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"The Dynamics of Bargaining Postures: The Role of a Third Party"

by

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The Dynamics of Bargaining Postures: The Role of a Third Party*

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Abstract

In many real world negotiations, from wage contract bargaining to product liability disputes, the bargaining parties often interact repeatedly and have the option of seeking outside judgement. This paper studies a model of repeated bargaining with a third party to analyze how and why bargaining postures endogenously evolve over time. A privately informed long-lived player bargains with a sequence of short-lived players, one at a time. Should the players fail to reach an agreement, an unbiased yet imperfect third party is called upon to make a judgement. The uninformed short-lived players learn through two channels: observed behavior of the informed player ("soft" information) and, if any, verdicts of the third party ("hard" information). The long-lived player wants to guard his private information by bargaining tough but at the expense of more information disclosure from the third party. As a result of the strategic use of these two sources of information, the players' bargaining postures change as the uninformed players' beliefs evolve. Interestingly, as third party information becomes more precise, the players adopt tough bargaining postures for a wider range of beliefs. Many repeated bargaining problems can be analyzed in this framework. In particular, the equilibrium dynamics provide an explanation for the puzzling contrast between the bargaining postures of Merck and Pfizer in their recent high-profile product liability litigations. The results also help us understand several other phenomena documented in the related literature.

1 Introduction

1.1 Overview

Recently, Merck and Pfizer, two of the largest pharmaceutical manufacturers, have been involved in a series of high-profile litigations surrounding their painkillers.[1] Despite the close similarity of their cases, the two firms have adopted contrasting approaches to the disputes. Merck contested every case in court. After losing the first case with a compensation verdict of $253 million in 2005, it continued to fight and won most of the cases that reached juries over the following two years. Subsequently, in 2007, the firm agreed to settle further 27,000 cases out of court for $4.85 billion in total, an amount far smaller than analysts and lawyers predicted at the time of the drug's withdrawal and, especially, after Merck's first defeat in court.[2] In sharp contrast, Pfizer sought to settle its disputes before taking any of them to court.[3] How do we reconcile these differences in bargaining postures?

Product liability litigations such as the above drugs cases in fact represent one of many important applications of bargaining and dispute resolutions that share several distinguishing characteristics. First, many negotiations feature repeated interactions between a large player and a pool of small players. For example, a landlord often contests with tenants over the amount of deposit to be returned and a firm routinely faces wage disputes with its employees. Second, and more importantly, these disputes are rarely resolved by the two sides alone. Amid the deadlock of a wage dispute, the bargaining parties often turn to a third party, such as an arbitrator or even a court. When traders disagree on the quality of goods or the terms of a deal, they hire an expert to make an assessment on their behalf. Even when transactions are conducted smoothly without outside intervention, the third party is usually in the shadow of the interaction.

In this paper, we develop a model of repeated bargaining with a third party, or an "expert". A long-lived player (e.g. firm) is in dispute with a sequence of short-lived players (e.g. customers, employees). The long-lived player has private information regarding his responsibility towards a transfer (e.g. damage compensation, wage increase) to each short-lived player. He is either "good" or "bad", with the bad type being more likely to be

[1]The drugs in dispute are Vioxx for Merck and Bextra and Celebrex for Pfizer. They all belong to the same class of painkillers known as COX-2 inhibitors.

[2]Source: New York Times, http://www.nytimes.com/2007/11/09/business/09merck.html

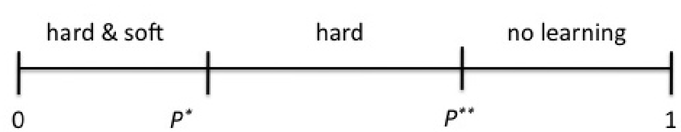
[3]Source: Wall Street Journal (May 2, 2008).

responsible. For instance, a firm privately knows whether or not its profits are high enough to warrant extra wages and a drug company has better knowledge about its own product's link to the user's health problem. In each period, a new short-lived player enters the game with a claim and makes a demand. If his demand is accepted, the short-lived player receives the corresponding amount from the long-lived player and leaves the game. But, if the long-lived player rejects the demand, an expert is called upon to make a decision on their behalf, at a cost to each party. An expert verdict is publicly observable, and so are the details of an agreement.

A critical aspect of this model is that expert verdicts are informative, unbiased but nonetheless imperfect. The "quality" of an expert is measured by the parameter $q \in (\frac{1}{2}, 1)$. With probability q, he correctly rules a responsible (or non-responsible) long-lived player to be indeed responsible (or not responsible). The expert is informative since $q > \frac{1}{2}$, imperfect since $q < 1$ and unbiased since, independently of the true state, he makes a mistake with probability $1 - q$ and the quality q is fixed over time. For example, it could be that an expert is drawn independently in each period from a pool of experts.

Learning of the uninformed short-lived players arises from two sources: the informed long-run player's equilibrium actions and the decisions made by the expert, if any. We interpret the former as "soft" information and the latter as "hard" information. The interplay between these two sources of information is the key innovative feature of our model that generates new, interesting equilibrium dynamics of bargaining postures and enables us to analyze several empirical observations. We interpret the public belief about the long-lived player being the good type as his "reputation". It turns out that the players' bargaining postures, as well as the interplay between the two sources of information, are characterized by two threshold levels of reputation, as illustrated by Figure 1 below:

Figure 1: Two thresholds



When reputation is above the upper threshold p^{**}, the short-lived player makes a low demand and both types of the long-lived player accept it for sure. Here, the short-lived player takes a weak bargaining posture, while the long-lived player takes a tough bargaining posture by never accepting any higher demand. This is where the full benefits of successful reputation building are reaped by the bad long-lived player. There is no learning on the part of the short-lived players in this region either from soft or hard information, because the long-lived player plays a pooling strategy and an agreement is reached without the expert.

When reputation is intermediate, the short-lived players makes a large demand that the good type cannot accept since this type expects a lower transfer from expert verdict. The bad type mimics the good type and also rejects the demand for sure, with the prospect of a high continuation payoff (or a low expected transfer) in the neighboring high reputation region. Rejection leads to an arrival of hard information which, in expectation, will reduce the bad type's reputation. Here, learning occurs but only through hard information since the long-lived player's pooling behavior conveys no soft information. The two parties adopt incompatible tough bargaining postures and, hence, an agreement cannot be reached without the third party.

When reputation is below the lower threshold p^*, the bad long-lived player faces a slim prospect of a high continuation payoff (or a low expected transfer) as the upper threshold is far away, that is, many pieces of good luck (favorable expert verdicts) are needed to reach the region where only low demands are made. Thus, he is willing to accommodate the tough bargaining posture of the short-lived player. In equilibrium, the bad long-lived player builds reputation by randomizing between tough and weak bargaining postures and, hence, hard information arrives only occasionally. Here, the impact of hard information is reduced in the presence of soft information and, in fact, soft information can sometimes revert the adverse effects of hard information. We fully characterize an open interval within this low reputation region where a brave rejection enhances reputation even after an unfavorable expert verdict.

Our results provide an explanation of the contrasting bargaining postures adopted by the two aforementioned pharmaceutical companies. Merck may have suffered a damage to its reputation by losing the first case in court, but winning many other cases would eventually take it to the high reputation region, where cases are settled for a low amount. Nonetheless, the *ex ante* benefit from building reputation from a low level is very small, and it is costly to repeatedly resort to the court. This may well have been the reason behind Pfizer's decision.

We also examine the effects of the expert quality q and the discount factor on the players' bargaining postures. It is shown that, as q goes to 1, the low reputation region is completely squeezed out by the intermediate region, while the high reputation region shrinks (yet remain present). Since the players resort to the expert with probability 1 in the intermediate region, our result may sound counterintuitive: why is a bad long-lived player more willing to go to expert when the expert will find him out almost surely? The reason lies in the conflict of interests between the forward-looking and myopic parties. When the expert is very precise, the short-lived player's expected payoff from expert verdict increases, and this makes the demand too high for the long-lived player to tolerate. The long-lived player is willing to take even a small chance of expert error; after all, a single mistake will greatly enhance his reputation when hard information is very precise. We show that, as the long-lived player becomes extremely patient, the low reputation region disappears and the high reputation region remains unchanged.

We characterize the exact payoff gain from reputation in all Markov equilibria. For low prior beliefs, the *ex ante* benefit is small, in contrast to the result of standard reputation games (e.g. Fudenberg and Levine [8]). It is shown that all equilibria are characterized by two threshold levels of reputation which determine the payoff bounds. Furthermore, in any equilibrium, there are two constant levels of demand that could be accepted. Interestingly, this fact is consistent with observations documented in the legal literature.

1.2 Contributions

Theory of repeated bargaining This paper develops a repeated bargaining model that explains how and why bargaining postures change over time. Related works on repeated bargaining include Schmidt [17] and Hart and Tirole [10]. To focus on the repeated interaction, we follow the literature and consider a simple bargaining protocol within each transaction. What distinguishes our paper from other repeated bargaining models is the presence of a third party. This feature is important for many applications. Real life transactions seldom involve just a pair of bargainers; the "background" of the bargaining matters. Experts, institutions and other third parties often influence the outcome of a transaction indirectly or directly. Note that the involvement of a third party is optional to the players in our model. The arrival of hard information is endogenous.[4] As we demonstrate in this paper, this realistic feature leads to distinct strategic implications and delievers new insights about bargaining postures that cannot be captured by the standard repeated bargaining models.

Theory of reputation Our results enrich the adverse selection theory of reputation initiated by Kreps, Milgrom, Roberts and Wilson [13] and later developed by, among many others, Fudenberg and Levine [8]. As Mailath and Samuelson [14] point out, in standard reputation models, no players actually *build* reputation in equilibrium; the privately informed player starts pooling with another type from the very beginning of the game and so "reputation springs to life". Furthermore, even though reputation can increase the equilibrium payoff, reputation can always be built.

In many applications, these features are not completely realistic. By introducing the interplay between soft and hard information, we show non-degenerate equilibrium dynamics in which reputation can be built and maintained but not always. In our equilibrium, the bad type attempts to gradually build up his reputation when it is low, but he can successfully do so only with a probability strictly less than 1. Reputation may move up or down and also, with a strictly positive probability, the bad type will reveal himself and hence fail to build reputation. It is worth noting that the bad type reveals himself only when he voluntarily gives up reputation building; hard information from the expert, due to its imperfectness, can never lead to full revelation.

Another related work is by Bar-Isaac [3] who considers a repeated signaling model in which the quality of a seller is imperfectly revealed in each period should he decide to produce.[5] In his model, the buyers purchase the seller's product at a price equal to the seller's expected quality and, thus, a buyer's response is a continuous and monotone function of the seller's reputation level. In contrast, the short-lived players in our model are strategic as in the other reputation models mentioned above. Indeed, we show that the short-lived players essentially make one of just two demands in any Markov equilibrium, even though they are allowed to choose any distribution over the real line. This prediction is actually important since it clarifies some puzzling observations about the demand distribution documented in

[4] Deterministic arrival of hard information has been studied in education signaling models (e.g. Kremer and Skryzpacz [12]).

[5] See also Bar-Isaac and Tadelis [4] for a comprehensive survey of economic models on signaling and reputation.

the litigation literature (see Section 5). We also note that the equilibrium value function in our model displays discontinuity, as similarly encountered by Mailath and Samuelson [14]. Our equilibrium construction, however, is entirely different from theirs.

Repeated settlement and litigation An important application of our analysis is found in repeated litigations with long-lived defendants. The economic significance of such disputes are often strikingly large. For instance, in the year of 2007 alone, the US experienced 177 federal securities fraud claims, where auditors and underwriters often face lawsuits repeatedly, with a total disclosure loss of $151 billion.[6] Also, in the US, the claimed damages involved in medical malpractice litigations totaled $28.7 billion in 2004 and, around the same period, the corresponding amount was about €2.4 billion in Italy.[7]

Our repeated bargaining model enables us to piece together several empirical observations identified in the aforementioned product liability litigations, regarding the long-run relationships between the strength of a case and settlement outcome (Alexander [2]) and between the strength of a case and trial rate (Palmrose [16]). We shall discuss our contribution to this literature in closer detail in Section 5.

1.3 Plan

The rest of the paper is organized as follows. The next section describes a model of repeated bargaining with a third party. In Section 3, we construct an equilibrium of the game and, also, conduct comparative static analysis. Section 4 then presents general characterization results. Finally, we offer some concluding remarks in Section 5. All technical proofs are relegated to Appendix.

2 The model

2.1 Description

We consider a discrete time model. Periods are indexed by $t = 1, 2, \ldots$. A single long-lived player 1 faces an infinite sequence of short-lived players 2, with a new player 2 entering in every period. Each player 2 brings a claim to player 1.

[6] Source: Stanford law school securities class action clearinghouse, http://securities.stanford.edu.
[7] Source: OECD [15].

Player 1 privately knows his type $\theta \in \{G, B\}$, where G stands for good and B for bad. Type B is responsible for each claim, while type G is not. The assumption is valid in applications, such as the aforementioned Merck/Pfizer cases, where the long-lived player faces repeated disputes all related to some foregone act.[8]

The stake involved in each claim, denoted by $H > 0$, is fixed and commonly known. Alternatively, H could be the expectation of a fixed distribution of uncertain stakes. Each player 1-player 2 pair attempt to settle their dispute via voluntary bargaining. Should they fail to reach an agreement, they call upon an external third party, an expert, an arbitrator or a court, to determine whether player 1 is responsible or not. Both players are committed to obey the third party's suggestion: player 1, if judged to be responsible, should pay H to player 2, and player 2 should receive no transfer otherwise. Seeking a third party incurs a cost $c_i > 0$ to player i, regardless of the verdict. We shall henceforth refer to the third party as an "expert".

The expert is informative, unbiased but imperfect: independently of the true type of player 1, he makes an error with probability $1 - q$, where $q \in \left(\frac{1}{2}, 1\right)$ is common knowledge. Specifically, when player 1 is responsible (or not responsible), the expert will incorrectly rule that the player is not responsible (or responsible) and hence owes nothing (or H) to player 2 with probability $1 - q$. We shall interpret q as the "quality" of the expert. Furthermore, we shall assume that the quality of expert judgement is independent of history and, hence, the expert is non-strategic.[9]

The timing of the stage game in period t is as follows. Player 2 makes a take-it-or-leave-it demand $s_t \in \mathbb{R}$, which player 1 can either accept or reject. If s_t is accepted, then player 1 transfers s_t to player 2; if the demand is rejected, an expert is called upon to make a judgement. At the end of a period, player 2 leaves the game forever.

Note that if player 1 is of type B his expected transfer to player 2 under expert resolution is equal to qH; if he is of type G the corresponding amount is $(1 - q)H$. To focus on interesting cases, it is assumed throughout that $c_1 + c_2 < qH - (1 - q)H = (2q - 1)H$.

An expert verdict is publicly observable, and so are the details of an agreement.[10] The

[8] See Section 5 for further discussion on this assumption.

[9] See Section 5 for further discussion on this last assumption.

[10] As discussed in Section 5, our results are robust to the possibility of (endogenous) confidentiality agreements. We also note that our assumption is consistent with many cases of actual settlement bargaining as, for example, in the securities/auditor cases studied by Alexander [2] and Palmrose [16]. The details of any negotiation process (such as the value of rejected demands) are usually private information known

first player 2 holds a prior belief, $p_1 \in (0,1)$, that player 1 is good. Later short-lived players update their beliefs from this prior and the public history that they observe. Let $p_t \in [0,1]$ denote player 2's posterior belief that player 1 is good in period t. This will be sometimes referred to as player 1's "reputation."

2.2 Strategies, payoffs and equilibrium notion

A (behavioral) strategy of player 1 is a mapping from the set of all possible histories that he can observe at the beginning of each period and the set of all possible demands from player 2 to probability distributions over the set $\{A, R\}$, where A and R denote acceptance and rejection, respectively.

A (behavioral) strategy of player 2 in period t is a mapping from the set of all possible histories that he can observe over preceding $t-1$ periods to probability distributions over all possible demands, $\mathbb{R}$.

We focus on perfect Bayesian equilibria in Markov strategies in which any relevant past history can be summarized by the level of belief that it induces. A Markov strategy for type θ player 1, r^θ, is

$$r^\theta : [0,1] \times \mathbb{R} \to [0,1]$$

such that $r^\theta(p,s)$ is the probability with which type θ rejects the demand $s \in \mathbb{R}$ at belief $p \in [0,1]$.

The Markovian property renders irrelevant the period in which player 2 makes entry and, hence, we shall write a Markov strategy for player 2, d, simply as

$$d : [0,1] \to \triangle(\mathbb{R})$$

such that $d(p) \in \triangle(\mathbb{R})$ for any $p \in [0,1]$.

If (r^B, r^G, d) is a Makrov strategy profile, we write type θ's discounted average expected payment at belief p as $V^\theta(p)$ with discount factor $\delta \in (0,1)$. This involves player 1's transfers to player 2s as well as expert costs. Note that we have already surpressed the dependence of V^θ on the strategy profile and the discount factor. Following the reputation literature, our focus below will be on interesting equilibrium behavior of the bad type. Thus, when the

only to the negotiating parties. But, once a deal is struck, the terms of the deal often enter the public domain.

meaning is clear, we shall refer to $V^B(p)$ simply as $V(p)$. Player 2 maximizes his expected stage game payoff while player 1 minimizes his (discounted average) expected payment.

A strategy profile (r^B, r^G, d), together with a system of beliefs, forms a Makrov perfect Bayesian equilibrium if the usual conditions are satisfied. See, for instance, Fudenberg and Tirole [9] for a formal definition. We will invoke a natural restriction of beliefs: when the type is revealed, the game proceeds as if it has complete information. Note also that the Markov property of the complete information game implies that the bad type, because his type is known, will accept a demand equal to the best that he could expect from expert verdict, that is, $qH + c_1$.[11]

3 Equilibrium analysis

3.1 Equilibrium behavior of the good type

If a dispute goes to the expert, good player 1, in expectation, incurs total payment $(1 - q)H + c_1$. It is then natural that this player 1 should not agree to pay anything above this amount from bargaining with player 2. We shall assume throughout that, regardless of past history, the good type accepts a demand if and only if it does not exceed $(1 - q)H + c_1$, and any observation of deviation from this behavior reveals the bad type. Other similar strategies with different cutoff levels can also constitute an equilibrium but, as shall be clear from our analysis and the equilibrium construction, they do not alter our findings.

The cutoff behavior of the good type appears similar to the irrational type's behavior in the bargaining model of Abreu and Gul [1]. However, it is important to note that, in our model, this behavior emerges as part of an equilibrium rather than as an assumption often imposed in standard reputation models. Whenever we henceforth refer to player 1 without mentioning his type, we shall mean the bad type.

3.2 First intuition

Let us first spell out some intuition. On the one hand, if player 2's posterior belief (on the good type) is high, his expected payoff from resolving the dispute via the expert is low and, moreover, he has to pay a cost to obtain a verdict. Thus, when the belief is sufficiently

[11] We would have a folk theorem type result if the Markov property is not imposed.

high, player 2 should make a low demand that will be accepted by both types of player 1 and the dispute is resolved without expert intervention.

If the posterior belief is low, on the other hand, player 2 expects to win a large transfer if the case goes to the expert. The corresponding expectation of good player 1 is still low and, therefore, player 2 should make a large demand that the good type will not tolerate. How should the bad type respond?

If the bad type accepts this demand, he reveals his type and consequently his future transfers will be high. He cannot therefore accept it with probability 1; otherwise, the equilibrium belief following rejection must be 1, and the bad type would mimic the good type by rejecting the demand. The bad type should also be reluctant to reject the large demand for sure. An expert verdict is imperfect but nonetheless informative ($q > \frac{1}{2}$). Thus, it will hurt his reputation on average. Moreover, from a very low reputation level, the bad type needs many pieces of good luck (favorable expert verdicts) in order to reach a level of reputation high enough that player 2 begins to make low demands.

This suggests that, when his reputation is very low, the bad type should play a mixed strategy: he rejects the high demand with an interior probability. The role of randomization here is to mitigate the effect of a non-favorable expert verdict. Since the good type rejects the demand for sure and the bad type rejects it only occasionally, the act of rejection will itself enhance player 1's reputation and may even overturn the effect of a non-favorable verdict.

Nonetheless, when reputation is sufficiently close to the point beyond which player 2 finds optimal to make a low demand, the bad type may still wish to fully mimic the good type, reject the high demand with probability 1 and count on the chance that expert verdict favors him. If he is lucky, his reputation will enter the region in which player 2 makes only a small demand.

These arguments suggest that the equilibrium can be characterized by two threshold beliefs that quantify the "low" and "high" reputation regions. This is indeed the case.

3.3 Formal description

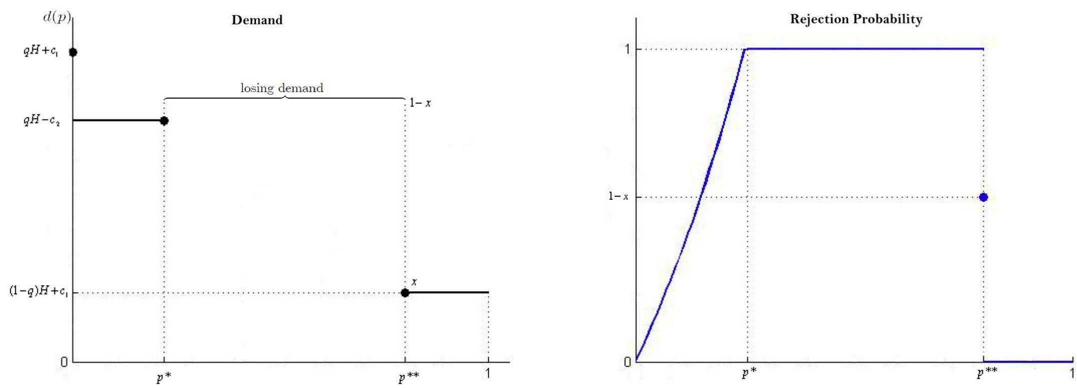
We now formally describe the equilibrium characterized by two threshold beliefs, $0 < p^* < p^{**} < 1$, confirming our previous intuition.[12] As is usual in a bargaining game, player 2 can

[12] In Section 4 below, we characterize the key properties of all Markov perfect Bayesian equilibria.

make a demand which he knows will be rejected for sure; let us refer to such a demand as a *losing demand*.

Figure 2 below illustrates the equilibrium strategies of the two players around three belief "regions". The left panel describes player 2's demand as a function of the belief; the right panel illustrates bad player 1's rejection probability of the equilibrium demand at each belief level.

Figure 2: Equilibrium strategies



The low reputation region, $(0, p^*)$. This is a region of learning through both soft and hard information. Player 2 makes a high demand, equal to $qH - c_2$, which the good type will reject for sure.[13] The bad type responds to such a demand by randomization; the rejection probability is monotonically increasing in p over this region such that, at the lower threshold p^*, it becomes exactly 1. The act of rejection itself leads to reputation building, and the subsequent expert signal will also lead to learning from player 2. A favorable verdict enhances reputation further, while a non-favorable verdict brings reputation back down.

The intermediate reputation region, (p^*, p^{**}). This is a region of learning through hard information alone. Here, player 2 makes a losing demand which both types reject with probability 1. Player 2 does not learn from player 1's act of rejection *per se*; rather, the learning takes place only through the realization of expert verdict.

[13]The amount of the high demand, $qH - c_2$, at low reputation levels turns out to be a general equilibrium property. See Section 4.

The high reputation region, $(p^{**}, 1)$. This is a region of no learning. The full benefit of reputation is obtained. Player 2 makes a low demand, equal to $(1-q)H + c_1$, that both types accept for sure.

The behavior at the two thresholds p^* and p^{**} are critical in the equilibrium construction. In particular, at p^{**}, player 2 mixes between $(1-q)H + c_1$ and a higher losing demand; both types of player 1 accept (reject) the low (high) demand for sure. Of course, in order to display reputational considerations, player 1 has to sufficiently care about the future. Let $\bar{\delta} = \frac{c_1+c_2}{(2q-1)H+c_1+c_2}$. We next state the equilibrium formally.

Proposition 1 *For any $\delta > \bar{\delta}$, the following is the outcome of a Markov perfect Bayesian equilibrium. There exist two thresholds, $0 < p^* < p^{**} < 1$, such that:*

- *If $p = 0$, player 2 demands $qH + c_1$ with probability 1; player 1 (the bad type) accepts it with probability 1.*

- *If $p \in (0, p^*]$, player 2 demands $qH - c_2$ with probability 1; player 1 rejects it with probability $r(p)$, where*
$$r(p) = \frac{p}{p^*}\frac{1-p^*}{1-p} \leq 1.$$

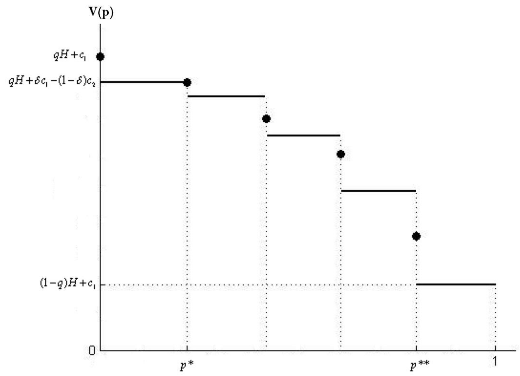
- *If $p \in (p^*, p^{**})$, player 2 makes a losing demand; player 1 rejects it with probability 1.*

- *If $p = p^{**}$, player 2 demands $(1-q)H + c_1$ with probability x and makes a higher losing demand with probability $1-x$ for some $x \in [0, 1)$; player 1 accepts $(1-q)H + c_1$ with probability 1 and rejects the other demand with probability 1.*

- *If $p \in (p^{**}, 1]$, player 2 demands $(1-q)H + c_1$ with probability 1; player 1 accepts it with probability 1.*

Figure 3 below illustrates bad player 1's equilibrium (discounted average) expected payment as a function of the belief. Indeed, the expected payment is decreasing in reputation; however, it is a discontinuous step function with a finite number of jumps. The key element of the equilibrium construction lies in devising continuation payments that provide correct incentives. Before providing the details of construction, let us first consider the distinguishing features of reputation in our model.

Figure 3: Equilibrium payments



The equilibrium displays some interesting features beyond the threshold dynamics. First, if the game starts with a prior belief in the low reputation region, player 1's equilibrium payment converges to $V(0)$ (the payment under complete information) as the discount factor goes to 1. *The gain from reputation building is small.* Second, starting from any interior prior, the posterior reaches the high reputation region $(p^{**}, 1)$ and then remains there forever with an interior probability. *Reputation can be built.* Third, player 1 will also fail to build reputation with a positive probability; this happens in the low reputation region $(0, p^*)$ where player 1 randomizes and reveals his type occasionally. *Reputation can be lost.* Finally, in the low reputation region where both soft and hard information are present, *soft information can overturn hard information* when their forces pull in opposite directions. In particular, when p is low enough relative to p^*, even after a non-favorable expert verdict the subsequent posterior at the beginning of the next period will be higher than the current period's initial level. Our next Proposition summarizes these findings formally.

Proposition 2 (Equilibrium properties)

- *"The gain from reputation building is small."*

  Suppose that $p_1 \in (0, p^*)$*. Then, the reputation gain is* $V(0) - V(p_1) = (1-\delta)(c_1 + c_2)$*, where* $V(p_1) = qH + \delta c_1 - (1-\delta)c_2$ *and* $V(0) = qH + c_1$.

- *"Reputation can be built."*

Suppose that $p_1 \in (0, p^{**}]$. Then, the probability with which the equilibrium posterior reaches the region $(p^{**}, 1)$ is positive.

- *"Reputation can be lost."*

  Suppose that $p_1 \in (0, p^{**}]$. Then, the probability with which the equilibrium posterior falls to 0 is positive.

- *"Soft information can overturn hard information."*

  Suppose that $p_t \in \left(0, \frac{p^*(1-q)}{p^*(1-q)+(1-p^*)q}\right)$. Suppose also that, in this period t, player 1 rejects player 2's demand and the subsequent expert verdict is non-favorable. Then, we have

$$p_{t+1} = \frac{p^*(1-q)}{p^*(1-q) + (1-p^*)q} > p_t.$$

We next discuss how the equilibrium responds to shifts in some key parameters. Of particular interest is how the thresholds change in response to increased patience and expert quality. We report limit results for technical reasons.

Proposition 3 (Comparative statics)

- *As δ goes to 1, p^* goes to 0; p^{**} is independent of δ.*
- *As q goes to 1, p^* goes to 0; p^{**} goes to $\frac{H-c_1-c_2}{H}$.*

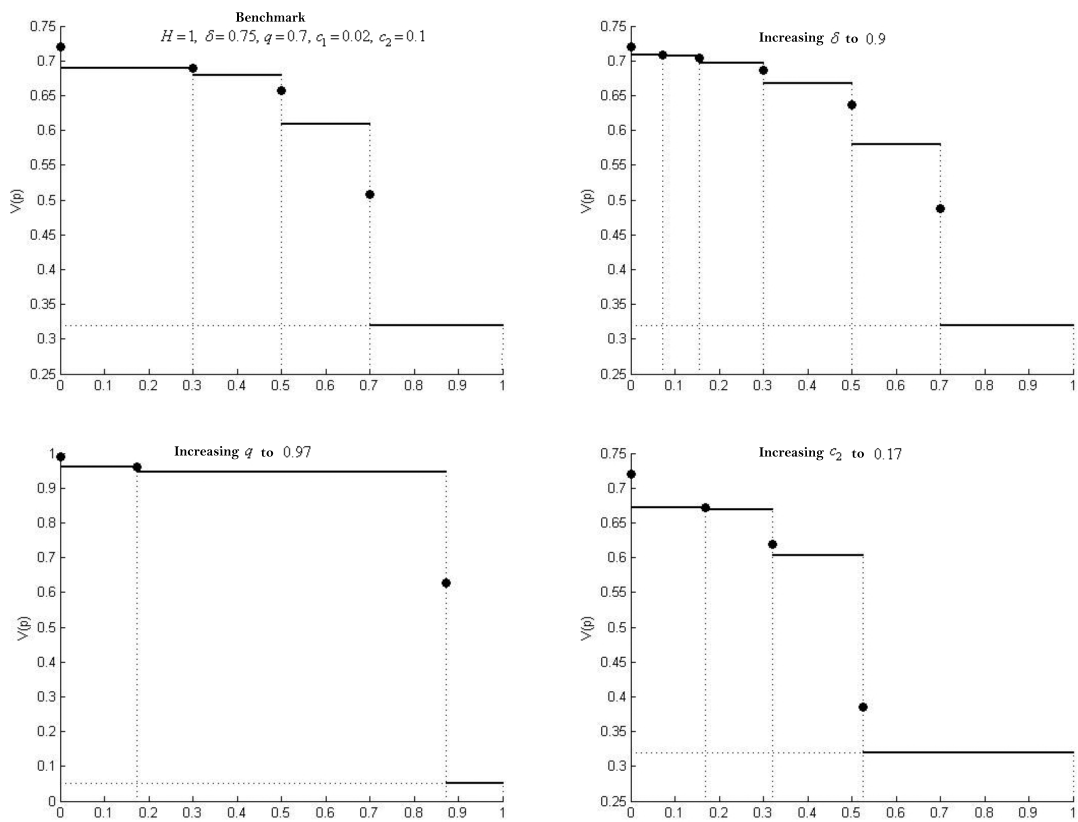
The impact of increased patience falls only on the lower threshold, p^*, which decreases. Thus, it expands the region in which player 1 fully mimics the good type and rejects the equilibrium demands for sure, thereby relying solely on expert verdicts. Although expert resolution, on average, worsens reputation, a more patient long-lived player is willing to try his luck *earlier*, in an effort to move into the no-learning region above the upper threshold, p^{**}, where he incurs only a small amount of transfer.

As the expert quality increases, the intermediate reputation region also expands. But here, this effect is achieved by a reduced lower threshold *and* an increased upper threshold, p^{**} (whose corresponding limit is less than 1). This first implies that the no-learning region shrinks, and we may interpret this as suggesting that reputation is indeed more difficult to build when the expert is more accurate.

While this last observation is intuitive, the fact that the intermediate region expands as q goes to 1 is somewhat surprising. This says that, when the expert is very precise, the parties will almost always resort to external intervention, rather than making voluntary agreements and saving on expert costs. Why is this? The reason is that when q is very large a single piece of good luck is all that is needed for player 1 to jump into the no-learning region and reap the full benefits of reputation. Given this, what player 2 asks for at low levels of reputation is too much for player 1 to accept.

Figure 4 illustrates the comparative static results in terms of payments with specific parameter values. The top left-hand side graph here represents the benchmark case when $H = 1$, $\delta = 0.75$, $q = 0.7$, $c_1 = 0.02$ and $c_2 = 0.1$. The next three graphs, going from left to right in each row, demonstrate how the payments change after an increase in δ, q and, also, c_2, respectively.

Figure 4: Comparative statics



An increase in δ from 0.75 to 0.9 indeed expands the intermediate region by inducing more "steps"; the high reputation region and the corresponding payments remain the same but the lower threshold falls and the payment at the low reputation region is pushed up.

Raising the expert precision from 0.7 to 0.97 shows a more drastic change. The intermediate region is vastly expanded but it involves only one step. Both thresholds move, in opposite directions. It is more difficult to build reputation and reach the high reputation region; moreover, the payments during the reputation building process are also higher than the benchmark. However, should player 1 succeed in reaching beyond the (increased) upper threshold, the benefits will actually be greater (lower payments).

The final, bottom right-hand side, graph illustrates the effect of an increase in c_2 (from 0.1 to 0.17), the expert cost incurred by player 2. Here, at any p, player 1's payment is lower, or the same, compared to the benchmark. Thus, making the expert more costly to player 2 may improve the benefits of player 1's reputation building.

3.4 Details of construction

We now demonstrate the technique behind the equilibrium construction which we believe to be innovative and interesting in its own right. The key is to install correct incentives through continuation payments. It turns out that the right continuation payments take the form illustrated in Figure 3 above, and finding such values requires a recursive process. We will describe this process step by step.

Step 1 At $p = 0$, it is clearly mutually optimal for player 2 to demand $qH + c_1$ and player 1 to accept it. Once the posterior falls to 0, it remains at this level.

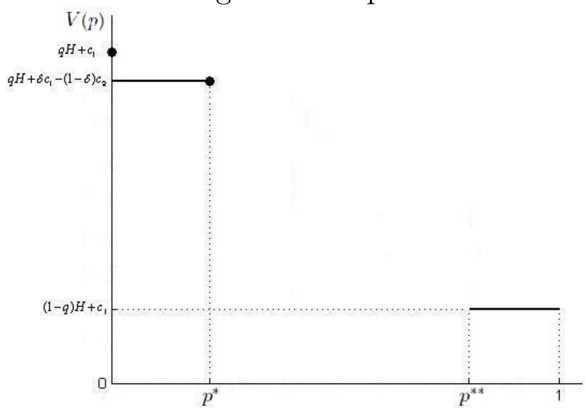
If p is sufficiently high, that is, at $p > p^{**}$ (we later define p^{**}), given the good type's behavior, it is mutually optimal for player 2 to demand $(1-q)H + c_1$ and (bad) player 1 to accept it.

If p is sufficiently low, that is, at $p \in (0, p^*]$ (we later define p^*), the proposed equilibrium strategies prescribe that player 2 demands $qH - c_2$ and player 1 is indifferent between rejecting and accepting it. Player 1's expected payment is then given by what he obtains from accepting and revealing his type. If player 1's type is revealed, the demand will be $qH + c_1$ in every period thereafter (and he is going to accept it) and, therefore, we have, for every $p \in (0, p^*]$,

$$V(p) = (1-\delta)(qH - c_2) + \delta(qH + c_1) = qH + \delta c_1 - (1-\delta)c_2. \tag{1}$$

In the next step, we shall construct continuation payments to support (1) as equilibrium payments and make player 1 indifferent. But first, let us summarize the continuation payment for $p \in [0, p^*] \cup (p^{**}, 1]$ in the following illustration.

Figure 5: Step 1



Step 2 We have to make player 1 indifferent between accepting and rejecting the demand $qH - c_2$ at $p \in (0, p^*]$. Let the rejection probability be such that right after the rejection, but before the expert verdict, the posterior belief is exactly p^* (therefore, at p^*, the rejection probability is 1).

What is the continuation payment from rejecting? In the current period, player 1 expects to spend $qH + c_1$ from going to the expert. As of the next period, the continuation payment depends on the outcome of expert verdict. If he obtains an unfavorable verdict (which happens with probability q), the posterior falls below p^* but then the continuation payment is given by equation (1) in Step 1 above. If he obtains a favorable verdict, the posterior improves to, say, p^1 with continuation payment $V(p^1)$.[14]

The indifference condition of player 1 at $p \in (0, p^*]$ then requires the following Bellman equation

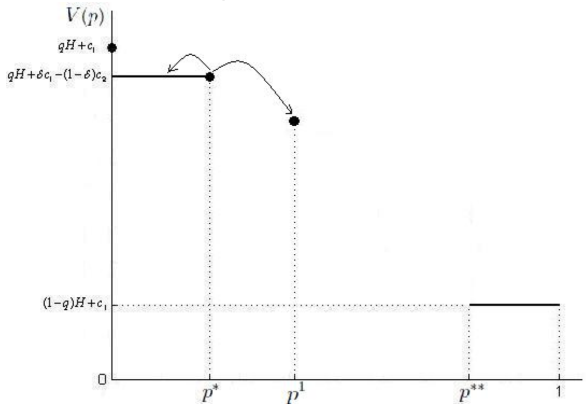
$$V(p) = (1-\delta)(qH + c_1) + \delta q \left[qH + \delta c_1 - (1-\delta)c_2\right] + \delta(1-q)V(p^1). \tag{2}$$

Equations (1) and (2) pin down $V(p^1)$.[15] Figure 6 below illustrates these arguments.

[14] By Bayesian updating, $p^1 = \frac{p^* q}{p^* q + (1-p^*)(1-q)}$.

[15] It is easy to check that $V(p^1) > (1-q)H + c_1$ when $\delta > \bar{\delta}$.

Figure 6: Step 2

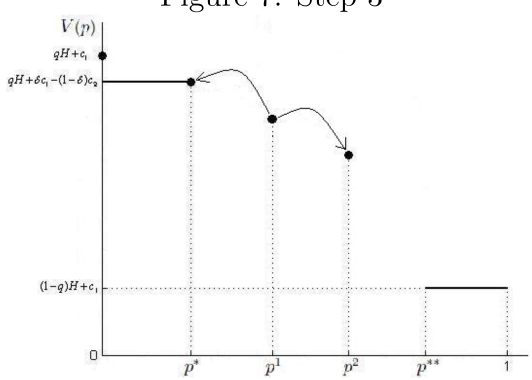


Step 3 We now turn to the continuation payment that supports $V(p^1)$ in equilibrium. At p^1, the proposed equilibrium requires player 1 to reject player 2's demand for sure. The current period's expected payment is $qH + c_1$. At the next period, if the expert verdict is favorable, the posterior belief improves to, say, p^2, with continuation payment $V(p^2)$; otherwise, the belief goes back to p^* and the continuation payment $V(p^*)$ is given by equation (1) above. The Bellman equation that supports $V(p^1)$ as an equilibrium payment is, therefore,

$$V(p^1) = (1-\delta)(qH + c_1) + \delta q V(p^*) + \delta(1-q)V(p^2), \tag{3}$$

and this delivers $V(p^2)$. Figure 7 below summarizes these arguments.

Figure 7: Step 3

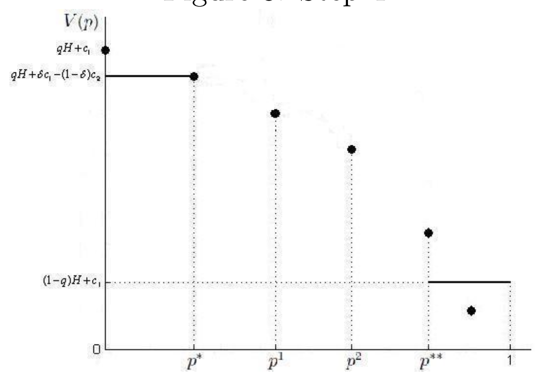


Step 4 By similar arguments, we can derive $V(p^3)$ that supports $V(p^2)$ in equilibrium and so forth, and put together the following recursive equation to characterize $V(p^n)$ for any integer n:[16]

$$V(p^n) = (1-\delta)(qH + c_1) + \delta q V(p^{n-1}) + \delta(1-q)V(p^{n+1}). \tag{4}$$

Starting from the two initial conditions $V(p^*)$ and $V(p^1)$, the solution to this second-order difference equation can easily be shown to be strictly decreasing and also divergent. Therefore, eventually, $V(p^n)$ will drop below $(1-q)H+c_1$, the lowest possible continuation payment in equilibrium. This is illustrated in Figure 8 below.

Figure 8: Step 4



Let N be the smallest integer such that $V(p^N) > (1-q)H + c_1$. Note that $V(p^N)$ is needed in order to support $V(p^{N-1})$ as an equilibrium continuation payment, but we cannot use $V(p^{N+1})$ to support $V(p^N)$ if the former is less than $(1-q)H + c_1$. Recall that the recursive arguments here are based on player 1 rejecting player 2's demand for sure. This

[16]Note that the unbiased expert assumption, that the quality of expert judgement, q, is symmetric across player 1 types, implies that the posterior updated from p^n following a non-favorable verdict is exactly p^{n-1}. It is straightforward to verify that, for any integer n, if

$$p^n = \frac{p^{n-1}q}{p^{n-1}q + (1-p^{n-1})(1-q)},$$

then

$$p^{n-1} = \frac{p^n(1-q)}{p^n(1-q) + (1-p^n)q}.$$

implies that, at p^N, player 1 cannot reject player 2's demand with probability 1 (except in the degenerate case where $V(p^{N+1}) = (1-q)H + c_1$ exactly).

The critical aspect of the equilibrium, therefore, is that, at p^N, player 2 has to randomize such that some demand is accepted while others are rejected. Indeed, player 2 will be indifferent exactly at $p^N = p^{**}$.[17] If his demand is rejected, player 2's expected payoff from going to the expert is $p^{**}(1-q)H + (1-p^{**})qH - c_2$. Therefore, from the indifference condition $p^{**}(1-q)H + (1-p^{**})qH - c_2 = (1-q)H + c_1$, we uniquely find

$$p^{**} = \frac{(2q-1)H - c_1 - c_2}{(2q-1)H} \in (0,1).$$

From $p^N = p^{**}$, we backtrack to find p^*; N consecutive unfavorable verdicts from p^{**} gives p^*. The exact mixing probability that supports $V(p^N)$, or $V(p^{**})$, is computed in Appendix. Since we now have p^* and p^{**}, we can also trace the entire continuation payment schedule, which takes the form illustrated in Figure 3 above. These details also appear in Appendix.

4 Some general properties of an equilibrium

The equilibrium constructed in the previous section exhibits a particular behavioral pattern. We now turn to the question of whether any aspects of the equilibrium apply more generally to an equilibrium. Our next results characterize some general properties of a Markov perfect Bayesian equilibrium, while maintaining the following condition:

(C) The good type, regardless of past history, accepts a demand if and only if it does not exceed $(1-q)H + c_1$; acceptance of a demand strictly greater than $(1-q)H + c_1$ reveals that player 1 is bad, both on and off the equilibrium path.

We start by examining the equilibrium strategies. The first result states that bad player 1 must use a cutoff strategy: there is a cutoff level of demand associated with each posterior belief such that any larger demand is rejected while any lower demand accepted.

Lemma 1 ***(Cutoff strategy)*** *Fix any δ and any Markov perfect Bayesian equilibrium. Also, fix any posterior p, and consider a demand $s > (1-q)H + c_1$. The following is true on or off the equilibrium path:*

[17] We later show that the accepted demand here must be $(1-q)H + c_1$.

(1) *If type B accepts s with a positive probability, then he must accept any $s' < s$ with probability 1.*

(2) *If type B rejects s with a positive probability, then it must reject any $s' > s$ with probability 1.*

Next, we obtain a suprising property regarding player 2's demand in any equilibrium. There are only two constant demand levels that could be accepted with postive probability in equilibrium. Any other demands must be either off the equilibrium path or offered and rejected in equilibrium. This general property has been confirmed in the particular equilibrium constructed above.

Proposition 4 *Fix any $\delta > \bar{\delta}$ and any Markov perfect Bayesian equilibrium. Suppose that, in equilibrium before player 1 reveals his type, player 2 makes a demand which player 1 accepts with a positive probability. Then, the demand is either $(1-q)H + c_1$ or $qH - c_2$.*

The demand $(1-q)H + c_1$ follows from the assumption on the good type's behavior. Let us argue that the only other acceptable equilibrium demand is $qH - c_2$. Suppose to the contrary that a higher demand is acceptable. Then, the acceptance must occur for sure. This is because, otherwise, player 2 could profitably deviate by demanding slightly less and the deviation would be met with sure acceptance (since player 1 plays a cutoff strategy). Player 2 clearly has no incentive to demand anything less than $qH - c_2$ (but greater than $(1-q)H + c_1$) since only the bad type would accept such a demand and the expected payoff under expert resolution conditional on player 1 being bad is $qH - c_2$.

Our final result examines player 1's expected equilibrium payments in any Markov perfect Bayesian equilibrium. Let $p^{**} = \frac{(2q-1)H - c_1 - c_2}{(2q-1)H}$ be the upper threshold belief as defined in the equilibrium construction in Section 3 above.

Proposition 5 *Suppose that $\delta > \bar{\delta}$. For any Markov perfect Bayesian equilibrium, there exists $p^* \in (0, p^{**})$ such that the following properties hold:*

- $V(0) = qH + c_1$.
- *For any $p \in (0, p^*)$, $V(p) = qH + \delta c_1 - (1-\delta)c_2$.*
- *For any $p \in (p^*, p^{**}]$, $V(p) \in [(1-q)H + c_1, qH + \delta c_1 - (1-\delta)c_2]$.*

- *For any* $p \in (p^{**}, 1]$, $V(p) = (1-q)H + c_1$.

Thus, we are able to obtain payment bounds for any equilibrium and, moreover, establish that the lower bound must be achieved when reputation is sufficiently high while the upper bound is met when reputation is sufficiently low. The proof which utilizes a novel argument with "downward induction on beliefs" can be found in Appendix.

5 Concluding discussion

In this section, we discuss the key assumptions of the model as well as some related work.

5.1 Robustness to the (un)observability of demands

We have assumed that accepted demands are publicly observable while rejected demands are not. Our equilibrium in Section 3 is robust to the (un)observability of the details of bargaining.

It is straightforward to see that the equilibrium continues to be valid when rejected demands are also publicly observable. Even though we have assumed that short-lived players do not observe previously rejected demands, it is common knowledge in equilibrium that the rejected demands must always be $qH - c_2$. Thus, it does not depend on whether this amount is observable or not.

We can also incorporate unobservability of accepted demands into our model. As is often the case in litigations, consider the two bargaining sides themselves choosing whether the amount of transfer will be publicly observable or confidential, should there be an agreement. Even with this modification to the model our equilibrium is robust under the following natural specification of belief upon observing a confidential agreement: player 2 assigns probability 1 to the bad type. This equilibrium survives the refinements such as the intuitive criterion. After all, it is natural that the good type who is innocent has nothing to hide. This eliminates any benefit of confidentiality.[18]

[18] See Daughety and Reinganum [6][7] for two-period litigation models with endogenous confidentiality agreements.

5.2 Relation between type and responsibility

Recall that we have conducted the analysis under the assumption that type B (G) is always responsible (not responsible) for each claim. Our analysis remains the same by instead assuming the following structure. For each claim, type G is responsible with probability $q' < \frac{1}{2}$ and type B is responsible with a probability $1 - q'$. Player 1 knows his type, but not his responsibility for each dispute due to some randomness (usual in medical malpractice cases, for instance). An expert makes a judgement on player 1's responsibility for each case with precision q''. It is readily verified that this is isomorphic to the model above with expert quality $q = q'q'' + (1 - q')(1 - q'')$.

5.3 Behavior of the good type

We have focused on equilibria in which the good type accepts a demand if and only if it does not exceed $(1 - q)H + c_1$. This is the maximum expected payment that the good type can guarantee himself since he always has the option of rejecting an offer. Indeed, using the same technique in Section 3, we can easily construct other equilibria where the good type's cutoff is D, for any $D < (1-q)H + c_1$. The indifference condition that pins down p^{**} is then $p^{**}(1 - q)H + (1 - p^{**})qH - c_2 = D$; the lower threshold p^* will be adjusted accordingly. The two levels of demand made by player 2 in the equilibrium will be $qH - c_2$ and D. The higher level remains the same, as well as the payoff benefit of reputation building for very small priors. Clearly, this modification to the good type's behavior does not add any new insights.

5.4 Other assumptions and extensions

In our model, the bargaining within each period takes a simple format: the uninformed player makes a take-it-or-leave-it offer. Such simplicity allows us to focus on the long-lived player's dynamic incentives, as done also in Schmidt [17], Daughety and Reinganum [6][7] and others. The one-sided offer by the uninformed player however rules out complex signaling effects. Spier [18] considers settlement bargaining between a single pair of defendant and plaintiff under more complex bargaining procotols.

Our analysis considers the case in which the long-lived player takes one of two possible types. As mentioned earlier, this fits a number of applications, including product liability litigations in which a sequence of disputes originate from the same act that a firm has, or is believed to have, already undertaken. Extending our analysis to the case of multiple types, nonetheless, offers an interesting direction for future research.

The stake (or the distribution thereof) in each dispute is assumed to be common knowledge. This also seems to be a reasonable description of many applications. For instance, in securities class actions, the stakes can be traced to the loss in share value. Introducing private information over the magnitude of the stake, in addition to private information on responsibility, will significantly complicate the analysis beyond the scope of the present paper.

We also assume that the the quality of the third party is constant. We could alternatively think of the model with an expert being drawn from a pool of experts with average quality q in each period.[19]

5.5 Further contribution to the legal literature

Alexander [2] studies repeated securities class action lawsuits involving underwriters behind similar claims of fraud in computer-related IPOs. She finds that, beyond very few exceptions, "the cases settled at an apparent 'going rate' of approximately one quarter of the potential damages... a strong case in this group appears to have been worth no more than a weak one" (Alexander [2], p.500). Thus, the *merit* of a case, or "the parties' estimates of the strength of the case" (Alexander [2], p500), does not appear to matter for settlement. Alexander suggests that reputation may play a role here because the securities class actions often involve long-lived defendants.[20] Our results support this observation; this is exactly what happens in the low reputation region where settlements occur and, moreover, the *amount* of settlement is constant over this interval of merits.

We also clarify the puzzle. Although the settlement amount, conditional on agreement, is independent of merit, settlement is nevertheless meritorious in that the settlement *rate* (i.e. the likelihood of settlement) is strictly decreasing in merit over the low reputation region. This is confirmed by Studdert and Mello [19] who find that, in medical malpractice

[19]Another possibility is to simply assume that the expert becomes more precise over time as evidence and verdicts accrue. But, this will make the problem less tractable.

[20]For instance, Alexander [2] notices that "two prominent investment banking firms stated in their own prospectuses that in 1986 they were involved in 60 and 73 lawsuits, respectively, over public offerings they had underwritten" (Alexander [2], p.558). Also, see Palmrose [16].

litigations, merits do indeed affect the settlement rate. Furthermore, it is observed that cases favoring neither party, or "close calls", are more likely to go to court (see Palmrose [16] and Studdert and Mello [19]). Such cases can be interpreted as corresponding to reputation levels over, or close to, the intermediate region in our equilibrium, where the bad defendant rejects plaintiffs' demands and proceed to trial for sure, or with a very high probability. Here, the conflict between the defendant's long-run interest and the plaintiff's short-run interest leads to the low settlement rate. The short-lived plaintiffs demand a relatively high compensation based on his estimate of the case's strength. However, the long-lived defendant is forward looking; the high reputation region is within reach and thus he will only accept a low demand, an amount less than what the plaintiffs are willing to offer.

6 Appendix

6.1 Omitted proofs of Section 3

Proof of Proposition 1

Our proof of this result is based on the following construction. We first need some notation. Let

$$\begin{aligned}\Phi^1(p) &= \frac{pq}{pq+(1-p)(1-q)}\\ \Phi^{-1}(p) &= \frac{p(1-q)}{p(1-q)+(1-p)q}.\end{aligned}$$

That is, when the belief is p, if both types of player 1 go to expert and the verdict is not liable (or liable), then the increased (or decreased) updated belief is equal to $\Phi^1(p)$ (or $\Phi^{-1}(p)$). Notice that $\Phi^{-1}\left(\Phi^1(p)\right) = p$ for any p.

Furthermore, for any positive integer k, define $\Phi^k(p)$ recursively such that $\Phi^2(p) = \Phi^1\left(\Phi^1(p)\right)$, $\Phi^3(p) = \Phi^1\left(\Phi^2(p)\right)$ and, hence, $\Phi^k(p) = \Phi^1\left(\Phi^{k-1}(p)\right)$. In other words, when the initial belief is p, if both types of player 1 go to expert k consecutive times and the verdict favors player 1 on each occasion, then the posterior belief updated from p is $\Phi^k(p)$, Similarly, we define $\Phi^{-k}(p)$ as the posterior reached from p after k successive non-favorable expert decisions for player 1. Also, let $\Phi^0(p) \equiv p$.

Next, let $\bar{\delta}$ solve the following:

$$[1-\bar{\delta}q][qH+\bar{\delta}c_1-(1-\bar{\delta})c_2]=(1-\bar{\delta})(qH+c_1)+\bar{\delta}(1-q)[(1-q)H+c_1]. \qquad (5)$$

It is straightforward to observe that such $\bar{\delta}$ must belong to $(0,1)$.

Fix any $\delta > \bar{\delta}$, and consider the profile (r^B, r^G, d) below, where $p^*, p^{**} \in (0,1)$ and player 2's randomization probability at p^{**}, x, are to be defined later.

First, player 2's strategy, d, is such that:

- At $p=0$, it demands $qH+c_1$ with probability 1;
- At any $p \in (0, p^{**})$, it demands $qH-c_2$ with probability 1;
- At $p=p^{**}$, it demands $(1-q)H+c_1$ with probability x and $qH-c_2$ with probability $1-x$;
- At any $p \in (p^{**}, 1]$, it demands $(1-q)H+c_1$ with probability 1.

Second, type G player 1's strategy, r^G, is such that, for any p, it accepts a demand s if and only if $s \le (1-q)H+c_1$.

Third, we define type B player 1's strategy, r^B:

- At $p=0$, it accepts a demand s if and only if $s \le qH+c_1$;
- At any $p \in (0, p^*]$,
  - it rejects any $s > qH-c_2$ with probability 1;
  - it accepts any $s < qH-c_2$ with probability 1;
  - it rejects $qH-c_2$ with probability $r(p)$, where $r(p)$ satisfies

    $$p^* = \frac{p}{p+(1-p)r(p)},$$

    and therefore,

    $$r(p) = \frac{p}{p^*}\frac{1-p^*}{1-p} \le 1.$$

    (Notice that $r(p^*)=1$.)

- At any $p \in (p^*, p^{**}]$,
  - it rejects any $s > \max\{\xi(p), (1-q)H + c_1\}$ with probability 1;
  - it accepts any $s \leq \max\{\xi(p), (1-q)H + c_1\}$ with probability 1, where $\xi(p)$ here is defined later.
- At any $p \in (p^{**}, 1]$,
  - it rejects any $s > (1-q)H + c_1$ with probability 1;
  - it accepts any $s \leq (1-q)H + c_1$ with probability 1.

Finally, the belief is updated by Bayes' rule and the equilibrium strategies whenever possible. We also assume that the posterior belief assigns probability 1 to type B after an acceptance of a demand higher than $(1-q)H + c_1$.

We now define p^*, p^{**} and x. Along the way, the equilibrium payment of type B, $V(p)$, will also be obtained, as well as $\xi(p)$ for $p \in (p^*, p^{**}]$.

Defining p^{**} At the upper threshold level of belief, p^{**}, player 2 must be indifferent between demanding $(1-q)H + c_1$, which is accepted with probability 1, and demanding $qH - c_2$, which is rejected with probability 1. Thus, it is computed from the equation

$$(1-q)H + c_1 = p^{**}((1-q)H - c_2) + (1-p^{**})(qH - c_2),$$

which yields

$$p^{**} \equiv \frac{(2q-1)H - c_1 - c_2}{(2q-1)H} \in (0,1). \tag{6}$$

Defining p^* At p^*, type B is indifferent between accepting and rejecting $qH - c_2$. Let $V_0 \equiv V(p^*)$ and $V_n \equiv V(\Phi^n(p^*))$. Then, since acceptance of the equilibrium demand leads to revelation, we first have

$$V_0 = (1-\delta)(qH - c_2) + \delta(qH + c_1) = qH + \delta c_1 - (1-\delta)c_2. \tag{7}$$

Rejection, on the other hand, yields the following:

$$V_0 = (1-\delta)(qH + c_1) + \delta q V_0 + \delta(1-q)V_1, \tag{8}$$

where the current period expected payment equals $qH + c_1$, the next period continuation expected payment following a favorable verdict (which takes place with probability $1-q$) is V_1 and the corresponding payment following a non-favorable verdict is also V_0 (since type B randomizes at any $p < p^*$).

Note here that, since we assume $(2q-1)H > c_1 + c_2$, $V_0 > (1-q)H + c_1$ and that, since $\delta > \bar{\delta}$, $V_1 > (1-q)H + c_1$ (see (5) above for the definition of $\bar{\delta}$).

Next, consider the equilibrium payment V_n (at $p = \Phi^n(p^*)$) for any integer $n \geq 1$. Here, since the equilibrium demand is rejected for sure, the continuation payment must satisfy the following recursive structure:

$$V_n = (1-\delta)(qH + c_1) + \delta q V_{n-1} + \delta(1-q)V_{n+1}. \tag{9}$$

Define $N = \sup\{n \in \mathbb{Z} : V_n > (1-q)H + c_1\}$, where $\mathbb{Z}$ denotes the set of integers; i.e. N is the largest integer n such that $V_n > (1-q)H + c_1$.

Then, given Claim 1 below, define $p^* = \Phi^{-N}(p^{**}) \in (0,1)$. Since $V_1 > (1-q)H + c_1$, N must be positive and, hence, $p^* < p^{**}$ as required by the equilibrium.

Claim 1 (1) *V_n is strictly decreasing in n.*

(2) *N is finte.*

Proof. (1) Notice that $V_0 < qH$ and V_0 is a convex combination of $qH + c_1$ and V_1. Then $V_1 < V_0$. Suppose $V_n < V_{n-1} < \cdots < V_0 < qH$. From (9), V_n is a convex combination of $qH+c_1$, V_{n-1}, and V_{n+1}, and hence $V_{n+1} < V_n$. The monotonicity of V_n follows by induction.

(2) Suppose to the contrary that N is infinite. That is, $V_n > (1-q)H+c_1$ for all n. Then, since V_n is strictly decreasing, V_n converges to V_∞ such that $(1-q)H + c_1 \leq V_\infty < qH + c_1$. But, from (9), it follows that $V_\infty = qH + c_1$. This is a contradiction. ■

Defining x At p^{**}, player 2 demands $(1-q)H + c_1$ with probability x and $qH - c_2$ with probability $1-x$; both types of player 1 accept the first demand with probability 1 and reject the second demand with probability 1. This implies that the equilibrium posterior at the next period must be such that:

- if $(1-q)H + c_1$ is accepted then the posterior remains at p^{**};
- if a demand is rejected, followed by a favorable verdict to player 1, then the posterior moves up to $\Phi^1(p^{**})$; and

- if a demand is rejected, followed by a non-favorable verdict to player 1, then the posterior moves down to $\Phi^{-1}(p^{**})$.

Thus, we have

$$V(p^{**}) \equiv V_N = x\left[(1-\delta)((1-q)H + c_1) + \delta V_N\right] + (1-x)X, \tag{10}$$

where V_N is given by the second-order difference equation (9) with the two initial conditions V_0 and V_1 as in (7) and (8) above, and

$$X \equiv (1-\delta)(qH + c_1) + \delta q V_{N-1} + \delta(1-q)((1-q)H + c_1). \tag{11}$$

Claim 2 *There exists a unique $x \in [0,1)$ that satisfies (10).*

Proof. Simple computation shows that

$$x = \frac{X - V_N}{X - (1-\delta)((1-q)H + c_1) - \delta V_N}.$$

Note first that $V_N \leq X$. This follows from comparing (11) above to the recursive equation

$$V_N = (1-\delta)(qH + c_1) + \delta q V_{N-1} + \delta(1-q)V_{N+1},$$

where, by assumption, $V_{N+1} \leq (1-q)H + c_1$. Also, we have $V_N > (1-\delta)((1-q)H + c_1) + \delta V_N$ because, again by assumption, $V_N > (1-q)H + c_1$. Thus, $x \in [0,1)$. ■

Equilibrium payments At this juncture, we characterize the equilibrium expected payments of type B. The following is clear:

- For any $p \leq p^*$, $V(p) = V_0$.
- For any $p = \Phi^n(p^*)$ with an integer $1 \leq n \leq N$, $V(p) = V_n$; in particular, $V(p^{**}) = V_N$.
- For any $p > p^{**}$, $V(p) = (1-q)H + c_1$.

We now pin down payments when $p \in (p^*, p^{**})$ but $p \neq \Phi^n(p^*)$ for any integer $1 \leq n \leq N$.

Claim 3 *Fix any integer $n \in [1, N]$ and any $p, p' \in (\Phi^{n-1}(p^*), \Phi^n(p^*))$. Then, we have*

$$V(p) = V(p') < V_0.$$

Proof. Consider the following recursive structure: for any integer k,

$$W_k = (1 - \delta)(qH + c_1) + \delta q W_{k-1} + \delta(1 - q)W_{k+1}$$

such that $W_0 = V_0$ and $W_{N+1} = (1 - q)H + c_1$, where N is defined as above.

Note that we have

$$\Phi^{-n}(p) = \Phi^{-n}(p') < p^* \text{ and } \Phi^{-n+1}(p) = \Phi^{-n+1}(p') > p^*;$$
$$\Phi^{N-n+1}(p) = \Phi^{N-n+1}(p') > p^{**} \text{ and } \Phi^{N-n}(p) = \Phi^{N-n}(p') < p^{**}.$$

Thus, it is straightforward to see that

$$W_n = V(p) = V(p').$$

Also, from the same arguments for Claim 1 above, we can show that W_k is strictly decreasing. ■

Defining $\xi(p)$ for $p \in (p^*, p^{**}]$ Recall that, in specifying player 1's equilibrium strategy earlier, we had deferred the definition of $\xi(p)$ at $p \in (p^*, p^{**}]$. Fix any $p \in (p^*, p^{**}]$, and define $\xi(p)$ as satisfying

$$(1 - \delta)\xi(p) + \delta(qH + c_1) = V(p),$$

where $V(p)$ is the equilibrium payment computed above. It is easily seen that $\xi(p) < qH - c_2$.

It remains to be shown that the profile (r^B, r^G, d) defined above, together with the stated beliefs, constitutes a Markov perfect Bayesian equilibrium.

First, given r^B and r^G, and the definition of p^{**}, it is straightforward to establish optimality of player 2 strategy, d. In particular, note that it is never optimal for player 2 to make a demand $s \in ((1 - q)H + c_1, qH - c_2)$.

Second, we check optimality of r^G, the strategy of type G. This is clear since player 2 never makes a demand less than $(1 - q)H + c_1$, which is precisely the amount that this type expects to pay in total in case the dispute goes to the expert in any period.

Finally, we check optimality of r^B.

- It is straightforward to check optimality of r^B at $p = 0$.

- Fix any $p \in (0, p^*]$. Suppose first that the demand, s, is less than $qH - c_2$. If type B accepts this demand, the continuation payment amounts to

$$(1 - \delta)s + \delta(qH + c_1) < V_0,$$

while, since rejected demands are not observable, the continuation payment from rejecting continues to be V_0. Thus, accepting any $s < qH - c_2$ for sure is optimal. A symmetric argument establishes that rejecting any $s > qH - c_2$ for sure is optimal. The rejection probability $r(p)$, supports the indifference conditions captured by (7) and (8) above.

- Fix any $p \in (p^*, p^{**})$. Here, by Claims 1 and 3 above, we have $V(p) < V_0$, and accepting the demand $qH - c_2$ yields precisely $V_0 = (1 - \delta)(qH - c_2) + \delta(qH + c_1)$ due to revelation. Thus, rejecting the equilibrium demand, $qH - c_2$, is optimal.

- Consider $p = p^{**}$. If type B accepts the equilibrium demand $qH - c_2$, he reveals his type and, hence, obtains a continuation payment V_0. If he rejects this demand, on the other hand, he obtains

$$(1 - \delta)(qH + c_1) + \delta q V_{N-1} + \delta(1 - q)((1 - q)H + c_1) \equiv X < V_0,$$

where the last inequality can be obtained from the proof of Claim 3 above. Thus, it is optimal to reject $qH - c_2$.

Next, consider the demand $(1 - q)H + c_1$. Rejection, again, yields a continuation payment X, while acceptance leads to a payment $(1 - \delta)((1 - q)H + c_1) + \delta V_N$. Since $V_N < X$ and $(1 - q)H + c_1 < X$, acceptance is optimal.

- Fix any $p \in (p^{**}, 1]$. Since player 2 plays a pure strategy here, accepting the equilibrium demand $(1 - q)H + c_1$ cannot reduce the equilibrium posterior. Thus, accepting yields a continuation payment $(1 - q)H + c_1$. On the other hand, rejection yields, at best, a continuation payment

$$(1 - \delta)(qH + c_1) + \delta\left((1 - q)H + c_1\right),$$

implying the optimality of acceptance.

Proof of Proposition 2

When $p_1 = 0$, it is known that player 1 is the bad type. Therefore, $V(0) = qH + c_1$. If $p_1 \in (0, p^*)$, player 1 plays mixed strategies and, hence, his payment is obtained from assuming that he agrees with player 2 on the first case and reveals his type, i.e. $V(p_1) = (1-\delta)(qH-c_2)+\delta V(0) = qH+\delta c_1-(1-\delta)c_2$. Therefore, the reputation gain is $(1-\delta)(c_1+c_2)$ if $p_1 \in (0, p^*)$. The next three properties follow directly from the equilibrium construction.

Proof of Proposition 3

1. We have already established that p^{**} is independent of δ (see (6) above). By definition, p^* is the posterior belief after N consecutive non-favorable expert decisions starting from p^{**}. Therefore, to show p^* goes to 0 as δ goes to 1, it suffices to establish that $N(\delta)$ goes to ∞ as δ goes to 1.

We first note that that $V(p^n) - V(p^0) \to 0$ as δ goes to 1 for any fixed n. This follows directly from the difference equation (4) and its initial conditions. Since $V(p^0) > (1 - q)H + c_1$ even when $\delta \to 1$, $N(\delta)$ goes to ∞ by definition.

2. It is immediate from the definition of p^{**} that $p^{**} \to \frac{H-c_1-c_2}{H}$ as $q \to 1$. By equation (2) in the main text, $V(p^1) \to -\infty$ as $q \to 1$. Therefore, $N \to 1$ as $q \to 1$ and, hence, p^* becomes the posterior probability obtained after a single non-favorable expert decision starting from p^{**}, that is, $p^* = \frac{p^{**}(1-q)}{p^{**}(1-q)+(1-p^{**})q}$. Given the limit of p^{**}, it follows immediately that $p^* \to 0$ as $q \to 1$.

6.2 Omitted proofs of Section 4

Proof of Lemma 1

(1) If s is accepted, the continuation (discounted average expected) payment from accepting s must be at least as good as that from rejecting it.

Since rejected demands are not observable, rejecting any demand results in the same continuation payment. Also, by (C), accepting any demand strictly above $(1 - q)H + c_1$ leads to the same continuation payment at the next period (equal to $qH + c_1$). Then, accepting any $s' \in ((1 - q)H + c_1, s)$ must be strictly better than rejecting it since it yields a lower immediate payment.

On the other hand, accepting a demand $s' \leq (1-q)H + c_1$ needs not lead to revelation but the continuation payment at the next period must still be bounded above by $qH + c_1$ and, hence, the same arguments imply that such a demand must also be accepted for sure.

(2) If s is rejected, the continuation payment from rejecting s must be at least as good as that from accepting it. Rejecting s or s' results in identical expected payments, both in the current period and each forthcoming period; on the other hand, while accepting s' and s yield the same continuation payment as of the next period, accepting $s' > s$ involves a strictly higher stage expected payment than accepting s. Thus, any $s' > s$ must be rejected for sure.

Proof of Proposition 4

The proof is by contradiction. We consider the following cases.

Case 0. $s < (1-q)H + c_1$ or $s > qH + c_1$.

Any demand $s < (1-q)H + c_1$ is dominated by $(1-q)H + c_1$ since type G accepts $(1-q)H + c_1$ and player 2's stage payoff from type B is $qH - c_2 > (1-q)H + c_1$ should he reject $(1-q)H + c_1$. Therefore, in equilibrium, player 2 will not demand $s < (1-q)H + c_1$. This contradicts the assumption that s is demanded in equilibrium.

If type B accepts a demand $s > qH + c_1$, by (C), he will reveal his type and the subsequent payment is $qH + c_1$ each period. If he rejects s, his current period expected payment is $qH + c_1$ while future expected payments are bounded above by $qH + c_1$. Therefore, $s > qH + c_1$, if demanded, will be rejected by type B for sure. This contradicts the assumption that s is accepted.

Case 1. $s \in ((1-q)H + c_1, qH - c_2)$.

But then, player 2 can profitably deviate by not demanding s and, instead, demanding any $s' > qH + c_1$. By (C), type G rejects both s and s' for sure; from Case 0 above, we know that type B must also reject s' for sure. But player 2 expects to earn $qH - c_2 > s$ from type B by seeking an expert and, therefore, would strictly prefer to have s' rejected than to have s accepted. This is a contradiction.

Case 2. $s \in (qH - c_2, qH + c_1]$ and type B rejects s with probability $r \in (0, 1)$.

But then, consider player 2 deviating by demanding $s - \epsilon > qH - c_2$ instead of s for some small $\epsilon > 0$. By Lemma 1, such a demand must be accepted by type B for sure; by

(C), type G rejects $s-\epsilon$. The deviation payoff then amounts to

$$p((1-q)H-c_2)+(1-p)(s-\epsilon),$$

while the corresponding equilibrium payoff is

$$p((1-q)H-c_2)+(1-r)(1-p)s+r(1-p)(qH-c_2).$$

Thus, such a deviation is profitable if $\epsilon < r(s-qH+c_2)$. This is a contradiction.

Case 3. $s \in (qH-c_2, qH+c_1]$ and type B accepts s with probability 1.

Let r^B be the given equilibrium strategy of type B, and let $s^* > qH-c_2$ denote the supremum of demands that it accepts with probability 1 at p; that is, $s^* = \sup\{s : r^B(p,s)=0\}$.

Then, by Lemma 1, $r^B(p,s')=0$ for any $s' \in (qH-c_2, s^*)$, and $r(p,s'')=1$ for any $s'' \in (s^*,\infty)$. Therefore, player 2's payoff is s' by demanding s' and $qH-c_2 < s^*$ by demanding s''. However, both s' and s'' are dominated by $s^*-\frac{s^*-s'}{2}$ which is accepted for sure, yielding a payoff of $s^*-\frac{s^*-s'}{2} > qH-c_2$. Therefore, given our arguments against Cases 0 and 1 above, player 2 will not make a demand other than $(1-q)H+c_1$ or s^* in equilibrium.

Suppose now that player 2 demands s^* with a positive probability. We shall show that this is impossible.

On the one hand, if player 2's equilibrium strategy demands s^* with a positive probability, type B must accept it with probability 1 by the same argument as in Case 2; otherwise, player 2 could profitably deviate by demanding $s^*-\epsilon$ instead of s^* for some small enough $\epsilon > 0$.

On the other hand, type B has an incentive to deviate by rejecting s^* if $\delta > \frac{c_1+c_2}{(2q-1)H+c_1+c_2}$. As we have already established, in equilibrium, the demand can only be either $(1-q)H+c_1$ or s^*, where the former demand is accepted for sure by both types and the latter is accepted for sure by type B while rejected for sure by type G. It then follows that the equilibrium posterior at the next period after observing rejection in the current period must be 1.

Thus, the deviation results in each subsequent player 2 demanding $(1-q)H+c_1$ and, hence, the continuation payment

$$(1-\delta)(qH+c_1)+\delta((1-q)H+c_1). \tag{12}$$

But, in equilibrium, acceptance of s^* results in revelation (condition (C)) and, hence, the continuation payment

$$(1-\delta)s^* + \delta(qH + c_1). \tag{13}$$

Since $s^* > qH - c_2$ and $\delta > \frac{c_1+c_2}{(2q-1)H+c_1+c_2}$, (13) exceeds (12) and, therefore, the deviation is profitable. This is a contradiction.

Proof of Proposition 5

We proceed to prove each claim of Proposition 5 in turn. Fix any $\delta > \frac{c_1+c_2}{(2q-1)H+c_1+c_2}$, as required by Proposition 4, and any Markov perfect Bayesian equilibrium. Also, for ease of exposition, let $\overline{V} = qH + \delta c_1 - (1-\delta)c_2$. We proceed with the following Lemmata.

Lemma 2 *For any $p \in (0,1)$, $V(p) \in \left[(1-q)H + c_1, \overline{V}\right]$.*

Proof. First of all, the lower bound is immediate since, with condition (C), any demand less than $(1-q)H + c_1$ is strictly dominated for player 2 and thus will never occur in equilibrium.

Next, we establish the upper bound. Let us consider two cases in turn.

First, suppose that every *equilibrium* demand of player 2 is accepted by type B. Then, player 2 must play pure strategy (given the assumption that each equilibrium demand is accepted, player 2 cannot randomize between a low demand and a high demand).

Then, by Proposition 4, the equilibrium demand is either $qH - c_2$ or $(1-q)H + c_1$. If the demand is $(1-q)H + c_1$, by condition (C), no belief updating occurs and, therefore, $V(p) = (1-q)H + c_1 < \overline{V}$. If the demand is $qH - c_2$, type B reveals himself and hence by the Markov property

$$V(p) = (1-\delta)(qH - c_2) + \delta(qH + c_1) = \overline{V}.$$

Second, suppose that, at p, some equilibrium demand is rejected with a positive probability. Let s_* be the infimum of these demands that are rejected by type B at p. By Lemma 1, all demands below s_* will be accepted and all demands above s_* will be rejected by this type.

Note that type B's equilibrium payment, $V(p)$, is bounded above by rejecting all demands. In particular, given the definition of s_*, the upper bound equals the continuation payment from rejecting an *equilibrium* demand $s_* + \epsilon$, for some $\epsilon \geq 0$.

But, at the same time, since $s_* + \epsilon$ occurs and is rejected in equilibrium, type H's equilibrium payment at p is bounded above by the continuation payment from accepting $s_* + \epsilon$. Therefore, it must be that

$$V(p) \leq (1-\delta)(s_* + \epsilon) + \delta(qH + c_1),$$

where $qH + c_1$ is the maximum possible continuation payment.

Now, by the definition of s_*, we can take $\epsilon \to 0$ and, hence, obtain

$$V(p) \leq (1-\delta)s_* + \delta(qH + c_1). \tag{14}$$

From (14), we are done if $s_* \leq qH - c_2$. We simply note that it is impossible that $s_* > qH - c_2$. The reasoning is as follows. Suppose not. By the definiton of s_*, there exists an equilibrium demand $s \geq s_*$ such that s is rejected and player 2 obtains a payoff of $qH - c_2$. But, by the definition of s_*, any $s_* - \epsilon > qH - c_2$ will be accepted by type B which gives player 2 a payoff of $s_* - \epsilon > qH - c_2$. Therefore, s cannot be demanded in equilibrium. This is a contradiction. ■

Lemma 3 *Let* $p^{**} = \frac{(2q-1)H - c_1 - c_2}{(2q-1)H}$. *For any* $p \in (p^{**}, 1)$, $(1-q)H + c_1$ *is demanded and accepted for sure.*

Proof. By demanding $(1-q)H + c_1$, player 2 obtains a payoff of at least

$$(1-q)H + c_1 \tag{15}$$

since the good type accepts it and he can obtain $qH - c_2 > (1-q)H + c_1$ if the bad type ever rejects the demand. Note that all lower demands are strictly dominated by $(1-q)H + c_1$.

By demanding $qH - c_2$, player 2 obtains at most

$$p((1-q)H - c_2) + (1-p)(qH - c_2) \tag{16}$$

since type G will reject it, leading to expected payoff of $(1-q)H - c_2$ for player 2, and $qH - c_2$ is player 2's expected payoff regardless of type B's response. Note that all demands in $((1-q)H + c_1, qH - c_2)$ are weakly dominated by $qH - c_2$, because type G rejects the demand and player 2's payoff is lower than $qH - c_2$ if type B ever accepts it.

Now, by Lemma 1 and Proposition 4, any demand greater than $qH - c_2$ is rejected by both types for sure, which gives player 2 a payoff of $p((1-q)H - c_2) + (1-p)(qH - c_2)$.

Therefore, we only need to compare (15) with (16). Since $p > p^{**}$, the former is larger, implying that $(1-q)H + c_1$ must be demanded for sure.

Then, since player 2 plays a pure strategy here, and by (C), accepting the equilibrium demand $(1-q)H + c_1$ cannot reduce the equilibrium posterior. Thus, accepting yields a continuation payment $(1-q)H + c_1$ to type B. On the other hand, rejection yields, at best, a continuation payment

$$(1-\delta)(qH + c_1) + \delta\left((1-q)H + c_1\right),$$

implying that $(1-q)H + c_1$ is accepted for sure. ■

In order to pin down our final claim, we first need the following Lemma.

Lemma 4 *Consider the state space* $P \subset [0,1]$ *such that* $P = P_1 \cup P_2 \cup P_3$. *Let* $V(p)$ *be the discounted average expected payment at* p *(with discount factor* $0 < \delta < 1$*).*

At any $p \in P_3$*, with probability* $1-q$ *the immediate payment is* 0 *and the new state becomes* $p' = \Phi^1(p)$*; with probability* q*, the payment is* H *and the new state becomes* $p'' = \Phi^{-1}(p)$*, where* $\Phi^1(\cdot)$ *and* $\Phi^{-1}(\cdot)$ *are as defined in the proof of Proposition 1 above. If* $p \in P_1$*,* $V(p) = v_1 > 0$*; If* $p \in P_2$*,* $V(p) = v_2 > 0$*.*

We then have the following: If $qH \geq \min\{v_1, v_2\}$*, then* $V(p) \geq \min\{v_1, v_2\}$ *for any* $p \in P_3$*.*

Proof. Suppose not. Let $v_3 = \inf_{p \in P_3} V(p)$. Then, by assumption, $v_3 < \min\{v_1, v_2\}$. For any small $\varepsilon > 0$, there exists $p^\varepsilon \in P_3$ such that $V(p^\varepsilon) < v_3 + \varepsilon$. We know that

$$\begin{aligned} V(p^\varepsilon) &= (1-\delta)qH + \delta((1-q)V(p') + qV(p'')) \\ &\geq (1-\delta)qH + \delta \min\{V(p'), V(p'')\}. \end{aligned}$$

Therefore,

$$\begin{aligned} \min\{V(p'), V(p'')\} &\leq \delta^{-1}(V(p^\varepsilon) - (1-\delta)qH) \\ &\leq \delta^{-1}[v_3 + \varepsilon - (1-\delta)v_3 + (1-\delta)v_3 - (1-\delta)qH] \\ &< v_3 + \delta^{-1}[\varepsilon + (1-\delta)(v_3 - qH)]. \end{aligned}$$

Taking ε to 0, we have $\min\{V(p'), V(p'')\} < v_3 + \delta^{-1}(1-\delta)(v_3 - qH)$. However, we know that, by assumption, $v_3 < \min\{v_1, v_2\} \leq qH$. It then follows that $\min\{V(p'), V(p'')\} < v_3$. This contradicts the definition of v_3. ■

We are now ready to complete the proof of Proposition 5 with the following Lemma.

Lemma 5 *There exists* $p^* \in (0, p^{**})$ *such that, for any* $p \in (0, p^*)$, $V(p) = \overline{V}$.

Proof. We shall follow a series of steps.

Step 1. Fix any $p < p^{**}$, and suppose that player 2 demands $(1-q)H+c_1$ in equilibrium. Then, type B must reject this demand with a positive probability, and hence the equilibrium posterior belief after a rejection but before the expert verdict does not exceed p.

Proof of Step 1. Suppose to the contrary that player 1 accepts the demand for sure. Player 2's payoff will be $(1-q)H+c_1$. We shall argue that $(1-q)H+c_1$ is strictly dominated and cannot be an equilibrium demand.

Consider another demand $qH - c_2$. If player 1 is type G, then he will reject it and player 2's payoff will be $(1-q)H - c_2$; if player 1 is type B, then whether or not he rejects $qH - c_2$, player 2 will earn $qH - c_2$ in expectation. Therefore, player 2's expected payoff is $p(1-q)H + (1-p)qH - c_2$. Since $p < p^{**}$, this amount is greater than $(1-q)H + c_1$. That is, $qH - c_2$ dominates $(1-q)H + c_1$.

Since $(1-q)H+c_1$ is rejected with positive probability, all higher demands are rejected for sure by Lemma 1. It follows that in this case rejection reduces the posterior belief.

Step 2. Fix any $p < p^{**}$. One of the following holds:

(a) $V(p) = \overline{V}$; or

(b) player 1 weakly prefers to reject any equilibrium demand and the equilibrium posterior immediately after the rejection (before the expert verdict) does not exceed p.

Proof of Step 2. There are two cases to consider.

Case 1: $(1-q)H + c_1$ is demanded with a positive probability in equilibrium.

Then, by Step 1, (b) holds.

Case 2: $(1-q)H + c_1$ is demanded with probability 0 in equilibrium.

In this case only $qH - c_2$ can be possibly accepted by Proposition 4.

- If type B's equilibrium strategy prescribes that $qH - c_2$ be rejected for sure, then the belief will not change after rejection; hence, (b) holds.

- If it prescribes that $qH - c_2$ be accepted with a positive probability, then all demands greater than $(1-q)H + c_1$ but less than $qH - c_2$ is going to be accepted for sure, and they are dominated by $qH - c_2$ for player 2 (because only type B accepts these demands).

Now, there are two possibilities here.

First, if $qH - c_2$ is not demanded in equilibrium by player 2, then all equilibrium demands are rejected and, therefore, belief never changes; hence, (b) holds.

Second, if $qH - c_2$ is demanded in equilibrium with a positive probability by player 2, then type B's continuation payment from rejecting any demand is higher than or equal to that from accepting $qH - c_2$. The latter amounts to

$$(1-\delta)(qH - c_2) + \delta(qH + c_1) = \overline{V}.$$

But, since $V(p) \leq \overline{V}$ by Lemma 2, it must be that $V(p) = \overline{V}$; hence, (a) holds.

At this point, for any positive integer k, let $p_k = \Phi^{-k}(p^{**})$, as defined in the proof of Proposition 1 above.

Step 3. Fix any $p \in [p_{k+1}, p_k)$, and suppose that

$$\overline{V} \geq (1-\delta^k)qH + \delta^k(1-q)H + c_1.$$

Then, we have

$$V(p) \geq \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\}.$$

Proof of Step 3. We employ induction. First, consider any $p \in [p_1, p^{**})$. By Step 2, we have either $V(p) = \overline{V}$ or an equilibrium demand is rejected and so $V(p)$ is given by the continuation payment from the rejection.

In the latter case, clearly, $V(p) \geq (1-\delta)(qH + c_1) + \delta((1-q)H + c_1)$. Thus,

$$V(p) \geq \min\{(1-\delta)qH + \delta(1-q)H + c_1, \overline{V}\}.$$

Next, assume that, for any $p \in [p_k, p_{k-1})$,

$$V(p) \geq \min\{(1-\delta^k)qH + \delta^k(1-q)H + c_1, \overline{V}\}.$$

We want to show that, for any $p \in [p_{k+1}, p_k)$,

$$V(p) \geq \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\}.$$

Again, given Step 2 above, consider the continuation payment when any equilibrium demand here is rejected such that the posterior immediately after rejection does not go above p.

Rejection results in the current period expected payment of $qH + c_1$. If the subsequent expert verdict is favorable, the next period's posterior belongs to $[p_k, p_{k-1})$ and, hence, the corresponding continuation payoff must be at least $\min\{(1-\delta^k)qH + \delta^k(1-q)H + c_1, \overline{V}\}$, by assumption.

If the expert verdict is not favorable then the next period's posterior must belong to $[p_{k+2}, p_{k+1})$. By Lemma 4 (taking $P_3 = [p_{k+2}, p_{k+1})$, $P_1 = [p_k, p_{k-1})$, $P_2 = \{p : V(p) = \overline{V}\} \backslash (P_1 \cup P_3)$), the corresponding continuation payment must also be bounded below by $\min\{(1-\delta^k)qH + \delta^k(1-q)H + c_1, \overline{V}\}$.

Thus, we have

$$\begin{aligned} V(p) &\geq \min\{(1-\delta)(qH + c_1) + \delta\left[(1-\delta^k)qH + \delta^k(1-q)H + c_1\right], \overline{V}\} \\ &= \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\}, \end{aligned}$$

and induction closes the proof of Step 3.

Now, let K be the largest integer such that $\overline{V} \geq (1-\delta^K)qH + \delta^K(1-q)H + c_1$. Then, Step 3 immediately implies that, for any $p \in [p_{k+1}, p_k)$, $k \geq K$, we must have

$$V(p) \geq \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\} = \overline{V}.$$

Since, by Lemma 2 we already know that $V(p) \leq \overline{V}$ for any $p \in (0,1)$, it follows that $V(p) = \overline{V}$ for any $p < p_K$. ■

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