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"Information Acquisition and the Excess Refund Puzzle"

by

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Information Acquisition and the Excess Refund Puzzle*

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Abstract

A buyer can learn her value for a *returnable experience good* by trying it out, with the option of returning the good for whatever refund the seller offers. Sellers tend to offer a "no questions asked" refund for such returns, a money back guarantee. The refund is often too generous, generating inefficiently high levels of returns. We present two versions of a model of a returnable goods market. In the *Information Acquisition Model,* consumers are ex ante identical and uninformed of their private values for the good. The firm then offers a generous refund in order to induce the consumers to learn their values by purchasing and trying the good out, rather than by doing costly research prior to purchasing. In the *Screening Model,* some consumers have negligible costs of becoming informed about their values prior to purchasing, and always do so; other consumers have prohibitive costs of acquiring pre-purchase information and always stay uninformed. The firm's optimal screening menu may then contain only a single contract, one that specifies a generous refund, and hence a high purchase price, in order to weaken the incentive constraint of the informed consumers.

KEYWORDS: *information acquisition, refunds, money back guarantees, returnable experience goods*
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1. Introduction

Many products are returned to the seller soon after their purchase. This is especially true in the United States, where about six percent of all purchased products are returned for an annual total of more than one trillion dollars.[1] Return levels are especially high in internet and catalog retailing.[2]

The refunds generating these returns are generous. Full "money back" refunds of the original purchase price, sometimes lowered by a small "restocking fee" that is charged the consumer, are typical. Sellers lose money on returns; the refund they pay for a return almost always exceeds their salvage value for it.[3] Retailers have been estimated to lose up to twenty-five percent of their sales on returns.[4] The presence of such generous refunds suggests that return levels are inefficiently high. Why do firms offer refunds for returned goods that exceed their own salvage values for them?

This **excess refund puzzle** has little to do with the refunds specified by warranties against product failure. Many returned products are not defective, are not claimed to be defective, and can not be easily verified to be defective in any case. Instead, many are returned by consumers who learn soon after purchasing that they do not value the product more than the refund given for a return.[5] Clothing is returned because it is found not to fit or flatter; nuts and bolts are returned because they are found to be wrong for the job at hand; a silver-colored DVD player is returned because a spouse finds it ugly. This observation suggests that the excess refund puzzle should be examined within a model in which consumers learn about a product by purchasing it.

A basic *learning-by-purchasing* model consists of firms selling a good to consumers

[1] Rogers and Tibben-Lembke (1999), p. 7-9.

[2] A NFO Interactive survey shows that in 2000, twenty percent of internet shoppers who purchased a product online in the first six months of the year returned it within six months. According to Forrester Research, the value of internet returns after the 2000 Christmas season was nearly 600 million dollars. According to Hammond and Kohler (2002), 12-35% of clothing purchased from catalogs is returned.

[3] The seller's salvage value for a return that is to be discarded is zero (e.g., restaurant food). If a return is to be resold, the salvage value is still low because it is equal to the (often marked down) resale price less the cost of refurbishing, repackaging, restocking, and storing the good for resale.

[4] "Returns Don't Need to Cost So Much," *Internet Retailer,* www.internetretailer.com, May 23, 2002.

[5] According to e-BuyersGuide.com's 1999 "Return to Sender" Shoppers' Expressions survey, 17 percent of those who returned a product purchased online said it (apparel) did not fit, 15 percent said they simply did not want the product, and 16 percent said the wrong product was delivered. Another 27 percent said the returned products were of poor quality or damaged.

who can learn their personal values for it only by obtaining and using it on a trial basis. However, if all parties are risk neutral, this basic model does not generate excessive refunds. Efficiency then requires the refund to equal the seller's salvage value for a return; only in this case will the consumer return the good precisely when she learns her value is less than the seller's salvage value, as allocative efficiency prescribes. Thus, since competitive equilibria are efficient,[6] competitive refunds are not excessive. Neither are monopoly refunds: as we shall show, a monopoly seller in this basic model extracts rent by charging a high price rather than promising a distortionary refund.

On the other hand, excessive refunds do arise in the basic model if the consumers are made risk averse. Because a consumer's value for the good is unverifiable, a second-best efficient outcome consists of firms offering excessive refunds that partially insure consumers against the risk of realizing a low value.[7] In our view, however, the applicability of an explanation based on risk aversion is limited. It is implausible that consumers are significantly risk averse with respect to many products for which returns are prevalent, such as clothing, books, and even home electronics, that cost little relative to personal wealth. What is needed, then, is an explanation that does not depend on risk aversion.

Given that the learning-by-purchasing model is inconsistent with the observation of excessive refunds when consumers are risk neutral, what is wrong with it? We suggest that it is the assumption that a consumer can learn her value for the good *only* by trying it out. Consumers in reality often have other ways of acquiring this information. Before they decide to purchase a good, consumers often do research to learn about its features, and which are important to them. They read product reviews, consult experts and friends, study their needs, and so forth. Given this second channel for information acquisition, a consumer chooses between learning her value by conducting prior research, or by purchasing the good to try it out. The smaller the refund offered for a return, the more attractive the consumer finds the prior research option.

We are thus led to a *learning-by-researching-or-purchasing* model. Its central premise is that consumers can privately learn their values in one of two ways, by conducting prior research or by purchasing the good and trying it out. We refer to the consumers who learn their values prior to purchasing as *informed*, and to the remainder as *uninformed.* The prior research option may be costly for a consumer, and possibly only some may

[6] This is easily proved, and is the special case $\lambda = 0$ of our Proposition 1.

[7] Although we have found no reference for it, this result is not hard to show. Che (1996) studies the basic learning-by-purchasing model with risk averse consumers, but restricts attention to the monopoly case and requires refunds to equal the purchase price, as we discuss below.

choose it. A firm may or may not want to encourage the prior information acquisition. It may also want to offer a menu of contracts from which the informed and uninformed will make different choices.

In order to disentangle the information acquisition and screening effects, we restrict attention to two polar versions of the model. In *Model IA (Information Acquisition),* all consumers have the same, intermediate cost of becoming informed. They all thus make the same information acquisition decision. This removes the screening role of a refund, allowing us to focus on the use of refunds for dissuading consumers from acquiring prior information.

We take the opposite tack in *Model SC (Screening).* In this version of the model, some consumers have a negligible or even negative cost of acquiring prior information, and so always become informed. The remaining consumers find it impossible to become informed prior to purchasing. This removes the information acquisition decision from the model, allowing us to focus on the role of refunds for screening the uninformed from the informed consumer types.

The addition of the prior research option does not change some of the results of the basic learning-by-purchasing model. In particular, in both versions of the model we find that efficiency still requires refunds to equal the seller's salvage value for a return. Since we also find that competitive equilibria are still efficient, competitive refunds are still not excessive.[8] Excessive refunds do arise, however, if the good is sold by a monopoly. It may be a monopoly retailer or, under an alternative interpretation, a monopoly wholesaler or manufacturer selling to a competitive retail sector.

We can now give a preview of the main results for a monopoly seller, in each version of the model.

Model IA

The seller in Model IA may or may not want to choose a refund contract that induces the consumers to stay uninformed, depending on which of two opposing forces prevails. The seller benefits when they stay uninformed because they then receive no information rents, and the cost of acquiring prior information is not incurred. On the other hand, when the consumers become informed, the seller benefits by not incurring the net cost

[8] Thus, we do not find that excessive refunds are due to competitive pressure, contrary to views expressed in the retailing literature. E.g., Bayles (2000) writes, "Reverse Logistics as a Competitive Weapon: Returns started spinning out of control back in the late 1980s, when many retailers began using returns as a competitive weapon in the battle to win market share."

of producing those units of the good that would have been returned if the consumers had remained uninformed. The former force is stronger if the consumers' cost of prior information acquisition lies in an intermediate range. The firm then offers an excessive refund in order to deter them from becoming informed. A full refund of the purchase price is optimal in some cases.

Model SC

The seller in this version of the model offers, in principle, a menu containing a refund and a no-refund contract. The informed consumers choose the no-refund contract, and the uninformed choose the refund contract. In order to deter the informed consumers from choosing the refund contract, it must specify a purchase price greater than that of the no-refund contract. When this incentive constraint binds, both contracts specify the same purchase price, which is equivalent to the seller offering the same contract to all consumers. (The informed just ignore its refund provision.) In order for the refund contract to specify a purchase price as high as that of the no-refund contract without deterring the uninformed from purchasing, the refund must sometimes exceed the seller's salvage value for a return. Excessive refunds thus arise when the incentive constraint of the informed and the participation constraint of the uninformed both bind. Again, even a full refund of the purchase price is optimal in some cases.

1.1. Related Literature

Davis et al. (1995) and Che (1996) present early learning-by-purchasing models. Davis et al. (1995) assume consumers are risk neutral, and show that a monopoly prefers to offer a full money-back refund, rather than no refund at all, if it has a high salvage value for a return. Che (1996) assumes consumers are risk averse, and shows that a monopoly also prefers to offer a full refund rather than no refund if the consumers are risk averse enough. These papers do not consider partial refunds, and so do not address the excess refund puzzle. It seems clear that excessive refunds would be generated if partial refunds were to be allowed in Che (1996), yielding an explanation based on risk aversion. On the other hand, we conjecture that if partial refunds were allowed in Davis et al. (1995), optimal refunds would be too small rather than too large.[9,10]

Courty and Li (2000) present a screening model somewhat related to our Model SC. It has a different purpose, namely, to shed light on when menus of contracts are actually used, such as an airline's menu of business (refundable) and economy (less-refundable) tickets. Unlike in our model, all consumers stay uninformed of their values prior to purchasing. A consumer's private type is the distribution from which her value will be drawn after purchasing. The value distribution of a "high" type is greater than that of a "low" type either in the sense of first-order stochastic dominance, or in the sense of being a mean-preserving spread. The main result is that the refunds high type consumers obtain are equal to the seller's salvage value (which is the production cost of the good); the refunds the low types obtain may bear any relationship to the salvage value. If the optimal menu ever contains just one contract, the refund it specifies is equal to the salvage value. The model thus sheds little light on the excess refund puzzle.

Turning to Model IA, it can be viewed as a contribution to the literature on mechanisms that prevent, encourage, or determine information acquisition, such as Cremer and Khalil (1992), Lewis and Sappington (1997), Cremer et al. (1998a,b), and Bergemann and Välimäki (2002). It also relates to studies of how much information a seller should directly provide buyers about their personal values, such as Lewis and Sappington (1994), Bergemann and Pesendorfer (2002), and Eso and Szentes (2004).

More narrowly, Model IA can be viewed as an exploration of an early suggestion made by Barzel (1982) that sellers may sometimes want to prevent buyers from acquiring information. That suggestion is also formalized recently in Barzel et al. (2004) in a model of IPO policies. An underwriter "stabilizes" an IPO by promising to agree to buy back a certain fraction of the shares from the buying investors at the IPO price. This is analogous to a stochastic contract in our framework that randomizes between a zero and a full refund. Barzel et al. (2004) show that if the underwriter wants to deter buyers from acquiring information, its optimal stabilization policy pays the full refund with positive probability.[11]

[9] This is because the consumers in Davis et al. (1995) benefit from the good during the trial period. This should create a downward force on refunds, since large refunds aggravate the moral hazard of consumers purchasing the good only to return it after use during the trial period.

[10] Marvel and Peck (1995) study refunds in a less related context. They show that a wholesaler might offer a retailer a refund for units of its good left unsold; this induces the retailer to stock enough of the good when it faces uncertain demand.

[11] See Remark 3 in the Appendix of Barzel et. al. (2004).

The retailing literature deals with return policies under the rubric of reverse logistics (Rogers and Tibben-Lembke, 1998). None of it to our knowledge bears on the excess refund puzzle. The study most related to Model SC seems to be Heiman et al. (2002), which shows how menus consisting of a full-refund contract, a no-refund contract, and an unbundled money back guarantee (essentially a pure insurance contract) can be used to screen consumer types that have different value distributions, roughly as in Courty and Li (2000). Regarding Model IA, the most relevant paper seems to be Heiman et al. (2001), which informally compares the relative merits of pre-purchase product demonstrations to money back guarantees as ways to reduce consumer uncertainty. Neither it nor any other study we have seen in the retailing literature considers the possibility that firms may not want consumers to acquire information.

1.2. Structure of the Paper

The environment is described in Section 2. Models SC and IA are studied in Sections 3 and 4, respectively; Model SC is studied first because it provides the building blocks for Model IA. In both cases the efficient, competitive, and monopoly contracts are characterized. The analysis is applied to a monopoly wholesaler, rather than a monopoly retailer, in Section 5. Concluding remarks are in Section 6. Appendices A and B contain the proofs for Sections 3 and 4, respectively. Appendix C contains the calculations for the examples.

2. Environment

A discrete returnable good is to be sold to a unit mass (continuum) of potential buyers. We consider a competitive market, but devote attention to a monopolized market. We refer to a seller as a firm and the buyers as consumers, having in mind a retailer and its customers. Under an alternative interpretation discussed in Section 5, the seller is a wholesaler or manufacturer that sells its good to a competitive retail sector, and offers refunds to the retailers for the goods that the consumers return to them.

2.1. Consumers

Each consumer wants at most one unit of the good. Her value for it, v, is drawn from a distribution F that has a positive and differentiable density, f, on $[0, 1]$, with mean $\bar{v}$. An *informed consumer* knows her value for the good when she decides whether to purchase it, and an *uninformed consumer* does not. No consumer's value is observed by another party.

An uninformed consumer who purchases the good learns her value for it during an initial trial period. The good gives her no benefit if she returns it at the end of the trial period. The consumer bears a return cost of $t \geq 0$ if she tries the good and then returns it to the seller.

A consumer with value v who purchases the good for price p receives utility $v - p$ if she keeps it, gross of any cost she might have borne to become informed. If she instead returns the good for a refund $\hat{r}$, her utility is $\hat{r} - t - p$.

We focus on two versions of the model that differ in how the number of informed consumers is determined.

Model SC (Screening). In this version an exogenously given fraction $\lambda \in (0, 1)$ of the consumers are informed. In essence, these consumers have a negligible or even negative cost of doing prior research to become informed. The remaining $1 - \lambda$ consumers are necessarily uninformed, and so can learn their values only by trying the good out. Whether a consumer is informed is independent of her value.

Model IA (Information Acquisition). In this version all consumers are ex ante identical and uninformed. Once she knows the set of contracts available in the market, each consumer chooses whether to pay an *information cost,* $c \in (0, 1)$, in order to become informed ("acquire information").

An encompassing model would allow the consumers to be arbitrarily heterogeneous in their information costs. Consumers with very high or low information costs would be like those of Model SC, and consumers with intermediate information costs would be like those of Model IA. By restricting attention to Models SC and IA, we are able to isolate the two forces at work, screening and information acquisition.

2.2. Firm

The firm's constant unit cost of procuring the good is $k \in [0, 1)$. This is either the cost of directly producing the good, or of obtaining it from a wholesaler.

The *gross salvage value* to the firm of a returned good is denoted by $\hat{s}$. We assume it is no greater than the cost of obtaining a new unit: $\hat{s} \leq k$. This is obviously the case when a returned good is simply discarded. It is also the case when a returned good is resold, as then the salvage value is equal to the cost k that is saved when a returned rather than a new unit is used to make a sale, less the refurbishing, restocking, and storing costs that are required to resell a returned good.[12]

We also assume $\hat{s} \geq t$: the salvage value of the good is no less than the consumer's cost of trying and returning it. Most of the results would also hold if $\hat{s} < t$, but the proofs would differ slightly.

The *(net) salvage value* of the good is its salvage value less the consumer's cost of trying and returning it: $s \equiv \hat{s} - t$. In terms of the net salvage value, the parameter assumptions $0 \leq t \leq \hat{s} \leq k$ become

Assumption 1. $k - s \geq t \geq 0$ and $s \geq 0$.

2.3. Contracts

The *gross refund* paid by the firm for a return is $\hat{r}$. The *(net) refund* the consumer receives is the gross benefit less the cost of trying and returning, $r \equiv \hat{r} - t$. We assume the gross refund cannot be negative, which is equivalent to $r \geq -t$. A *refund contract* is a pair (p, r) consisting of the purchase price p and the net refund r.

A firm should never offer a gross refund greater than the purchase price. Unlike the possibly significant cost t of returning the good after trying it, a consumer's cost of returning the good immediately after purchasing it is presumably negligible. Hence, offering a refund greater than the price would create a money pump in which consumers would purchase and return large numbers of the good, creating a big loss for the firm. We accordingly require $p \geq \hat{r}$, which is equivalent to $p \geq r + t$.

We thus deem a contract (p, r) to be *feasible* if it satisfies the following condition:

(FE) $0 \leq r + t \leq p$.

A contract with a zero refund takes the form $(p, -t)$, since its gross refund is $\hat{r} = r + t = 0$. Of course, any contract with a nonpositive net refund will generate no returns, and hence be equivalent to a contract with a zero refund. We thus refer to any contract (p, r) with $r \leq 0$ as a *no-refund contract.*

A *full (money-back) refund* contract is one with $\hat{r} = p$, or rather, $(p, r) = (p, p - t)$.

[12] The alternative case, $\hat{s} > k$, is less plausible, though it might hold if $\hat{s}$ is the price at which the firm can sell the good in a separate, distinct market.

2.4. Payoffs

An uninformed consumer returns the good if and only if she learns that her value is less than the net refund offered for a return. Her induced value,

$$V_u(r) \equiv \int_0^1 \max(v, r) dF(v). \tag{1}$$

is the most she would be willing to pay for the bundled good and refund option. Her expected utility from purchasing according to the terms of a contract (p, r) is $V_u(r) - p$.

The probability that an uninformed consumer returns the good is $F(r)$. The expected profit of the firm when an uninformed consumer chooses a contract (p, r) is thus

$$\pi_u(p, r) \equiv p - k + (s - r)F(r), \tag{2}$$

Turning to the informed consumers, note that they do not care about the refund. An informed consumer purchases the good only if she knows she will keep it, since the refund is not more than the price. She purchases the good only if her value exceeds the price. Her gross expected utility when offered price p is

$$V_i(p) \equiv \int_p^1 (v - p) dF(v). \tag{3}$$

Her net expected utility is $V_i(p) - c$ if she paid c to learn her value. The firm's expected profit from offering the good for price p to an informed consumer is thus

$$\pi_i(p) \equiv (p - k)(1 - F(p)).$$

Assumption 2. $\pi_i(\cdot)$ has a unique maximizer, p_I, and $\pi_i'(p) \gtreqless 0$ as $p \lesseqgtr p_I$.

3. Model SC

In this section we characterize in turn the efficient, competitive, and monopoly contracts in Model SC.

3.1. Efficient Contracts

Efficient Contracts for the Informed

It is efficient to procure the good for an informed consumer if and only if her value for it exceeds the procurement cost, i.e., $v \geq k$. This outcome would be achieved if she were to be offered any feasible contract of the form $(p, r) = (k, r)$. The amount of the promised refund is irrelevant, as an informed consumer who purchases the good never returns it.

Efficient Contracts for the Uninformed

If an uninformed consumer obtains the good and learns her value is v, a surplus of s or v is generated depending on whether she returns the good. Efficiency requires the good to be returned if $v < s$. The resulting gross surplus is $\max(v, s)$. The expectation of this is $V_u(s)$, where $V_u(\cdot)$ is defined in (1). Hence, the maximal expected surplus generated by giving an uninformed consumer the good is

$$S_u^* \equiv V_u(s) - k. \tag{4}$$

We assume it is efficient to procure the good for an uninformed consumer:

Assumption 3. $S_u^* > 0$.

If an uninformed consumer purchases the good according to the terms of a contract (p, r), the resulting outcome is efficient if and only if $r = s$. The refund cannot be greater or less than the salvage value, for then the consumer would inefficiently return or keep the good when her value is between r and s. In addition, the purchase price cannot be too high: $p \leq V_u(s)$ is required in order for an uninformed consumer to purchase.

Among the efficient contracts for an uninformed consumer that give both parties nonnegative payoffs, (k, s) is the best for the consumer, as it gives the firm zero profit. The best for the firm is $(V_u(s), s)$, which extracts the full surplus S_u^*.

Achieving Efficiency

In equilibrium, each consumer who purchases the good chooses her most preferred contract in the market. The resulting outcome is efficient if and only if the informed choose a contract with price k, and the uninformed choose a contract with refund s.

The primary example of an efficient contract is (k, s). If it is the only contract offered, an efficient outcome is achieved . Every informed consumer purchases the good if her value is greater than k, and never returns it. Every uninformed consumer purchases the good, and returns it if she learns $v < s$. The firms make zero profit.

Efficiency can also be achieved by a menu of contracts of the form $\{(k, r), (p, s)\}$, provided the informed choose (k, r) and the uninformed choose (p, s). In general, many such incentive compatible and individually rational menus exist. But in any case, efficiency is achieved only if the uninformed choose a contract that specifies the refund to be the salvage value. Efficiency precludes the paying of excessive refunds.

3.2. Competitive Contracts

As Rothschild and Stiglitz (1976) proved, competition among firms for consumers with privately known types may yield an inefficient outcome. Here, whether a consumer is informed or uninformed is her privately known type. If competitive equilibria were to be inefficient, perhaps competition could generate excessive refunds. However, as we now show, in our model competitive equilibria are efficient.

Assuming the presence of multiple firms, define a *competitive menu of contracts* to be a set of refund contracts such that (a) each operating firm offers one or more of them; (b) each contract is chosen by a positive mass of consumers; (c) each firm makes nonnegative profit; and (d) no firm or entrant can offer a new contract that would attract consumers away from the menu and make positive profit.

Observe that the efficient singleton menu $\{(k, s)\}$ is a competitive menu. The contract (k, s) gives zero profit to any firm that offers it, whether it is chosen by an informed or an uninformed consumer. Its efficiency implies that no other contract can both attract a consumer and yield positive profit.

Other menus of contracts are also competitive, such as the outcome-equivalent menu $\{(k, 0), (k, s)\}$ from which the informed choose either contract. But they all achieve a zero-profit efficient outcome:

Proposition 1. *Every competitive menu of contracts achieves an efficient outcome, and every contract in it earns zero profit. In particular,* (k, s) *is in the menu and chosen by all uninformed consumers.*

The proof of Proposition 1 is in the Appendix , and is fairly simple. At its heart is the observation that (k, s) is a surplus-maximizing contract for either type of consumer. It also generates the same profit regardless of which type of consumer chooses it, as does any contract with a refund equal to the salvage value. Hence, a putative inefficient equilibrium can always be destabilized by an entrant offering a contract that specifies a refund equal to the salvage value, and a price slightly higher than k. Such a contract is guaranteed to make a profit, no matter which types it attracts – it is impervious to the adverse selection that makes equilibria inefficient in Rothschild and Stiglitz (1976). A standard undercutting argument then shows that all equilibria are efficient.

Any refund paid for a return in a competitive equilibrium is thus equal to the salvage value of the good. Competitive pressure does not account for excessive refunds.

3.3. Monopoly Contracts

Assume now there is only one firm. We consider first its optimal menu of contracts that induces the uninformed to purchase. Without loss of generality, we assume the menu contains two contracts, one selected by each type of consumer. We can also assume the contract meant for the informed is a no-refund contract; these consumers do not care about refunds, and giving them no refund maximally weakens the incentive constraint of the uninformed. Such a menu can be written as (p_i, p_u, r), where p_i is the price specified by the no-refund contract and (p_u, r) is the refund contract.

The firm's optimal no-exclusion menu solves the following program:

$$
\begin{aligned}
&\text{(P)} \quad \max_{p_i, p_u, r} \lambda \pi_i(p_i) + (1-\lambda)\pi_u(p_u, r) \\
&\quad \text{subject to} \\
&\quad (\text{IR}_u) \quad V_u(r) - p_u \geq 0, \\
&\quad (\text{IC}_u) \quad V_u(r) - p_u \geq \bar{v} - p_i, \\
&\quad (\text{IC}_i) \quad p_u \geq p_i, \\
&\quad (\text{FE}) \quad 0 \leq r + t \leq p_u.
\end{aligned}
$$

Constraint (IR_u) is the individual rationality constraint insuring that the uninformed purchase. The incentive constraint (IC_u) requires an uninformed consumer to prefer (p_u, r) to the no-refund contract, which gives her utility $V_u(0) - p_i = \bar{v} - p_i$. Incentive constraint ($\text{IC}_i$) requires an informed consumer to prefer the no-refund contract; she does not care about the refund, and so prefers the contract with the lower price.

As the first step in solving (P), consider the relaxed problem obtained by removing both incentive constraints. Recall that p_I maximizes $\pi_i(\cdot)$, and contract $(V_u(s), s)$ maximizes $\pi_u(p_u, r)$ subject to (IR_u). Hence, the solution to this relaxed problem, the *first-best menu,* is

$$M^{FB} \equiv (p_I, V_u(s), s).$$

When $p_I \in [\bar{v}, V_u(s)]$, the menu M^{FB} satisfies both incentive constraints: (IC_u) holds because $0 \geq \bar{v} - p_I$, and (IC_i) holds because $V_u(s) \geq p_I$. Furthermore, the firm cannot gain by excluding the uninformed in this case, as it would lose the profit S_u^* from each of them without being able to extract more from the informed. The menu M^{FB} is therefore the firm's optimal menu in this case.

When instead $p_I < \bar{v}$, the first-best menu violates the uninformed's incentive constraint (IC_u), since the no-refund contract with price p_I gives the uninformed positive utility. The constraint is optimally restored by making the no-refund contract less attractive by raising p_i above p_I, and by making the refund contract more attractive by lowering p_u below $V_u(s)$. The refund remains equal to the salvage value. This is because raising the refund is an inefficient way to give the uninformed rent. The firm's profit on an uninformed consumer is the surplus generated by the transaction less the rent she must be given to satisfy her incentive constraint, and hence is maximized by setting the refund equal to the salvage value to maximize the surplus, and lowering the non-distortionary price p_u to give the consumer the required rent.

We have thus obtained the firm's optimal scheme when $p_I \leq V_u(s)$. The following proposition, proved in the Appendix , summarizes.

Proposition 2. *If $p_I \leq V_u(s)$, the firm does not exclude the uninformed, and its optimal menu satisfies $r = s$. This optimal menu is M^{FB} if $p_I \in [\bar{v}, V_u(s)]$. If $p_I < \bar{v}$, then $p_i \in (p_I, \bar{v}]$ and $p_u = V_u(s) + p_i - \bar{v}$.*

Excessive refunds are thus possible only when $p_I > V_u(s)$. In this case the first-best menu violates the informed consumers' incentive constraint, (IC_i), since the price $V_u(s)$ in the refund contract is less than the p_I of the no-refund contract. As we shall prove, the constraint is optimally restored by making the no-refund contract more attractive by lowering p_i below p_I, and by making the refund contract less attractive by raising p_u above $V_u(s)$. But raising p_u will cause the uninformed to refrain from purchasing – unless the refund is raised as well. This generates an excessive refund.

However, lowering p_i causes a loss in profit on the informed that may outweigh the profit obtained from the uninformed. If so, the firm should simply offer the no-refund contract with price p_I that maximizes its profit on the informed. The uninformed then will not purchase, since $p_I > V_u(s)$ implies $p_I > \bar{v}$. The firm does not prefer this no-exclusion strategy if the informed consumers are only a small fraction of the population. The following theorem gives the details.

Theorem 1. *If $p_I > V_u(s)$, then $\bar{\lambda} \in (0, 1]$ exists such that the firm does not exclude the uninformed if $\lambda < \bar{\lambda}$. In this case the optimal menu satisfies*

$$p_i = p_u = V_u(r) < p_I, \text{ and } r \geq s.$$

Furthermore, if $s > 0$ or $\pi_i'(\bar{v}) > \frac{1-\lambda}{\lambda}$, then $r > s$.

In addition to showing the optimality of excessive refunds, Theorem 1 also shows that the firm can achieve its optimal profit by offering just one contract. Because the two contracts in the optimal no-exclusion menu specify the same purchase price, the firm achieves the same outcome by offering just one contract, $(V_u(r), r)$. This is in contrast to the case of Proposition 2, since then $p_i < p_u$ except in exceptional cases. Thus, within the context of Model SC, a firm observed to sell a good for one price without a refund and for a higher price with a promised refund, is not offering an excessive refund. But a firm observed to always sell its product with a refund may indeed be offering an excessive refund.

We note in passing that each case in Proposition 2 and Theorem 1 holds for some parameters. For example, if $k > 0$ and F is uniform, we have the case $p_I > V_u(s)$ of Theorem 1.[13]

We end this section with an example showing that a firm in Model SC may optimally offer a full money-back refund, as in reality many do.

Example 1. *Let F be the uniform distribution, and let $\lambda = .5$, $k = .45$, $s = .2$, and $t = .245$. As noted above, this case is that of Theorem 1. Calculations presented in the Appendix show that an optimal strategy for the firm is to offer a single contract, $(p, r) = (.545, .3)$. This contract does not exclude the uninformed. It is a full refund contract: the gross refund is $\hat{r} = r + t = .545 = p$.*

4. Model IA

We now consider Model IA, studying in turn efficient and monopoly contracts.

4.1. Efficient Contracts

For the same reasons as in Model SC, if it is efficient for consumers to stay uninformed, they must be given a contract of the form (p, s), with the price p low enough that they purchase. If instead it is efficient for them to become informed, they must be given a contract specifying k as the purchase price. Determining whether they should become informed requires a comparison of social benefits and costs.

The social benefit of a consumer becoming informed is that the procurement cost of the good can be saved when her value turns out to be less than k, as she should then not be given the good. The social cost of her becoming informed is the information cost c, and the expected opportunity cost of the unrealized net salvage value.

Formally, the expected surplus created if the consumers become informed is

$$S_i^*(c) \equiv \int_k^1 (v - k) dF(v) - c = V_i(k) - c. \tag{5}$$

[13] If $F(v) = v$ and $k \in (0, 1)$, then $p_I = \frac{k+1}{2} > \frac{s^2+1}{2} = V_u(s)$. If $F(v) = v^2$, each case can occur: $p_I < \bar{v}$ if $k = s < .25$; $p_I \in [\bar{v}, V_u(s)]$ if $.25 \leq k = s \leq .26649$; and $p_I > V_u(s)$ if $.26649 < k = s$.

On the other hand, the surplus created if they stay uninformed is $S_u^* = V_u(s) - k$. Whether they should become informed depends on which surplus is greater. Equating the two and solving for c yields the *social value of pre-purchase information:*

$$c^* \equiv V_i(k) - S_u^* = \int_s^k F(v)dv. \tag{6}$$

Efficiency requires the consumer to stay uninformed only if $c \geq c^*$.

As in Model SC, efficiency is achieved if (k, s) is the only contract available. To prove this, we now need only to show that this contract induces efficient information acquisition. It does so because it gives a consumer all the surplus that can be generated given her information choice: she obtains utility $V_i(k) - c = S_i^*(c)$ if she becomes informed, and $V_u(s) - k = S_u^*$ if she does not. Each consumer thus acquires information efficiently if offered (k, s).

Other contracts also achieve efficiency. If $c \leq c^*$, any contract specifying a purchase price of k and a refund less than the salvage value achieves an efficient outcome, since lowering the refund only increases the incentive to take the efficient action of becoming informed. If $c > c^*$, contracts generally exist that specify a greater price and achieve efficiency. The price cannot, however, be so high as to induce the consumers to become informed or refrain from purchasing.

Despite this multiplicity, efficiency requires that any refund ever paid be equal to the salvage value, and hence precludes the paying of excessive refunds.

Furthermore, competitive refunds are also not excessive, because again a competitive equilibrium is efficient. In particular, if $c > c^*$ the competitive equilibrium consists of all firms offering (k, s). We omit a formal statement and proof of this, as the argument is the standard one of Bertrand undercutting. Adverse selection is not an issue now, since the consumers are ex ante identical.

4.2. Monopoly Contracts

We show now that a monopoly firm offers an excessive refund if the consumers' information cost is not too low or high. The firm raises the refund above the salvage value so that it can charge a higher price without triggering information acquisition.

Consider first a consumer's decision to acquire information. When offered a contract (p, r), she becomes informed if $V_i(p) - c \geq V_u(r) - p$. Using (1) and (3) and integrating by parts, this becomes

$$c \leq \int_r^p F(v)\, dv. \tag{7}$$

The expression on the right of this inequality is the consumer's value, when offered the contract, for the pre-purchase information. She acquires the information only if her cost of doing so is less than her value for it.

Now, recall the contract $(V_u(s), s)$ that would yield profit S_u^* if the consumers were to stay uninformed. When offered it, a consumer's value for information is

$$\int_{s}^{V_u(s)} F(v)\,dv \equiv \bar{c}. \tag{8}$$

So the contract induces her to stay uninformed if her information cost exceeds $\bar{c}$. Comparing (6) to (8), we see that $\bar{c}$ exceeds the efficient critical cost c^*. Thus, when $c \geq \bar{c}$ the maximal surplus is greater when consumers stay uninformed: $S_u^* > S_i^*(c)$. Furthermore, the firm's profit if it offers any contract that induces consumers to become informed is at most the maximal total surplus $S_i^*(c)$. The firm is therefore best off offering $(V_u(s), s)$ to obtain profit S_u^*. This proves the following:

Lemma 1. *If* $c \geq \bar{c}$, *the firm's unique optimal contract is* $(V_u(s), s)$, *and the consumers stay uninformed.*

When the information cost is less than $\bar{c}$, the firm must take into account the possibility that the contract it chooses to offer may induce information acquisition. In order to determine the firm's optimal offer, we derive separately its optima within the sets of contracts that do and do not induce the consumers to become informed.

Inducing Consumers to Stay Uninformed

A contract that induces consumers to stay uninformed fails to satisfy (7). Thus, if we define a price $P(r,c)$ by[14]

$$\int_{r}^{P(r,c)} F(v)\,dv \equiv c, \tag{9}$$

a consumer is content to stay uninformed if and only if the following *information acquisition constraint* holds:

$$(\text{IA}_u) \quad p \leq P(r,c).$$

It is easy to show that $P(r,c)$ increases in both arguments. Hence, the greater is the refund or the information cost, the more the purchase price can be raised without

[14]Let $P(r,0) = 0$ for $r < 0$, as any nonpositive price satisfies (9) if $r \leq 0$ and $c = 0$. Then P is continuous on $[-t,\infty) \times \mathbb{R}_+$.

triggering information acquisition. The contract $(V_u(s), s)$ that extracts the full surplus from the uninformed fails to satisfy this constraint precisely when $c < \bar{c}$.

The maximal profit obtainable while inducing the consumers stay uninformed is

$$\begin{aligned}
(\mathrm{P}_u) \quad & \Pi_u(c) \equiv \max_{p,r} \; p - k - (r-s)F(r) \\
& \text{subject to } (\mathrm{IA}_u), \\
& (\mathrm{IR}_u) \quad p \leq V_u(r), \\
& (\mathrm{FE}) \quad 0 \leq r + t \leq p.
\end{aligned}$$

The following proposition states that if the information cost is less than $\bar{c}$, but the firm can still make profit without inducing information acquisition,[15] it does so optimally by offering a contract for which the information acquisition constraint binds and the refund is excessive.

Proposition 3. *If $c < \bar{c}$ and $\Pi_u(c) > 0$, any solution (p^*, r^*) of (P_u) satisfies $p^* = P(r^*, c)$, and $r^* > s$.*

We explain the rationale for the excess refund result of Proposition 3 by comparing its "low c" case to the "high c" case of Lemma 1. In the latter case, a consumer's threat of becoming informed is not credible. This allows the firm, given any refund, to set the purchase price equal to a consumer's induced value for the good, $V_u(r)$. The firm's marginal benefit if it then raises the refund is the amount that doing so allows this price to be raised:[16]

$$MB^H \equiv V_u'(r) = F(r).$$

On the other hand, in the low c case the information constraint binds, and so the firm sets the price equal to $P(r, c)$ in order to deter information acquisition. The firm's marginal benefit from raising the refund in this case is

$$MB^L \equiv P_r(r, c) = \frac{F(r)}{F(P(r,c))}.$$

Note that $MB^L > MB^H$ (as $P(r, c) < 1$) : the firm's marginal benefit from raising the refund is strictly greater when it must deter information acquisition. The net cost of providing the refund option is $(r - s)F(r)$ in both cases, and so raising the refund has

[15] A solution of (P_u) that yields nonpositive profit is not relevant, since then the firm optimally chooses a contract that induces consumers to become informed. See Lemma B6 in the Appendix.

[16] The fact that $V_u'(r) = F(r)$ is intuitive. An increase of Δr in the refund increases the consumer's induced value by the increase in the expected refund payment, $F(r)\,\Delta r$.

the same marginal cost in both cases. The optimal refund in the low c case is therefore greater than the optimal refund in the high c case. The latter refund is the salvage value, by Lemma 1, and so the former refund must exceed the salvage value.

Inducing Consumers to Become Informed

We now turn to the firm's optimal contract that induces the consumers to become informed. We can restrict attention to no-refund contracts, since the informed do not return the good. The consumers then choose to become informed only if the purchase price exceeds $P(0,c)$. This yields another information acquisition constraint,

$$(\text{IA}_i) \quad p \geq P(0,c).$$

Even if this is satisfied, the consumers will still stay uninformed if their payoff from becoming informed, $V_i(p) - c$, is negative. This yields the individual rationality constraint

$$(\text{IR}_i) \quad p \leq P_i(c),$$

where $P_i(\cdot)$ is the inverse of $V_i(\cdot)$.

These two constraints are necessary and sufficient for the contract to induce the consumers to become informed. The optimal price solves the program

$$(\text{P}_i) \quad \Pi_i(c) \equiv \max_p \ \pi_i(p) \text{ subject to } (\text{IA}_i) \text{ and } (\text{IR}_i).$$

For small enough c, neither constraint binds and the optimal price is just p_I. For higher c, one of the constraints binds, and so determines the solution. Which one binds depends on whether p_I is greater than the mean value $\bar{v}$, as the following proposition shows. The two alternative cases are depicted together in Figure 1.

Proposition 4. *Program (P_i) has a solution if and only if $c \in [0, V_i(\bar{v})]$. If $p_I \geq \bar{v}$, the solution is*

$$p^*(c) = \begin{cases} p_I & \text{if } c \leq V_i(p_I) \\ P_i(c) & \text{if } c \geq V_i(p_I). \end{cases} \tag{10}$$

If $p_I < \bar{v}$, the solution is

$$p^*(c) = \begin{cases} p_I & \text{if } c \leq \int_0^{p_I} F(v)dv \\ P(0,c) & \text{if } c \geq \int_0^{p_I} F(v)dv. \end{cases} \tag{11}$$

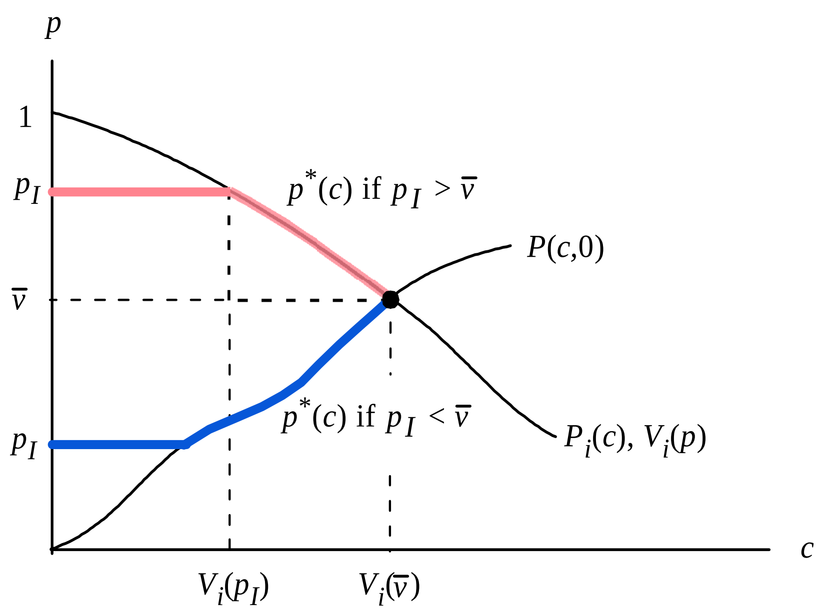

Figure 1: The two possible forms of a solution to (P_i).

The Optimal Contract

The proof of the following theorem shows that a critical $\underline{c}$ exists such

$$\Pi_i(c) \gtreqless \Pi_u(c) \text{ as } c \lesseqgtr \underline{c}.$$

The firm's optimal strategy is thus to deter consumers from becoming informed precisely when c exceeds $\underline{c}$. The optimal refund is excessive in a range of cases because $\underline{c}$ is strictly less than $\bar{c}$: when c is between these two critical levels, the firm induces consumers to stay uninformed by offering a refund greater than the salvage value.

Theorem 2. *Unique numbers* $0 \leq \underline{\underline{c}} \leq \underline{c} < \bar{c}$ *exist such that the firm's optimal contracts and the consumers' information acquisition decisions are the following:*

1. *if* $c < \underline{\underline{c}}$*, the firm offers a no-refund contract with price* p_I*, and the consumers become informed;*

2. *if* $c \in (\underline{\underline{c}}, \underline{c})$*, the firm offers a no-refund contract with price* $P_i(c)$*, and the consumers become informed;*

3. *if* $c \in (\underline{c}, \bar{c})$*, the firm offers a refund contract with refund* $r > s$ *and price* $P(r, c)$*, and the consumers stay uninformed; and*

4. *if* $c \geq \bar{c}$, *the firm offers* $(p, r) = (V_u(s), s)$, *and the consumers stay uninformed.*

Furthermore, (*a*) $\underline{\underline{c}} = 0$ *if and only if* $t = 0$ *and* $s = k$, *and* (*b*) $\underline{\underline{c}} = \underline{c}$ *if* $p_I \leq \bar{v}$.

We end this section with an example again showing that the firm may optimally offer a full refund. This occurs when the transaction cost t is high enough that the feasibility constraint in (P_u) binds, so that the gross refund is equal to the purchase price. The example also shows that $\underline{c}$ may be more or less than c^*. This implies that the firm may induce too little or too much information acquisition. When $\underline{c} < c < c^*$, the firm offers a refund that deters the consumers from becoming informed, whereas a benevolent social planner would not; the opposite is true when $c^* < c < \underline{c}$.

Example 2. *Let* F *be the uniform distribution,* $s = .125$, *and* $k = .375$. *Then Assumptions* $1-3$ *are satisfied for* $t \leq .25$. *Given these parameters, in an efficient outcome the consumers become informed only if their information cost is no greater than* $c^* = .0625$. *The following claims are true.* (a) *If* $c = .1$ *and* $t \in [.228, .25]$, *the consumers stay uninformed and the firm optimally offers a full refund.* (b) *If* $t = .23$, *then* $\underline{c} > c^*$. (c) *If* $t = 0$, *then* $\underline{c} < c^*$.

5. Monopoly Wholesaler

Rather than being a monopoly retailer, an alternative interpretation in either model is that the firm is a monopoly wholesaler or manufacturer that sells to a competitive retail sector. The questions then center on the price and refund the wholesaler offers retailers. The results of both models still hold, assuming the returns of a retailer to the wholesaler are the goods returned to it by consumers.[17] This is because the competition between retailers drives their profits to zero, and they hence simply pass through to consumers the price and refund set by the wholesaler. It is then as though the wholesaler deals directly with the consumers.

To be specific, let our firm be a wholesaler that has cost k for producing a unit of the good, and salvage value $\hat{s}$ for each return. Its decision variables are a price p and a gross refund $\hat{r}$ to offer retailers. Let t_R be a retailer's cost of returning the good to the wholesaler, and let t continue to be the consumer's cost of trying and returning a good to a retailer. A retailer's gross salvage value for a consumer return is then the

[17] The return to the wholesaler of unsold goods is beyond our scope here. Unsold goods do not arise in this paper because of the absence of aggregate demand uncertainty.

net refund it obtains from the wholesaler for the return: $\hat{s}_R = \hat{r} - t_R$. A retailer's net salvage value for a return is thus

$$s_R = \hat{s}_R - t = \hat{r} - t_R - t.$$

A retailer's cost of procuring the good is the price it pays the wholesaler: $k_R = p$.

A retailer sells the good to consumers for a price p_R and a gross refund $\hat{r}_R$, which amounts to what we have called a contract (p_R, r_R) with net refund $r_R = \hat{r}_R - t$. A competitive retail equilibrium in either model, as was discussed in the previous sections, consists of each retailer offering the zero-profit contract $(p_R, r_R) = (k_R, s_R)$.[18] Hence, in terms of the wholesaler's decision variables, the consumers face contract $(p_R, r_R) = (p, r)$, where $r = \hat{r} - t_R - t$. Given the wholesaler's choice of (p, r), the consumers have exactly the same choice problem as in the previous sections. The wholesaler's profit from an informed consumer is $\pi_i(p)$, as the retailers make zero profit, an informed consumer never returns the good, and she buys if and only her value exceeds $p_R = p$. Since every return to a retailer is returned to the wholesaler, the wholesaler's probability of a return from an uninformed consumer is the same as a retailer's, $F(r)$. Thus, letting $s = \hat{s} - t_R - t$, the wholesaler's profit on an uninformed consumer is

$$p - k + (\hat{s} - \hat{r})F(r) = p - k + (s - r)F(r) = \pi_u(p, r).$$

These are the same profit expressions as in the previous sections, and so their results hold unchanged with the wholesaler as the firm, except that now the cost of trying and returning is the sum $t_R + t$.

6. Conclusions

We have provided a possible explanation for the prevalence of generous return policies for consumer goods. Rather than starting from the premise that consumers are risk averse, our explanation is based on the premise that at least some consumers are able to learn about their personal values for a good without trying it out. In either version of the model, a seller with market power promises a refund that is no less, and is sometimes more, than its salvage value for a return. Such refunds are excessive in so far as they generate an inefficiently high number of returns.

Refunds have a screening function in Model SC. The consumers in it are of two types, those who are ex ante informed of their values, and those who are uninformed and can

[18] There may be other competitive retail equilibria, but as they are all efficient and give retailers zero profit, the argument can be adapted to hold for them as well.

learn their values for the good only by trying it out. By offering an excessive refund, the firm is able to charge a higher price, and it chooses to do this if the price it would like to charge the informed consumers in isolation is sufficiently high. An excessive refund is promised in order to weaken the informed type's incentive constraint. This screening can occur without the use of a menu of contracts, since both types of consumer pay the same price for the good when the incentive constraint of the informed types binds.

Refunds play a different role in Model IA. Here, they serve to deter consumers from becoming informed of their values before purchasing, thereby eliminating information rents. The refund is not excessive if the consumers' cost of acquiring information is so high that it can be ignored. Otherwise the information acquisition constraint binds, which causes any refund that is ever paid to be excessive. However, a caveat to this excessive refund result is that for some parameter values, the firm does not offer a refund when a benevolent social planner would (see Example 2).

Our explanations for excessive refunds also apply to the refunds a monopoly wholesaler offers retailers for the returns that they receive from consumers. To the extent that wholesalers are more likely than retailers to have market power, the model may be at least as applicable to the excessive refunds offered by wholesalers to retailers as it is for those offered by retailers to consumers.

The hypotheses developed here await empirical study. Future work will hopefully produce the data and the empirical tests to determine the relative merits of the screening and information acquisition (and risk aversion) rationales for refunds.

Appendices

Appendices A and B contain the proofs omitted from Sections 3 and 4 for Models SC and IA, respectively. Some lemmas in Appendix A are again used in Appendix B. The calculations for each section's example are collected in Appendix C.

A. Proofs for Section 3

Given a promised refund r, the surplus generated when an uninformed consumer purchases, $[V_u(r) - p] + \pi_u(p, r)$, is

$$V_u(r) + (s - r)F(r) - k = sF(r) + \int_r^1 v dF(v) - k \equiv S_u(r). \tag{A1}$$

Lemma A1. *The unique maximizer of $S_u(r)$ is s, yielding $S_u(s) = S_u^*$. Furthermore, $S_u'(r) \gtreqless 0$ as $r \lesseqgtr s$.*

Proof. Follows from $S_u'(r) = (s - r)f(r)$ and $S_u(s) = V_u(s) - k = S_u^*$. ■

Proof of Proposition 1. Let $(\tilde{p}, \tilde{r})$ be a contract in a competitive menu chosen by some informed consumers. These consumers are indifferent between $(\tilde{p}, \tilde{r})$ and the no-refund contract $(\tilde{p}, 0)$. For any $\varepsilon > 0$, they would prefer the contract $(\tilde{p} - \varepsilon, 0)$. An entrant offering this contract would attract all the informed consumers and earn a profit of $\tilde{p} - \varepsilon - k$ on each of them, and on any uninformed consumers the contract might attract. Since this entrant cannot earn positive profit, we conclude that $\tilde{p} \leq k$. This implies that every firm makes nonpositive profit on the informed consumers.

Now let (p, r) be a contract in the menu chosen by some uninformed consumers. As the firm offering it makes nonnegative profit overall, and nonpositive profit on the informed, it must make nonnegative profit on (p, r) when it is chosen by an uninformed consumer: $\pi_u(p, r) \geq 0$. If $r \neq s$, the surplus generated when an uninformed consumer chooses (p, r) is not the maximal amount, $S_u^* = S_u(s)$, by Lemma A1. Thus, an entrant could offer a contract of the form (p', s), with p' set so that $V_u(s) - p' > V_u(r) - p$ and $\pi_u(p', s) > \pi_u(p, r)$. This new contract would attract all the uninformed, and earn positive profit on them. It would also earn positive profit if an informed consumer chose it, since its realized profit does not depend on whether the good is returned. This is a contradiction, since an entrant should not be able to make positive profit. This proves $r = s$. An undercutting argument like that above now proves $p \leq k$. But since we have already shown $\pi_u(p, r) \geq 0$ and $r = s$, we conclude that $p = k$.

Every firm thus makes zero profit on the uninformed, and so must also make zero profit on the informed. Hence, $\tilde{p} = k$. We conclude that every contract in the menu makes zero profit, and an efficient outcome is achieved because the informed choose contracts of the form $(k, \tilde{r})$, and the uninformed choose (k, s). ■

Proof of Proposition 2. We can assume $p_I < \bar{v}$, since the proof for case $p_I \in [\bar{v}, V_u(s)]$ is in the text. Consider the relaxed problem obtained from (P) by deleting constraints (IC_i) and (FE). This relaxed program has just two constraints, (IC_u) and (IR_u), and they can be written as one, $V_u(r) - p_u \geq \bar{U}(p_i)$, where

$$\bar{U}(p_i) \equiv \max(0, \bar{v} - p_i). \tag{A2}$$

This combined constraint binds, as otherwise p_u could be profitably raised. Using this binding combined constraint to substitute for p_u in $\pi_u(p_u, r)$, and using (A1), we can write the relaxed program as

$$\text{(Pa)} \quad \max_{p_i, r} \; \left[\lambda \pi_i(p_i) - (1-\lambda)\bar{U}(p_i)\right] + (1-\lambda) S_u(r). \tag{A3}$$

A triple (p_i, p_u, r) solves the relaxed program obtained from (P) by deleting (IC_i) and (FE) if and only if (p_i, r) solves (Pa) and $p_u = V_u(r) - \bar{U}(p_i)$.

By Lemma A1, the second term in (A3) is uniquely maximized by $r = s$. Denote the first term as $A(p_i)$, and note that

$$A'(p_i) = \begin{cases} \lambda \pi_i'(p_i) + 1 - \lambda & \text{for } p_i < \bar{v} \\ \lambda \pi_i'(p_i) & \text{for } p_i > \bar{v}. \end{cases}$$

Since $p_I < \bar{v}$, Assumption 2 and $0 < \lambda < 1$ imply

$$A'(p_i) \begin{cases} > 0 & \text{for } p_i \leq p_I \\ < 0 & \text{for } p_i > \bar{v}. \end{cases}$$

All maximizers of $A(\cdot)$ are thus in $(p_I, \bar{v}]$. Hence, $\bar{U}(p_i) = \bar{v} - p_i$. We have thus shown that any solution (p_i, p_u, r) of the relaxed program obtained by deleting (IC_i) and (FE) from (P) satisfies $r = s$, $p_i \in (p_I, \bar{v}]$, and

$$p_u = V_u(s) + p_i - \bar{v}. \tag{A4}$$

We now show that the relaxed program and (P) have the same solutions. We do this by showing that any solution, say (p_i, p_u, r), of the relaxed program satisfies the neglected constraints (IC_i) and (FE). Constraint (IC_i) holds because (A4) and $V_u(s) \geq \bar{v}$

imply $p_u \geq p_i$. To establish (FE), note that $p_i > s + t$, since $p_i > p_I > k$ and, by Assumption 1, $k \geq s + t$. Hence, $p_u > s + t$, and so (FE) holds.

This completes the proof, except for showing that the firm cannot do better by excluding the uninformed. Any contract that excludes them must have a price $p \geq \bar{v}$. This price is greater than p_I, since $p_I < \bar{v}$ in the present case. Thus, lowering p to p_I increases the profit obtained on the informed and, as a bonus, profitably attracts the uninformed too. Excluding the uninformed is therefore not optimal. ■

Lemma A2. *If $t > 0$, then $r^v \in (s, 1)$ exists such that $V_u(r^v) = r^v + t$ and, for all $r \geq -t$,*

$$V_u(r) \gtreqless r + t \iff r \lesseqgtr r^v.$$

If $t = 0$, we let $r^v = 1$ and have $V_u(r) > r$ for $r < r^v$, and $V_u(r) = r$ for $r \geq r^v$.

Proof. Note that $V'(r) = F(r) < 1$ for all $r < 1$. By Assumptions 1 and 3, $V_u(s) > k \geq s + t$. Hence, $V_u(r) > r + t$ for $r \leq s$. For $r \geq 1$, $V_u(r) = r \leq r + t$. So

$$r^v \equiv \sup\{r \mid V_u(r) > r + t\}$$

is well-defined and satisfies the stated properties. ■

Proof of Theorem 1. The proof is in two steps. In the first we characterize the optimal no-exclusion menu. In the second we show that this menu is better than the no-refund contract with price p_I that excludes the uninformed if λ is small.

Step 1. Consider the relaxed program obtained by deleting (IC_u) from (P). In this program (IR_u) binds, as otherwise p_u could be raised profitably. So $p_u = V_u(r)$. Substitute this into the relaxed program and use (A1) to obtain

$$\text{(Pb)} \qquad \max_{p_i, r} \lambda \pi_i(p_i) + (1 - \lambda) S_u(r)$$
$$\text{subject to}$$
$$(\text{IC}_i') \quad p_i \leq V_u(r),$$
$$(\text{FE}') \quad 0 \leq r + t \leq V_u(r).$$

If (IC_i') were not to bind in (Pb), its solution would be (p_I, s), since p_i uniquely maximizes $\pi_i(\cdot)$, s uniquely maximizes $S_u(\cdot)$, and (p_I, s) satisfies (FE$'$) by Assumptions 1 and 3. But then (IC_i') would imply $p_I \leq V_u(s)$, contrary to hypothesis. So (IC_i') binds in

(Pb), and its solution satisfies $p_i = V_u(r)$. We can thus replace p_i by $V_u(r)$ and discard (IC_i'). Finally, by Lemma A2, $r^v \in (s, 1]$ exists such that (FE′) is equivalent to

$$(\text{FE}'') \quad -t \leq r \leq r^v.$$

Hence, (p_i, p_u, r) solves (Pb) if and only if r solves the program

$$(\text{Pb}') \quad \max_r \lambda \pi_i(V_u(r)) + (1-\lambda) S_u(r)$$
$$\text{subject to } (\text{FE}''),$$

and $p_i = p_u = V_u(r)$. Furthermore, $p_i = p_u$ implies that the neglected constraint (IC_u) holds, since $V_u(r) \geq \bar{v}$ for all r. This shows that (p_i, p_u, r) solves the original program (P) if and only if r solves (Pb′), and $p_i = p_u = V_u(r)$.

Denote the objective function of (Pb′) as $M(r)$. It is a continuous function, with a right derivative on $[0, 1)$ given by

$$M'(r) = \lambda \pi_i'(V_u(r))F(r) + (1-\lambda)(s-r)f(r). \tag{A5}$$

For $r < 0$, $M'(r) = 0$. For $r \in [0, s)$, since $V_u(\cdot)$ is nondecreasing and $V_u(s) < p_I$ here, we have $V_u(r) < p_I$. Assumption 2 thus implies $\pi_i'(V_u(r)) > 0$, and hence $M'(r) > 0$, for $r \in [0, s)$. This shows that a solution of (Pb′) satisfies $r \geq s$. This inequality is strict if $s > 0$, for then

$$M'(s) = \lambda \pi_i'(V_u(s))F(s) > 0.$$

If $s = 0$, then $M'(s) = 0$ and

$$M''(s) = \left[\lambda \pi_i'(\bar{v}) - (1-\lambda)\right] f(0),$$

using $V_u(0) = \bar{v}$. So $M''(s) > 0$ if $\pi_i'(\bar{v}) > \frac{1-\lambda}{\lambda}$, and this again yields $r > s$.

It remains to show that a solution $(p_i, p_u, r) = (V_u(r), V_u(r), r)$ of (Pb) satisfies $p_u < p_I$. Assume the opposite. Then $V_u(r) \geq p_I > V_u(s)$. This implies, since $V_u(\cdot)$ is nondecreasing, that $r > s$, and hence $(s-r)f(r) < 0$. Also, $V_u(r) \geq p_I$ and Assumption 2 imply $\pi_i'(V_u(r)) \leq 0$. Thus, in light of (A5), $M'(r) < 0$. So r must be a left corner solution of (Pb′): $r = -t$. This contradicts $r > s$, by Assumption 1. Hence, $p_u < p_I$.

Step 2. Write the optimal no-exclusion menu of Step 1 as a function of λ,

$$(p_i, p_u, r) = (V_u(r^*), V_u(r^*), r^*),$$

where $r^* = r^*(\lambda)$ solves program (Pb′). Lemma A1 implies $r^*(0) = s$. Denote the value function of (Pb′) as

$$\Pi_{sc}(\lambda) \equiv \lambda \pi_i(V_u(r^*(\lambda))) + (1-\lambda) S_u(r^*(\lambda)).$$

By the maximum theorem, $\Pi_{sc}(\cdot)$ is continuous on $[0,1]$. Define

$$X(\lambda) \equiv \Pi_{sc}(\lambda) - \lambda \pi_i(p_I).$$

The firm does not exclude the uninformed when $X(\lambda) > 0$, since $(p_I, 0)$ is the optimal exclusion contract. Note that

$$X(0) = \Pi_{sc}(0) = S_u(s) = S_u^*.$$

So Assumption 3 implies $X(0) > 0$. Thus, as $X(\cdot)$ is continuous, $\bar{\lambda} \in (0,1]$ exists such that $X(\lambda) > 0$ for all $\lambda \in [0, \bar{\lambda}]$. ■

B. Proofs for Section 4

Recall that Proposition 3 is about

$$\begin{aligned}
(\mathrm{P}_u) \quad & \Pi_u(c) \equiv \max_{p,r} \; p - k - (r-s)F(r) \\
& \text{subject to} \\
& (\mathrm{IA}_u) \qquad p \leq P(r,c), \\
& (\mathrm{IR}_u) \qquad p \leq V_u(r), \\
& (\mathrm{FE}) \qquad 0 \leq r + t \leq p.
\end{aligned}$$

Lemma B1. *$\Pi_u(c)$ is well-defined and continuous at any $c \geq 0$.*

Proof. For any $c \geq 0$, the constraint set of (P_u) is non-empty, as it contains $(p, r) = (0, -t)$. The constraint set is closed because its defining functions are continuous. It is bounded, since any feasible (p, r) satisfies $r \in [-t, r^v]$, by (FE), (IR_u), and Lemma A2, and $p \in [0, V_u(r^v)]$ by (FE), (IR_u), and the monotonicity of $V_u(\cdot)$. So $\Pi_u(\cdot)$ is well-defined on $\mathbb{R}_+$. As the constraint set is a continuous correspondence in c, $\Pi_u(\cdot)$ is continuous by the maximum theorem. ■

The next two lemmas, as well as Lemma A2, establish properties of the constraint set of (P_u), the shaded area in Figure 2. (The figure is drawn for the case of a relatively high $c < \bar{c}$, so that the indicated crossing points satisfy $r^{vp} < r^v$. The opposite holds if c is smaller.) Proposition 3 will be proved by showing that the solution of (P_u) is on the indicated heavy line. Also shown are two iso-profit curves; the higher one corresponds to the case unconstrained by (IA_u), $\pi_u(p, r) = \pi_u(V_u(s), s) = S_u^*$.

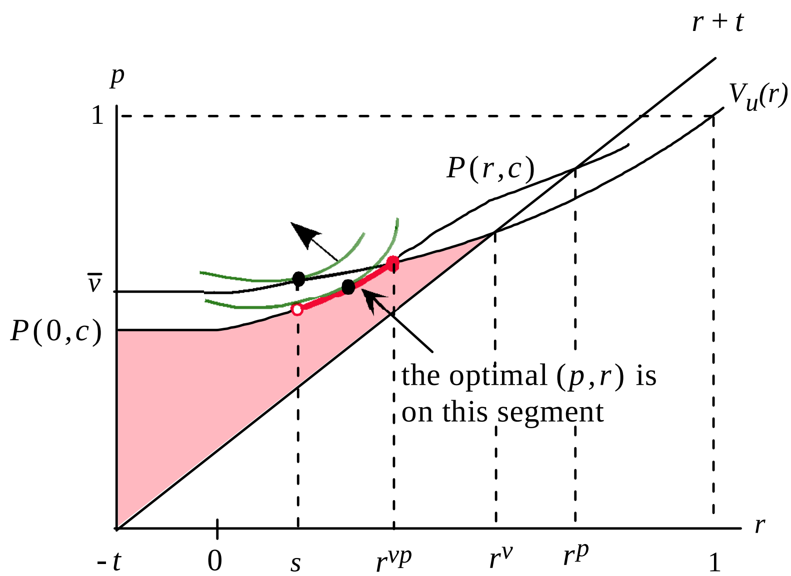

Figure 2: The constraint set of program (P_u).

Lemma B2. *For any* $c \in [0, \bar{c})$, *a unique* $r^{vp} \in (s, 1]$ *exists such that* $P(r^{vp}, c) = V_u(r^{vp})$ *and, for all* $r \in [-t, 1]$,

$$P(r, c) \lesseqgtr V_u(r) \iff r \lesseqgtr r^{vp}.$$

Proof. First consider $c \in (0, \bar{c})$. Since $P(s, \bar{c}) = V_u(s)$, and $P(s, \cdot)$ is increasing, $P(s, c) < V_u(s)$. But $P(1, c) = 1 + c > 1 = V_u(1)$. By continuity, $r^{vp} \in (s, 1)$ exists such that $P(r^{vp}, c) = V_u(r^{vp})$. For any $(\hat{r}, \hat{c})$, we have the derivatives $P_r(\hat{r}, \hat{c}) = F(\hat{r})/F(P(\hat{r}, \hat{c}))$ and $V_u'(\hat{r}) = F(\hat{r})$, and hence $P_r(\hat{r}, \hat{c}) \geq V_u'(\hat{r})$. Since $r^{vp} < 1$, $P(r^{vp}, c) = V_u(r^{vp}) < 1$. So $P_r(r^{vp}, c) > V_u'(r^{vp})$. Therefore $P(r, c) > V_u(r)$ if $r > r^{vp}$, and $P(r, c) < V_u(r)$ if $r < r^{vp}$.

Now consider $c = 0$. Since $P(r, 0) = \max(r, 0)$, and $V_u(r) > r$ for $r < 1$, and $V_u(1) = 1$, the lemma's claim holds with $r^{vp} = 1$. ■

Lemma B3. *For* $c \geq t$, $P(r, c) \geq r + t$ *for all* $r \geq -t$. *For* $c < t$, *there exists* $r^p \in [-t, 1)$ *such that* $P(r^p, c) = r^p + t$ *and*

$$P(r, c) \gtreqless r + t \iff r \lesseqgtr r^p.$$

Proof. Since $F(\cdot) \leq 1$ and $c \geq 0$,

$$P(r, c) - r \geq \int_r^{P(r,c)} F(v)dv = c$$

for any r. Thus, $P(r,c) \geq r+t$ for all r if $c \geq t$. Now assume $c < t$. Then for $r \geq 1$, $P(r,c) = r+c < r+t$. Since $P(-t,c) \geq 0$, we have $P(r,c) \geq r+t$ at $r=-t$. Since $P(\cdot,\cdot)$ is continuous, this proves the existence of $r^p \in [-t,1)$ such that $P(r^p,c) = r^p + t$. Because $P(\cdot,c)$ is constant on $[-t,0]$, $P_r(r,c) < 1$ for $r < 0$. For $r \geq 0$, $P_r(r,c) = F(r)/F(P(r,c))$.Thus, if $r \in [0,1)$ and $P(r,c) = r+t$, then $P_r(r,c) < 1$ (as $t > 0$ since $c < t$). So the graph of $P(\cdot,c)$ crosses that of $r+t$ from above at r^p, and the two curves do not cross at any other r. ■

Lemma B4. *If $c < \bar{c}$, any solution (p^*, r^*) of (P_u) satisfies $p^* = P(r^*, c)$.*

Proof. Assume $p^* < P(r^*,c)$. Then, as the only other upper bound on the price is (IR_u), it binds: $p^* = V_u(r^*)$. Hence, $V_u(r^*) < P(r^*,c)$, and so Lemma B2 implies $r^* > r^{vp} > s$. The firm's profit is

$$V_u(r^*) - k - (r^* - s)F(r^*) = S_u(r^*).$$

Similarly, if the firm were to instead choose (p,r) with $p = V_u(r)$ and $r \in [r^{vp}, r^*)$, its profit would be $S_u(r)$, which exceeds $S_u(r^*)$ by Lemma A1. This contradiction proves $p^* = P(r^*,c)$. ■

Proof of Proposition 3. By Lemma B4, $p^* = P(r^*,c)$. It remains to show $r^* > s$. Lemma B4 also implies we can substitute $P(r,c)$ for p in (P_u). That is, defining

$$A(r) \equiv P(r,c) - k - (r-s)F(r), \tag{B1}$$

we have $\Pi_u(c) = A(r^*)$, and r^* solves the program

$$\max_r \; A(r) \text{ subject to } -t \leq r \leq \min(r^v, r^{vp}, r^p), \tag{B2}$$

using Lemmas A2, B2, and B3. For $r \leq 0$, $A(r) = P(0,c) - k$. For $r > 0$,

$$\begin{aligned} A'(r) &= P_r(r,c) - F(r) + (s-r)f(r) \\ &= \left[\frac{1}{F(P(r,c))} - 1\right] F(r) + (s-r)f(r). \end{aligned}$$

The first term on the right is positive for any $r \leq s$, since then $P(r,c) < P(s,\bar{c}) = V_u(s) < 1$. The second term is nonnegative for any $r \leq s$. Hence, $A'(r) > 0$ for any $r \in [0,s]$. The solution thus satisfies $r^* > s$ or $r^* = \min(r^v, r^{vp}, r^p)$. Since $\min(r^v, r^{vp}) > s$ by

Lemmas A2 and B2, we have $r^* > s$ or $r^* = r^p$. Thus, $r^* \leq s$ would imply $r^* = r^p \leq s$, and so

$$\begin{aligned}\Pi_u(c) &= P(r^p, c) - k + (s - r^p)F(r^p) \\ &= r^p + t - k + (s - r^p)F(r^p) \\ &\leq r^p + t - k + (s - r^p) \\ &= s + t - k \leq 0,\end{aligned}$$

where the second equality comes from $P(r^p, c) = r^p + t$, and the last inequality from Assumption 1. This contradiction of $\Pi_u(c) > 0$ proves $r^* > s$. ■

Proof of Proposition 4. The constraint set of (P_i) is

$$C \equiv \{p \in \mathbb{R} | \, P(0, c) \leq p \leq P_i(c)\} \, .$$

It is nonempty if and only if $P(0, c) \leq P_i(c)$. Note that $P_i(\cdot) = V_i^{-1}(\cdot)$ is decreasing, with $P_i(0) = 1$ and $P_i(V_i(\bar{v})) = \bar{v}$. Also, $P(0, \cdot)$ is increasing, with $P(0, 0) = 0$ and, by Lemma B7, $P(0, V_i(\bar{v})) = \bar{v}$. (This verifies the accuracy of the curves $P(c, 0)$ and $P_i(c)$ shown in Figure 1). Hence, $C \neq \varnothing$ if and only if $c \in [0, V_i(\bar{v})]$; program (P_i) has a solution if and only if this is the case. Assumption 2 implies the solution is p_I if $P(0, c) \leq p_I \leq P_i(c)$, and that otherwise it is whichever price is closer to p_I, $P(0, c)$ or $P_i(c)$. The solution is thus the $p^*(c)$ defined by (10) and (11). ■

Lemma B5. $\Pi_u(0) = \Pi_i(0)$ *if* $t = k - s = 0$. *Otherwise,* $\Pi_u(0) < \Pi_i(0)$.

Proof. Suppose $t = k - s = 0$. Then, when $c = 0$, a consumer can be induced to stay uninformed if and only if the contract offers a full refund. That is: with $t = 0$, (FE) is $p \geq r$. Since $P(r, 0) = r$ for all $r \geq 0$, (IA_u) becomes $p \leq r$. The two constraints together are $p = r$. By Lemma A2, $r^v = 1$, and so (IR_u) amounts to $r \leq 1$. Hence, $\Pi_u(0) = \max_{0 \leq p \leq 1} \pi_u(p, p)$. Since $\pi_u(p, p) = \pi_i(p) + (s - k)F(p)$, we have

$$\Pi_u(0) = \max_{0 \leq p \leq 1} \{\pi_i(p) + (s - k)F(p)\} \, . \tag{B3}$$

Thus, in this case $\Pi_u(0) = \max \pi_i(p) = \pi_i(p_I)$. By Proposition 4, $\Pi_i(0) = \pi_i(p_I)$ too. Hence, $\Pi_u(0) = \Pi_i(0)$.

Now suppose $t = 0$ and $k - s > 0$. Then, as (B3) relies only on $t = 0$, it still holds. Let $\hat{p}$ maximize the expression shown on the right of (B3). Then $\hat{p} \neq p_I$, as $\pi_i'(p_I) = 0$ and

$s \neq k$ imply p_I does not satisfy the first order condition. Hence, $\pi_i(\hat{p}) < \pi_i(p_I) = \Pi_i(0)$. Therefore, since $s < k$,

$$\Pi_u(0) = \pi_i(\hat{p}) + (s-k)F(\hat{p}) \leq \pi_i(\hat{p}) < \Pi_i(0).$$

Lastly, suppose $t > 0$. Then, if $c = 0$, no contract with a positive price induces a consumer to stay uninformed; the only contract that does so is $(0, -t)$. (If $t > 0$, then (FE), (IA_u), and $P(r, 0) = \max(r, 0)$ together imply $(p, r) = (0, -t)$.) Hence, $\Pi_u(0) = \pi_u(0, -t) = -k$. Since $\Pi_i(0) = \pi_i(p_I) > 0$, this proves $\Pi_i(0) > \Pi_u(0)$. ■

Lemma B6. *(i)* $\Pi_i(\cdot)$ *is continuous and nonincreasing on* $[0, V_i(\bar{v})]$. *(ii)* $\Pi_u(\cdot)$ *is continuous and increasing on* $[0, \bar{c}]$, *and* $\Pi_u(c) = S_u^*$ *for* $c \geq \bar{c}$. *(iii) For all* $c \geq 0$, $\max(\Pi_u(c), \Pi_i(c)) > 0$.

Proof. *(i)* $\Pi_i(\cdot)$ is continuous on its domain by the maximum theorem. It is nonincreasing because the constraint set of (P_i) shrinks as c increases, since $P(0, c)$ increases and $P_i(c)$ decreases in c.

(ii) Lemma B1 shows $\Pi_u(\cdot)$ is continuous, and Lemma 1 shows $\Pi_u(c) = S_u^*$ for $c \geq \bar{c}$. Let $0 \leq c_1 < c_2 < \bar{c}$. Denote program ($\text{P}_u$) as ($\text{P}_u i$) when $c = c_i$. Let (p_i, r_i) solve ($\text{P}_u i$). Since $P(r, c_1) < P(r, c_2)$ for all r, (p_1, r_1) is in the constraint set of ($\text{P}_u 2$). Hence, $\Pi_u(c_1) \leq \Pi_u(c_2)$. If this were an equality, (p_1, r_1) would solve both programs, and so Lemma B4 would imply $p_1 = P(r_1, c_i)$ for both $i = 1$ and $i = 2$, contrary to $P(\cdot, c_1) < P(\cdot, c_2)$. Thus, $\Pi_u(\cdot)$ increases on $[0, \bar{c}]$.

(iii) Consider a contract (p, s), with price $p < V_u(s)$. The consumers will surely accept it. If they stay uninformed, this contract yields profit $p - k$. If they become informed, it yields the lower profit $\pi_i(p) = (p - k)(1 - F(p))$. Hence,

$$\max(\Pi_u(c), \Pi_i(c)) \geq \max_{p \leq V_u(s)} \pi_i(p).$$

Thus, since $k < 1$ and $V_u(s) > k$ imply $\max_{p \leq V_u(s)} \pi_i(p) > 0$, $\max(\Pi_u(c), \Pi_i(c)) > 0$. ■

Lemma B7. $P(0, V_i(\bar{v})) = \bar{v}$.

Proof. This is a consequence of (9) and

$$\begin{aligned}\int_0^{\bar{v}} F(v)dv &= \bar{v}F(\bar{v}) - \int_0^{\bar{v}} v dF(v) \\ &= \bar{v}F(\bar{v}) - \left[\bar{v} - \int_{\bar{v}}^1 v dF(v)\right] \\ &= \int_{\bar{v}}^1 (v - \bar{v}) dF(v) = V_i(\bar{v}). \; ■\end{aligned}$$

Lemma B8. *For any* $c \in [0, V_i(\bar{v})]$, $\Pi_u(c) > \Pi_i(c)$ *if* $p^*(c) = P(0, c)$. *In particular,* $\Pi_u(V_i(\bar{v})) > \Pi_i(V_i(\bar{v}))$.

Proof. This is implied by Lemma B6 if $\Pi_i(c) \leq 0$. So assume $\Pi_i(c) > 0$. This, and

$$(p^*(c) - k)(1 - F(p^*(c))) = \Pi_i(c),$$

imply $p^*(c) - k > 0$. This in turn yields $p^*(c) > 0$, and so $1 - F(p^*(c)) < 1$. Hence,

$$p^*(c) - k > (p^*(c) - k)(1 - F(p^*(c))).$$

Since $p^*(c) = P(0, c)$, the consumers are indifferent about becoming informed when faced with contract $(p^*(c), 0)$. So $(p, r) = (p^*(c), 0)$ is in the constraint set of (P_u). Hence,

$$\Pi_u(c) \geq p^*(c) - k.$$

The three displayed expressions imply $\Pi_u(c) > \Pi_i(c)$. Lemma B7 now implies $\Pi_u(V_i(\bar{v})) > \Pi_i(V_i(\bar{v}))$. ■

Proof of Theorem 2. Recall the $\bar{c}$ defined by (8), and note it is positive by Assumption 3. Part 4 of the theorem is proved in the text as Lemma 1. Here we prove the unique existence of the indicated $\underline{\underline{c}}$ and $\underline{c}$, and parts $1-3$ of the theorem, separately for three mutually exhaustive cases of the parameters. Part (a) of the theorem follows from the combined analyses of the cases, and (b) is proved in Case 2.

Case 1: $t = 0$ and $s = k$.

By Lemmas B5 and B6, $\Pi_u(0) = \Pi_i(0)$ and $\Pi_u(c) > \Pi_i(c)$ for all $c \in (0, V_i(\bar{v})]$. The optimal contract is thus a refund contract that induces consumers to stay uninformed, for all $c > 0$. If $c < \bar{c}$, the firm offers a refund contract with refund $r > s$ and price $P(r, c)$, by Proposition 3. This proves the theorem, with $0 = \underline{\underline{c}} = \underline{c}$. ■

Case 2: $t > 0$ or $s < k$, and $p_I \leq \bar{v}$.

Because $t > 0$ or $s < k$, Lemma B5 implies $\Pi_u(0) < \Pi_i(0)$. By Lemma B8, $\Pi_u(V_i(\bar{v})) > \Pi_i(V_i(\bar{v}))$. By Lemma B6, $\Pi_u(\cdot) - \Pi_i(\cdot)$ is continuous and increasing on $[0, V_i(\bar{v})]$. A unique $0 < \underline{c} < V_i(\bar{v})$ therefore exists such that

$$\Pi_u(c) - \Pi_i(c) \lesseqqgtr 0 \text{ as } c \lesseqqgtr \underline{c}. \tag{B4}$$

The firm thus offers a a refund contract if $c > \underline{c}$, and this contract satisfies the properties stated in parts 3 and 4 of the theorem by Proposition 3 and Lemma 1. Because $\Pi_u(\bar{c}) >$

$\Pi_i(\bar{c})$, $\underline{c} < \bar{c}$. When $c \in [0, \underline{c})$, (B4) implies the firm offers a no-refund contract that induces the consumers to acquire information. The price in this no-refund contract is the $p_i^*(c)$ of Proposition 4, which is either p_I or $P(0,c)$ because $p_I \leq \bar{v}$. But it cannot be $P(0,c)$, since then Lemma B8 would imply $\Pi_u(c) > \Pi_i(c)$, contrary to $c < \underline{c}$ and (B4). The firm thus offers a no-refund contract with price p_I for all $c < \underline{c}$. This proves parts 1 and 2 of the theorem, with $\underline{\underline{c}} \equiv \underline{c}$. ■

Case 3: $t > 0$ or $s < k$, and $p_I > \bar{v}$.

The proof is the same as in the previous case, except that now, when $c < \underline{c}$, Proposition 4 implies that the price in the no-refund contract is p_I for $c \leq V_i(p_I)$, but $P_i(c)$ for $c \geq V_i(p_I)$. This proves parts 1 and 2 of the theorem, with $\underline{\underline{c}} \equiv \min(\underline{c}, V_i(p_I))$. ■

C. Calculations for Examples 1 and 2

Calculations for Example 1. Here, F is uniform and $\lambda = .5$, $k = .45$, $s = .2$, and $t = .245$. Hence,

$$p_I = \arg\max_p (p-k)(1-p) = \frac{1}{2}(k+1) = .725,$$

$$V_u(r) = \int_0^1 \max(v,r)dv = \frac{1}{2}\left(1+r^2\right).$$

So $V_u(s) = 0.52$. That Assumptions $1-3$ hold is immediate. As $p_I > V_u(s)$, we are in the case of Theorem 1.

We first derive the optimal no-exclusion menu. Recall that we can take it to be a single contract of the form $(V_u(r^*), r^*)$, where r^* maximizes

$$\begin{aligned} M(r) &= \lambda\pi_i(V_u(r)) + (1-\lambda)[V_u(r) - k + (s-r)F(r)] \\ &= .5\left[.5\left(1+r^2\right)-k\right]\left[1-.5\left(1+r^2\right)\right] + .5\left[.5\left(1+r^2\right)-k+(s-r)r\right] \end{aligned}$$

subject to $r \in [-t, r^v]$. (See Step 1 of the proof of Theorem 1 above.) The point r^v is determined by $V_u(r) = r + t$, which yields $r^v = .3$. As shown in Step 1 of the proof of Theorem 1, the solution r^* is in the interval $(s, r^v] = (.2, .3]$. On this interval,

$$\begin{aligned} M'(r) &= .5\left[1+k-(1+r^2)\right] + .5\left[.5(1+r^2)-k+s-r\right] \\ &= .5\left[.7-r-.5r^2\right] > 0. \end{aligned}$$

The solution is thus the upper corner: $r^* = r^v = .3$. The optimal no-exclusion contract is accordingly $(V_u(r^*), r^*) = (.545, .3)$, and it yields profit $M(.3) = .054113$.

The optimal contract that excludes the uninformed is the no-refund contract with price $p_I = .725$. It yields profit

$$\lambda \pi_i(p_I) = .5\left(\frac{1-k}{2}\right)^2 = .037813 < M(.3).$$

The firm's optimal strategy is thus to offer the refund contract $(.545, .3)$, rather than to exclude the uninformed. ■

Calculations for Example 2.

In this example F is uniform, and so the curves in Figure 2 are given by $V_u(r) = \frac{1}{2}\left(1+r^2\right)$ and $P\left(r,c\right) = \sqrt{r^2+2c}$ for $r \geq 0$. In all three parts, $s = .125$ and $k = .375$. Assumptions 1 and 3 are then satisfied, and so is Assumption 1 if $t \in [0, k-s] = [0, .25]$.

Part (a). Fix $c = .1$. We find a range of costs t for which a full refund is optimal.

Recall from Proposition 3 that when $c < \bar{c}$, a solution (p^*, r^*) to (P_u) generating positive profit satisfies $p^* = P(r^*, c)$. Repeating (B2), the optimal r^* solves the program

$$\max_r \; A(r) \text{ subject to } -t \leq r \leq \min(r^v, r^{vp}, r^p), \tag{C1}$$

where $A(r) = P(r,c) - k - (r-s)F(r)$. Here, for $r \geq 0$ we have

$$A(r) = \sqrt{r^2 + .2} - .375 - (r - .125)r.$$

This $A(r)$ attains its global maximum at $r = .39215$, and increases on $[0, .39215]$. The solution of (C1) is thus $r^* = \min(r^v, r^{vp}, r^p)$ if this is not greater than $.39215$.

The intersection points are determined by

$$\frac{1}{2}\left(1+r^2\right) = \sqrt{r^2+2c} \implies r^{vp} = \sqrt{1-2\sqrt{2c}} = .32492,$$

$$r + t = \sqrt{r^2+2c} \implies r^p = \frac{2c-t^2}{2t} = \frac{.2-t^2}{2t},$$

$$r + t = \frac{1}{2}\left(1+r^2\right) \implies r^v = 1 - \sqrt{2t}.$$

It is easily verified that $r^p = \min(r^v, r^{vp}, r^p)$ if $t \geq .22787$, and so in this case the solution of (B2) is $r^* = r^p$. This is a full refund contract because the gross refund is $r^p + t = P(r^p, c) = p^*$.

For $t \geq .22787$, the profit from inducing the consumers to stay uninformed is

$$\Pi_u(.1) = A(r^p) = \sqrt{\left(\frac{.2-t^2}{2t}\right)^2 + .2} - .375 - \left[\left(\frac{.2-t^2}{2t}\right) - .125\right]\left(\frac{.2-t^2}{2t}\right).$$

This must be compared to $\Pi_i(c)$. Since

$$p_I = \arg\max_p (p-k)(1-p) = \frac{1}{2}(k+1) = .6875,$$

and this exceeds $\bar{v} = .5$, Proposition 4 implies $\Pi_i(c) = \pi_i(\min(p_I, P_i(c))$, where

$$P_i(c) = V_i^{-1}(c) = 1 - \sqrt{2c}.$$

Since $P_i(.1) = .55279 < p_I$, $\Pi_i(.1) = \pi_i(.55279) = .079509$. By a numerical calculation, $\Pi_u(.1) > \Pi_i(.1)$ if $t \in [.22787, .25]$. Thus, for each t in this interval the optimal contract is the full refund contract $(P(r^p, c), r^p)$. ■

Part (b). Fix $t = .23$. From the above, if $c = .1$ then $r^p = \min(r^v, r^{vp}, r^p)$. As c decreases, r^p decreases, r^v does not change, and r^{vp} increases. Hence, $r^p = \min(r^v, r^{vp}, r^p)$ for any $c < .1$ Since the efficient critical cost is

$$c^* = \int_s^k F(v)dv = \frac{1}{2}\left(k^2 - s^2\right) = .0625,$$

$r^p = \min(r^v, r^{vp}, r^p)$ when $c = c^*$. So, by the above arguments, the optimal profit from inducing the consumers to stay uninformed when $c = c^*$ is $A(r^p)$, where

$$r^p = \frac{2c^* - t^2}{2t} = .15674.$$

Hence,

$$\Pi_u(c^*) = A(.15674) = .093911.$$

We now compare this to $\Pi_i(c^*) = \pi_i(\min(p_I, P_i(c^*))$. Since $P_i(c^*) = 1 - \sqrt{2c^*} = .6464$ and $p_I = .6875$, we have

$$\Pi_i(c^*) = \pi_i(.64645) = .095971.$$

So $\Pi_u(c^*) < \Pi_i(c^*)$. By continuity, $\Pi_u(\cdot) < \Pi_i(\cdot)$ on an interval above c^*, and hence $\underline{c} > c^*$. ■

Part (c). Now fix $t = 0$. This does not change the calculations $c^* = .0625$ and $\Pi_i(c^*) = .095971$ made above for part (b). The intersection point r^{vp} when $c = c^*$ is

$$r^{vp} = \sqrt{1 - 2\sqrt{2c^*}} = .54120.$$

The other two constraints do not bind when $t = 0$ (as $r^v = 1$ and $r^p = \infty$). Thus $\Pi_u(c^*)$ is determined by $\Pi_u(c^*) = \max_{r \leq r^{vp}} A(r)$. Since the global maximizer of $A(\cdot)$ is $r = .39215 < .54120$,

$$\Pi_u(c^*) = A(.39215) = .11503.$$

Thus, $\Pi_u(c^*) > \Pi_i(c^*)$. So $\Pi_u(\cdot) > \Pi_i(\cdot)$ on an interval below c^*, and hence $\underline{c} < c^*$. ■

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