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“Pro-Cyclical Unemployment Benefits?
Optimal Policy in an Equilibrium Business Cycle Model”

by

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Pro-cyclical Unemployment Benefits? Optimal Policy in an Equilibrium Business Cycle Model

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Abstract

We study the optimal provision of unemployment insurance (UI) over the business cycle. We consider an equilibrium Mortensen-Pissarides search and matching model with risk-averse workers and aggregate shocks to labor productivity. Both the vacancy creation decisions of firms and the search effort decisions of workers respond endogenously to aggregate shocks as well as to changes in UI policy. We characterize the optimal history-dependent UI policy. We find that, all else equal, the optimal benefit is decreasing in current productivity and decreasing in current unemployment. Optimal benefits are therefore lowest when current productivity is high and current unemployment is high. The optimal path of benefits reacts non-monotonically to a productivity shock. Following a drop in productivity, benefits initially rise in order to provide short-run relief to the unemployed and stabilize wages, but then fall significantly below their pre-recession level, in order to speed up the subsequent recovery. Under the optimal policy, the path of benefits is pro-cyclical overall. As compared to the existing US UI system, the optimal history-dependent benefits smooth cyclical fluctuations in unemployment and deliver non-negligible welfare gains.

Keywords: Unemployment Insurance, Business Cycles, Optimal Policy, Search and Matching

JEL Codes: E24, E32, H21, J65

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1 Introduction

In 2009, the United States paid out \$76 billion in unemployment benefits. The size of the unemployment insurance (UI) system raises concerns about how the benefit policy should respond to changes in unemployment and productivity. Unemployment benefits provide insurance to the unemployed, but may distort their search behavior and firms' vacancy creation decisions, possibly exacerbating the negative effects of a drop in productivity. To determine the optimal benefit policy in the presence of aggregate shocks we use a general equilibrium search and matching model in the style of Mortensen and Pissarides (1994). The advantage of a general equilibrium approach is that it enables us to capture the effects of policy changes on both firms' vacancy creation and worker search behavior.

Our model features free entry of firms and endogenous worker search effort decisions. Wages are determined by Nash bargaining and therefore respond to both aggregate productivity and the UI policy. The vacancy posting decisions of firms respond to the UI policy because changes in the worker outside option affect wages, and therefore the returns to posting a vacancy. Worker search effort decisions respond to the UI policy for two reasons: first, benefits directly affect the value of being unemployed; second, benefits affect the aggregate job-finding rate, and therefore the returns to search, through their effect on vacancy posting. Our general equilibrium approach acknowledges that the fluctuations in the returns to search are themselves endogenous and, in particular, respond to changes in policy.

We consider the optimal policy choice of a benevolent, utilitarian government that is allowed to change unemployment benefit levels in response to aggregate shocks and to run deficits in some states of nature, as long as it balances its budget on average. We solve for the optimal state-contingent UI policy and find that it prescribes for benefits to rise immediately following a drop in productivity. Subsequently, however, it prescribes a persistent decline in benefits below their pre-recession level. Thus, the response of benefits to a negative shock is non-monotonic: it is positive in the short run (4-6 weeks after the shock) but negative in the longer run (2-10 quarters after the shock). We find that the optimal path of benefits is pro-cyclical overall.

The features of the optimal benefits can be explained as follows. Higher benefits translate into a lower job-finding rate through both lower returns to posting a vacancy and lower returns to search effort. Immediately after a negative productivity shock hits, the social returns to job creation are low, so the government is more concerned with providing short-term relief for the unemployed and slowing the decline of wages than with inducing high job finding. It therefore raises benefits temporarily, triggering a decrease in both vacancy creation and worker search effort. Subsequently, since the shock is mean-reverting, the government expects an economic recovery and would like to stimulate job finding, which requires lowering benefits.

Our paper contributes to the literature on the design of optimal UI policy in response to aggregate economic conditions. While a huge literature (see, for example, Baily (1978), Shavell and Weiss (1979), and Hopenhayn and Nicolini (1997) for seminal contributions) has analyzed the insurance-incentives trade-off involved in optimal UI provision, most of this literature has bypassed the optimal response of benefits to aggregate shocks. Recently, several studies (Kiley (2003) , Sanchez (2008), Andersen and Svarer (2010, 2011), Kroft and Notowidigdo (2010), Landais, Michaillat, and Saez (2010)) have examined the optimal design of a state-contingent policy. The focus of this emerging literature is the notion that the moral hazard distortion resulting from unemployment insurance depends on the underlying state of the economy. In particular, an argument can be made for countercyclical unemployment benefits if unemployment benefits distort job search incentives less in recessions than in booms. Our paper reassesses the desirability of such state-contingent policies in a general equilibrium framework.¹ Our result that the optimal benefit path is pro-cyclical is new to this literature.

Our paper is not the first to analyze the design of optimal unemployment insurance in equilibrium search models. A number of studies, such as Fredriksson and Holmlund (2001), Coles and Masters (2006), and Lehmann and van der Linden (2007), study optimal UI design in models with endogenous job creation and wage bargaining. The contribution of our paper is to introduce aggregate productivity

¹Andersen and Svarer (2010) and Landais, Michaillat, and Saez (2010) also consider models with endogenously determined vacancy creation but assume rigid wages, implicitly assuming that changes in UI benefits leave wages unaffected.

shocks into such optimal policy analysis.

The paper is organized as follows. We present the model in section 2. Section 3 describes the optimal policy. We describe how we calibrate the model to US data in section 4. We report our results in section 5. Finally, we conclude in section 6.

2 Model description

2.1 Economic Environment

We consider an infinite-horizon discrete-time model. The economy is populated by a unit measure of workers and a larger continuum of firms.

Agents. In any given period, a worker can be either employed (matched with a firm) or unemployed. Workers are risk-averse expected utility maximizers and have expected lifetime utility

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(x_t) - I_t c(s_t)],$$

where \mathbb{E}_0 is the period-0 expectation operator, $\beta \in (0, 1)$ is the discount factor, x_t denotes consumption in period t , s_t denotes search effort exerted in period t , and I_t is an indicator variable equal to 1 if the worker is unemployed and zero otherwise. The within-period utility of consumption $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice differentiable, strictly increasing, strictly concave, and satisfies $u'(0) = \infty$. The cost of search effort $c : [0, 1] \rightarrow \mathbb{R}$ is twice differentiable, strictly increasing, strictly convex, and satisfies $c'(1) = \infty$. An unemployed worker produces h units of the consumption good via home production. We assume that there do not exist private insurance markets and that workers cannot save or borrow.

Firms are risk-neutral and maximize profits. We assume that workers and firms have the same discount factor β . A firm can be either matched to a worker or vacant. A firm posting a vacancy incurs a flow cost k .

Production. The economy is subject to aggregate shocks to labor productivity. Specifically, a matched worker-firm pair produces output z_t , where z_t is stochastic. We assume that $\ln z_t$ follows

an AR(1) process

$$\ln z_t = \rho \ln z_{t-1} + \sigma_\varepsilon \varepsilon_t,$$

where $0 \leq \rho < 1$, $\sigma_\varepsilon > 0$, and ε_t are independent and identically distributed standard normal random variables. We will write $z^t = \{z_0, z_1, \dots, z_t\}$ to denote the history of shocks up to period t .

Matching. Job creation occurs through a matching function. We assume that the number of new matches in period t equals

$$M(S_t(1 - L_{t-1}), v_t),$$

where $1 - L_{t-1}$ is the unemployment level in period $t - 1$, S_t is the average search effort exerted by unemployed workers in period t , and v_t is the measure of vacancies posted in period t . The quantity $S_t(1 - L_{t-1})$ represents the measure of efficiency units of worker search.

The matching function M exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and has the property that the number of new matches cannot exceed the number of potential matches: $M(S(1 - L), v) \leq \min\{S(1 - L), v\}$. We define

$$\theta_t = \frac{v_t}{S_t(1 - L_{t-1})}$$

to be the market tightness in period t . We define the functions

$$f(\theta) = \frac{M(S(1 - L), v)}{S(1 - L)} = M(1, \theta) \quad \text{and}$$

$$q(\theta) = \frac{M(S(1 - L), v)}{v} = M\left(\frac{1}{\theta}, 1\right)$$

where $f(\theta)$ is the job-finding probability per efficiency unit of search and $q(\theta)$ is the probability of filling a vacancy. By the assumptions on M made above, the function $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ . For an individual worker exerting search effort s , the probability of finding a job is $sf(\theta)$. Note that, when workers choose the amount of search effort s , they take as given the aggregate job-finding probability $f(\theta)$.

We assume that existing matches are exogenously destroyed with a constant job separation probability δ . Thus, any worker of the L_{t-1} workers employed in period $t-1$ has a probability δ of becoming unemployed. Given these assumptions, the law of motion for employment is:

$$L_t(z^t) = (1 - \delta) L_{t-1}(z^{t-1}) + S_t(z^t) f(\theta_t(z^t)) (1 - L_{t-1}(z^{t-1})) \quad (1)$$

2.2 Government policy

The US UI system is financed by payroll taxes on firms and is administered at the state level. However, under the provisions of the Social Security Act, each state can borrow from a federal unemployment insurance trust fund, provided it meets certain federal requirements. Motivated by these features of the UI system, we assume that the government in the model economy can insure against aggregate shocks by buying and selling claims contingent on the aggregate state and is required to balance its budget only in expectation. Further, we assume that the price of a claim to one unit of consumption in state z_{t+1} after a history z^t is equal to the probability of z_{t+1} conditional on z^t ; this would be the case, e.g., in the presence of a large number of out-of state risk-neutral investors with the same discount factor.

The government levies a constant lump sum tax τ on firm profits, and distributes unemployment benefits b_t to unemployed workers. We allow the benefit policy to depend on the entire history of past aggregate shocks; thus the policy $b_t = b_t(z^t)$ must be measurable with respect to z^t . Benefits must be the same across all unemployed workers at a point in time. They are constrained to be non-negative: the government cannot tax home production. Since we assumed that the government has access to financial markets in which a full set of state-contingent claims is traded, its budget constraint collapses to a present-value budget constraint

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{L_t(z^t) \tau - (1 - L_t(z^t)) b_t(z^t)\} \geq 0 \quad (2)$$

2.3 Timing

The government commits to a policy $(\tau, b_t(\cdot))$ once and for all before the period-0 shock realizes. Within each period t , the timing is as follows. The economy enters period t with some level of employment L_{t-1} . The aggregate shock z_t then realizes. Firms observe the aggregate shock and decide how many vacancies to post, at cost k per vacancy. At the same time, workers choose their search effort s_t at the cost of $c(s_t)$. Together, these determine the market tightness θ_t , and $S_t f(\theta)(1 - L_{t-1})$ unemployed workers find jobs. At the same time, a fraction δ of the existing L_{t-1} matches are exogenously destroyed. All the workers who are now employed produce z_t and receive a bargained wage w_t . Firms receive profits $z_t - w_t - \tau$. Workers who are unemployed consume their home production plus unemployment benefits, $h + b_t$.

2.4 Worker value functions

A worker entering period t employed retains his job with probability $1 - \delta$ and loses it with probability δ . If he retains his job, he consumes his wage $w_t(z^t)$ and proceeds as employed to period $t + 1$. If he loses his job, he consumes his home production plus benefits, $h + b_t(z^t)$, and proceeds as unemployed to the next period.

A worker entering period t unemployed first chooses search effort s_t and suffers the disutility $c(s_t)$. He finds a job with probability $s_t f(\theta_t(z^t))$ and remains unemployed with the complementary probability. If he finds a job, he earns the wage $w_t(z^t)$ and proceeds as employed to period $t + 1$. If he remains unemployed, he consumes his home production plus benefits, $h + b_t(z^t)$, and proceeds as unemployed to the next period.

Denote by $W_t(z^t)$ the value after a history z^t for a worker who enters period t employed. Similarly, denote by $U_t(z^t)$ the value of an unemployed worker. The Bellman equations for employed and

unemployed workers are then:

$$W_t(z^t) = (1 - \delta) [u(w_t(z^t)) + \beta \mathbb{E}_t W_{t+1}(z^{t+1})] + \delta [u(h + b_t(z^t)) + \beta \mathbb{E}_t U_{t+1}(z^{t+1})] \quad (3)$$

$$U_t(z^t) = \max_{s_t} -c(s_t) + s_t f(\theta_t(z^t)) [u(w_t(z^t)) + \beta \mathbb{E}_t W_{t+1}(z^{t+1})] \\ + (1 - s_t f(\theta_t(z^t))) [u(h + b_t(z^t)) + \beta \mathbb{E}_t U_{t+1}(z^{t+1})] \quad (4)$$

A worker's surplus from being employed, as opposed to unemployed, in period t is

$$\Delta_t(z^t) = [u(w_t(z^t)) + \beta \mathbb{E}_t W_{t+1}(z^{t+1})] - [u(h + b_t(z^t)) + \beta \mathbb{E}_t U_{t+1}(z^{t+1})] \quad (5)$$

2.5 Firm value functions

A matched firm retains its worker with probability $1 - \delta$. In this case, the firm receives the output net of wages and taxes, $z_t - w_t(z^t) - \tau$, and then proceeds into the next period as a matched firm. If the firm loses its worker, it gains nothing in the current period and proceeds into the next period unmatched.

A firm that posts a vacancy incurs a flow cost k and finds a worker with probability $q(\theta_t(z^t))$. If the firm finds a worker, it gets flow profits $z_t - w_t(z^t) - \tau$ and proceeds into the next period as a matched firm. Otherwise, it proceeds unmatched into the next period.

Denote by $J_t(z^t)$ the value of a firm that enters period t matched to a worker, and denote by $V_t(z^t)$ the value of an unmatched firm posting a vacancy. These value functions satisfy the following Bellman equations:

$$J_t(z^t) = (1 - \delta) [z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1})] + \delta \beta \mathbb{E}_t V_{t+1}(z^{t+1}) \quad (6)$$

$$V_t(z^t) = -k + q(\theta_t(z^t)) [z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1})] + (1 - q(\theta_t(z^t))) \beta \mathbb{E}_t V_{t+1}(z^{t+1}) \quad (7)$$

The firm's surplus from employing a worker in period t is denoted

$$\Gamma_t(z^t) = z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1}) - \beta \mathbb{E}_t V_{t+1}(z^{t+1}) \quad (8)$$

2.6 Wage bargaining

We assume that wages are determined according to Nash bargaining: the wage is chosen to maximize a weighted product of the worker's surplus and the firm's surplus. Specifically, the worker-firm pair chooses the wage $w_t(z^t)$ to maximize

$$\Delta_t(z^t)^\xi \Gamma_t(z^t)^{1-\xi}, \quad (9)$$

where $\xi \in (0, 1)$ is the worker's bargaining weight.

2.7 Equilibrium given policy

In this section, we define the equilibrium of the model, taking as given a government policy $(\tau, b_t(\cdot))$ and characterize its properties.

2.7.1 Equilibrium definition

Taking as given an initial condition (z_{-1}, L_{-1}) , we define an equilibrium given policy:

Definition 1 *Given a policy $(\tau, b_t(\cdot))$ and an initial condition (z_{-1}, L_{-1}) an equilibrium is a sequence of z^t -measurable functions for wages $w_t(z^t)$, search effort $S_t(z^t)$, market tightness $\theta_t(z^t)$, employment $L_t(z^t)$ and value functions $\{W_t(z^t), U_t(z^t), J_t(z^t), V_t(z^t), \Delta_t(z^t), \Gamma_t(z^t)\}$ such that:*

1. *The value functions satisfy the worker and firm Bellman equations (3), (4), (5), (6), (7), (8)*
2. *Optimal search: The search effort S_t solves the maximization problem in (4) for s_t*
3. *Free entry: The value $V_t(z^t)$ of a vacant firm is zero for all z^t*
4. *Nash bargaining: The wage maximizes equation (9)*
5. *Law of motion for employment: Employment satisfies (1)*
6. *Budget balance: Tax revenue and benefits satisfy (2)*

2.7.2 Characterization of equilibrium

First, we derive the law of motion for the surplus from being employed. From equations (3) and (4), we obtain:

$$\begin{aligned} \Delta_t(z^t) &= u(w_t(z^t)) - u(h + b_t(z^t)) \\ &\quad + \beta \mathbb{E}_t [c(S_{t+1}(z^{t+1})) + (1 - \delta - S_{t+1}(z^{t+1})) f(\theta_{t+1}(z^{t+1}))] \Delta_{t+1}(z^{t+1}) \end{aligned} \quad (10)$$

Optimal search implies the necessary first-order condition for S_t :

$$c'(S_t) = f(\theta_t(z^t)) \Delta_t(z^t) \quad (11)$$

Substituting (11) into (10) we get:

$$\begin{aligned} \frac{c'(S_t(z^t))}{f(\theta_t(z^t))} &= u(w_t(z^t)) - u(h + b_t(z^t)) \\ &\quad + \beta \mathbb{E}_t \left[c(S_{t+1}(z^{t+1})) + (1 - \delta - S_{t+1}(z^{t+1})) f(\theta_{t+1}(z^{t+1})) \frac{c'(S_{t+1}(z^{t+1}))}{f(\theta_{t+1}(z^{t+1}))} \right] \end{aligned} \quad (12)$$

Next, we derive the law of motion for the firm's surplus from hiring. By the free-entry condition, the value $V_t(z^t)$ of a firm posting a vacancy must be zero. Equations (6) and (7) then simplify to:

$$J_t(z^t) = (1 - \delta) [z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1})] \quad (13)$$

$$0 = -k + q(\theta_t(z^t)) [z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1})]; \quad (14)$$

which together imply

$$J_t(z^t) = (1 - \delta) \frac{k}{q(\theta_t(z^t))} \quad (15)$$

$$\Gamma_t(z^t) = \frac{k}{q(\theta_t(z^t))} \quad (16)$$

Equations (13) and (15) imply that $\Gamma_t(z^t)$ follows the law of motion $\Gamma_t(z^t) = z_t - w_t(z^t) - \tau + \beta(1 - \delta)\mathbb{E}_t\Gamma_{t+1}(z^{t+1})$, or

$$\frac{k}{q(\theta_t(z^t))} = z_t - w_t(z^t) - \tau + \beta(1 - \delta)\mathbb{E}_t\frac{k}{q(\theta_{t+1}(z^{t+1}))} \quad (17)$$

Finally, the first-order condition with respect to $w_t(z^t)$ for the Nash bargaining problem (9) is

$$\xi u'(w_t(z^t))\Gamma_t(z^t) = (1 - \xi)\Delta_t(z^t) \quad (18)$$

Substituting (16) and (11) into (18) and using the fact that $f(\theta) = \theta q(\theta)$, we rewrite (18) equivalently as

$$\xi u'(w_t(z^t))k\theta_t(z^t) = (1 - \xi)c'(S_t(z^t)) \quad (19)$$

Note that the sequences $\{w_t(z^t), S_t(z^t), \theta_t(z^t), L_t(z^t)\}$ pin down the worker and firm values $\{W_t(z^t), U_t(z^t), \Delta_t(z^t), J_t(z^t), \Gamma_t(z^t)\}$ through equations (3), (4), (11), (15) and (16)). Therefore, the equilibrium given a policy and an initial condition is fully characterized by a wage sequence $\{w_t(z^t)\}$, a search effort sequence $\{S_t(z^t)\}$, a market tightness sequence $\{\theta_t(z^t)\}$ and an employment sequence $\{L_t(z^t)\}$ satisfying: the law of motion (12) for the worker surplus, the law of motion (17) for the firm's surplus, the bargaining solution (19), the law of motion (1) for employment, and the budget constraint (2).

3 Optimal policy

We assume that the government is utilitarian: it chooses a policy to maximize the period-0 expected value of worker utility, taking the equilibrium conditions as constraints. In order to focus on the history-dependence of optimal benefits, we assume that the government is choosing the benefit schedule but taking the tax rate as given.

Definition 2 *A policy $b_t(z^t)$ is feasible if there exists a sequence of z^t -measurable functions $\{w_t(z^t), S_t(z^t), \theta_t(z^t), L_t(z^t)\}$ such that (1), (12), (17), (19) hold for all z^t , and the government budget*

constraint (2) is satisfied.

Definition 3 *The optimal policy is a policy $b_t(z^t)$ that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \{L_t(z^t) u(w_t(z^t)) + (1 - L_t(z^t)) u(h + b_t(z^t)) - (1 - L_{t-1}(z^{t-1})) c(S_t(z^t))\} \quad (20)$$

over the set of all feasible policies.

The government's problem can be written as one of choosing a policy $b_t(z^t)$ as well as functions $\{w_t(z^t), S_t(z^t), \theta_t(z^t), L_t(z^t)\}$ to maximize (20) subject to (1), (12), (17), (19) holding for all z^t , and subject to the government budget constraint (2). We find the optimal policy by solving the system of necessary first-order conditions for this problem.

The optimal b_t will depend not only on the current productivity z_t and the current unemployment level $1 - L_{t-1}$, but also on the entire history of past aggregate shocks. In the appendix, we show that the optimal policy is of the form $b_t(z_t, 1 - L_{t-1}, \mu_{t-1}, \gamma_{t-1})$, where μ_{t-1} and γ_{t-1} are the Lagrange multipliers on the constraints (12) and (17), respectively, in the maximization problem (20). In other words, the quadruple $(z_t, 1 - L_{t-1}, \mu_{t-1}, \gamma_{t-1})$ is a sufficient state variable that captures the dependence of b_t on the history z^t . The fact that the z_t and $1 - L_{t-1}$ are not sufficient reflects the fact that the optimal policy is time-inconsistent: the optimal benefit b_t may differ from the optimal $b_{t'}$ even if $z_t = z_{t'}$, $L_{t-1} = L_{t'-1}$. Intuitively, the government might want to induce firms to post vacancies - and workers to search - by promising low unemployment benefits, but has an ex post incentive to provide higher benefits, so as to smooth worker consumption, after employment outcomes have realized. Including the multipliers μ_t, γ_t as state variables in the optimal policy captures exactly this trade-off. Note that we assume throughout the paper that the government can fully commit to its policy. In the appendix we explain the method used to solve for the optimal policy.

4 Calibration

We calibrate the model to verify that it captures salient features of the US labor market, and is thus a useful one for studying optimal policy design. We normalize mean productivity to one. We assume a benefit scheme that mimics the benefit extension provisions currently in place within the US policy. In the US, local and federal employment conditions trigger automatic 13-week and 26-week extensions (for example if a state's unemployment rate increases above 6% it triggers a 13-week extension of benefits). In the model we assume that the level of benefits automatically increases if productivity falls one or two standard deviations below average level. In order to map the value of the extension into a level we compute the present discounted value of the benefit extensions assuming that the weekly job finding rate falls by one half of a percentage point for each standard deviation drop in productivity. Thus, we set normal benefit levels to 0.4, and 0.42 and 0.44 when productivity is one and two standard deviations below the mean respectively. We pick the tax rate τ so that the government balances its budget if the unemployment rate is 5.5%.

We assume log utility: $u(c) = \ln c$. For the cost of search, we assume the functional form

$$c(s) = A \ln \left(\frac{1}{1-s} \right) \quad (21)$$

This functional form satisfies all the assumptions made on the search cost function; in particular, it implies that the optimal search effort will always be less than 1 for any $A > 0$.

For the matching function, we follow den Haan, Ramey, and Watson (2000) and pick

$$M(S(1-L), v) = \frac{S(1-L)v}{[S^\chi(1-L)^\chi + v^\chi]^{1/\chi}}$$

This matching technology satisfies all the assumptions made earlier, in particular the assumption that the implied job-finding rate is always less than one. We have:

$$f(\theta) = \frac{\theta}{(1+\theta^\chi)^{1/\chi}}$$

$$q(\theta) = \frac{1}{(1 + \theta x)^{1/\chi}}$$

The model period is taken to be 1 week. We set the discount factor $\beta = 0.99^{1/12}$, implying a yearly discount rate of 0.96. Following Shimer (2005), labor productivity z_t is taken to mean real output per person in the non-farm business sector. This measure of productivity is taken from the data constructed by the BLS and the parameters for the shock process are estimated, at the weekly level, to be $\rho = 0.9895$ and $\sigma_\varepsilon = 0.0034$. The job separation parameter δ is set to 0.0081 to match the average weekly job separation rate.² We use the Hagedorn and Manovskii (2008) estimate of the costs of vacancy creation and set $k = 0.58$.

This leaves four parameters to be calibrated: the matching function parameter χ , the coefficient of the search cost function A , the value h of home production, and the worker bargaining weight ξ . We jointly calibrate these four parameters to simultaneously match four data targets: (1) the average vacancy-unemployment ratio; (2) the standard deviation of vacancy-unemployment ratio; (3) the average weekly job-finding rate; and (4) the elasticity of unemployment duration with respect to benefits. The first three of these targets are directly measured in the data. For the elasticity of unemployment duration with respect to benefits, $\epsilon_{u,b}$, we use micro estimates reported by Meyer (1990) and target an elasticity of 0.9. The table below reports the calibrated parameters.

Table 1: Internally Calibrated Parameters

Parameter	Value	Target	Data	Model
h Home production	0.475	Average $S\theta$	0.634	0.634
ξ Bargaining power	0.247	St. dev of $\ln(S\theta)$	0.259	0.259
χ Matching parameter	0.428	Average $Sf(\theta)$	0.139	0.139
A Disutility of search	0.037	$\epsilon_{u,b}$	0.9	0.9

5 Results

In order to illustrate the mechanism, in Figure 1 we plot the optimal benefit policy $b(z, 1 - L, \mu, \gamma)$ as a function of z and $1 - L$, keeping μ and γ fixed at their average values. The optimal benefit is decreasing

²See Hagedorn and Manovskii (2008) on how to obtain the weekly estimates for the job finding rate and the job separation rate from monthly data.

in productivity z and decreasing in unemployment $1 - L$. The intuition for this result is that the optimal benefit is lower in states of the world when the marginal social benefit of job creation is higher, because lower benefits are used to encourage search effort by workers and vacancy creation by firms. The marginal social benefit of job creation is higher when z is higher, since the output of an additional worker-firm pair is then higher. The marginal social benefit is also higher when current employment is lower. As a consequence, optimal benefits are lowest, all else equal, when current productivity is high and current employment is low, i.e. at the beginning of an economic *recovery*. This shape of the policy function also implies that during a recession, there are two opposing forces at work - low productivity and high unemployment - which give opposite prescriptions for the response of optimal benefits. This gives an ambiguous prediction for the overall cyclical behavior of benefits.

In order to understand the overall behavior of the optimal policy, in Figures 2 and 3 we analyze the response of the economy to a negative productivity shock under the optimal policy and compare it to the response under the current policy. In Figure 2 we plot the response of the optimal policy when productivity drops by 1% after a long sequence of productivity held at 1. Benefits initially jump up, but then fall for about two quarters following the shock, and slowly revert to their pre-shock level. Unemployment rises in response to the drop in productivity and continues rising for about one quarter before it starts to return to its pre-shock level. Note that the rise in unemployment is significantly lower than under the current benefit policy. Wages also fall more gradually under the optimal policy than they do under the current policy. In Figure 3 we plot the response of other key labor market variables. As compared to the current benefit policy, the optimal policy results in a faster recovery of the market tightness θ , as well as search effort S .

The intuition for the policy response is that the government would like to provide immediate insurance against the shock and, expecting future productivity to rise, would like to induce a recovery in vacancy creation and search effort. Thus, benefits respond positively to the initial drop in productivity but negatively to the subsequent rise in unemployment - consistent with the prediction of Figure 1. The

initial rise in benefits smooths the fall in wages. The subsequent benefit decline ameliorates the rise in unemployment.

Finally, we investigate how the economy behaves over time under the optimal policy. To this end, we simulated the model both under the current benefit policy and under the optimal policy. Table 2 reports the summary statistics, under the optimal policy, for the behavior of unemployment benefits b and per-period benefit expenditures $b \cdot (1 - L)$. The key observation is that, over a long period of time, the correlation of optimal benefits with productivity is positive: benefits are pro-cyclical in the long run and, in particular, negatively correlated with the unemployment rate. Moreover, this result is not driven by any balanced budget requirement, since we allow the government to run deficits in recessions.

Tables 3 and 4 report the moments of key labor market variables when the model is simulated under the current policy and the optimal policy, respectively. These results corroborate our earlier intuition that the benefit policy serves to smooth the cyclical fluctuations in unemployment.

We compute the expected welfare gain from switching from the current policy to the optimal policy. We find that implementing the optimal policy results in a non-negligible 0.276% welfare gain as measured in consumption equivalent variation terms.

6 Conclusion

We analyzed the design of an optimal UI system in the presence of aggregate shocks in an equilibrium search and matching model. Our main findings are that optimal benefits respond non-monotonically to productivity shocks and are pro-cyclical overall, counter to previous results in the literature. In the context of the current recession, our results suggest that the government, conditional on choosing to extend the duration of benefits, should have lowered their level over the course of the recession. We also find that the optimal benefit policy, in addition to providing insurance to unemployed workers, results in the smoothing of unemployment over the business cycle.

References

- ANDERSEN, T. M., AND M. SVARER (2010): “Business Cycle Dependent Unemployment Insurance,” *Economics Working Papers 2010-16*, School of Economics and Management, University of Aarhus.
- (2011): “State Dependent Unemployment Benefits,” *Journal of Risk and Insurance*, 42(1), 1–20.
- BAILY, M. N. (1978): “Some aspects of optimal unemployment insurance,” *Journal of Public Economics*, 10(3), 379–402.
- COLES, M., AND A. MASTERS (2006): “Optimal Unemployment Insurance in a Matching Equilibrium,” *Journal of Labor Economics*, 24(1), 109–138.
- DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): “Job Destruction and Propagation of Shocks,” *American Economic Review*, 90(3), 482–498.
- FREDRIKSSON, P., AND B. HOLMLUND (2001): “Optimal Unemployment Insurance in Search Equilibrium,” *Journal of Labor Economics*, 19(2), 370–99.
- HAGEDORN, M., AND I. MANOVSKII (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98(4), 1692–1706.
- HOPENHAYN, H. A., AND J. P. NICOLINI (1997): “Optimal Unemployment Insurance,” *Journal of Political Economy*, 105(2), 412–38.
- KILEY, M. T. (2003): “How Should Unemployment Benefits Respond to the Business Cycle?,” *The B.E. Journal of Economic Analysis & Policy*, 3(1), 1–20.
- KROFT, K., AND M. J. NOTOWIDIGDO (2010): “Does the Moral Hazard Cost of Unemployment Insurance Vary With the Local Unemployment Rate? Theory and Evidence,” *Economics working paper*, University of Chicago Booth Graduate School of Business.
- LANDAIS, C., P. MICHAILLAT, AND E. SAEZ (2010): “Optimal Unemployment Insurance over the Business Cycle,” *Working paper*, National Bureau of Economic Research.
- LEHMANN, E., AND B. VAN DER LINDEN (2007): “On the Optimality of Search and Matching Equilibrium When Workers Are Risk Averse,” *Journal of Public Economic Theory*, 9(5), 867–884.
- MEYER, B. D. (1990): “Unemployment Insurance and Unemployment Spells,” *Econometrica*, 58(4), 757–82.
- MORTENSEN, D. T., AND C. A. PISSARIDES (1994): “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, 61(3), 397–415.
- SANCHEZ, J. M. (2008): “Optimal state-contingent unemployment insurance,” *Economics Letters*, 98(3), 348–357.
- SHAVELL, S., AND L. WEISS (1979): “The Optimal Payment of Unemployment Insurance Benefits over Time,” *Journal of Political Economy*, 87(6), 1347–62.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.

A Solving for the optimal policy

The government is maximizing

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{L_t(z^t) u(w_t(z^t)) + (1 - L_t(z^t)) u(h + b_t(z^t)) - (1 - L_{t-1}) c(S_t(z^t))\} \quad (22)$$

subject to the conditions (1), (12), (17), (19) holding for all z^t , and subject to the government budget constraint (2).

Let $\pi(z^t)$ be the probability of history $z^t = \{z_0, z_1, \dots, z_t\}$ given the initial condition z_{-1} . Denote the Lagrange multipliers on (1), (12), (17), (19) by $\beta^t \pi(z^t) \lambda_t(z^t)$, $\beta^t \pi(z^t) \mu_t(z^t)$, $\beta^t \pi(z^t) \gamma_t(z^t)$, $\beta^t \pi(z^t) \phi_t(z^t)$, respectively, and denote by η the Lagrange multiplier on (2). In what follows, we suppress the dependence on z^t for notational simplicity. The first-order conditions for $b_t, S_t, w_t, \theta_t, L_t$, respectively are:

$$(1 - L_t - \mu_t) u'(h + b_t) - \eta(1 - L_t) = 0 \quad (23)$$

$$(1 - L_{t-1}) (\lambda_t f'(\theta_t) - c'(S_t)) + \phi_t (1 - \xi) c''(S_t) + \frac{c''(S_t)}{f(\theta_t)} [\mu_{t-1} (1 - \delta - S_t f'(\theta_t)) - \mu_t] = 0 \quad (24)$$

$$(L_t + \mu_t) u'(w_t) - \gamma_t - \phi_t \xi k \theta_t u''(w_t) = 0 \quad (25)$$

$$\lambda_t S_t (1 - L_{t-1}) f'(\theta_t) + (\mu_t - (1 - \delta) \mu_{t-1}) c'(S_t) \frac{f'(\theta_t)}{(f(\theta_t))^2} + (\gamma_t - (1 - \delta) \gamma_{t-1}) \frac{k q'(\theta_t)}{(q(\theta_t))^2} - \phi_t \xi k u'(w_t) = 0 \quad (26)$$

$$\mathbb{E}_t [\lambda_{t+1} (1 - \delta - S_{t+1} f'(\theta_{t+1})) + c(S_{t+1})] - \lambda_t + u(w_t) - u(h + b_t) + \eta(\tau + b_t) = 0 \quad (27)$$

To find the optimal policy, we first guess η and solve the above system of difference equations (23)-(27) and (1), (12), (17), (19) for the optimal policy vector

$$\Omega(z^t) = (b_t(z^t), S_t(z^t), w_t(z^t), \theta_t(z^t), L_t(z^t); \lambda_t(z^t), \mu_t(z^t), \gamma_t(z^t), \phi_t(z^t))$$

Then we iterate on η until the resulting policy satisfies the budget constraint.

Observe that the only period- $t-1$ variables that enter the period- t first-order conditions are $L_{t-1}, \mu_{t-1}, \gamma_{t-1}$, and no variables from periods prior to $t-1$ enter the period- t first-order conditions. This implies that the quadruple $(z_t, L_{t-1}, \mu_{t-1}, \gamma_{t-1})$ is a sufficient state variable for the history of shocks z^t up to and including period t . Specifically, let

$$\Psi : (z, L, \mu, \gamma) \mapsto (b, S, w, \theta, L', \lambda, \mu', \gamma', \phi)$$

be a function that satisfies

$$(1 - L' - \mu') u'(h + b) - \eta(1 - L') = 0 \quad (28)$$

$$(1 - L) (\lambda f(\theta) - c'(S)) + \phi(1 - \xi) c''(S) + \frac{c''(S)}{f(\theta)} [\mu(1 - \delta - Sf(\theta)) - \mu'] = 0 \quad (29)$$

$$(L' + \mu') u'(w) - \gamma' - \phi \xi k \theta u''(w) = 0 \quad (30)$$

$$\lambda S(1 - L) f'(\theta) + (\mu' - (1 - \delta) \mu) c'(S) \frac{f'(\theta)}{(f(\theta))^2} + (\gamma' - (1 - \delta) \gamma) \frac{kq'(\theta)}{(q(\theta))^2} - \phi \xi k u'(w) = 0 \quad (31)$$

$$\mathbb{E}_{z'|z} [\lambda' (1 - \delta - S' f(\theta')) + c(S')] - \lambda + u(w) - u(h + b) + \eta(\tau + b) = 0 \quad (32)$$

as well as

$$L' = (1 - \delta) L + S f(\theta) (1 - L) \quad (33)$$

$$\begin{aligned} \frac{c'(S)}{f(\theta)} &= u(w) - u(h + b) \\ &+ \beta \mathbb{E}_{z'|z} \left[c(S') + (1 - \delta - S' f(\theta')) \frac{c'(S')}{f(\theta')} \right] \end{aligned} \quad (34)$$

$$\frac{k}{q(\theta)} = z - w - \tau + \beta(1 - \delta) \mathbb{E}_{z'|z} \frac{k}{q(\theta(z', L', \mu', \gamma'))} \quad (35)$$

$$\xi u'(w) k \theta = (1 - \xi) c'(S) \quad (36)$$

Then the sequence defined by

$$\Omega(z^t) = \Psi((b_t(z^t), S_t(z^t), w_t(z^t), \theta_t(z^t), L_t(z^t); \lambda_t(z^t), \mu_t(z^t), \gamma_t(z^t), \phi_t(z^t)))$$

satisfies the system (23)-(27) and (1), (12), (17), (19).

To find the optimal policy given η , we therefore solve the system of functional equations (28)-(36). We use spectral projection methods to solve this system, using Chebyshev polynomials as our basis. The details of the computation are in a supplementary appendix, available by request.

B Tables and figures

Table 2: Optimal benefit behavior

	Benefits	Benefit expenditures
	b	$b \cdot (1 - L)$
Mean	0.403	0.022
Standard deviation	0.024	0.034
Correlation with z	0.504	-0.360
Correlation with $1 - L$	-0.136	0.717

Table 3: Model statistics simulated under the current US policy

	z	$1 - L$	$v / (1 - L)$	$Sf(\theta)$	v	w	S
Mean	1	0.059	0.634	0.139	0.035	0.953	0.760
Standard Deviation	0.013	0.128	0.259	0.170	0.130	0.011	0.090
	z	-0.784	0.826	0.773	0.717	0.983	0.626
	$1 - L$	1	-0.846	-0.837	-0.609	-0.703	-0.772
	$v / (1 - L)$	-	1	0.994	0.903	0.734	0.938
Correlation	$Sf(\theta)$	-	-	1	0.912	0.674	0.970
Matrix	v	-	-	-	1	0.630	0.892
	w	-	-	-	-	1	0.512
	S	-	-	-	-	-	1

Note: Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

Table 4: Model statistics simulated under the optimal policy

	z	$1 - L$	$v / (1 - L)$	$Sf(\theta)$	v	w	S
Mean	1	0.055	0.647	0.140	0.035	0.953	0.776
Standard Deviation	0.013	0.028	0.056	0.034	0.033	0.011	0.012
	z	-0.876	0.762	0.742	0.539	0.924	0.627
	$1 - L$	1	-0.895	-0.885	-0.659	-0.667	-0.823
	$v / (1 - L)$	-	1	0.999	0.925	0.456	0.982
Correlation	$Sf(\theta)$	-	-	1	0.933	0.428	0.987
Matrix	v	-	-	-	1	0.198	0.956
	w	-	-	-	-	1	0.281
	S	-	-	-	-	-	1

Note: Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

Figure 1: Optimal policy

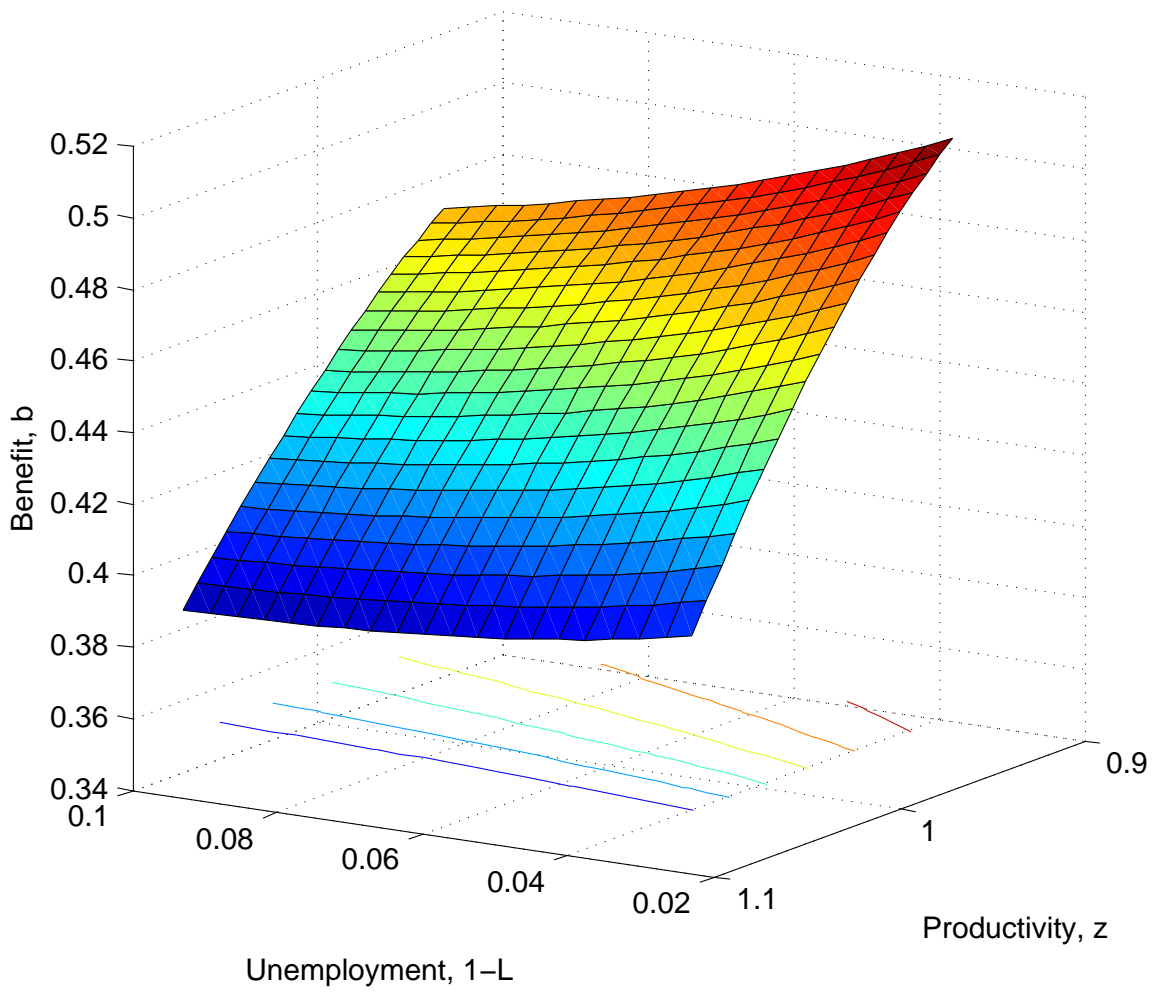


Figure 2: Responses to 1% drop in productivity

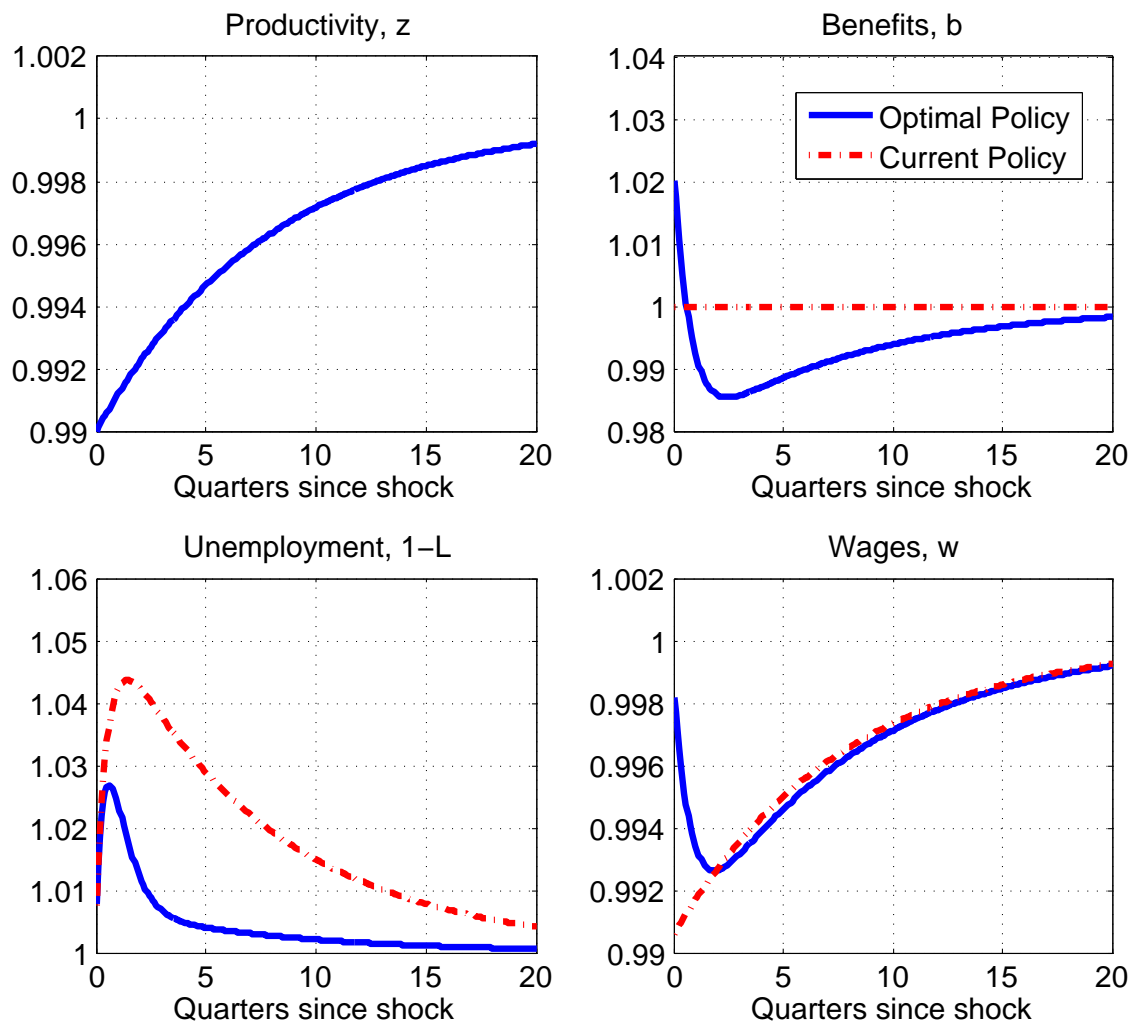


Figure 3: Responses to 1% drop in productivity

