PIER Working Paper 08-040

"Worker Replacement"

by

Guido Menzio and Espen Moen

http://ssrn.com/abstract=1291130

Worker Replacement

GUIDO MENZIO   ESPEN MOEN[*]

University of Pennsylvania   Norwegian School of Management

Hoover Institution

September 2008

Abstract

We consider a frictional labor market in which firms want to insure their senior employees against income fluctuations and, at the same time, want to recruit new employees to fill their vacant positions. Firms can commit to a wage schedule, i.e. a schedule that specifies the wage paid by the firm to its employees as function of their tenure and other observables. However, firms cannot commit to the employment relationship with any of their workers, i.e. firms can dismiss workers at will. We find that, because of the firm's limited commitment, the optimal schedule prescribes not only a rigid wage for senior employees, but also a downward rigid wage for new hires. Moreover, we find that, while the rigidity of the wage of senior workers does not affect the allocation of labor, the rigidity of the wage of new hires magnifies the response of unemployment and vacancies to negative shocks to the aggregate productivity of labor.

JEL Codes: E24, E32, J64.

Keywords: Competitive Search, Risk Sharing, Unemployment, Business Cycles.

1

1 Introduction

1.1 Motivation

Imagine a firm that needs to fill a vacant position in its accounting department. Since the economy is in a severe recession, the firm can attract some candidates by offering a low hiring wage. However, since workers dislike income fluctuations, the firm continues to offer to its senior accountants the same high wage that they received before the economy entered the recession. Now, imagine that two candidates show up at the firm's door. Then, the firm has the incentive to fill the vacancy with the first candidate, and to replace one of the senior accountants with the second candidate. And if the firm is unable to commit to the employment relationship with its workers, it will replace one of the senior accountants.

In this paper, we want to build a model of the worker replacement problem described in the previous paragraph. Then, we want to use the model to understand how the replacement problem affects the design of the optimal wage schedule of the firm. That is, the schedule that specifies the wage paid by the firm to its employees conditional on their tenure and other observable characteristics of the firm and of the aggregate economy. Finally, we want to use the model to understand how the replacement problem (through its effect on the wage schedule) affects the response of unemployment, vacancies and other labor market variables to aggregate productivity shocks.

1.2 Summary

We consider an economy populated by risk-neutral firms and by risk-averse workers who do not have access to the credit and the insurance markets. At the beginning of each period, new firms enter the labor market and post a wage schedule, i.e. a schedule that specifies the wage paid by the firm to its employees, conditional on their tenure, the firm's productivity, and the aggregate conditions of the economy. After having observed the schedule posted by new and old firms, unemployed workers choose where to apply for a job. We assume that firms can commit to their wage schedule. However, we assume that firms cannot commit to the employment relationship with any of their workers, i.e. firms can dismiss workers at will. Moreover, in order to keep the model tractable, we assume that firms are productive for two periods only.

In this environment, the firm has two goals. First, the firm wants to insure its workers against income fluctuations. In order to do this efficiently, the firm should offer to its senior workers a wage that is independent from its productivity and from the aggregate conditions of the economy. Also, the firm should employ its senior workers as long as the gains from trade are positive. Second, the firm wants to recruit new workers to fill its vacant positions. In order to do this efficiently, the firm should offer to its junior workers a wage that positively depends on its productivity and on the aggregate conditions of the economy. However, because the firm has limited commitment, it cannot insure its senior employees efficiently and, at the same time, recruit new employees efficiently. For example, consider a situation in which the firm's productivity falls so much that the efficient hiring wage is lower than the efficient insurance wage. In this situation, if the schedule were to prescribe the efficient insurance wage for senior employees and the efficient hiring wage for junior employees, the firm would replace all the senior workers for whom there is a qualified substitute among the pool of new applicants.

In the first part of the paper, we study how the optimal schedule resolves the tension between efficient insurance provision and efficient recruitment caused by the firm's limited commitment. We find that, in the second period of the firm's life, the optimal schedule divides the firm's productivity space into three regions. If the firm's productivity falls in the highest region, the optimal schedule prescribes the efficient hiring wage for junior employees and the efficient insurance wage for senior employees. In this region, the firm does not replace its senior employees with new hires, because the efficient hiring wage is greater than the efficient insurance wage. If the firm's productivity falls in the intermediate region, the optimal schedule prescribes the same wage for junior and senior employees. This wage is greater than the efficient hiring wage and smaller than the efficient insurance wage, but it guarantees that senior workers will not be replaced with new hires. When the firm's productivity falls in the lowest region, the design of the optimal schedule varies depending on the parameters of the model. For example, if the search frictions in the labor market are sufficiently strong, the schedule prescribes that senior employees should be paid the efficient insurance wage, and that junior employees should be paid somewhat less than the efficient hiring wage. Given these prescriptions, the firm replaces its senior employees with a small,

3

but positive probability.

In the second part of the paper, we study the effect of the firm's limited commitment on the response of unemployment and vacancies to aggregate productivity shocks. We find that limited commitment magnifies the response of unemployment and vacancies to small negative productivity shocks. In contrast, we find that limited commitment does not affect the response of unemployment and vacancies to positive productivity shocks.

When a negative productivity shock hits the economy, the wage offered by old firms to junior employees does not fall as much as it would have fallen under full commitment, and the wage offered to senior employees falls more than under full commitment. While the distortion on the wage of senior employees does not affect the allocation of labor, the distortion on the wage of junior employees implies that too many workers apply for a job at one of the old firms. For this reason, not enough new firms enter the labor market, not enough new vacancies open, and too many workers remain unemployed. In contrast, when a positive productivity shock hits the economy, the wages offered by old firms to junior and senior employees are the same as under full commitment. For this reason, the number of firms that enter the labor market, the number of vacancies that open, and the number of workers who remain unemployed are the same as under full commitment.

Our findings are based on the assumption that a firm can commit to its wage schedule, but cannot commit to the employment relationship with any of its workers. This assumption is common in the literature (e.g. Thomas and Worrall 1988, MacLeod and Malcomson 1989, Rudanko 2006) and can be justified in several ways. For example, a third party (e.g. a court of law, a workers' union) may not be able to verify whether a worker that leaves the firm does so voluntarily, because of poor performance, because his job ceased to exist, or because the firm replaced him with another worker. In contrast, a third party may be able verify whether a worker has been employed by the firm and how much he has been paid. For this reason, enforcing the firm's promises about employment may be extremely difficult, while enforcing the firm's promises about wages may be much easier. For the same reason, enforcing severance payments may be difficult as well.

There is also some empirical evidence that supports our assumption. For example, Bewley (1999) finds that many firms set up an explicit pay structure, i.e. a system tying pay to

individual output, qualification, and seniority. However, Bewley does not report of any firms that explicitly promise to their employees that they will not be replaced with new hires. These observations suggest that firms are able to commit to their wage schedule but are unable to commit to their employment relationships.

1.3 Related Literature

Our paper contributes to the literature that studies the properties of the optimal employment contract between a risk-neutral firm and a risk-averse worker who cannot access the insurance and the credit markets. Azariadis (1975) and Bailey (1974) study the properties of the optimal contract when both the firm and the worker are able to commit to their employment relationship. They find that the optimal contract prescribes a wage that remains constant over time and across states of the world. Holmstrom (1983) and Beaudry and DiNardo (1991) study the properties of the optimal contract when only the firm can commit to the employment relationship. They find that the optimal contract prescribes a downward rigid contract. Finally, Thomas and Worrall (1988) and Rudanko (2006) study the properties of the optimal contract when neither the firm nor the worker is able to commit to the employment relationship. They find that the optimal contract prescribes a wage that remains constant as long as it induces the two parties to stay together. In contrast to ours, these papers abstract from the replacement problem because they either assume that the firm can employ at most one worker at a time (as in Thomas and Worrall 1988 and in Rudanko 2006) or that the firm can commit to the employment relationship with its employees (as in Holmstrom, 1983). Therefore, in contrast to ours, these papers do not predict any rigidity in the hiring wage of successive cohorts of workers (which is the kind of wage rigidity that magnifies unemployment fluctuations).[1]

---

[1] Independently from us, Snell and Thomas (2007) develop a related theory of wage rigidity. They consider an economy populated by risk-neutral firms and risk-averse workers who cannot access the insurance and credit markets. Under the assumption that firms cannot wage discriminate between workers with different tenure, they find that the optimal contract prescribes a wage that does not respond to productivity shocks as much as a spot market wage. Moreover, under the assumption that new firms cannot enter the labor market, they find that there is involuntary unemployment, and that involuntary unemployment is countercyclical. There are two key differences between this paper and ours. First, in our paper, the link between the wage of senior and junior employees is not exogenous, but emerges endogenously as the optimal solution of the replacement problem. Second, in our paper, new firms are allowed to enter the labor market. Moreover, the rigidity of the hiring wage of old firms magnifies the response of unemployment to productivity shocks only because it has an effect of the entry of new firms.

Our paper also relates to the literature that uses search models of the labor market to measure the contribution of different types of shocks to the cyclical volatility of unemployment, vacancies and workers' transition rates. Using Pissarides' (1985) model, Shimer (2005) finds that aggregate productivity shocks account for a negligible fraction of the cyclical volatility of US unemployment and vacancies.[2] Using a version of Pissarides' model in which the wage of new hires is assumed to be rigid, Hall (2005) finds that aggregate productivity shocks account for almost all of the cyclical volatility of US unemployment and vacancies. Similarly, using a search model in which firms are not allowed to adjust the wage of new hires in every period, Gertler and Trigari (2006) find that aggregate productivity shocks cause large movements in the unemployment rate. In our paper, we find that the optimal solution to the worker-replacement problem is such that the wage of new hires is downward rigid. In this sense, our paper provides a partial theoretical foundation to the assumptions adopted by Hall (2005) and by Gertler and Trigari (2006).[3]

2 The Model

Time is discrete and continues forever. The economy is populated by a continuum of workers with measure 1. Each worker is endowed with an indivisible unit of labor. Each worker maximizes the von Neumann-Morgenstern function $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where $c_t \in \mathbb{R}_+$ is the worker's consumption in period t, $u : \mathbb{R}_+ \to \mathbb{R}$ is a twice differentiable, strictly increasing and strictly concave function, and $\beta \in (0, 1)$ is the discount factor. The economy is also populated by a continuum of firms with positive measure. Each firm can enter the labor market by paying the investment cost $I > 0$. When it is in the labor market, the firm operates a

---

[2]Hagedorn and Manovskii (2008) argue that the findings in Shimer (2005) are very sensitive to the strategy used to calibrate the Pissarides' model. Menzio and Shi (2008a) and Ramey (2008) argue that the Pissarides' model provides a biased measure of the contribution of aggregate productivity shocks to the cyclical volatility of the US labor market, because it fails to endogenize the workers' transition from employment to unemployment and across employers.

[3]Menzio (2005), Kennan (2008) and Moen and Rosen (2008) develop alternative theories of wage rigidity. According to Menzio (2005), a firm does increase its hiring wage in response to a positive productivity shock, because it does not want to reveal to its senior employees that the gains from trade have increased. According to Kennan (2008), wages are determined as the outcome of an asymmetric information bargaining game between the firm and the worker. If an expansion is associated with an increase in the extent of the firm's private information, the firm's profits will be more procyclical, and the worker's wage will be less procyclical than under symmetric information. Moen and Rosen (2008) consider a labor market with search frictions, adverse selection and moral hazard. They find that the worker's share of the surplus from an employment relationship is countercyclical.

technology that turns labor into output according to the production function $\min\{n, n_t\}y_t$, where $n > 0$ is the measure of jobs at the firm, $n_t \geq 0$ is the measure of workers employed by the firm in period t, and $y_t \geq 0$ is the firm's productivity in period t. After operating its technology for two periods, the firm exits the labor market. A firm maximizes the von Neumann-Morgenstern function $\sum_{t=0}^{\infty} \beta^t \pi_t$, where $\pi_t \in \mathbb{R}$ is the firm's profit in period t.

Each period is divided into four stages: separation, entry, search, and production. During the separation stage, an employed worker exogenously moves into unemployment with probability $\delta \in (0, 1)$. Moreover, during the separation stage, an employed worker can voluntarily move into unemployment, and a firm can voluntarily dismiss any of its employees.

During the second stage, a firm chooses whether to enter the labor market or not. If the firm enters the labor market, it first draws its idiosyncratic productivity $y_1 \in Y$ from the probability distribution $\Psi_1(y_1|x)$, where $x \in X$ is the aggregate productivity of the economy, $X = \{x_1, x_2, ...x_{N(x)}\}$ and $Y = \{y_1, y_2, ...y_{N(y)}\}$. Then, the firm announces its wage schedule $\omega_1 = \{w_1, w_{i,2}\}$. The first element of the schedule specifies the wage paid by the firm to a worker who is employed in the current period, $w_1 \in \mathbb{R}_+$. The second element of the schedule is a function $w_{i,2} : X \times Y \times [0, 1] \to \mathbb{R}_+$ that specifies the wage paid by the firm to a worker who is employed in the next period, given that the worker has tenure $i \in \{1, 2\}$, the aggregate productivity of the economy[4] is $\hat{x} \in X$, the firm's idiosyncratic productivity is $\hat{y}_2 \in Y$, and the outcome of a lottery is $\hat{\sigma}_2 \in [0, 1]$. It is useful to denote with $\hat{s}_2$ the vector $\{\hat{x}, \hat{y}_2, \hat{\sigma}_2\}$, and with S the set $X \times Y \times [0, 1]$.

During the third stage, a worker gets the opportunity of searching for a job with a probability that depends on his recent employment history. In particular, if the worker was unemployed at the beginning of the period, he has the opportunity to search with probability 1, otherwise he cannot search. If the worker has the opportunity to search, he sends an application for a particular job to a particular firm.[5] The worker's application is successful with probability $\lambda(q)$, where q is the expected number of applications submitted for the job, and $\lambda : \mathbb{R}_+ \to [0, 1]$ is a twice differentiable function such that $\lambda'(q) < 0$, $\lambda(0) = 1$

---

and $\lambda(\infty) = 0$. Conversely, the firm finds a successful applicant for the job with probability $\eta(q) = \lambda(q)q$., where $\eta : \mathbb{R}_+ \to [0, 1]$ is a twice differentiable function such that $\eta'(q) > 0$, $\eta''(q) < 0$, $\eta(0) = 0$ and $\eta(\infty) = 1$. If the firm finds a successful applicant, it has the opportunity to hire him and to dismiss any other worker who might have been holding the job. We find it convenient to assume that the elasticity $\epsilon_\eta(q)$ of the job-filling probability $\eta(q)$ with respect to q is such that $\eta(q)\epsilon_\eta(q)/(1{-}\epsilon_\eta(q))$ is a strictly decreasing function.[6]

During the last stage, an unemployed worker produces and consumes $b > 0$ units of output. A worker employed at a new firm[7] produces y_1 units of output and consumes w_1 of them. A worker employed at an old firm produces y_2 units of output, consumes w_2 of them, and then moves into unemployment. As implied by the notation, we assume that workers can neither borrow nor save.[8]

At the end of the production stage, nature draws next period's aggregate productivity $\hat{x}$ from the probability distribution $\Omega(\hat{x}|x)$. Then, for each new firm, nature draws next period's idiosyncratic productivity $\hat{y}_2$ from the probability distribution $\Psi_2(\hat{y}_2|\hat{x})$. And finally, for each new firm, nature draws the realization $\hat{\sigma}_2$ of next period's lottery from the uniform distribution over the interval $[0, 1]$.

3 Conditions and Definition of Equilibrium

At the beginning of each period, the state of the economy can be summarized by the current realization of aggregate productivity and by the distribution of old firms across different wage schedules and employment levels. In this paper, we are interested in equilibria in which the agents' values and strategies depend on the state of the economy only through the realization of aggregate productivity, and not through the entire distribution of old firms.[9]

---

[6]Our assumptions on $\lambda(q)$ and $\eta(q)$ are satisfied by many of the standard matching processes. For example, our assumptions are satisfied by the urn-ball matching process, $\lambda(q) = q^{-1}(1 - \exp(-q))$ and $\eta(q) = 1 - \exp(-q)$, and by the telephone-line matching process, $\lambda(q) = (1 + q)^{-1}$ and $\eta(q) = (1 + q)^{-1}q$.

[7]Througout the paper, we refer to the firms that have entered the labor market in the current period as new firms; and we refer to the firms that entered the labor market in the previous period as old firms.

[8]This assumption is common in the literature on labor contracts, e.g. Azariadis (1975), Holmstrom (1983), Thomas and Worrall (1988), Beaudry and DiNardo (1991), Boldrin and Horvath (1995), Burdett and Coles (2003), Rudanko (2006).

[9]As we shall see in the next pages, the equilibrium conditions of our model define a system of functional equations in which all of the equations depends on the state of the economy only through the aggregate state, and not through the distribution of firms across employment states. Therefore, if this system of functional equations admits a solution, there will be an equilibrium in which the agents' values and strategies are

In these equilibria, we can denote with $U(x)$ the lifetime utility of an unemployed worker whenever aggregate productivity is x. Similarly, whenever aggregate productivity is x, we can denote with $W_1(\omega_1; x)$ the lifetime utility of a worker employed by a new firm that offers the schedule ω_1, and with $W_2(w_2; x)$ the lifetime utility of a worker employed by an old firm that offers him the wage w_2. These value functions are measured at the beginning of the production stage. Finally, we can denote with $q(W; x)$ the expected queue of applicants attracted by a job of type W, where W is the applicant's lifetime utility if he successfully matches with the job.[10]

3.1 Equilibrium Conditions

3.1.1 Worker's Problem

Consider a worker who has the opportunity to apply for a job at the beginning of the search stage. If the worker does not apply to any job, he enters the production stage unemployed. If the worker applies to a job W, he succeeds in matching with the job with probability $\lambda(q(W; x))$, and he fails with probability $1 - \lambda(q(W; x))$. If he succeeds, his lifetime utility is W. If he fails, he enters the production stage unemployed. Therefore, the worker's lifetime utility at the beginning of the search stage is

$$Z(x) = U(x) + \max_{W \geq U(x)}\{\lambda(q(W; x))(W - U(x))\}. \tag{E1}$$

Consider an unemployed worker at the beginning of the production stage. In the current period, the worker produces and consumes b units of output. In the next period, the worker enters the search stage unemployed and has the opportunity to apply for a job. Therefore, the worker's lifetime utility $U(x)$ is equal to

$$U(x) = u(b) + \beta \mathbb{E}_{\hat{x}|x}[Z(\hat{x})]. \tag{E2}$$

Throughout the paper, $\mathbb{E}_{\chi|x}$ denotes the expectation of the variable χ conditional on x.

Next, consider a worker who is employed by an old firm at the beginning of the production stage. In the current period, the worker consumes w_2 units of output. In the next period, the worker enters the search stage unemployed and has the opportunity to apply for a job. Therefore, the worker's lifetime utility $W_2(w_2; x)$ is equal to

$$W_2(w_2; x) = u(w_2) + \beta\mathbb{E}_{\hat{x}|x}[Z(\hat{x})]. \tag{E3}$$

We denote with $\mu(w; x)$ the probability that an old firm finds a successful applicant for a job that offers the value $W_2(w; x)$, i.e. $\mu(w; x) = \eta(q(W_2(w; x); x))$.

Finally, consider a worker who is employed by a new firm at the beginning of the production stage. In the current period, the worker consumes w_1 units of output. In the next separation stage, the worker becomes unemployed with probability d and remains employed with probability $1 - d$, where $d = \delta$ if $w_{2,2}$ is greater than b and smaller than $\hat{y}_2$, and $d = 1$ otherwise. If the worker enters the next search stage unemployed, he does not have the opportunity to apply for a job. If the worker is still employed at the beginning of the next search stage, he is replaced by a new hire with probability $\rho\mu(w_{1,2}; \hat{x})$, where $\rho = 1$ if $w_{2,2}$ is smaller than $w_{1,2}$, and $\rho = 0$ otherwise. With probability $1 - \rho\mu(w_{1,2}; \hat{x})$, the worker remains employed until the next production stage. Therefore, the worker's lifetime utility $W_1(\omega_1; x)$ is equal to

$$\begin{aligned} W_1(\omega_1; x) = \quad & u(w_1) + \beta\mathbb{E}_{\hat{s}_2|x}[dU(\hat{x}) + (1 - d)\rho\mu(w_{1,2}; \hat{x})U(\hat{x})] + \\ & \beta\mathbb{E}_{\hat{s}_2|x}[(1 - d)(1 - \rho\mu(w_{1,2}; \hat{x}))W_2(w_{2,2}, \hat{x})]. \end{aligned} \tag{E4}$$

For the sake of brevity, the previous expression omits the dependence of $w_{1,2}$, $w_{2,2}$, d and ρ on $\hat{s}_2$.

3.1.2 Firm's Problem

Consider a new firm at the beginning of the production stage. The wage schedule offered by the firm is $\omega_1 = \{w_1, w_{i,2}\}$. The measure of workers employed by the firm is n_1. In the current period, the firm's profits are $n_1(y_1 - w_1)$. In the next period, the firm loses a fraction d of its employees during the separation stage. During the search stage, the firm receives $q(W_2(w_{1,2}; \hat{x}); \hat{x})$ applications for each of its $n - n_1(1 - d)$ vacant positions. Moreover, if $w_{1,2}$

10

is smaller than $w_{2,2}$, the firm receives $q(W_2(w_{1,2}; \hat{x}); \hat{x})$ applications for each of its $n_1(1 - d)$ filled positions. During the production stage, the firm employs $n_1(1 - d)(1 - \rho\mu(w_{1,2}; \hat{x}))$ senior workers and $[n - n_1(1 - d)(1 - \rho)] \mu(w_{1,2}; \hat{x})$ junior workers. The firm pays the wage $w_{2,2}$ to all of its senior workers and the wage $w_{1,2}$ to all of its junior workers. Therefore, the firm's lifetime profits $F_1(\omega_1, n_1; s_1)$ are equal to

$$
\begin{aligned}
F_1(\omega_1, n_1; s_1) = \quad & n_1(y_1 - w_1) + \\
& \beta \mathbb{E}_{\hat{s}_2 | x} \left\{ [n_1(1 - d)(1 - \rho\mu(w_{1,2}; \hat{x}))] (\hat{y}_2 - w_{2,2}) \right\} + \\
& \beta \mathbb{E}_{\hat{s}_2 | x} \left\{ [n - n_1(1 - d)(1 - \rho)] \mu(w_{1,2}; \hat{x})(\hat{y}_2 - w_{1,2}) \right\}.
\end{aligned}
\tag{E5}
$$

For the sake of brevity, the previous expression omits the dependence of $w_{1,2}$, $w_{2,2}$, d and ρ on $\hat{s}_2$.

Next, consider a new firm at the entry stage. First, the firm realizes its state s_1, where $s_1 = \{x, y_1, \sigma_1\}$. Then, the firm chooses which wage schedule to offer. If the firm chooses to offer the wage schedule ω_1, its profits at the beginning of the production stage are $F_1(\omega_1, n_1; s_1)$, where n_1 is equal to $n\eta(q(W_1(\omega_1; x); x))$. Therefore, conditional on s_1, the lifetime profits of a newly created firm are

$$
\begin{aligned}
F(s_1) = \max_{\omega_1} \quad & F_1(\omega_1, n_1; s_1), \\
\text{s.t.} \quad & n_1 = n\eta(q(W_1(\omega_1; x); x)).
\end{aligned}
\tag{E6}
$$

3.1.3 Queue Length

During the search stage, a worker chooses where to send an application. The benefit of applying to a job W is $\lambda(q(W; x))W + (1 - \lambda(q(W; x)))U(x)$. The opportunity cost of applying to a job W is $Z(x)$. When the benefit is strictly smaller than the cost, the worker's optimal search strategy is to not send his application to the job W. When the benefit is equal to the cost, the worker is indifferent between sending his application there or somewhere else.

If the value W is offered by a positive number of jobs, the queue length $q(W; x)$ is consistent with the worker's optimal search strategy if and only if

$$
Z(x) \geq \lambda(q(W; x))W + (1 - \lambda(q(W; x)))U(x),
\tag{E7}
$$

and $q(W; x) \geq 0$, with complementary slackness. If there is no job offering the value W,

the queue length $q(W; x)$ is consistent with the worker's optimal search strategy if and only if $\lambda(q(W; x))W + (1 - \lambda(q(W; x)))U(x)$ is smaller or equal than $Z(x)$. Following most of the literature on competitive search (e.g. Moen 1997, Acemoglu and Shimer 1999, Menzio 2007, Garibaldi and Moen 2008), we restrict attention to equilibria in which the queue length $q(W; x)$ satisfies condition (E7) for all $W's$.

3.1.4 Market Clearing and Free Entry

In equilibrium, the market for job applications clears, i.e. the measure of applications received by new and old firms is equal to the measure of applications sent by unemployed workers. When new firms do not receive any applications, this market clearing condition determines the value of searching $Z(x)$. When new firms receive some applications, this market clearing condition determines the number of new firms. In this case, the value of searching $Z(x)$ is determined by the firm's free entry condition

$$\mathbb{E}_{y_1|x}[F(s_1)] = I. \tag{E8}$$

In this paper, we restrict attention to equilibria in which new firms receive a positive number of applications.

3.2 Definition of Equilibrium

The previous paragraphs motivate the following definition of equilibrium.

Definition 1: *A Recursive Equilibrium with Entry (REE) consists of a search value function $Z : X \to \mathbb{R}$, an unemployment value function $U : X \to \mathbb{R}$, a firm's profit function $F : S \to \mathbb{R}$, a policy function $\omega_1 : S^2 \to \mathbb{R}_+^3$, and a queue length function $q : \mathbb{R} \times X \to \mathbb{R}_+$. These functions satisfy the following conditions:*
(i) For all $x \in X$, $Z(x)$ satisfies equation (E1);
(ii) For all $x \in X$, $U(x)$ satisfies equation (E2);
(iii) For all $s \in S$, $F(s)$ is the maximum of (E6), and $\omega(s)$ is the associated maximizer;
(iv) For all $W \in \mathbb{R}$ and all $x \in X$, $q(W; x)$ satisfies equation (E7);
(v) For all $x \in X$, $F(s)$ satisfies equation (E8).

4 Micro Consequences of the Replacement Problem

Because of limited commitment, firms face a trade-off between the goal of insuring their senior employees against income fluctuations, and the goal of recruiting new workers efficiently. To illustrate this point, consider a firm that enters the labor market, draws a relatively high realization of the idiosyncratic component of productivity, and hires workers at a relatively high wage. In the second period of activity, this firm is hit by a negative shock to the idiosyncratic component of productivity. In order to insure the income of its senior employees, this firm would like to offer them the same wage as in the previous period. In order to efficiently recruit junior employees, this firm would like to offer them a relatively low wage. However, given these wages, the firm would have the incentive to replace senior employees with new hires. In this section, we characterize the optimal resolution to this trade-off between insurance and recruitment.

4.1 Necessary Conditions for Optimality

Consider an old firm at the beginning of the period. Let $s_2 = \{x_2, y_2, \sigma_2\}$ denote, respectively, the current realization of the aggregate component of productivity, the realization of the firm-specific component of productivity, and the realization of the sunspot. Let n_1 denote the number of senior workers currently employed by the firm. Let $\omega_1 = \{w_1, w_{i,2}\}$ denote the firm's wage schedule. We find it convenient to let ψ_2 denote the tuple $\{n_1, w_1, s_2\}$; to let v_1 denote the wage that the schedule ω_1 prescribes for junior workers in state s_2; and to let v_2 denote the wage that the schedule ω_1 prescribes for senior workers in state s_2.

If $v_2 \leq v_1$ and $b \leq v_2 \leq y_2$, the sum of senior workers' lifetime utility and firm's profits is equal to

$$
\begin{aligned}
V_k(v_1, v_2; \psi_2) = \quad & n_1 \left[(1-\delta)u(v_2) + \Phi(x_2)\right]/u'(w_1) + \\
& [n_1(1-\delta)](y_2 - v_2) + [n - n_1(1-\delta)]\mu(v_1; x_2)(y_2 - v_1),
\end{aligned}
\tag{C1}
$$

where $\Phi(x)$ is defined as to $\delta u(b) + \beta \mathbb{E}_{\hat{x}|x}[Z(\hat{x})]$. The first term on the right hand side of (C1) is the product of the number of senior workers, n_1, and the lifetime utility of each of these workers, $[(1-\delta)u(v_2) + \Phi(x_2)]/u'(w_1)$. The reader should notice that the first term is measured in units of output because the worker's utility $(1-\delta)u(v_2) + \Phi(x_2)$ is divided by the worker's marginal utility of consumption, $u'(w_1)$. The second term on the right hand

side of (C1) is the number of senior workers employed by the firm at the production stage, $n_1(1 - \delta)$, times the profits created by each one of these workers, $y_2 - v_2$. The last term on the right hand side of (C1) is the number of junior workers employed by the firm at the production stage, $[n - n_1(1 - \delta)]\, \mu(v_1; x_2)$, times the profits created by each one of these workers, $y_2 - v_1$. Clearly, the second and third terms are also measured in units of output. We denote with V_k^* the maximum of $V_k(v_1, v_2)$ with respect to v_1 and v_2, subject to the constraints $v_2 \leq v_1$ and $b \leq v_2 \leq y_2$.

If $v_1 < v_2$ and $b \leq v_2 \leq y_2$, the sum of senior workers' lifetime utility and firm's profits is equal to

$$
\begin{aligned}
V_r(v_1, v_2; \psi_2) = \quad & n_1 \left\{ (1 - \delta) \left[\mu(v_1; x_2)u(b) + (1 - \mu(v_1; x_2))u(v_2) \right] + \Phi(x_2) \right\} / u'(w_1) + \\
& [n_1(1 - \delta)](1 - \mu(v_1; x_2))(y_2 - v_2) + n\mu(v_1; x_2)(y_2 - v_1).
\end{aligned}
$$
$$\tag{C2}$$

The first term on the right hand side of (C2) is the product of the number of senior workers and the lifetime utility of each of these workers. The second term on the right hand side of (C2) measures the profits created by senior workers; and the last term measures the profits created by junior workers. The reader should notice that the terms on the right hand side of (C2) are different from those on the right hand side of (C1) because, when v_1 is smaller than v_2, the firm has the incentive to replace senior workers with junior hires. We denote with V_r^* the maximum of $V_r(v_1, v_2)$ with respect to v_1 and v_2, subject to the constraints $v_1 < v_2$ and $b \leq v_2 \leq y_2$.

Lemma 1 establishes that, if a wage schedule solves the firm's problem (E6) in period $t - 1$, then it also maximizes the sum of senior workers' lifetime utility and firm's profits in state s_2 of period t. In other words, Lemma 1 establishes that an optimal schedule is ex-post (constrained) efficient.

Lemma 1: *Let* $\omega_1^* = \{w_1^*, w_{i,2}^*\}$ *be a schedule that solves the firm's problem (E6) for* $s_1 = \{x_1, y_1, \sigma_1\}$. *(i) If* $w_{2,2}^*(s_2) \leq w_{1,2}^*(s_2)$ *and* $b \leq w_{2,2}^*(s_2) \leq y_2$, *then* $V_k(w_{1,2}^*(s_2), w_{2,2}^*(s_2); \psi_2)$ *is equal to* $\max \{ V_k^*(\psi_2), V_r^*(\psi_2) \}$. *(ii) If* $w_{1,2}^*(s_2) < w_{2,2}^*(s_2)$ *and* $b \leq w_{2,2}^*(s_2) \leq y_2$, *then* $V_r(w_{1,2}^*(\psi_2), w_{2,2}^*(\psi_2); s_2)$ *is equal to* $\max \{ V_k^*(\psi_2), V_r^*(\psi_2) \}$. *(iii) If and only if* $y_2 \geq b$, $w_{2,2}^*(s_2)$ *belongs to the interval* $[b, y_2]$.

Proof: In the Appendix. ∎

4.2 The Best Schedule With and Without Worker Replacement

Lemma 1 suggests a simple procedure for characterizing the prescriptions of the optimal schedule in state $s_2 = \{x_2, y_2, \sigma_2\}$, $y_2 \geq b$. First, we characterize the wages $\{v_1, v_2\}$ that maximize the sum of the senior workers' lifetime utility and the firm's profits, subject to the no-replacement constraint, $v_2 \leq v_1$, and the individual rationality constraint, $b \leq v_2 \leq y_2$. These wages can be interpreted as the prescriptions of the best schedule that does not induce the firm to replace senior workers with junior hires. Second, we characterize the wages $\{v_1, v_2\}$ that maximize the sum of the senior workers' lifetime utility and the firm's profits, subject to the replacement constraint, $v_1 < v_2$, and the individual rationality constraint. These wages can be interpreted as the prescriptions of the best schedule that induces the firm to replace senior workers with junior hires. Finally, we identify the prescriptions of the optimal schedule by comparing the values, V_k^* and V_r^*, associated to the best schedule with and without worker replacement. In the following pages, we carry out this procedure for w_1 and y_2 greater than $z(x_2)$, where $z(x_2)$ is the consumption equivalent of the worker's flow value of searching, i.e. $u(z(x)) = Z(x) - \beta \mathbb{E}_{\hat{x}|x}[Z(\hat{x})]$.[11]

4.2.1 A Useful Benchmark: Full Commitment

As a preliminary step, we find it useful to consider the following hypothetical question: "If the firm could commit to not replace senior workers with junior hires in state s_2, what wages would the optimal schedule prescribe?" The answer to this hypothetical question is the solution to the following maximization problem

$$V_c^*(\psi_2) = \max_{v_1, v_2} \quad n_1[(1-\delta)u(v_2) + \Phi(x_2)]/u'(w_1) +$$
$$n_1(1-\delta)(y_2 - v_2) + [n - n_1(1-\delta)]\,\mu(v_1; x_2)(y_2 - v_1), \qquad \text{(C3)}$$
$$\text{s.t.} \quad b \leq v_2 \leq y_2.$$

The solution to the maximization problem (C3) with respect to v_1 is equal to the wage that maximizes the firm's expected profits from a vacant position, i.e. $w^h(s_2) = \arg\max \mu(v_1; x)(y_2 - v_1)$. Given the properties of the job-filling probability function, it fol-

---

[11]When $w_1 < z(x_2) \leq y_2$, it is immediate to verify that the optimal schedule prescribes the first-best hiring wage $w^h(s_2)$ for junior workers, and the first-best insurance wage $w^i(w_1, s_2)$ for senior workers (w^h and w^i are defined in the next subsection). When $b \leq y_2 \leq z(x_2)$, the optimal schedule prescribes that junior workers should be offered a wage smaller than $z(x_2)$, and that senior workers should be offered the wage $w^i(w_1, s_2)$.

lows that the wage $w^h(s_2)$ is strictly greater than the worker's flow value of searching, $z(x_2)$, and strictly smaller than the firm's productivity of labor, y_2. Given the properties of the job-filling probability function, it also follows that the wage $w^h(s_2)$ is strictly increasing with respect to the firm's productivity y_2. In the remainder of the paper, we refer to $w^h(s_2)$ as the first-best hiring wage.

The solution to the maximization problem (C3) with respect to v_1 is equal to the wage that maximizes the sum of the lifetime utility of a senior worker and the profits that this worker generates for the firm, i.e. $w^i(w_1, s_2) = \arg\max[u(v_2)/u'(w_1) + \Phi(x_2) + y_2 - v_2]$, s.t. $b \leq v_2 \leq y_2$. Given the concavity of the utility function, it follows that $w^i(w_1, s_2)$ is equal to the firm's productivity for all y_2 smaller than w_1; and it is equal to w_1 for all y_2 greater than w_1. In the remainder of the paper, we refer to $w^i(w_1, s_2)$ as the first-best insurance wage.

Given the properties of w^h and w^i, it follows that there exists a $k_1(w_1, x_2)$ such that if the firm's productivity y_2 is smaller than k_1, the first-best hiring wage is strictly smaller than the first-best insurance wage. And if the firm's productivity is greater than k_1, the first-best hiring wage is strictly greater than the first-best insurance wage. The properties of the wages w^h and w^i are established in Lemma 2 and illustrated in Figure 1.

Lemma 2: *Denote with $w^h(s_2)$ and $w^i(w_1, s_2)$ the wages that solve the maximization problem (C3). (i) The wage $w^h(s_2)$ is strictly increasing with respect to y_2; it is strictly greater than $z(x_2)$; and it is strictly smaller than y_2. (ii) The wage $w^i(w_1, s_2)$ is equal to $\min\{w_1, y_2\}$.*

Proof: In the Appendix.　∎

4.2.2　The Best Schedule Without Worker Replacement

Next, we need to characterize the wages that maximize the sum of the senior workers' lifetime utility and the firm's profits, subject to the no-replacement constraint and the individual rationality constraint. Formally, we need to characterize the solution to the following maximization problem

$$
\begin{aligned}
V_k^*(\psi_2) = \max_{v_1, v_2} \quad & n_1[(1-\delta)u(v_2) + \Phi(x_2)]/u'(w_1) + \\
& n_1(1-\delta)(y_2 - v_2) + [n - n_1(1-\delta)]\,\mu(v_1; x_2)(y_2 - v_1), \qquad \text{(C4)} \\
\text{s.t.} \quad & v_2 \leq v_1, \; b \leq v_2 \leq y_2.
\end{aligned}
$$

Notice that the maximization problem (C4) is a version of the commitment problem

(C3) with the addition of the no-replacement constraint. Also, notice that the solution to the commitment problem does satisfy the no-replacement constraint when the firm's productivity is greater than k_1, and it violates it when the firm's productivity is smaller than k_1. Therefore, if y_2 is greater than k_1, the solution to the maximization problem (C4) is equal to the first-best hiring wage, $w^h(s_2)$, and the first-best insurance wage, $w^i(w_1, s_2)$. Instead, if the firm's productivity y_2 is smaller than k_1, we can prove that the solution to the maximization problem (C4) is equal to a wage, $w^f(\psi_2)$, that the firm offers to both senior and junior employees. Moreover, if the firm's productivity y_2 is greater than $k_2(n_1, w_1, x_2)$ and smaller than k_1, $w^f(\psi_2)$ is strictly greater than the first-best hiring wage and strictly smaller than the first-best insurance wage. If the firm's productivity y_2 is smaller than k_2, $w^f(\psi_2)$ is equal to y_2. The properties of the solution to the maximization problem (C4) are established in Lemma 3 and illustrated in panel (a) of Figure 2.

Lemma 3: *Let $v_1^k(\psi_2)$ and $v_2^k(\psi_2)$ denote the wages that solve the maximization problem (C4). (i) For all y_2 in the interval between $z(x_2)$ and $k_2(n_1, w_1, x_2)$, $v_1^k(\psi_2)$ and $v_2^k(\psi_2)$ are equal to y_2. (ii) For all y_2 in the non-empty interval between $k_2(n_1, w_1, x_2)$ and $k_1(w_1, x_2)$, $v_1^k(\psi_2)$ and $v_2^k(\psi_2)$ are equal to $w^f(\psi_2)$, where $w^f(\psi_2) > w^h(s_2)$ and $w^f(\psi_2) < w^i(w_1, s_2)$. (iii) For all y_2 greater than $k_1(w_1, x_2)$, $v_1^k(\psi_2)$ is equal to $w^h(s_2)$, and $v_2^k(\psi_2)$ is equal to $w^i(w_1, s_2)$.*

Proof: In the Appendix. ■

4.2.3 The Best Schedule With Worker Replacement

Finally, we need to characterize the wages that maximize the sum of the senior workers' lifetime utility and the firm's profits, subject to the replacement constraint and the individual rationality constraint. Formally, we need to characterize the solution to the following maximization problem

$$V_r^*(\psi_2) = \max_{v_1, v_2} \quad n_1\{(1-\delta)\left[\mu(v_1; x_2)u(b) + (1-\mu(v_1; x_2))u(v_2)\right] + \Phi(x_2)\}/u'(w_1) +$$
$$n_1(1-\delta)(1-\mu(v_1; x_2))(y_2 - v_2) + n\mu(v_1; x_2)(y_2 - v_1), \qquad \text{(C5)}$$
$$\text{s.t.} \quad v_1 < v_2, \ b \le v_2 \le y_2.$$

When v_1 is smaller than v_2, a marginal increase in the wage offered to junior hires not only affects the firm's expected profits from a vacant position, but it also lowers the employment

17

probability of senior workers. Therefore, the solution to the maximization problem (C5) with respect to v_1 is smaller than the first-best hiring wage. Also, when v_1 is smaller than v_2, a marginal increase in v_2 exclusively affects the sum between the lifetime utility of a senior worker and the profits created by each of them. Therefore, the solution to the maximization problem (C5) with respect to v_2 is equal to the first-best insurance wage. The properties of the solution to the maximization problem (C5) are established in Lemma 4 and are illustrated in panel (b) of Figure 2.

Lemma 4: *Let $v_1^r(\psi_2)$ and $v_2^r(\psi_2)$ denote the wages that solve the maximization problem (C5). (i) For all y_2 in the interval between $z(x_2)$ and $k_1(w_1, x_2)$, $v_1^r(\psi_2)$ is greater than $z(x)$ and smaller than $w^h(s_2)$. (ii) For all y_2 in the interval between $z(x_2)$ and $k_1(w_1, x_2)$, $v_2^r(\psi_2)$ is equal to $w^i(w_1, s_2)$.*

Proof: In the Appendix. ∎

4.3   The Optimal Schedule

Now, we are in the position to characterize the prescriptions of the optimal schedule ω_1^* in state s_2. If the firm's productivity y_2 is greater than k_1, the best schedule without worker replacement specifies the wages $\{v_1^k(\psi_2), v_2^k(\psi_2)\}$, where $\{v_1^k(\psi_2), v_2^k(\psi_2)\}$ is equal to $\{w^h(s_2), w^i(w_1, s_2)\}$. Given these wages, senior workers have the same probability of employment and the same consumption as under full commitment. Given these wages, each of the firm's vacant positions attracts the same number of applicants and pays the same wage as under full commitment. Therefore, the sum of the senior workers' lifetime utility and the firm's profits under the no-replacement schedule is the same as under full commitment. In turn, this implies that the no-replacement schedule is optimal if the firm's productivity y_1 is greater than k_1.

If the firm's productivity y_2 is smaller than k_1, the best schedule without worker replacement prescribes the wages $\{v_1^k(\psi_2), v_2^k(\psi_2)\}$, where $v_i^k(\psi_2) = w^f(\psi_2)$. Given these wages, senior workers have the same probability of employment, but lower consumption than under full commitment. Given these wages, each of the firm's vacant positions attracts more applicants and pays higher wages than under full commitment. In contrast, if the firm's productivity y_2 is smaller than k_1, the best schedule with worker replacement prescribes the

18

wages $\{v_1^r(\psi_2), v_2^r(\psi_2)\}$, where $v_1^r(\psi_2) \leq w^h(s_2)$ and $v_2^r(\psi_2) = w^i(w_1, s_2)$. Given these wages, senior workers have the same consumption as under full commitment, but a lower probability of employment. Given these wages, each of the firm's vacant positions attracts fewer applicants and pays lower wages than under full commitment. Therefore, when the firm's productivity y_2 is smaller than k_1, both the no-replacement schedule and the replacement schedule introduce some distortions with respect to the full commitment benchmark and, in general, either one of them may be optimal.

However, when the firm's productivity y_2 is sufficiently close to k_1, the ranking between the two schedules is unambiguous. On the one hand, when y_2 gets closer and closer to k_1, the wage prescribed by the no-replacement schedule becomes closer and closer to both the full commitment insurance and hiring wages. Therefore, when y_2 gets closer and closer to k_1, the sum of the senior workers' lifetime utility and the firm's profits under the no-replacement schedule gets closer and closer to the full commitment benchmark. On the other hand, the sum of the senior workers' lifetime utility and the firm's profits under the replacement schedule remains bounded away from the full commitment benchmark even when y_2 converges to k_1. In fact, if $v_1^r(\psi_2)$ converges to a wage greater than $z(x_2)$, the schedule generates too much employment risk for senior workers. If $v_1^r(\psi_2)$ converges to a wage smaller than $z(x_2)$, the schedule does not attract enough unemployed workers to the firm's vacant positions.

If there are enough frictions in the matching process,[12] we can construct examples in which the replacement schedule is optimal when the firm's productivity y_2 is smaller than some critical threshold k_3. In these examples, when y_2 is smaller than k_3, the no-replacement schedule prescribes that both senior and junior employees should be paid y_2. In contrast, the replacement schedule prescribes that senior employees should be paid y_2, and that junior employees should be paid a wage between $z(x_2)$ and y_2. In these examples, the additional profits made by the firm under the replacement schedule are worth more than the additional employment security enjoyed by senior workers under the no-replacement schedule. Such an example is illustrated in Figure 3.

Theorem 1: (Micro Consequences of the Replacement Problem) *Let* $\omega_1^* = \{w_1^*, w_{i,2}^*\}$ *be*

---

[12]For example, in the job-filling probability function $\eta(q) = Aq/(1+q)$, the extent of the matching frictions increases as the parameter A falls.

a schedule that solves the firm's problem (E6) for $s_1 = \{y_1, x_1, \sigma_1\}$. (i) For all y_2 greater than $k_1(w_1^*, x_2)$, $\{w_{1,2}^*(s_2), w_{2,2}^*(s_2)\}$ is equal to $\{v_1^k(\psi_2), v_2^k(\psi_2)\}$, where $v_1^k(\psi_2) = w^h(s_2)$ and $v_2^k(\psi_2) = w^i(w_1^*, s_2)$. (ii) There exists an $\epsilon > 0$ such that, for all y_2 between $k_1(w_1^*, x_2) - \epsilon$ and $k_1(w_1^*, x_2)$, $\{w_{1,2}^*(s_2), w_{2,2}^*(s_2)\}$ is equal to $\{v_1^k(\psi_2), v_2^k(\psi_2)\}$, where $v_i^k(\psi_2) = w^f(\psi_2)$.

Proof: In the Appendix.　■

5　Macro Consequences of the Replacement Problem

In this section, we want to understand how firm's limited commitment affects the response of unemployment and vacancies to aggregate productivity shocks. We find that limited commitment magnifies the response of both vacancies and unemployment to negative shocks to aggregate productivity. However, we find that limited commitment has no influence on the way in which vacancies and unemployment respond to positive shocks.

5.1　Environment

In every period, the economy is either in the recessionary state x_b, or in the expansionary state x_g. When the economy is in the recessionary state, the productivity of new firms is $y_{1,b}$, and the productivity of old firms is $y_{2,b}$. When the economy is in the expansionary state, the productivity of new firms is $y_{1,g}$, $y_{1,g} > y_{1,b}$, and the productivity of old firms is $y_{2,g}$, $y_{2,g} > y_{2,b}$. In the next period, the economy is in the same state as in the current period with probability p; and it is in a different state with probability $1 - p$, $p \to 1$. This simplifying assumption allows us to characterize the dynamics of the model in closed-form solution.[13]

We calibrate the productivity of new and old firms, y_1 and y_2, so that a worker employed by a new firm under the optimal wage schedule $\omega_1(x)$ is paid as much as a worker employed by an old firm under the first-best hiring wage $w^h(x)$, i.e. $w_1(x_i) = w^h(x_i)$ for $i = \{b, g\}$. Moreover, we calibrate the consumption equivalent of leisure, b, so that a worker employed by a new firm under the optimal wage schedule has the same lifetime utility as a worker employed by an old firm under the first-best hiring wage, i.e. $W_1(\omega_1(x_b); x_b) = W_2(w^h(x_b); x_b)$. Thanks

---

[13]Given the assumptions about the stochastic process for y, it follows that the state of the firm is completely summarized by the aggregate state of the economy, x, and by the firm's age, t. Moreover, given the assumptions about the stochastic process for x, it follows that the optimal wage schedule does not involve randomization. Therefore, in the following pages, we will omit the dependence of various variables from the firm's productivity, y_t, and from the realization of the sunspot, σ.

to this calibration, the model has realistic implications that the firm's hiring wage and the workers' lifetime utility are independent from the firm's age (even though, for the sake of tractability, we assume that the firm's life is finite).

5.2 Wage Dynamics

When the economy is in the recessionary state x_b, new firms offer the wage schedule $\omega(x_b) = \{w_1(x_b), w_{i,2}(\hat{x}; x_b)\}$. In the current period, the schedule prescribes the wage $w^h(x_b)$, i.e. $w_1(x_b) = w^h(x_b)$. If in the next period the economy is still in state x_b, the schedule prescribes the first-best hiring wage for junior workers and the first-best insurance wage for senior workers, i.e. $w_{1,2}(x_b; x_b) = w^h(x_b)$ and $w_{2,2}(x_b; x_b) = w^i(w_1(x_b), x_b)$. Since $w^h(x_b)$ is equal to $w^i(w_1(x_b); x_b)$, these prescriptions do not induce firms to replace senior workers with junior hires. Similarly, if in the next period the economy moves to the state x_g, the schedule prescribes the first-best hiring wage and the first-best insurance wage, i.e. $w_{1,2}(x_g; x_b) = w^h(x_g)$ and $w_{2,2}(x_g; x_b) = w^i(w_1(x_b), x_g)$. Since $w^h(x_g)$ is greater than $w^i(w_1(x_b), x_g)$, these prescriptions do not induce worker replacement. Overall, when the economy is in the recessionary state, new firms offer the same wage schedule that they would offer if they had full commitment.

When the economy is in the expansionary state x_h, new firms offer the wage schedule $\omega(x_g) = \{w_1(x_g), w_{i,2}(\hat{x}; x_g)\}$. In the current period, the schedule prescribes the wage $w^h(x_g)$, i.e. $w_1(x_g) = w^h(x_g)$. If in the next period the economy is still in state x_g, the schedule prescribes the first-best hiring wage for junior workers and the first-best insurance wage for senior workers, i.e. $w_{1,2}(x_g; x_g) = w^h(x_g)$ and $w_{2,2}(x_g; x_g) = w^i(w_1(x_g), x_g)$. Since $w^h(x_g)$ is equal to $w^i(w_1(x_g), x_g)$, these prescriptions do not induce worker replacement. In contrast, if the economy moves to the state x_b, the first-best insurance wage and the first-best hiring wage would induce firms to replace senior workers with junior hires. In this case, as long as aggregate productivity shocks are sufficiently small, the schedule prescribes the wage w^f for both junior and senior employees, where $w^f > w^h(x_b)$. Overall, the schedule $\omega(x_g)$ coincides with the full commitment benchmark if the economy remains in the expansionary state, but it differs when the economy transits to the recessionary state.

Lemma 5: *In a Recursive Equilibrium with Entry: (i) The value of searching $Z(x)$ is such*

that $Z(x_g) > Z(x_b)$, and the first-best hiring wage $w^h(x)$ is such that $w^h(x_g) > w^h(x_b)$. (ii) The wage schedule $\omega_1(x_b) = \{w_1(x_b), w_{i,2}(\hat{x}; x_b)\}$ is such that $w_{1,2}(\hat{x}; x_b) = w^h(\hat{x})$ and $w_{1,2}(\hat{x}; x_b) = w^h(x_b)$. (iii) The wage schedule $\omega_1(x_g) = \{w_1(x_g), w_{i,2}(\hat{x}; x_g)\}$ is such that $w_{i,2}(x_g; x_g) = w^h(x_g)$ for $i = 1, 2$. Moreover, there exists an $\epsilon > 0$ such that, if $\|(y_{1g}, y_{2,g}) - (y_{1,b}, y_{2,b})\| < \epsilon$, then $w_{i,2}(x_b; x_b) = w^f$ for $i = \{1, 2\}$, where $w^f > w^h(x_b)$.

The proof of this lemma is straightforward and is omitted for the sake of brevity. However, all details are available upon request.

5.3 Vacancy and Unemployment Dynamics

Consider a period in which the economy transitions from the expansionary to the recessionary state. In this period, old firms offer the wage w^f to both junior and senior workers. Given these wages, old firms receive q_2 applications for each of their vacant positions, $q_2 = q(W_2(w^f; x_b))$, and they do not receive applications for any of their filled positions. Moreover, in this period, new firms offer the wage schedule $\omega_1(x_b)$. Given this schedule, new firms receive q_1 applications for each of their vacant positions, $q_1 = q(W_2(w^h(x_b); x_b))$.[14]

The number of firms that enter the labor market is such that the measure of applications submitted by the workers is equal to the measure of applications received by the firms. In particular, the measure of applications received by new firms is equal to $f_1 n q_1$, where f_1 denotes the number of new firms in the labor market. The measure of applications received by old firms is equal to $f_2[n - n_1(1 - \delta)]q_2$, where f_2 denotes the number of old firms in the labor market. Finally, the measure of applications submitted by workers is equal to the measure of unemployed workers at the beginning of the period, i.e. $1 - f_2 n_1$. Therefore, the number of firms that enter the labor market is

$$f_1 = (nq_1)^{-1} \{1 - f_2 n_1 - f_2[n - n_1(1 - \delta)]q_2\}. \tag{GE1}$$

At the search stage, new firms have $f_1 n$ vacancies, and old firms have $f_2[n - n_1(1 - \delta)]$ vacancies. Therefore, at the search stage, the vacancy rate is

$$\nu = f_1 n + f_2[n - n_1(1 - \delta)]. \tag{GE2}$$

---

[14]Perhaps, it is useful to remind the reader that, in a Recursive Equilibrium with Entry, the optimal wage schedule posted by new firms does not depend on the prescriptions of the wage schedule offered by old firms.

At the production stage, new firms employ $f_1 n \eta(q_1)$ workers, and old firms employ $f_2 n_1(1-\delta)$ senior workers as well as $f_2[n - n_1(1-\delta)]\eta(q_2)$ junior workers. Therefore, at the production stage, the unemployment rate is

$$v = 1 - f_1 n \eta(q_1) - f_2 \left\{ n_1(1-\delta) + [n - n_1(1-\delta)]\eta(q_2) \right\}. \tag{GE3}$$

If firms could commit to not replace workers, how differently would vacancies and unemployment respond to the negative shock to the aggregate component of productivity? In order to answer this question, notice that, under full commitment, old firms would offer the wage $w^h(x_b)$ to their junior employees and the wage $w^h(x_g)$ to their senior employees. And new firms would offer the wage schedule $\omega_1(x_b)$. Therefore, under full commitment, old firms would attract $q_2 - q_1$ fewer applicants for each of their vacant positions, while new firms would attract the same number of applicants for each of their vacant positions. As a result, under full commitment, Δ_f additional firms would enter the labor market, and Δ_ν additional vacancies would be opened, where

$$\begin{aligned}
\Delta_f &= (nq_1)^{-1} f_2[n - n_1(1-\delta)](q_2 - q_1) > 0, \\
\Delta_\nu &= q_1^{-1} f_2[n - n_1(1-\delta)](q_2 - q_1) > 0.
\end{aligned} \tag{GE4}$$

Under full commitment, new firms would employ $\Delta_f n \eta(q_1)$ additional workers, and old firms would employ $f_2[n - n_1(1-\delta)] (\eta(q_2) - \eta(q_1))$ fewer workers. Therefore, under full commitment, the unemployment rate would be $v + \Delta_v$, where

$$\Delta_v = f_2[n - n_1(1-\delta)][\eta(q_2) - \eta(q_1) - q_1^{-1}(q_2 - q_1)\eta(q_1)]. \tag{GE5}$$

Since the job-filling probability $\eta(q)$ is concave with respect to the queue length and $q_2 > q_1$, the increase in the measure of workers employed by new firms is greater than the decrease in the measure of workers employed by old firms. Therefore, Δ_v is strictly negative.[15]

From the previous discussion, it follows that the response of unemployment and vacancies to a negative shock to the aggregate component of productivity is magnified by the firms' limited commitment. In contrast, from the characterization of the wage schedule offered by new firms in state x_b, it follows that the response of unemployment and vacancies to a

---

[15] Since $\eta(q)$ is concave and $\eta(0)$ is equal to 0, it follows that $q_1^{-1}\eta(q_1)$ is strictly greater than $\eta'(q_1)$. Since $\eta(q)$ is concave and q_2 is greater than q_1, it follows that $(q_2 - q_1)^{-1} (\eta(q_2) - \eta(q_1))$ is smaller than $\eta'(q_1)$. Therefore, $q_1^{-1}(q_2 - q_1)\eta(q_1)$ is strictly greater than $\eta(q_2) - \eta(q_1)$ and $\Delta_u < 0$.

positive shock to the aggregate component of productivity is the same under full and limited commitment. These findings are summarized in the following theorem.

Theorem 2: (Macro Consequences of the Replacement Problem) *In a Recursive Equilibrium with Entry: (i) When the economy moves from state x_g to x_b, the increase in unemployment and the decline in vacancies are greater because of the firm's limited commitment. (ii) When the economy moves from state x_b to x_g, the decrease in unemployment and the increase in vacancies are unaffected by the firm's limited commitment.*

6 Conclusions

In this paper, we studied a labor market in which firms have two goals. First, a firm wants to insure its workers against income fluctuations. In order to do this efficiently, a firm should offer to its senior workers a wage that is independent from its productivity and from the aggregate conditions of the economy. Also, a firm should employ its senior workers as long as the gains from trade are positive. Second, a firm wants to recruit new workers to fill its vacant positions. In order to do this efficiently, a firm should offer to their junior workers a wage that depends on its productivity and on the aggregate conditions of the economy. However, since a firm cannot commit to not replace senior workers with new hires, it cannot simultaneously attain efficient insurance and efficient recruitment.

In the first part of the paper, we studied how the worker replacement problem affects the design of the firm's optimal wage schedule. We found that the optimal schedule divides the firm's productivity space into three regions. If the firm's productivity is in the highest region, the optimal schedule prescribes the efficient hiring wage for junior employees and the efficient insurance wage for senior employees. In this region, the firm has no incentive to replace its senior employees, because the efficient hiring wage is greater than the efficient insurance wage. If the firm's productivity falls in the intermediate region, the optimal schedule prescribes the same wage for junior and senior employees. This wage is greater than the efficient hiring wage and smaller than the efficient insurance wage, but it guarantees that senior workers will not be replaced with new hires. When the firm's productivity falls in the lowest region, the optimal schedule may induce the firm to replace its senior employees with new hires. When this is the case, the schedule prescribes that senior workers should receive the efficient

24

insurance wage, and junior employees should be paid strictly less than the efficient hiring wage.

In the second part of the paper, we studied how the replacement problem affects (through its effect on the design of the optimal wage schedule) the response of unemployment and vacancies to aggregate productivity shocks. We found that the replacement problem magnifies the response of unemployment and vacancies to small negative productivity shocks. In contrast, we found that the replacement problem does not affect the response of unemployment and vacancies to positive productivity shocks.

Appendix

Proof of Lemma 1: (i) Let $\omega_1^* = \{w_1^*, w_{i,2}^*\}$ denote the wage schedule that solves the firm's profit maximization problem (E6) for $s_1 = \{x_1, y_1, \sigma_1\}$. Let v_1^* denote $w_{1,2}^*(s_2)$ for $s_2 \in S^*$, where S^* is equal to $\{(x, y, \sigma) : x = x_2, y = y_2, \sigma \in [\overline{\sigma}_2, \underline{\sigma}_2]\}$. Similarly, let v_2^* denote $w_{2,2}^*(s_2)$ for $s_2 \in S^*$. Suppose that $v_2^* \leq v_1^*$ and $v_2^* \in [b, y_2]$.

Next, let $\omega_1 = \{w_1, w_{i,2}\}$ denote an alternative schedule such that: (i) for all $s_2 \notin S^*$, $w_{i,2}(s_2)$ is equal to $w_{i,2}^*(s_2)$; (ii) for all $s_2 \in S^*$, $w_{i,2}(s_2)$ is equal to v_i, $v_i \in \mathbb{R}_+$; (iii) $W_1(\omega_1; x_1) = W_1(\omega_1^*; x_1)$. In words, the alternative schedule ω_1 is such that: (i) in all states $s_2 \notin S^*$, the wages for junior and senior workers are the same as under the optimal schedule ω_1^*; (ii) in all states $s_2 \in S^*$, the wages for junior and senior workers may be different than under the schedule ω_1^*; (iii) in its first period of activity, the firm offers the same lifetime utility as under the schedule ω_1^*. The last property implies that, in its first period of activity, the firm employs the same number of workers as under the schedule ω_1^*.

First, consider the case in which $v_2 \leq v_1$ and $v_2 \in [b, y_2]$. In this case, the difference between the firm's profits under the optimal schedules, ω_1^*, and under the alternative schedule, ω_1, is equal to

$$
\begin{aligned}
F_1(n_1, \omega_1^*; s_1) - F_1(n_1, \omega_1; s_1) = \\
n_1(w_1 - w_1^*) + \beta \Pr(S^*) n_1(1 - \delta)(v_2 - v_2^*) + \\
\beta \Pr(S^*)[n - n_1(1 - \delta)][\mu(v_1^*; x_2)(y_2 - v_1^*) - \mu(v_1; x_2)(y_2 - v_1)] \geq 0,
\end{aligned}
\tag{A1}
$$

where the wage w_1 is

$$
w_1 = u^{-1}\left(u(w_1^*) + \beta \Pr(S^*)(1 - \delta)(u(v_2^*) - u(v_2))\right).
\tag{A2}
$$

After substituting (A2) into (A1), we obtain that

$$
\begin{aligned}
\lim_{\overline{\sigma}_2 \to \underline{\sigma}_2} \left[(F_1(n_1, \omega_1^*; s_1) - F_1(n_1, \omega_1; s_1))/\beta \Pr(S^*)\right] = \\
n_1(1 - \delta)\left[(u(v_2^*) - u(v_2))/u'(w_1^*) + v_2 - v_2^*\right] + \\
[n - n_1(1 - \delta)][\mu(v_1^*; x_2)(y_2 - v_1^*) - \mu(v_1; x_2)(y_2 - v_1)] \geq 0.
\end{aligned}
\tag{A3}
$$

It is immediate to verify that the limit above is equal to the difference between $V_k(v_1^*, v_2^*; \psi_2)$ and $V_k(v_1, v_2; \psi_2)$.

Next consider the case in which $v_1 < v_2$ and $v_2 \in [b, y_2]$. In this case, the difference between

the firm's profits under the optimal schedule, ω_1^*, and under the alternative schedule, ω_1, is equal to

$$
\begin{aligned}
F_1(n_1, \omega_1^*; s_1) - F_1(n_1, \omega_1; s_1) = {} & n_1(w_1 - w_1^*) + \\
& \beta \Pr(S^*) n_1(1 - \delta)[\mu(v_1; x_2)v_1 + (1 - \mu(v_1; x_2))v_2 - v_2^*] + \\
& \beta \Pr(S^*)[n - n_1(1 - \delta)][\mu(v_1^*; x_2)(y_2 - v_1^*) - \mu(v_1; x_2)(y_2 - v_1)] \geq 0,
\end{aligned}
\tag{A4}
$$

where the wage w_1 is

$$
w_1 = u^{-1}\left(u(w_1^*) + \beta \Pr(S^*)(1 - \delta)\left[u(v_2^*) - \mu(v_1; x_2)u(b) - (1 - \mu(v_1; x_2))u(v_2)\right]\right). \tag{A5}
$$

After substituting (A5) into (A4), we obtain that

$$
\begin{aligned}
\lim_{\overline{\sigma}_2 \to \underline{\sigma}_2} \left[(F_1(n_1, \omega_1^*; s_1) - F_1(n_1, \omega_1; s_1))/\beta \Pr(S^*)\right] = {} \\
n_1(1 - \delta)\left[u(v_2^*) - \mu(v_1; x_2)u(b) - (1 - \mu(v_1; x_2))u(v_2)\right]/u'(w_1^*) + \\
n_1(1 - \delta)[\mu(v_1; x_2)v_1 + (1 - \mu(v_1; x_2))v_2 - v_2^*] + \\
[n - n_1(1 - \delta)][\mu(v_1^*; x_2)(y_2 - v_1^*) - \mu(v_1; x_2)(y_2 - v_1)] \geq 0.
\end{aligned}
\tag{A6}
$$

It is immediate to verify that the limit above is equal to the difference between $V_k(v_1^*, v_2^*; \psi_2)$ and $V_r(v_1, v_2; \psi_2)$.

Finally, notice that inequality (A3) holds for all wages $\{v_1, v_2\}$ such that $v_2 \leq v_1$ and $b \leq v_2 \leq y_2$. Moreover, notice that inequality (A6) holds for all wages $\{v_1, v_2\}$ such that $v_1 < v_2$ and $b \leq v_2 \leq y_2$. Therefore, inequalities (A3) and (A6) imply that

$$
V_k(v_1^*, v_2^*; \psi_2) = \max\{V_k(\psi_2), V_r(\psi_2)\}. \tag{A7}
$$

(ii) We omit the proof of part (ii) because it is similar to the proof of part (i). We omit the proof of part (iii) because it is straightforward. However, all details are available upon request. ∎

Proof of Lemma 2: The maximization problem (C3) can be written as

$$
\begin{aligned}
V_c^*(\psi_2) = {} & [n - n_1(1 - \delta)] \max_{v_1} \{\mu(v_1, x_2)(y_2 - v_1)\} + \\
& [n_1(1 - \delta)] \max_{b \leq v_2 \leq y_2} \left\{[u(v_2) + (1 - \delta)^{-1}\Phi(x_2)]/u'(w_1) + y_2 - v_2\right\}.
\end{aligned}
\tag{A8}
$$

(i) Consider the first maximization problem in (A8). The maximand of this problem is continuous with respect to v_1; it is equal to zero for $v_1 \leq z(x_2)$; it is strictly positive for

$v_1 \in (z(x_2), y_2)$; and it is smaller or equal than zero for all $v_1 \geq y_2$. Therefore, the maximizer of this problem exists and belongs to the open interval between $z(x_2)$ and y_2.

Over the interval $(z(x_2), y_2)$, the derivative of the maximand with respect to v_1 is

$$d[\mu(v_1, x_2)(y_2 - v_1)]/dv_1 =$$
$$\eta'(q(W_2(v_1; x_2)))q'(W_2(v_1; x_2))W_2'(v_1; x_2)(y_2 - v_1) - \eta(q(W_2(v_1; x_2))), \quad \text{(A9)}$$

where $q(W_2(v_1; x_2))$ is defined by the solution to the equation

$$Z(x_2) = u(b) + \beta \mathbb{E}_{\hat{x}}[Z(\hat{x})] + \lambda(q)[u(v_1) - u(b)]. \quad \text{(A10)}$$

After differentiating (A10) with respect to v_1, we can rewrite (A9) as

$$d[\mu(v_1, x_2)(y_2 - v_1)]/dv_1 =$$
$$\frac{\eta'(q(W_2(v_1; x_2)))q(W_2(v_1; x_2))}{1 - \epsilon_\eta(q(W_2(v_1; x_2)))} \frac{u'(v_1)(y_2 - v_1)}{u(v_1) - u(b)} - \eta(q(W_2(v_1; x_2))). \quad \text{(A11)}$$

Equation (A10) implies that $q(W_2(v_1; x))$ is strictly increasing with respect to v_1. Since $\eta'(q)q/(1 - \epsilon_\eta(q))$ is decreasing in q, the first term on the right hand side of (A11) is strictly decreasing with respect to v_1. Also, since $\eta(q)$ is strictly increasing with respect to q, the second term on the right hand side of (A11) is strictly increasing with respect to v_1. Therefore, the maximand of the first problem in (A1) is strictly concave with respect to v_1 over the interval $(z(x_2), y_2)$.

From the previous observations, it follows that the maximizer of the first problem in (A8) is the unique solution to the first order condition

$$\frac{\eta'(q(W_2(v_1; x_2)))q(W_2(v_1; x_2))}{1 - \epsilon_\eta(q(W_2(v_1; x_2)))} \frac{u'(v_1)(y_2 - v_1)}{u(v_1) - u(b)} - \eta(q(W_2(v_1; x_2))) = 0. \quad \text{(A12)}$$

Since the left hand side of (A12) is strictly increasing in y_2, the maximizer is a function $w^h(s_2)$ that is strictly increasing with respect to the firm's productivity y_2.

(ii) The reader can immediately verify that $\min\{w_1, y_2\}$ is the unique solution to the second maximization problem in (A8). ∎

28

Proof of Lemma 3: The maximization problem (C4) can be written as

$$V_k^*(\psi_2) = \max_{v_1, v_2} \quad [n - n_1(1-\delta)] \, \mu(v_1, x_2)(y_2 - v_1) +$$

$$n_1(1-\delta) \left\{ [u(v_2) + (1-\delta)^{-1} \Phi(x_2)] / u'(w_1) + y_2 - v_2 \right\}, \tag{A13}$$

$$\text{s.t.} \quad b \leq v_2 \leq v_1 \leq y_2.$$

First, notice that the objective function in (A13) is continuous with respect to $\{v_1, v_2\}$ and the the feasible set is non-empty and compact. Therefore, a solution to the maximization problem (A13) exists. Second, notice that the objective function is strictly concave with respect to $\{v_1, v_2\}$ and the feasible set is convex. Therefore, the solution to the maximization problem (A13) is unique. Moreover, the solution $\{v_1^k(\psi_2), v_2^k(\psi_2)\}$ and the multiplier $\phi^r(\psi_2)$ satisfy the following necessary and sufficient conditions for optimality with respect to $\{v_1, v_2, \phi\}$:

(i) the wage v_1 is such that

$$[n - n_1(1-\delta)] \, [\mu'(v_1, x_2)(y_2 - v_1) - \mu(v_1, x_2)] + \phi \geq 0 \tag{A14}$$

and $v_1 \leq y_2$, with complementary slackness;

(ii) the wage v_2 is such that

$$n_1(1-\delta) \, [u'(v_2)/u'(w_1) - 1] - \phi \leq 0 \tag{A15}$$

and $b \leq v$, with complementary slackness;

(iii) the multiplier ϕ is such that $\phi \geq 0$ and $v_1 - v_2 \geq 0$, with complementary slackness.

For all $y_2 \geq k_1(w_1, x_2)$, it is immediate to verify that the triple $\{w^h(s_2), w^i(w_1, s_2), 0\}$ satisfies the necessary and sufficient conditions for optimality (i)–(iii) with respect to $\{v_1, v_2, \phi\}$. For all $y_2 < k_1(w_1, x_2)$, it is immediate to verify that the optimality conditions (i)–(iii) are satisfied by the triple $\{w^f(\psi_2), w^f(\psi_2), \phi^f\}$, where ϕ^f is equal to $n_1(1-\delta)[u'(v_2)/u'(w_1) - 1]$ and w^f is such that

$$\frac{n - n_1(1-\delta)}{n_1(1-\delta)} \, [\mu'(w^f, x_2)(y_2 - w^f) - \mu(v_1, x_2)] + \frac{u'(w^f) - u'(w_1)}{u'(w_1)} \geq 0 \tag{A16}$$

and $w^f \leq y_2$, with complementary slackness.

Denote with $k_2(n_1, w_1, s_2)$ the level of firm-specific productivity such that

$$-\frac{n - n_1(1-\delta)}{n_1(1-\delta)} \mu(k_2, x_2) + \frac{u'(k_2) - u'(w_1)}{u'(w_1)} \geq 0. \tag{A17}$$

29

Clearly, $k_2(n_1, w_1, s_2)$ is strictly greater than $z(x_2)$ and strictly smaller than $k_1(w_1, x_2)$. For all $y_2 \leq k_2(n_1, w_1, s_2)$, the left hand side of (A16) is strictly decreasing with respect to v_1, and it is strictly positive at $v_1 = y_2$. Therefore, for all $y_2 \leq k_2(n_1, w_1, s_2)$, the wage $w^f(\psi_2)$ is equal to y_2. For all y_2 in the interval $(k_2(n_1, w_1, s_2), k_1(w_1, x_2))$, the first term on the left hand side of (A16) is strictly decreasing with respect to v_1, and it is equal to zero at $v_1 = w^h(s_2)$, where $w^h(s_2) < w_1$. For all y_2 in the interval $(k_2(n_1, w_1, s_2), k_1(w_1, x_2))$, the second term on the left hand side of (A16) is strictly decreasing with respect to v_2, and it is equal to zero at $v_1 = w_1$. Moreover, for all y_2 between $k_2(n_1, w_1, s_2)$ and $k_1(w_1, x_2)$, the sum of the first and the second term on the left hand side of (A16) is strictly negative at $v_1 = y_2$. Therefore, for all y_2 between $k_2(n_1, w_1, s_2)$ and $k_1(w_1, x_2)$, $w^f(\psi_2)$ is strictly greater than the first-best hiring wage $w^h(s_2)$, and it is strictly smaller than the first-best insurance wage $w^i(w_1, s_2) = \min\{w_1, y_2\}$. ∎

Proof of Lemma 4: Without the replacement constraint $v_1 < v_2$, the maximization problem (C5) can be written as

$$V_r^*(\psi_2) = \max_{v_1} \quad n\mu(v_1; x_2)(y_2 - v_1) + n_1(1-\delta)(u(b) + \Phi(x_2))/u'(w_1) +$$
$$n_1(1-\delta)(1-\mu(v_1; x_2)) \max_{b \leq v_2 \leq y_2} \{(u(v_2) - u(b))/u'(w_1) + y_2 - v_2\}. \tag{A18}$$

First, consider the inner maximization problem in (A18). As proved in Lemma 4.2, the maximizer of this problem is the first-best insurance wage $w^i(w_1, s_2)$. Next, consider the outer maximization problem in (A18). The maximand of this problem is continuous with respect to v_1; it is constant for all $v_1 \leq z(x_2)$; and it is strictly decreasing for all $v_1 \geq w^h(s_2)$. Therefore, the maximizer of this problem exists and and belongs to the open interval between $z(x_2)$ and $w^h(s_2)$. Finally, notice that the maximizer of the inner problem is greater than the maximizer than the outer problem because $w^h(s_2) < w^i(w_1, s_2)$ for all y_2 between $z(x_2)$ and $k_1(w_1, x_2)$. Therefore, the solution to the maximization problem (A15) is also the solution to the maximization problem (C5). ∎

Proof of Theorem 1: From the proof of Lemma 2, it follows immediately that the value function $V_c^*(\psi_2)$ is equal to $\max_{v_1, v_2} V_k(v_1, v_2; \psi_2)$ subject to $b \leq v_1, v_2 \leq y_2$. The objective function $V_k(v_1, v_2; \psi_2)$ is continuous with respect to v_1, v_2 and y_2. The feasible set $\Gamma_c =$

$\{v_1, v_2 : b \leq v_1, v_2 \leq y_2\}$ is non-empty, bounded and continuous with respect to y_2. Therefore, by the Theorem of the Maximum, the value function $V_c^*(\psi_2)$ is continuous with respect to y_2.

The value function $V_k^*(\psi_2)$ is equal to $\max_{v_1, v_2} V_k(v_1, v_2; \psi_2)$ subject to $b \leq v_2 \leq v_1 \leq y_2$. The objective function $V_k(v_1, v_2; \psi_2)$ is continuous with respect to v_1, v_2 and y_2. The feasible set $\Gamma_k = \{v_1, v_2 : b \leq v_2 \leq v_1 \leq y_2\}$ is non-empty, bounded and continuous with respect to y_2. Therefore, by the Theorem of the Maximum, the value function $V_k^*(\psi_2)$ is continuous with respect to y_2. Moreover, since $\Gamma_k \subset \Gamma_c$, $V_k^*(\psi_2)$ is smaller or equal to $V_c^*(\psi_2)$. And since $\{v_1^k(\psi_2), v_2^k(\psi_2)\}$ is equal to $\{w^h(s_2), w^i(w_1, s_2)\}$ for all $y_2 \geq k_1(w_1, x_2)$, $V_k^*(\psi_2)$ is equal to $V_c^*(\psi_2)$ for all $y_2 \geq k_1(w_1, x_2)$.

From the proof of Lemma 4, it follows immediately that the value function $V_r^*(\psi_2)$ is smaller or equal to $\overline{V}_r^*(\psi_2)$, where $\overline{V}_r^*(\psi_2)$ is defined as $\max_{v_1} V_r(v_1, w^h(w_1, s_2); \psi_2)$ subject to $z(x_2) \leq v_1 \leq y_2$. The objective function $V_r(v_1, w^h(w_1, s_2); \psi_2)$ is continuous with respect to v_1 and y_2. The feasible set $\Gamma_r = \{v_1 : z(x_2) \leq v_1 \leq y_2\}$ is non-empty, bounded and continuous with respect to y_2. Therefore, by the Theorem of the Maximum, the value function $\overline{V}_r^*(\psi_2)$ is continuous with respect to ψ_2. Moreover, the difference between $V_c^*(\psi_2)$ and $\overline{V}_r^*(\psi_2)$ is equal to

$$V_c^*(\psi_2) - \overline{V}_r^*(\psi_2) =$$
$$\min_{z(x_2) \leq v_1 \leq y_2} n_1(1-\delta)\mu(v_1; x_2) \left[(u(w^i(w_1, s_2)) - u(b))/u'(w_1) + v_1 - w^i(w_1, s_2)\right] +$$
$$[n - n_1(1-\delta)][\mu(w^h(s_2))(y_2 - w^h(s_2)) - \mu(v_1)(y_2 - v_1)].$$
$$\text{(A19)}$$

From the concavity of $u(w)$, it follows that $u(w^i) - u(b)$ is strictly greater than $u'(w^i)(w^i - b)$. From $w^i \leq w_1$ and $b \leq z(x_2) \leq v_1$, it follows that $u(w^i) - u(b)$ is strictly greater than $u'(w_1)(w^i - v_1)$. Therefore, the first term on the right hand side of (A19) is strictly greater than zero for all $v_1 > z(x_2)$, and it is equal to zero for $v_1 = z(x_2)$. Moreover, the second term on the right hand side of (A19) is strictly greater than zero for all $v_1 \neq w^h(s_2)$, and it is equal to zero for $v_1 = w^h(s_2)$. Since $w^h(s_2) > z(x_2)$, it follows that the right hand side of (A19) is strictly positive. Hence, $V_c^*(\psi_2) > \overline{V}_r^*(\psi_2)$.

For all $y_2 \geq k_1(w_1, x_2)$, the value function $V_k^*(\psi_2)$ is equal to $V_c^*(\psi_2)$, and the value function $\overline{V}_r^*(\psi_2)$ is strictly smaller than $V_c^*(\psi_2)$. Moreover, since the value functions $V_k^*(\psi_2)$, $V_c^*(\psi_2)$

and $\overline{V}_r^*(\psi_2)$ are continuous with respect to y_2, there exists an $\epsilon > 0$ such that, if y_2 is between $k_1(w_1, x_2) - \epsilon$ and $k_1(w_1, x_2)$, then $V_k^*(\psi_2)$ is strictly greater than $\overline{V}_r^*(\psi_2)$. Since $V_r^*(\psi_2) \leq \overline{V}_r^*(\psi_2)$, the previous observations imply that the optimal schedule ω_1^* prescribes the wages $\{v_1^k(s_2), v_2^k(s_2)\}$ for all y_2 greater than $k_1(w_1, x_2) - \epsilon$. ∎
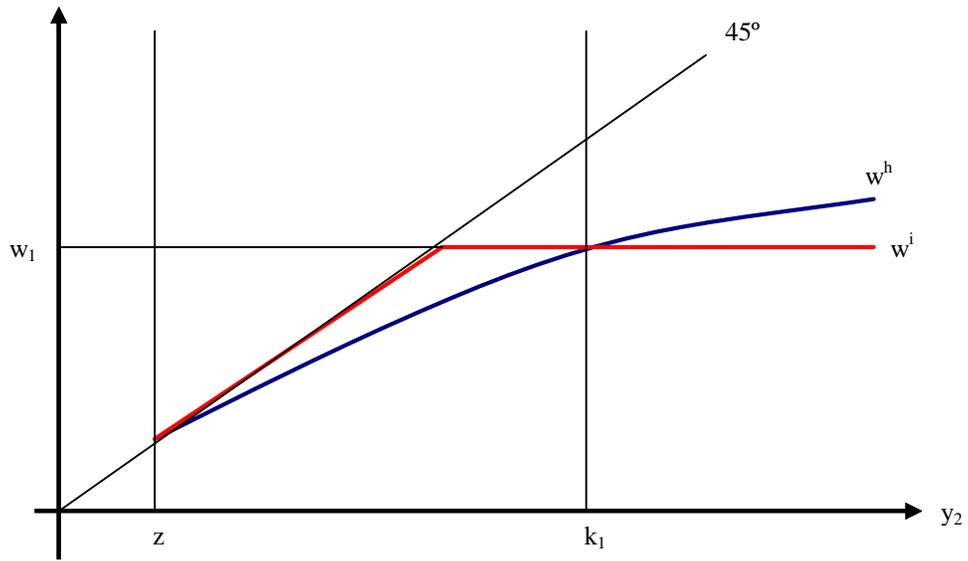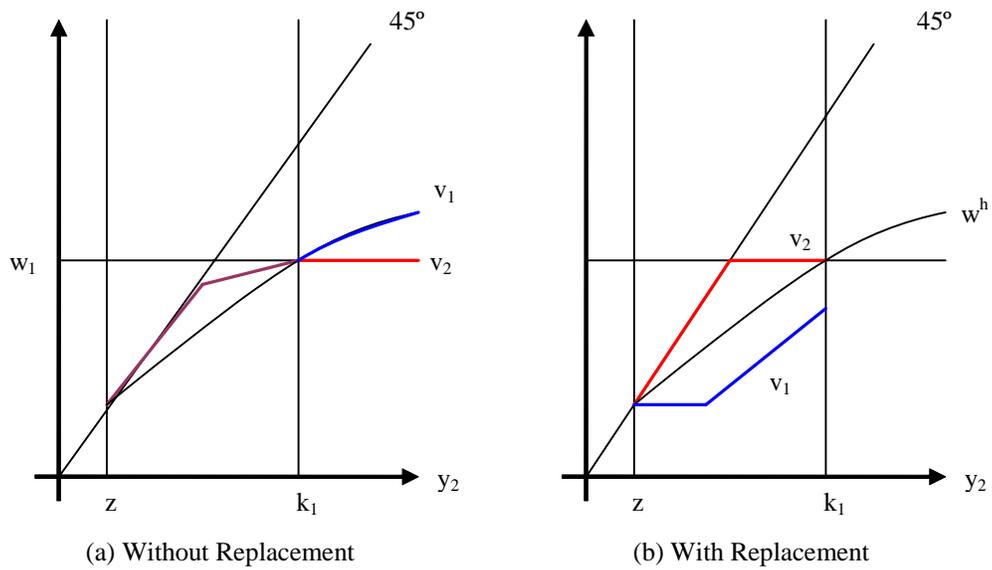
Figure 1: Full Commitment Benchmark

(a) Without Replacement

(b) With Replacement

Figure 2: Best Schedule With and Without Worker Replacement
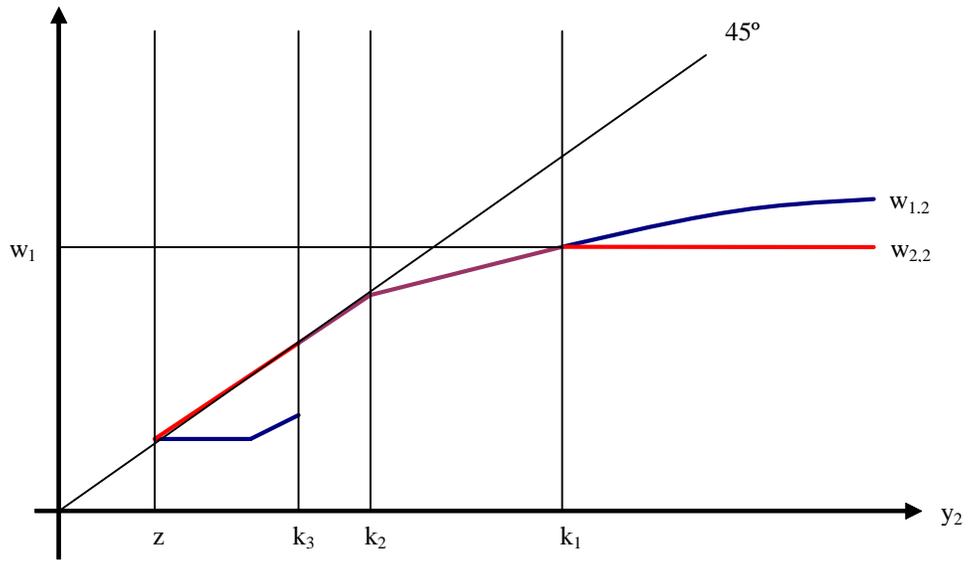
Figure 3: Optimal Wage Schedule

References

[1] Acemoglu, D., and R. Shimer. 1999. Efficient Unemployment Insurance. *J.P.E.* 107: 893–928.

[2] Azariadis, C. 1975. Implicit Contracts and Underemployment Equilibria. *J.P.E.* 83: 1183–202.

[3] Baily, M. 1974. Wages and Employment under Uncertain Demand. *Rev. Econ. Studies* 41: 37–50.

[4] Beaudry, P., and J. DiNardo. 1991. The Effect of Implicit Contracts on the Movement of Wages Over the Business Cycle: Evidence from Micro Data. *J.P.E.* 99: 665–688.

[5] Bewley, T. 1999. *Why Wages Don't Fall during a Recession*. Harvard University Press, Cambridge, MA.

[6] Boldrin, M., and M. Horvath. 1995. Labor Contracts and Business Cycles. *J.P.E.* 103: 972–1004.

[7] Burdett, K., and M. Coles. 2003. Equilibrium Wage-Tenure Contracts. *Econometrica* 71: 1377–404.

[8] Garibaldi, P., and E. Moen. 2008. Industry Dynamics and Search in the Labor Market. Manuscript, Norwegian School of Management.

[9] Gertler, M., and A. Trigari. 2006. Unemployment Fluctuations with Staggered Nash Bargaining. Manuscript, NYU.

[10] Hagedorn, M., and I. Manovskii. 2008. The Cyclical Behavior of Unemployment and Vacancies Revisited. *A.E.R.*

[11] Hall, R. 2005. Employment Fluctuations with Equilibrium Wage Stickiness. *A.E.R.* 95: 50–65.

[12] Holmstrom, B. 1983. Equilibrium Long-Term Contracts. *Q.J.E.* 98: 23–55.

[13] Kennan, J. 2008. Private Information, Wage Bargaining and Employment Fluctuations. Manuscript, University of Winsonsin-Madison.

[14] MacLeod, B., and J. Malcomson. Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment. *Econometrica* 57: 447–80.

[15] Menzio, G. 2005. High-Frequency Wage Rigidity. Manuscript, Univ. Pennsylvania.

[16] ——. 2007. A Theory of Partially Directed Search. *J.P.E.* 115: 748–69.

[17] Menzio, G., and S. Shi. 2008a. Efficient Search on the Job and the Business Cycle. PIER Working Paper.

[18] ——. 2008b. A Tractable Model of Search on the Job and Aggregate Fluctuations. Manuscript, Univ. Pennsylvania.

[19] Moen, E. 1997. Competitive Search Equilibrium. *J.P.E.* 105: 694–723.

[20] Moen, E., and Å. Rosen, 2008. Incentives in Competitive Search Equilibrium. Working paper, Norwegian School of Management.

[21] Pissarides, C. 1985. Short-Run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages. *A.E.R.* 75: 676–90.

[22] Ramey, G. 2008. Exogenous vs. Endogenous Separation. Manuscript, U.C. San Diego.

[23] Rudanko, L. 2006. Labor Market Dynamics under Long Term Wage Contracting and Incomplete Markets. Manuscript, Univ. Chicago.

[24] Shimer, R. 2005. The Cyclical Behavior of Unemployment and Vacancies. *A.E.R.* 95: 25–49.

[25] Snell, A., and J. Thomas. 2007. Labor Contracts, Equal Treatment, and Wage-Unemployment Dynamics. Manuscript, Univ. of Edinburgh.

[26] Thomas, J., and T. Worrall. 1988. Self-Enforcing Wage Contracts. *Rev. Econ. Studies* 55: 541–53.