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"Does Competitive Pricing Cause Market Breakdown under Extreme Adverse Selection?"

by

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Does Competitive Pricing Cause Market Breakdown under Extreme Adverse Selection?*

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Abstract: We study market breakdown in a finance context under extreme adverse selection with and without competitive pricing. Adverse selection is extreme if for any price there are informed agent types with whom uninformed agents prefer not to trade. Market breakdown occurs when no trade is the only equilibrium outcome. We present a necessary and sufficient condition for market breakdown. If the condition holds, then trade is not viable. If the condition fails, then trade can occur under competitive pricing. There are environments in which the condition holds and others in which it fails.

Keywords: Adverse selection, market breakdown, separation, competitive pricing.

***JEL* Classification Numbers:** D40, D82, D83, G12, G14.

1 Introduction

The presence of adverse selection can cause severe inefficiencies. This is most starkly illustrated by Akerlof's (1970) example where adverse selection leads to market breakdown (i.e., no trade is the unique equilibrium outcome). This possibility of market breakdown is particularly salient in financial markets, since private information is a major concern when reallocating risk. Glosten (1989) has shown that market breakdown can arise in competitive financial markets under adverse selection. Specifically, market breakdown occurs in a CARA-normal environment,

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with the notion of competition requiring *separation*, i.e., the equilibria are informationally efficient in the sense that the investor's payoff-relevant private information is fully revealed.[1] Since the distribution of private information has unbounded support, adverse selection is *extreme*: for any price there are informed investor types with whom uninformed agents prefer not to trade. This extreme adverse selection is necessary for market breakdown to arise in Glosten's (1989) model of a competitive market (Hellwig, 1992).

Glosten (1989) has shown that a monopoly market maker can sometimes facilitate trade when no trade is possible under his notion of competition. Competition in Glosten (1989) (and many other models) leads to pricing that precludes cross-subsidization among trades, which we refer to as *competitive pricing*. This raises the possibility that competitive pricing is a contributing factor in the market breakdown found under extreme adverse selection, as it can be under non-extreme adverse selection (see Glosten and Milgrom (1985) and Leach and Madhavan (1993)).

We study market breakdown in a finance context under extreme adverse selection with and without competitive pricing. We find that competitive pricing is *not* a contributing factor in market breakdown: If trade is *viable* (in the sense that market makers do not lose money in expectation) then trade can also occur under competitive pricing. The key to this finding is that competitive pricing does not require informational efficiency.

Our environment generalizes Glosten (1989). There is a single informed, risk-averse strategic trader (with CARA, i.e., constant absolute risk aversion, preferences) and risk neutral market makers. The informed trader can act either as a buyer or as a seller; there are no restrictions on order sizes. There is a two-dimension adverse selection problem in which the informed trader has private information about the expected payoff of the risky asset as well as about his endowment. In Glosten (1989), both of these random variables are normally distributed and the informed trader's private information can be summarized by a one-dimensional normally-distributed type. Following Biais, Martimort, and Rochet (2000), we make no parametric distribution assumptions, so that the summarizing one-dimensional type need not be normally distributed (in fact, there are no essential restrictions on the one-dimensional type distribution beyond symmetry and finite variance). This generalization from Glosten's (1989) environment is important, because market breakdown under competitive pricing cannot occur in his environment, but can occur in the generalization.

[1] The literature extending Glosten's (1989) result (i.e., Bhattacharya and Spiegel (1991), Spiegel and Subrahmanyam (1992), and Bhattacharya, Reny, and Spiegel (1995)) also focuses on separating competitive equilibria in CARA-normal environments.

We view the unbounded type space as an idealization of the adverse selection problem caused by large, but bounded type space. The model with unbounded type space should thus be the limit of models with bounded type spaces. Unfortunately, as we discuss in remarks 4.1 and 5.2, the nature of this limit model is unclear. Consequently, like Hellwig (1992), we study extreme adverse selection as the limit case of a sequence of markets in which bounded supports of the distribution of the informed trader's information become arbitrary large.

We identify a condition, the *market breakdown condition*, under which trade is not viable (theorem 4.1). Moreover, if the condition does not hold, then trade can occur under competitive pricing (theorem 4.2). The condition relates the distribution of the informed trader's information to a simple measure of the gains from trade. It is satisfied for a class of fat-tailed distributions, including Pareto (theorem 4.3), while it fails for thin-tailed distributions (theorem 4.4), such as normal (Glosten's (1989) case). The market breakdown condition is thus not vacuous—there are environments in which the condition holds and others in which it fails.

After describing the environment in the next section, we study non-extreme adverse selection (i.e., bounded support distributions of the informed trader's information) in section 3. In addition to being an important benchmark, this analysis underpins our limit analysis. We define the central notions of competitive and viable trading schedules in Subsection 3.1, and introduce the market breakdown condition and discuss its relation to Glosten's (1994) breakdown condition in Subsection 3.2. In subsection 4.1, we describe the nature of the limit analysis, and present the main results (theorems 4.1–4.4)) in subsection 4.2, with the proof of theorem 4.1 in section 6 and of theorem 4.2 in section 5. Technical details are relegated to the appendix.

2 Information Structure and Preferences

We consider a market for a risky asset in which risk neutral market makers provide liquidity to an informed trader who, depending on his private information, may wish to buy or sell the risky asset. Following Glosten (1989, 1994) we refer to the informed trader as the investor. Let $x \in \mathbb{R}$ denote the quantity of the risky asset traded by the investor, with $x > 0$ corresponding to a purchase and $x < 0$ to a sale. The corresponding monetary transfer is denoted by $m \in \mathbb{R}$, with $m < 0$ representing an amount received by the investor and $m > 0$ an amount payed by the investor.

The final value of the risky asset is $v = t + \varepsilon$. The investor privately observes t and his endowment ω of of the risky asset before trade takes place. The random variables (t, ω) describing the investor's private information are uncorrelated and elliptically distributed (see Fang, Kotz, and Ng, 1990) with variances $\sigma_t^2 > 0$ and

$\sigma_\omega^2 > 0$. The random variable ε, realized after trade, is normally distributed with variance $\sigma_\varepsilon^2 > 0$ and independent of (t, ω). For simplicity we assume t, ε, and ω all have zero mean.

When engaging in a trade x resulting in a monetary transfer m, the investor's final wealth is $w = (x + \omega)(t + \varepsilon) - m$. (For simplicity the risk-free rate and the investor's initial money holdings are assumed to be zero.) The investor has CARA preferences with risk aversion parameter $\gamma > 0$. As ε is normally distributed this yields, as usual, a convenient quadratic representation $U(x, m \mid t, \omega)$ of the investor's preferences over $(x, m) \in \mathbb{R}$ conditional on his private information. Defining

$$r \equiv \gamma \sigma_\varepsilon^2 > 0, \tag{1}$$

such a representation is given by (see Biais, Martimort, and Rochet, 2000)[2]

$$\mathscr{U}(x, m \mid t, \omega) = (t - r\omega)x - rx^2/2 - m.$$

While the private information of the investor is two-dimensional, his preferences depend on this information only through the one-dimensional variable $t - r\omega$, which reflects a blend of the investor's informational and hedging motives for trade. Setting

$$\theta \equiv E[v \mid t - r\omega] = E[t + \varepsilon \mid t - r\omega],$$

the linear conditional expectation property of elliptically distributed random variables (Hardin, 1982) implies

$$\theta = \frac{t - r\omega}{b}, \tag{2}$$

where

$$b \equiv \frac{\sigma_t^2 + r^2 \sigma_\omega^2}{\sigma_t^2} > 1. \tag{3}$$

Conditional on θ, the investor's preferences over trade-transfer-pairs are thus described by the utility function

$$\mathscr{U}(x, m \mid \theta) = b\theta x - \frac{1}{2} r x^2 - m. \tag{4}$$

Market makers are risk neutral and maximize expected trading profits. It suffices for our purposes to consider aggregate trading profits $m - vx$. Conditional on θ, expected aggregate trading profits are given by

$$\mathscr{V}(x, m \mid \theta) = m - \theta x. \tag{5}$$

[2]This quadratic representation holds for all distributions of t and ω, since these variables are known to the investor. The quadratic representation does rely on CARA preferences and normality of the noise term ε. We could avoid the normality assumption on ε by assuming the investor has mean-variance preferences.

The above assumptions on the information structure and traders' preferences are as in Glosten (1989), with the important exception that we do not restrict the random variables (t,ω) describing the investor's private information to be normally distributed. We assume these variables are elliptically distributed without imposing further parametric assumptions.[3] This yields payoff functions (4) and (5) identical to those arising in Glosten's (1989) environment (and used in Hellwig's (1992) analysis of Glosten's competitive model) while dispensing with normality of θ.

The distribution of θ is, however, not completely arbitrary, as (2) determines θ as a function of the elliptically distributed random variables (t,ω) and the underlying parameters γ and σ_ε^2. In particular, it is immediate from our assumptions on (t,ω) that the distribution function of θ, denoted by F, is symmetric and has a finite variance. The following result (proved in appendix A) shows that the additional requirement that F has a density decreasing in the absolute value of θ suffices to ensure consistency with the underlying environment.

Lemma 2.1 *For any $b > 1$ and $r > 0$ and any symmetric distribution function F with finite variance and density decreasing in $|\theta|$, there exist $\gamma > 0$ and random variables (t,ω,ε) satisfying the assumptions introduced above such that F is the distribution function of θ.*

Our analysis is conducted in the reduced form environment, with the investor's private information summarized by his one-dimensional type θ and payoff functions given by $u(x,\theta)-m$ for the investor and $m-v(x,\theta)$ for market makers, where (see (4) and (5))

$$u(x,\theta) = b\theta x - \frac{1}{2}rx^2, \tag{6}$$

$$v(x,\theta) = \theta x. \tag{7}$$

The parameters of this environment are r, b, and the distribution function F of θ. We assume $r > 0$ and $b > 1$, as required by the underlying environment (see (1) and (3)). For F we assume the following (which adds some conditions on the density to the conditions of Lemma 2.1).

Assumption 2.1 *The distribution function F of θ is symmetric with finite variance and connected support Θ. It possesses a strictly positive and, for all $\theta \neq 0$, twice continuously differentiable density f, which is decreasing in $|\theta|$.*

We can interpret the reduced form environment with payoffs (6) and (7) in terms of the economic considerations of the underlying structural environment.

[3] Foster and Viswanathan (1993) similarly extend Kyle's (1985) model by considering elliptically rather than normally distributed random variables.

The type θ is the expected payoff of the risky asset conditional on the investor's marginal valuation of the risky asset at $x = 0$. The parameter b is a measure of the investor's informational advantage in predicting the asset's final payoff conditional on his type θ being commonly known, whereas the parameter r measures the strengths of the investor' hedging motive for trade (caused by his risk aversion and the variability of his initial endowment).

As the investor's preferences over (x,m) are quasilinear in m, the surplus resulting from type θ trading quantity x of the risky asset is given by

$$s(x,\theta) = u(x,\theta) - v(x,\theta) = (b-1)\theta x - \frac{1}{2}rx^2. \tag{8}$$

The surplus is maximized by the trading quantity

$$q^{FB}(\theta) = \frac{b-1}{r}\theta \tag{9}$$

with resulting (first best) surplus[4]

$$s^{FB}(\theta) = \frac{(b-1)^2}{2r}\theta^2. \tag{10}$$

Note that first best quantities are increasing and unbounded in θ, and assumption 2.1 implies that ex ante first best surplus is finite.

3 Non-Extreme Adverse Selection

Assumption 2.1 implies that the support of F is either given by $\mathbb{R}$ or by a bounded interval of the form $[-\tau, \tau]$, where $\tau > 0$. We say adverse selection is *extreme* when for any price there are investor types with whom market makers prefer not to trade. Given our parameterization, this condition can be written as: for all $x \neq 0$ and all $p \in \mathbb{R}$, there exists $\theta \in \Theta$ such that $(p-\theta)x < 0$. Extreme adverse selection thus arises if and only if $\Theta = \mathbb{R}$. We study here non-extreme adverse selection (the bounded case), as it is both a benchmark and a central tool in our investigation of extreme adverse selection.

[4] The first best quantity as given by (9) is defined with respect to the reduced form environment. It maximizes surplus under the constraint that trades are measurable with respect to θ. This is to be distinguished from the first best allocation of the underlying structural environment, which provides complete insurance for the informed agent's endowment shock ω.

3.1 Competitive and Viable Market Structures

Rather than providing an explicit game theoretic model of the trading process, we model the trading decision of the investor as the solution to the problem of choosing an optimal trade when faced with a *price schedule* $p : \mathbb{R} \to \mathbb{R}$, specifying a price per unit of the risky asset as a function of the investor's trade. The solution to this maximization problem results in a *trading schedule* $q : [-\tau, \tau] \to \mathbb{R}$ specifying a trade for each type of the investor in the support of the type distribution F.

In line with models of competitive market making such as Kyle (1985) and Rochet and Vila (1994), we suppose that competition between market makers results in a price schedule under which market makers obtain zero expected profits *conditional on the quantity traded*. Let $E[\cdot]$ denote the expectation with respect to F.

Definition 3.1 *Suppose* $\Theta = [-\tau, \tau]$. *A price schedule p and a trading schedule q are* competitive *if p* implements *q,*

$$q(\theta) \in \operatorname*{argmax}_{x \in \mathbb{R}} u(x, \theta) - p(x)x, \quad \forall \theta \in \Theta, \tag{11}$$

and both the zero-profit *condition,*

$$p(x) = E[\theta \mid q(\theta) = x], \quad \forall x \in q(\Theta), \tag{12}$$

and the sequentiality *condition,*

$$p(x) \in \Theta, \quad \forall x \in \mathbb{R}, \tag{13}$$

are satisfied.

The sequentiality condition (13) insists that for *all* possible quantities, the price schedule specify a price consistent with zero profits, reflecting competition between market makers with some common belief over the possible types of the investor who might have chosen such a quantity. It is thus akin to a Kreps and Wilson (1982)-sequentiality requirement.[5] We refer to price and trading schedules satisfying the implementability (11) and the zero-profit (12) conditions, but not necessarily the sequentiality condition, as *zero-profit*.

[5] A trading schedule is competitive in our sense if and only if it is a sequential equilibrium outcome of a signaling game in which the investor chooses a quantity of the risky asset to trade and the market makers then compete a la Bertrand to take the other side of the trade (see Kreps (1990, Section 17.3) for an extended discussion in the context of Spence (1973)-job market signaling). The analysis of Gale and Hellwig (2004), which studies a general equilibrium model of an insurance market with adverse selection, provides an alternative "micro-foundation" for our definition of competitive trading schedules.

Our notion of competition does not impose separation, where a trading schedule is said to be *separating* if it is one-to-one. In contrast to Glosten (1989) (and the other papers mentioned in footnote 1) we allow for pooling (i.e., different types of the investor choosing the same quantity) in a competitive trading schedule. Requiring pricing to be competitive in the sense of definition 3.1 thus eliminates the possibility of cross-subsidization among quantities, while not handicapping competition by imposing the additional requirement of informational efficiency (in the sense that prices must be equal to the expected value of the risky asset conditional on the investor's type).[6] This allows us to address on the question whether, as suggested in Glosten (1989, 1994), cross-subsidization among quantities plays an essential role in avoiding market breakdown.

Rather than considering a particular model of the trading process as an alternative to competitive pricing, we consider *all* market structures in which market makers obtain non-negative aggregate profits. This is captured in the following definition.

Definition 3.2 *Suppose $\Theta = [-\tau, \tau]$. A price schedule p and a trading schedule q are* viable *if p implements q and*

$$E[(p(q(\theta)) - \theta)q(\theta)] \geq 0.$$

Note that zero-profit and so, in particular, competitive price and trading schedules, are viable. Glosten's (1994) model of a discriminatory limit order market results in price and trading schedules which are viable, but not competitive in the sense of definition 3.1 (as there is cross-subsidization among different trade sizes) even though market makers obtain zero expected trading profits. Screening models in which market makers post price schedules (Biais, Martimort, and Rochet, 2000) result in viable, but not competitive, price and trading schedules in which market makers obtain strictly positive expected trading profits.

Because the investor's preferences satisfy single-crossing ($u_{x\theta} > 0$, where subscripts denote partial derivatives), requiring a trading schedule q be implemented

[6] As a property of trading schedules, separation is not a natural implication of competition per se, but a common and convenient assumption that allows researchers to study informational efficiency. If the market is modeled as signaling game, refinements in the spirit of those proposed by Kohlberg and Mertens (1986) and Cho and Kreps (1987) provide formal support for such a focus on separation. See Gale (1992, 1996) for a related Walrasian approach to competition in markets with adverse selection yielding similar conclusions. While Kohlberg and Mertens's (1986) strategic stability has an abstract continuity motivation, the "intuitive" motivations for some of its implications seem less persuasive (Mailath, Okuno-Fujiwara, and Postlewaite, 1993). See also Laffont and Maskin (1990) who consider a financial market signalling model (more akin to the model in Leland and Pyle (1977) than to the one we consider) and argue that separating trading schedules will not be observed when they are interim inefficient in the set of competitive trading schedules (as is always the case in our environment, see section 5.2).

by some price schedule p imposes significant structure on the trading schedule. Standard results from mechanism design (Rochet (1987, Proposition 1) and Milgrom and Segal (2002, Theorem 2)) imply the following:

Lemma 3.1 *Suppose $\Theta = [-\tau, \tau]$. There exists a price schedule p such that p implements q if and only if q is increasing ($\theta \leq \theta' \Rightarrow q(\theta) \leq q(\theta')$).*

Lemma 3.2 *Suppose $\Theta = [-\tau, \tau]$. If a price schedule p implements a trading schedule q, then*

$$R(\theta) - R(\theta') = \int_{\theta'}^{\theta} u_\theta(q(\tilde{\theta}), \tilde{\theta}) d\tilde{\theta} = b \int_{\theta'}^{\theta} q(\tilde{\theta}) d\tilde{\theta} \tag{14}$$

for all $\theta, \theta' \in [-\tau, \tau]$, where $R(\theta) = u(q(\theta), \theta) - p(q(\theta))q(\theta)$ is the rent function.

3.2 Market Breakdown

Given either viability or competition, the natural definition of market breakdown under non-extreme adverse selection is that the only viable or competitive trading schedule is the *no-trade trading schedule* given by $q(\theta) = 0$ for all $\theta \in \Theta$. Since every economically interesting trading schedule must be viable, we reserve the term *market breakdown* for the case where the only viable trading schedule is the no-trade schedule. If there is no market breakdown in this sense, but the only *competitive* trading schedule is the no-trade trading schedule, we say competition causes market breakdown.

To develop our intuition for the non-extreme adverse selection case, it is helpful to momentarily consider the environment with more general investor payoffs u satisfying single crossing $u_{x\theta} > 0$ and strict concavity $u_{xx} < 0$. Glosten (1994) then provides a simple intuition for the condition characterizing market environments in which market breakdown occurs. Trade can occur when there is a price at which market makers are willing to engage in a small transaction with the non-empty pool of investor types willing to engage in that transaction. If no such price can be found, market breakdown results. In particular, proposition 5 in Glosten (1994) suggests that the condition

$$E[\tilde{\theta} \mid \tilde{\theta} < \theta] \leq u_x(0, \theta) \leq E[\tilde{\theta} \mid \tilde{\theta} > \theta], \quad \forall \theta \in (-\tau, \tau) \tag{15}$$

is necessary and sufficient for market breakdown when the support of F is bounded.[7] Because of single crossing, the set of investor types willing to sell the risky asset at

[7] Glosten does not consider the case in which condition (15) holds with equality and imposes a regularity condition on price schedules (see corollary 1 and the subsequent discussion in Glosten (1994)). Using the techniques developed in Nöldeke and Samuelson (2007), it is straightforward to show that (15) is in fact necessary and sufficient for market breakdown when $u_{x\theta} > 0$ and $u_{xx} < 0$.

a price $p = u_x(0,\theta)$ is given by $\tilde{\theta} < \theta$, and so the left inequality in (15) precludes small trades $x < 0$; similarly, the right inequality precludes small trades $x > 0$.

In our environment, F is symmetric and the investor's marginal willingness to pay for the risky asset is given by $u_x(x,\theta) = b\theta - rx$, and so condition (15) is equivalent to

$$(b-1)\theta \le e(\theta), \quad \forall \theta \in [0,\tau), \tag{16}$$

where $e : [0,\tau) \to \mathbb{R}$ is the *mean excess function* (or *residual life function*) of the distribution function F,

$$e(\theta) = E[\tilde{\theta} - \theta \mid \tilde{\theta} > \theta].$$

Because the right side vanishes as θ approaches the upper bound τ and the left-hand-side is clearly strictly positive for all $\theta > 0$ (recall we have assumed $b > 1$), this condition is never satisfied, and so market breakdown does not arise with bounded support. Moreover, competition does not cause market breakdown, as there exist competitive trading schedules satisfying $q(\theta) \neq 0$ for all $\theta \neq 0$ (lemma 5.1 below) so that almost all types trade a non-zero quantity, a situation we refer to as a *liquid market*.

However, as discussed by Hellwig (1992), this does not rule out the possibility that as the support becomes arbitrarily large, the quantities transacted become arbitrarily small for most types. As we will demonstrate below, for distributions with unbounded support the counterpart to (16) may hold and it is exactly in these circumstances that market breakdown becomes an issue.

4 Extreme Adverse Selection

4.1 Capturing Extreme Adverse Selection

We study extreme adverse selection as the limit of a sequence of environments in which bounded supports of the distribution of θ become arbitrarily large. Let F^* be a distribution with *unbounded support* satisfying assumption 2.1. We refer to F^* as the *limit distribution function* and say that for $\tau > 0$ a distribution function F is a τ*-truncation* of F^* if it is obtained from F^* by conditioning on $\theta \in [-\tau,\tau]$:

$$F(\theta) = \begin{cases} 1, & \text{if } \theta > \tau, \\ \dfrac{F^*(\theta) - F^*(-\tau)}{F^*(\tau) - F^*(-\tau)}, & \text{if } \theta \in [-\tau,\tau], \\ 0, & \text{if } \theta < -\tau. \end{cases}$$

Let $\{\tau_n\}$ denote a strictly positive sequence satisfying $\tau_n \to \infty$ as $n \to \infty$. The τ_n-truncation of F^* is denoted by F_n. Observe that $\{F_n\}$ converges weakly to F^*.[8,9]

Definition 4.1 *A sequence* $\{(\tau_n, q_n, p_n)\}$ *is* viable *(respectively,* competitive *or* zero-profit*) if for all n,* (p_n, q_n) *is viable (resp., competitive or zero-profit) given the type distribution* F_n.

The counterpart to our notion of market breakdown in the bounded case is that *all* viable (and thus all competitive) sequences converge to a closed market in the sense of the following definition.

Definition 4.2 *A viable sequence* $\{(\tau_n, q_n, p_n)\}$ converges to a closed market *if for all* $\theta \in \mathbb{R}$,

$$\lim_{n\to\infty} q_n(\theta) = 0.$$

If there are viable sequences not converging to a closed market, which merely requires the existence of some type for which $q_n(\theta)$ does not converge to zero, it is of interest of ask whether there are sequences which converge to a well-defined limit. The following definition adds the requirement that in such a limit all types but $\theta = 0$ trade a non-zero-quantity, corresponding to our notion of a liquid market for the bounded case.

Definition 4.3 *A viable sequence* $\{(\tau_n, q_n, p_n)\}$ converges to a liquid market *if for all* $\theta \neq 0$, $\lim_{n\to\infty} q_n(\theta)$ *exists and*

$$\lim_{n\to\infty} q_n(\theta) \neq 0.$$

We will show that the two definitions exhaust all relevant possibilities.

Remark 4.1 (Extreme Adverse Selection as an Idealization) If one views (as we do) the unbounded type space as an idealization of a situation with large, but bounded type space, the model with unbounded type space should be the limit of models with bounded type spaces. The nature of this limit model is unclear. We discuss here one problem which indicates why adopting the definitions of competitive and viable schedules to the unbounded support case is unsatisfactory. For all

[8] We work with truncations to simplify notation. Our analysis applies essentially unchanged to sequences of distributions with bounded supports $\{F_n\}$ converging weakly to F^*, provided each F_n satisfies assumption 2.1 and $\sup_n \int |\theta|^\alpha dF_n(\theta) < \infty$ for some $\alpha > 2$, so that the relevant moments converge (Chung, 1974, Theorem 4.5.2).

[9] Any τ-truncation of F^* satisfies assumption 2.1 and is thus consistent with the underlying environment of the information structure and preferences (since r is fixed, the underling preference parameter γ and variance σ_ε^2 can be taken as fixed).

τ, the *no-trade trading schedule* given by $q(\theta) = 0$ for all $\theta \in [-\tau, \tau]$, is viable. However, there is no price schedule implementing the limit of no-trade trading schedules as $\tau \to \infty$: Given any price schedule $p : \mathbb{R} \to \mathbb{R}$ and any $x > 0$, type θ can ensure the payoff $[b\theta - p(x)]x - rx^2/2$ by choosing x. For sufficiently large θ this expression will be strictly positive, precluding $q(\theta) = 0$ as the optimal choice for such a type θ.[10] We discuss another possible formulation of the limit model in remark 5.2.

♦

4.2 The Main Results

The counterpart to condition (16) for the limit distribution F^* is

$$(b-1)\theta \leq e^*(\theta), \qquad \forall \theta \geq 0, \tag{17}$$

where

$$e^*(\theta) \equiv E^*[\tilde{\theta} - \theta \mid \tilde{\theta} > \theta],$$

is the mean excess function for the limit distribution and E^* denotes expectation with respect to F^*. If this *market breakdown condition* holds, then there is market breakdown in the sense that every viable sequence converges to a closed market.

Theorem 4.1 *If the market breakdown condition* (17) *holds, then every viable sequence converges to a closed market.*

Our next result shows that the market breakdown condition is *necessary* as well as sufficient for market breakdown. Indeed, it shows much more: competition is *not* a cause of market breakdown.

Theorem 4.2 *If the market breakdown condition* (17) *fails, then for every sequence* $\{\tau_n\}$ *with* $\tau_n \to \infty$, *there exists an associated competitive sequence converging to a liquid market.*

Glosten (1994) has suggested that a necessary ingredient for a market structure to avoid market breakdown (when it can be avoided) is a "small-trade spread." Under such a spread, all types in a neighborhood of the zero type do not trade,

[10] One could follow Glosten (1989) in interpreting the non-existence of any viable (resp. competitive) trading schedule in a model with unbounded support as market breakdown. In our view, such an approach is only warranted if the non-existence corresponds to the convergence of every sequence of viable (resp. competitive) trading schedules to the no-trade trading schedule. Except for the special case of the separating competitive trading schedules studied by Hellwig (1992) (see section 5.1), no such result is available.

precluding convergence to a liquid market. Since we have convergence to a *liquid* market, and not just the absence of market breakdown, when (17) fails, theorem 4.2 shows that, in our setting at least, a small-trade spread is not needed to avoid market breakdown.

The proofs of the two theorems are logically independent and we find it pedagogically convenient to prove them in reverse order. We prove theorem 4.2 in section 5, where we explicitly construct the associated competitive sequence for any sequence of truncations $\tau_n \to \infty$. In section 6, we prove theorem 4.1.

In the remainder of this section we discuss circumstances under which the market breakdown condition (17) will or will not hold. In particular, are there limit distributions F^* for which (17) holds, i.e., can extreme adverse selection indeed cause market breakdown?[11]

If $b > 2$, the answer is no: Hellwig (1992) constructs a competitive sequence converging to a liquid market (see lemma 5.2 below).

For any $b < 2$, on the other hand, there are limit distributions (satisfying assumption 2.1) for which (17) holds. One example of such a distribution is obtained via symmetrization of a translated Pareto distribution (the proof is in appendix C):

Theorem 4.3 *Suppose $b < 2$. The market breakdown condition holds for the symmetric distribution whose distribution function F^* is given by*

$$F^*(\theta) = 1 - \frac{1}{2}(\theta+1)^{-\beta}, \qquad \theta \geq 0,$$

if $2 < \beta \leq b/(b-1)$.[12]

Remark 4.2 It is straightforward to verify that for the distribution in theorem 4.3, decreases in the parameter β induce first-order-stochastic shifts in the first-best surplus. Since the market-breakdown condition fails for small values of β but not for large values, a change in the type distribution leading to a first-order-stochastic dominant *increase* in the distribution of first-best gains from trade may thus cause a liquid market to *close*.

♦

[11]This question is of particular interest, as (to the best of our knowledge) all previous examples where no trade is the only viable outcome in a financial market context (see Glosten and Milgrom (1985), Leach and Madhavan (1993), Glosten (1994)) rely on the existence of a mass of *risk neutral* informed investors, with risk neutrality precluding gains from trade between these investors and market makers.

[12]The distribution F^* satisfies assumption 2.1. It has finite variance because $\beta > 2$. Since $b < 2$ implies $2 < b/(b-1)$, there are $\beta > 2$ satisfying $\beta \leq b/(b-1)$.

More generally, any limiting distribution F^* such that $e^*(\theta)/\theta$ is a decreasing function satisfying

$$\lim_{\theta\to\infty} \frac{e^*(\theta)}{\theta} \geq (b-1) \tag{18}$$

will satisfy (17). Conversely, if the limit appearing in (18) is well-defined, (18) is clearly necessary for market breakdown. The following result builds on this observation to obtain a more explicit necessary condition for market breakdown. The result requires that the *proportional hazard rate* of the distribution function F^*,

$$g^*(\theta) \equiv \frac{\theta f^*(\theta)}{1-F^*(\theta)},$$

have a well-defined limit as $\theta \to \infty$. This mild regularity condition ensures that $\lim_{\theta\to\infty} e^*(\theta)/\theta$ exists. The distributions commonly studied in economics satisfy this property.[13]

Theorem 4.4 *Suppose the limit of $g^*(\theta)$ as $\theta \to \infty$ (which may be infinite) exists and suppose for $k \geq 2$, the k^{th} moment of F^* is finite. Then for all $b \geq k/(k-1)$, the market breakdown condition (17) fails and there exist competitive sequences converging to a liquid market.*

When the regularity condition on the proportional hazard rate holds, theorem 4.4 implies, in particular, that for any given $b > 1$ the market breakdown condition fails if all moments of F^* exist. Hence, a necessary condition for the market breakdown condition (17) is that F^* has fat tails. In terms of the underlying structural environment, it also follows that if all moments of (the limit distributions) of t and ω are finite (such as in Glosten's (1989) environment, where these variables are normally distributed), the market breakdown condition fails.

Remark 4.3 Suppose, as for many commonly studied distributions, the density f^* of F^* is log-concave on $\mathbb{R}_+$.[14] By An (1998, proposition 1), $e^*(\theta)$ is decreasing in θ, providing a simple proof that the market breakdown condition will *not* hold for sufficiently large θ in this case. As log-concavity of the density implies that the hazard rate (and thus the proportional hazard rate) is increasing and that all moments of F^* exist (An, 1998, corollary 1), this result is a special case of theorem 4.4.

[13] It is satisfied by any distribution F^* with a truncation from below possessing an increasing proportional hazard rate. See van den Berg (1994) for an extensive discussion of distributions possessing an increasing proportional hazard rate.

[14] Bagnoli and Bergstrom (2005) contains a list of parametric families of distribution functions with log-concave densities. Note that f^* will be log-concave on $\mathbb{R}_+$ whenever it is obtained by symmetrizing the log-concave density function of a distribution with support $\mathbb{R}_+$.

♦

5 Competitive Trading Schedules and Liquid Markets

In this section, we prove theorem 4.2. We analyze separating competitive schedules in section 5.1. These schedules converge to liquid markets for $b > 2$. When $b \leq 2$, the distortions required by separation result in the separating competitive schedules converging to a closed market (even when the market breakdown condition fails). Reducing these distortions requires pooling some types, and as an intermediate step we analyze tail-pooling schedules in section 5.2. We prove theorem 4.2 in section 5.3 using *semi-pooling* competitive trading schedules. In these schedules, the distortions implied by separation are ameliorated by pooling the right set of types of investors.

Because our environment is symmetric, we can restrict attention to symmetric competitive and zero-profit trading and price schedules, where a trading schedule is *symmetric* if $q(-\theta) = -q(\theta)$ for all $\theta \in [0, \tau]$ and a corresponding definition applies for price schedules. Note that a symmetric trading schedule satisfies $q(0) = 0$ and that (from lemma 3.1) negative types sell the risky asset ($q(\theta) \leq 0$ for $\theta < 0$) and positive types buy the risky asset ($q(\theta) \geq 0$ for $\theta > 0$). We specify such trading schedules only for positive types (and the corresponding implementing price schedules only for positive quantities) with the extension to negative types (and negative quantities) then given by symmetry. Since the trading schedules for positive types maximize $u(x, \theta) - p(x)x$ over $x \geq 0$, it is immediate that no type finds it profitable to choose a quantity specified for a type of a different sign.

5.1 Separating Trading Schedules

Here we present some preliminary results on separating zero-profit trading schedules. While these results are essentially in Hellwig (1992), technical differences preclude our simply appealing to his paper. A slight modification of the arguments in Mailath (1987) yields the following lemma (see appendix D for the proof of the first statement, the second statement follows from the discussion after the lemma).

Lemma 5.1 *Suppose $\Theta = [-\tau, \tau]$. For all $\bar{x} \in (0, q^{FB}(\tau)]$, there exists a unique symmetric separating zero-profit trading schedule $q : [-\tau, \tau] \to \mathbb{R}$ satisfying $q(\tau) = \bar{x}$. Furthermore, a separating zero-profit trading schedule is competitive if and only if it is symmetric and satisfies $q(\tau) = q^{FB}(\tau)$. Hence, there is a unique separating competitive trading schedule for every $\tau > 0$.*

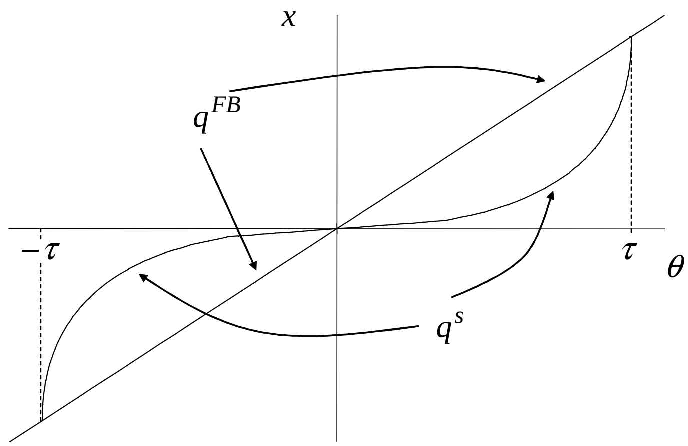

Figure 1: The separating competitive trading schedule q^s for $b \leq 2$. The trading schedule q^s is tangential to the θ-axis at $\theta = 0$.

Figures 1 and 2 illustrate the separating competitive trading schedule. As usual, in a separating trading schedule, imposing the sequentiality condition determines the behavior of the "worst" types. Among positive types the worst belief the market makers can hold is $\theta = \tau$, while among negative types the worst belief is $\theta = -\tau$. Since each type receives his or her type as the price in a separating zero profit price schedule ($p(q(\theta)) = \theta$), the worst types cannot be disciplined in a separating competitive trading schedule and so choose their "first-best" quantity, $q^{FB}(\theta)$. Due to the incentive constraints, the quantities for all types in the intervals $(-\tau, 0)$ and $(0, \tau)$ are distorted from their first best level towards zero. For a given support of the type distribution, the degree of distortion is determined by the trade-off between the incentive to mislead the market and the increased cost of lowered diversification, i.e. the parameters b and r in the investor's utility function. Note that, as illustrated in the figures, the structure of the separating competitive trading schedules is different for the cases $b \leq 2$ and $b > 2$.

The behavior of the investor in a separating competitive trading schedule depends on the characteristics of the distribution of the private information in a limited and particular way. The value of the boundary type completely determines the separating competitive trading schedule, with other characteristics of the distribution function irrelevant. On the other hand, again as usual, increasing the severity of adverse selection by increasing τ has a significant impact on the separating competitive trading schedule. In particular, competitive sequences with separating trading schedules converge to a liquid market if $b > 2$ and converge to a closed

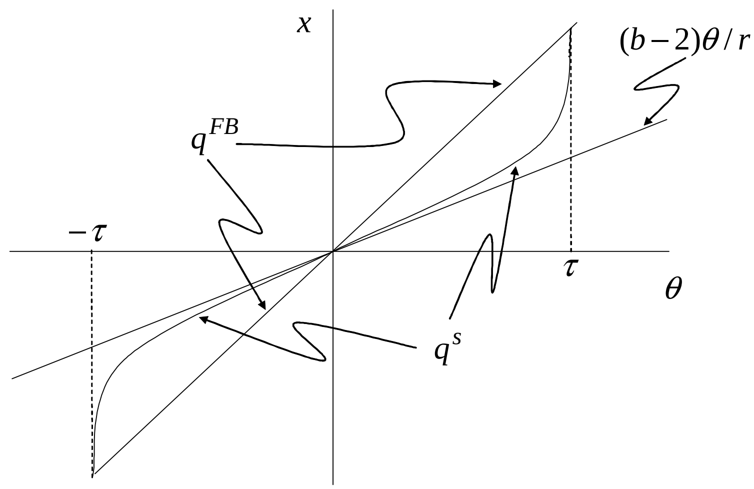

Figure 2: The separating competitive trading schedule q^s for $b > 2$. The trading schedule q^s is tangential to the line $q = (b-2)\theta/r$ at $\theta = 0$.

market if $b \leq 2$ (again, see appendix D for the proof):

Lemma 5.2 *Suppose $\{(\tau_n, q_n, p_n)\}$ is a competitive sequence with q_n separating for each n. If $b \leq 2$, the competitive sequence converges to a closed market. If $b > 2$, the competitive sequence converges to a liquid market; in particular $q_n(\theta) \to (b-2)\theta/r$ for all θ.*

Lemma 5.2 implies that for all sequences $\{\tau_n\}$ satisfying $\tau_n \to \infty$ there is an associated competitive sequence converging to a liquid market when $b > 2$.[15] To show that the failure of (17) is sufficient for the existence of competitive sequences converging to a liquid market, it thus suffices to consider the case $b \in (1, 2]$ for the remainder of this section.

Remark 5.1 (Discontinuity at infinity) For $b > 2$, the separating competitive trading schedules q_n and price schedules p_n converge pointwise to the linear equilibrium in Glosten (1989). The additional separating competitive trading schedules identified by Glosten (1989) for the limit market environment are eliminated as potential limit outcomes by the sequentiality condition, $p_n(x) \in [-\tau_n, \tau_n]$.

♦

[15]It is then an implication of theorem 4.1 that (17) must fail for $b > 2$.

5.2 Tail-pooling Schedules

For $b \leq 2$, separating competitive sequences converge to a closed market because the distortions required to separate all types become arbitrarily large as τ becomes arbitrarily large. A natural conjecture (see, for example, Hellwig (1992, footnote 3)) is that pooling extreme types eliminates the negative impact of requiring all types to separate. As we demonstrate in this subsection, this conjecture is correct in the sense that pooling extreme types does yield a *zero-profit* sequence that converges to a liquid market *if* the market breakdown condition fails. However, pooling types in the tail does not generate *competitive* sequences that converge to a liquid market, so the result obtained here falls short of theorem 4.2. This defect is rectified in the next subsection, where we construct a competitive sequence converging to a liquid market (when the market breakdown condition fails) that also converges to the same limit trading schedule in which extreme types are pooled.

Definition 5.1 *A symmetric trading schedule q is* tail-pooling *if there exists a* cut-off type $0 < \hat{\theta} \in \Theta$ *and a* pooling quantity $\hat{x} > 0$ *such that*

$$q(\theta) = \hat{x}, \quad \forall \theta > \hat{\theta}$$

and the restriction of q to $[-\hat{\theta}, \hat{\theta}]$ is separating.

Suppose the market breakdown condition (17) fails, so that for some $\hat{\theta} > 0$ we have $(b-1)\hat{\theta} > e^*(\hat{\theta})$. This raises the possibility of pooling all types $\theta > \hat{\theta}$ at a strictly positive quantity and price that allows market makers to break even. Accordingly, we construct a tail-pooling trading schedule q^t with cut-off type $\hat{\theta}$ for the limit case, $\Theta = \mathbb{R}$. The pooling quantity $\hat{x}$ maximizes $\hat{\theta}$'s payoff under zero-profit pricing ($p(\hat{x}) = E^*[\theta \mid \theta > \hat{\theta}] = \hat{\theta} + e^*(\hat{\theta})$), i.e.,

$$\hat{x} = \underset{x}{\operatorname{argmax}}(b\hat{\theta} - \hat{\theta} - e^*(\hat{\theta}))x - \frac{1}{2}rx^2 = \frac{((b-1)\hat{\theta} - e^*(\hat{\theta}))}{r} > 0. \tag{19}$$

The payoff to type $\hat{\theta}$ from choosing $\hat{x}$ is $U^*(\hat{\theta}) \equiv ((b-1)\hat{\theta} - e^*(\hat{\theta}))^2/r$, which is strictly positive from (19).

We now describe the separating component of q^t. Let $\bar{x} \in (0, \hat{x})$ be the quantity making type $\hat{\theta}$ indifferent between revealing his type at $\bar{x}$ and joining the pool, i.e., $\bar{x}$ is the (unique) quantity $\bar{x} \in (0, \hat{x})$ satisfying $u(\bar{x}, \hat{\theta}) - \hat{\theta}\bar{x} = U^*(\hat{\theta})$.[16] The separating component is given by the separating zero-profit schedule on $[-\hat{\theta}, \hat{\theta}]$ with initial value $q^t(\hat{\theta}) = \bar{x}$. By lemma 5.1, this schedule is unique.

[16] The quantity $\bar{x}$ is well-defined, because $U^*(\hat{\theta}) > 0$.

Consider now $\tau_n \to \infty$. We construct, for large n, a symmetric tail-pooling zero-profit trading schedule q_n with cut-off type $\hat{\theta}$ and pooling quantity $\hat{x}$, converging pointwise to q^t.

Suppose n is sufficiently large that $\hat{\theta} < \tau_n$. The zero-profit condition requires a price at the quantity $\hat{x}$ of

$$p_n(\hat{x}) = E_n\left[\tilde{\theta} \mid \tilde{\theta} > \hat{\theta}\right] = \hat{\theta} + e_n(\hat{\theta}). \tag{20}$$

The payoff to type $\hat{\theta}$ from choosing $\hat{x}$ is then given by

$$U_n(\hat{\theta}) = [(b-1)\hat{\theta} - e_n(\hat{\theta})]\hat{x} - \frac{1}{2}r\hat{x}^2.$$

Because $e_n(\hat{\theta}) \to e^*(\hat{\theta})$ (lemma B.2), $U_n(\hat{\theta})$ converges to $U^*(\hat{\theta}) > 0$. Thus, for sufficiently large n, the payoff $U_n(\hat{\theta})$ is strictly positive, and for such n we construct a symmetric zero-profit trading schedule.

Set $q_n(\theta) = \hat{x}$ for all $\theta \in (\hat{\theta}, \tau_n]$. To complete the specification of the trading schedule q_n, we proceed analogously to the limit case. Letting $\bar{x}_n \in (0, \hat{x})$ be the quantity making type $\hat{\theta}$ indifferent between revealing his type at $\bar{x}_n$ and joining the pool, there is a symmetric separating zero-profit trading schedule $q_n : [-\hat{\theta}, \hat{\theta}] \to \mathbb{R}$ satisfying the initial value $q_n(\hat{\theta}) = \bar{x}_n$.

The price schedule is determined for quantities in the range of q_n by using the zero profit condition, i.e., (20) and $p_n(q_n(\theta)) = \theta$ for all $\theta \in [-\hat{\theta}, \hat{\theta}]$. Standard arguments using the single-crossing property of u show that no type has an incentive to choose the quantity of another type. By specifying sufficiently unattractive prices for quantities outside the range of q_n, no type has an incentive to choose quantities outside the range, and so the symmetric tail-pooling schedule constructed in this way is implementable.

Clearly, $\bar{x}_n \to \bar{x}$, and so $q_n(\theta) \to q^t(\theta)$ for all θ. Since $q^t(\theta) = 0$ only if $\theta = 0$, we have proved the following lemma.

Lemma 5.3 *Suppose there exists $\hat{\theta} > 0$ satisfying $(b-1)\hat{\theta} > e^*(\hat{\theta})$. For every $\{\tau_n\}$ with $\tau_n \to \infty$, there exists an associated zero-profit sequence $\{(\tau_n, q_n, p_n)\}$ converging to a liquid market, with* $\lim_n q_n(\theta) = q^t(\theta)$ *for all θ.*

The sequence constructed in the proof of lemma 5.3 is *not* competitive: For any price schedule p_n implementing the trading schedule q_n constructed above, the payoff received by type τ_n is given by

$$U_n(\tau_n) = b[\tau_n - \hat{\theta}]\hat{x} + U_n(\hat{\theta}).$$

As $U_n(\hat{\theta})$ converges to a finite limit, it follows that $U_n(\tau_n)$ is of order $O(\tau_n)$, while from (10), $s^{FB}(\tau_n)$ is of order $O(\tau_n^2)$, so that eventually $U_n(\tau_n) < s^{FB}(\tau_n)$. If p_n is a competitive price schedule, type τ_n can obtain a payoff at least equal to $s^{FB}(\tau_n)$ by choosing $q^{FB}(\tau_n)$, and so for large n, p_n cannot implement q_n, a contradiction. Consequently, any implementing price schedule must violate the sequentiality condition for sufficiently large n.

This failure of the sequentiality condition is not an artefact of our particular construction of a tail pool: It can be shown that all tail-pooling *competitive* sequences converge to a closed market when $b \leq 2$, implying that tail-pooling competitive sequences must converge to a closed market under precisely the same conditions as separating competitive trading schedules. This is despite the fact that for every τ there exist competitive tail-pooling trading schedules interim Pareto-dominating the competitive separating trading schedule.[17] Every tail-pooling competitive sequence converges to a closed market when $b \leq 2$ because sustaining a tail-pool for large τ_n requires a large pooling quantity $\hat{x}_n$ (to ensure that type τ_n is willing to participate in the pool), which in turn requires the cutoff-type, $\hat{\theta}_n$, to also be large. That is, $\lim_n \hat{x}_n = \infty$ and $\lim_n \hat{\theta}_n = \infty$. But for $b \leq 2$ this implies convergence to a closed market for the same reason that separating competitive sequences converge to a closed market. The tail-pool zero-profit trading schedule of lemma 5.3 converges to a liquid market, on the other hand, because both the pooling quantity and cutoff type are bounded away from infinity as n gets large.

5.3 Semi-pooling Trading Schedules

The difficulties noted after the statement of lemma 5.3 are avoided by adjusting the construction of a tail-pooling trading schedule to allow sufficiently extreme types to separate.

Definition 5.2 *For $\Theta = [-\tau, \tau]$, a symmetric trading schedule q is* semi-pooling *if there exists a* pooling interval $(\hat{\theta}, \bar{\theta}]$ *where $0 < \hat{\theta} < \bar{\theta} < \tau$ and a pooling quantity $\hat{x} > 0$ such that*

$$q(\theta) = \hat{x}, \qquad \forall \theta \in (\hat{\theta}, \bar{\theta}]$$

and the restriction of q to $\theta \in [-\tau, -\bar{\theta}) \cup [-\hat{\theta}, \hat{\theta}] \cup (\bar{\theta}, \tau]$ is one-to-one.

A semi-pooling trading schedule differs from a tail-pooling trading schedule only in that the types $\theta \in (\bar{\theta}, \tau]$ do not choose the pooling quantity $\hat{x}$ but are instead separated.

In conjunction with lemma 5.2 the following result establishes theorem 4.2.

[17] That is, every non-zero type of the investor achieves a higher payoff under the former than under the later. As market makers obtain zero profits under any competitive trading schedule, this is the appropriate notion of interim Pareto-dominance.

Lemma 5.4 *Suppose $b \in (1,2]$ and there exists $\hat{\theta} > 0$ satisfying $(b-1)\hat{\theta} > e^*(\hat{\theta})$. For every sequence $\{\tau_n\}$ with $\tau_n \to \infty$, there exists an associated competitive sequence $\{(\tau_n, q_n, p_n)\}$ converging to a liquid market, with $\lim_n q_n(\theta) = q^t(\theta)$ for all θ, where q^t is the limit tail-pooling schedule from section 5.2.*

We now describe the construction of the competitive semi-pooling trading schedule q_n for sufficiently large n. Lemma D.1 in appendix D ensures that for large n, there exists a triple $(\bar{\theta}_n, \hat{x}_n, \hat{p}_n) \in \mathbb{R}^3$ satisfying the properties required in this construction. (Note that $\hat{p}_n$ is a number, while p_n is a pricing schedule.) The formal argument showing the convergence of such a sequence to a liquid market is provided in lemma D.2.

Our construction of the competitive semi-pooling schedule q_n is illustrated in in figure 3 for positive types. The solid line depicts the trading schedule. The pooling interval is given by $(\hat{\theta}, \bar{\theta}_n]$ with $\bar{\theta}_n < \tau_n$ and $\hat{\theta}$ satisfying the condition $(b-1)\hat{\theta} > e^*(\hat{\theta})$. The pooling quantity is $\hat{x}_n$. For $\theta > \bar{\theta}_n$ the trading schedule is identical to the unique separating competitive trading schedule on $[-\tau_n, \tau_n]$ illustrated in figure 1. Type $\bar{\theta}_n$ is indifferent between his trade in the separating competitive trading schedule, $q_n^s(\bar{\theta}_n)$, and trading the pooling quantity at the price

$$\hat{p}_n = E_n\left[\theta | \theta \in (\hat{\theta}, \bar{\theta}_n]\right] < b\hat{\theta}, \tag{21}$$

where the inequality will hold for n sufficiently large. The pooling quantity $\hat{x}_n$ satisfies the condition that it be the optimal quantity for type $\hat{\theta}$ taking as given the price $\hat{p}_n$:

$$\hat{x}_n = \underset{x}{\operatorname{argmax}}(b\hat{\theta} - \hat{p}_n)x - \frac{1}{2}rx^2 = \frac{(b\hat{\theta} - \hat{p}_n)}{r} > 0,$$

where the inequality is from the inequality in (21). Finally, for $0 \leq \theta \leq \hat{\theta}$, the trading schedule is given by the zero-profit separating trading schedule on $[-\hat{\theta}, \hat{\theta}]$ satisfying the initial condition $q(\hat{\theta}) = \underline{x}_n$ where $\underline{x}_n \in (0, \hat{x}_n)$ satisfies the indifference condition

$$u(\underline{x}_n, \hat{\theta}) - \hat{\theta}\underline{x}_n = \frac{(b\hat{\theta} - \hat{p}_n)^2}{r},$$

with the inequality in (21) ensuring that the quantity $\underline{x}_n$ is well-defined and strictly positive.

It is immediate from the construction of the semi-pooling trading schedule q_n that the trading schedule is implemented by a price schedule p_n specifying $p_n(\hat{x}_n) = \hat{p}_n$ and $p_n(q_n(\theta)) = \theta$ for all θ that are separated,[18] and sufficiently

[18] Note that we have ensured that type $\hat{\theta}$ (resp., $\bar{\theta}_n$) is indifferent between trading the pooling quantity $\hat{x}$ at price $\hat{p}$ to trading the quantity $\underline{x}_n$ at price $\hat{\theta}$ (resp., to trading the quantity $q_n^s(\bar{\theta}_n)$ at price $\bar{\theta}_n$).

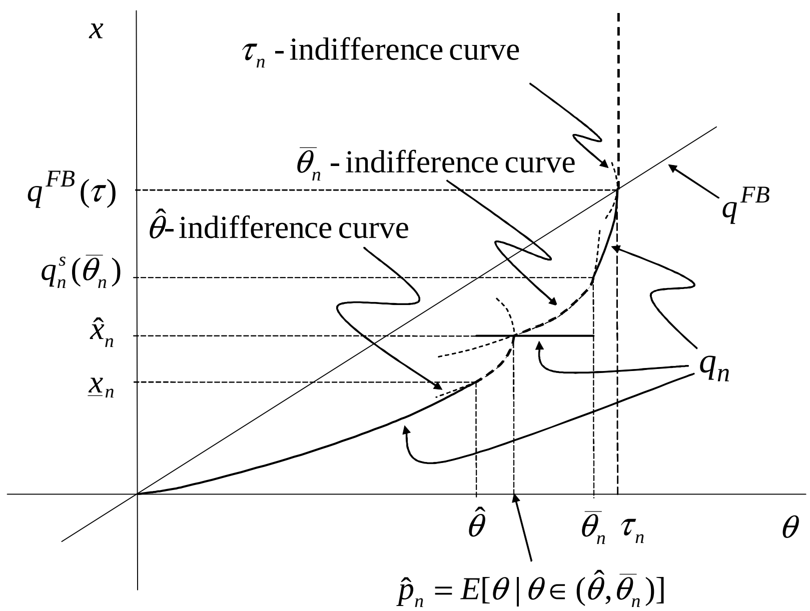

Figure 3: Semi-pooling competitive trading schedule with pooling quantity $\hat{x}_n$ and pooling interval $(\hat{\theta}, \bar{\theta}_n]$. The solid line depicts the trading schedule q_n.

unattractive prices for all quantities q outside the range of the trading schedule. Hence, q_n is a zero-profit trading schedule.

To verify that q_n is in fact competitive, and not just zero-profit, requires extending the zero-profit specification of the price schedule p_n to quantities not in the range of q_n. This extension is illustrated in figure 3 where the heavy dashed lines indicate the specification of the price schedule p_n outside the range of q_n. For quantities x in the interval $(\underline{x}_n, \hat{x}_n)$, the price $p_n(x)$ is set to make $\hat{\theta}$ indifferent between trading x at the price $p_n(x)$ and trading $\hat{x}_n$ at the price $\hat{p}_n$ (equivalently, trading $\underline{x}_n$ at the price $\hat{\theta}$); for quantities $x \in (\hat{x}_n, q_n^s(\bar{\theta}_n))$, the price $p_n(x)$ is set to make $\bar{\theta}_n$ indifferent between trading x at the price $p_n(x)$ and trading $\hat{x}_n$ at the price $\hat{p}_n$ (equivalently, trading $q_n^s(\bar{\theta}_n)$ at the price $\bar{\theta}_n$); and finally, for $x > q^{FB}(\tau_n)$, set $p_n(x) = \tau_n$. In addition to implementing q_n (this is immediate from the single-crossing property of u and footnote 18) the defined price function is increasing and continuous (and so satisfies the sequentiality condition).

In the proof of lemma D.2, we demonstrate that due to $b \leq 2$, the sequence $\bar{\theta}_n$ associated with the semi-pooling schedule q_n converges to infinity, implying that the sequence of pooling quantities $\{\hat{x}_n\}$ converges to the strictly positive limit $\hat{x}$ given in (19), and that the sequence of pooling prices $\{\hat{p}_n\}$ converges to $\hat{\theta} + e^*(\hat{\theta})$. The sequence of semi-pooling trading schedules thus converges pointwise to the same limit as the sequence of tail-pooling schedules constructed in section 5.2. In particular, the sequence converges to a liquid market, thus proving lemma 5.4.

Remark 5.2 We return to the issue raised in remark 4.1, namely the need to study extreme adverse selection as the limit of non-extreme adverse selection . As we saw in that remark, simply adopting the notion of viability to the unbounded case leads to a failure of continuity (more specifically, upper hemicontinuity). Remark 5.1 discussed a failure of lower hemicontinuity for the similar adoption of competitive schedules.

An alternative limit model abandons the requirement that a price schedule must attach a finite price to every quantity (so that market makers can refuse to execute some trades). This amounts to restricting the domain of p and the set of feasible trades in the definitions of competitive and viable schedules. Under these definitions, the limit tail-pooling schedule from section 5.2 is "competitive." While our analysis in this subsection does show that this schedule is the limit of competitive schedules for the bounded support models, it also shows that this result is non-trivial (due to the sequentiality condition). Moreover, we do not know if arbitrary "competitive" trading schedules can be approximated by competitive trading schedules for large, but bounded support.

♦

6 Convergence to Closed Markets

In this section, we prove theorem 4.1, i.e, that every viable sequence converges to a closed market if the market breakdown condition (17) holds. We present the key steps, relegating the more technical arguments to appendix E.

Suppose $\{(\tau_n, q_n, p_n)\}$ is a viable sequence. For type distribution F_n, denote the market makers' expected profit by

$$\Pi_n \equiv \int_{-\tau_n}^{\tau_n} [p_n(q_n(\theta)) - \theta] q_n(\theta)\, dF_n(\theta), \tag{22}$$

with viability implying $\Pi_n \geq 0$ for all n.

Since the market environment includes the possibility of both negative and positive types, as well as as market makers acting as both buyers and sellers, profits from trades on one side of the market can subsidize losses on the other. However, because the environment is symmetric, there is no loss of generality in assuming there is no such cross-subsidization in the limit. To be more precise, define, for all n,

$$\Pi_n^+ \equiv \int_0^{\tau_n} [p_n(q_n(\theta)) - \theta] q_n(\theta)\, dF_n(\theta).$$

Then, by lemma E.2, if $\{(\tau_n, q_n, p_n)\}$ is a viable sequence that does not converge to a closed market, we can assume without loss of generality that $q_n(0) = 0$ for all n, the existence of $\hat{\theta} > 0$ such that $q_n(\hat{\theta}) \to \hat{x} > 0$, and $\Pi_n^+ \to \Pi^+ \geq 0$. We will obtain a contradiction by showing that under (17), no viable sequence satisfies these three properties. To show this, we clearly can restrict attention to $\theta \geq 0$.

Because $\hat{x} > 0$ and the second moment of F^* exists, there exists $\theta^\dagger > \hat{\theta}$ such that

$$-\frac{r}{2}[1 - F^*(\hat{\theta})]\hat{x}^2 + 2\int_{\theta^\dagger}^{\infty} s^{FB}(\theta) f^*(\theta) d\theta < 0. \tag{23}$$

From lemma E.1, $q_n(\theta^\dagger)$ is bounded and there thus exists a subsequence $\{(\tau_m, q_m, p_m)\}$ of $\{(\tau_n, q_n, p_n)\}$ such that $q_m(\theta^\dagger) \to x^\dagger$. As every implementable trading schedule is increasing, we have $x^\dagger \geq \hat{x}$.

We now argue that if the market breakdown condition (17) holds, $\{\Pi_n^+\}$ is bounded above by a sequence converging to the left hand side of (23), contradicting the hypothesis $\Pi_n^+ \to \Pi^+ \geq 0$ and thus establishing theorem 4.1.

We begin by using lemma 3.2 and integration by parts to obtain

$$\Pi_n^+ = \int_0^{\tau_n} VS_n(q_n(\theta), \theta)\, dF_n(\theta), \tag{24}$$

where

$$VS_n(\theta) \equiv s(q_n(\theta), \theta) - b\frac{(1 - F_n(\theta))}{f_n(\theta)} q_n(\theta)$$

is the *virtual surplus*.

As $\{q_n(\theta)\}$ may be unbounded as a function of θ, we define $\tilde{q}_n(\theta) \equiv \min\{q_n(\theta), q_n(\theta^\dagger)\}$ and write the expression for Π_n^+ in (24) as

$$\Pi_n^+ = \int_0^{\tau_n} \widetilde{VS}_n(\theta)\, dF_n(\theta) + \int_0^{\tau_n} \left\{ VS_n(\theta) - \widetilde{VS}_n(\theta) \right\} dF_n(\theta), \tag{25}$$

where $\widetilde{VS}_n$ is the virtual surplus under the schedule $\tilde{q}_n$.

Defining

$$G_n \equiv \int_0^{\tau_n} [(b-1)\theta f_n(\theta) - b(1-F_n(\theta))]\, \tilde{q}_n(\theta) d\theta,$$

the quadratic nature of the surplus allows us to write the first integral in (25) as

$$\begin{aligned}
\int_0^{\tau_n} [s(\tilde{q}_n(\theta), \theta) f_n(\theta) - (1-F_n(\theta)) b \tilde{q}_n(\theta)] d\theta \\
&= G_n - \int_0^{\tau^n} \frac{r}{2} (\tilde{q}_n(\theta))^2\, dF_n(\theta) \\
&\leq G_n - \int_{\hat{\theta}}^{\tau_n} \frac{r}{2} (q_n(\hat{\theta}))^2\, dF_n(\theta) \\
&= G_n - \frac{r}{2}(1 - F_n(\hat{\theta}))(q_n(\hat{\theta}))^2
\end{aligned}$$

(where the inequality is an implication of $\theta^\dagger > \hat{\theta}$ and q_n increasing).

A calculation (see lemma E.3) shows that the second integral in (25) is, for large n, bounded above by

$$2 \int_{\theta^\dagger}^{\infty} s^{FB}(\theta)\, dF^*(\theta),$$

so that, for large n,

$$\Pi_n^+ \leq G_n + \left\{ -\frac{r}{2} \{1 - F_n(\hat{\theta})\} q_n(\hat{\theta})^2 + 2 \int_{\theta^\dagger}^{\infty} s^{FB}(\theta)\, dF^*(\theta) \right\}. \tag{26}$$

Since

$$\lim_n -\frac{r}{2}(1 - F_n(\hat{\theta}))(q_n(\hat{\theta}))^2 = -\frac{r}{2}(1 - F^*(\hat{\theta}))\hat{x}^2,$$

from (23), the term in braces in (26) is strictly negative for sufficiently large n.

It remains only to argue that G_n cannot dominate the other terms. It is here that the market-breakdown condition is used. By the next lemma, that integral does converge to zero, and so Π_n^+ is eventually negative, contradicting the viability of q_n.

Lemma 6.1 *Suppose $\tau_n \to \infty$ and $\{x_n^\dagger\}$ is a sequence of numbers converging to $x^\dagger \geq 0$. Let H_n be the value of the program*

$$\max_{q_n} \int_0^{\tau_n} \{(b-1)\theta f_n(\theta) - b(1-F_n(\theta))\} q_n(\theta)\, d\theta \tag{27}$$

subject to

$$q_n : [0, \tau_n] \to [0, x_n^\dagger] \text{ increasing}. \tag{28}$$

If $(b-1)\theta \leq e^*(\theta)$ for all $\theta \geq 0$, then $H_n \to 0$.

Proof. The constrained maximization problem described by (27) subject to (28) is a special case of the optimal auction design problem in Myerson (1981). Hence, there exists θ_n such that

$$H_n = x_n^\dagger \int_{\theta_n}^{\tau_n} \{(b-1)\theta f_n(\theta) - b(1-F_n(\theta))\}\, d\theta. \tag{29}$$

Suppose the sequence $\{H_n\}$ does not converge to zero. As $H_n \geq 0$ for all n, the first moment of F^* is finite, and $x_n^\dagger \to x^\dagger$, the sequence $\{H_n\}$ is bounded. We may thus assume (by taking an appropriate subsequence if necessary) $H_n \to H > 0$. Suppose, first, the associated sequence $\{\theta_n\}$ satisfying (29) is unbounded. Then, there exists a subsequence $\{\theta_m\}$ such that $\theta_m \to \infty$, implying

$$H_m \leq x_m^\dagger \int_{\theta_m}^{\tau_m} (b-1)\theta\, dF_m(\theta) \leq \frac{x_m^\dagger}{F^*(\tau_m) - F^*(-\tau_m)} \int_{\theta_m}^{\infty} (b-1)\theta\, dF^*(\theta) \to 0,$$

contradicting the hypothesis $H_n \to H > 0$.

Suppose, then, that the sequence $\{\theta_n\}$ satisfying (29) is bounded. Then there exists a subsequence $\{\theta_m\}$ such that $\theta_m \to \bar{\theta} \geq 0$. Performing an integration by parts on the first term in (29) and using lemma B.1, we obtain

$$H_m = x_m^\dagger (1 - F_m(\theta_m))[(b-1)\theta_m - e_m(\theta_m)]$$

and thus

$$H_m \to x^\dagger (1 - F^*(\bar{\theta}))[(b-1)\bar{\theta} - e^*(\bar{\theta})].$$

By assumption $(b-1)\bar{\theta} - e^*(\bar{\theta}) \leq 0$ holds for all $\bar{\theta} \geq 0$, contradicting the hypothesis $H_n \to H > 0$, finishing the proof. ∎

Appendices

A Proof of Lemma 2.1

Let $\hat{\theta}$ denote the random variable $b\theta$. Because the distribution of $\hat{\theta}$ is symmetric with density decreasing in $|\theta|$, it follows from Eaton (1981, proposition 1) that we may assume the existence of a random variable μ so that the distribution of $(\hat{\theta}, \mu)$ is rotation invariant and independent of ε. Let $\alpha \in (0, 2\pi)$ satisfy $\tan\alpha = \sqrt{b-1}$ and define random variables x and y as the solution to the equations

$$\begin{aligned} & \mu = x\cos\alpha + y\sin\alpha \\ \text{and} \qquad & \hat{\theta} = -x\sin\alpha + y\cos\alpha. \end{aligned}$$

Because (x, y) is a rotation of $(\hat{\theta}, \mu)$, the distribution of (x, y) is identical to the distribution of $(\hat{\theta}, \mu)$ and thus, in particular, rotation invariant. Let $t = y\cos\alpha$ and $\omega = x\sin\alpha/r$. As a linear transformation of (x, y), the random variables (t, ω) are elliptically distributed (Fang, Kotz, and Ng, 1990). Because elliptically distributed random variables possess the linear conditional expectation property (Hardin, 1982), (t, ω) have zero mean, and are uncorrelated, we have

$$E[t|t - r\omega] = \frac{\sigma_t^2}{\sigma_t^2 + r^2\sigma_\omega^2}(t - r\omega).$$

As $E[\nu|t - r\omega] = E[t|t - r\omega]$ and $(t - r\omega) = \hat{\theta} = b\theta$ holds by construction, this implies (2), provided the equality

$$b = \frac{\sigma_t^2 + r^2\sigma_\omega^2}{\sigma_t}$$

is satisfied. This in turn follows from

$$\frac{r\sigma_\omega}{\sigma_t} = \frac{\sigma_x \sin\alpha}{\sigma_y \cos\alpha} = \tan\alpha = \sqrt{b-1},$$

where the second equality uses the fact that the distribution of (x, y) is rotation invariant and the corresponding standard deviations thus satisfy $\sigma_x = \sigma_y$.

B Properties of the Mean Excess Function

For a distribution function F with support $[-\tau, \tau]$, let $e : [0, \tau) \to \mathbb{R}$ be the mean excess function defined by

$$e(\theta) = E[\tilde{\theta} - \theta | \tilde{\theta} > \theta] = \frac{1}{1 - F(\theta)} \int_\theta^\tau \tilde{\theta}\, dF(\tilde{\theta}) - \theta.$$

The mean excess function for F^* is $e^* : [0,\infty) \to \mathbb{R}$ defined by $e^*(\theta) = E^*[\tilde{\theta} - \theta|\tilde{\theta} > \theta]$.

Lemma B.1 *The mean excess function e satisfies*

$$e(\theta) = \frac{1}{1-F(\theta)} \int_\theta^\tau 1 - F\left(\tilde{\theta}\right) d\tilde{\theta},$$

and the mean excess function e^* satisfies

$$e^*(\theta) = \frac{1}{1-F^*(\theta)} \int_\theta^\infty 1 - F^*\left(\tilde{\theta}\right) d\tilde{\theta}.$$

Proof. This follows from integration by parts (for e^*, use the property that $\lim_{\theta\to\infty} \theta(1-F^*(\theta)) = 0$, an implication of the existence of the first moment of F^*). ∎

Lemma B.2 *Suppose $\tau_n \to \infty$. Then, the associated sequence of mean excess functions $\{e_n\}$ converges to e^* pointwise.*

Proof. The convergence of $e_n(\theta)$ to $e^*(\theta)$ follows from the convergence of F_n to F^* and of $\int |\theta|\, dF_n(\theta)$ to $\int |\theta|\, dF^*(\theta)$. ∎

C The Market Breakdown Condition

Proof of theorem 4.3. By construction, the mean excess function of the random variable $\theta+1$ is the one of a Pareto distributed random variable with parameter β, so that

$$e^*(\theta) = \frac{(\theta+1)}{(\beta-1)}.$$

Hence $e^*(\theta) \geq \theta/(\beta-1)$. Since $\beta \leq b/(b-1)$, this implies $e^*(\theta) \geq (b-1)\theta$. ∎

Proof of theorem 4.4. Let $\lim_{\theta\to\infty} g^*(\theta) = g$, where g is possibly infinite. The finiteness of the k^{th}-moment of F^* for $k \geq 2$ implies $g > k \geq 2$ (Lariviere, 2006, theorem 2).[19] Using lemma B.1 for the first equality and applying l'Hôpital's rule to get the second equality, we have

$$\lim_{\theta\to\infty} \frac{e^*(\theta)}{\theta} = \lim_{\theta\to\infty} \frac{1}{\theta(1-F^*(\theta))} \int_\theta^\infty (1-F^*(\tilde{\theta}))\, d\tilde{\theta})$$

[19] Lariviere (2006, theorem 2) assumes g^* is increasing, but the proof only uses g^* increasing to conclude that $\lim_{\theta\to\infty} g^*(\theta)$ exists.

$$= \lim_{\theta\to\infty} \frac{-(1-F^*(\theta))}{1-F^*(\theta)-\theta f^*(\theta)}$$

$$= \lim_{\theta\to\infty} \frac{-1}{1-g^*(\theta)} = \frac{1}{g-1}$$

and thus

$$\lim_{\theta\to\infty} \frac{e^*(\theta)}{\theta} < \frac{1}{k-1}. \tag{30}$$

For θ sufficiently large, (30) implies

$$e^*(\theta) < \frac{1}{k-1}\theta \leq (b-1)\theta,$$

where the second inequality uses the assumption $b \geq k/(k-1)$. ■

D Competitive Trading Schedules

Proof of lemma 5.1. Since a symmetric separating zero-profit trading schedule $q : [-\tau, \tau] \to \mathbb{R}$ satisfies $q(-\theta) = -q(\theta)$, it is enough to show the existence of a unique one-to-one function q^s with domain $[0, \tau]$ solving

$$\theta \in \operatorname*{argmax}_{\theta' \in [0,\tau]} \; u(q^s(\theta'), \theta) - \theta' q^s(\theta') \tag{31}$$

$$\text{and} \qquad q^s(\tau) = \bar{x}. \tag{32}$$

The differentiability of any one-to-one function q^s satisfying (31), the key property in the subsequent analysis, would follow from Mailath (1987, Theorem 2), except that belief monotonicity (his condition (2)) is not satisfied. Belief monotonicity requires that the marginal payoff to a change in the beliefs of the uninformed agents (here given by $-x$) never equals 0. However, since single crossing implies a strictly increasing solution to (31), at most $\theta = 0$ can choose $x = 0$, and so belief monotonicity holds for interior types. An examination of the arguments in Mailath (1987) reveals this is enough to obtain differentiability.

We verify existence and uniqueness directly. The maximization problem in (31) implies the first order condition

$$\frac{dq^s(\theta)}{d\theta} = \frac{q^s(\theta)}{(b-1)\,\theta - rq^s(\theta)}. \tag{33}$$

Letting $y(x) = (q^s)^{-1}(x)$ and rearranging, we have

$$xy' - (b-1)y = -rx. \tag{34}$$

Suppose $b \neq 2$. The linear function $rx/(b-2)$ is a particular solution to (34), and βx^{b-1} is a general solution to the homogeneous differential equation $xy' - (b-1)y = 0$. Adding these two yields the general solution

$$y(x) = \frac{r}{(b-2)}x + \beta x^{b-1} \tag{35}$$

(this is well-defined since $x \geq 0$), where β is chosen to satisfy the initial value implied by (32),

$$y(\bar{x}) = \tau. \tag{36}$$

Thus,

$$\beta = \bar{x}^{1-b}\left(\tau - \frac{r}{b-2}\bar{x}\right). \tag{37}$$

Suppose now $b = 2$. Rewrite (34) as $xy' = y - rx$, and differentiate, yielding $y' + xy'' = y' - r$. That is, $y'' = -r/x$. Integrating twice gives $y(x) = -r \int \log x + \alpha x + \kappa$, where α and κ are constants. Equation (34) is only satisfied if $\kappa = 0$.

Hence, the general solution is

$$y(x) = -rx\log x + rx + \alpha x \tag{38}$$

for $x > 0$ with $y(0) = 0$. The parameter α is chosen so that (36) holds.

It remains to verify the uniqueness claim (monotonicity can be verified by calculation). For all $\varepsilon \in (0, \bar{x})$, the equation $(b-1)y/x - r$ is Lipschitz in x for all $x \in [\varepsilon, \bar{x}]$, the initial value problem (34) and (36) has a unique solution on $[\varepsilon, \bar{x}]$. Letting $\varepsilon \to 0$ gives uniqueness on $[0, \bar{x}]$, and so the initial value problem (33) and (32) has the inverse of y as a unique solution. ∎

Proof of lemma 5.2. Fix n. From lemma 5.1, there is a unique symmetric separating competitive trading schedule q_n^s. For $b \neq 2$, the schedule is implicitly given by, for $\theta \geq 0$,

$$\theta = \frac{-r}{2-b}q_n^s(\theta) + \beta_n \left(q_n^s(\theta)\right)^{b-1},$$

where

$$\beta_n = \left(\frac{b-1}{r}\right)^{1-b} \frac{1}{2-b}\tau_n^{2-b}$$

(see (35) and (37)). For $b = 2$, the schedule is given by (see (38))

$$\theta = rq_n^s(\theta)\left[\log \tau_n - \log q_n^s(\theta)\right] + rq_n^s(\theta).$$

For $b < 2$, as $\tau_n \to \infty$, we have $\beta_n \to \infty$. In other words, for a fixed trade level $x > 0$, the type choosing that trade diverges. Equivalently, (since the trading schedules are ordered, with $q^s_n(\theta) > q^s_{n'}(\theta)$ for $0 \le \theta \le \tau_n$ if $\tau_n < \tau'_n$) the trade of any fixed type converges to 0. Similarly, for $b = 2$, as $\tau_n \to \infty$, for a fixed trade level $x > 0$, the type choosing that trade diverges. Hence, if $b \le 2$ every competitive sequence of separating trading schedules converges to a closed market.

Finally, for $b > 2$, $\beta_n \to 0$ as $\tau_n \to \infty$, and so $q_n(\theta) \to (b-2)\theta/r$ for all θ in every competitive sequence of separating trading schedules. ■

Lemma D.1 *Let $\hat{\theta} > 0$ satisfy $(b-1)\hat{\theta} > e^*(\hat{\theta})$. For any $\{\tau_n\}$ satisfying $\tau_n \to \infty$ there exists an associated sequence $\{(\bar{\theta}_n, \hat{x}_n, \hat{p}_n)\}$ with $(\bar{\theta}_n, \hat{x}_n, \hat{p}_n) \in \mathbb{R}^3$, satisfying for all n sufficiently large, $\bar{\theta}_n \in (\hat{\theta}, \tau_n)$,*

$$\hat{p}_n = E_n\left[\theta \mid \theta \in (\hat{\theta}, \bar{\theta}_n]\right] < b\hat{\theta}, \tag{39}$$

$$\hat{x}_n = \frac{(b\hat{\theta} - \hat{p}_n)}{r} > 0, \tag{40}$$

and

$$U^s_n(\bar{\theta}_n) = u(\hat{x}_n, \bar{\theta}_n) - \hat{p}_n \hat{x}_n, \tag{41}$$

where $U^s_n : [-\tau_n, \tau_n] \to \mathbb{R}$ is the payoff function associated with the unique separating competitive trading schedule on $[-\tau_n, \tau_n]$.

Proof of lemma D.1. Observe first that there exists N such that $\hat{\theta} < \tau_n$ for all $n \ge N$.

Consider any sequence $\{\bar{\theta}_n\}$ satisfying $\bar{\theta}_n \in (\hat{\theta}, \tau_n)$ for all $n \ge N$. For such n, determine $(\hat{p}_n, \hat{x}_n)$ by the equalities in (39) and (40). From lemma B.2, we have $e_n(\hat{\theta}) \to e^*(\hat{\theta})$ and thus,

$$(b-1)\hat{\theta} > e_n(\hat{\theta}). \tag{42}$$

for n large. Because

$$E_n\left[\theta \mid \theta \in (\hat{\theta}, \bar{\theta}_n]\right] < \hat{\theta} + e_n(\hat{\theta})$$

it is immediate from (42) that the inequality in (39) and, thus the inequality in (40), holds for all sufficiently large n.

It remains to argue that the sequence $\{\bar{\theta}_n\}$ can be chosen such that (41) holds for n large. Towards this end, note first that since $\hat{x}_n$ is the utility maximizing quantity for trader $\hat{\theta}$ facing a fixed price of $\hat{p}_n \ge \hat{\theta}$, and the trader captures the first best surplus at the price $\hat{\theta}$ when trading the quantity $q^{FB}(\hat{\theta})$, we have $u(\hat{x}_n, \hat{\theta}) - \hat{p}_n \hat{x}_n \le s^{FB}(\hat{\theta})$. Moreover, for n fixed, $\hat{p}_n$ and $\hat{x}_n$ are continuous functions of $\bar{\theta}_n \in [\hat{\theta}, \tau_n]$.

At the point $\bar{\theta}_n = \hat{\theta}$ we have $\hat{p}_n = \hat{\theta}$ and thus $\hat{x}_n = q^{FB}(\hat{\theta})$, implying that the right side of (41) is strictly larger than the left side (as $U_n^s(\theta) < s^{FB}(\theta)$ for all $\theta \in (0, \tau_n)$). As

$$u(\hat{x}_n, \bar{\theta}_n) - \hat{p}_n \hat{x}_n = (\bar{\theta}_n - \hat{\theta})\hat{x}_n + u(\hat{x}_n, \hat{\theta}) - \hat{p}_n \hat{x}_n \leq (\bar{\theta}_n - \hat{\theta})q^{FB}(\hat{\theta}) + s^{FB}(\hat{\theta}),$$

the right side of (41) increases linearly with $\bar{\theta}_n$. Consequently, because $U_n^s(\tau_n) = s^{FB}(\tau_n)$ is a quadratic function of τ_n, for n large the left side of (41) is strictly larger than the right side at $\bar{\theta}_n = \tau_n$. As both sides of (41) are continuous in $\bar{\theta}_n$ it then follows from the intermediate value theorem that there exists $\bar{\theta}_n \in (\hat{\theta}, \tau_n)$ such that (41) holds. ■

Lemma D.2 *The semi-pooling trading schedule constructed in section 5.3 converges to a liquid market.*

Proof of lemma D.2. Under q_n, the quantity traded by $\hat{\theta}$ is

$$\hat{x}_n > \frac{[(b-1)\hat{\theta} - e_n(\hat{\theta})]}{r}.$$

Let $\eta \equiv [(b-1)\hat{\theta} - e^*(\hat{\theta})]/2 > 0$. Since for large n, $\left|e_n(\hat{\theta}) - e^*(\hat{\theta})\right| < \eta$, the quantity traded by $\hat{\theta}$ is bounded below by

$$\frac{[(b-1)\hat{\theta} - e^*(\hat{\theta}) - \eta]}{r} = \frac{\eta}{r}.$$

It remains to argue that, for $\theta \neq 0$, $q_n(\theta)$ converges to a nonzero quantity.

We claim that $\bar{\theta}_n \to \infty$ as $n \to \infty$: If not, there exists a subsequence with $\bar{\theta}_n \to \bar{\theta} < \infty$. But, as $b \leq 2$, we then have $q_n^s(\bar{\theta}_n) \to 0$, and so $U_n^s(\bar{\theta}_n) \to 0$. However, $U_n^s(\bar{\theta}_n) = [2b\bar{\theta}_n - b\hat{\theta} - \hat{p}_n)]\hat{x}_n/2$, the utility from pooling. Since this latter term is no smaller than $(b-1)\bar{\theta}_n\hat{x}_n/2$, which is bounded away from zero, we have a contradiction.

Consequently, q_n converges pointwise to q^t. ■

E Convergence to Closed Markets

For any viable trading and price schedule pair (q_n, p_n), aggregate trading profits are

$$\pi_n(\theta) = [p_n(q_n(\theta)) - \theta]q_n(\theta),$$

the surplus function is given by

$$S_n(\theta) \equiv s(q_n(\theta),\theta) = R_n(\theta) + \pi_n(\theta),$$

where R_n is the rent function from lemma 3.2, and the virtual surplus function by

$$VS_n(\theta) \equiv \begin{cases} S_n(\theta) + b\frac{F_n(\theta)}{f_n(\theta)} q_n(\theta), & \text{if } \theta < 0, \\ S_n(\theta) - b\frac{1-F_n(\theta)}{f_n(\theta)} q_n(\theta), & \text{if } \theta > 0. \end{cases}$$

When we decorate a trading schedule, such as $\check{q}_n$, the corresponding functions defined above are similarly decorated.

Given F^* and a sequence $\{\tau_n\}$ satisfying $\tau_n \to \infty$, we assume that n is sufficiently large that

$$F^*(\tau_n) - F^*(-\tau_n) > \frac{1}{2}. \tag{43}$$

Lemma E.1 *Let $\{(\tau_n, q_n, p_n)\}$ be a viable sequence. Then for all $\theta \in \mathbb{R}$ the sequence $\{q_n(\theta)\}$ is bounded.*

Proof. Suppose there exists $\hat{\theta} \in \mathbb{R}$ such that $\{q_n(\hat{\theta})\}$ is unbounded above (the case in which $\{q_n(\hat{\theta})\}$ is unbounded below is analogous). There then exists a subsequence $\{q_m\}$ such that $q_m(\hat{\theta}) \to \infty$. For fixed $\theta^\dagger > \hat{\theta}$, since the trading schedules q_m are increasing (lemma 3.1), we have

$$q_m(\theta) \to \infty, \quad \forall \theta \in [\hat{\theta}, \theta^\dagger]. \tag{44}$$

Since $R_m(\theta) \geq 0$ and so $S_m(\theta) \geq \pi_m(\theta)$ for all θ, we have

$$\Pi_m \leq \int_{-\tau_m}^{\tau_m} S_m(\theta)\, dF_m(\theta). \tag{45}$$

For sufficiently large m, $-\tau_m < \hat{\theta} < \theta^\dagger < \tau_m$ and recalling (43), so

$$\begin{aligned} \int_{-\tau_m}^{\tau_m} S_m(\theta)\, dF_m(\theta) &= \frac{1}{F^*(\tau_m) - F^*(-\tau_m)} \int_{-\tau_m}^{\tau_m} S_m(\theta)\, dF^*(\theta) \\ &\leq 2\int_{-\tau_m}^{\tau_m} S_m(\theta)\, dF^*(\theta) \\ &\leq 2\left[\int_{-\tau_m}^{\tau_m} s^{FB}(\theta)\, dF^*(\theta) + \int_{\hat{\theta}}^{\theta^\dagger} S_m(\theta)\, dF^*(\theta)\right], \end{aligned}$$

where the first line follows from F_m being the τ_m-truncation of F^* and the second from (43).

Using (10) we have

$$\int_{-\tau_m}^{\tau_m} s^{FB}(\theta)\, dF^*(\theta) \to \frac{(b-1)^2}{2r}\sigma^2,$$

where σ^2 is the variance of F^*. From (8) and (44) we have $S_m(\theta) \to -\infty$ for all $\theta \in [\hat{\theta}, \theta^\dagger]$ and thus

$$\int_{\hat{\theta}}^{\theta^\dagger} S_m(\theta)\, dF^*(\theta) \to -\infty.$$

Hence, the right side of (45) converges to $-\infty$, and so $\Pi_m \to -\infty$, contradicting the hypothesis that $\{q_n\}$ is a viable sequence. ■

Lemma E.2 *If there exists a viable sequence not converging to a closed market, then there exists a viable sequence $\{(\tau_m, \check{q}_m, \check{p}_m)\}$ satisfying:*

1. *there exist $\hat{\theta} > 0$ and $\hat{x} > 0$ such that $\check{q}_m(\hat{\theta}) \to \hat{x}$ as $m \to \infty$,*

2. *$\check{q}_m(0) = 0$ for all m, and*

3. *there exists $\check{\Pi}^+ \geq 0$ such that*

$$\check{\Pi}_m^+ = \int_0^{\tau_n} \check{\pi}_n(\theta) dF_n(\theta) \to \check{\Pi}^+ \quad \textit{as} \quad m \to \infty.$$

Proof. The lemma is established in three steps, in which we sequentially construct the sequence, verifying at each step that the desired property holds. Denote by $\{(\tau_n, q_n, p_n)\}$ the viable sequence not converging to a closed market.

STEP 1 As the sequence $\{(\tau_n, q_n, p_n)\}$ does not converge to a closed market, there is a type θ^* such that $q_n(\theta^*)$ does not converge to zero. From lemma E.1, the sequence $\{q_n(\theta^*)\}$ is bounded, so there exists a subsequence $\{(\tau_m, q_m, p_m)\}$ of $\{(\tau_n, q_n, p_n)\}$ such that $q_m(\theta^*) \to x^* \neq 0$. If $x^* > 0$ and $\theta^* > 0$, then property 1 in the statement of the Lemma holds for the viable sequence $\{(\tau_m, q_m, p_m)\}$.

If $x^* > 0$ and $\theta^* \leq 0$, consider any $\hat{\theta} > 0 \geq \theta^*$. As q_m is increasing in θ for all m and $\{q_m(\hat{\theta})\}$ is bounded, there exists a subsequence $\{(\tau_k, q_k, p_k)\}$ of $\{(\tau_m, q_m, p_m)\}$ and an $\hat{x} \geq x^* > 0$ such that $q_k(\hat{\theta}) \to \hat{x}$, verifying property 1 for the viable sequence $\{(\tau_k, q_k, p_k)\}$.

If $x^* < 0$, define a new sequence $\{(\tau_m, q_m^\dagger, p_m^\dagger)\}$ by "flipping" $\{q_m\}$ and $\{p_m\}$, i.e., $q_m^\dagger(\theta) = -q_m(-\theta)$ for all θ and m, $p_m^\dagger(x) = -p_m(-x)$ for all x and m. This sequence then satisfies $q_m^\dagger(-\theta^*) \to -x^* > 0$ and is viable for $\{F_m\}$, because F_m is symmetric. Replacing θ^* by $-\theta^*$, x^* by $-x^*$, and $\{q_m\}$ by $\{q_m^\dagger\}$ in the arguments for the case $x^* > 0$ establishes property 1.

STEP 2 By step 1, we can now assume property 1 holds for the original sequence, i.e., there exists $\hat{\theta} > 0$ satisfying $q_n(\hat{\theta}) \to \hat{x} > 0$.

Let

$$\check{q}_n(\theta) = \begin{cases} \min[q_n(\theta), 0], & \text{if } \theta < 0 \\ 0, & \text{if } \theta = 0 \\ \max[q_n(\theta), 0], & \text{if } \theta > 0 \end{cases}$$

The trading sequence $\{\check{q}_n\}$ satisfies $\check{q}_n(0) = 0$ for all n and $\check{q}_n(\hat{\theta}) \to \hat{x} > 0$. We show next that $\check{q}_n$ is viable for F_n for each n, establishing the existence of a sequence $\{\tau_n, \check{q}_n, \check{p}_n)\}$ satisfying properties 1 and 2 in the statement of the lemma. Towards this end note, first, that as q_n is increasing so is $\check{q}_n$. Lemma 3.1 implies that, for all n, the trading schedule $\check{q}_n$ is implementable. To show that $\{\check{q}_n\}$ is viable, it suffices to show that

$$\check{R}_n(\theta) \leq R_n(\theta) \quad \text{and} \quad s(\check{q}_n(\theta), \theta) \geq s(q_n(\theta), \theta) \tag{46}$$

and thus $\check{\pi}_n(\theta) \geq \pi_n(\theta)$ holds for all θ.

Let $\underline{\theta}_n = \inf\{\theta \mid \check{q}_n(\theta) = 0\}$ (we do not exclude the possibility $\underline{\theta}_n = -\tau_n$) and $\bar{\theta}_n = \sup\{\theta \mid \check{q}_n(\theta) = 0\}$ (we do not exclude the possibility $\bar{\theta}_n = \tau_n$). For all $\theta \in (\underline{\theta}_n, \bar{\theta}_n)$ (46) holds because for those types $\check{R}_n(\theta) = 0 \leq R_n(\theta)$ and $s(\check{q}_n(\theta) = 0 \geq s(q_n(\theta), \theta)$, where the latter inequality follows from (8) and observing that $q_n(\theta)\theta \leq 0$ for all types in $(\underline{\theta}_n, \bar{\theta}_n)$. Consider then $\theta > \bar{\theta}_n$. By construction, we have $\check{q}_n(\theta) = q_n(\theta)$ and thus $s(\check{q}_n(\theta), \theta) = s(q_n(\theta), \theta)$. From (14) we have

$$\check{R}_n(\theta^\dagger) - \check{R}_n(\bar{\theta}_n) = R_n(\theta^\dagger) - R_n(\bar{\theta}_n), \quad \forall \theta^\dagger > \bar{\theta}_n,$$

implying (46) (because $\check{R}_n(\bar{\theta}_n) = 0 \leq R_n(\bar{\theta}_n)$). For $\theta < \underline{\theta}_n$, (46) follows from an analogous argument, establishing the viability of $\{\check{q}_n\}$.

STEP 3 Let $\{(\tau_n, q_n, p_n)\}$ be a viable sequence satisfying properties 1 and 2 in the statement of the lemma. We have the trivial identity

$$\Pi_n = \int_{-\tau_n}^{0} \pi_n(\theta) dF_n(\theta) + \int_{0}^{\tau_n} \pi_n(\theta) dF_n(\theta) \equiv \Pi_n^- + \Pi_n^+.$$

By hypothesis, $q_n(0) = 0$ and thus $R_n(0) = 0$ holds for all n, and so from lemma 3.2 and integrating by parts,

$$\Pi_n^- = \int_{-\tau_n}^{0} VS_n(\theta)\, dF_n(\theta)$$

and

$$\Pi_n^+ = \int_{0}^{\tau_n} VS_n(\theta)\, dF_n(\theta).$$

We next show that the sequences $\{\Pi_n^-\}$ and $\{\Pi_n^+\}$ are bounded so that there exists a subsequence $\{(\tau_m, q_m, p_m)\}$, $\Pi^- \in \mathbb{R}$, and $\Pi^+ \in \mathbb{R}$ such that $\Pi_m^- \to \Pi^-$ and $\Pi_m^+ \to \Pi^+$. Using $q_n(\theta) \geq 0$ for all $\theta \geq 0$ in the first inequality, we have for all n sufficiently large:

$$\begin{aligned}
\Pi_n^+ &\leq \int_0^{\tau_n} S_n(\theta)\, dF_n(\theta) \\
&\leq \int_0^{\tau_n} s^{FB}(\theta)\, dF_n(\theta) \\
&= \frac{1}{F^*(\tau_n) - F^*(-\tau_n)} \int_0^{\tau_n} s^{FB}(\theta)\, dF^*(\theta) \\
&< \frac{1}{F^*(\tau_n) - F^*(-\tau_n)} \int_0^{\infty} s^{FB}(\theta)\, dF^*(\theta) \\
&< 2\int_0^{\infty} s^{FB}(\theta)\, dF^*(\theta) = \frac{(b-1)^2}{2r}\sigma^2,
\end{aligned}$$

establishing that $\{\Pi_n^+\}$ is bounded above. An analogous argument shows that $\{\Pi_n^-\}$ is bounded above. Because $\{(\tau_n, q_n, p_n)\}$ is viable we have $\Pi_n = \Pi_n^- + \Pi_n^+ \geq 0$ for all n. It then follows from the fact that $\{\Pi_n^-\}$ (resp. $\{\Pi_n^+\}$) is bounded above that $\{\Pi_n^+\}$ (resp. $\{\Pi_n^-\}$) is bounded below.

Let $\{(\tau_m, q_m, p_m)\}$ be a subsequence of $\{(\tau_n, q_n, p_n)\}$ satisfying $\Pi_m^- \to \Pi^-$ and $\Pi_m^+ \to \Pi^+$. If $\Pi^+ \geq 0$, the sequence $\{(\tau_m, q_m, p_m)\}$ satisfies properties 1–3 in the statement of the lemma and so is the desired sequence $\{(\tau_m, \check{q}_m, \check{p}_m)\}$.

Finally, suppose $\Pi^+ < 0$. Then, because $\Pi_m \to \Pi^- + \Pi^+$, viability of the sequence $\{(\tau_m, q_m, p_m)\}$ implies $\Pi^- > 0$. Consider the "flipped" sequence $\{(\tau_m, q_m^\dagger, p_m^\dagger)\}$ defined by $q_m^\dagger(\theta) = -q_m(-\theta)$ for all θ and m, and $p_m^\dagger(x) = -p_m(-x)$ for all x and m. By construction, this sequence satisfies properties 2 and 3 in the statement of the lemma and, because of symmetry, is viable. We complete our argument by demonstrating that there is a subsequence $\{(\tau_k, \check{q}_k, \check{p}_k)\}$ of the flipped sequence $\{(\tau_m, q_m^\dagger, p_m^\dagger)\}$ also satisfying property 1. Suppose not. Then we must have, for the unflipped sequence, $q_m(\theta) \to 0$ for all $\theta < 0$. [If not, we can find a type $\check{\theta} < 0$ and a subsequence $\{q_k\}$ such that $q_k(\check{\theta}) \to \check{x} \neq 0$. Because $q_k(\check{\theta}) \leq 0$ holds for all k we must have $\check{x} < 0$, and so the flipped sequence satisfies property 1.] Let $0 < \varepsilon < \Pi^-$. As the second moment of F^* exists, there exists $\hat{\theta} < 0$ such that

$$2\int_{-\infty}^{\hat{\theta}} s^{FB}(\theta)\, dF^*(\theta) < \varepsilon.$$

Noting that for all m sufficiently large,

$$\Pi_m^- \leq 2\int_{-\infty}^{\hat{\theta}} s^{FB}(\theta)\, dF^*(\theta) + \int_{\hat{\theta}}^{0} S_m(\theta)\, dF_m(\theta),$$

and that the second integral on the right hand side converges to zero because $q_m(\theta) \to 0$ for all $\theta \in [\hat{\theta}, 0]$, we obtain a contradiction to the hypothesis $\Pi_m^- \to \Pi^- > \varepsilon$. ■

Lemma E.3 *For n sufficiently large,*

$$\int_0^{\tau_n} \left[VS_n(\theta) - \widetilde{VS}_n(\theta) \right] dF_n(\theta) \le 2 \int_{\theta^\dagger}^{\infty} s^{FB}(\theta)\, dF^*(\theta).$$

Proof. The integrand on the left is equal to zero for all $\theta \in (0, \theta^\dagger]$. For $\theta \ge \theta^\dagger$ we have $q_n(\theta) \ge q_n(\theta^\dagger) = \tilde{q}_n(\theta) \ge 0$ and thus

$$\begin{aligned} VS_n(\theta) - \widetilde{VS}_n(\theta) &= S_n(\theta) - \tilde{S}_n(\theta) - b\frac{1 - F_n(\theta)}{f_n(\theta)}[q_n(\theta) - \tilde{q}_n(\theta)] \\ &\le S_n(\theta) - \tilde{S}_n(\theta) \\ &\le s^{FB}(\theta) \end{aligned}$$

where the last inequality follows from the calculation

$$\begin{aligned} s(z,\theta) - s(x,\theta) &= (b-1)\theta(z-x) - \frac{1}{2}r(z^2 - x^2) \\ &= (b-1)\theta(z-x) - \frac{1}{2}r(z-x)^2 - \frac{1}{2}r\left(2zx - 2x^2\right) \\ &\le s^{FB}(\theta) - rx(z-x). \end{aligned}$$

It follows that

$$\begin{aligned} \int_0^{\tau_n} \left[VS_n(\theta) - \widetilde{VS}_n(\theta) \right] dF_n(\theta) &\le \int_{\theta^\dagger}^{\tau_n} s^{FB}(\theta)\, dF_n(\theta) \\ &\le 2 \int_{\theta^\dagger}^{\infty} s^{FB}(\theta)\, dF^*(\theta). \end{aligned}$$

■

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