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"Dynamic Disappointment Aversion:
Don't Tell Me Anything Until You Know For Sure"

by

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Dynamic Disappointment Aversion: Don't Tell Me Anything Until You Know For Sure*

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Abstract

We show that for a disappointment-averse decision maker, splitting a lottery into several stages reduces its value. To do this, we extend Gul's (1991) model of disappointment aversion into a dynamic setting while keeping its basic characteristics intact. The result depends solely on the sign of the coefficient of disappointment aversion. It can help explain why people often buy periodic insurance for moderately priced objects, such as electrical appliances and cellular phones, at much more than the actuarially fair rate.

Keywords: Disappointment aversion, recursive preferences, compound lotteries.

1. Introduction

Assume you are waiting to receive an important announcement. For concreteness, assume that you filled a betting ticket regarding the results of a horse race, which is taking place at the moment and will end in a short time. You have two ways of spending your time until the race ends. The first is to turn on the radio and hear the commentator describing "live" what is happening at the race. The second would be to sit back, wait patiently, and turn the radio on only once the results of the race are determined. Which one seems more appealing to you?

The answer to this question may depend on many factors, such as the amount of money you spent on the ticket, your financial condition, and most of all, on your personal character. A plausible answer might be: "I prefer to wait and hear only the final result. Being exposed to the resolution process bears the risk of perceiving intermediate outcomes as disappointing.

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Therefore, since I take disappointments hard, getting partial information in the middle of the process will only stress me further and cause me to suffer more on average."

A similar argument can be made by a Ph.D. candidate who has applied to a certain school. The final result is expected to arrive in a couple of months, but the student is aware that he is going to be judged sequentially until that date, according to many parameters (grades, preparation test results, recommendations, publication list). The student has no ability to affect the final outcome once he has submitted his application. Against the natural curiosity and psychological need to be involved and receive as much information as possible regarding his ranking relative to other candidates, a no less natural attitude can be adopted by the easily disappointed candidate who prefers to receive no information at all until the final decision has been made.

Gul (1991) suggests a model to study disappointment-averse individuals. According to his model, the decision maker divides the support of a certain lottery into two groups: the disappointing and the elating prizes. The threshold to this division is determined endogenously, as follows: equipped with a utility function over prizes, he calculates the expected utility of the lottery while uniformly assigning to all the disappointing outcomes a greater weight. The value is thus the certainty equivalent of the lottery where all prizes with a value higher than this number are considered elations and all prizes with lower value disappointments. (Mathematical definition below in (1).)

Gul's basic model is static, as all the decision maker cares about is the probability distribution over final outcomes. To study the effect of potential disappointment emerging from gradual exposure to risk, one needs to extend his model into a dynamic setting. This requires some additional assumptions regarding the way compound lotteries (*i.e.*, lotteries whose outcomes are tickets to other lotteries) are evaluated.

As we describe in detail below, we assume that the decision maker folds-back the probability tree and applies the same (static) preferences as in Gul in every stage. The implied model maintains preferences, now defined over a richer domain, to be fully determined by the pair (u, β)— a utility function over prizes and a coefficient of disappointment aversion that determines the additional weight given to the disappointing outcomes, which is thought of as a characteristic of the decision maker. Palacious-Huerta (1999) adopts this approach and by working out an example, demonstrated the tendency of a disappointment-averse individual to prefer getting information that is resolved all at once rather than gradually. The lottery used in his example, however, is very special and contains only two prizes. Thus the division to disappointment and elation is, in that example, obvious at every step of the folding-back process.

Our aims in this paper are to show that Palacious-Huerta's observation holds in the gen-

eral case and, in particular, to emphasize the linkage between the sign of the coefficient of disappointment aversion and the attitude toward the way in which uncertainty is resolved over time. We show that a disappointment-averse decision maker, that is, one who is characterized by the pair (u, β) and $\beta \geqslant 0$, will always prefer any compound lottery to be resolved in a single stage. The opposite is true if $\beta \leqslant 0$.

As an application, we demonstrate that disappointment-averse individuals are likely to purchase dynamic insurance contracts, such as periodic insurance for electrical appliances and cellular phones, at much more than actuarially fair rates. The reason is that in addition to the standard risk premium, they are willing to pay a premium to avoid being exposed to the gradual resolution of uncertainty. These two premia reinforce one another, and this aspect makes the individual more reluctant to take risks. While the gradual resolution premium is non-negative for $\beta \geqslant 0$, it is not an increasing function of β. This observation is valid in the general case and is independent of the specific insurance problem we consider. When β is extremely large, the gradual resolution premium converges to zero, which is its level when the DM is an expected utility maximizer ($\beta = 0$). Therefore, there is always an interior value of β in which the gradual resolution premium is maximized.

Dillenberger (2010) studies recursive preferences over compound lotteries and characterized preferences for one-shot resolution of uncertainty, that is, preferences to have any compound lottery resolved in a single stage. In this paper we give a direct proof that any disappointment-averse decision maker displays this property. It is remarkable that within the disappointment aversion class, only one parameter, β, accounts for preferences for one-shot resolution of uncertainty. This feature sheds light on the driving force behind the variety of applications that use Gul's preferences. (See, for example, Ang, Bekaert and Liu (2005) who use recursive disappointment aversion preferences to study a dynamic asset allocation problem.)

The remainder of the paper is organized as follows: In Section 2 we present the model and the statement of our main result. In Section 3 we give a complete mathematical proof of our result. In section 4 we apply our model to study an insurance problem. We conclude by suggesting extensions for the basic model.

2. The model and the main theorem

We consider an interval $X \subset \mathbb{R}$ of prizes. A lottery P is a vector of probabilities indexed by $x \in X$ such that $\sum_{x \in X} p_x = 1$, and we restrict to the case in which in any given lottery the number of possible prizes (*i.e.*, prizes with non-zero probability) is finite. To avoid complicating notation, we assume that $x \in X$ is both the prize and its perceived value. In

the context of this paper no generality is lost by this, and the same results hold if we assume a utility function $u : X \to \mathbb{R}$, replacing the prize x by the value $u(x)$.

The value of a lottery is a function that assigns to each lottery a number between the largest and the smallest $x \in X$ and that depends on a parameter $-1 < \beta < \infty$. β should be thought of as a property of the decision maker that captures his disappointment aversion, if $\beta > 0$, or elation seeking, if $-1 < \beta < 0$. (For $\beta = 0$ the value will simply be the standard expectation.) The value V is defined as follows: it is the unique solution of the equation

$$v = \frac{\sum\limits_{\{x:x>v\}} xp_x + (1+\beta) \sum\limits_{\{x:x\leq v\}} xp_x}{1 + \beta \sum\limits_{\{x:x\leq v\}} p_x} \tag{1}$$

As discussed in the introduction, this definition goes back to Gul (1991). Thus, when computing the value $V = V_\beta(P)$, if say $\beta > 0$, we average the prizes in such a way that disappointing prizes are given an extra weight. The number V is the unique number such that if the decision maker sets his disappointment-satisfaction threshold at V, then he is indifferent between carrying out the lottery and receiving V dollars.

We turn to the definition of the value of a two-stage lottery. Assume that one is given m lotteries, denoted $P^{(j)}$ for $j = 1, \ldots, m$. Each lottery $P^{(j)}$ is defined by the probabilities it assigns to the different $x \in X$, which we denote $p_x^{(j)}$. For the two-stage lottery, one is given probabilities $\alpha_1, \ldots, \alpha_m$ for gaining the lotteries $P^{(1)}, \ldots, P^{(m)}$ respectively. In the first stage a lottery $P^{(j)}$ is realized with probability α_j and then, in the second stage, a prize is obtained according to $P^{(j)}$.

Note that the probability distribution over final prizes induced by the two-stage lottery is the one in which a prize x is won with probability $\sum_{j=1}^m \alpha_j p_x^{(j)}$. The value of this reduced two-stage lottery is as defined in (1) above for a (one-stage) lottery, $V(P)$. This corresponds to the case where the decision maker is not exposed to the gradual resolution of uncertainty.

On the other hand, if the decision maker sees the results of the first stage of the lottery, then he or she will be disappointed or elated also with the results of this first stage. The value of the two stage lottery in this case will be the value of a lottery Q with prizes $V(P^{(j)})$ with probabilities α_j, for $j = 1 \ldots m$. (Notice that we now have a different set $Y = (V(P^{(j)}))_{j=1}^m \subset \mathbb{R}^m$ of prizes.) Notice that the same parameter β is used to decide the value of each lottery $P^{(j)}$ and the value of the lottery Q.

We now show that with the above definitions, a decision maker who is disappointment-averse prefers not to be exposed to the gradual resolution of uncertainty, and an elation-seeking decision maker will want to be involved and receive information as many times as possible during the resolution process, despite the fact that he has no possibility of affecting

the outcome. More precisely our main theorem reads as follows:

Theorem 1: Given m lotteries $P^{(j)} = (p_x^{(j)})_{x \in X}$, and numbers $0 \leq \alpha_j \leq 1$, $j = 1, \ldots, m$, such that $\sum_{j=1}^m \alpha_j = 1$, define the lotteries P and Q as follows:

P assigns probability $\sum_{j=1}^m \alpha_j p_x^{(j)}$ to the prize $x \in X$,

Q assigns probability α_j to the prize $V(P^{(j)})$.

Then, for $\beta \geq 0$ we have $V(P) \geq V(Q)$, and for $-1 < \beta \leq 0$ we have $V(P) \leq V(Q)$.

3. Proof of the main theorem

In order to prove Theorem 1, we need to first discuss the function $V = V_\beta(P)$. We fix $\beta > 0$ and omit the index β throughout this section. The case $\beta < 0$ is completely analogous, and we comment on it at the end of the proof.

Rearranging equation (1), we see that V is defined as the intersection of the function

$$f_P(v) = \sum_{\{x : x > v\}} p_x(x - v) + (1 + \beta) \sum_{\{x : x \leq v\}} p_x(x - v)$$

with the v-axis, that is, V is the solution of $f_P(v) = 0$. This function is continuous, decreasing, and linear on every interval $[x_i, x_{i+1}]$ that does not include points x with $p_x > 0$ in its interior. The slope of f_P at a point $v \in \mathbb{R}$ (with $p_v = 0$) is equal to $(-1 - \beta \sum_{x \leq v} p_x)$. If P is non trivial (assigning positive probability to more than one value) then one has $f_P(\min X) > 0$ and $f_P(\max X) < 0$.

Given two lotteries P and Q, showing that $V(P) \geq V(Q)$ is equivalent (since f_P is decreasing) to showing that $f_P(V(Q)) \geq 0$. Notice that by the definition of P and of f_P we have that $f_P(v) = \sum_{j=1}^m \alpha_j f_j(v)$ where we have denoted $f_{P^{(j)}}(v) = f_j(v)$.

For notational convenience we also denote the value of $P^{(j)}$ by $v_j = V(P^{(j)})$ and the value of Q by $w = V(Q)$, so that w is the solution for the equation

$$f_Q(w) = \sum_{\{j : v_j > w\}} \alpha_j (v_j - w) + (1 + \beta) \sum_{\{j : v_j \leq w\}} \alpha_j (v_j - w) = 0.$$

The above implies that

$$\sum \alpha_j v_j - w = \beta \sum_{\{j : v_j \leq w\}} \alpha_j (w - v_j). \tag{2}$$

In particular we see here that $w \leq \mathbb{E}(Q)$, the expected value of Q.

Proof of Theorem 1. We wish to show that $f_P(w) \geq 0$. We subtract from it $0 = \sum_{j=1}^{m} \alpha_j f_j(v_j)$, which does not change the expression, and regroup the terms as follows

$$
\begin{aligned}
f_P(w) &= \sum_{j=1}^{m} \alpha_j (f_j(w) - f_j(v_j)) \\
&= \sum_{\{j:v_j<w\}} \alpha_j \Bigg(\sum_{\{x:x>w\}} p_x^{(j)}(v_j - w) + \\
&\quad \sum_{\{x:v_j<x\leq w\}} p_x^{(j)}(\beta x + v_j - (1+\beta)w) + (1+\beta) \sum_{\{x:x\leq v_j\}} p_x^{(j)}(v_j - w) \Bigg) \\
&\quad + \sum_{\{j:v_j>w\}} \alpha_j \Bigg(\sum_{\{x:x>v_j\}} p_x^{(j)}(v_j - w) + \\
&\quad \sum_{\{x:w<x\leq v_j\}} p_x^{(j)}(-\beta x + (1+\beta)v_j - w) + (1+\beta) \sum_{\{x:x\leq w\}} p_x^{(j)}(v_j - w) \Bigg) \\
&= \left(\sum_{j=1}^{m} \alpha_j v_j - w \right) + \\
&\quad \beta \left[\sum_{\{j:v_j<w\}} \alpha_j \left(\sum_{\{x:v_j<x\leq w\}} p_x^{(j)}(x - w) + \sum_{\{x:x\leq v_j\}} p_x^{(j)}(v_j - w) \right) \right] + \\
&\quad \beta \left[\sum_{\{j:v_j>w\}} \alpha_j \left(\sum_{\{x:w<x\leq v_j\}} p_x^{(j)}(v_j - x) + \sum_{\{x:x\leq w\}} p_x^{(j)}(v_j - w) \right) \right].
\end{aligned}
$$

We already see that the first and third terms are nonnegative. We now use the relation (2) and substitute the first term by

$$
\left(\sum_{j=1}^{m} \alpha_j v_j - w \right) = \beta \sum_{\{j:v_j<w\}} \alpha_j (w - v_j) = \beta \sum_{\{j:v_j<w\}} \alpha_j \sum_{x\in X} p_x^{(j)}(w - v_j).
$$

The constant β appears as a coefficient in all three terms now, so we have that

$$
\begin{aligned}
f_P(w)/\beta = &\left[\sum_{\{j:v_j<w\}} \alpha_j \left(\sum_{\{x:v_j<x\leq w\}} p_x^{(j)}(x - v_j) + \sum_{\{x:x>w\}} p_x^{(j)}(w - v_j) \right) \right] \\
&+ \left[\sum_{\{j:v_j>w\}} \alpha_j \left(\sum_{\{x:w<x\leq v_j\}} p_x^{(j)}(v_j - x) + \sum_{\{x:x\leq w\}} p_x^{(j)}(v_j - w) \right) \right].
\end{aligned}
$$

It is evident that both expressions on the right-hand side are non negative. In particular we get that for $\beta > 0$, $f_P(w) \geq 0$, which in turn implies that $V(P)$, which is the zero of $f_P(v)$, satisfies $V(P) \geq w = V(Q)$. We did not use the fact that $\beta > 0$ anywhere in the derivation (which consisted of equalities only), and similarly we get that for $-1 < \beta < 0$ one has $f_P(w) \leq 0$, so that in the case of elation seeking we have $V(P) \leq w = V(Q)$. $\square$

4. Application, an insurance problem

Theorem 1 can be readily extended to compound lotteries with arbitrary (finite) number of stages. The decision maker evaluates any n-stage lottery by folding back the probability tree and applying the same V_β in each stage. A decision maker with such preferences prefers to replace each compound sub-lottery with its single-stage counterpart. Let Q^n be an n-stage lottery that induces the same probability distribution over final outcomes as P. The amount $V_\beta(P) - V_\beta(Q^n)$ is the gradual resolution premium, that is, the amount that the decision maker would pay to replace Q^n with P.[1] By Theorem 1, $\beta > 0$ implies $V_\beta(P) - V_\beta(Q^n) \geqslant 0$.

Understanding the effect of the gradual resolution premium, insurance companies, when offering dynamic insurance contracts, can require much greater premiums than the actuarially fair ones and still be sure of consumers' participation. This can help explain why people often buy periodic insurance for moderately priced objects, such as electrical appliances and cellular phones, at much more than the actuarially fair rates. An example is given by Tim Harford ("The Undercover Economist", *Financial Times*, May 13, 2006):

> "There is plenty of overpriced insurance around. A popular cell phone retailer will insure your \$90 phone for \$1.70 a week—nearly \$90 a year. The fair price of the insurance is probably closer to \$9 a year than \$90."

To illustrate, consider the following insurance problem: an individual with Gul's preferences, with a linear u and $\beta > 0$, owns an appliance (*e.g.,* a cellular phone) that he is about to use for n periods. The individual gets utility 1 in any period the appliance is used and 0 otherwise. In each period, there is an exogenous probability $(1-p)$ that the appliance will not work (it might be broken, fail to get reception, etc.). The individual can buy a periodic insurance policy, which guarantees the availability of the appliance, for a price $z \in (1-p, 1)$. Therefore, if he buys insurance for some period, he gets a certain utility of $(1-z)$, and otherwise he faces the lottery in which with probability p he gets 1, and with the remaining probability he gets 0. For simplicity, assume that the price of a replacement appliance is 0,

[1] In this section we keep assuming that u is linear. More generally the gradual resolution premium is the value x that solves: $u\left(u^{-1}\left(V_\beta(P)\right) - x\right) = V_\beta\left(Q^n\right)$.

so that the individual either carries it over from the last period or gets a new one for free in the beginning of any period.

Let P be the probability distribution over final outcomes (without insurance). Denote by X the total number of periods in which the appliance works. Since X is a binomial random variable, $\Pr(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$, for $k=0,...,n$. Applying Gul's formula, one obtains

$$V_\beta(P)=\frac{\sum_{k=h+1}^{n}\binom{n}{k}p^k(1-p)^{n-k}k+(1+\beta)\sum_{k=0}^{h}\binom{n}{k}p^k(1-p)^{n-k}k}{1+\beta\sum_{k=0}^{h}\binom{n}{k}p^k(1-p)^{n-k}}$$

where $h(p,\beta,n)$ is the unique natural number such that all prizes greater than it are elating and all those smaller than it are disappointing.

Let Q^n be the corresponding gradual (n-stage) lottery *as perceived by* the individual. By the n-stage folding back procedure, its value is:

$$V_\beta(Q^n)=\frac{1}{(1+\beta(1-p))^n}\sum_{k=0}^{n}\binom{n}{k}p^k(1-p)^{n-k}(1+\beta)^{n-k}k.$$

Using standard backward induction arguments, it can be shown that the individual will buy insurance for all n periods if $\beta>\frac{z-(1-p)}{(1-z)(1-p)}>0$. In that case, $z<1-\frac{V_\beta(Q^n)}{n}$. Nevertheless, if β is not too high,[2] we have $1-p<1-\frac{V_\beta(P)}{n}<z$, meaning that he would not buy insurance at all *if he could avoid being aware of the gradual resolution of uncertainty.*[3] This observation explains why and how the attractiveness of a lottery depends not only on the uncertainty embedded in it, but also on the way this uncertainty is resolved over time.

Since $V_\beta(P)$ decreases with β, the risk premium, $\mathrm{rp}(\beta\,|p,n):=np-V_\beta(P)$, is a strictly increasing function of β. The behavior of the gradual resolution premium, $\mathrm{grp}(\beta\,|p,n):=V_\beta(P)-V_\beta(Q^n)$ is more subtle. We have the following result:

Proposition 1: *In the insurance problem described above:*

(*i*) $\mathrm{grp}(\beta\,|p,n)>0\ \forall\beta\in(0,\infty)$

(*ii*) $\mathrm{grp}(0\,|p,n)=0$ *and* $\lim_{\beta\to\infty}\mathrm{grp}(\beta\,|p,n)=0$

(*iii*) *Single-peakness: There exists* $\beta^*(p,n)<\infty$ *such that either* $0<\beta<\beta'<\beta^*$ *or* $\beta^*<\beta'<\beta$ *implies*

$$\mathrm{grp}(\beta\,|p,n)<\mathrm{grp}(\beta'\,|p,n)<\mathrm{grp}(\beta^*\,|p,n)$$

See figure 1.

[2] The condition is: $1+\beta<\min\left\{\frac{p^n}{p^n+n(1-p)-1},\frac{p^nz}{(1-z)(1-p^n)-p(1-p^{n-1})-1}\right\}$.

[3] Nayyar (2004) termed such a situation an "insurance trap." Note that DM still acts rationally given that without insurance he is forced to be exposed to Q rather than to P.

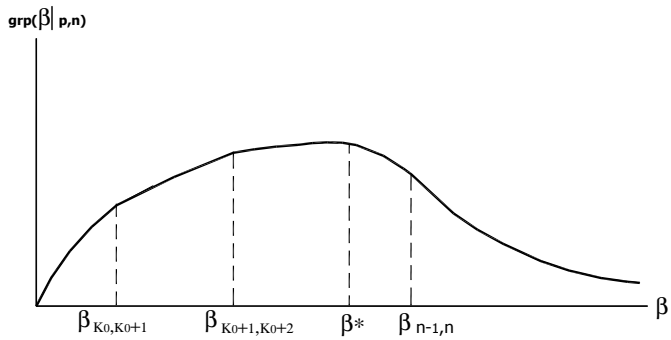

Figure 1: $\text{grp}(\beta\,|p,n)$. $\beta_{k,k+1}$ is the value of β where $h\,(\beta\,|p,n)$ decreases from $(n-k)$ to $(n-(k+1))$. $\text{grp}(\beta\,|p,n)$ is non-differentiable in each such $\beta_{k,k+1}$. k_0 is the smallest natural number that solves $\max\limits_{k'>n(1-p)} \frac{n-k'}{n}$

In its original context, a higher β implies greater disappointment aversion (as well as greater risk aversion). As we argued in the introduction, being averse to the gradual resolution of uncertainty can be interpreted as dynamic disappointment aversion. Under this interpretation, it seems intuitive to expect the gradual resolution premium to be an increasing function of β. This intuition is wrong and, in fact, item (ii) remains valid independent of the decision problem under consideration. To see this, note that $\text{grp}(\beta\,|p,n)$ is defined as the difference of two functions, both strictly decreasing in β. When $\beta = 0$, DM cares only about the expected value of the lottery. When β is sufficiently large, all prizes but 0 become elating, and hence the value of P converges to 0. Correspondingly, the value of the gradual lottery, Q^n, converges to the value of the worst sub-lottery that by itself approaches 0. Since $\text{grp}(\beta\,|p,n)$ is a continuous function and is strictly positive on the positive reals, there must exist a finite β, denoted β^* in figure 1, in which $\text{grp}(\beta\,|p,n)$ is maximized. Item (iii) sheds further light on the behavior of moderate disappointment-averse individuals. It suggests that $\beta^*\,(p,n)$ is unique, and that $\text{grp}(\beta\,|p,n)$ is single-peaked. Behaviorally speaking, moderately disappointment-averse individuals are more inclined to pay a higher premium than individuals who are either approximately disappointment-indifferent or extremely disappointment-averse.

5. Extensions

Until now we assume that the function V_β, which is applied recursively, does not change over time. Time independent V_β implies that the decision maker does not care when the uncertainty is resolved as long as all resolution happens in a single stage. We now briefly suggest how to incorporate preferences for the timing of resolution of uncertainty into our basic model. While we keep assuming that the decision maker evaluates compound lotteries using the folding-back procedure, we relax the assumption that V_β is time independent. In particular, we assume that for all t, V_{β_t} is a disappointment aversion function, in which $u_t = u$ for all t and β_t is positive and increasing with t (for example, $\beta_t = 1 - \frac{1}{t+1}$). Increasing β (while keeping u fixed) implies that the decision maker becomes more disappointment-averse (and more risk-averse) as the time of consumption gets closer.[4] Intuitively, an early bad signal is more likely to be corrected than a late signal, hence the decision maker is more sensitive to later signals.

For a given lottery P, consider the set of two stage lotteries that induce the same probability distribution over final outcomes. If $\beta_1 < \beta_2$, the decision maker faces a trade-off: since $\beta_t > 0$ for $t = 1, 2$, Theorem 1 implies that he is averse to the gradual resolution of uncertainty. But since $V_\beta(P)$ decreases in β, he prefers a lottery in which all resolution occurs in the first stage to a lottery in which all resolution happens in the second stage. Therefore, while a compound lottery that fully resolves in the first stage is unambiguously the most preferred, it is not clear whether the decision maker prefers a non-degenerate compound lottery to a lottery in which all resolution takes place in the second stage. An increase in the distance between β_1 and β_2 favors the gradually resolved lottery, whereas the one-shot aspect dominates as the compound lottery becomes more degenerate (for example, when one of the second stage nodes is obtained with probability $1 - \varepsilon$, and $\varepsilon > 0$ is small enough).[5]

A different extension is a model in which the sequence of βs, instead of being exogenously determined, evolve endogenously as a function of the history of disappointments and elations

[4] Theorem 5 in Gul (1991) establishes that the risk attitude of two decision makers who have the same u can be ranked solely by comparing their different βs. If only changes in risk aversion were concerned, we could, alternatively, fix $\beta_t = \beta$ for all t, and make the concavity of u_t increases with t.

[5] A similar tradeoff between early and one-shot resolution of uncertainty was studied in Köszegi and Rabin (2009). Köszegi and Rabin study a model in which utility additively depends on both current consumption and on recent changes in (rational) beliefs about present and future consumption, where the latter component displays loss aversion. Denote by $\gamma_{t,T}$ the weight that is given to changes in period $t < T$ beliefs about consumption in period T, and assume that the sequence $\{\gamma_{t,T}\}$ is increasing with t. Under this assumption, Köszegi and Rabin provide a set of results that identify the tradeoff that the decision maker faces. In stating these results, however, they confine their attention to the case in which consumption happens only in the last period and is binary. In our setup, this corresponds to compound lotteries over only two monetary prizes. With the modification of the sequence $\{\beta_t\}$ described in the text, we can generate similar predictions regarding information preferences without restricting the possible outcomes to take only two values.

in prior stages. Such model can address experimental evidence that suggest that the way in which risk unfolds over time affects risk attitudes (see, for example, Thaler and Johnson (1990)). For a model along these lines, see Dillenberger and Rozen (2010).

6. Appendix

Proof of Proposition 1

Let $\triangle V(\beta\,|p,n)$:=grp$(\beta\,|p,n)$, and for $k=2,3,...,n-1$, denote $\triangle V(\beta\,|p,n)$ with $h(\beta\,|p,n)=n-k$ by $\triangle V^{(k)}(\beta\,|p,n)$. Some calculations show that

$$\begin{aligned}&\triangle V^{(k)}(\beta\,|p,n)\\&=np\beta(1-p)\frac{-(1-p)^{k-1}\left(-\beta\left(\sum_{j=0}^{n-(k+1)}\binom{j+k-2}{j}p^{j}\right)+p^{n-k}\left(\binom{n-2}{n-(k+1)}\beta+\binom{n-1}{n-k}\right)\right)+1}{(1+\beta(1-p))\left(\beta\left(\sum_{j=k-1}^{n-1}\binom{j}{j-(k-1)}p^{j-(k-1)}\right)(1-p)^{k}+1\right)}\end{aligned}$$

The denominator of $\triangle V^{(k)}(\beta\,|p,n)$ is always positive, whereas the coefficient $np\beta(1-p)$ is strictly positive for $\beta>0$. At $\beta=0$ the numerator is equal to $1-\binom{n-1}{n-k}(1-p)^{k-1}p^{n-k}$ which is positive since $\binom{n-1}{n-k}(1-p)^{k-1}p^{n-k}$ is simply the probability of $n-k$ successes in $n-1$ trials of a Bernoulli random variable with parameter p. We then note that the nominator is also increasing with β. Indeed, this is the case if $\left(\sum_{j=0}^{n-(k+1)}\binom{j+k-2}{j}p^{j}\right)>p^{n-k}\binom{n-2}{n-(k+1)}$, which is true since $p<1$ and $\sum_{j=0}^{n-(k+1)}\binom{j+k-2}{j}=\binom{n-2}{n-k-1}$. Therefore, item (i) is implied. Since $\beta=0$ implies expected utility, the first part of item (ii) is immediate. For the second part of item (ii), observe that as β increases, the value of the sequential lottery $(V(Q^{n}))$ is (smoothly) strictly decreasing and converges to 0, the value of the worst prize in its support. The value of the one stage lottery $(V(\widehat{p}))$ is affected in two ways when β increases. First, given a threshold $h(\beta\,|p,n)$, the value is (smoothly) strictly decreasing with β. Second, $h(\beta\,|p,n)$ itself is a decreasing step-function of β. For β large enough, all prizes but 0 are elated and the value of the lottery is given by $\frac{\sum_{k=1}^{n}\binom{n}{k}p^{k}(1-p)^{n-k}k}{1+\beta(1-p)^{n}}\underset{\beta\to\infty}{\longrightarrow}0$.

To show the existence of β^{*} (item (iii)), pick $\beta'>0$ such that grp$(\beta'\,|p,n)=\epsilon>0$. Since $\lim_{\beta\to\infty}$ grp$(\beta\,|p,n)=0$, there exists $\overline{\beta}:=\max\left\{\beta\,\middle|\,\text{grp}(\beta\,|p,n)=\frac{\epsilon}{2}\right\}$ and $\overline{\beta}<\infty$. Thus grp$(\beta\,|p,n)$ is a continuous function on the compact interval $\left[0,\overline{\beta}\right]$, and hence achieves its maximum on this domain. For single-peakness, we have the following two claims:

Claim 1: $\forall k=2,3,...,n-1$, $\triangle V^{(k)}(\beta\,|p,n)$ *is either strictly increasing or single-peaked on* $(0,\infty)$.

Proof: By differentiating $\triangle V^{(k)}(\beta\,|p,n)$ with respect to β, one gets:

$$\frac{\partial}{\partial\beta}\triangle V^{(k)}(\beta\,|p,n)$$
$$= np\frac{C\beta^2+\left(2Ap^k(1-p)^k-2p^n\binom{n-2}{n-k-1}(1-p)^k\right)\beta+\left((1-p)p^k-p^n\binom{n-1}{n-k}(1-p)^k\right)}{p^k(-\beta+p\beta-1)^2\left(B\beta(-p+1)^k+1\right)^2}$$

Where C is some constant, and $A:=\left(\sum_{j=0}^{n-(k+1)}\binom{j+k-2}{j}p^j\right)$.

The roots of $\frac{\partial}{\partial\beta}\triangle V^{(k)}(\beta\,|p,n)$ are the roots of the second-degree polynomial in β that appears in the nominator.

Evaluated at $\beta=0$, this polynomial is equal to $\left(p^k-pp^k-p^n\binom{n-1}{n-k}(1-p)^k\right)$. Note that

$$\left(p^k-pp^k-p^n\binom{n-1}{n-k}(1-p)^k\right)>0\Longleftrightarrow 1>\binom{n-1}{n-k}p^{n-k}(1-p)^{k-1}$$

which is true as claimed before.

In addition, the slope of that polynomial at $\beta=0$ is equal to the coefficient of β, $2Ap^k(1-p)^k-2p^n\binom{n-2}{n-k-1}(1-p)^k$, which is positive since $\left(\sum_{j=0}^{n-(k+1)}\binom{j+k-2}{j}p^j\right)>p^{n-k}\binom{n-2}{n-k-1}$.

To summarize, both the slope and the intercept of the polynomial in the nominator are positive at $\beta=0$. Therefore, if $C\geq 0$ then $\frac{\partial}{\partial\beta}\triangle V^{(k)}(\beta\,|p,n)$ has no positive roots, and otherwise it has exactly one positive root.‖

Note that $\triangle V(\beta\,|p,n)$ is a continuous function that is not differentiable in the points where $h(\beta\,|p,n)$ changes. For $k=2,3,...,n-1$, let $\beta_{k,k+1}$ be the value of β where $h(\beta\,|p,n)$decreases from $(n-k)$ to $(n-(k+1))$. Using the same notation as above, we claim that at the switch point, the slope of the resolution premium decreases.

Claim 2: $\lim\limits_{\beta\to_-\beta_{k,k+1}}\frac{\partial}{\partial\beta}\triangle V^{(k)}(\beta\,|p,n)>\lim\limits_{\beta\to_+\beta_{k,k+1}}\frac{\partial}{\partial\beta}\triangle V^{(k+1)}(\beta\,|p,n)$

Proof: Apart from at $\beta=0$, where $\triangle V^{(k)}(0\,|p,n)=\triangle V^{(k+1)}(0\,|p,n)=0$, it can be shown that the two curves cross at exactly one more point, given by

$$\beta_{k,k+1}=\frac{np-(n-k)}{\left(\sum_{j=0}^{n-(k+1)}(n-k-j)\binom{j+k-1}{j}p^j\right)(1-p)^{k+1}}$$

Note that $\beta_{k,k+1}>0$ iff $p>\frac{n-k}{n}$. To prove the claim it will be sufficient to show that $\frac{\partial}{\partial\beta}\triangle V^{(k)}(0\,|p,n)<\frac{\partial}{\partial\beta}\triangle V^{(k+1)}(0\,|p,n)$, since this implies that at $\beta_{k,k+1}$, $\triangle V^{(k+1)}(\beta\,|p,n)$

crosses $\triangle V^{(k)}(\beta\,|p,n)$ from above. Now

$$\frac{\partial}{\partial\beta}\triangle V^{(k)}(0\,|p,n) = np\frac{\left(p^k - pp^k - p^n\binom{n-1}{n-k}(1-p)^k\right)}{p^k}$$

and

$$\frac{\partial}{\partial\beta}\triangle V^{(k+1)}(0\,|p,n) = np\frac{\left(p^{k+1} - pp^{k+1} - p^n\binom{n-1}{n-k-1}(1-p)^{k+1}\right)}{p^{k+1}}.$$

Therefore,

$$\begin{aligned}
&\frac{\partial}{\partial\beta}\triangle V^{(k+1)}(0\,|p,n) > \frac{\partial}{\partial\beta}\triangle V^{(k)}(0\,|p,n)\\
\iff &\frac{1}{p^k}n(-p+1)^k p^n\left(p\binom{n-1}{-k+n} + p\binom{n-1}{-k+n-1} - \binom{n-1}{-k+n-1}\right) > 0\\
\iff &p\binom{n-1}{-k+n} + p\binom{n-1}{-k+n-1} - \binom{n-1}{-k+n-1} > 0\\
\iff &p > \frac{\binom{n-1}{-k+n-1}}{\binom{n-1}{-k+n} + \binom{n-1}{-k+n-1}} = \frac{(n-k)}{n}.\|
\end{aligned}$$

To complete the proof, we verify that both claims above are also valid for the two extreme cases: $k=1$ (where only the best prize, n, is elation) and $k=n$ (only the worst prize, 0, is disappointment).

$k=1$: Using the same notation as above we have:

$$\triangle V^{(1)}(\beta\,|p,n) = np\beta\left(\sum_{j=0}^{n-2}p^j\right)(p-1)^2\frac{\beta+1}{(1+(1-p)\beta)(1+(1-p^n)\beta)}$$

and

$$\frac{\partial}{\partial\beta}\triangle V^{(1)}(\beta\,|p,n) = n(1-p)(p-p^n)\frac{(1-pp^n)\beta^2+2\beta+1}{(-\beta+p^n\beta-1)^2(-\beta+p\beta-1)^2} > 0$$

for all $\beta \geq 0$ so $\triangle V^{(1)}(\beta\,|p,n)$ is strictly increasing with β (claim 1).

For the second claim, similar calculations establish that:

$$\frac{\partial}{\partial\beta}\triangle V^{(2)}(0\,|p,n) > \frac{\partial}{\partial\beta}\triangle V^{(1)}(0\,|p,n) \iff p > \frac{n-1}{n}$$

so claim 2 follows as well.

$k = n$:

$$\triangle V^{(n)}(\beta\,|p,n) = np^2\beta\,(1-p)\,\frac{\left(\sum_{j=1}^{n-1}\binom{n-1}{j}p^{j-1}(-1)^{j-1}\right)}{(1+\beta\,(1-p))\,(1+\beta\,(1-p)^n)}$$

Let $C = \left(\sum_{j=1}^{n-1}\binom{n-1}{j}p^{j-1}(-1)^{j-1}\right)$, so:

$$\frac{\partial}{\partial\beta}\triangle V^{(n)}(\beta\,|p,n) = Cnp^2\,(p-1)\,\frac{\beta^2\,(1-p)^{n+1}-1}{(\beta\,(-p+1)^n+1)^2\,(-\beta+p\beta-1)^2}$$

which is clearly single peaked *on* $(0,\infty)$ (claim 1), and, again by similar calculations:

$$\frac{\partial}{\partial\beta}\triangle V^{(n)}(0\,|p,n) > \frac{\partial}{\partial\beta}\triangle V^{(n-1)}(0\,|p,n) \Longleftrightarrow p > \frac{1}{n}$$

which is claim 2.

Combining claim 1 and claim 2 ensures that $\triangle V(\beta\,|p,n)$is single-peaked on $(0,\infty)$. $\square$

References

[1] Ang, Andrew, Geert Bekaert, and Jun Liu (2005), "Why Stocks May Disappoint." *Journal of Financial Economics*, 76: 471-508.

[2] Dillenberger, David (2010), "Preferences for One-Shot Resolution of Uncertainty and Allais-Type Behavior." *Econometrica,* forthcoming.

[3] Dillenberger, David and Kareen Rozen (2010), "Disappointment Cycles." Unpublished Manuscript, University of Pennsylvania.

[4] Köszegi, Botond and Matthew Rabin (2009), "Reference-Dependent Consumption Plans." *American Economic Review*, 99: 909–936.

[5] Gul, Faruk (1991), "A Theory of Disappointment Aversion." *Econometrica*, 59: 667-686.

[6] Nayyar, Shivani (2004), "Disappointment Aversion and the Insurance Trap." Unpublished manuscript.

[7] Palacious-Huerta, Ignacio (1999), "The Aversion to the Sequential Resolution of Uncertainty." *Journal of Risk and Uncertainty*, 18: 249-269

[8] Thaler, Richard and Eric Johnson (1990), "Gambling with the House Money and Trying to Break Even: The Effects of Prior Outcomes on Risky Choice." *Management Science*, 36: 643–660.