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PIER Working Paper 11-004

"History-Dependent Risk Attitude"

by

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http://ssrn.com/abstract=1761417

History-Dependent Risk Attitude*

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February 2011

Abstract

We propose a model of history-dependent risk attitude (HDRA), allowing the attitude of a decision-maker (DM) towards risk at each stage of a T-stage lottery to evolve as a function of his history of disappointments and elations in prior stages. We establish an equivalence between the existence of an HDRA representation and two documented cognitive biases. First, the DM's risk attitudes are reinforced by prior experiences: he becomes more risk averse after suffering a disappointment and less risk averse after being elated. Second, the DM displays a primacy effect: early outcomes have the strongest effect on risk attitude. Furthermore, the DM lowers his threshold for elation after a disappointing outcome and raises it after an elating outcome; this makes disappointment more likely after elation and vice-versa, leading to statistically reversing risk attitudes. "Gray areas" in the elation-disappointment assignment are connected to optimism and pessimism in determining endogenous reference points.

Keywords: history-dependent risk attitude, statistically reversing risk attitudes, reinforcement effect, primacy effect, endogenous reference dependence, betweenness, optimism, pessimism
JEL Codes: D03, D81, D91

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*First version June 2010. This paper generalizes a previous version that circulated under the title "Disappointment Cycles." We are grateful to Simone Cerreia-Vioglio for helpful conversations. We also benefitted from comments and suggestions by Ben Polak, Wolfgang Pesendorfer, Larry Samuelson, and seminar participants at Boston University, Johns Hopkins University, Northwestern University, University of British Columbia, University of Pennsylvania, and Yale University.

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Once bitten, twice shy. — Proverb

1. Introduction

Consider two people in a casino, Alice and Bob, who have the same total wealth and the same utility function over monetary prizes. Alice has already won several games at the roulette wheel, while Bob lost at those games. Will Alice's and Bob's attitudes toward further risk be the same, or might they depend on whether they had previously won or lost? Consider now a third person, Carol, who has just won \$100 in an even-chance lottery between \$100 and \$x. Could Carol's attitude to future risk depend on whether x, the alternate outcome, corresponded to winning or losing a thousand dollars?

There is experimental evidence that the way in which risk unfolds over time affects risk attitudes; and moreover, that individuals are affected by unrealized outcomes, a phenomenon known as *counterfactual thinking*.[1] Thaler and Johnson (1990) suggest that individuals become more risk averse after negative experiences and less risk averse after positive ones. Post, van den Assem, Baltussen, and Thaler (2008) suggest that individuals are more willing to take risks after extreme realizations. The latter two studies consider settings of pure chance—suggesting that the effects therein are not the result of learning about oneself or one's environment. In a more general context, Malmendier and Nagel (2010) study how personal experiences of macroeconomic shocks affect financial risk-taking. Controlling for wealth, income, age, and year effects, they find that for up to three decades later, "households with higher experienced stock market returns express a higher willingness to take financial risk, participate more in the stock market, and conditional on participating, invest more of their liquid assets in stocks." In the context of professional sports, Rao (2009) shows that "a majority of the [NBA basketball] players...significantly change their behavior in response to hit streaks by taking more difficult shots" but that "controlling for shot conditions, players show no evidence of ability changing as a function of past outcomes."

In this paper, we propose a model of history-dependent risk attitude (HDRA) over multi-stage lotteries that permits risk attitudes to be shaped by prior experiences. To ease exposition, we begin by describing our HDRA model in the simple setting of T-stage temporal lotteries (the model is later extended to stochastic decision trees, in which intermediate actions can be taken). A realization of a temporal lottery is another temporal lottery, which is one stage shorter. In the HDRA model, the DM categorizes each realization of a temporal lottery as an elating or disappointing outcome. At each stage, the DM's history is the preceding sequence of elations and disappoint-

[1] The collection Roese and Olson (1995) offers a comprehensive overview of the counterfactual thinking literature.

ments. Each possible history h corresponds to a preference relation over one-stage lotteries, which comes from an admissible set of preferences. These one-stage preferences are rankable in terms of their risk aversion. For example, an admissible collection could be a class of expected utility preferences with a Bernoulli function $u(x) = \frac{x^{1-\rho_h}}{1-\rho_h}$, where the coefficient of relative risk aversion ρ_h is history dependent. More generally, we require each of the one-stage preferences to be members of the betweenness class (Chew (1989), Dekel (1986)), a broad class of preferences that includes expected utility and Gul (1991)'s model of disappointment aversion.

Given the history assignment for each realization, the DM calculates the value of a T-stage lottery recursively; that is, starting at the T-th stage and proceeding backwards, each one-stage lottery is replaced with its appropriate, history-dependent certainty equivalent. The key assumption of the HDRA model is that the history assignment for all sublotteries is *internally consistent*: if a sublottery is considered *elating* (*disappointing*), then its value should indeed be weakly larger than (strictly smaller than) the value of the sublottery from which it emanates. In a multistage setting, this imposes a fixed point requirement on the assignment of histories. Internal consistency builds on a related notion of elation and disappointment for one-stage lotteries: a DM is elated if the prizes he receives from a lottery is at least as good as the lottery itself, while he is disappointed if the prize he receives is worse than the lottery itself (see, for example, Chew (1989) or Gul (1991)).

We do not place further restrictions on how risk aversion should depend on the history. Nonetheless, we show that the HDRA model (in particular, the internal consistency requirement) predicts two well-documented cognitive biases; and that these biases are sufficient conditions for an HDRA representation to exist. First, in accordance with the experimental evidence cited above, the DM's risk attitudes are *reinforced* by prior experiences: he becomes less risk averse after positive experiences and more risk averse after negative ones. Second, and perhaps more surprisingly, the DM displays *primacy effects*: his risk attitudes are disproportionately affected by early realizations. In particular, the earlier the DM is disappointed, the more risk averse he becomes. Sequencing biases, especially the primacy effect, are robust and long-standing experimental phenomena (early literature includes Anderson (1965)); and several empirical studies argue that early experiences may shape financial or cultural attitudes (e.g. Guiso, Sapienza and Zingales (2004) or Alesina and Fuchs-Schündeln (2007)). The primacy effect also has implications for the optimal sequencing of information to manipulate behavior. For example, we study how a financial advisor trying to convince a DM to invest in a risky asset should deliver mixed news.

HDRA also has predictions for the DM's endogenous reference levels. In particular, the model predicts *statistically reversing risk attitudes*. The DM increases the threshold for elation after positive experiences and lowers it after negative experiences. This makes disappointment more likely after elation, and vice-versa. The psychological literature, in particular Parducci (1995) and Smith, Diener, and Wedell (1989), provides support for the prediction that elation thresholds increase (decrease) after positive (negative) experiences.[2]

For some lotteries, there may be more than one internally consistent assignment of histories given an admissible set of preferences. The DM's history assignment is revealed by his choice behavior over temporal lotteries, as seen in the axiomatic foundations we provide. Because the DM's assignment may affect his utility from a temporal lottery, we view an *optimist* as a DM who always selects the "most favorable" interpretation (when more than one assignment is possible) and a *pessimist* as a DM who always selects the "least favorable" interpretation. This notion of optimism and pessimism is distinct from previous notions which identify optimism and pessimism with the choice of (distorted) beliefs; for example, see Bénabou and Tirole (2002), Chateauneuf, Eichberger, and Grant (2007), or Epstein and Kopylov (2007).

Applied work suggests that changing risk aversion helps explain several empirical phenomena. Barberis, Huang, and Santos (2001) allow risk aversion to depend on prior stock market gains and losses à la the experimental evidence of Thaler and Johnson (1990), and show that their model is consistent with the well-documented equity premium and excess volatility puzzles. Gordon and St-Amour (2000) study bull and bear markets, allowing risk attitudes to vary stochastically by introducing a state-dependent CRRA parameter in a discounted utility model. They show that countercyclical risk aversion best explains the cyclical nature of equity prices, suggesting that "future work should address the issue of determining the factors that underline the movements in risk preferences" which they identified. In this work, we lay theoretical foundations for a a class of models under which such shifts in risk attitude may arise. In particular, we find that the simple requirement of internal consistency provides sufficient structure to make predictions on how risk aversion changes with prior experiences.

In many theories of choice over temporal lotteries, risk aversion could depend on the passage of time, wealth effects or habit formation in consumption; see Kreps and Porteus (1978), Chew and Epstein (1989), Segal (1990), and Rozen (2010), among others. We study how risk attitudes are affected by the past, independently of such effects. In the HDRA model, risk attitudes are affected by "what might have been." This means that our model relaxes consequentialism (see Machina (1989)), an assumption that is maintained by all the papers above. Our type of history dependence is conceptually distinct from models where contemporaneous and future beliefs affect

[2] Summarizing these works, Schwarz and Strack (1998) observe that "an extreme negative (positive) event increased (decreased) satisfaction with subsequent modest events....Thus, the occasional experience of extreme negative events facilitates the enjoyment of the modest events that make up the bulk of our lives, whereas the occasional experience of extreme positive events reduces this enjoyment."

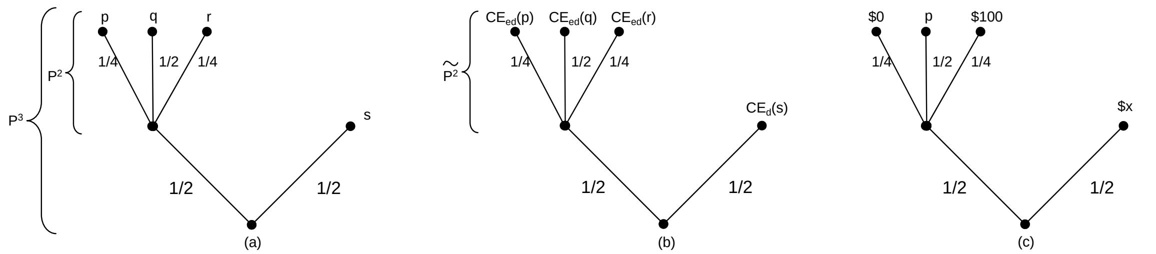

Figure 1: The three stage lottery P^3 is seen in (a), and in (b) all the terminal one-stage lotteries in P^3 are replaced with their appropriate certainty equivalents under the history assignment. Figure (c) corresponds to the specification in the numerical example.

contemporaneous utility (that is, dependence of utility on "what might be" in the future). This literature includes Caplin and Leahy (2001), Epstein (2008), and Köszegi and Rabin (2009).[3]

1.1. Illustration

Consider the three stage lottery P^3 visualized (informally) in Figure 1(a), where $p, q, r,$ and s are one-stage lotteries. In the first stage of P^3, there is an even chance that the DM faces the lottery s (the right branch) or faces an additional stage of risk before learning which lottery governs his winnings (the left branch). Under the two-stage lottery P^2 in the left branch, the DM faces each of the lotteries p and r with probability $1/4$, and faces lottery q with probability $1/2$.

The DM evaluates P^3 using the class of CRRA expected utility preferences. That is, the DM's Bernoulli function is given by $\frac{x^{1-\rho_h}}{1-\rho_h}$ under history h. The set of possible histories is $H = \{0, e, d, ee, ed, de, dd\}$. For example, 0 is the initial history (before resolution of risk), e corresponds to having been elated once, and de corresponds to having first been disappointed and then elated. By the reinforcement effect, the DM's history-dependent coefficient of relative risk aversion ρ_h satisfies $\rho_e < \rho_d$ in stage one and both $\rho_{ee} < \rho_{ed}$ and $\rho_{de} < \rho_{dd}$ in stage two (the DM is more risk averse after a disappointment than after an elation). By the primacy effect, $\rho_{ed} < \rho_{de}$ (the DM is more risk averse the earlier he is disappointed). As we will later show, the reinforcement and primacy effects arise endogenously as a result of our internal consistency condition.

Suppose that the DM considers the right branch disappointing (d), the left branch P^2 elating (e), and, within P^2, considers r elating (ee) but p and q disappointing (ed). To verify that this is

[3]In Köszegi and Rabin (2009), given any fixed current belief over consumption, utility is not affected by prior history (how that belief was formed). Their model, which presumes the DM is loss averse over changes in successive beliefs, could be generalized to include historical differences in beliefs, which would then affect utility values but not actual risk aversion due to their assumption of additive separability; we conjecture that one could relax additive separability to find choices of parameters and functional forms for their model that replicate the primacy effect and the reinforcement effect predicted by HDRA.

an internally consistent history assignment, we must check that the conditions below are satisfied. For any one-stage lottery ℓ and history h, denote by $CE_h(\ell)$ the certainty equivalent of ℓ using the CRRA coefficient ρ_h. Proceeding backwards, we begin by replacing each of the one-stage lotteries p, q, r, and s with their history-dependent certainty equivalent, as seen in Figure 1(b). Then, conditional on the left branch P^2 being elating, verifying that p, q, and r have internally consistent assignments requires checking that

$$CE_{ed}(p),\, CE_{ed}(q) \;<\; CE_e\left(\widetilde{P^2}\right) \;\leq\; CE_{ee}(r),$$

where $\widetilde{P^2}$ is the one-stage lottery seen in Figure 1(b) where p, q, and r in the two-stage lottery P^2 are replaced with their respective certainty equivalents.

Second, checking that s and P^2 have internally consistent assignments requires checking that

$$CE_d(s) < \; CE_0\left(\widetilde{P^3}\right) \;\leq\; CE_e\left(\widetilde{P^2}\right),$$

where $\widetilde{P^3}$ is the recursively-constructed one-stage lottery where $CE_d(s)$ and $CE_e(\widetilde{P^2})$ are each received with probability $1/2$.

As a simple example, consider the following specification of P^3. Suppose $0 = \rho_e < \rho_0 < \rho_d = 1/2$ (only these coefficients are relevant for this example). As pictured in Figure 1(c), let p, r, and s be degenerate lotteries giving \$0, \$100, and \$$x$, respectively. Let q be a lottery whose support consists of prizes between \$25 and \$50. We demonstrate that for any $x < 25$, the history assignment described above is the unique internally consistent assignment for this specification of P^3. Indeed, within P^2, receiving 0 must be disappointing and receiving 100 must be elating. Moreover, if P^2 is elating, then q, whose certainty equivalent under any history must be between 25 and 50, must be disappointing. To see this, denote by z the relevant certainty equivalent of q. Under $\rho_e = 0$, the consistency condition for q to be disappointing is

$$\frac{1}{4}\cdot 0 + \frac{1}{2}\cdot z + \frac{1}{4}\cdot 100 > z \text{ if and only if } z < 50.$$

Finally, to see that P^2 is indeed elating, note that $CE_0(\widetilde{P}_2)$ must be larger than 25, which is the certainty equivalent evaluated using a higher CRRA coefficient $(\frac{1}{2})$ and in the worst case when q gives \$25 with probability one. Therefore, for $x < 25$, the DM is elated when he doesn't receive x and subsequently views q as a disappointment.

Ceteris paribus, it can similarly be shown that for $x > 50$, the DM is disappointed when he doesn't receive x and subsequently views q as an elation. In the second stage, receiving the lowest prize (0) is always disappointing and receiving the highest prize (100) is always elating. Hence the probability of being elated after a disappointment is $3/4$; and similarly, the probability of being disappointed after an elation is $3/4$. This illustrates the property of statistically reversing risk attitudes under HDRA: disappointment is more likely after elation, and vice versa.

More generally, for any given lottery q, there will be a cutoff $\underline{x} \geq 25$ such that for any $x < \underline{x}$, there is a unique internally consistent assignment under which P^2 is elating and q is disappointing; and there is a cutoff $\bar{x} \leq 50$ such that for $x > \bar{x}$, there is a unique internally consistent assignment under which P^2 is disappointing and q is elating. For $x \in [\underline{x}, \bar{x}]$, both history assignments are internally consistent. This means that both $CE_e(\widetilde{P^2}) \geq CE_0(\widetilde{P^3}) > x$ (where $\widetilde{P^2}$ and $\widetilde{P^3}$ correspond to the case where P^2 is elating) and $x \geq CE_0(\widetilde{P^3}') > CE_d(\widetilde{P^2}')$ (where $\widetilde{P^2}'$ and $\widetilde{P^3}'$ correspond to the case where P^2 is disappointing). An optimist, who always takes the "most favorable" view of risk, would view P^2 as elating as soon as possible (i.e., for all $x > \underline{x}$); while a pessimist, who always takes the "least favorable" view of risk, would view P^2 as disappointing for as long as possible (i.e., for all $x < \bar{x}$). In particular, the optimist prefers the lottery P^3 to receiving $\$x$ for sure, while the pessimist is more risk averse and prefers the sure prospect of $\$x$ to P^3.

The remainder of this paper is organized as follows. Section 2 formalizes the domain of temporal lotteries. Section 3 provides a primer on the class of preferences over one-stage lotteries satisfying the betweenness axiom. Section 4 formalizes the HDRA model and contains our main results for temporal lotteries. Implications of HDRA are studied in Section 5. Section 6 extends our model and results to a setting where intermediate actions are possible. Axiomatic foundations for HDRA are provided in Section 7. Section 8 discusses directions for further research.

2. Framework: T-stage lotteries

We begin by studying the simple setting of T-stage lotteries; in Section 6 we extend the setting to stochastic decision trees.

Let $X = [w,b] \subset R$ be a bounded interval of monetary prizes, where w is the worst prize and b is the best prize. The set of all simple lotteries (i.e., having a finite number of outcomes) over X is denoted $\mathscr{L}(X)$, or simply $\mathscr{L}^1$. Elements of $\mathscr{L}^1$ are one-stage lotteries. We reserve lowercase letters for one-stage lotteries; typical elements of $\mathscr{L}^1$ are denoted p, q, or r. The probability of a monetary outcome x under p is denoted $p(x)$. A typical element p has the form $\langle p(x_1), x_1; \ldots, p(x_m), x_m \rangle$. The degenerate lottery $\delta_x \in \mathscr{L}^1$ gives the prize x with probability one. For $\alpha \in (0,1)$ and any two lotteries $p, q \in \mathscr{L}^1$, the lottery $\alpha p + (1-\alpha) q \in \mathscr{L}^1$ denotes the convex combination of p and q, giving prize x with probability $\alpha p(x) + (1-\alpha) q(x)$.

For $T \geq 2$, the set of T-stage lotteries, $\mathscr{L}^T$, is defined by the inductive relation $\mathscr{L}^T = \mathscr{L}\left(\mathscr{L}^{T-1}\right)$. A typical element P^T of $\mathscr{L}^T$ has the form

$$P^T = \left\langle \alpha_1, P_1^{T-1}; \ldots; \alpha_m, P_m^{T-1} \right\rangle,$$

where each $P_j^{T-1} \in \mathscr{L}^{T-1}$ is a $(T-1)$-stage lottery. If P_j^{T-1} is the outcome of P^T, then all remaining uncertainty is resolved according to P_j^{T-1}. To simplify notation, we use the superscript T only when $T \geq 3$. The degenerate lottery $\delta_x^T \in \mathscr{L}^T$ gives the lottery δ_x^{T-1} with probability one (i.e., x is received with probability one after T stages).

To avoid redundancy, our notation for any t-stage lottery implicitly assumes that the elements in the support are distinct.

3. Preliminaries: recursive preferences

The primitive in our model is a preference relation $\succeq$ over the set of T-stage compound lotteries ($\mathscr{L}^T$) which has a recursive structure. As a preliminary step, we need to discuss the single-stage preferences ($\succeq_1$ over $\mathscr{L}^1$) that will be applied recursively.

3.1. Singe-stage preferences

In this paper, we confine our attention to a class of single-stage preferences that are continuous, monotone (with respect to the relation of first-order stochastic dominance), and satisfy the following betweenness property: for all $p, q \in \mathscr{L}^1$ and $\alpha \in [0,1]$, $p \succeq_1 q$ implies $p \succeq_1 \alpha p + (1-\alpha) q \succeq_1 q$. The betweenness axiom is a weakened form of the vNM-independence axiom. It implies neutrality toward randomization among equally-good lotteries; this retains the linearity of indifference curves in expected utility theory but relaxes the assumption that they are parallel.

Our two leading examples of single-stage preferences in this class are:

Example 1 (Expected utility). *For this class, the value of a lottery p is simply $\mathbb{E}(\cdot; u) = \sum_x p(x) u(x)$, where $u : X \to \mathbb{R}$ is a utility function over monetary prizes.*

Example 2 (Disappointment aversion). *Gul (1991) proposes a theory of disappointment aversion. The disappointment aversion value of a lottery p, $V(p; \beta, u)$, is the unique v that solves*

$$v = \frac{\sum_{\{x|u(x) \geq v\}} p(x) u(x) + (1+\beta) \sum_{\{x|u(x)<v\}} p(x) u(x)}{1 + \beta \sum_{\{x|u(x)<v\}} p(x)}, \tag{1}$$

where $\beta \in (-1,\infty)$ and $u : X \to \mathbb{R}$ is increasing. (The term in the denominator normalizes the weights on the prizes so that they sum to one.)

According to Gul's model, the support of any nondegenerate lottery is divided into two groups, the elating outcomes (for which $u(x) \geq v$) and the disappointing outcomes (for which $u(x) < v$), where the threshold $v = V(p;\beta,u)$ is determined endogenously. The DM then values lotteries by taking their "expected utility," except that disappointing outcomes get a uniformly greater (or smaller) weight that depends on the value of a single parameter β, the *coefficient of disappointment aversion*.[4] Gul's model was first intended to explain the Allais paradox. In a dynamic setting, it proved a useful model to address the equity premium puzzle (Ang, Bekaert and Liu, 2005), a statistically significant negative correlation between volatility and private investment (Aizenman and Marion, 1999), and observed asset-pricing behavior (Routledge and Zin, 2010).

More generally, Chew (1989) and Dekel (1986) show that a preference relation $\succeq_1$ satisfies continuity, monotonicity, and betweenness if and only if there exists a utility representation, V, where each $V(p)$ is defined implicitly as the unique $v \in [0,1]$ that solves

$$\sum_x p(x)\, u(x,v) = v, \tag{2}$$

where $u : X \times [0,1] \to [0,1]$ is a (local utility) function which is continuous in both arguments, strictly increasing in the first argument, and satisfies $u(w,v) = 0$ and $u(b,v) = 1$ for all $v \in [0,1]$. The local utility function $u(x,v)$ can be interpreted as the value of the prize x relative to a reference utility level v.[5]

3.2. Folding back T-stage lotteries

We now discuss how these single-stage preferences can be extended recursively to the richer domain of T-stage lotteries using the folding back approach proposed by Segal (1990). To illustrate, consider a two-stage lottery $P = \langle \alpha_1, p_1; \alpha_2, p_2; \alpha_3, p_3 \rangle$. First, each lottery p_i in the support of P is replaced with its certainty equivalent; that is, the element $CE(p_i) \in X$ satisfying $V(\delta_{CE(p_i)}) = V(p_i)$. The value of the resulting one-stage, "folded back" lottery $\langle \alpha_1, CE(p_1); \alpha_2, CE(p_2); \alpha_3, CE(p_3) \rangle$ is calculated using the function V and assigned to be the utility of the original lottery P. The value

[4]When $\beta > 0$, disappointing outcomes are overweighted and the DM is called *disappointment averse.* When $\beta < 0$, disappointing outcomes are underweighted and the DM is called *elation seeking*. When $\beta = 0$, the model reduces to the model of expected utility.

[5]A prize x is a disappointing outcome of p if $u(x,V(p)) < V(p)$ (and is elating otherwise). Within the betweenness class, $u(x,V(p)) < V(p)$ if and only if $p \succ_1 \delta_x$ (see Dekel (1986)). Therefore, the elating outcomes of p are precisely those prizes that are preferred to p, and the disappointing outcomes are the prizes that are inferior to p.

of the temporal lottery is thus calculated by applying V recursively. For T-stage lotteries, the procedure is analogous. Lotteries in the last stage are replaced with their certainty equivalent, resulting in a $(T-1)$-stage lottery; and this procedure is repeated until a one-stage lottery results, whose value is then calculated using V.

The "folding back" procedure does not require that the same V be used throughout. For example, V may vary with the passage of time. More generally, the value of a T-stage lottery can be calculated by folding back using a potentially different V at each node.

4. History-dependent risk attitude

We now describe our model of history-dependent risk attitude over T-stage lotteries. In this model, the DM endogenously categorizes, in an internally consistent manner, each sublottery as an elating or disappointing outcome of the sublottery from which it emanates. The value of a T-stage lottery is calculated by folding back, where the DM's valuation of a sublottery is determined by the sequence of elating or disappointing outcomes leading to it.

A t-stage lottery P^t is a *sublottery* of P^T if there is a sequence $P^{t+1}, P^{t+2}, \ldots, P^T$ such that for every $t' \in \{t, \ldots, T-1\}$, $P^{t'} \in \text{supp}\, P^{t'+1}$. By convention, P^T is a sublottery of itself. There may be more than one sequence of sublotteries within P^T which leads to the t-stage sublottery P^t when $t < T-1$. To unambiguously identify a particular sublottery P^t in the T-stage lottery P^T, we use the notation $P^{t,T}$ to denote a lottery P^t that is part of the sequence $(P^t, P^{t+1}, \ldots, P^T)$ of sublotteries leading to it.

We now formalize the notion of histories within T-stage lotteries. The initial history—i.e., prior to any resolution of risk—is empty (0). If a sublottery is degenerate—i.e., it leads to some sublottery with probability one—then the DM is not exposed to risk at that stage and his history is unchanged. If a sublottery is nondegenerate, each sublottery in its support may be an elating (e) or disappointing (d) outcome. The set of all possible histories is given by

$$H = \bigcup_{t=1}^{T} \{0\} \times \{e, d\}^{T-t}.$$

For each sublottery $P^{t,T}$, the *history assignment* $a(\cdot)$ assigns a history $h \in H$. The initial assignment (that is, $a(P^T) = 0$) is always implicit; e.g., within the two-stage lottery $P = \langle \alpha, p; 1-\alpha, q \rangle$, we write $a(p) = e$ rather than $a(p) = 0e$ if p is elating. If outcome $j \in \{e, d\}$ occurs after history h, the updated history is hj, implicitly assuming the resulting history is in H (i.e., h is a nonterminal history). The length of a history h is the total number of e and d outcomes in h.

Each history $h \in H$ corresponds to a utility function $V_h : \mathscr{L}^1 \to \mathbb{R}$ over one-stage lotteries. The DM applies V_h during the folding back procedure to value any sublottery $P^{t,T}$ whose history assignment is h.[6] To study how risk attitudes are shaped by prior experiences, we would like the utility functions after each history to be rankable in terms of their risk aversion. We thus define the following comparative measure of risk aversion.[7]

Definition 1. *We say that V_h is more risk averse than $V_{h'}$, denoted $V_h >_{RA} V_{h'}$, if for any $x \in X$ and any nondegenerate $p \in \mathscr{L}^1$, $V_h(p) \geq V_h(\delta_x)$ implies that $V_{h'}(p) > V_{h'}(\delta_x)$.*

Accordingly, we assume that the collection $\mathscr{V} := \{V_h\}_{h \in H}$ satisfies the following properties.

Definition 2. *We say that the collection $\mathscr{V}$ is admissible if (i) each V_h represents a preference relation over $\mathscr{L}^1$ that is continuous, monotone, and satisfies betweenness; and (ii) all the elements in $\mathscr{V}$ are rankable in terms of risk aversion: for any $h, h' \in H$ either $V_h >_{RA} V_{h'}$ or $V_{h'} >_{RA} V_h$.*[8]

Each utility representation in an admissible collection has an equivalent representation of the betweenness form (2) in Section 3. In particular, any collection of utilities of the betweenness class that differ in only a single parameter of risk aversion would form an admissible class. One example of an admissible collection consists of expected CRRA utility with a history-dependent risk aversion parameter: $\mathscr{V} = \{\mathbb{E}(\cdot; \frac{x^{1-\rho_h}}{1-\rho_h})\}_{h \in H}$. Another example of an admissible collection consists of disappointment aversion with a history-dependent disappointment aversion parameter and history-independent utility over prizes: $\mathscr{V} = \{V(\cdot; \beta_h, u)\}_{h \in H}$, where $V(\cdot; \beta_h, u)$ is given by (1) in Section 3. This is because Gul (1991, Proposition 5) shows that the DM becomes increasingly risk averse as the disappointment aversion coefficient increases, holding the utility over prizes constant.

Our model of history-dependent risk attitude (HDRA), defined below, places restrictions on the history assignments permitted in the folding back procedure.

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[6]Starting backwards, the DM calculates the certainty equivalent of each one-stage sublottery $P^{1,T}$, denoted $CE_{a(P^{1,T})}(p)$ with $V_{a(P^{1,T})}$. That is, $CE_{a(P^{1,T})}(p)$ is the value x that solves $V_{a(P^{1,T})}(\delta_x) = V_{a(P^{1,T})}(p)$. Next, the DM considers each two-stage sublottery $P^{2,T}$, denoting $P^2 = \langle \alpha_1, P_1^{1,T}; \ldots, \alpha_m, P_m^{1,T} \rangle$. The DM uses $V_{a(P^{2,T})}$ to calculate the certainty equivalent of the "folded back" one-stage lottery in which each p_i in the support of $P^{2,T}$ is replaced with its certainty equivalent calculated above; that is, $\langle \alpha_1, CE_{a(P_1^{1,T})}(P_1^{1,T}); \ldots; \alpha_m, CE_{a(P_m^{1,T})}(P_m^{1,T}) \rangle$. Continuing in this manner, the T-stage lottery is reduced to a one-stage lottery (over the certainty equivalents of its continuation sublotteries) whose value is calculated using V_0, since $a(P^T) = 0$. In the text, we will use the notation $CE(P^{t,T}; a, \mathscr{V})$ to denote the certainty equivalent of $P^{t,T}$ as calculated above; that is, $CE(P^{t,T}; a, \mathscr{V}) = CE_{a(P^{t,T})}(\widetilde{P^{t,T}})$, where $\widetilde{P^{t,T}}$ is the folded-back version of $P^{t,T}$ constructed in the folding back procedure.

[7]For nonexpected utility theories, there are stronger notions of comparative risk aversion; see, for example, Chew and Mao (1995). The definition we provide is standard for expected utility and is also used by Gul (1991, Definition 4) for disappointment aversion preferences.

[8]For our results, it would suffice that V_h and $V_{h'}$ are rankable only when h, h' are histories of the same length.

Definition 3 (History-dependent risk attitude, HDRA). *An HDRA utility representation over $\mathscr{L}^T$ consists of an admissible collection $\mathscr{V} := \{V_h\}_{h\in H}$ of utility functions over one-stage lotteries and a history assignment, a, satisfying for each $P^T \in \mathscr{L}^T$,*

1. *Sequential assignment. The DM assigns histories to all sublotteries of P^T sequentially:*

   (i) *if $P^{t+1,T}$ is nondegenerate and $P^{t,T} \in \operatorname{supp} P^{t+1,T}$ then $a(P^{t,T}) \in \{a(P^{t+1,T})\} \times \{e,d\}$;*

   (ii) *if $P^{t+1,T}$ is degenerate and $P^{t,T} \in \operatorname{supp} P^{t+1,T}$ then $a(P^{t,T}) = a(P^{t+1,T})$.*

2. *Folding back. The utility of P^T is calculated by folding back using a and the family $\mathscr{V} := \{V_h\}_{h\in H}$. We denote by $V(P^T;a,\mathscr{V})$ the value of P^T and denote by $CE(P^{t,T};a,\mathscr{V})$ the certainty equivalent of a sublottery $P^{t,T}$, as calculated in the folding back procedure.*

3. *Internal consistency. If $P^{t,T} \in \operatorname{supp} P^{t+1,T}$ is an elating (disappointing) outcome of a nondegenerate sublottery $P^{t+1,T}$, then the certainty equivalent of $P^{t,T}$ must be weakly larger than (strictly smaller than) the certainty equivalent of $P^{t+1,T}$ in P^T.*

We identify a DM with an HDRA representation by the pair $(\mathscr{V},a)$ satisfying the above.

Observe that the HDRA representation is ordinal in nature: the ranking over T-stage lotteries induced by the HDRA model is invariant to increasing, potentially different transformations of each of the utilities in the collection $\mathscr{V}$. This is because the HDRA model takes into account only the certainty equivalents of sublotteries after each history h, which may be found using any increasing transformation of the utility V_h.

To illustrate HDRA, consider the case of expected CRRA utility, $\mathscr{V} = \{\mathbb{E}(\cdot;\frac{x^{1-\rho_h}}{1-\rho_h})\}_{h\in H}$, over two-stage lotteries. When $T=2$, sequential history assignment is trivially satisfied. There are three risk aversion coefficients, $\{\rho_0,\rho_e,\rho_d\}$. For any one-stage lottery $p\in\mathscr{L}^1$ and $h\in\{0,e,d\}$, $CE_h(p)$ satisfies $\mathbb{E}(p;\frac{x^{1-\rho_h}}{1-\rho_h}) = \frac{CE_h(p)^{1-\rho_h}}{1-\rho_h}$. If a two-stage lottery P is degenerate (i.e., $P=\langle 1,p\rangle$) then $V(P;a,\mathscr{V}) = \mathbb{E}(p;\frac{x^{1-\rho_0}}{1-\rho_0})$. For any nondegenerate two-stage lottery $P = \langle\alpha_1,p_1;\ldots;\alpha_j,p_j;\ldots;\alpha_m,p_m\rangle$, the HDRA representation assigns to each one-stage lottery p in the support of P a history $a(p)\in\{e,d\}$, and the value of P is given by

$$V(P;a,\mathscr{V}) = \sum_{j=1}^{m}\alpha_j\frac{[CE_{a(p_j)}(p_j)]^{1-\rho_0}}{1-\rho_0}. \tag{3}$$

Moreover, the history assignment is internally consistent. If $a(p_j)=e$ then $CE(p_j;a,\mathscr{V}) \geq CE(\tilde{P};a,\mathscr{V})$; and if $a(p_j)=d$ then $CE(p_j;a,\mathscr{V}) < CE(\tilde{P};a,\mathscr{V})$, where $\tilde{P}$ in each case refers to the one stage lottery obtained from P when each p_j is replaced with its certainty equivalent $CE(p_j;a,\mathscr{V})$ under the history assignment a.

In the HDRA model, the DM's risk attitudes depend on the prior sequence of disappointments and elations, but not on the "intensity" of those experiences. That is, the DM is affected only by his general impressions of past experiences. This simplification of histories can be viewed as an extension of the notions of elation and disappointment for one-stage preferences in the betweenness class (see Section 3). This specification permits us to study endogenous reference dependence under the simplest departure from history independence.

Note that in the definition of HDRA, we assumed a DM considers an outcome of a nondegenerate lottery elating if its certainty equivalent is at least as large as the certainty equivalent of the lottery from which it emanates. Alternatively, one could redefine HDRA so that an outcome is disappointing if its certainty equivalent is at least as small as that of the parent lottery; or even introduce a third assignment, neutral (n), which treats the case of equality differently than elation or disappointment.[9] How equality is treated may affect the value of a lottery; but in either case, equality is possible only in a measure zero set of lotteries. Generically, a nonempty history consists of a sequence of strict elations and disappointments.

4.1. The reinforcement effect and the primacy effect

In this section we show that the existence of an HDRA representation implies regularity properties on $\mathscr{V}$ that are related to well-known cognitive biases; and that in turn, these properties imply the existence of HDRA.

As discussed in the introduction, experimental evidence suggests that an individual's risk attitudes depend on how prior uncertainty resolved. In particular, the literature suggests that people's risk attitudes are reinforced by prior experiences: they become less risk averse after positive experiences and more risk averse after negative ones. This effect is captured in the following definition.

Definition 4. *The collection $\mathscr{V}$ displays the reinforcement effect if $V_{hd} >_{RA} V_{he}$ for all h.*

A body of evidence also suggests that individuals are affected by the position of items in a sequence. One well-documented cognitive bias is the *primacy effect*, in which early observations have a strong effect on later judgments. In our setting, the order in which elations and disappointments occur affect the DM's risk attitude. The reinforcement effect suggests that after an initial elation, a disappointment increases the DM's risk aversion; and that after an initial disappointment,

[9] If $a(p|P) = n$, then for a generic perturbation of p to p' in P, resulting in a perturbed lottery P', $a(p|P') \neq n$. Our comparative statics on risk aversion would extend (see Lemma 4).

an elation reduces the DM's risk aversion. A primacy effect would further suggest that the shift in attitude from the initial realization has a lasting and disproportionate effect. Future elations or disappointments can only mitigate but not overpower the first impression, as in the following definition.

For any t, let d^t (or e^t) denote t repetitions of d (or e). The history hed^t, for example, corresponds to experiencing one elation and t successive disappointments after the history h, under the implicit assumption that the resulting history is in H.

Definition 5. *The collection $\mathscr{V}$ displays the weak primacy effect if $V_{hde} >_{\text{RA}} V_{hed}$ for all h. The collection displays the strong primacy effect if $V_{hde^t} >_{\text{RA}} V_{hed^t}$ for all h and $t \geq 1$.*

The combination of the reinforcement effect and the strong primacy effect implies strong restrictions on the collection $\mathscr{V}$; these are formalized in the following result and seen in Figure 2.

We refer below to the lexicographic order on histories of the same length as the ordering where $\tilde{h}$ precedes h if it precedes it alphabetically. Since d comes before e, this is interpreted as "the DM is disappointed earlier in $\tilde{h}$ than in h."

Proposition 1. *The following statements are equivalent:*

(i) $\mathscr{V}$ displays the reinforcement effect and the strong primacy effect;

(ii) For $h, \tilde{h}$ of the same length, $V_{\tilde{h}} >_{\text{RA}} V_h$ if $\tilde{h}$ precedes h lexicographically.

Assuming that $V_{hd} >_{\text{RA}} V_h >_{\text{RA}} V_{he}$ for all $h \in H$, conditions (i) and (ii) are also equivalent to:

(iii) For any h, h', h'', we have $V_{hdh''} >_{\text{RA}} V_{heh'}$.

Condition (ii) of Proposition 1 says, comparing histories of the same length, that the DM's risk aversion is greater when he has been disappointed earlier. This implies that the DM's risk aversion after any continuation $\tilde{h}$ is no greater than if he were to be consistently disappointed thereafter, and no less than if he were to be consistently elated thereafter. To show the lexicographic ordering across the rows in Figure 2, note that the first row from the bottom $(V_d >_{\text{RA}} V_e)$ follows directly from the reinforcement effect. The reinforcement effect also implies the left and right portions of the second row ($V_{ed} >_{\text{RA}} V_{ee}$ and $V_{dd} >_{\text{RA}} V_{de}$) while the weak primacy effect implies that $V_{de} >_{\text{RA}} V_{ed}$. Alternating the use of the reinforcement and strong primacy effects, one obtains each of the rows in Figure 2. Under the additional assumption $V_{hd} >_{\text{RA}} V_h >_{\text{RA}} V_{he}$, which says that an elation reduces (and a disappointment increases) the DM's risk aversion *relative to his initial*

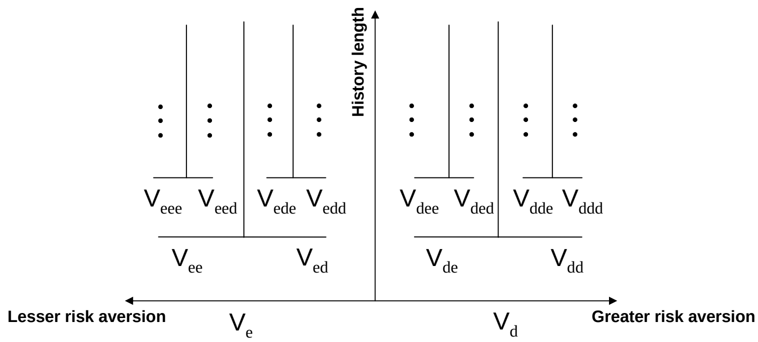

Figure 2: Starting from the bottom, each row depicts the risk aversion rankings $>_{\mathrm{RA}}$ of the V_h for histories of length $t = 1,2,3,\ldots,T-1$. The reinforcement effect and the primacy effect imply the lexicographic ordering in each row (Proposition 1). The assumption $V_{hd} >_{\mathrm{RA}} V_h >_{\mathrm{RA}} V_{he}$ for all $h \in H$ implies the vertical lines and consecutive row alignment.

level, one obtains the condition (iii), represented graphically in the vertical lines and consecutive row alignment in Figure 2. In words, condition (iii) says that whatever happens afterwards, the DM's risk aversion is always lower after an elation than it would have been, had he instead been disappointed at that same point in time. Along a realized path, however, condition (iii) imposes no restriction on how current risk aversion compares to risk aversion two or more periods ahead *when the continuation path* consists of both elating and disappointing outcomes: e.g., one can have both $\rho_h < \rho_{hed}$ or $\rho_h > \rho_{hed}$.

4.1.1. Necessary and sufficient conditions for HDRA

The following results link the two cognitive biases mentioned above to necessary and sufficient conditions for the existence of an HDRA representation.

Theorem 1 (Necessary conditions for HDRA). *In an HDRA representation $(\mathscr{V}, a)$, the collection $\mathscr{V}$ must display the reinforcement effect and the weak primacy effect. Under the additional assumption that $V_{hd} >_{RA} V_h >_{RA} V_{he}$, the collection $\mathscr{V}$ also displays the strong primacy effect (and is ordered as in Figure 2).*

Theorem 2 (Sufficient conditions for HDRA). *If the collection $\mathscr{V}$ displays the reinforcement effect and the strong primacy effect, then an HDRA representation $(\mathscr{V}, a)$ exists.*

Observe that on the set of two-stage lotteries, $\mathscr{L}^2$, the reinforcement effect is by itself necessary and sufficient for an HDRA representation, as there are too few stages for the primacy effect to apply. Similarly, on $\mathscr{L}^3$, the reinforcement effect and the weak primacy effect are both necessary and sufficient. Finally, note that to have the most concise statement of our results, the admissibility requirement rules out the standard case of history independent single-stage preferences. However, for the standard model, the existence of an internally consistent assignment is trivial; indeed, any assignment is consistent because history does not affect valuations.

Theorems 1 and 2 are proved in the appendix. There we provide an algorithm for finding an internally consistent history assignment for two-stage lotteries, which can be used recursively to prove existence of the HDRA representation for T-stage lotteries when the one-stage preferences satisfy betweenness. To illustrate how the algorithm works for two-stage lotteries, consider the simple example $P = \langle \alpha_1, \delta_b; \alpha_2, p; \alpha_3, \delta_w \rangle$. It is evident that δ_b should be elating, and that δ_w should be disappointing. If it is internally consistent to call p a disappointment, then the algorithm is complete. If it is not, then certainty equivalent of p under disappointment must be larger than that of P given the history assignment. By the reinforcement effect, $CE_e(p) > CE_d(p)$. But then viewing p as an elation would result in an internally consistent assignment of P *if* the certainty equivalent of p under elation is larger than that of P given the new assignment. We show that the latter property is implied by betweenness: if a prize is elating in a one-stage lottery, and that prize is increased (resulting in a modified lottery), then the increased prize is also elating in the modified lottery. The idea behind this algorithm can be generalized to any two-stage lottery, as well as more stages.

To see why the reinforcement effect is necessary in the case $T = 2$, assume by contradiction that $V_e >_{\text{RA}} V_d$. Then, for any nondegenerate $p \in \mathscr{L}^1$, $CE_d(p) > CE_e(p)$. Consider the lottery $P = \langle \alpha, p; 1-\alpha, \delta_x \rangle$. For p to be an elation in P, internal consistency requires $CE_e(p) > x$; for p to be a disappointment in P, internal consistency requires $CE_d(p) < x$. But then there cannot be an internally consistent assignment for any $x \in (CE_e(p), CE_d(p))$. Note that this particular argument depends only on the monotonicity of the certainty equivalents with respect to prizes.

We now sketch the argument for the necessity of the weak primacy effect in the case $T = 3$. Consider a three-stage lottery of the form $Q^3 = \langle \alpha, Q; 1-\alpha, \delta_x^2 \rangle$ and assume by contradiction that $V_{ed} >_{\text{RA}} V_{de}$. Hence, $CE_{ed}(q) < CE_{de}(q)$ for any nondegenerate $q \in \mathscr{L}^1$. Extending the idea above, a contradiction to internal consistency would arise if *for every internally consistent assignment within Q*, the certainty equivalent of Q after elation is smaller than that after disappointment. By betweenness and continuity, we construct Q, with q in its support, such that: (1) q must be elating when Q is disappointing, (2) q must be disappointing when Q is elating, and (3) the probability of q is sufficiently high so that $CE_d(\tilde{Q}) \approx CE_{de}(q) > CE_{ed}(q) \approx CE_e(\tilde{Q}')$ (where $\tilde{Q}$ and $\tilde{Q}'$ are the folded-back versions of Q given the assignments d and e, respectively) under the only possible internally consistent assignments of Q given each of V_d and V_e. As above, no internally consistent assignment of Q^3 would exist for $x \in (CE_e(\tilde{Q}'), CE_d(\tilde{Q}))$. Essentially, if it is not true that $V_{de} >_{\text{RA}} V_{ed}$ then an elating outcome received after a disappointment may overturn the assignment of the initial outcome as a disappointment. The intuition for the strong primacy effect is similar but requires a more complex construction.

5. Implications

In this section we discuss two phenomena that arise under HDRA, statistically reversing risk attitudes and the possibility of "gray areas" where two DM's, facing the same information and having the same $\mathscr{V}$, may disagree on which outcomes are elating or disappointing (the history assignment a) based on their optimistic or pessimistic tendencies. Further implications are studied in Section 6, in a richer setting where intermediate actions may be taken while uncertainty resolves.

5.1. Statistically reversing risk attitudes

Theorem 1 says that a DM with an HDRA representation displays the reinforcement effect. We now discuss an implication of this in our dynamic setting.

Greater risk aversion means that a one-stage lottery has a lower certainty equivalent. Thus, using our notions of disappointment and elation, for any nondegenerate $p \in \mathscr{L}^1$, and h, h' such that $V_h >_{\text{RA}} V_{h'}$, whenever a prize x is (1) disappointing in p under V_h, then it is disappointing in p under $V_{h'}$, and (2) elating in p under $V_{h'}$, then it is elating in p under V_h. Because of this feature, the reinforcement effect means that a DM who has been elated is not only less risk averse than a DM who has been disappointed, but also has a higher elation threshold. In other words, the reinforcement effect implies that after a disappointment, the DM is more risk averse and "settles for less"; whereas after an elation, the DM is less risk averse and "raises the bar." This leads to *statistically reversing risk attitudes*: disappointment is more likely after elation, and vice versa.

A simple example of this phenomenon was discussed in Section 1.1 using CRRA preferences. We now show that, depending on the initial realization, the DM can display different paths of reversal even within the same lottery. The following example uses disappointment aversion with linear utility over prizes to abstract from wealth effects.

Example 3. *Consider the three-stage lottery in Figure 3. The DM first learns whether or not he wins \$100; then, his additional winnings are determined by* $\langle \frac{1}{4}, \delta_0; \frac{1}{2}, p; \frac{1}{4}, \delta_{100} \rangle$, *where* p *is a lottery whose support consists of prizes between \$25 and \$50. Suppose the DM has history-dependent disappointment aversion preferences, where* $u(x) = x$, $\beta_e = 0$, $\beta_d = 2$, *and the other* β_h *are aligned as in Figure 2. First, note that the left sublottery (after winning \$100) must be elating,*

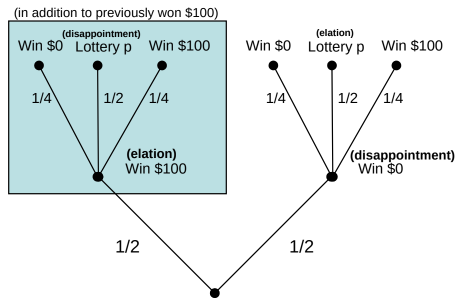

Figure 3: Let p be any lottery whose support consists of prizes between \$25 and \$50. Using disappointment aversion (with $\beta_e = 1$, $\beta_d = 2$, and $u(x) = x$), the figure shows the only internally consistent history assignment for the given three-stage lottery: p is a disappointment after first winning \$100, and an elation otherwise.

and the right sublottery (after winning \$0) must be disappointing, because the worst outcome on the left dominates the best outcome on the right. Now consider the folded-back sublottery $\langle\frac{1}{4},0;\frac{1}{2},x;\frac{1}{4},100\rangle$ *that results if* p *is replaced with a prize* x. *Using* $\beta_e = 0$, *any prize* x *smaller than \$50 is a disappointment; while using* $\beta_d = 2$, *any prize* x *larger than \$25 is an elation. But the certainty equivalent of* p *is always between \$25 and \$50 by monotonicity. Thus, the only consistent assignment of* p *is as a disappointment after winning \$100 and as an elation otherwise.*

Under the assumption that $V_{hd} >_{\text{RA}} V_h >_{\text{RA}} V_{he}$ for all h, HDRA implies condition (iii) in Proposition 1 (visualized in Figure 2). That condition says that after an elation, the DM's greatest possible degree of risk aversion in the future decreases; and conversely, after a disappointment, the DM's lowest possible degree of risk aversion in the future increases. However, in a finite horizon setting, this does not imply that the DM's mood swings moderate in intensity with experience. For example, this means that the collection of CRRA risk aversion parameters used *need not* satisfy $|\rho_{ed} - \rho_e| \geq |\rho_{ede} - \rho_{ed}| \geq |\rho_{eded} - \rho_{ede}| \cdots$. That is, the intensity of reversals in risk attitude may well persist.

5.2. Is the glass half full or half empty?

Consider the two stage lottery $\langle \alpha, p; 1-\alpha, \delta_x \rangle$ and suppose that $CE_e(p) > x > CE_d(p)$. Under this assumption, it would be consistent for the lottery p to be either an elation or a disappointment. The moral of this example is that while the collection $\mathscr{V}$ and history assignment a can be pinned down uniquely by choice behavior (as shown by the axiomatization in Section 7), one cannot fully reconstruct the DM's preference relation from only the information contained in $\mathscr{V}$. Predicting the DM's behavior in such "gray areas" as above requires a theory of how the DM assigns histories (as

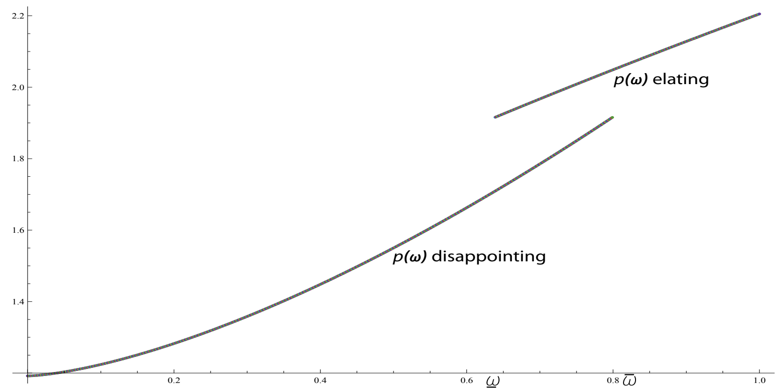

Figure 4: The set of possible HDRA utilities of $P(\omega)$ are pictured on the vertical axis for each $\omega \in (0,1)$ on the horizontal axis, given CRRA utilities $V_h = \mathbb{E}(\cdot; \frac{x^{1-\rho_h}}{1-\rho_h})$, where $\rho_e = 0$, $\rho_0 = 1/4$, $\rho_d = 1/2$. The sublottery $p(\omega)$ can be viewed as an elation or a disappointment in the range $[\underline{\omega}, \overline{\omega}]$.

seen later, such a theory has testable predictions for his preference relation over $\mathscr{L}^T$).

A dictionary definition of optimism is "An inclination to put the most favorable construction upon actions and events or to anticipate the best possible outcome."[10] In our setting, optimism and pessimism may be understood in terms of this multiplicity of internally consistent history assignments, where the optimist always selects the most favorable one and the pessimist selects the least favorable one.

Definition 6. *We say that a DM is an optimist if for every $P^T \in \mathscr{L}^T$ he selects the sequential and internally consistent history assignment a that maximizes his HDRA utility $V(P^T; a, \mathscr{V})$. Similarly, we say the DM is a pessimist if for every $P^T \in \mathscr{L}^T$ he selects the sequential and internally consistent assignment a that minimizes his HDRA utility $V(P^T; a, \mathscr{V})$. Given the same $\mathscr{V}$, we say that one DM is more optimistic than another if his HDRA utility is higher for every $P^T \in \mathscr{L}^T$.*

The optimist and pessimist agree on fundamentals (that is, the collection $\mathscr{V}$ of utilities to apply after each history), but they take a different perspective on what outcomes are disappointing and elating. This approach differs from most models of optimism and pessimism, which typically view optimism in terms of attaching higher probability to positive events. Under HDRA, probabilities are objective and unchanging, but endogenous reference dependence allows the DM to select an internally consistent view of the unfolding risk according to his optimistic or pessimistic tendency. This manifests itself in the DM's risk atittude over T-stage lotteries; indeed, note that given the same $\mathscr{V}$, whenever a pessimist prefers a lottery P^T to a degenerate outcome, so does the optimist.

[10] "optimism." Merriam-Webster Online Dictionary. 2010. http://www.merriam-webster.com (14 June 2010).

To illustrate, consider Figure 4, which depicts for each $\omega \in (0,1)$ all possible HDRA values of the two-stage lottery $P(\omega) = \langle \frac{1}{3}, \delta_1; \frac{1}{3}, \delta_2; \frac{1}{3}, p(\omega) \rangle$ where $p(w) = \langle \omega, 3; 1-\omega, 0 \rangle$ using the example of a CRRA collection $\mathscr{V}$. An increase in ω is a first-order stochastic improvement of the risky sublottery $p(\omega)$. While $p(\omega)$ is unambiguously elating (disappointing) for high (low) values of ω, there is an intermediate range $[\underline{\omega}, \overline{\omega}]$ where $p(\omega)$ can be viewed either as an elation or as a disappointment. The certainty equivalents of the other sublotteries are independent of their history assignment because they are degenerate. At the same time, the reinforcement effect implies $CE_e(p(\omega)) > CE_d(p(\omega))$. Because HDRA utility is increasing in the certainty equivalents, viewing $p(\omega)$ as an elation gives higher utility. The optimist thus views $p(\omega)$ as an elation as soon as possible (for all $\omega \geq \underline{\omega}$). On the other hand, the pessimist views $p(\omega)$ as a disappointment for as long as possible (for all $\omega \leq \overline{\omega}$). More generally, a DM may have a cutoff $\omega^* \in [\underline{\omega}, \overline{\omega}]$ at which $p(\omega)$ switches from a disappointment to an elation. If one DM is more optimistic than another, then his cutoff ω^* must be lower.

It is easy to see that the reinforcement effect implies that a DM with an HDRA representation may violate first-order stochastic dominance on T-stage lotteries: for example, if the probability α of p is very high, the lottery $\langle \alpha, p; 1-\alpha, \delta_w \rangle$ may be preferred to $\langle \alpha, p; 1-\alpha, \delta_b \rangle$; the "thrill of winning" outweighs the "pain of losing." The above example suggests, however, that both the optimist and pessimist satisfy the following regularity property related to first-order dominance, stated for simplicity for $T = 2$.

Proposition 2. *Let $\succeq$ be the preference relation represented by the DM's HDRA utility on $\mathscr{L}^2$, and let $>_{FOSD}$ denote the first-order stochastic dominance relation on $\mathscr{L}^1$. Fix any prizes $x_1, \ldots, x_{m-1}$ and probabilities $\alpha_1, \ldots, \alpha_m$. If the DM is an optimist or a pessimist, then*[11]

$$\langle \alpha_1, \delta_{x_1}; \ldots; \alpha_{m-1}, \delta_{x_{m-1}}; \alpha_m, p \rangle \succ \langle \alpha_1, \delta_{x_1}; \ldots; \alpha_{m-1}, \delta_{x_{m-1}}; \alpha_m, q \rangle \text{ whenever } p >_{FOSD} q. \quad (4)$$

The idea behind Proposition 2 is that fixing p as either an elation or a disappointment, the utility of $P(\omega)$ is increasing in ω; and viewing p as an elation gives strictly higher utility for each ω. The fact that the other sublotteries are degenerate ensures that the history assignment of p does not affect their value. Consider instead a lottery $\langle \alpha, p; 1-\alpha, q \rangle$, where both p, q are risky and $p >_{FOSD} q$. For each V_h, the certainty equivalent of p is larger than that of q by first-order dominance; hence it would always be consistent to label p as an elation and q as a disappointment. However, if $CE_e(q) > CE_d(p)$ then it would also be consistent to label p as a disappointment and q

[11] More generally, any DM whose history assignment a applies a cutoff for viewing the risky lottery as an elation (as in the above discussion) will also satisfy Property (4).

as an elation; and if the probability $1-\alpha$ of q is sufficiently high, the optimist may achieve a higher HDRA utility by doing so. The intuition is that by viewing a high probability, riskier prospect as an elation (if it is consistent to do so), the optimist puts a "positive spin" on the uncertainty. (A similar feature applies for the pessimist.)

6. HDRA with intermediate choices

We now extend the HDRA model to the setting of stochastic decision trees. Roughly speaking, a stochastic decision tree is a lottery over choice sets of shorter stochastic decision trees. In each choice set, the DM can choose the continuation stochastic decision tree. Formally, for any set Z, let $K(Z)$ be the set of finite, nonempty subsets of Z. A one-stage stochastic decision tree is simply a one-stage lottery. The set of one-stage stochastic decision trees is $\mathscr{D}^1 = \mathscr{L}^1$, with typical elements p, q. For $t = 2, .., T$, the set of finite, nonempty sets of $(t-1)$-stage stochastic decision trees is given by $\mathscr{A}^{t-1} = K(\mathscr{D}^{t-1})$, with typical elements A^{t-1}, B^{t-1}. Then the set of t-stage stochastic decision trees is the set of lotteries over finite choice sets of $(t-1)$-stage stochastic decision trees. Formally, the set of t-stage stochastic decision trees is $\mathscr{D}^t = \mathscr{L}(\mathscr{A}^{t-1})$, with typical elements P^t, Q^t. Our domain is thus $\mathscr{D}^T = \mathscr{L}(\mathscr{A}^{T-1})$. (Our previous domain of T-stage lotteries can be seen as the case in which all choice sets are degenerate.)

The realization of a t-stage stochastic decision tree is a choice set, which is categorized by the DM as either elating or disappointing. The set of possible histories, H, is the same as before, with the understanding that histories now refer to choice sets. The set of admissible collections of utilities is also the same. We abuse notation and identify the stochastic decision tree $P^T \in \mathscr{D}^T$ with a degenerate choice set, denoted $A^T := \{P^T\}$. Moreover, in analogy to the discussion in Section 4, to uniquely identify a choice set A^t in a stochastic decision tree P^T we use the notation $A^{t,T}$ to denote the choice set A^t which arises from the sequence of choices and realizations $(A^t, P^{t+1}, A^{t+1}, P^{t+2}, \cdots, A^T)$, where $A^\tau \in \text{supp } P^{\tau+1}$ and $P^{\tau+1} \in A^{\tau+1}$.

The folding back procedure may be extended to this richer domain in a way that the history assignment of a choice set determines the one-stage utility V_h applied to value each folded back stochastic decision tree inside it, and the value of the choice set itself is the maximal value of those stochastic decision trees.[12]

[12]Formally, the certainty equivalent of each set of one-stage stochastic decision trees is determined by

$$V_h\left(\delta_{CE_h(A^{1,T})}\right) = \max_{p \in A^{1,T}} V_h(p), \text{ where } h = a(A^{1,T}).$$

Fold back each $P^2 \in A^2$, by replacing each realization (a choice set, $A_j^{1,T}$) with its certainty equivalent calculated

The definition of HDRA is then almost the same as before.

Definition 7 (HDRA with intermediate choices). *An HDRA utility representation over T-stage lotteries consists of an admissible collection $\mathscr{V} := \{V_h\}_{h\in H}$ of utility functions over one-stage lotteries and a history assignment a satisfying, for each $A^T \in \mathscr{D}^T$,*

1. *Sequential assignment. The DM assigns histories to all realizations of stochastic decision subtrees of $\mathscr{D}^T$. Let $a(A^T) := \beta_0$ and, recursively, for $t < T$:*

   (i) If $P^{t+1,T}$ is nondegenerate, $A^{t,T} \in \text{supp } P^{t+1,T}$, and $P^{t+1,T} \in A^{t+1,T}$ then $a(A^{t,T}) \in a(A^{t+1,T}) \times \{e,d\}$.

   (ii) If $P^{t+1,T}$ is degenerate, $A^{t,T} \in \text{supp } P^{t+1,T}$, and $P^{t+1,T} \in A^{t+1,T}$ then $a(A^{t,T}) = a(A^{t+1,T})$.

2. *Folding back. The DM calculates the utility of the stochastic decision tree by folding back. We denote by $V(A^T; a, \mathscr{V})$ the utility of A^T. We denote by $CE(A^{t,T}; a, \mathscr{V})$ and $CE(P^{t,T}; a, \mathscr{V})$ the certainty equivalents of a choice set $A^{t,T}$ and subtree $P^{t,T}$ as calculated in the folding back procedure.*

3. *Internal consistency. For each nondegenerate $P^{t+1,T}$, if $A^{t,T} \in \text{supp } P^{t+1,T}$ is an elating (disappointing) outcome in $P^{t+1,T}$, then $CE(A^{t,T}; a, \mathscr{V})$ must be weakly larger than (strictly smaller than) $CE(P^{t+1,T}; a, \mathscr{V})$.*

Observe that the DM is dynamically consistent under HDRA with intermediate actions. From any future choice set, the DM anticipates selecting the best stochastic decision tree. That choice leads to an internally consistent history assignment of that choice set. Thus, when reaching a choice set, the single-stage utility she uses to value the choices therein is the one she anticipated using, and her choice is precisely her anticipated choice.

Internal consistency is a stronger requirement than before, because it takes optimal choices into account. However, our previous results on the restrictions that internal consistency imposes on how history affects risk attitude extend to the model of HDRA with intermediate actions.

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above, to get the folded back lottery:

$$\widetilde{P^2} = \langle \alpha_1, CE_{a(A_1^{1,T})}(A_1^{1,T}); \ldots; \alpha_m, CE_{a(A_m^{1,T})}(A_m^{1,T}) \rangle.$$

The certainty equivalent of a choice set $A^{2,T}$ of two-stage stochastic decision trees is determined by its best element:

$$V\left(\delta_{CE_h(A^{2,T})}\right) = \max_{\widetilde{P^2} \in A^{2,T}} V_h(\widetilde{P^2}), \text{ where } h = a(A^{2,T}).$$

Continuing in this manner, the T-stage stochastic decision tree is reduced to a one-stage lottery (over the certainty equivalents of its continuation subtrees) whose value is calculated using V_0.

Theorem 3 (Extension of previous results). *The conclusions of Theorems 1 and 2 (necessary and sufficient conditions for HDRA) also hold for HDRA with intermediate actions.*

One may wonder whether a DM with an HDRA representation with intermediate actions would satisfy a version of the weak axiom of revealed preference (WARP), modified to this setting. Suppose that within the stochastic decision tree A^T, the DM would select a subtree $P^{t,T}$ from the choice set $A^{t,T}$. Now consider a stochastic decision tree B^T which is identical to A^T except that the choice set $A^{t,T}$ is replaced with a subset $B^{t,T}$ containing the original choice: $P^{t,T} \in B^{t,T} \subset A^{t,T}$. Would the DM still select $P^{t,T}$ from $B^{t,T}$? The answer is that if $P^{t,T}$ is chosen from $A^{t,T}$ under an internally consistent history assignment for A^T, then the same history assignment (modified so that $a(A^{t,T}) = a(B^{t,T})$) would also be internally consistent for B^T, and the DM's choice from every subset would be the same. That is, there is always an internally consistent history assignment satisfying WARP. Moreover, it is easy to see that both optimism and pessimism are consistent with selecting a WARP-consistent assignment: if an assignment is utility maximizing (or minimizing) in A^T, the same assignment would be utility maximizing (or minimizing) in B^T.

6.1. Implications

Actions that can be taken while risk unfolds may arise naturally in various settings. Under HDRA, the DM's risk taking behavior may depend on his history: his risk attitudes are reinforced by prior experiences and he displays primacy effects. For example, the reinforcement effect suggests that a basketball player might attempt more difficult shots after a string of successful ones. Rao (2009), for example, finds evidence to this effect within the NBA, and uses this as an explanation for the "hot hand" fallacy, which is the belief that a winning streak indicates future success (even in independent events).[13]

The biases predicted by HDRA, particularly the primacy effect, may also be exploited by agents who can manipulate the presentation of information to affect the DM's behavior. For example, consider a financial advisor trying to sell the DM a risky investment. The DM has an HDRA representation with disappointment aversion preferences, where $u(x) = x$ and the DM's disappointment aversion coefficients are strictly positive and ordered as in Figure 2. The risky investment, which requires an initial payment of I, is an even chance gamble between $I+U$ and $I-D$. The DM knows that the upside, U, and downside, D, are independently and uniformly distributed on

[13] Unlike previous studies, such as Gilovich, Vallone and Tversky (1985), Rao controls for shot difficulty (i.e., taking more or less difficult shots after successes or failures) and shows that risk taking behavior —but not ability—is affected by previous outcomes.

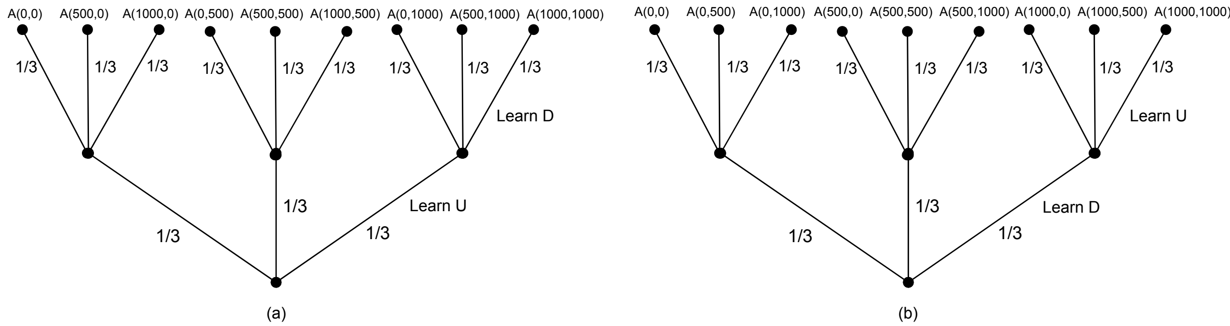

Figure 5: The stochastic decision tree that the DM faces when the financial advisor (a) reveals the upside first and the downside later or (b) reveals the downside first and the upside later. For given U,D, the choice set $A(D,U) = \{\delta_I, \langle .5, I+U; .5, I-D\rangle\}$ corresponds to either investing or not.

$\{0,500,1000\}$. The financial advisor receives a commission whenever the DM invests and is informed about the true values of U and D. The DM may consult with the financial advisor at no cost to learn U and D, and may choose whether or not to invest based on the information provided. The financial advisor is obligated to tell the truth about U and D, but can reveal this information in any order.[14]

It is straightforward to check that without any information, the DM prefers not to invest. Hence the DM chooses to consult with the advisor. Suppose the financial advisor has some good news and some slightly worse news: the upside is high ($U = 1000$) but the downside is moderate ($D = 500$).[15] How should she reveal this? Since the financial advisor knows the DM's preferences, she can predict his choice based on how she provides information about U and D. For $U = 1000$ and $D = 500$, the DM will invest if the disappointment aversion coefficient used to evaluate the investment is smaller than one. The primacy effect suggests that conveying the best news first increases the DM's inclination to invest. This can be formalized by applying HDRA to the stochastic decision trees in Figure 5, which describe the DM's problem when the financial advisor reveals U or D first. For a wide range of disappointment aversion coefficients (for example, if $\beta_h \in [.5, 1.5]$ for all h and $\beta_{ed} < 1 < \beta_{de}$), the DM is immediately disappointed when $D = 500$ is mentioned first, and wouldn't invest even upon hearing $U = 1000$; while the DM is immediately elated when $U = 1000$ is mentioned first, and invests even upon hearing $D = 500$. Therefore, the financial advisor should

[14]We assume for simplicity that the DM accepts the information the financial advisor gives in that order, without making inferences (this is an assumption often made in the context of framing effects); relaxing this assumption is interesting but beyond the scope of this example.

[15]This is the only case in which manipulation is possible. It is clear that when learning $U = 0$, the DM would never invest; when learning $D = 0$, the DM would always invest; and that the DM would not invest if $U = D$ and he has strictly positive disappointment aversion coefficients.

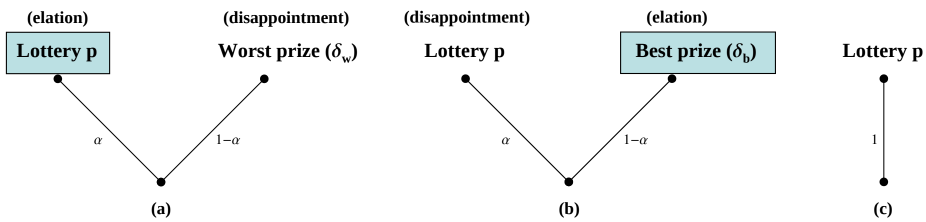

Figure 6: As the lottery p is varied, (a) corresponds to the objects inducing the relation $\succeq_{e,\alpha}$, (b) corresponds to $\succeq_{d,\alpha}$; and (c) corresponds to $\succeq_0$.

reveal the best news first to minimize the DM's subsequent risk aversion and ensure he invests.

By contrast, if an agent were trying to maximize the DM's risk aversion (for example, if the agent is selling insurance), then the agent should tell the worst news first to ensure a purchase.

7. Axiomatic foundations for HDRA on two-stage lotteries

In this section, we present axioms necessary and sufficient for a preference $\succeq$ on the set of two-stage lotteries, $\mathscr{L}^2$, to have an HDRA representation. This simplified setting allows for the clearest exposition of the underlying ideas; we discuss the extension to more stages in Section 7.1.

To motivate our axiomatization, note that in some two-stage lotteries, which history to assign to each realization can be determined by a quick inspection. For example, this is true of all the lotteries depicted in Figure 6. In the lottery in Figure 6(b), which has the form $\langle \alpha, p; 1-\alpha, \delta_b \rangle$, receiving the lottery p is disappointing compared to receiving the best monetary prize (b) with certainty.[16] How should the DM compare the two-stage lotteries $P = \langle \alpha, p; 1-\alpha, \delta_b \rangle$ and $Q = \langle \alpha, q; 1-\alpha, \delta_b \rangle$, which both have this form? Both p and q are disappointing in P and Q, respectively, and are received with the same probability α. According to HDRA, V_d must be applied to evaluate the certainty equivalents of p and q, while the certainty equivalent of δ_b is simply b. Therefore, the preference over P and Q should be determined by the utilities of p and q according to V_d.

We define $\succeq_{e,\alpha}$ on $\mathscr{L}^1$ by $p \succeq_{e,\alpha} q$ if $\langle \alpha, \delta_w; 1-\alpha, p \rangle \succeq \langle \alpha, \delta_w; 1-\alpha, q \rangle$. Similarly, we define $\succeq_{d,\alpha}$ on $\mathscr{L}^1$ by $p \succeq_{d,\alpha} q$ if $\langle \alpha, \delta_b; 1-\alpha, p \rangle \succeq \langle \alpha, \delta_b; 1-\alpha, q \rangle$ and $\succeq_0$ on $\mathscr{L}^1$ by $p \succeq_0 q$ if $\langle 1, p \rangle \succeq \langle 1, q \rangle$. These relations are induced from preferences over the objects in Figure 6. Our first axiom requires that these induced relations form an admissible class (see Definition 2). We say that two preferences $\succeq$ and $\hat{\succeq}$ are *rankable in terms of risk aversion* if either (i) $p \succeq \delta_x$ implies $p \hat{\succ} \delta_x$ for all nondegenerate $p \in \mathscr{L}^1$, or (ii) $p \hat{\succeq} \delta_x$ implies $p \succ \delta_x$ for all nondegenerate $p \in \mathscr{L}^1$.

[16]Except in the knife-edge case $p = \delta_b$; however then the certainty equivalent is b regardless of how p is labeled.

Axiom A (Admissibility). The induced relations $\succeq_{e,\alpha}$, $\succeq_{d,\alpha}$, and $\succeq_0$ are continuous and monotone preference relations which satisfy betweenness and are rankable in terms of risk aversion.

Our next axiom says that "no news" does not affect the DM's attitude toward risks. If he knows that his monetary winnings will be determined by a one-stage lottery p, he does not care whether the uncertainty in p is resolved now or later. Hence the DM's risk attitude is not affected by the mere passage of time, but rather only by previous disappointments and elations.

Axiom TN (Time neutrality). For all $p \in \mathscr{L}^1$, $\langle p(x_1), \delta_{x_1}; p(x_2), \delta_{x_2}; ...; p(x_m), \delta_{x_n} \rangle \sim \langle 1, p \rangle$.

Recall that the procedure of folding back a two-stage lottery involves replacing each one-stage lottery with its certainty equivalent. In the HDRA model, p is an elation in $\langle \alpha, \delta_w; 1-\alpha, p \rangle$ for each $\alpha \in (0,1)$. We say that a prize x is an α-*elation certainty equivalent* of p if it solves $\langle \alpha, \delta_w; 1-\alpha, p \rangle \sim \langle \alpha, \delta_w; 1-\alpha, \delta_x \rangle$ (the α-*disappointment certainty equivalent* is defined analogously). By Axiom A, it is clear that there exists a unique α-elation certainty equivalent for each p (and similarly for disappointment). The next axiom says these certainty equivalents depend only on the history assignment of p, independently of the probability with which it occurs.

Axiom CE (Uniform certainty equivalence). Take any $p \in \mathscr{L}^1$, $x \in X$, and $z \in \{w, b\}$. If $\langle \alpha, \delta_z; 1-\alpha, p \rangle \sim \langle \alpha, \delta_z; 1-\alpha, \delta_x \rangle$ for some $\alpha \in (0,1)$, then $\langle \alpha', \delta_z; 1-\alpha', p \rangle \sim \langle \alpha', \delta_z; 1-\alpha', \delta_x \rangle$ for all $\alpha' \in (0,1)$.

We define $CE_{e,\succeq}(p)$, the *elation certainty equivalent* of p, as the value solving $\langle \alpha, \delta_w; 1-\alpha, p \rangle \sim \langle \alpha, \delta_w; 1-\alpha, \delta_{CE_{e,\succeq}(p)} \rangle$ for all $\alpha \in (0,1)$. The *disappointment certainty equivalent* of p, $CE_{d,\succeq}(p)$, is analogously defined.

HDRA requires that if a one-stage lottery p is elating in a two-stage lottery P, then it must indeed be preferred to P as a whole. HDRA assumes the certainty equivalent of a lottery p in P is affected only by its assigned history h. This motivates the following definitions. Consider $P = \langle \alpha_1, p_1; \ldots; \alpha_j, p_j; \ldots \alpha_m, p_m \rangle$. We say p_j *is elating* in P if

$$P \sim \langle \alpha_1, p_1; \ldots; \alpha_j, \delta_{CE_{e,\succeq}(p_j)}; \ldots \alpha_m, p_m \rangle \preceq \langle 1, \delta_{CE_{e,\succeq}(p_j)} \rangle.$$

Similarly, we say p_j *is disappointing* in P if

$$P \sim \langle \alpha_1, p_1; \ldots; \alpha_j, \delta_{CE_{d,\succeq}(p_j)}; \ldots \alpha_m, p_m \rangle \succ \langle 1, \delta_{CE_{d,\succeq}(p_j)} \rangle.$$

Our final axiom says that the preference $\succeq$ always allows the DM to categorize a realization p_j of a two-stage lottery P according to one of the possibilities above.

Axiom CAT (Categorization). For any nondegenerate $P \in \mathscr{L}^2$ and any $p \in \text{supp } P$, p is either elating or disappointing in P.

These axioms are equivalent to an HDRA representation on two-stage lotteries.

Theorem 4 (Representation). *$\succeq$ on $\mathscr{L}^2$ satisfies Axioms A, TN, CE, and CAT if and only if it admits a history-dependent disappointment aversion (HDRA) representation $(\mathscr{V}, a)$.*

In the theorem above, the underlying single-stage preference after each history h is uniquely determined (each V_h in $\mathscr{V}$ is unique up to increasing transformation), and the history assignment is uniquely determined for each $P \in \mathscr{L}^2$, except in knife-edge (measure zero) cases that two decompositions would give P the same value.

7.1. Extending to three or more stages

With an appropriate modification of the axioms, Theorem 4 can be extended to represent preferences over (arbitrary) T-stage lotteries. In this section, we highlight the required changes for the case $T = 3$; the more general case is similarly analyzed.

We must first generalize the sets of compound lotteries for which the history assignment of any final stage lottery, p, is unambiguous. For example, consider a lottery of the form $\langle \alpha, \delta_w^2; 1-\alpha, P \rangle$, where P is of the form $\langle \alpha', \delta_b; 1-\alpha', p \rangle$. Here, the lottery p must have the history assignment $h = ed$. We may define an induced preference $\succeq_{ed,(\alpha,\alpha')}$ on $\mathscr{L}^1$, and similarly for other possible history assignments. Axiom A requires that these induced preference relations are rankable in terms of risk aversion and each satisfy continuity, monotonicity, and betweenness. The definition of the certainty equivalent of a sublottery is extended analogously; for example, for $h = ed$, $CE_{ed,\succeq}(p)$ is the value solving

$$\langle \alpha, \delta_w^2; 1-\alpha, \langle \alpha', \delta_b, 1-\alpha_w'^2; 1-\alpha, \langle \alpha', \delta_b, 1-\alpha', \delta_{CE_{ed,\succeq}(p)} \rangle \rangle$$

for all α, α'. Axiom CE then says that conditional on each history, the certainty equivalent of a sublottery is independent of the probability with which it is received.

For any given single-stage lottery p, there are three compound lotteries in which the only nondegenerate sublottery is the one where p is fully resolved. Axiom TN requires that the DM be indifferent among these lotteries. Formally, for all $p \in \mathscr{L}^1$,

$$\langle p(x_1), \delta^2_{x_1}; p(x_2), \delta^2_{x_2}; ...; p(x_m), \delta^2_{x_m} \rangle \sim$$
$$\langle 1, \langle p(x_1), \delta_{x_1}; p(x_2), \delta_{x_2}; ...; p(x_m), \delta_{x_m} \rangle \rangle \sim \langle 1, \langle 1, p \rangle \rangle.$$

Lastly, Axiom CAT requires that a sublottery can be replaced by a degenerate lottery that gives the history-dependent certainty equivalent of this sublottery for sure, with the consistency condition taking into account the history assignment of the sublottery. For example, if a one-stage lottery $p \in \text{supp}\, P = \langle \alpha', \delta_b; 1-\alpha', p \rangle$ is replaced by its certainty equivalent after history $h = ed$, then it must be the case that

$$\langle \alpha, \delta^2_w; 1-\alpha, P \rangle \sim \langle \alpha, \delta^2_w; 1-\alpha, \langle \alpha', \delta_b, 1-\alpha', \delta_{CE_{ed,\succeq}(p)} \rangle \rangle \succ \langle \alpha, \delta^2_w; 1-\alpha, \delta^2_{CE_{ed,\succeq}(p)} \rangle \rangle$$

8. Conclusion and directions for future research

We propose and axiomatize a model of history dependent risk attitude, in which prior disappointments and elations endogenously affect the DM's view of future risks. The HDRA model predicts that the DM satisfies two documented cognitive biases: the reinforcement effect and the primacy effect. In addition, the DM raises the threshold for elation after positive experiences but is willing to "settle for less" after negative ones, making disappointment more likely after elation and vice-versa.

To study endogenous reference dependence under the minimal departure from recursive history-independent preferences, HDRA posits the categorization of each sublottery as either elating or disappointing. The DM's risk attitudes depend on the prior sequence of disappointments or elations, but not on the "intensity" of those experiences. We are also interested in extending the HDRA model to permit such dependence. That extension raises several questions, beginning with how to define the intensity of elation or disappointment and how internal consistency is to be understood. The testable implications of such a model depend on whether it is possible to identify the extent to which a realization is disappointing, as that designation depends on the extent to which other options are elating or disappointing.

Finally, this paper considers a finite-horizon model of decision-making. In an infinite-horizon setting, our methods easily extend to prove that the reinforcement and primacy effects remain necessary. However, our methods do not immediately extend to ensure the existence of an infinite-horizon internally consistent history assignment. One possible way to embed our finite-horizon HDRA preferences into an infinite-horizon economy is through the use of an overlapping generations model. In the context of asset pricing, for example, it would be interesting to study whether the pattern of attitudes toward risk implied by HDRA is consistent with observed movements of asset prices.

A. Appendix

A.1. Proofs for Section 4

Proof of Proposition 1. It is clear that (ii) implies (i), as both the reinforcement effect and the strong primacy effect respect the lexicographic ordering. Proving that (i) implies (ii) follows from alternating applications of the reinforcement and strong primacy effects starting from the tail of the history. To illustrate, observe that $V_{eed} >_{\mathrm{RA}} V_{eee}$ by the reinforcement effect with $h = ee$; $V_{ede} >_{\mathrm{RA}} V_{eed}$ by the strong primacy effect with $h = e$; $V_{edd} >_{\mathrm{RA}} V_{ede}$ by the reinforcement effect with $h = ed$; $V_{dee} >_{\mathrm{RA}} V_{edd}$ by the strong primacy effect with $h = 0$; and so on and so forth.

Now assume that $V_{hd} >_{\mathrm{RA}} V_h >_{\mathrm{RA}} V_{he}$ for all h. Let $|h|$ denote the length of history h (the number of e's and d's). To see that (iii) implies (i), note that the reinforcement effect is implied by taking $h' = h'' = 0$; and that the strong primacy effect is implied by taking $h' = d^t$ and $h'' = e^t$. To see that (ii) implies (iii), if h' is not entirely consisting of d's and h'' is not entirely consisting of e's then we know $V_{hed^{|h'|}} >_{\mathrm{RA}} V_{heh'}$ and $V_{hdh''} >_{\mathrm{RA}} V_{hde^{|h''|}}$. Using the strong primacy effect to combine these bounds delivers the result if $|h''| > |h'|$; so suppose that $|h''| > |h'|$ (the other argument is symmetric). Then repeated use of the assumption that $V_{\hat{h}d} >_{\mathrm{RA}} V_{\hat{h}}$ implies $V_{hed^{|h''|}} >_{\mathrm{RA}} V_{hed^{|h'|}}$, and using the strong primacy effect again completes the proof. ■

Recall that $CE_h(p)$ solves $V_h\left(\delta_{CE_h(p)}\right) = V_h(p)$.

Lemma 1. *If $\bigcap_{\tau=0}^{t}(CE_{de^\tau}(p), CE_{ed^\tau}(p)) \neq \emptyset$ for all $p \in \mathscr{L}^1$, then $CE_{de^{t+1}}(p) \leq CE_{ed^{t+1}}(p)$ for all $p \in \mathscr{L}^1$.*

Proof. Fix $\widehat{p} = \langle 0.5, w; 0.5, b\rangle$ and let $\bigcap_{\tau=0}^{t}(CE_{de^\tau}(\widehat{p}), CE_{ed^\tau}(\widehat{p})) := \left(CE_{\underline{h}}, CE_{\overline{h}}\right)$. Let p be a lottery such that $\operatorname{supp} p \subset \left(CE_{\underline{h}}, CE_{\overline{h}}\right)$. Define $P^2 = \langle \varepsilon, \delta_w; \varepsilon, \delta_b; 1-2\varepsilon, p\rangle$, $P^3 = \langle \varepsilon, \delta_w^2; \varepsilon, \delta_b^2; 1-2\varepsilon, P^2\rangle$, and continuing inductively, $P^{t+1} = \langle \varepsilon, \delta_w^{t+1}; \varepsilon, \delta_b^{t+1}; 1-2\varepsilon, P^t\rangle$. That is, in each stage $1,\ldots,t$ the lottery P^{t+1} gives the worst and the best outcome, both with probability ε and the continuation with the remaining probability. At period t, the continuation lottery is the lottery p. Note the following:

(i) For all h, $CE_h(p) \subset \left(CE_{\underline{h}}, CE_{\overline{h}}\right)$. This is by monotonicity.

(ii) Fixing h, h', $\lim_{\varepsilon \to 0} CE_h\left(\langle \varepsilon, w; \varepsilon, b; 1-2\varepsilon, CE_{h'}(p)\rangle\right) = CE_{h'}(p)$.

(iii) By betweenness, for all τ, p is elating in $P^2 = \langle \varepsilon, \delta_w; \varepsilon, \delta_b; 1-2\varepsilon, p\rangle$ when P^2 is evaluated under h_{de^τ} and p is disappointing in P^2 when P^2 is evaluated under h_{ed^τ}.

Assume by contradiction that $CE_{h_{ed^{\tau+1}}}(p) < CE_{h_{de^{t+1}}}(p)$. Pick $\varepsilon > 0$ small enough and apply (ii) and (iii) repeatedly to show that if for $\tau = 1$ h_e is used, then for $\tau > 1$, the only consistent set of continuation histories are h_{ed^τ}; and if for $\tau = 1$ h_d is used, then for $\tau > 1$, the only consistent set of continuation histories are $h_{de^{\tau}}$. But, again, for $\varepsilon > 0$ small enough there exists $\delta > 0$ such that the first continuation value (evaluated with h_e) is less than $CE_{h_{ed^{t+1}}}(p) + \delta$ and the first continuation value (evaluated with h_d) is greater than $CE_{h_{de^{t+1}}}(p) - \delta$. Now pick $x \in \left(CE_{h_{ed^{t+1}}}(p) + \delta, CE_{h_{de^{t+1}}}(p) - \delta\right)$ and note that for $\left\langle \alpha, P^{t+1}; 1-\alpha, \delta_x^{t+1} \right\rangle$ an internally consistent HDRA history assignment cannot exist. Since the utilities are ranked in risk aversion, the proof is complete. ■

Proof of Theorem 1. We prove each statement separately.

(i) *The reinforcement effect.* Suppose by contradiction that $V_d >_{\text{RA}} V_e$. Then for any $p \in \mathscr{L}^1$, $CE_e(p) < CE_d(p)$. Choose any $x \in (CE_e(p), CE_d(p))$. Let P^t be the t-stage lottery which has no uncertainty until stage $t-1$ and delivers p at stage $t-1$ with probability one. Observe that the T-stage lottery $\langle \alpha, P^{T-1}; 1-\alpha, \delta_x^{T-1} \rangle$ would have no internally consistent assignment. To show, for example, that $V_{ed} >_{\text{RA}} V_{ee}$, modify the above to $\langle \alpha, \langle \gamma, P^{T-2}; 1-\gamma, \delta_x^{T-2} \rangle; 1-\alpha, \delta^T, b \rangle$, where $x \in (CE_{ee}(p), CE_{ed}(p))$ if by contradiction $V_{ee} >_{\text{RA}} V_{ed}$. Analogously, one shows $V_{hd} >_{\text{RA}} V_{he}$ by constructing the appropriate initial history.

(ii) *Weak primacy effect.* Given $V_{hd} >_{\text{RA}} V_{he}$, shown in (i), $V_{hde} >_{\text{RA}} V_{hed}$ follows from the risk aversion ranking and Lemma 1 for the case $t = 0$.

(iii) *Strong primacy effect.* By strong induction. The initial step is true by part (ii). Assume it is true for $\tau \leq t-1$. Note that $V_{hd} >_{\text{RA}} V_h >_{\text{RA}} V_{he}$ and $V_{hde^{\tau}} >_{\text{RA}} V_{hed^{\tau}}$ for all $\tau \leq t-1$ imply $\bigcap_{\tau=0}^{t} (CE_{de^{\tau}}(p), CE_{ed^{\tau}}(p)) \neq \emptyset$ for all $p \in \mathscr{L}^1$. Hence the strong primacy effect for $\tau \leq t$ follows from the risk aversion ranking and Lemma 1.

■

Fix any utility $V : \mathscr{L}^1 \to \mathbb{R}$ over one-stage lotteries. For any $p \in \mathscr{L}^1$, the assignment of prizes to being either elating or disappointing is called an elation disappointment decomposition (EDD). Let $e(p) := \{x \in \text{supp } p \mid V(\delta_x) > V(p)\}$, $n(p) := \{x \in \text{supp } p \mid V(\delta_x) = V(p)\}$ and $d(p) := \{x \in \text{supp } p \mid V(\delta x) < V(p)\}$.

Lemma 2. *Take* $p = \langle p(x_1), x_1; \ldots; p(x_j), x_j; \ldots; p(x_m), x_m \rangle$ *and* $p' = \langle p(x_1'), x_1'; \ldots; p(x_j), x_j; \ldots; p(x_m), x_m \rangle$.

1. *If* $x_1 \notin d(p)$ *and* $x_1' > x_1$ *then* $x_1' \in e(p')$.

2. *If* $x_1 \notin e(p)$ *and* $x_1' < x_1$ *then* $x_1' \in d(p')$.

Proof. We prove statement (1), since the proof of (2) is analogous. If $x_1 \notin d(p)$ then $\delta_{x_1} \succeq p$. Note that p can be written as the convex combination of the lotteries δ_{x_1} (with weight $p(x_1)$) and $p_{-1} = \langle \frac{p(x_2)}{1-p(x_1)}, x_2; \ldots; \frac{p(x_m)}{1-p(x_1)}, x_m \rangle$ (with weight $1-p(x_1)$). By betweenness, this implies that $\delta_{x_1} \succeq p_{-1}$. Since $x_1' > x$, monotonicity implies $\delta_{x_1'} \succ \delta_{x_1} \succeq p_{-1}$, and thus that $\delta_{x_1'} \succ p_{-1}$. But then again by betweenness and the fact that $p(x_1) \in (0,1)$, x_1' must be strictly preferred to the convex combination of $\delta_{x_1'}$ (with weight $p(x_1)$) and p_{-1} (with weight $1-p(x_1)$). But that convex combination is p', meaning that $x_1' \in e(p')$. ■

Lemma 3. *Suppose that for any nondegenerate $p \in \mathscr{L}^1$, $CE_e(p) > CE_d(p)$. Then for any nondegenerate $P \in \mathscr{L}^2$, a consistent decomposition (using only strict elation and disappointment for nondegenerate lotteries in its support) exists.*

Proof. Consider $P = \langle \alpha_1, p_1; \ldots; \alpha_m, p_m \rangle$. Suppose for simplicity that all p_i are nondegenerate (if $p_i = \delta_x$ is degenerate, then $CE_e(p_i) = CE_d(p_i)$, so the algorithm can be run on the nondegenerate sublotteries, with the degenerate ones labeled ex-post according to internal consistency). Without loss of generality, suppose that the indexing in P is such that $p_1 \in \arg\max_{i=1,\ldots,m} CE_e(p_i)$, $p_m \in \arg\min_{i=2,\ldots,m} CE_d(p_i)$, and $CE_e(p_2) \geq CE_e(p_3) \geq \cdots \geq CE_e(p_{m-1})$. A consistent decomposition is constructed by the following algorithm (consistency means that all p_i set as elations (disappointments) have $CE_{e(d)}(p_i)$ weakly larger (strictly smaller) than the certainty equivalent of P calculated by folding back using this assignment). Set $a^1(p_1) = e$ and $a^1(p_j) = d$ for all $i > 1$. Let CE^1 be the certainty equivalent of P when it is folded back under a^1; if CE^1 is consistent with a^1, the algorithm and proof are complete. If not, consider $i = 2$. If $CE_d(p_2) \geq CE^1$, then set $a^2(p_2) = e$ and $a^2(p_i) = a^1(p_i)$ for all $i \neq 2$ (if $CE_d(p_2) < CE^1$, let $a^2(p_i) = a^1(p_i)$ for all i). Let CE^2 be the resulting certainty equivalent of P when it is folded back under a^2. If CE^2 is consistent with a^2, the algorithm and proof are complete. If not, move to $i = 3$, and so on and so forth, so long as $i \leq m-1$. Notice from Lemma 2 that if $CE_d(p_i) \geq CE^{i-1}$, then $CE_e(p_i)) > CE^i$. Moreover, notice that if $CE_e(p_i) > CE^i$, then for any $j < i$, $CE_e(p_j) \geq CE_e(p_i) > CE^i$, so previously switched assignments remain strict elations; also, because $CE^i \geq CE^{i-1}$ for all i, previous disappointments remain disappointments. If the final step of the algorithm reaches $i = m-1$, notice that $CE_d(p_m)$ is the lowest disappointment certainty equivalent, therefore the lowest value among $\{CE_{a^{m-1}(p_j)}(p_j)\}_{j=1,\ldots,m}$. Hence, the final constructed decomposition a^{m-1} is consistent with CE^{m-1}. ■

Proof of Theorem 2. By Lemma 3 and the reinforcement effect, an internally consistent (strict) elation-disappointment decomposition exists for any nondegenerate $P \in \mathscr{L}^2$, using any initial V_h. By induction, suppose that for any $(t-1)$-stage lottery an internally consistent history assignment exists, using any initial V_h. Consider a t-stage nondegenerate lottery $P^t = \langle \alpha_1, P_1^{t-1}; \ldots; \alpha_m, P_m^{t-1} \rangle$. Notice that the algorithm in Lemma 3 for $\mathscr{L}^2$ only uses the fact that $CE_e(p) > CE_d(p)$ for any nondegenerate $p \in \mathscr{L}^1$. But the same algorithm can be used to construct an internally consistent history assignment for P^t if for any $P^{t-1} \in \mathscr{L}^{t-1}$, $CE_e(P^{t-1}) > CE_d(P^{t-1})$. While there may be multiple consistent decompositions of P^{t-1} using each of V_e and V_d, the strong primacy effect ensures this strict inequality regardless of the chosen decomposition. Indeed, starting with V_e, the tree is folded back using higher certainty equivalents sublottery by sublottery, and evaluated using a less risk averse single-stage utility, as compared to starting with the more risk averse V_d. As in Lemma 3, the history for any degenerate sublottery can be assigned ex-post according to what is consistent; its certainty equivalent is not affected by the assignment of e or d. ■

Here we study the possibility of a third assignment of neutrality (n). The set of histories is extended in the obvious manner and an admissible class additionally requires each V_{hn} to be continuous, monotone, satisfy betweenness, and be rankable in terms of risk aversion with respect to any other element of $\mathscr{V}$. For simplicity the characterization of β_{hn} is given for $h = 0$; the generalization is immediate.

Lemma 4. *Suppose there is a nondegenerate r that is neutral in $P = \langle \alpha_1, r; \ldots; \alpha_j, \delta_{x_j}; \ldots \alpha_m, \delta_{x_m} \rangle$ in $\mathscr{L}^2$. For any r', define $P(r') = \langle \alpha_1, r'; \ldots; \alpha_j, \delta_{x_j}; \ldots \alpha_m, \delta_{x_m} \rangle$. There is an open neighborhood N_r of r such that if there is (1) nondegenerate $r' \in N_r$ strictly elating in $P(r')$, then $V_n >_{RA} V_e$; and (2) nondegenerate $r'' \in N_r$ disappointing in $P(r'')$, then $V_d >_{RA} V_n$. Moreover, at least one of (1) or (2) holds for any small enough open neighborhood.*

Proof. Suppose that for some nondegenerate r,

$$P = \langle p_1, r; p_2, \delta_x; p_3, \delta_y \rangle \sim \langle p_1, \delta_{CE_n(r)}; p_2, \delta_x; p_3, \delta_y \rangle,$$

where $V_n(\delta_{CE_n(r)}) = V_n(r)$. Let $0 < \gamma \leq \min_{h \in \{e,d\}} |CE_h(r) - CE_n(r)|$ ($\neq 0$ by the ranking of risk aversion). Pick r^ε such that

$$\max\{|CE_e(r^\varepsilon) - CE_e(r)|, |CE_d(r^\varepsilon) - CE_d(r)|, |CE_n(r^\varepsilon) - CE_n(r)|\} < \frac{\gamma}{6}.$$

Suppose first that $r^\varepsilon >_1 r$. Then it cannot be that $\langle p_1, r^\varepsilon; p_2, \delta_x; p_3, \delta_y \rangle \sim \langle p_1, \delta_{CE_n(r)}; p_2, \delta_x; p_3, \delta_y \rangle$. To see this, first note that the RHS is indifferent to $\langle 1, \delta_{CE_n(r)} \rangle$. If r^ε is neutral then the LHS is indifferent to $\langle 1, \delta_{CE_n(r^\varepsilon)} \rangle$, but indifference then contradicts monotonicity and $CE_n(r^\varepsilon) > CE_n(r)$. So by construction we know that $CE_n(r) \notin \{CE_e(r^\varepsilon), CE_n(r^\varepsilon), CE_d(r^\varepsilon)\}$.

Suppose that r^{ε} is a strict elation. We claim $CE_e(r^{\varepsilon}) > CE_n(r)$. Suppose otherwise. We know

$$\langle p_1, \delta_{CE_n(r)}; p_2, \delta_x; p_3, \delta_y \rangle \sim \delta_{CE_n(r)} \succeq \delta_{CE_e(r^{\varepsilon})} \succ \langle p_1, \delta_{CE_e(r^{\varepsilon})}; p_2, \delta_x; p_3, \delta_y \rangle.$$

But if the prize $CE_e(r^{\varepsilon})$ is elating in the single-stage lottery, and it is improved to $CE_n(r)$, then as shown in Lemma 2, it must remain elating, a contradiction to being neutral. The same argument says that if r^{ε} is a disappointment then $CE_d(r^{\varepsilon}) < CE_n(r)$. Given the choice of r^{ε} in the γ-neighborhood above, this implies the desired conclusion. ■

A.2. Proof of Theorem 3

The proof of necessity is analogous to that of Theorem 1. The proof of sufficiency is analogous to that of Theorem 2, with two additions of note. First, since the reinforcement and strong primacy effects imply the certainty equivalent of each stochastic decision tree in a choice set increases when evaluated as an elation, the certainty equivalent of the choice set (the maximum of those values) also increases when viewed as an elation (relative to being viewed as a disappointment). Second, if the certainty equivalent of a choice set is the same when viewed as an elation and as a disappointment, the best option in both choice sets must be a degenerate continuation. Then its history assignment may be made ex-post according to internal consistency. ■

A.3. Proof of Theorem 4

Step 1: Evident elation or disappointment. For any $\alpha \in (0,1)$, define the sets

$$\begin{aligned}
\mathscr{L}_{d,\alpha} &:= \{\langle \alpha, \delta_b; 1-\alpha, p \rangle \mid p \in \mathscr{L}^1\}, \\
\mathscr{L}_{e,\alpha} &:= \{\langle \alpha, \delta_w; 1-\alpha, p \rangle \mid p \in \mathscr{L}^1\}, \text{ and} \\
\mathscr{L}_0 &:= \{\langle 1, p \rangle \mid p \in \mathscr{L}^1\}
\end{aligned}$$

which consist of all the lotteries of the form in Figure 6 in the main text.

Consider the restriction of $\succeq$ to $\mathscr{L}_{e,\alpha}$. This induces $\succeq_{e,\alpha}$. Note that by Axiom CE and the definition of $CE_{e,\succeq}(\cdot)$, for any p and $\alpha \in (0,1)$,

$$\langle \alpha, \delta_w; 1-\alpha, p \rangle \sim \langle \alpha, \delta_w; 1-\alpha, \delta_{CE_{e,\succeq}(p)} \rangle.$$

Then by definition of $\succeq_{e,\alpha}$, $p \sim_{e,\alpha} \delta_{CE_{e,\succeq}(p)}$. But this means that $CE_{e,\succeq}(p)$ is the certainty equivalent of p under $\succeq_{e,\alpha}$. Since $CE_{e,\succeq}(p)$ is independent of α by Axiom CE, it must be that for each

α and α', $\succeq_{e,\alpha}$ and $\succeq_{e,\alpha'}$ are the same preference, denoted $\succeq_e$ for simplicity. By Axiom A, $\succeq_e$ is a continuous, monotone preference relation satisfying betweenness; it is thus representable by a Chew-Dekel utility $V_e : \mathscr{L}^1 \to \mathbb{R}$.

The case of $\succeq$ restricted to $\mathscr{L}_{d,\alpha}$ is analogous, so that $CE_{d,\succeq}(p)$ is the certainty equivalent of p according to a Chew-Dekel utility $V_d : \mathscr{L}^1 \to \mathbb{R}$. Similarly for $\succeq_0$, it is represented by a Chew-Dekel utility $V_0 : \mathscr{L}^1 \to \mathbb{R}$. By Axiom A, V_e, V_d, and V_0 are rankable in terms of risk aversion.

Step 2: Endogenous neutrality, elation or disappointment. Consider any nondegenerate $P = \langle \alpha_1, p_1; \cdots; \alpha_m, p_m \rangle$. By Axiom CAT, for every $j = 1, 2, \ldots, m$, p_j is elating or disappointing in P. Beginning with $j = 1$, this implies that

$$P \sim P^{(1)} = \langle \alpha_1, \delta_{CE_{a(1),\succeq}(p_1)}; \alpha_2, p_2 \cdots; \alpha_m, p_m \rangle \text{ for some } a(1) \in \{e, d\}.$$

Now, notice by Axiom CAT that p_2 is elating or disappointing in $P^{(1)}$. Hence

$$P \sim P^{(1)} \sim P^{(2)} = \langle \alpha_1, \delta_{CE_{a(1),\succeq}(p_1)}; \alpha_2, \delta_{CE_{a(2),\succeq}(p_2)} \cdots; \alpha_m, p_m \rangle \text{ for some } a(2) \in \{e, d\}.$$

By repeatedly applying categorization in this manner,

$$P \sim P^{(1)} \sim P^{(2)} \sim \cdots \sim P^{(m-1)} \sim P^{(m)} = \langle \alpha_1, \delta_{CE_{a(1),\succeq}(p_1)}; \alpha_2, \delta_{CE_{a(2),\succeq}(p_2)} \cdots; \alpha_m, \delta_{CE_{a(m),\succeq}(p_m)} \rangle,$$

where each $a(j) \in \{e, d\}$. Moreover, by Axiom CAT and use of transitivity, if $a(j) = e$ then $\delta_{CE_{e,\succeq}(p_j)} \succeq P^{(m)}$, and if $a(j) = d$ then $P^{(m)} \succ \delta_{CE_{d,\succeq}(p_j)}$.[17] By Axiom TN, V_0 also represents $\succeq$ restricted to lotteries where resolution of risk occurs in the first stage, such as in a folded back lottery. Hence the utility of P may be given by $V_0(\langle \alpha_1, \delta_{CE_{a(1),\succeq}(p_1)}; \alpha_2, \delta_{CE_{a(2),\succeq}(p_2)} \cdots; \alpha_m, \delta_{CE_{a(m),\succeq}(p_m)} \rangle)$.
■

[17]Notice that the construction of $P^{(m)}$ may have been path-dependent (potentially more than one of the relations holds). But for any path of construction, either (i) $P^{(m)}$ has at least one $CE_{d,\succeq}$ and one $CE_{e,\succeq}$ or (ii) $P^{(m)}$ consists entirely of $CE_{e,\succeq}$'s, all of which are indifferent to $P^{(m)}$.

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