirical results presented in Asplund and Nocke (2000, 2005).

As noted above, our paper is novel in considering how entrepreneurs decide which market to enter and departs from the existing literature in that our cross-industry predictions derive not from comparative statics exercises on a single-market model, but rather represent the equilibrium outcomes of a multi-market model.[2] A related literature considers the question of which members of the population should become entrepreneurs in a given market. Kihlstrom and Laffont (1979) consider the role of attitudes toward risk in entrepreneurial decision-making, which lies at the heart of the Knightian theory of entrepreneurship. We abstract from this aspect in our model of entrepreneurship and assume that entrepreneurs are risk-neutral profit maximizers. We also abstract from wealth constraints and imperfections in the capital market, which are explored in Evans and Jovanovic (1989). Holmes and Schmitz (1990) distinguish between entrepreneurial and managerial tasks to develop a Schultzian theory of entrepreneurship. In our model, these tasks are inseparable, and so a business cannot be transferred from an entrepreneur to a manager. Our theory of entrepreneurship is most closely related to that of

[2] While the problem may be viewed as a problem of matching entrepreneurs to heterogeneous markets, it differs from that analyzed in the standard matching literature (e.g., Roth and Sotomayor (1990)) in at least two respects. First, this is a matching problem with externalities since each entrepreneur's value of entering one market depends on the entry decisions of other entrepreneurs. Second, the number of entrepreneurs that are matched to one market is not fixed but endogenous.

Lucas (1978), where different agents have different levels of entrepreneurial talent. In Lucas's model, however, entrepreneurs are not free to choose between different markets, and thus he does not develop the sorting implications investigated here. Our paper is loosely connected to Rosen's (1981) model of "superstars" in which he explores the relationship between talent and earnings.[3] As in Lucas's model, all agents behave as price takers and compete in the same market, and so the issue of self-selection into different markets does not arise.

The plan of this paper is as follows. In the next section, we present the baseline model where a population of heterogeneous entrepreneurs decides which market to enter. This model is analyzed in section 3. There, we present the central sorting result of the paper: the larger is the market, the more capable are its entrepreneurs. We also show that the number of active entrepreneurs (and thus product variety) may actually be smaller in a larger market. In section 4, we analyze a stochastic dynamic extension of the baseline model and show that the churning rate of entrepreneurial firms is greater in larger markets. In section 5, we consider two further extensions of our baseline model. First, we allow fixed costs to differ across markets. Second, we explore the implications of our theory for regional or international trade by allowing entrepreneurs to export their output to other markets. We conclude in section 6.

2 The Baseline Model of Entry

We consider a model of N imperfectly competitive markets which differ in their size, S. Markets are labeled in decreasing order of market size: $S_1 > S_2 > ... > S_N$. Our preferred interpretation is that these are independent geographical markets within the same industry, and so S_i may be thought of as the mass of consumers living in market i, which we take as given. While we will henceforth adopt this interpretation, the reader may keep in mind an alternative interpretation, namely that different markets represent different industries. In that case, S_i may be thought of as a measure of aggregate sales in industry i.[4] To isolate the effect of market size, we assume that markets are identical in all other respects (but see section 5.1 for a generalization), which is a more realistic assumption under our preferred interpretation.

There is a population of (potential) entrepreneurs, each of whom may decide to enter one of the N markets, and to sell only in that market. To avoid multiplicity of equilibria and integer problems, we assume that this population forms a continuum of mass M. Each (potential) entrepreneur has a unique "idea": the know-how to produce one unique product.[5] The quality of the entrepreneur's idea varies with her entrepreneurial talent. The entrepreneur's type is denoted by c, which may be the post-entry marginal cost of the entrepreneurial firm. Alternatively, an entrepreneur's type c may be inversely related to the perceived quality of her product. In any event, lower c's will be associated with better entrepreneurs. Any heterogeneity

[3] Rosen (1982) analyzes the optimal assignment of talent to hierarchical positions within an organization, and the implications for the distribution of earnings.

[4] For instance, suppose each consumer has a two-tier utility function, where the first tier utility function is over different goods (produced in different industries), and the second tier over different varieties of the same good. Then, if the first tier utility function is Cobb-Douglas, each consumer will spend a *fixed* fraction of his income on the varieties offered in industry i. In this case, industry sales are exogenously fixed by consumer preferences.

[5] Our results would remain unchanged if we were to assume that each entrepreneur can produce the same fixed number of products.

amongst entrepreneurs is assumed to be captured by this one-dimensional type; firms are symmetric in all other respects. In the pool of potential entrants, the distribution of types is given by the cumulative distribution function $G(\cdot)$ with support $[0,1]$.

If an entrepreneur decides to enter a market, she has to pay a fixed production cost $\phi > 0$. Since each entrepreneur offers a unique differentiated product, she faces a downward-sloping residual demand curve. The gross profit of a type-c entrepreneur in market i is given by

$$S_i \Pi(c; h(\mu_i)) \geq 0.$$

The (Borel) measure μ_i summarizes the distribution of entrepreneurial types in market i. For any interval A, the number $\mu_i(A)$ thus gives the mass of entrepreneurs active in market i whose types fall into the interval A. The "intensity of competition" in market i depends on the (endogenous) distribution of entrepreneurial types and is summarized by $h(\mu_i) \in \mathbb{R}$. Here, gross profits are proportional to market size for a given population of entrants: this holds quite generally in models of competition whenever firms produce at constant marginal costs and an increase in market size means a replication of the population of consumers (leaving the distribution of consumers' tastes and incomes unchanged).[6]

We impose the following assumptions on the reduced-form profit function $S\Pi(c; h(\mu))$.

(MON) *There is a* $\overline{c}(\mu) \in (0,1]$ *such that* $\Pi(c; h(\mu)) = 0$ *for all* $c \in (\overline{c}(\mu), 1]$, whereas for $c < \overline{c}(\mu)$, $\Pi(c; h(\mu))$ is strictly decreasing in both arguments, c and $h(\mu)$.

That is, entrepreneurial firms with higher marginal cost c have lower gross profits. Moreover, a change in the distribution of active entrepreneurs that increases the toughness of competition (so that $h(\mu)$ increases) decreases profits. We allow for the possibility that sufficiently inefficient entrepreneurs (those with marginal costs $c > \overline{c}(\mu)$) cannot make positive gross profits.

(DOM) *If* $\mu'([0,c]) \geq \mu([0,c])$ *for all* $c \in (0,1]$*, then* $h(\mu') \geq h(\mu)$*. If, in addition, the inequality is strict for some* $c < \overline{c}(\mu)$*, then* $h(\mu') > h(\mu)$*.*

This assumption says that competition is more intense (in that $h(\mu)$ is larger) if the mass of active firms is larger, and the entrepreneurs are more efficient. It ensures that additional entry of entrepreneurs reduces profits, and hence the value of an entrant.

(CON) *The functions* $\Pi(c; h(\mu))$ *and* $h(\mu)$ *are continuous.*[7]

While (MON) ensures that an increase in the intensity of competition $h(\mu)$ reduces the profits of all entrepreneurs, not all types are likely to be affected to the same extent.

(COMP) *For* $h(\mu') > h(\mu)$*, the profit ratio* $\Pi(c; h(\mu'))/\Pi(c; h(\mu))$ *is strictly decreasing in* c *on* $[0, \overline{c}(\mu))$*.*

Condition (COMP) says that any change in the distribution of active types that makes competition more intense (and reduces the profits of all types), causes the gross profit of less efficient types to fall by a larger fraction than that of more efficient types.[8] This property will play a key role for the central sorting result.

To ensure that, in equilibrium, there is a positive mass of entrants in each market and some entrepreneurs (obviously, the least capable ones) do not enter any market, we assume that

[6] This (standard) assumption can easily be relaxed, as discussed in Nocke (2003).

[7] We endow the set of Borel measures on $[0,1]$ with the topology of weak* convergence.

[8] This is equivalent to assuming that (the absolute value of) the elasticity of the gross profit function with respect to c increases as the market becomes more competitive.

unbounded entry drives profits down to zero,

$$\lim_{\lambda \to \infty} S\Pi(c; h(\lambda\mu)) = 0 \text{ for all } c \in (0, 1],$$

and that the total mass of potential entrepreneurs, M, is "sufficiently large". Further, we posit that the fixed cost ϕ is "sufficiently small" so that entering an "empty" market is preferred to not entering any market:

$$S_N \Pi(1; h(\mu^0)) > \phi,$$

where μ^0 is the "null measure", i.e., $\mu^0([0, 1]) \equiv 0$.

Formally, the model may be viewed as an *anonymous game* with a continuum of players. An entrepreneur's pure strategy s is a mapping $s : [0, 1] \to \{0, 1, ..., N\}$, where $s(c) = 0$ means "do not enter", and $s(c) = i$, $i = 1, ..., N$, stands for "enter market i". We seek the pure-strategy Nash equilibrium of this game.

Our assumptions on the reduced-form profit function hold in many standard models of symmetric and non-localized competition. It is straightforward to verify this for the linear demand model described below. In the appendix, we show that the homogeneous-good Cournot model (with a finite number of firms which differ in their (constant) marginal costs) satisfies all of our assumptions on the reduced-form profit function (under very mild restrictions on demand).

Example 1 (Linear Demand) *There is a continuum of identical consumers (of mass $8S$) with utility function*

$$U = \int_0^n \left(x(k) - x^2(k) - 2\sigma \int_0^n x(k)x(l)dl \right) dk + H,$$

where $x(k)$ is the consumption of variety $k \in [0, n]$, and H the consumption of the Hicksian composite commodity (the price of which is normalized to one). The parameter $\sigma \in (0, 1)$ measures the substitutability between different varieties. The linear-quadratic utility function gives rise to the well-known linear demand system.[9] *A type-c entrepreneur has marginal cost c, independently of output. As we show in the appendix, the equilibrium profit of a type-c entrepreneur is given by*

$$S\Pi(c; h(\mu)) = \begin{cases} S(\bar{c}(\mu) - c)^2 & \text{if } c \leq \bar{c}(\mu), \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

where

$$\bar{c}(\mu) = \frac{1 + \sigma \int_0^{\bar{c}(\mu)} z\mu(dz)}{1 + \sigma\mu([0, \bar{c}(\mu)])}.$$

denotes the marginal type such that all less efficient types make zero sales even when pricing at marginal cost. The intensity of price competition $h(\mu)$ is negatively related to this marginal type $\bar{c}(\mu)$.

[9] The discrete version of the utility function goes back to Bowley (1924). The linear demand system is widely used in oligopoly models; see Vives (1999).

3 Endogenous Sorting of Entrepreneurs

In this section, we show that the unique equilibrium exhibits sorting of entrepreneurs: the most capable entrepreneurs all enter the largest market, less capable entrepreneurs enter the next largest market, and so on. This implies that the mass of active entrepreneurs and, hence, product variety may actually be smaller in a larger market.

Since the fixed cost ϕ is assumed to be "small", and the size of the pool of entrepreneurs, M, "large", any equilibrium has the following features: there is a positive mass of entrepreneurs in each market and a positive mass of entrepreneurs who prefer not to enter any market. Denote by μ_i the measure of entrepreneurs who decide to enter market i in equilibrium. Then, we must have the following ordering:

$$h(\mu_1) > h(\mu_2) > ... > h(\mu_N).$$

That is, the larger is the market, the larger is $h(\mu)$, and so the more intense is competition. To see this, suppose otherwise that there are markets i and $j > i$ such that $h(\mu_i) \leq h(\mu_j)$. This implies that *all* entrepreneurs (who are sufficiently efficient so as to make positive sales) strictly prefer to enter market i rather than market j:

$$S_i\Pi(c; h(\mu_i)) \geq S_i\Pi(c; h(\mu_j)) > S_j\Pi(c; h(\mu_j)), \;\; c \in [0, \bar{c}(\mu_j))$$

where the first inequality follows from $h(\mu_i) \leq h(\mu_j)$ and the second inequality from $S_i > S_j$. However, if all entrepreneurs preferred market i over market j, no entrepreneur would decide to enter market j, and so we could not have $h(\mu_i) \leq h(\mu_j)$.

Consider the ratio of gross profits of a type-c entrepreneur,

$$\Psi_{ij}(c) \equiv \frac{S_i\Pi(c; h(\mu_i))}{S_j\Pi(c; h(\mu_j))}. \tag{2}$$

Suppose that entrepreneurial type c_{ij} is indifferent between entering market i and market $j > i$, and so $\Psi_{ij}(c_{ij}) = 1$. Since $h(\mu_i) > h(\mu_j)$, condition (COMP) implies that the profit ratio $\Psi_{ij}(c)$ is strictly decreasing in c for $c < \bar{c}(\mu_i)$. Hence, all entrepreneurs more capable than c_{ij} strictly prefer to enter (the larger but more competitive) market i, while less capable entrepreneurs strictly prefer to enter (the smaller but less competitive) market j. Since there can be at most one entrepreneur (with positive profit) who is indifferent between the two markets, we obtain the central sorting result of this paper.

Proposition 1 *There exists a unique equilibrium. In equilibrium, there are marginal types* $0 \equiv c_0 < c_1 < ... < c_N$ *such that (almost) all entrepreneurs of type* $c \in [c_{i-1}, c_i)$ *enter market* i*, while (almost) all entrepreneurs of type* $c \in [c_N, 1]$ *do not enter any market. Hence, each entrepreneur in a given market is more capable than any entrepreneur in a smaller market.*

Proof. See appendix. ■

The proposition shows that the relationship between the characteristics of a market and the talents of its entrepreneurs takes a surprisingly extreme form: the larger is the market, the more talented are its entrepreneurs in that each entrepreneur in a large market is more efficient than any entrepreneur in a smaller market. Consequently, the total mass of entrepreneurs in market i whose types fall into the interval $[0, z]$ is given by

$$\mu_i([0,z]) = \begin{cases} 0 & \text{if } z < c_{i-1} \\ M\left[G(z) - G(c_{i-1})\right] & \text{if } z \in [c_{i-1}, c_i) \\ M\left[G(c_i) - G(c_{i-1})\right] & \text{if } z \geq c_i \end{cases}$$

The sorting result obtains since more capable entrepreneurs are better off in a larger and endogenously more competitive market whereas less capable entrepreneurs are better off in a smaller and hence less competitive market. To obtain a better intuition, suppose that a type-c entrepreneur produces at constant marginal cost $k(c)$, where $k'(c) > 0$. Gross profit then takes the form $S\Pi(c; h(\mu)) = [p(c; h(\mu)) - k(c)]\, q(c; h(\mu), S)$. Suppose type c_{ij} is indifferent between entering market i and (the smaller) market $j > i$. Intuitively, one would expect that type c_{ij} would charge a lower price in the endogenously more competitive market i than in market j, i.e., $p(c_{ij}; h(\mu_i)) < p(c_{ij}; h(\mu_j))$. Indeed, as the following proposition shows, condition (COMP) is *equivalent* to requiring that an increase in the toughness of price competition (an increase in $h(\mu)$) results in lower equilibrium prices for all types with positive sales.[10]

Proposition 2 *Suppose (entrepreneurial) firms have constant marginal costs* $k(c)$*, where* $k'(c) > 0$*, so that the gross profit of a type-c firm can be written as* $S\Pi(c; h(\mu)) = [p(c; h(\mu)) - k(c)]q(c; h(\mu), S)$*, where* $p(c; h(\mu))$ *is equilibrium price, and* $q(c; h(\mu), S)$ *equilibrium output. Then, assumption (COMP) holds if and only if, for* $c \in [0, \bar{c}(\mu))$*, the equilibrium price* $p(c; h(\mu))$ *is decreasing in the intensity of price competition* $h(\mu)$*.*

Proof. See appendix. ■

Since entrepreneurial type c_{ij} is indifferent between entering market i and market j, but would charge a lower price in the more competitive market i, it follows that she would sell a larger quantity in that market:

$$q(c_{ij}; h(\mu_i), S_i) > q(c_{ij}; h(\mu_j), S_j). \tag{3}$$

Do entrepreneurs who are marginally more capable than type c_{ij} prefer to enter the smaller or the larger market? From the envelope theorem, the additional profit from a marginal decrease in c is equal to $k'(c)q$, which is increasing in output q. From (3), it follows that an entrepreneur who is slightly more efficient than type c_{ij} strictly prefers to enter market i rather than the smaller market j, while a slightly less talented entrepreneur strictly prefers to enter the smaller market. Hence, more talented entrepreneurs sort into larger markets than less talented entrepreneurs.

As in Lucas (1978), the size distribution of firms in a given market is determined by the underlying distribution of entrepreneurial talent (namely, the distribution function $G(\cdot)$). While our theory does not impose testable restrictions on the size distribution within a given market, it does allow us to make predictions across markets. Suppose we measure firm size by output

[10] Note, however, that the Dixit-Stiglitz model (with a continuum of firms with constant marginal costs) does not have this property; there, firms' markups are completely independent of the state of competition, and so $\Pi(c; h(\mu'))/\Pi(c; h(\mu))$ is independent of c, violating (COMP).

$q(c;h(\mu),S)$, and assume that output is decreasing in the entrepreneur's type c (which indeed it is if marginal costs are increasing in c). As discussed above any entrepreneur, who is indifferent between entering two markets would produce a greater output in the larger market. Since the more talented entrepreneurs enter the larger market, and the less talented ones the smaller market, our model predicts that firms located in larger markets are larger than those in smaller markets.

Let us now reconsider the relationship between market size and the number of entrants. Following Bresnahan and Reiss (1991), a number of researchers have found that the ratio between the number of firms and market size is smaller in larger markets. This finding has been interpreted as evidence for the existence of the price competition effect: an increase in market size typically leads to more entry, and then the price competition effect implies a fall in price-cost margins. Hence, in larger markets, market size has to increase by a larger amount so as to sustain an additional firm in the market. The existing studies have implicitly assumed that the distribution of entrants' efficiency levels does not vary across markets. In particular, they have not allowed for self-selection of entrepreneurs at the entry stage.

In our model, the sorting effect may reinforce the price competition effect: since more efficient firms self-select into larger markets, entry causes price-cost margins to fall "at a much faster rate" with market size than without sorting. In fact, the sorting effect may be so strong that a larger market may have fewer entrepreneurs and, hence, less product variety to offer than a smaller market. This counterintuitive relationship may arise since competition in a market may be more intense for two reasons: (i) there is a larger population of active entrepreneurs, and (ii) the active entrepreneurs are more efficient. Moreover, the endogenous intensity of price competition changes continuously with market size, whereas the average efficiency of its entrepreneurs may change discontinuously. Suppose that the difference in size between markets i and $j > i$ is small. Then, competition in market i is not much more intense than in market j in the sense that $h(\mu_i) - h(\mu_j)$ is small. But entrepreneurs in market i are much more capable than those in j, and so we may have $\mu_i([0,1]) = M\left(G(c_i) - G(c_{i-1})\right) < M\left(G(c_j) - G(c_{j-1})\right) = \mu_j([0,1])$ even though $h(\mu_i) > h(\mu_j)$. This is illustrated in the following numerical example.

Example 2 (Linear Demand) *Consider the linear demand example. Suppose there are only two markets, $N = 2$, and entrepreneurial types are uniformly distributed on the unit interval, i.e., $G(c) = c$ for $c \in [0,1]$. In the unique equilibrium, the marginal types c_1 and $c_2 > c_1$ are determined by*

$$S_1 \left(\frac{1 + \sigma M c_1^2/2}{1 + \sigma M c_1} - c_1 \right)^2 = S_2 \left(\frac{1 + \sigma M \left(c_2^2 - c_1^2\right)/2}{1 + \sigma M (c_2 - c_1)} - c_1 \right)^2,$$

$$\frac{S_2}{8} \left(\frac{1 + \sigma M \left(c_2^2 - c_1^2\right)/2}{1 + \sigma M (c_2 - c_1)} - c_2 \right)^2 = \phi.$$

Assume that $\phi = 1/8$, $\sigma M = 2$, $S_1 = 50$, and $S_2 = 20$. The marginal types are then given by $c_1 \approx 0.360$ and $c_2 \approx 0.606$. As expected, the total mass of entrepreneurial firms is larger in the larger market: $\mu_1([0,1])/M = c_1 \approx 0.360$, whereas $\mu_2([0,1])/M = (c_2 - c_1) \approx 0.246$. Assume now that the smaller market 2 is larger, $S_2 = 30$. In this case, $c_1 \approx 0.300$ and $c_2 \approx 0.609$. Perhaps surprisingly, there are more (but less capable) entrepreneurs in the smaller market:

$\mu_1([0,1])/M \approx 0.300 < 0.309 \approx \mu_2([0,1])/M$. *More generally, whenever the difference in size between any two markets (as measured by* $S_i - S_{i+1}$*) is sufficiently large, there is a positive correlation between market size and the number of entrepreneurs (or product variety) across markets. If, however, the differences are sufficiently small (in that* $S_1 - S_N$ *is small), there is a* negative *cross-sectional correlation.*

Empirical Evidence. Following Sveikauskas (1975) and Henderson (1986), there is an empirical literature on productivity differences across cities and regions. A robust finding is that total factor productivity is higher in larger cities or more densely populated regions. In a recent paper using Japanese data, Davis and Weinstein (2001) find that, ceteris paribus, a doubling of region size raises productivity by 3.5 percent. Syverson (2004) shows that cement plants are more efficient in more densely populated U.S. metropolitan areas. The urban and regional economics literature has traditionally attributed these productivity differences to *agglomeration externalities*. The present model suggests that a different force may be at work: more productive firms may endogenously select into larger markets. By failing to account for self-selection, however, the empirical literature may overestimate the role of externalities.

4 The Dynamic Model with Entry and Exit

Rates of firm turnover differ substantially across industries. These differences are similar from one country to another, and stable over time. While these cross-industry differences are not yet very well understood, there is a small number of models that attempt to relate firm turnover to observable industry characteristics. For example, Hopenhayn (1992) and Lambson (1991) consider the effect of sunk costs in a dynamic model with price-taking firms. Asplund and Nocke (2005) analyze the impact of market size and sunk costs in a dynamic model of imperfect competition, and show that turnover rates are positively related to market size. In these single-industry models, firms are ex-ante identical, and it is implicitly assumed that the distribution of entrants' characteristics (such as entrepreneurial "talent") are identical across industries.

In this section, we re-examine the relationship between market size and turbulence, but take a different approach from the existing literature: we analyze the impact of market size on firm turnover in a multi-market model where the distribution of entrants' capabilities may vary endogenously across markets.

The Dynamic Model. We assume that time is discrete, firms have an infinite horizon and a common discount factor $\delta \in [0, 1)$.[11] In each period, there is a mass M of "young" entrepreneurs whose current types are distributed according to $G(\cdot)$. Knowing her current type, a young entrepreneur decides whether to enter a market and if so, which one of the N markets. As before, each entrepreneur can enter at most one market. To generate turbulence, we assume that the quality of an entrepreneur's idea changes stochastically over time. This may be due to shocks to consumers' tastes for the entrepreneur's product. Specifically, with probability $\alpha \in (0, 1)$, the entrepreneur will be of the same efficiency as in the last period, whereas with the remaining probability $1 - \alpha$ she gets a new draw of her type from a continuous and strictly increasing distribution function $F(\cdot)$. A currently more efficient entrepreneur is more likely

[11] If the probability of the entrepreneur's (physical) death in a period is γ and the factor of time preference is $\widetilde{\delta}$, the effective discount factor becomes $\delta = \widetilde{\delta}(1 - \gamma)$.

to be efficient in the future than a currently less efficient entrepreneur, and this persistence is measured by the probability α.

The timing in each period is as follows. At the first stage, young entrepreneurs (potential entrants) and old entrepreneurs (incumbents) "learn" the realization of their current types. At the second stage, young entrepreneurs make their entry decisions and incumbents decide whether or not to exit the market. Entrepreneurs who decide not to be active take up an outside option, normalized to zero. Re-entry after exit is not possible. We assume that only young entrepreneurs are (geographically) mobile, which implies that old entrepreneurs cannot switch from one market to another. At the third and final stage, the active entrepreneurs in a given market compete and obtain a gross profit $S\Pi(c; h(\mu))$, which depends on their current type c, the endogenous intensity of price competition $h(\mu)$, and the size of their market, S. Moreover, active entrepreneurs pay a fixed per-period cost $\phi > 0$, which ensures that the least efficient entrants will not enter any market and that sufficiently inefficient incumbents will decide to leave the market. We impose the same structure on the gross profit function as in the baseline model without trade: (MON), (CON), (DOM), and (COMP) are assumed to hold.

Stationary Equilibrium. We confine attention to stationary equilibria in which the entrepreneurial entry and exit strategies, and hence the distribution of active types in each market, are time-independent.

Let $V(c; h(\mu), S)$ denote the value (at stage 2) of a type-c entrepreneur in a market of size S, where the distribution of types is given by μ. Since the entrepreneur has the option to leave the market, this value satisfies

$$V(c; h(\mu), S) = \max\{\overline{V}(c; h(\mu), S), 0\},$$

where

$$\overline{V}(c; h(\mu), S) = S\Pi(c; h(\mu)) - \phi + \delta \left\{ \alpha V(c; h(\mu), S) + (1-\alpha) \int_0^1 V(u; h(\mu), S) F(du) \right\}$$

is the value conditional on staying in the market in the current period, and behaving optimally thereafter. It is straightforward to see that $\overline{V}(c; h(\mu), S)$ is continuous and decreasing in c on $[0, \overline{c}(\mu))$. Let $c^*(\mu, S)$ denote the optimal exit policy: all more efficient types prefer to stay in the market, all less efficient types prefer to leave the market and take up the outside option, i.e., $\overline{V}(c^*(\mu, S); h(\mu), S) = 0$. If $c^*(\mu, S) < \overline{c}(\mu)$ (as we assume for the moment), $c^*(\mu, S)$ is unique.

The conditional value can be re-written as

$$\overline{V}(c; h(\mu), S) = \frac{1}{1-\delta\alpha} \{ S\Pi(c; h(\mu)) - \phi \; + \frac{\delta(1-\alpha) F(c^*(\mu, S))}{1 - \delta\alpha - \delta(1-\alpha) F(c^*(\mu, S))} \times \int_0^{c^*(\mu,S)} [S\Pi(u; h(\mu)) - \phi] \, F(du) \},$$

and so $\overline{V}(c; h(\mu), S) \approx \{S\Pi(c; h(\mu)) - \phi\} / [1 - \delta\alpha]$ if $\delta(1-\alpha)/(1-\delta) \approx 0$. That is, as the parameter of cost persistence, α, goes to one (or the discount factor δ goes to zero), a firm's value – conditional on staying in the market for another period – becomes proportional to its current gross profit. We obtain the following result.

Proposition 3 *Suppose that costs are persistent over time (or the discount factor is small) such that $\delta(1-\alpha)/(1-\delta)$ is sufficiently small. Then, the unique stationary equilibrium of the dynamic model exhibits sorting of entrepreneurs by types. That is, there exist marginal types $0 \equiv c_0 < c_1 < ... < c_N$ such that (almost) all entrepreneurs of type $c \in [c_{i-1}, c_i)$ enter market i, while (almost) all entrepreneurs of type $c \in [c_N, 1]$ do not enter any market.*

Proof. See appendix. ■

Note that while the sorting result applies to new entrants, it is no longer true that any entrepreneur in a larger market is more talented than all entrepreneurs in smaller markets. But, as we will show below, a weaker result holds: the least efficient entrepreneur in a larger market is more talented than the least efficient entrepreneur in a smaller market.

From now on, let us assume that the unique stationary equilibrium exhibits sorting of types, as it indeed does under the condition of proposition 3. For any markets i and j, there exists a unique type c_{ij} such that $\overline{V}(c_{ij}; h(\mu_i), S_i) = \overline{V}(c_{ij}; h(\mu_j), S_j) \geq 0$. In the stationary equilibrium, the total mass of entrants per period is equal to the total mass of exiting firms:

$$[G(c_i) - G(c_{i-1})]M = (1-\alpha)[1 - F(c^*(\mu_i, S_i))]\mu_i([0,1]).$$

The total mass of entrepreneurs active in market i is then given by

$$\mu_i([0,1]) = \frac{[G(c_i) - G(c_{i-1})]M}{(1-\alpha)[1 - F(c^*(\mu_i, S_i))]}.$$

While the value of the least efficient entrant in the smallest market is zero, it is strictly positive in any other market $i < N$, $\overline{V}(c_i; h(\mu_i), S_i) > 0 = \overline{V}(c_N; h(\mu_N), S_N)$. Since the value of the least efficient incumbent (who is just indifferent between exiting and staying in the market) is zero, it follows that the marginal incumbent is less efficient than the least efficient entrant in that market (except for the smallest market):

$$c^*(\mu_i, S_i) > c_i \text{ for } i = 1, ...N-1, \text{ and } c^*(\mu_N, S_N) = c_N. \tag{4}$$

In each period, a share

$$\theta_i \equiv (1-\alpha)[1 - F(c^*(\mu_i, S_i))] \tag{5}$$

of entrepreneurs exit market i. Given our simple stochastic process, the probability of exit is independent of the entrepreneurial type (within the same market), and so θ_i is equal to each incumbent's probability of exit in market i. We will henceforth use θ as our measure of firm turnover. As equation (5) shows, turnover rate θ varies across markets if different exit policies $c^*(\mu, S)$ are used in different markets: the tougher is the exit policy (the smaller is $c^*(\mu, S)$), the higher is the turnover rate θ.

We now claim that the marginal incumbent in market i is more efficient in larger markets, i.e., $c^*(\mu_i, S_i)$ is increasing in i. To see this, recall that (i) entrepreneurial type c_i, $i < N$, is indifferent between entering the larger market i and the smaller market $i+1$, i.e., $\overline{V}(c_i; h(\mu_i), S_i) = \overline{V}(c_i; h(\mu_{i+1}), S_{i+1})$, and that (ii) from equation (4), the marginal incumbent in market $i < N$ is less efficient than the least efficient entrant in that market, i.e.,

$c^*(\mu_i, S_i) > c_i$. From the single-crossing property of $\overline{V}$, it then follows that entrepreneurial type $c^*(\mu_i, S_i)$ is better off in the less competitive market $i+1$ than in market i, and so

$$0 = \overline{V}(c^*(\mu_i, S_i); h(\mu_i), S_i) < \overline{V}(c^*(\mu_i, S_i); h(\mu_{i+1}), S_{i+1}).$$

Since $\overline{V}(c^*(\mu_{i+1}, S_{i+1}); h(\mu_{i+1}), S_{i+1}) = 0$, and the value function $\overline{V}$ is strictly decreasing in its first argument, it follows that $c^*(\mu_{i+1}, S_{i+1}) > c^*(\mu_i, S_i)$.

The equilibrium exit policy is tougher in larger markets, and so the turnover rate θ_i must also be higher in larger markets. We summarize our result on turnover in the following proposition.

Proposition 4 *Suppose the stationary equilibrium exhibits sorting by types: for any two markets i and j, there exists a unique type c_{ij} such that $\overline{V}(c_{ij}; h(\mu_i), S_i) = \overline{V}(c_{ij}; h(\mu_j), S_j) \geq 0$. Then, the marginal entrepreneur is less efficient in smaller markets, i.e., $c^*(\mu_i, S_i)$ is increasing in i. Hence, the equilibrium turnover rate is larger in larger markets, i.e., θ_i is decreasing in i.*

The proposition implies that the range of efficiency levels of firms within a market is smaller in larger markets.[12] Moreover, in smaller markets, the distribution of entrepreneurial types is shifted towards less efficient types in the sense of first-order stochastic dominance since entrants are less efficient in smaller markets, and incumbents' exit policy is less tough.

There is a close link between firm turnover and the age distribution of businesses. Intuition suggests that markets with higher turnover rates have on average younger firms. Let the period-t age of a firm that entered in period $t^e \leq t$ be given by $t - t^e + 1$. Then, in stationary equilibrium, the average firm age in market i is equal to $1/\theta_i$. Furthermore, the share of firms active in market i whose age is less than or equal to $y \geq 1$ is given by

$$\frac{\sum_{t=0}^{y-1}(1-\theta_i)^t}{\sum_{t=0}^{\infty}(1-\theta_i)^t} = 1 - (1-\theta_i)^y.$$

For $y > 1$, this expression is increasing in θ_i. Since the turnover rate θ_i is decreasing in i, we obtain the following result.

Corollary 1 *Suppose the condition of proposition 4 holds. Then, in stationary equilibrium, firms are on average older in smaller markets. Specifically, the age distribution of firms in smaller markets first-order stochastically dominates that in larger markets.*

It is straightforward to show that the (conditional) value of *any* type is the same across markets if firms behave as price takers or compete *à la* Dixit-Stiglitz. In this case, sorting of firms does not obtain, and the turnover rate and age distribution do not vary across markets.

Empirical Evidence. How can our predictions be tested empirically? The magnitude of the underlying fluctuations in the pattern of demand (or technology) is likely to vary greatly across industries. As pointed out by Sutton (1997), this factor may be of primary importance, but it is very difficult to measure it or to control for its impact empirically. This causes a serious problem for any empirical test of cross-industry predictions on firm turnover. Fortunately,

[12] This is consistent with the empirical evidence on cement plants; see Syverson (2004).

an attractive feature of our theory is that its predictions on turnover rates can be tested by comparing turnover rates of local service firms in different-sized local markets within the same industry. This should control for many of those factors that would otherwise differ across industries. This is the route taken in Asplund and Nocke (2000), where we use data on driving schools in Sweden. Estimating the probability of exit in a Probit model, we find some supportive evidence for the prediction that turnover rates are higher in larger municipalities. In more recent work, Asplund and Nocke (2005), we analyze the age distribution of hairdressers in Sweden. Using non-parametric tests, we find that the age distribution of firms in smaller markets first-order stochastically dominates the age distribution in larger markets, as predicted by our theory.

5 Robustness and Extensions

In this section, we consider two extensions of our baseline model. First, we investigate the robustness of our predictions by allowing markets to differ not only in their size but also in the level of fixed costs. Second, we extend the baseline model by allowing entrepreneurs to export their product from their home market to all other markets at a unit transport cost or tariff. To keep this section short and to the point, we analyze these extensions for the case of the linear demand model. However, we will also briefly remark on what properties of the reduced-form profit function are sufficient to obtain our results.

5.1 Sorting of Entrepreneurs when Markets Differ in Size and Fixed Costs

So far, we have assumed that markets differ only in their size but are identical otherwise. This served to make our point most forcefully: everything else equal, the most capable entrepreneurs will enter the largest market, while less capable entrepreneurs will self-select into smaller markets. In empirical applications, however, markets may differ not only in their size but also along other dimensions. This problem arises in particular under the alternative interpretation where different markets represent different industries (rather than different local markets within the same industry). We now show that our sorting result continues to hold when fixed costs are non-negatively related to market size.

Suppose that demand in each market i is linear, and so the gross profit of a type-c entrepreneur who produces at marginal cost c in market i is given by

$$\begin{aligned} S_i\Pi(c;h(\mu_i)) &= [p(c;h(\mu_i)) - c]\, q(c;h(\mu_i), S_i) \\ &= \begin{cases} S_i\left(\overline{c}(\mu_i) - c\right)^2 & \text{if } c \leq \overline{c}(\mu_i), \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

(Recall that the intensity of price competition in market i, $h(\mu_i)$, is inversely related to the marginal type $\overline{c}(\mu_i)$.) We assume that fixed costs are weakly larger in larger markets, i.e., $\phi_{i+1} \geq \phi_i$ for any $i < N$. For simplicity, we posit that differences in fixed costs across markets are not too large (relative to differences in market size) so that a positive mass of entrepreneurs will enter each market in equilibrium.

Consider an entrepreneur of type c_{ij} who is indifferent between entering market i and the smaller market $j > i$,

$$\begin{aligned} \psi(c_{ij}) &\equiv [S_i\Pi(c_{ij}; h(\mu_i)) - \phi_i] - \left[S_j\Pi(c_{ij}; h(\mu_j)) - \phi_j\right] \\ &= \left[S_i\left(\overline{c}(\mu_i) - c_{ij}\right)^2 - \phi_i\right] - \left[S_j\left(\overline{c}(\mu_j) - c_{ij}\right)^2 - \phi_j\right] = 0. \end{aligned}$$

We now want to show that all entrepreneurial types more capable than c_{ij} will then strictly prefer to enter the larger market i, i.e., $\psi'(c_{ij}) < 0$. To see this, note that

$$\begin{aligned} \psi'(c_{ij}) &= -q(c_{ij}; h(\mu_i), S_i) + q(c_{ij}; h(\mu_j), S_j) \\ &= -2S_i\left(\overline{c}(\mu_i) - c_{ij}\right) + 2S_j\left(\overline{c}(\mu_i) - c_{ij}\right), \end{aligned} \tag{6}$$

which is negative if and only if type c_{ij} would produce a larger output in market i than in the smaller market j. We distinguish between two cases. (i) If competition is not more intense in market i than in the smaller market j, i.e., $\overline{c}(\mu_i) \geq \overline{c}(\mu_j)$, it follows from (6) and $S_i > S_j$ that $\psi'(c_{ij}) < 0$. (ii) If competition is more intense in the larger market i, i.e., $\overline{c}(\mu_i) < \overline{c}(\mu_j)$, then from (COMP) and proposition 2, we know that type c_{ij} would charge a lower price in the more competitive market i, i.e.,

$$p(c_{ij}; h(\mu_i)) = \frac{\overline{c}(\mu_i) + c_{ij}}{2} < \frac{\overline{c}(\mu_j) + c_{ij}}{2} = p(c_{ij}; h(\mu_j)).$$

By definition, type c_{ij} would make the same net profit in both markets. Since fixed costs are weakly larger in market i, it follows that her gross profit in market i would be larger than or equal to that in the smaller market j. But since she would charge a lower price in market i, we can conclude that she would produce a larger quantity in that market, and so $\psi'(c_{ij}) < 0$. Hence, our sorting result of proposition 1 extends to the case where fixed costs and market size are positively related.

Proposition 5 *Suppose fixed costs are weakly larger in larger markets, i.e., $\phi_{i+1} \geq \phi_i$ for all $i \in \{1, ..., N-1\}$. Then, in equilibrium there are marginal types $0 \equiv c_0 < c_1 < ... < c_N$ such that (almost) all entrepreneurs of type $c \in [c_{i-1}, c_i)$ enter market i, while (almost) all entrepreneurs of type $c \in [c_N, 1]$ do not enter any market. Hence, each entrepreneur in a given market is more capable than any entrepreneur in a smaller market.*

While we have derived this result for our static baseline model, it is straightforward to see that the same sorting result applies to our dynamic model as long as $\delta(1-\alpha)/(1-\delta)$ is sufficiently small. Furthermore, this implies that our previous result on turbulence (proposition 4) extends as well to the case where fixed costs are weakly increasing with market size.

What assumption on the reduced-form profit function gives rise to this sorting result? As should be clear from our discussion above, the key step consists in showing that the marginal type c_{ij} would produce a larger output in the larger market i than in market j. If competition is (endogenously) at least as intense in the larger market i (case (ii) above), then this follows immediately from our earlier assumption (COMP). Otherwise (case (i) above), we need an additional assumption on the reduced-form profit function, namely that the equilibrium output of a firm is decreasing as more firms (or more efficient firms) enter, and price competition is more intense, i.e., $q(c; h(\mu), S)$ is decreasing in $h(\mu)$. In terms of the reduced-form profit function, this is equivalent to assuming that for $h(\mu') > h(\mu)$, the profit difference $\Pi(c; h(\mu)) - \Pi(c; h(\mu'))$ is decreasing in c. It can easily be verified that this property holds not only for the linear demand model, but also for the Dixit-Stiglitz CES-model, and for the Cournot model (provided quantities are strategic substitutes).

Turning to the empirical application, for many small service industries, the rental price of the office space is a major component of a firm's fixed cost (and the one that is most likely to vary across geographical markets). Furthermore, we would expect rental prices of office space to be higher in larger cities/municipalities than in smaller ones. Indeed, in Asplund and Nocke (2005), where we investigate firm turnover amongst hair salons in Sweden, we find a strong positive correlation between land values (as a proxy for rents) and market size (measured as the population living in the postal area).[13]

5.2 Trade between Markets

Thus far, we have assumed that an entrepreneur can sell her product only in the local market she chooses to locate production in. In this section, we extend the baseline model by allowing entrepreneurs to export their goods to other geographical markets (countries or regions) at a unit transport cost or tariff. We show that the central sorting result continues to hold. However, depending on the size of transport costs, no entrepreneur may decide to enter the smallest market(s). In fact, if transport costs are sufficiently small, then all entrepreneurs will enter the largest market.

As in the baseline model, an entrepreneur will locate production in a single (geographical) market. However, she can now sell her product in all other markets but has to incur a unit transport cost or tariff t. Assuming that a type-c entrepreneur produces at constant marginal cost c, the unit cost of *selling* in a "foreign" market is then equal to $c+t$.[14] Since entrepreneurs can set different prices in the home and foreign markets, a type-c entrepreneur sets the same price (or quantity) as a foreign type-$(c + t)$ entrepreneur in the foreign entrepreneur's home market. There are no additional fixed costs associated with exports. For simplicity, we assume that the fixed cost of production, ϕ, is the same in all markets, and that demand is linear (i.e., the reduced-form profit function is given by equation (1)).[15]

[13] Of course, in a cross-industry study, there is no reason to believe that fixed costs are positively correlated with industry size. However, one may envisage the following empirical strategy. First, following Sutton (1991), by using industry sales as a proxy for market size and engineering estimates (if available) as a proxy for fixed costs. Second, by splitting the sample into four (or more) subsamples, according to whether market size is large or small and whether fixed costs are large or small. While our theory remains silent when comparing large markets with small fixed costs and small markets with high fixed costs, it allows us to make predictions on entrepreneurial efficiency and churning when comparing large markets with high fixed costs and small markets with low fixed costs.

[14] It can be shown that the same results obtain with "iceberg-type" transport costs, where a type-c firm's marginal cost of selling in a foreign market is τc with $\tau > 1$.

[15] It is straightforward to embed this model in a general equilibrium model. For instance, we may assume that – apart from entrepreneurial ability – there is a single input, labor. In addition to the differentiated products, there is a homogeneous good, which is produced in all countries in a perfectly competitive industry, using a constant-returns-to-scale technology. The wage rate is thus determined in the homogeneous good industry, and

We need to distinguish between the distribution of entrepreneurial types *producing* in a given market, summarized by the (Borel) measure $\widehat{\mu}_i$ on $[0,1]$, and the distribution of types *selling* in that market, summarized by measure μ_i on $[0,1+t]$. Since a foreign type-$(c+t)$ entrepreneur behaves like a home type-c entrepreneur, the mass of entrepreneurs selling in market i whose types fall into the interval A is given by

$$\mu_i(A) = \widehat{\mu}_i(A) + \sum_{j \neq i} \widehat{\mu}_j(A - t).$$

As in the baseline model without trade, price competition is more intense in larger markets.

Lemma 1 *The larger is the market, the more intense is price competition amongst firms* selling *in that market:*

$$h(\mu_1) > ... > h(\mu_k) \geq ... \geq h(\mu_N),$$

where $k \in \{1, ..., N\}$ is the largest integer such that $\widehat{\mu}_k([0,1]) > 0$ (i.e., k is the smallest market in which a positive mass of entrepreneurs locate).

Proof. See appendix. ■

In contrast to the baseline model without trade, an entrepreneur will not necessarily locate her production in the market in which it can make the largest profit from domestic sales. Instead, she will prefer to locate production in market i rather than in market j if

$$S_i \left[\Pi(c; h(\mu_i)) - \Pi(c+t; h(\mu_i))\right] > S_j \left[\Pi(c; h(\mu_j)) - \Pi(c+t; h(\mu_j))\right],$$

i.e., if the profit increase resulting from avoiding the transport cost for sales in market i is larger than the corresponding profit increase for sales in market j. Nevertheless, the central sorting result carries over to our model with trade.

Proposition 6 *In the model with trade between markets, the equilibrium exhibits sorting of entrepreneurs by capabilities. In equilibrium, there exists a market $k \in \{1, ..., N\}$ and marginal types $0 \equiv \widehat{c}_0 < \widehat{c}_1 < ... < \widehat{c}_k$ such that (almost) all entrepreneurs of type $c \in [\widehat{c}_{i-1}, \widehat{c}_i)$ enter market i, while (almost) all entrepreneurs of type $c \in [\widehat{c}_k, 1]$ do not enter any market.*

Proof. See appendix. ■

Observe that consumers may enjoy a larger product variety in a smaller market, *even if the mass of entrepreneurs locating in the smaller market is smaller.* To see this, note that the more competitive is the market, the more talented entrepreneurs must be in order to make positive sales. Hence, entrepreneurs located in a large market (who are very efficient) will make positive export sales in smaller markets (provided transport costs are not too large); in contrast, an entrepreneur who is located in a small market may not be efficient enough to make export sales in a large market where competition is more intense.

Another empirical prediction of our model is that exporters are (on average) more efficient than non-exporters. This is for two reasons. First, consider firms located in the same market: in order to being able to profitably export to market i, a firm's marginal cost has to be less

is the same in all countries.

than $\overline{c}(\mu_i) - t$. Second, note that more efficient firms locate in larger markets and attempt to export to smaller and endogenously less competitive markets. For example, suppose there are only two markets, a large market 1 and a small market 2. In equilibrium, $\overline{c}(\mu_1) < \overline{c}(\mu_2)$. To profitably export to market 1, a firm's marginal cost has to be less than $\overline{c}(\mu_1) - t$, while the upper bound on marginal costs for exports to market 2 is $\overline{c}(\mu_2) - t > \overline{c}(\mu_1) - t$. Since the more efficient firms endogenously locate in market 1, they are more likely to export. Indeed, there is strong empirical evidence supporting this prediction; see Bernard and Jensen (1999).

In contrast to our baseline model without trade, assuming that the fixed cost ϕ is sufficiently small, and the mass M of potential entrepreneurs sufficiently large no longer ensures that a positive mass of entrepreneurs locate in each market. Small market may solely rely on "imports" since entrepreneurs may find it optimal to locate production in larger markets where domestic sales are larger, and then export to other markets. The extent to which this may happen depends on the magnitude of transport costs. If transport costs are small, then each entrepreneur either enters the largest market or does not enter any market.

Proposition 7 *Suppose transport cost t is "sufficiently small". Then, in equilibrium, (almost) all entrepreneurs of type $c \in [0, \widehat{c}_1)$ enter market 1, while (almost) all entrepreneurs of type $[\widehat{c}_1, 1]$ do not enter any market.*

Proof. See appendix. ■

In the limit as transport costs go to zero, the most capable entrepreneurs enter the largest market, less capable entrepreneurs do not enter any market, and no entrepreneur enters any market other than the largest. Intuitively, the existence of a transport cost implies that, by entering a larger rather than a smaller market, an entrepreneur is more efficient in the larger (and endogenously more competitive) market, and less efficient in the smaller (and less competitive) market. The marginal increase in profit from sales in market i from having slightly lower marginal costs in that market is equal to an entrepreneur's output in market i. If transport costs are small, the intensity of price competition is approximately the same in all markets, and so the home market output for *any* type is greater in larger markets. Consequently, all entrepreneurs prefer to enter a larger rather than a smaller market when transport costs are sufficiently small. As transport costs become small, firms locate production in the market that allows them to minimize total transport costs, and this is the largest market.

Over the last twenty years, fears have been expressed by smaller countries that (symmetric) trade liberalization may lead to de-location of firms and even to de-industrialization; see Baldwin and Robert-Nicoud (2000). In our simple model, this fear seems to be well-grounded, even though it does not mean that trade liberalization has negative welfare implications.

Under what assumption on the reduced-form profit function do the predictions obtain? Proposition 6 requires two assumptions in addition to those of the baseline model, namely that for $h(\mu') > h(\mu)$, (i) the difference $\Pi(c; h(\mu)) - \Pi(c; h(\mu'))$ is decreasing in c, and (ii) the ratio $[\Pi(c; h(\mu')) - \Pi(c + t; h(\mu'))] / [\Pi(c; h(\mu)) - \Pi(c + t; h(\mu))]$ is decreasing in c. Assumption (i) is the same assumption we required in section 5.1; it holds not only for the linear demand model, but also in the Cournot model when quantities are strategic substitutes (as is commonly assumed). Assumption (ii) holds in the linear demand model, and in the Cournot model provided demand is downward-sloping. To prove proposition 7, only a technical regularity condition is required, namely that the partial derivatives of $\Pi(c; h(\mu))$ with respect to c and h are locally continuous in both arguments (for $c < \overline{c}(\mu)$).

6 Conclusion

The aim of this paper has been to present a simple theory of entrepreneurial entry and exit, where (young) entrepreneurs decide *which* market to enter. We have obtained a striking sorting result: in equilibrium, the most talented entrepreneurs all choose to enter the largest market, less talented entrepreneurs enter the next largest market, and so on. The larger the market, the more efficient are thus its entrants. This result follows naturally from properties of standard models of imperfect competition. It may provide an alternative explanation for the empirical finding that factor productivity is greater in larger cities or regions. In fact, in a recent empirical study using French data, Combes, Duranton, and Gobillon (2003) show that a large fraction of the observed spatial wage disparities is due to sorting of more talented workers into larger towns.[16] Reconsidering the relationship between market size and the number of firms, we have shown that the sorting effect may reinforce the price competition effect. In fact, the sorting effect may be so strong that the number of active firms (and hence product variety) is not necessarily larger in larger markets.

Our sorting result continues to hold when entrepreneurs can export their goods or services from one market (region, country) to another. However, in this case, no entrepreneur may decide to enter the smallest market(s). For sufficiently small transport costs, all active entrepreneurs locate in the largest market. This illustrates that a symmetric reduction in trade barriers may lead to a de-location of firms at the expense of small regions or countries.

In the dynamic extension of our model, we have shown that the churning rate of entrepreneurs is higher in larger markets (provided entrepreneurial efficiency levels do not change at too fast a rate), and so the life span of firms is shorter. Consequently, the age distribution of firms in larger markets is shifted towards younger firms. This is consistent with the empirical evidence on local service industries in Sweden, as shown in Asplund and Nocke (2000, 2005).

As discussed in the introduction, our theory abstracts from several issues in the economics of entrepreneurship, such as the role of risk and liquidity constraints. Moreover, we have assumed that each entrepreneur cannot enter more than one market. Our theory therefore only applies to those industries where the entrepreneurial span of control has strongly diminishing returns across different markets.[17] Also, we have assumed that all entrepreneurs are completely mobile and may decide to enter any one market. While this may be an extreme assumption, it allows us to analyze a benchmark case without having to make assumptions on the initial distribution of potential entrants over geographical locations. In any event, even if a fraction of entrepreneurs

[16] Our theory is concerned with the sorting of entrepreneurs rather than workers but can easily be extended to allow for sorting of workers. Indeed, if there are complementarities between the capabilities of entrepreneurs and those of (skilled) workers, then more talented workers will follow the more talented entrepreneurs into larger markets.

[17] In the extreme case, where a firm could enter any number of markets (and, on the cost side, these entry decisions are completely independent from one another), our results on the relationship between efficiency and market size would be reversed: the most efficient firms enter all markets, while less efficient firms only enter the larger markets.

are not mobile, the intuition for the sorting result should still hold for all those entrepreneurs who are mobile, and thus imply that entrepreneurial firms in larger markets are more efficient.

Appendix

The Cournot Model with Heterogeneous Firms. Here, we show that our assumptions on the reduced-form profit function $S\Pi(c; h(\mu))$ are satisfied in a homogenous goods Cournot model, where firms differ in their (constant) marginal costs. We will be interested in the properties of *extremal Cournot equilibria* (i.e., of the equilibria with the smallest and largest industry output); see Vives (1999).

There is a population of N active firms. Firm i's (constant) marginal cost is denoted by c_i. An increase in market size means a replication of the population of consumers (leaving the distribution of consumers' tastes and incomes unchanged), so that inverse demand $P(X)$ depends only on the ratio $X = Q/S$ between aggregate output Q and market size S. Throughout, we make the following smoothness assumption:

(A) $P(\cdot)$ is twice differentiable and $P'(\cdot) < 0$.

We claim that, in any extremal Cournot equilibrium, each firm's output and profit are proportional to market size S. Consequently, equilibrium price is independent of market size. To see this, note that firm i's first-order condition for profit maximization is given by

$$P\left(\frac{\sum_j q_j}{S}\right) - c + \frac{q_i}{S} P'\left(\frac{\sum_j q_j}{S}\right) = 0, \tag{7}$$

where q_j is firm j's quantity. Since output and market size enter only through the ratio q_j/S, this ratio must be independent of market size in any (extremal) equilibrium.

Hence, the equilibrium gross profit of a firm with marginal cost c can be written as $S\Pi(c; Q/S)$, where equilibrium industry output Q is a function of the vector of marginal costs $\mathbf{c} \equiv (c_1, c_2, ..., c_N)$, and is proportional to market size, $Q = S \cdot h(\mathbf{c})$. Any change in the underlying distribution of marginal costs that increases equilibrium industry output Q can be thought of representing an increase in the intensity of competition $h(\mathbf{c})$. (Below, we will verify that an increase in Q does indeed reduce the gross profit of any firm with positive output.) To simplify notation, we will henceforth set $S = 1$, and analyze the properties of $\Pi(c; Q)$.

The assumptions on the reduced-form profit function in the main text involve two types of profit comparisons: (i) holding fixed the distribution of marginal costs (i.e., for any given equilibrium), we compare the profit of firms with different marginal costs; and (ii) we compare the profit of different firms with variations in the distribution of marginal costs (i.e., across equilibria). To the extent that the Cournot equilibrium is not unique, our comparative statics results will apply to both the smallest and the largest equilibrium.

Let $q(c; Q)$ denote the equilibrium output of a type-c firm when equilibrium industry output is Q. (The function $q(c; \cdot)$ is sometimes called the backward-reaction function.) From the first-order condition (7), the equilibrium output of a type-c firm is equal to

$$q(c; Q) = \begin{cases} -\frac{P(Q)-c}{P'(Q)} & \text{if } c \leq \bar{c}(Q) \equiv P(Q), \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Conditional on industry output Q, the equilibrium profit of a type-c firm can then be written as

$$\Pi(c;Q) = \begin{cases} -\frac{[P(Q)-c]^2}{P'(Q)} & \text{if } c \leq \bar{c}(Q) \equiv P(Q), \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Consider now the (continuous) function

$$g(Q;\mathbf{c}) \equiv -\frac{\sum_{j=1}^{N} \max\{P(Q)-c_j, 0\}}{P'(Q)}.$$

A Cournot equilibrium is then a solution to the equation $g(Q;\mathbf{c}) = Q$. We are interested in extremal Cournot equilibria, i.e., in the smallest and largest solutions to the above equation, denoted $Q_L(\mathbf{c})$ and $Q_H(\mathbf{c})$, respectively.

We now make two observations. First, we claim that $\Pi(c;Q)$ is decreasing in Q on $(0, \bar{c}(Q))$ if and only if the second-order condition for profit maximization holds strictly for any firm with positive equilibrium output, i.e.,

$$2P'(Q) + q(c;Q)P''(Q) < 0, \ \forall c < P(Q), \ Q > 0. \tag{10}$$

To see this, note that, from (9),

$$\frac{\partial \Pi(c;Q)}{\partial Q} = -q(c;Q)\left\{2P'(Q) + q(c;Q)P''(Q)\right\},$$

where we have made use of (8). We henceforth assume that (10) holds. Second, we claim that in any extremal Cournot equilibrium, an increase in the vector $\mathbf{c}$ of marginal costs results in a weakly smaller equilibrium output, i.e., $Q_L(\mathbf{c})$ and $Q_H(\mathbf{c})$ are weakly increasing in $\mathbf{c}$. To see this, note that $g(Q;\mathbf{c})$ is weakly decreasing in $\mathbf{c}$. The claim then follows from corollary 1 in Milgrom and Roberts (1994). Further, if $\mathbf{c}' > \mathbf{c}$ with a strictly inequality for at least one firm i with $c_i < P(Q_k(\mathbf{c}))$, $k \in \{L, H\}$, then $Q_k(\mathbf{c}') < Q_k(\mathbf{c})$. The two observations jointly imply that assumption (DOM) holds in any extremal Cournot equilibrium.

We now claim that assumption (MON) holds in any extremal equilibrium. To see this, observe that, from (9), $\Pi(c;Q) = 0$ for $c \geq \bar{c}(Q)$, and $\Pi(c;Q)$ is decreasing in c on $(0, \bar{c}(Q))$. Moreover, as shown above, $\Pi(c;Q)$ is decreasing in Q on $(0, \bar{c}(Q))$.

Next, we claim that assumption ($COMP$) holds in the Cournot model. We need to show that for $Q' > Q$, the profit ratio $\Pi(c;Q')/\Pi(c;Q)$ is decreasing in c for all $c < \bar{c}(Q)$. From (9), we have

$$\frac{\Pi(c;Q')}{\Pi(c;Q)} = \frac{P'(Q)}{P'(Q')} \frac{[P(Q')-c]^2}{[P(Q)-c]^2}.$$

It is straightforward to verify that this profit ratio is decreasing in c if and only if $P(Q') < P(Q)$, which we assumed to hold.

As regards our continuity assumption (CON), it is straightforward to see from (9) that $\Pi(c;Q)$ is continuous in c and Q. Moreover, as long as a small change in the vector of marginal costs does not affect the number of Cournot equilibria, it follows immediately from the equation $g(Q;\mathbf{c}) = Q$ that equilibrium industry output Q varies continuously with the vector of marginal costs.

Note that the Cournot equilibrium is unique under the (standard) assumption that $2P'(Q) + QP"(Q) < 0$ for any $Q > 0$. Hence, sufficient conditions for all of our assumptions on the profit function to hold in the Cournot model are: $P'(Q) < 0$ and $2P'(Q)+QP''(Q) < 0$ for any $Q > 0$.

The Linear Demand Example. Here, we derive the profit function for the linear demand example. The representative consumer's utility function is given by

$$\begin{aligned} U &= \int_0^n \left(x(k) - x^2(k) - 2\sigma \int_0^n x(k)x(l)dl \right) dk + H \\ &= \int_0^n x(k)dk - \int_0^n x^2(k)dk - 2\sigma \left(\int_0^n x(k)dk \right)^2 + H. \end{aligned}$$

Let Y denote the consumer's income (which we assume is "sufficiently large" so that the consumer will consume a positive amount of the Hicksian composite commodity). Utility maximization then implies $H = Y - \int_0^n p(k)x(k)dk$, and

$$1 - 2x(k) - 4n\sigma\widehat{x} = p(k), \tag{11}$$

where

$$\widehat{x} \equiv \frac{1}{n} \int_0^n x(k)dk$$

is the average consumption over all varieties. Equation (11) gives each consumer's inverse demand for variety k.

Consider now the maximization problem of the firm producing variety k. Since all consumers are identical and firms have constant marginal costs of production, we can think of the firm choosing the average output per consumer, $x(k)$. (Note that in models of monopolistic competition with a continuum of firms, price and quantity competition yield the same equilibrium allocation.) There are $8S$ consumers, and so the firm's total output will be $q(k) = 8Sx(k)$. The entrepreneur's problem is given by:

$$\max_{x(k)} \left[1 - 2x(k) - 4n\sigma\widehat{x} - c(k)\right] x(k).$$

Dropping arguments for notational simplicity, the first-order condition yields

$$x = \frac{1 - 4n\sigma\widehat{x} - c}{4}.$$

Taking averages, we obtain

$$\widehat{x} = \frac{1 - \widehat{c}}{4(1 + n\sigma)},$$

where $\widehat{c}$ is the average marginal cost of all firms with positive output. Hence, the output of a firm with marginal cost c is

$$q(c) = 8Sx = 8S \left(\frac{1 - n\sigma\frac{1-\widehat{c}}{(1+n\sigma)} - c}{4} \right) = 2S \left(\frac{1 + n\sigma\widehat{c}}{1 + n\sigma} - c \right),$$

provided

$$c \leq \overline{c} \equiv \frac{1 + n\sigma\widehat{c}}{1 + n\sigma},$$

and $q(c) = 0$ otherwise. Hence, $\overline{c}$ denotes the marginal firm's type: all firms with lower marginal costs make positive sales, while all less efficient firms make zero sales. The equilibrium price of a firm with marginal cost $c \leq \overline{c}$ is then (from (11)) given by

$$p(c) = 1 - 2x - 4n\sigma\widehat{x} = \frac{1}{2}\left(\frac{1 + n\sigma\widehat{c}}{1 + n\sigma} + c\right).$$

Finally, the firm's profit is

$$\Pi(c) = [p(c) - c]\, q(c) = S\left(\frac{1 + n\sigma\widehat{c}}{1 + n\sigma} - c\right)^2$$

if $c \leq \overline{c}$, and $\Pi(c) = 0$ otherwise.

The expressions in the main text then follow by noting that $n = \mu([0, \overline{c}])$ and $n\widehat{c} = \int_0^{\overline{c}} c\mu(dc)$.

Proof of proposition 2. Note that (COMP) holds if and only if

$$\frac{\Pi_1(c; h(\mu'))}{\Pi(c; h(\mu'))} < \frac{\Pi_1(c; h(\mu))}{\Pi(c; h(\mu))} \;\; \forall c \in \left[0, \overline{c}(\mu')\right),\; h(\mu') > h(\mu).$$

Applying the envelope theorem, this inequality is equivalent to $p(c; h(\mu')) < p(c; h(\mu))$. ■

Proof of proposition 1. In the main text, we have shown that if an entrepreneur of type c_{ij} is indifferent between entering market i and a smaller market $j > i$, then all more efficient types strictly prefer to enter the larger market i, while all less efficient types strictly prefer to enter market j. Since we assume that entry and fixed costs are sufficiently small, and the mass of potential entrepreneurs M sufficiently large, a positive mass of entrepreneurs must enter each market, while a positive mass of entrepreneurs does not enter any market. Hence, in equilibrium, there exist marginal types $\{c_i\}_{i=0}^N$ such that $c_i < c_{i+1}$,

$$c_0 \equiv 0, \tag{E$_0$}$$

$$S_i\Pi(c_i; h(\mu_i)) = S_{i+1}(c_i; h(\mu_{i+1})),\; i = 1, ..., N-1, \tag{E$_i$}$$

$$S_N\Pi(c_N; h(\mu_N)) = \phi. \tag{E$_N$}$$

As is well known, there always exists a pure-strategy Nash equilibrium in games with a continuum of atomless players and a countable and finite set of actions; see, for instance, theorems 1 and 2 in Mas-Colell (1984), or corollary 1 in Khan and Sun (1995). (Furthermore, if the distribution of entrepreneurial types has no mass points, as we assume, then there exists a "symmetric" pure-strategy equilibrium in which all entrepreneurs of the same type c choose the same action, i.e., enter the same market.) We now want to prove uniqueness of equilibrium. For this, suppose there exist marginal types $\{\widetilde{c}_i\}_{i=0}^N \neq \{c_i\}_{i=0}^N$ satisfying (E$_0$) to (E$_N$). Assume, for instance, that $\widetilde{c}_N < c_N$. For (E$_N$) to hold, we thus have $h(\widetilde{\mu}_N) > h(\mu_N)$. The last observation in turn implies that $\widetilde{c}_{N-1} < c_{N-1}$. To see this, suppose instead that $\widetilde{c}_{N-1} \geq c_{N-1}$; however, from $\widetilde{c}_N < c_N$ and (DOM), it would then follow that $h(\widetilde{\mu}_N) \leq h(\mu_N)$, contradicting our finding that $h(\widetilde{\mu}_N) > h(\mu_N)$. Observe now that, for given measures μ_i and μ_{i+1}, the marginal type c_i is *uniquely* defined by (E_i), where uniqueness follows from (COMP); furthermore, c_i is decreasing in $h(\mu_i)$ and increasing in $h(\mu_{i+1})$. Hence, $\widetilde{c}_{N-1} < c_{N-1}$ and $h(\widetilde{\mu}_N) > h(\mu_N)$ imply that $h(\widetilde{\mu}_{N-1}) > h(\mu_{N-1})$. Following the same steps of argument, we obtain that $\widetilde{c}_i < c_i$ and $h(\widetilde{\mu}_i) > h(\mu_i)$ for all $i \in \{1, ..., N\}$. However, $\widetilde{c}_1 < c_1$ and $\widetilde{c}_0 = c_0 = 0$ imply that $h(\widetilde{\mu}_1) < h(\mu_1)$, contradicting $h(\widetilde{\mu}_1) > h(\mu_1)$. Hence, we cannot have $\widetilde{c}_N < c_N$. A similar reasoning yields that we cannot have $\widetilde{c}_N > c_N$. We therefore conclude that $\widetilde{c}_N = c_N$. Suppose now that $\widetilde{c}_N = c_N$ and $\widetilde{c}_{N-1} < c_{N-1}$. It then follows that $h(\widetilde{\mu}_{N-1}) > h(\mu_{N-1})$. As before, it is straightforward to show that this leads to a contradiction. Applying these arguments to all $i \in \{1, ..., N\}$, we find that $\widetilde{c}_i = c_i$ for all $i \in \{0, 1, ..., N\}$, proving uniqueness of equilibrium. ■

Proof of proposition 3. The proof is similar to that of proposition 1. The first step consists in showing that, in equilibrium, the distribution of active entrepreneurs is larger (in the sense of representing more intense competition) in larger markets: $h(\mu_i) > h(\mu_j)$ for any markets i, $j > j$. The proof of this assertion proceeds as before. The remaining steps are slightly more involved. Since we assume that each market is sufficiently large relative to entry and fixed costs (so that each market is non-empty in equilibrium) and since the conditional value is continuous in c, for any two markets i, $j > j$, there exists some type, say c_{ij}, who his indifferent between entering markets i and j: $\overline{V}(c_{ij}; h(\mu_i), S_i) = \overline{V}(c_{ij}; h(\mu_j), S_j)$. Similarly, there exists a unique type, say $\hat{c}_{ij}$, who would make the same (current) profit in both markets: $S_i\Pi(\hat{c}_{ij}; h(\mu_i)) = S_j\Pi(\hat{c}_{ij}; h(\mu_j))$. Assumption (COMP) ensure that the profit ratio $\Pi(c; h(\mu_i))/\Pi(c; h(\mu_j))$ is decreasing in c on $[0, \overline{c}(\mu_i))$. If $c_{ij} \leq \hat{c}_{ij}$, then it is straightforward to see that the ratio of conditional values, $\overline{V}(c; h(\mu_i), S_i)/\overline{V}(c; h(\mu_j), S_j)$, is decreasing in c at $c = c_{ij}$; this holds independently of the level of $\delta(1-\alpha)/(1-\delta)$. In this case, any type more efficient than c_{ij} strictly prefers to enter market i, whereas all less efficient types prefer to enter the smaller market j. Now, if c_{ij} is (much) larger than $\hat{c}_{ij}$, then the ratio of conditional values may not be monotonically decreasing in c. By assuming that $\delta(1-\alpha)/(1-\delta)$ is small, we ensure that c_{ij} is close to $\hat{c}_{ij}$, and hence that $\overline{V}(c; h(\mu_i), S_i)/\overline{V}(c; h(\mu_j), S_j)$ is decreasing in c at $c = c_{ij}$. The asserted sorting result follows then immediately. Uniqueness of equilibrium can be shown in a way similar to the proof of proposition 1. Note that the assumption that $\delta(1-\alpha)/(1-\delta)$ is small implies that the marginal incumbent $c^*(\mu, S)$ (who is just indifferent between exiting and staying in the market) makes a positive gross profit, $S\Pi(c^*(\mu, S); h(\mu)) > 0$, and hence $c^*(\mu, S) < \overline{c}(\mu)$ (as we posited before). ■

Proof of lemma 1. Suppose the assertion were not true. Then, there exist markets i and j such that $i < j \leq k$, $h(\mu_i) \leq h(\mu_j)$, and $\hat{\mu}_j([0,1]) > 0$. We now show that, in this case, all entrepreneurs who choose to locate in market j would be strictly better off by locating in market i instead, contradicting $\hat{\mu}_j([0,1]) > 0$. Entrepreneurial type c strictly prefers to locate in market i rather than in market j if and only if she can make a larger profit by producing in market i and exporting to market j rather than doing the reverse, i.e.,

$$S_i\Pi(c; h(\mu_i)) + S_j\Pi(c+t; h(\mu_j)) > S_j\Pi(c; h(\mu_j)) + S_i\Pi(c+t; h(\mu_i)),$$

or

$$\frac{\Pi(c; h(\mu_i)) - \Pi(c+t; h(\mu_i))}{\Pi(c; h(\mu_j)) - \Pi(c+t; h(\mu_j))} > \frac{S_j}{S_i}, \tag{12}$$

Under linear demand, and assuming $\Pi(c+t; h(\mu_j)) > 0$, this inequality becomes

$$\frac{\overline{c}(\mu_i) - c - t/2}{\overline{c}(\mu_j) - c - t/2} > \frac{S_j}{S_i}.$$

Since $S_i > S_j$, the right-hand side of this inequality is smaller than one. However, since $h(\mu_i) \leq h(\mu_j)$ implies $\overline{c}(\mu_i) \geq \overline{c}(\mu_j)$, the left-hand side of the inequality is larger than or equal to one. Hence, the inequality holds for all types c such that $\Pi(c+t; h(\mu_j)) > 0$, and so all of these types are strictly better off by producing in market i. Moreover, for those (inefficient) types for which $\Pi(c+t; h(\mu_j)) = 0$, the left-hand side of (12) must be less than or equal to one, as $h(\mu_i) \leq h(\mu_j)$, and so all of these types also strictly prefer to produce in market i rather than in market j. But this contradicts the assumption that $h(\mu_i) \leq h(\mu_j)$ and $\hat{\mu}_j([0,1]) > 0$. ■

Proof of proposition 6. Suppose that entrepreneurial type c_{ij} is indifferent between locating in the larger market i and the smaller market j, i.e.,

$$\frac{\Pi(c; h(\mu_i)) - \Pi(c+t; h(\mu_i))}{\Pi(c; h(\mu_j)) - \Pi(c+t; h(\mu_j))} = \frac{S_j}{S_i} \text{ for } c = c_{ij}. \tag{13}$$

Under linear demand, and assuming $\Pi(c_{ij}+t; h(\mu_i)) > 0$, the equality becomes

$$\frac{\overline{c}(\mu_i) - c - t/2}{\overline{c}(\mu_j) - c - t/2} = \frac{S_j}{S_i} \text{ for } c = c_{ij}.$$

Since competition is endogenously more intense in the larger market (lemma 1), $\overline{c}(\mu_i) < \overline{c}(\mu_j)$, the left-hand side is strictly decreasing in c, and so all entrepreneurial types more capable than c_{ij} strictly prefer to locate in the larger market i rather than in the smaller market j. If $\Pi(c_{ij}+t; h(\mu_i)) = 0 < \Pi(c_{ij}+t; h(\mu_j))$, the equation becomes

$$\frac{[\overline{c}(\mu_i) - c]^2}{2t\left[\overline{c}(\mu_j) - c - t/2\right]} = \frac{S_j}{S_i} \text{ for } c = c_{ij}.$$

Since $\overline{c}(\mu_i) < \overline{c}(\mu_j)$ and $\overline{c}(\mu_j) - c - t > 0$, it can easily be verified that the left-hand side is strictly decreasing in c, implying that all entrepreneurial types more capable than c_{ij} strictly prefer to locate in the larger market i. The same conclusion holds if $\Pi(c_{ij}+t; h(\mu_j)) = 0$. In this case, the left-hand side of equation (13) becomes $\Pi(c; h(\mu_i))/\Pi(c; h(\mu_j))$, which by assumption (COMP) is also strictly decreasing in c.

Proof of Proposition 7. Let $\hat{c}_1^t$ denote the marginal entrant if entrepreneurs can choose only between entering market 1 and not entering any market. Formally, $\hat{c}_1^t$ is defined by

$$S_1\Pi(\hat{c}_1^t; h(\mu_1^t)) + \sum_{j=2}^{N} S_j\Pi(\hat{c}_1^t + t; h(\mu_j^t)) = \phi, \tag{14}$$

where μ_1^t is such that $\mu_1^t([0,z]) = \mu_j^t([t, z+t]) = MG(\min\{z, \hat{c}_1^t\})$, $j \neq 1$, for any interval $[0,z]$. (Our assumptions ensure that $\hat{c}_1^t$ does indeed exist.) Then, we claim that

$$S_i\Pi(c; h(\mu_i^t)) + \sum_{j\neq i} S_j\Pi(c; h(\mu_j^t)) < S_1\Pi(c; h(\mu_1^t)) + \sum_{j\geq 2} S_j\Pi(c+t; h(\mu_j^t)) \tag{15}$$

for any $c < \hat{c}_1^t$ and $i = 2, ..., N$. Note that (14) and (15) imply that no type in $(\widehat{c}_1, 1]$ would find it profitable to enter *any* market, and that all types in $[0, \hat{c}_1^t)$ strictly prefer to enter the largest market than any other market. Hence, we prove the assertion by showing that (14) implies (15), provided the transport cost t is sufficiently small. Observe that (14) implies (15) if

$$-\frac{S_1\left[\Pi(c+t; h(\mu_1^t)) - \Pi(c; h(\mu_1^t))\right]}{t} > -\frac{S_i\left[\Pi(c+t; h(\mu_i^t)) - \Pi(c; h(\mu_i^t))\right]}{t} \tag{16}$$

for $i = 2, ..., N$ and $c < \hat{c}_1^t$. Note also that $h(\mu_i^t) \to h(\mu_1^0)$ as $t \to 0$: in the limit as transport costs vanish, the distribution of entrepreneurial types selling in a market is the same for all markets. Thus, (16) holds for small enough t if[18]

$$-S_1 \frac{\partial \Pi(c; h(\mu_1^0))}{\partial c} > S_i \frac{\partial \Pi(c; h(\mu_1^0))}{\partial c} \text{ for } i = 2, ..., N,\ c < \hat{c}_1^t,$$

i.e., if in the limit as transport costs vanish an entrepreneur of type c would produce a larger quantity in market 1 than in any smaller market. The inequality can be rewritten as

$$S_1 > \max_{i>1} S_i,$$

which holds by definition. ■

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[18] We assume here that the partial derivatives of $\Pi(c; h(\mu_1^0))$ with respect to both c and h exist and are continuous. It can easily be verified that this holds in the linear demand case (as well as in the Cournot model).

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