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"Disappointment Cycles"

by

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Disappointment Cycles*

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Abstract

We propose a model of history dependent disappointment aversion (HDDA), allowing the attitude of a decision-maker (DM) towards disappointment at each stage of a T-stage lottery to evolve as a function of his history of disappointments and elations in prior stages. We establish an equivalence between the existence of an HDDA representation and two documented cognitive biases. First, the DM overreacts to news: after suffering a disappointment, the DM lowers his threshold for elation and becomes more risk averse; similarly, after an elating outcome, the DM raises his threshold for elation and becomes less risk averse. This makes disappointment more likely after elation and vice-versa, leading to statistically cycling risk attitudes. Second, the DM displays a primacy effect: early outcomes have the strongest effect on risk attitude. "Gray areas" in the elation-disappointment assignment are connected to optimism and pessimism in determining endogenous reference points.

Keywords: history dependent disappointment aversion, disappointment cycles, overreaction to news, primacy effect, endogenous reference dependence, optimism, pessimism
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Once bitten, twice shy. — Proverb

1. Introduction

Consider two people in a casino who have the same total wealth and the same utility for monetary prizes. One person has already won several games at the roulette wheel; the other person lost at those games. Will these individuals' attitudes toward further risk be the same, or might they depend on whether they had previously won or lost? Consider now a third person who has just won \$100 in a lottery between \$100 and \$x. Even though his winnings are \$100 in both cases, could his attitude to future risk depend on whether x, the alternate outcome, corresponded to winning or losing a thousand dollars?

There is experimental evidence that the way in which risk unfolds over time affects risk attitudes (e.g., Thaler and Johnson (1990), Gneezy and Potters (1997), Bellemare, Krause, Kröger, and Zhang (2005), and Post, van den Assem, Baltussen, and Thaler (2008)); and moreover, that individuals are affected by unrealized outcomes, a phenomenon known as *counterfactual thinking* (e.g., Kahneman and Tversky (1982), Kahneman and Miller (1986), and Medvec, Madey and Gilovich (1995)). Thaler and Johnson (1990) suggest that individuals become more risk averse after negative experiences and less risk averse after positive ones. Post, van den Assem, Baltussen, and Thaler (2008) suggest that individuals are more willing to take risks after extreme realizations. The latter two studies consider settings of pure chance—suggesting that the effects therein are psychological in origin, and are not the result of learning about oneself or one's environment. However, a psychological effect on risk attitude may potentially exist in more general contexts; for example, among NBA basketball players, Rao (2009) shows that "a majority of the players...significantly change their behavior in response to hit streaks by taking more difficult shots" but that "controlling for shot conditions, players show no evidence of ability changing as a function of past outcomes."

In this paper, we propose a model of history-dependent disappointment aversion (HDDA) over T-stage lotteries that permits risk attitudes to be shaped by prior experiences. Our building block is Gul (1991)'s model of disappointment aversion for one-stage lotteries, in which a decision maker (DM) categorizes monetary outcomes of a lottery as elating or disappointing, and calculates his "expected utility" of a lottery while uniformly overweighting the disappointing outcomes. A prize is elating (disappointing) if its utility is weakly larger than (strictly smaller than) the utility of the lottery as a whole. For a fixed utility over monetary prizes, the DM's risk aversion is increasing in his *disappointment aversion coefficient* (the additional weight he places on disappointing outcomes). We extend Gul's idea to a multi-stage setting with history dependence, analyzing how disappointments and elations affect the evolution of risk attitude (by way of the disappointment aversion coefficient).

To ease exposition, we begin by describing our HDDA model in the simple setting of temporal lotteries, in which no intermediate choices may be taken while risk unfolds; later, the model is extended in a dynamically consistent manner to the setting of stochastic decision trees, with our results carrying over. In the HDDA model, the DM endogenously characterizes each realization of a temporal lottery (itself a sublottery) as elating or disappointing. At each stage, the DM's history is the preceding sequence of elations and disappointments. Each possible history corresponds to a (potentially different) disappointment aversion coefficient. The HDDA model consists of a continuous and increasing utility function over monetary prizes, a set of potential disappointment aversion coefficients, and a history assignment mapping sublotteries to those coefficients. The value of a lottery is determined recursively using the model of disappointment aversion and the appropriate disappointment aversion coefficient for each history, with the requirement that the history assignment for all sublotteries be *internally consistent*. Internally consistency requires that if a sublottery is considered elating (disappointing), then its value should indeed be weakly larger than (strictly smaller than) the value of the sublottery from which it emanates.

We do not place an explicit restriction on how histories map to disappointment aversion coefficients (that is, how risk aversion should depend on the history). Nonetheless, we show that the HDDA model predicts two well-documented cognitive biases; and that these biases are sufficient conditions for an HDDA representation to exist. First, in accordance with the experimental evidence cited above, the DM *overreacts to news*: he becomes less risk averse after positive experiences and more risk averse after negative ones. Second, the DM displays *primacy effects*: his risk attitudes are disproportionately affected by early realizations. Sequencing biases, especially the primacy effect, are robust and long-standing experimental phenomena (early literature includes Anderson (1965)). The primacy effect has implications for the optimal sequencing of information to manipulate behavior; for example, we study how a financial advisor trying to convince a DM to invest in a risky asset should deliver mixed news.

HDDA also has predictions for the DM's endogenous reference levels. In particular, the model predicts *disappointment cycles*. The DM increases the threshold for elation after positive experiences and lowers it after negative experiences. This makes disappointment more likely after elation, and vice-versa, leading to statistically cycling risk attitudes. The psychological literature, in particular Parducci (1995) and Smith, Diener, and Wedell (1989), provides support for the prediction that elation thresholds increase (decrease) after positive (negative) experiences.[1]

[1] Summarizing these works, Schwarz and Strack (1998) observe that "an extreme negative (positive) event increased

For some lotteries, there may be more than one internally consistent assignment of histories. The DM's history assignment is revealed by preferences. Because the DM's choice of assignment may affect his utility from a temporal lottery, we view an *optimist* as a DM who always chooses the "most favorable" interpretation (when more than one interpretation is possible) and a *pessimist* as a DM who always chooses the "least favorable" interpretation. This notion of optimism and pessimism is distinct from previous notions which identify optimism and pessimism with the choice of (distorted) beliefs; for example, see Bénabou and Tirole (2002), Chateauneuf, Eichberger, and Grant (2007), or Epstein and Kopylov (2007).

Applied work suggests that changing risk aversion helps to understand several empirical phenomena. Barberis, Huang, and Santos (2001) allow risk aversion to depend on prior stock market gains and losses à la the experimental evidence of Thaler and Johnson (1990), and show that their model is consistent with the well-documented equity premium and excess volatility puzzles. Routledge and Zin (2010) address the same empirical phenomena, proposing a generalization of Gul's model for one-stage lotteries that allows for a subset of lotteries to be valued under expected utility theory. Applying their model recursively (and without history dependence), they show that the preference parameters and transition probabilities between low and high states of the economy can be calibrated to generate effective risk aversion that is countercyclical.

In the HDDA model, risk attitudes are affected by "what might have been." In many theories of choice over temporal lotteries, risk aversion could depend on the passage of time, wealth effects or habit formation in consumption; see Kreps and Porteus (1978), Chew and Epstein (1989), Segal (1990), Dillenberger (2010), and Rozen (2010), among others. We study how risk attitudes are affected by the past, independently of such effects as above. Our type of history dependence is conceptually distinct from models where contemporaneous and future beliefs affect contemporaneous utility (that is, dependence of utility on "what might be" in the future). This literature includes Caplin and Leahy (2001), Epstein (2008), and Köszegi and Rabin (2009).[2]

The remainder of this paper is organized as follows. Section 2 formalizes the domain of temporal lotteries. Section 3 provides a primer on the model of disappointment aversion of Gul (1991). Section 4 formalizes the HDDA model and contains our main results for temporal lotteries. Im-

(decreased) satisfaction with subsequent modest events....Thus, the occasional experience of extreme negative events facilitates the enjoyment of the modest events that make up the bulk of our lives, whereas the occasional experience of extreme positive events reduces this enjoyment."

[2]In Köszegi and Rabin (2009), given any fixed current belief over consumption, utility is not affected by prior history (how that belief was formed). Their model, which presumes the DM is loss averse over changes in successive beliefs, could be generalized to include historical differences in beliefs, which would then affect utility values but not actual risk aversion due to their assumption of additive separability; we conjecture that one could relax additive separability to find choices of parameters and functional forms for their model that replicate the primacy effect and overreaction to news predicted by HDDA.

plications of HDDA are studied in Section 5. Section 6 extends our model and results to a setting where intermediate actions are possible. Axiomatic foundations for HDDA are provided in Section 7. Section 8 discusses directions for further research.

2. Framework: T-stage lotteries

We begin by studying the simple setting of T-stage lotteries; in Section 6 we extend the setting to stochastic decision trees.

Let $X = [w,b] \subset R$ be a bounded interval of monetary prizes, where w is the worst prize and b is the best prize. The set of all simple lotteries (i.e., having a finite number of outcomes) over X is denoted $\mathscr{L}(X)$, or simply $\mathscr{L}^1$. Elements of $\mathscr{L}^1$ are one-stage lotteries. We reserve lowercase letters for one-stage lotteries; typical elements of $\mathscr{L}^1$ are denoted p, q, or r. The probability of a monetary outcome x under p is denoted $p(x)$. A typical element p has the form $\langle p(x_1), x_1; \ldots, p(x_m), x_m \rangle$. The degenerate lottery $\delta_x \in \mathscr{L}^1$ gives the prize x with probability one.

Two-stage lotteries are simple lotteries over $\mathscr{L}^1$. The set of two-stage lotteries is denoted $\mathscr{L}(\mathscr{L}(X))$, or simply $\mathscr{L}^2$. A typical element $P \in \mathscr{L}^2$ has the form

$$P = \langle \alpha_1, p_1; \ldots; \alpha_m, p_m \rangle,$$

where for every j, $p_j \in \mathscr{L}^1$, and $\alpha_j \in [0,1]$, with $\sum_{j=1}^m \alpha_j = 1$.

For $T \geq 3$, the set of T-stage lotteries, $\mathscr{L}^T$, is defined by the inductive relation $\mathscr{L}^T = \mathscr{L}\left(\mathscr{L}^{T-1}\right)$. A typical element P^T of $\mathscr{L}^T$ has the form

$$P^T = \left\langle \alpha_1, P_1^{T-1}; \ldots; \alpha_m, P_m^{T-1} \right\rangle,$$

where each $P_j^{T-1} \in \mathscr{L}^{T-1}$ is a $(T-1)$-stage lottery. If P_j^{T-1} is the outcome of P^T, then all remaining uncertainty is resolved according to P_j^{T-1}. To simplify notation, we use the superscript T only when $T \geq 3$. The degenerate lottery $\delta_x^T \in \mathscr{L}^T$ gives the lottery δ_x^{T-1} with probability one (i.e., x is received with probability one after T stages).

To avoid redundancy, our notation for any t-stage lottery implicitly assumes that the elements in the support are distinct.

3. Disappointment aversion

The model of disappointment aversion proposed by Gul (1991) characterizes a DM's preferences over the set of one-stage lotteries, $\mathscr{L}^1$. For any lottery $p \in \mathscr{L}^1$, consider its certainty equivalent $CE(p)$. This is the monetary prize which the DM considers to be indifferent to p itself. The lottery p may contain prizes in its support which are weakly preferred to p; receiving such a prize is considered an *elating* outcome. The lottery p may also contain prizes in its support which are worse than p; receiving such a prize is a *disappointing* outcome. Gul's model of disappointment aversion considers a DM who values lotteries by taking their "expected utility," except that disappointing outcomes get a uniformly greater weight that depends on the value of a single parameter β, the *coefficient of disappointment aversion*. Since $CE(p)$ depends on all the prizes in the support of p, the division into elating and disappointing outcomes (known as the *elation-disappointment decomposition*), is determined endogenously, as seen in the utility representation below.

Formally, the disappointment aversion model consists of an increasing and continuous utility function over prizes $u : X \to \mathbb{R}$ and a *disappointment aversion coefficient* $\beta \in (-1, \infty)$ such that the value of a lottery p, $V(p; u, \beta)$, uniquely solves

$$V(p; u, \beta) = \frac{\sum_{\{x | u(x) \geq V(p;u,\beta)\}} p(x) u(x) + (1+\beta) \sum_{\{x | u(x) < V(p;u,\beta)\}} p(x) u(x)}{1 + \beta \sum_{\{x | u(x) < V(p;u,\beta)\}} p(x)}. \tag{1}$$

The term in the denominator normalizes the weights on the prizes so that they sum to one. When $\beta > 0$, disappointing outcomes are overweighted and the DM is called *disappointment averse.* When $\beta < 0$, disappointing outcomes are underweighted and the DM is called *elation seeking*. When $\beta = 0$, the model reduces to the model of expected utility.

In Gul's model, risk aversion is captured by both the concavity of u (as in expected utility) and the value of β. In particular, Gul (1991, Proposition 3) shows that the DM is risk averse if and only if u is concave and $\beta \geq 0$. Unlike expected utility, the model of disappointment aversion partially disentangles risk attitude and the shape of utility over monetary outcomes. Fixing a concave utility u over prizes, a larger (positive) disappointment aversion coefficient increases a DM's risk aversion (and disappointment aversion) and strictly reduces his utility from any risky lottery. (See Gul (1991, Proposition 5)).

3.1. Folding back T-stage lotteries

The primitive of our model is a preference relation over the set of T-stage lotteries. The model of disappointment aversion can be extended to this richer domain recursively (see Artstein-Avidan and Dillenberger (2010) or Dillenberger (2010)) using the folding back approach proposed by Segal (1990). To illustrate, consider a two-stage lottery $P = \langle \alpha_1, p_1; \alpha_2, p_2; \alpha_3, p_3 \rangle$. First, each lottery p_i in the support of P is replaced with its certainty equivalent under disappointment aversion; that is, the value $CE(p_i)$ satisfying $u(CE(p_i)) = V(p_i; u, \beta)$. The value of the resulting one-stage, "folded back" lottery $\langle \alpha_1, CE(p_1); \alpha_2, CE(p_2); \alpha_3, CE(p_3) \rangle$ is calculated using $V(\cdot\,; u, \beta)$ and assigned to be the utility of the original lottery P. The value of the temporal lottery is thus calculated by applying disappointment aversion recursively. For T-stage lotteries, the procedure is analogous. Lotteries in the last stage are replaced with their certainty equivalent under disappointment aversion, resulting in a $(T-1)$-stage lottery; and this procedure is repeated until a one-stage lottery results, whose value is calculated using disappointment aversion.

The "folding back" procedure does not require that the same disappointment aversion coefficient be used throughout. For example, the extent of disappointment aversion may vary with the passage of time. More generally, the value of a T-stage lottery can be calculated by folding back using an arbitrary combination of disappointment aversion coefficients.

4. History dependent disappointment aversion

We propose a model of history dependent disappointment aversion over T-stage lotteries, in which the DM endogenously *categorizes* each sublottery as an elating or disappointing outcome of the sublottery from which it emanates. The value of a T-stage lottery P^T is calculated by folding back, where the disappointment aversion coefficient assigned to a sublottery is determined by the sequence of elating or disappointing outcomes leading to it. In analogy to Gul (1991), the DM's categorization must be *internally consistent*: for example, if a sublottery P^t is considered an elating outcome of the sublottery P^{t+1}, then the value of P^t should indeed be larger than that of P^{t+1}.

We begin by formalizing the notion of histories within T-stage lotteries. A t-stage lottery P^t is a *sublottery* of P^T if there is a sequence $P^{t+1}, P^{t+2}, \ldots, P^T$ such that for every $t' \in \{t, \ldots, T-1\}$, $P^{t'} \in \operatorname{supp} P^{t'+1}$. By convention, P^T is a sublottery of itself. The initial history—i.e., prior to any resolution of risk—is empty (0). If a sublottery P^{t+1} is degenerate—i.e., leads to some P^t with probability one—then the DM is not exposed to risk at that stage and his history is unchanged. If a sublottery P^{t+1} is nondegenerate, each sublottery P^t in its support may be an elating (e) or disappointing (d) outcome of P^{t+1}. The set of all possible histories is given by

$$H = \bigcup_{t=1}^{T} \{0\} \times \{e, d\}^{T-t}.$$

For each $P^T \in \mathscr{L}^T$, the *history assignment* $a(\cdot\,|P^T)$ assigns a history $h \in H$ to each sublottery of P^T. The DM's history assignments for all $P^T \in \mathscr{L}^T$ are simply denoted a. The initial assignment (that is, $a(P^T|P^T) = 0$) is always implicit; e.g., within the two-stage lottery $P = \langle \alpha, p; 1-\alpha, q \rangle$, we write $a(p|P) = e$ rather than $a(p|P) = 0e$ if p is elating. If outcome $j \in \{e,d\}$ occurs after history h, the updated history is hj, implicitly assuming the resulting history is in H (i.e., h is a nonterminal history). The length of a history h (denoted $|h|$) is the total number of e and d outcomes.

Each history corresponds to a disappointment aversion coefficient in the collection $B := \{\beta_h\}_{h\in H}$. We may define the folding back procedure for a DM who has a utility u, history assignment a, and a collection of disappointment aversion coefficients B. Starting backwards, calculate the certainty equivalent of each one-stage sublottery p using Equation (1) and $\beta_{a(p|P^T)}$; that is, $CE_{a(p|P^T)}(p) = u^{-1}(V(p;u,\beta_{a(p|P^T)}))$. Next, consider each two-stage sublottery $P = \langle \alpha_1, p_1; \ldots, \alpha_m, p_m \rangle$ and use $\beta_{a(P|P^T)}$ to calculate the certainty equivalent of the "folded back" one-stage lottery in which each p in the support of P is replaced with its certainty equivalent calculated above; that is, $\langle \alpha_1, CE_{a(p_1|P^T)}(p_1); \ldots; \alpha_m, CE_{a(p_m|P^T)}(p_m) \rangle$. Continuing in this manner, the T-stage lottery is reduced to a one-stage lottery (over the certainty equivalents of its continuation sublotteries) whose value is calculated using β_0, since $a(P^T|P^T) = 0$.

Our model of history dependent disappointment aversion (HDDA), defined below, places restriction on the history assignments permitted in the folding back procedure.

Definition 1 (History dependent disappointment aversion, HDDA). *An HDDA utility representation over T-stage lotteries consists of an increasing and continuous utility over monetary prizes $u : X \to \mathbb{R}$, a collection of disappointment aversion coefficients $B = \{\beta_h\}_{h\in H}$, and a history assignment a satisfying, for each $P^T \in \mathscr{L}^T$,*

1. *Sequential assignment. The DM assigns histories to all sublotteries of P^T sequentially*

   (i) if P^{t+1} is nondegenerate and $P^t \in supp\, P^{t+1}$ then $a(P^t|P^T) \in \{a(P^{t+1}|P^T)\} \times \{e,d\}$;

   (ii) if P^{t+1} is degenerate and $P^t \in supp\, P^{t+1}$ then $a(P^t|P^T) = a(P^{t+1}|P^T)$.

2. *Folding back. The utility of P^T is calculated by folding back using u, a, and B. We let $V(P^t;u,a,B|P^T)$ denote the value of any sublottery P^t of P^T calculated as such, simply writing $V(P^T;u,a,B)$ for the value of P^T.*

3. *Internal consistency. Within P^T, if $P^t \in supp\, P^{t+1}$ is an elating (disappointing) outcome of a nondegenerate sublottery P^{t+1}, then the value of P^t calculated above must be weakly larger than (strictly smaller than) the value of P^{t+1} in P^T. For example, if $a(P^{t+1}|P^T) = h$ and $a(P^t|P^T) = he$, then $V(P^t;u,a,B|P^T) \geq V(P^{t+1};u,a,B|P^T)$.*

We identify a DM with an HDDA representation by the triple (u,a,B) satisfying the above conditions.

In the above, we assumed a DM considers an outcome of a nondegenerate lottery elating if its value is at least as large as the value of the lottery from which it emanates. Alternatively, one could redefine HDDA so that an outcome is disappointing if its value is at least as small as that of the parent lottery; or even introduce a third assignment, neutral (n), which treats the case of equality differently than elation or disappointment.[3] Unlike in the one-stage model of disappointment aversion, how equality is treated affects the value of the lottery; but in either case, equality is possible only in a measure zero set of lotteries. Generically, a nonempty history consists of a sequence of strict elations and disappointments.

To illustrate how HDDA is determined, consider the case of two-stage lotteries, where sequential history assignment is trivially satisfied. There are three disappointment aversion coefficients, $B=\{\beta_0,\beta_e,\beta_d\}$. For any one-stage lottery $p\in\mathscr{L}^1$ and $h\in\{0,e,d\}$, $CE_h(p)$ is the disappointment aversion certainty equivalent of p using u and β_h. If a two-stage lottery P is degenerate (i.e., $P=\langle 1,p\rangle$) then $V(P;u,a,B)$ is simply the disappointment aversion value of p under β_0, or $V(p;u,\beta_0)$. For any nondegenerate two-stage lottery $P=\langle\alpha_1,p_1;\ldots;\alpha_j,p_j;\ldots;\alpha_m,p_m\rangle$, the HDDA representation assigns to each one-stage lottery p in the support of P a history $a(p|P)\in\{e,d\}$, and the value of P is given by

$$V\left(P;u,a,B\right)=\frac{\sum_{\{j\,|\,a\left(p_j|P\right)=e\}}\alpha_j u\left(CE_e(p_j)\right)+(1+\beta_0)\sum_{\{j\,|\,a\left(p_j|P\right)=d\}}\alpha_j u\left(CE_d(p_j)\right)}{1+\beta_0\sum_{\{j\,|\,a\left(p_j|P\right)=d\}}\alpha_j}. \qquad (2)$$

Moreover, the history assignment is internally consistent. If $a(p_j|P)=e$ then $u\left(CE_e(p_j)\right)\geq V\left(P;u,a,B\right)$; and if $a(p_j|P)=d$ then $u\left(CE_d(p_j)\right)<V\left(P;u,a,B\right)$.

4.1. Overreaction to news and the primacy effect

We say that an HDDA representation using the collection of disappointment aversion coefficients B has *endogenous reference dependence* if $\beta_{he}\neq\beta_{hd}$ for all h.[4] In this section we show that under endogenous reference dependence, the existence of an HDDA representation implies regularity

[3] If $a(p|P)=n$, then for any perturbation of p to p' in P, resulting in a perturbed lottery P', $a(p|P')\neq n$. That is, in each neighborhood of a neutral point, there is an elation or a disappointment. If sufficiently close to a neutral point there is an elation, then $\beta_e\leq\beta_n$. If sufficiently close to a neutral point there is a disappointment, then $\beta_n\leq\beta_d$. (Refer to Lemma 4).

[4] Relaxing this condition to be satisfied for only some h would lead to more cumbersome conditions.

properties on B that are related to well-known cognitive biases; and that in turn, these properties imply the existence of HDDA.[5]

As discussed in the introduction, experimental evidence suggests that an individual's risk attitudes depend on how prior uncertainty resolved. In particular, the literature suggests that people *overreact* to news received: they become less risk averse after positive experiences and more risk averse after negative ones. Since risk aversion is increasing in the disappointment aversion coefficient, this effect is captured in the following definition.

Definition 2. *The collection of coefficients B overreacts to news if $\beta_{he} < \beta_{hd}$ for all h.*

A body of evidence also suggests that individuals are affected by the position of items in a sequence. One well-documented cognitive bias is the *primacy effect*, in which early observations have a strong effect on later judgments. In our setting, the order in which elations and disappointments occur might affect the DM's risk attitude. Overreaction to news suggests that after an initial elation, a disappointment increases the DM's risk aversion; and that after an initial disappointment, an elation reduces the DM's risk aversion. A primacy effect would further suggest that the shift in attitude from the initial realization has a lasting and disproportionate effect. Future elations or disappointments can only mitigate but not overpower the first impression, as in the following definition.

For any t, let d^t (or e^t) denote t repetitions of d (or e). The history hed^t, for example, corresponds to experiencing one elation and t successive disappointments after the history h, under the implicit assumption that the resulting history is in H.

Definition 3. *The collection of coefficients B displays a weak primacy effect if $\beta_{hed} \leq \beta_{hde}$ for all h. The collection B displays a strong primacy effect if $\beta_{hed^t} \leq \beta_{hde^t}$ for all h and $t \geq 1$.*

The combination of overreaction to news and the strong primacy effect imply strong restrictions on the collection of disappointment aversion coefficients B; these are formalized in the following result and visualized in Figure 1. We refer below to the lexicographic order on histories of the same length as the ordering where $\tilde{h}$ precedes h if it precedes it alphabetically. Since d comes before e, this is interpreted as "the DM is disappointed earlier in $\tilde{h}$ than in h."

[5] It is easy to check that if $\beta_0 = \beta_{he} = \beta_{hd}$ for every h (as in the standard history-independent recursive disappointment aversion), then an HDDA representation exists: it is trivial for history assignments to be internally consistent because the assignment does not affect value.

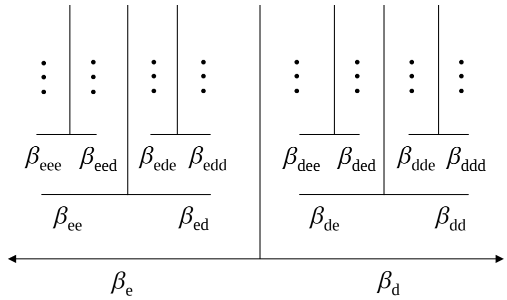

Figure 1: Starting from the bottom, each row corresponds to the set of applicable disappointment aversion coefficients for a stage $t = 2, 3, \ldots, T$. Overreaction to news and the primacy effect imply the lexicographic ordering of disappointment aversion coefficients in each row (Proposition 1). The assumption $\beta_h \in [\beta_{he}, \beta_{hd}]$ for all $h \in H$ implies the vertical lines and consecutive row alignment.

Proposition 1. *The following statements are equivalent:*

(i) B satisfies overreaction to news and the strong primacy effect (with strict inequalities);

(ii) For $h, \tilde{h}$ of the same length, $\beta_h < \beta_{\tilde{h}}$ if $\tilde{h}$ precedes h lexicographically.

Under the assumption $\beta_h \in [\beta_{he}, \beta_{hd}]$ for all $h \in H$, conditions (i) and (ii) are also equivalent to:

(iii) For any h, h', h'', we have $\beta_{heh'} < \beta_{hdh''}$.

Condition (ii) of Proposition 1 says, comparing histories of the same length, that the DM's risk aversion is greater when he has been disappointed earlier. This implies that for all $h, \tilde{h}$,

$$\beta_{he^{|\tilde{h}|+1}} \leq \beta_{he\tilde{h}} \leq \beta_{hed^{|\tilde{h}|}} < \beta_{hde^{|\tilde{h}|}} \leq \beta_{hd\tilde{h}} \leq \beta_{hd^{|\tilde{h}|+1}},$$

meaning that the DM's risk aversion after any continuation $\tilde{h}$ is no greater than if she were to be consistently disappointed thereafter, and no less than if she were to be consistently elated thereafter. To show the lexicographic ordering across the rows in Figure 1, note that the first row from the bottom ($\beta_e < \beta_d$) follows directly from overreaction to news. Overreaction to news also implies the left and right portions of the second row ($\beta_{ee} < \beta_{ed}$ and $\beta_{de} < \beta_{dd}$) while the primacy effect (with strict inequalities) implies that $\beta_{ed} < \beta_{de}$. Alternating the use of overreaction to news and the strong primacy effect, one obtains each of the rows in Figure 1. Under the additional assumption $\beta_h \in [\beta_{he}, \beta_{hd}]$, which says that an elation reduces (and a disappointment increases) the DM's risk aversion *relative to his initial level*, one obtains the condition (iii), represented graphically in the vertical lines and consecutive row alignment in Figure 1. In words, condition (iii) says that whatever happens afterwards, the DM's risk aversion is always lower after an elation than it would have been had she instead been disappointed at that same point in time.

The following results link the two cognitive biases mentioned above to necessary and sufficient conditions for the existence of an HDDA representation.

Theorem 1 (Necessary conditions for HDDA). *If the HDDA representation (u,a,B) has endogenous reference dependence, then the collection B overreacts to news and displays a weak primacy effect. If in addition $\beta_h \in [\beta_{he}, \beta_{hd}]$ and $\beta_{hed^t} \neq \beta_{hde^t}$ for all h and t, then the collection B also displays a strong primacy effect (and is ordered as in Figure 1).*

Theorem 2 (Sufficient conditions for HDDA). *If the collection of disappointment aversion coefficients B overreacts to news and displays a strong primacy effect (with strict inequalities), then for any continuous and strictly increasing utility over prizes $u : X \to \mathbb{R}$, an HDDA representation (u,a,B) exists.*

Observe that on the set of two-stage lotteries, $\mathscr{L}^2$, overreaction to news is by itself necessary and sufficient for an HDDA representation with endogenous reference dependence, as there are too few stages for the primacy effect to apply. Similarly, on $\mathscr{L}^3$, overreaction to news and the weak primacy effect are both necessary and sufficient.

Theorems 1 and 2 are proved in the appendix. There we provide an algorithm for finding an internally consistent history assignment for two-stage lotteries, which can be used recursively to prove existence of the HDDA representation for T-stage lotteries. Let us illustrate why overreaction to news is necessary under endogenous reference dependence. Suppose $T = 2$ and consider the lottery $P = \langle \alpha, p; 1-\alpha, \delta_x \rangle$. For p to be an elation in P, internal consistency requires $u(CE_e(p)) > u(x)$; for p to be a disappointment in P, internal consistency requires $u(CE_d(p)) < u(x)$. If $CE_d(p) > CE_e(p)$, then there cannot be an internally consistent assignment for any $x \in (CE_e(p), CE_d(p))$. Then it must be, by endogenous reference dependence, that $CE_d(p) < CE_e(p)$; and by the properties of disappointment aversion, this implies $\beta_e < \beta_d$. To sketch the proof that the weak primacy effect is necessary under $T = 3$, consider a three-stage lottery $Q^3 = \langle \alpha, Q; 1-\alpha, \delta_x^2 \rangle$. Assuming by contradiction that $\beta_{de} < \beta_{ed}$, we construct a two-stage lottery Q such that $CE_d(Q) > CE_e(Q)$ under the only possible internally consistent assignment of Q given each of β_e and β_d. But then, as above, no internally consistent assignment of Q^3 would exist. Essentially, if $\beta_{de} < \beta_{ed}$ then an elating outcome received after a disappointment may overturn the assignment of the initial outcome as a disappointment. The intuition for the strong primacy effect is similar but requires a more complex construction.

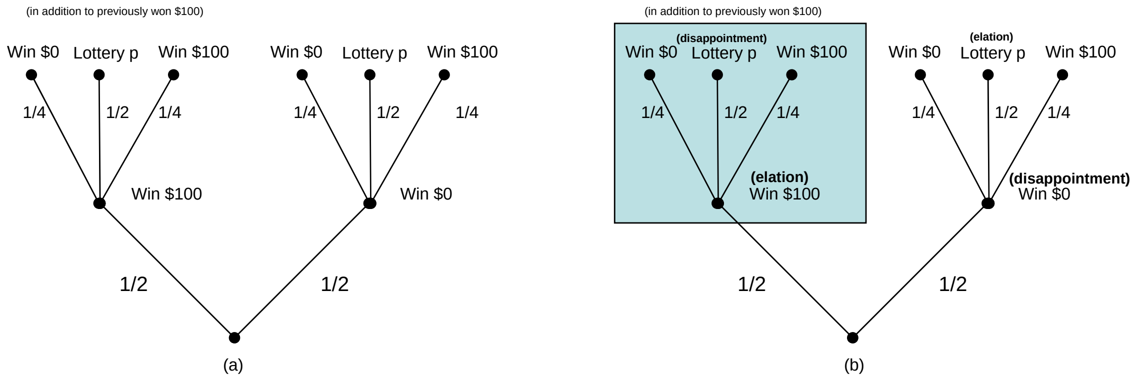

Figure 2: Let p be any lottery whose support consists of prizes between \$25 and \$$33\frac{1}{3}$. Then figure (b) shows the only internally consistent history assignment for the three-stage lottery shown in (a), using $\beta_e = 1$, $\beta_d = 2$, and $u(x) = x$. The lottery p is a disappointment after first winning \$100, and an elation otherwise. Note that with $u(x) = x$, there cannot be any wealth effects on risk aversion.

5. Implications

In this section we discuss two phenomena that arise under HDDA, statistically cycling disappointment attitudes and the possibility of "gray areas" where two DM's, facing the same information and having the same u and B, may disagree on which outcomes are elating or disappointing (the history assignment a) based on their optimistic or pessimistic tendencies. Further implications are studied in Section 6, in a richer setting where intermediate actions may be taken while uncertainty resolves.

5.1. Disappointment cycles

Theorem 1 says that a DM with an HDDA representation overreacts to news. To illustrate the implications of this, consider the three-stage lottery P^3 shown in Figure 2(a). We assume $u(x) = x$, $\beta_e = 1$, $\beta_d = 2$, and any choice of the other disappointment aversion coefficients satisfying overreaction to news and the weak primacy effect.

Suppose that p is a lottery whose support consists of prizes between \$25 and \$$33\frac{1}{3}$. Then, the two-stage sublottery P_ℓ on the left—where the DM immediately accrues \$100—must be elating in P^3, while the two-stage sublottery P_r on the right must be disappointing in P^3. Indeed, this is the only internally consistent history assignment because the worst outcome in P_ℓ dominates the best outcome in P_r. Therefore, P_ℓ is evaluated using β_e, and P_r is evaluated using β_d.

Whether or not the DM wins \$100, his additional winnings are determined by the same two-stage lottery, $P = \langle \frac{1}{4}, \delta_0; \frac{1}{2}, p; \frac{1}{4}, \delta_{100} \rangle$. Because $u(x) = x$, there are no wealth effects: incrementing all the prizes in P by 100 simply raises its HDDA utility by 100, without affecting which outcomes are elating or disappointing. To calculate the value of P using HDDA, we fold it back by replacing p with its certainty equivalent calculated using the appropriate history assignment. Within each of P_ℓ and P_r, we must determine whether p is an elation or a disappointment. Consider the lottery $\langle \frac{1}{4}, 0; \frac{1}{2}, x; \frac{1}{4}, 100 \rangle$ that would result if p is replaced with a prize x. Using disappointment aversion, it is easy to show that using $\beta_e = 1$, any prize x smaller than $\$33\frac{1}{3}$ is a disappointment in this lottery; while using $\beta_d = 2$, any prize x larger than \$25 is an elation. But the certainty equivalent of p, evaluated using any β, is always between \$25 and $\$33\frac{1}{3}$. Therefore, the only consistent assignment of p is as a disappointment after winning \$100 and as an elation otherwise. That is, the DM's disappointment attitude must cycle, as shown in Figure 2(b).

This example suggests a more general prediction of HDDA. Note that in Gul (1991)'s original model, the disappointment aversion coefficient affects both risk aversion and the elation threshold. Because the utility of the lottery determines its certainty equivalent, increasing β lowers the DM's elation threshold: for any $p \in \mathscr{L}^1$ and $\beta' < \beta$, if a prize x is (1) disappointing in p under β then it is disappointing in p under β', and (2) elating in p under β' then it is elating in p under β. Because of this feature, overreaction to news means, in our dynamic setting, that a DM who has been elated is not only less risk averse than a DM who has been disappointed, but also has a higher elation threshold. In other words, overreaction to news implies that after a disappointment, the DM is more risk averse and "settles for less"; whereas after an elation, the DM is less risk averse and "raises the bar." As is in the example above, this leads to *statistically cycling disappointment attitudes*: disappointment is more likely after elation, and vice versa.

Under the assumption that $\beta_h \in [\beta_{he}, \beta_{hd}]$ for all h, HDDA implies condition (iii) in Proposition 1 (visualized in Figure 1). That condition says that after an elation, the DM's greatest possible degree of risk aversion in the future decreases; and conversely, after a disappointment, the DM's lowest possible degree of risk aversion in the future increases. However, in a finite horizon setting, this does not imply that the DM's mood swings moderate in intensity with experience (for example, $|\beta_{ed} - \beta_e| \geq |\beta_{ede} - \beta_{ed}| \geq |\beta_{eded} - \beta_{ede}| \cdots$). That is, the intensity of disappointment cycles may well persist.

5.2. Is the glass half full or half empty?

Consider the two stage lottery $\langle \alpha, p; 1-\alpha, \delta_x \rangle$ and suppose that $u(CE_e(p)) > u(x) > u(CE_d(p))$. Under this assumption, it would be consistent for the lottery p to be either an elation or a disappointment. The moral of this example is that while u, the collection B, and history assignment a can be pinned down uniquely by choice behavior (as shown by the axiomatization in Section 7),

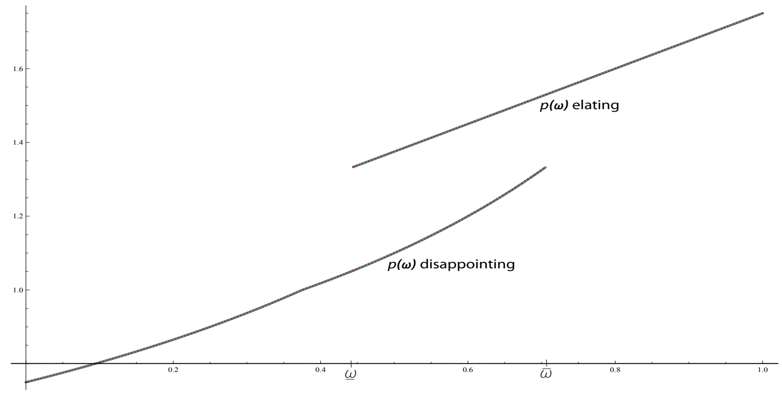

Figure 3: The set of possible HDDA utilities of $P(\omega)$ are pictured on the vertical axis for each $\omega \in (0,1)$ on the horizontal axis, given $\beta_e = 0$, $\beta_0 = 1$, $\beta_d = 2$, and $u(x) = x$. The sublottery $p(\omega)$ can be viewed as an elation or a disappointment in the range $[\underline{\omega}, \overline{\omega}]$.

one cannot fully reconstruct the DM's preference relation from only the information contained in u and B. Predicting the DM's behavior in such "gray areas" as above requires a theory of how the DM assigns histories (as seen later, such a theory has testable predictions for his preference relation over $\mathscr{L}^T$).

A dictionary definition of optimism is "An inclination to put the most favorable construction upon actions and events or to anticipate the best possible outcome."[6] In our setting, optimism and pessimism may be understood in terms of this multiplicity of internally consistent history assignments, where the optimist always chooses the most favorable one and the pessimist chooses the least favorable one.

Definition 4. *We say that a DM is an optimist if for every $P^T \in \mathscr{L}^T$ he chooses the sequential and internally consistent history assignment a that maximizes his HDDA utility $V(P^T; u, a, B)$. Similarly, we say the DM is a pessimist if for every $P^T \in \mathscr{L}^T$ he chooses the sequential and internally consistent assignment a that minimizes his HDDA utility $V(P^T; u, a, B)$. Given the same utility over prizes u and disappointment aversion coefficients B, we say that one DM is more optimistic than another if his HDDA utility is higher for every $P^T \in \mathscr{L}^T$.*

The optimist and pessimist agree on fundamentals (their utility from monetary prizes and their disappointment aversion coefficients), but they take a different perspective on what outcomes are disappointing and elating. This approach differs from most models of optimism and pessimism, which typically view optimism in terms of attaching higher probability to positive events. Under HDDA, probabilities are objective and unchanging, but endogenous reference dependence allows

[6] "optimism." Merriam-Webster Online Dictionary. 2010. http://www.merriam-webster.com (14 June 2010).

the DM to select an internally consistent view of the unfolding risk according to his optimistic or pessimistic tendency.

To illustrate, consider Figure 3, which depicts for each $\omega \in (0,1)$ all possible HDDA values of the two-stage lottery $P(\omega) = \langle \frac{1}{3}, \delta_1; \frac{1}{3}, \delta_2; \frac{1}{3}, p(\omega)\rangle$ where $p(w) = \langle \omega, 3; 1-\omega, 0\rangle$. An increase in ω is a first-order stochastic improvement of the risky sublottery $p(\omega)$. While $p(\omega)$ is unambiguously elating (disappointing) for high (low) values of ω, there is an intermediate range $[\underline{\omega}, \overline{\omega}]$ where $p(\omega)$ can be viewed either as an elation or as a disappointment. The certainty equivalents of the other sublotteries are independent of their history assignment because they are degenerate. At the same time, overreaction to news implies $CE_e(p(\omega)) > CE_d(p(\omega))$. Because HDDA utility is increasing in the certainty equivalents, viewing $p(\omega)$ as an elation gives higher utility. The optimist thus views $p(\omega)$ as an elation as soon as possible (for all $\omega \geq \underline{\omega}$). On the other hand, the pessimist views $p(\omega)$ as a disappointment for as long as possible (for all $\omega \leq \overline{\omega}$). More generally, a DM may have a cutoff $\omega^* \in [\underline{\omega}, \overline{\omega}]$ at which $p(\omega)$ switches from a disappointment to an elation. If one DM is more optimistic than another, then his cutoff ω^* must be lower.

It is easy to see that overreaction to news implies that a DM with an HDDA representation may violate first-order stochastic dominance on T-stage lotteries: for example, if the probability α of p is very high, the lottery $\langle \alpha, p; 1-\alpha, \delta_w\rangle$ may be preferred to $\langle \alpha, p; 1-\alpha, \delta_b\rangle$; the "thrill of winning" outweighs the "pain of losing." The above example suggests, however, that both the optimist and pessimist satisfy the following regularity property related to first-order dominance, stated for simplicity for $T = 2$.

Proposition 2. *Let $\succeq$ be the preference relation represented by the DM's HDDA utility on $\mathscr{L}^2$, and let $>_{FOSD}$ denote the first-order stochastic dominance relation on $\mathscr{L}^1$. Fix any prizes $x_1, \ldots, x_{m-1}$ and probabilities $\alpha_1, \ldots, \alpha_m$. If the DM is an optimist or a pessimist, then*[7]

$$\langle \alpha_1, \delta_{x_1}; \ldots; \alpha_{m-1}, \delta_{x_{m-1}}; \alpha_m, p\rangle \succ \langle \alpha_1, \delta_{x_1}; \ldots; \alpha_{m-1}, \delta_{x_{m-1}}; \alpha_m, q\rangle \text{ whenever } p >_{FOSD} q. \quad (3)$$

The idea behind Proposition 2 is that fixing p as either an elation or a disappointment, the utility of $P(\omega)$ is increasing in ω; and viewing p as an elation gives strictly higher utility for each ω. The fact that the other sublotteries are degenerate ensures that the history assignment of p does not affect their value. Consider instead a lottery $\langle \alpha, p; 1-\alpha, q\rangle$, where both p, q are risky and $p >_{FOSD} q$. Fixing any β, the certainty equivalent of p is larger than that of q by first-order dominance; hence it would always be consistent to label p as an elation and q as a disappointment.

[7]More generally, any DM whose history assignment a applies a cutoff for viewing the risky lottery as an elation (as in the above discussion) will also satisfy Property (3).

However, if $CE_e(q) > CE_d(p)$ then it would also be consistent to label p as a disappointment and q as an elation; and if the probability $1-\alpha$ of q is sufficiently high, the optimist may achieve a higher HDDA utility by doing so. The intuition is that by viewing a high probability, riskier prospect as an elation (if it is consistent to do so), the optimist puts a "positive spin" on the uncertainty. (A similar feature applies for the pessimist.)

6. HDDA with intermediate choices

We now extend the HDDA model to the setting of stochastic decision trees. Roughly speaking, a stochastic decision tree is a lottery over choice sets of shorter stochastic decision trees. In each choice set, the DM can choose the continuation stochastic decision tree. Formally, for any set Z, let $K(Z)$ be the set of finite, nonempty subsets of Z. A one-stage stochastic decision tree is simply a one-stage lottery. The set of one-stage stochastic decision trees is $\mathscr{D}^1 = \mathscr{L}^1$, with typical elements p, q. For $t = 2, .., T$, the set of finite, nonempty sets of $(t-1)$-stage stochastic decision trees is given by $\mathscr{A}^{t-1} = K(\mathscr{D}^{t-1})$, with typical elements A^{t-1}, B^{t-1}. Then the set of t-stage stochastic decision trees is the set of lotteries over finite choice sets of $(t-1)$-stage stochastic decision trees. Formally, the set of t-stage stochastic decision trees is $\mathscr{D}^t = \mathscr{L}(\mathscr{A}^{t-1})$, with typical elements P^t, Q^t. Our domain is thus $\mathscr{D}^T = \mathscr{L}(\mathscr{A}^{T-1})$. (Our previous domain of T-stage lotteries can be seen as the case in which all choice sets are degenerate.)

The realization of a t-stage stochastic decision tree is a choice set, which is categorized by the DM as either elating or disappointing. The set of possible histories, H, is the same as before, with the understanding that histories now refer to choice sets. We abuse notation and identify the stochastic decision tree $P^T \in \mathscr{D}^T$ with a degenerate choice set, denoted $A^T := \{P^T\}$.

The folding back procedure may be extended to this richer domain in a way that the history assignment of a choice set determines the disappointment aversion coefficient applied to each stochastic decision tree inside it, and the value of the choice set itself is the maximal value of those stochastic decision trees. Formally, the certainty equivalent of each set of one-stage stochastic decision trees is determined by

$$u\left(CE_{a(A^1|P^T)}(A^1)\right) = \max_{p \in A^1} V(p; u, \beta_{a(A^1|P^T)}).$$

Fold back each $P^2 \in A^2$, by replacing each realization (a choice set, A^1_j) with its certainty equivalent, as calculated above, to get the sublottery

$$\widetilde{P^2} = \langle \alpha_1, CE_{a(A^1_1|P^T)}(A^1_1); \ldots; \alpha_m, CE_{a(A^1_m|P^T)}(A^1_m) \rangle.$$

The certainty equivalent of a set of two-stage stochastic decision trees, A^2, is determined by

$$u\left(CE_{a(A^2|P^T)}(A^2)\right) = \max_{\left\{\widetilde{P^2}:P^2\in A^2\right\}} V(\widetilde{P^2};u,\beta_{a(A^2|P^T)}).$$

Continuing in this manner, the T-stage stochastic decision tree is reduced to a one-stage lottery (over the certainty equivalents of its continuation subtrees) whose value is calculated using β_0, since $a(A^T|P^T) = 0$.

The definition of HDDA is almost the same as before.

Definition 5 (HDDA with intermediate choices). *An HDDA utility representation over T-stage stochastic decision trees consists of an increasing and continuous utility over monetary prizes $u: X \to \mathbb{R}$, a collection of disappointment aversion coefficients $B = \{\beta_h\}_{h\in H}$, and a history assignment a satisfying, for each $P^T \in \mathscr{D}^T$,*

1. *Sequential assignment. The DM assigns histories to all realizations of stochastic decision subtrees of $\mathscr{D}^T$. Let $a(A^T|P^T) := \beta_0$ and, recursively, for $t < T$:*

   (i) if P^{t+1} is nondegenerate and $A^t \in$supp $P^{t+1} \in A^{t+1}$ then $a(A^t|P^T) \in a(A^{t+1}|P^T) \times \{e,d\}$.

   (ii) if P^{t+1} is degenerate and and $A^t \in$supp $P^{t+1} \in A^{t+1}$ then $a(A^t|P^T) = a(A^{t+1}|P^T)$.

2. *Folding back. The DM calculates the utility of the stochastic decision tree by folding back. We let $V(A^t;u,a,B|P^T)$ and $V(P^t;u,a,B|P^T)$ denote, respectively, the value of any choice set A^t in P^T and subtree P^t of P^T, as calculated above, simply writing $V(P^T;u,a,B)$ for the value of P^T.*

3. *Internal consistency. Within P^T, for each nondegenerate P^{t+1}, if $A^t \in$supp P^{t+1} is an elating (disappointing) outcome in P^{t+1}, then the value of A^t must be weakly larger than (strictly smaller than) the value of P^{t+1} in A^T.*

Observe that the DM is dynamically consistent under HDDA with intermediate actions. From any future choice set, the DM anticipates selecting the best stochastic decision tree. That choice leads to an internally consistent history assignment of that choice set. Thus, when reaching a choice set, the disappointment aversion coefficient she uses to value the choices therein is the one she anticipated using, and her choice is precisely her anticipated choice.

Internal consistency is therefore a stronger requirement than before, because it takes optimal choices into account. However, our previous results on the restrictions that internal consistency imposes on how history affects disappointment aversion extend to the model of HDDA with intermediate actions.

Theorem 3 (Extension of previous results). *The conclusions of Theorems 1 and 2 (necessary and sufficient conditions for HDDA) also hold for HDDA with intermediate actions.*

6.1. Implications

Actions that can be taken while risk unfolds may arise naturally in various settings. Under HDDA, the DM's risk taking behavior may depend on her history—she overreacts to news, and satisfies primacy effects. For example, overreaction to news suggests that a basketball player might attempt more difficult shots after a string of successful ones. Rao (2009), for example, finds evidence to this effect within the NBA, and uses this as an explanation for the "hot hand" fallacy, which is the belief that a winning streak indicates future success (even in independent events).[8]

The biases predicted by HDDA, particularly the primacy effect, may also be exploited by agents who can manipulate the presentation of information to affect the DM's behavior. For example, consider a financial advisor trying to sell a risky investment to a DM who has an HDDA representation (u,a,B), with utility over monetary prizes $u(x)=x$ and strictly positive disappointment aversion coefficients ordered as in Figure 1. The risky investment, which requires an initial payment of I, is an even chance gamble between $I+U$ and $I-D$. The DM knows that the upside, U, and downside, D, are independently and uniformly distributed on $\{0,500,1000\}$. The financial advisor receives a commission whenever the DM invests and is informed about the true values of U and D. The DM may consult with the financial advisor at no cost to learn U and D, and may choose whether or not to invest based on the information provided. The financial advisor is obligated to tell the truth about U and D, but can reveal this information in any order.[9]

It is straightforward to check that without any information, the DM prefers not to invest. Hence the DM chooses to consult with the advisor. Suppose the financial advisor has some good news and some slightly worse news: the upside is high ($U=1000$) but the downside is moderate ($D=500$).[10] How should she reveal this? Since the financial advisor knows the DM's preferences, she can

[8] Unlike previous studies, such as Gilovich, Vallone and Tversky (1985), Rao controls for shot difficulty (i.e., taking more or less difficult shots after successes or failures) and shows that risk taking behavior —but not ability—is affected by previous outcomes.

[9] We assume for simplicity that the DM accepts the information the financial advisor gives in that order, without making inferences (this is an assumption often made in the context of framing effects); relaxing this assumption is interesting but beyond the scope of this example.

[10] This is the only case in which manipulation is possible. It is clear that when learning $U=0$, the DM would never invest; when learning $D=0$, the DM would always invest; and that the DM would not invest if $U=D$ and he has strictly positive disappointment aversion coefficients.

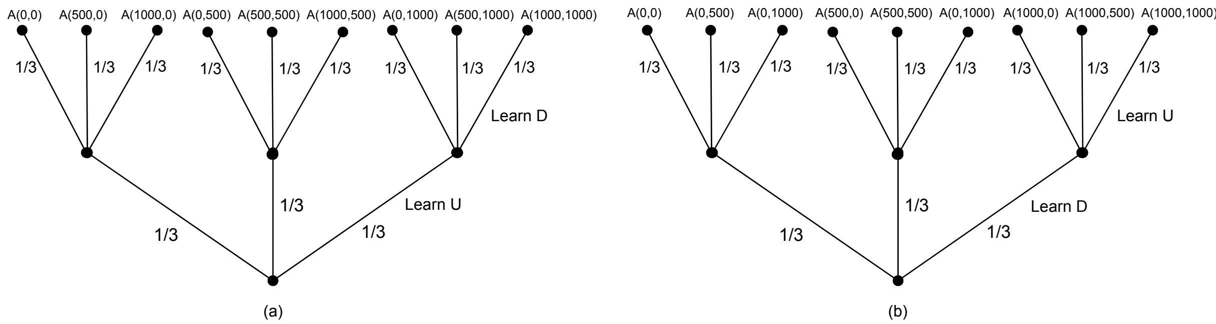

Figure 4: The stochastic decision tree that the DM faces when the financial advisor (a) reveals the upside first and the downside later or (b) reveals the downside first and the upside later. For given U, D, the choice set $A(D,U) = \{\delta_I, \langle .5, I+U; .5, I-D \rangle\}$ corresponds to either investing or not.

predict his choice based on how she provides information about U and D. For $U = 1000$ and $D = 500$, the DM will invest if the disappointment aversion coefficient used to evaluate the investment is smaller than one. The primacy effect suggests that conveying the best news first increases the DM's inclination to invest. This can be formalized by applying HDDA to the stochastic decision trees in Figure 4, which describe the DM's problem when the financial advisor reveals U or D first. For a wide range of disappointment aversion coefficients (for example, if $\beta_h \in [.5, 1.5]$ for all h and $\beta_{ed} < 1 < \beta_{de}$), the DM is immediately disappointed when $D = 500$ is mentioned first, and wouldn't invest even upon hearing $U = 1000$; while the DM is immediately elated when $U = 1000$ is mentioned first, and invests even upon hearing $D = 500$. Therefore, the financial advisor should reveal the best news first to minimize the DM's subsequent risk aversion and ensure he invests.

7. Axiomatic foundations for HDDA on two-stage lotteries

In this section, we present axioms necessary and sufficient for a preference $\succeq$ on the set of two-stage lotteries, $\mathscr{L}^2$, to have an HDDA representation (with uniquely identified u, B, and history assignment a). This simplified setting allows for the clearest exposition of the underlying ideas; we discuss the extension to more stages in Section 7.1.

For two-stage lotteries, an HDDA representation consists of an increasing and continuous utility over prizes $u : X \to \mathbb{R}$, an internally consistent and sequential history assignment a, and disappointment aversion coefficients $B = \{\beta_0, \beta_e, \beta_d\}$. The value of degenerate lottery $P = \langle 1, p \rangle$ is given by $V(p; u, \beta_0)$; and the value of a nondegenerate lottery $P = \langle \alpha_1, p_1; \ldots; \alpha_j, p_j; \ldots; \alpha_m, p_m \rangle$

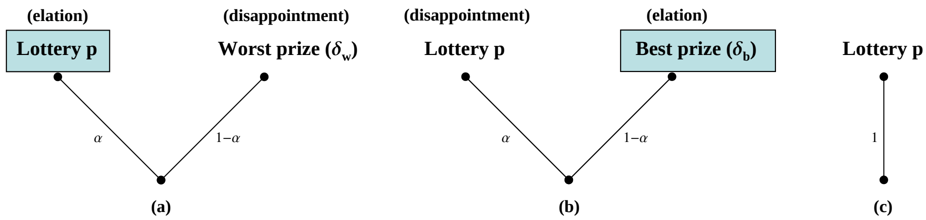

Figure 5: As the lottery p is varied, (a) corresponds to the objects inducing the relation $\succeq_{e,\alpha}$, (b) corresponds to $\succeq_{d,\alpha}$; and (c) corresponds to $\succeq_0$.

is given by

$$V(P;u,a,B) = \frac{\sum_{\{j \mid a(p_j|P)=e\}} \alpha_j u\left(CE_e(p_j)\right) + (1+\beta_0)\sum_{\{j \mid a(p_j|P)=d\}} \alpha_j u\left(CE_d(p_j)\right)}{1+\beta_0 \sum_{\{j \mid a(p_j|P)=d\}} \alpha_j},$$

where for each $h \in \{e,d\}$, $CE_h(p)$ is the certainty equivalent of p calculated using (1) with u and β_h. Recall further that internal consistency means, for example, that if p is elating in P (i.e., $a(p|P)=e$) then it should indeed be that $CE_e(p)$ is weakly larger than $V(P;u,a,B)$.

In some two-stage lotteries, which history to assign to each realization can be determined by a quick inspection. For example, this is true of all the lotteries depicted in Figure 5. In the lottery in Figure 5(b), which has the form $\langle \alpha, p; 1-\alpha, \delta_b \rangle$, receiving the lottery p is disappointing compared to receiving the best monetary prize (b) with certainty.[11] How should the DM compare the two-stage lotteries $P = \langle \alpha, p; 1-\alpha, \delta_b \rangle$ and $Q = \langle \alpha, q; 1-\alpha, \delta_b \rangle$, which both have this form? Both p and q are disappointing in P and Q, respectively, and are received with the same probability α. According to HDDA, β_d must be applied to evaluate both p and q in the representation according to disappointment aversion, and the value of δ_b is fixed at $u(b)$. Therefore, the preference over P and Q should be determined by the utilities of p and q according to disappointment aversion, using u and β_d.

We define $\succeq_{e,\alpha}$ on $\mathscr{L}^1$ by $p \succeq_{e,\alpha} q$ if $\langle \alpha, \delta_w; 1-\alpha, p \rangle \succeq \langle \alpha, \delta_w; 1-\alpha, q \rangle$. Similarly, we define $\succeq_{d,\alpha}$ on $\mathscr{L}^1$ by $p \succeq_{d,\alpha} q$ if $\langle \alpha, \delta_b; 1-\alpha, p \rangle \succeq \langle \alpha, \delta_b; 1-\alpha, q \rangle$ and $\succeq_0$ on $\mathscr{L}^1$ by $p \succeq_0 q$ if $\langle 1, p \rangle \succeq \langle 1, q \rangle$. These relations are induced from preferences over the objects in Figure 5.

Our first axiom requires that these relations, induced from $\succeq$, each has a one-stage disappointment aversion representation. Axioms for one-stage disappointment aversion are well-known and provided in Gul (1991).

[11] Except in the knife-edge case that $p = \delta_b$; however in that case the utility of the entire lottery is $u(b)$ regardless of how p is labeled, and so not affecting Axiom DA.

Axiom DA (Disappointment aversion). The relations $\succeq_{e,\alpha}$, $\succeq_{d,\alpha}$, and $\succeq_0$ (induced from $\succeq$) have a disappointment aversion representation with common utility over prizes u (up to affine transformations).

As shown in Gul (1991), in any representation of the form (1), β is unique and u is unique up to affine transformation. The requirement of a common u captures the idea that how risk unfolds affects risk attitude through disappointment attitudes but not through the DM's actual utility over monetary prizes.[12] Axiom DA does allow the disappointment aversion coefficients after each history to differ.

Our next axiom says that "no news" does not affect the DM's attitude toward risks. If he knows that his monetary winnings will be determined by a one-stage lottery p, he does not care whether the uncertainty in p is resolved now or later. Hence the DM's risk attitude is not affected by the mere passage of time, but rather only by previous disappointments and elations.

Axiom TN (Time neutrality). For all $p \in \mathscr{L}^1$, $\langle p(x_1), \delta_{x_1}; p(x_2), \delta_{x_2}; \ldots; p(x_m), \delta_{x_n} \rangle \sim \langle 1, p \rangle$.

Recall that the procedure of folding back a two-stage lottery involves replacing each one-stage lottery with its certainty equivalent. In HDDA, p is an elation in $\langle \alpha, \delta_w; 1-\alpha, p \rangle$ for each $\alpha \in (0,1)$. We say that a prize x is an α*-elation certainty equivalent* of p if it solves $\langle \alpha, \delta_w; 1-\alpha, p \rangle \sim \langle \alpha, \delta_w; 1-\alpha, \delta_x \rangle$ (the α*-disappointment certainty equivalent* is defined analogously). By Axiom DA, it is clear that there exists a unique α-elation certainty equivalent for each p (and similarly for disappointment). The next axiom says these certainty equivalents depend only on the history assignment of p, independently of the probability with which it occurs.

Axiom CE (Uniform certainty equivalence). Take any $p \in \mathscr{L}^1$, $x \in X$, and $z \in \{w, b\}$. If $\langle \alpha, \delta_z; 1-\alpha, p \rangle \sim \langle \alpha, \delta_z; 1-\alpha, \delta_x \rangle$ for some $\alpha \in (0,1)$, then $\langle \alpha', \delta_z; 1-\alpha', p \rangle \sim \langle \alpha', \delta_z; 1-\alpha', \delta_x \rangle$ for all $\alpha' \in (0,1)$.

We define $CE_{e,\succeq}(p)$, the *elation certainty equivalent* of p, as the value solving $\langle \alpha, \delta_w; 1-\alpha, p \rangle \sim \langle \alpha, \delta_w; 1-\alpha, \delta_{CE_{e,\succeq}(p)} \rangle$ for all $\alpha \in (0,1)$. The *disappointment certainty equivalent* of p, $CE_{d,\succeq}(p)$, is analogously defined.

[12] In particular, a behavioral implication of this assumption is that the DM's Arrow-Pratt measures of risk aversion after each history are the same (see Gul (1991, Theorem 4)), assuming twice-differentiability of u. Since we only know from the fact that u is increasing on the bounded interval X that it is differentiable except on a measure-zero set, unchanging u implies a constant relative marginal rate of substitution (MRS) condition where the derivatives exist. Our proof of the representation takes this route, showing that Axiom DA can be broken up into a requirement of disappointment aversion on each of these sets and such an MRS condition.

Recall that in the one-stage theory of disappointment aversion, each prize is categorized as an elating, neutral, or disappointing outcome; and if a prize is considered elating, for example, then it must be preferred to the lottery as a whole. In analogy, HDDA requires that if a one-stage lottery p is elating in a two-stage lottery P, then it must indeed be preferred to P as a whole. To study the minimal departure that permits history dependence, HDDA assumes the certainty equivalent of a lottery p in P is affected only by its assigned history h (and therefore the same as calculated within $\mathscr{L}_h(\alpha)$). This motivates the following definitions. Consider $P = \langle \alpha_1, p_1; \dots; \alpha_j, p_j; \dots \; \alpha_m, p_m \rangle$. We say p_j *is elating* in P if

$$P \sim \langle \alpha_1, p_1; \dots; \alpha_j, \delta_{CE_{e,\succeq}(p_j)}; \dots \; \alpha_m, p_m \rangle \preceq \langle 1, \delta_{CE_{e,\succeq}(p_j)} \rangle.$$

Similarly, we say p_j *is disappointing* in P if

$$P \sim \langle \alpha_1, p_1; \dots; \alpha_j, \delta_{CE_{d,\succeq}(p_j)}; \dots \; \alpha_m, p_m \rangle \succ \langle 1, \delta_{CE_{d,\succeq}(p_j)} \rangle.$$

Our final axiom says that the preference $\succeq$ always allows the DM to categorize a realization p_j of a two-stage lottery P according to one of the possibilities above.

Axiom CAT (Categorization). For any nondegenerate $P \in \mathscr{L}^2$ and any $p \in \text{supp } P$, p is either elating or disappointing in P.

These axioms are equivalent to an HDDA representation on two-stage lotteries.

Theorem 4 (Representation). *$\succeq$ on $\mathscr{L}^2$ satisfies Axioms DA, TN, CE, and CAT if and only if it admits a history-dependent disappointment aversion (HDDA) representation with some continuous and increasing utility over monetary prizes $u : X \to \mathbb{R}$, history assignment a and a set of disappointment aversion coefficients $B = \{\beta_0, \beta_e, \beta_d\}$.*

In the theorem above, the disappointment aversion coefficients are unique; u is unique up to positive affine transformation; and with endogenous reference dependence, the history assignment is uniquely determined for each $P \in \mathscr{L}^2$ except in knife-edge cases that two decompositions would give P the same value.

7.1. Extending to three or more stages

With an appropriate modification of the axioms, Theorem 4 can be extended to represent preferences over (arbitrary) T-stage lotteries. In this section, we highlight the required changes for the case $T = 3$; the more general case is similarly analyzed.

We must first generalize the sets of compound lotteries for which the history assignment of any final stage lottery, p, is unambiguous. For example, consider a lottery of the form $\langle \alpha, \delta_w^2; 1-\alpha, P \rangle$, where P is of the form $\langle \alpha', \delta_b; 1-\alpha', p \rangle$. Here, the lottery p must have the history assignment $h = ed$. We may define an induced preference $\succeq_{ed,(\alpha,\alpha')}$ on $\mathscr{L}^1$, and similarly for other possible history assignments. Axiom DA requires that these induced preference relations have disappointment aversion representations, and that the utility over prizes u is common (up to affine transformation) in all these representations. The definition of the certainty equivalent of a sublottery is extended analogously; for example, for $h = ed$, $CE_{ed}(p)$ is the value solving

$$\langle \alpha, \delta_w^2; 1-\alpha, \langle \alpha', \delta_b, 1-\alpha_w'^2; 1-\alpha, \langle \alpha', \delta_b, 1-\alpha', \delta_{CE_{ed}(p)} \rangle \rangle$$

for all α, α'. Axiom CE then says that conditional on each history, the certainty equivalent of a sublottery is independent of the probability with which it is received.

For any given single-stage lottery p, there are three compound lotteries in which the only nondegenerate sublottery is the one where p is fully resolved. Axiom TN requires that the DM be indifferent among these lotteries. Formally, for all $p \in \mathscr{L}^1$,

$$\begin{gathered} \langle p(x_1), \delta_{x_1}^2; p(x_2), \delta_{x_2}^2; \ldots; p(x_m), \delta_{x_n}^2 \rangle \sim \\ \langle 1, \langle p(x_1), \delta_{x_1}; p(x_2), \delta_{x_2}; \ldots; p(x_m), \delta_{x_n} \rangle \rangle \sim \langle 1, \langle 1, p \rangle \rangle. \end{gathered}$$

Lastly, Axiom CAT requires that a sublottery can be replaced by a degenerate lottery that gives the history dependent certainty equivalent of this sublottery for sure, with the consistency condition taking into account the history assignment of the sublottery. For example, if a one-stage lottery $p \in \operatorname{supp} P = \langle \alpha', \delta_b; 1-\alpha', p \rangle$ is replaced by its certainty equivalent after history $h = ed$, then it must be the case that

$$\langle \alpha, \delta_w^2; 1-\alpha, P \rangle \sim \langle \alpha, \delta_w^2; 1-\alpha, \langle \alpha', \delta_b, 1-\alpha', \delta_{CE_{ed}(p)} \rangle \rangle \succ \langle \alpha, \delta_w^2; 1-\alpha, \delta^2_{CE_{ed}(p)} \rangle \rangle$$

8. Conclusion and directions for future research

We propose and axiomatize a model of history dependent disappointment aversion, in which risk attitudes depend endogenously on prior disappointments and elations. The HDDA model predicts that the DM satisfies two well-documented cognitive biases, overreaction to news and the primacy effect, as well as disappointment cycles; the DM raises the threshold for elation after positive experiences but is willing to "settle for less" after negative ones, making disappointment more likely after elation and vice-versa.

To study endogenous reference dependence under the minimal departure from recursive disappointment aversion, HDDA posits the categorization of each sublottery as either elating or disappointing. The DM's risk attitudes depend on the prior sequence of disappointments or elations, but not on the "intensity" of those experiences. We are also interested in extending the HDDA model to permit such dependence. That extension raises several questions, beginning with how to define the intensity of elation or disappointment and how internal consistency is to be understood. The testable implications of such a model depend on whether it is possible to identify the extent to which a realization is disappointing, as that designation depends on the extent to which other options are elating or disappointing.

A. Appendix

A.1. Proofs for Section 4

Proof of Proposition 1. It is clear that (ii) implies (i), as both overreaction to news and the strong primacy effect respect the lexicographic ordering. Proving that (i) implies (ii) follows from alternating applications of overreaction to news and the strong primacy effect (with strict inequalities) starting from the tail of the history. To illustrate, observe that $\beta_{eee} < \beta_{eed}$ by overreaction to news with $h = ee$; $\beta_{eed} < \beta_{ede}$ by the strong primacy effect with $h = e$; $\beta_{ede} < \beta_{edd}$ by overreaction to news with $h = ed$; $\beta_{edd} < \beta_{dee}$ by the strong primacy effect with $h = 0$; and so on and so forth.

Now assume that $\beta_h \in [\beta_{he}, \beta_{hd}]$ for all h. To see that (iii) implies (i), note that overreaction to news is implied by taking $h' = h'' = 0$; and that the strong primacy effect is implied by taking $h' = d^t$ and $h'' = e^t$. To see that (ii) implies (iii), observe that we know $\beta_{heh'} < \beta_{hed^{|h'|}}$ and $\beta_{hde^{|h''|}} < \beta_{hdh''}$. Using the strong primacy effect to combine these bounds delivers the result if $|h''| > |h'|$; so suppose that $|h''| > |h'|$ (the other argument is symmetric). Then repeated use of the assumption that $\beta_{\hat{h}} \leq \beta_{\hat{h}d}$ implies $\beta_{hed^{|h'|}} \leq \beta_{hed^{|h''|}}$, and using the strong primacy effect again completes the proof. ■

Lemma 1. *If $\bigcap_{\tau=0}^{t} (\beta_{ed^\tau}, \beta_{de^\tau}) \neq \emptyset$, then $\beta_{ed^{t+1}} \leq \beta_{de^{t+1}}$.*

Proof. Let $\bigcap_{\tau=0}^{t} (\beta_{ed^\tau}, \beta_{de^\tau}) = \left(\underline{\beta}, \overline{\beta}\right)$. For each $p \in \mathscr{L}^1$ and $\beta \in (-1, \infty)$, define $CE_\beta(p) = u^{-1}(V(p; u, \beta))$. Let

$$CE_{\underline{\beta}} := CE_{\underline{\beta}}(\langle 0.5, \delta_w; 0.5, \delta_b \rangle) \text{ and } CE_{\overline{\beta}} := CE_{\overline{\beta}}(\langle 0.5, \delta_w; 0.5, \delta_b \rangle).$$

Let p be a lottery such that supp $p \subset \left(CE_{\overline{\beta}}, CE_{\underline{\beta}}\right)$. Define $P^2 = \langle \varepsilon, \delta_w; \varepsilon, \delta_b; 1-2\varepsilon, p \rangle$, $P^3 = \langle \varepsilon, \delta_w^2; \varepsilon, \delta_b^2; 1-2\varepsilon, P^2 \rangle$, and continuing inductively, $P^{t+1} = \langle \varepsilon, \delta_w^{t+1}; \varepsilon, \delta_b^{t+1}; 1-2\varepsilon, P^t \rangle$. That is, in each stage $1, ..., t$ the lottery P^{t+1} gives the worst and the best outcome, both with probability ε and the continuation with the remaining probability. At period t, the continuation lottery is the lottery p. Note the following:

(i) For all β, $CE_\beta(p) \subset \left(CE_{\overline{\beta}}, CE_{\underline{\beta}}\right)$. This is by monotonicity of the functional (1).

(ii) Fixing β, β', $\lim_{\varepsilon \to 0} V\left(\langle \varepsilon, w; \varepsilon, b; 1-2\varepsilon, CE_{\beta'}(p) \rangle; u, \beta\right) = V\left(p; u, \beta'\right) = u(CE_{\beta'}(p))$.

(iii) For all τ, p is elating in $P^2 = \langle \varepsilon, \delta_w; \varepsilon, \delta_b; 1-2\varepsilon, p \rangle$ when P^2 is evaluated under β_{de^τ} and p is disappointing in P^2 when P^2 is evaluated under β_{ed^τ}.

Assume by contradiction that $\beta_{ed^{t+1}} > \beta_{de^{t+1}}$. Then $CE_{\beta_{ed^{t+1}}}(p) < CE_{\beta_{de^{t+1}}}(p)$. Pick $\varepsilon > 0$ small enough and apply (ii) and (iii) repeatedly to show that if for $\tau = 1$ β_e is used, then the only consistent set of continuation betas is β_{ed^t}; and if for $\tau = 1$ β_d is used, then the only consistent set of continuation betas is β_{de^t}. But, again, for $\varepsilon > 0$ small enough and

$$\delta = \frac{u(CE_{\beta_{de^{t+1}}}(p)) - u(CE_{\beta_{ed^{t+1}}}(p))}{3} > 0,$$

the first continuation value (evaluated with β_e) is less than $CE_{\beta_{ed^{t+1}}}(p) + \delta$ and the first continuation value (evaluated with β_d) is greater than $CE_{\beta_{de^{t+1}}}(p) - \delta$. To complete the proof, pick $x \in \left(CE_{\beta_{ed^{t+1}}}(p) + \delta, CE_{\beta_{de^{t+1}}}(p) - \delta\right)$ and note that for $\left\langle \alpha, P^{t+1}; 1-\alpha, \delta_x^{t+1} \right\rangle$ an internally consistent HDDA history assignment cannot exist. ■

Proof of Theorem 1. We prove each statement separately. Note that by endogenous reference dependence, $\beta_{hd} \neq \beta_{he}$ for every a.

(i) *Overreaction to news.* Suppose by contradiction that $\beta_e > \beta_d$. Then for any $p \in \mathscr{L}^1$, $CE_e(p) < CE_d(p)$. Choose any $x \in (CE_e(p), CE_d(p))$. Let P^t be the t-stage lottery which has no uncertainty until stage $t-1$ and delivers p at stage $t-1$ with probability one. Observe that the T-stage lottery $\langle \alpha, P^{T-1}; 1-\alpha, \delta_x^{T-1} \rangle$ would have no internally consistent assignment. To show, for example, that $\beta_{ee} < \beta_{ed}$, modify the above to $\langle \alpha, \langle \gamma, P^{T-2}; 1-\gamma, \delta_x^{T-2} \rangle; 1-\alpha, \delta^T, b \rangle$. Analogously, one shows $\beta_{he} < \beta_{hd}$ by constructing the appropriate initial history.

(ii) *Weak primacy effect.* Given $\beta_{he} < \beta_{hd}$, shown in (i), $\beta_{hed} < \beta_{hde}$ follows immediately from Lemma 1 for the case $t = 0$.

(iii) *Strong primacy effect.* By induction. The initial step is true by part (ii). Assume it is true for $t-1$. Note that $\beta_h \in [\beta_{he}, \beta_{hd}]$ and $\beta_{hed^\tau} < \beta_{hde^\tau}$ for all $\tau \leq t-1$ imply $\bigcap_{\tau=0}^{t-1} (\beta_{ed^\tau}, \beta_{de^\tau}) \neq \emptyset$. Hence the strong primacy effect follows from Lemma 1 for t as well.

■

Fix any continuous and increasing u and $\beta \in (-1, \infty)$. For any $p \in \mathscr{L}^1$, the endogenous assignment of prizes to being either elating or disappointing is called an elation disappointment decomposition (EDD). Let $e(p) := \{x \in \text{supp } p \,|\, u(x) > V(p; u, \beta)\}$, $n(p) := \{x \in \text{supp } p \,|\, u(x) = V(p; u, \beta)\}$ and $d(p) := \{x \in \text{supp } p \,|\, u(x) < V(p; u, \beta)\}$.

Lemma 2. *Take* $p = \langle p(x_1), x_1; \ldots; p(x_j), x_j; \ldots; p(x_m), x_m \rangle$ *and* $p' = \langle p(x_1'), x_1'; \ldots; p(x_j), x_j; \ldots; p(x_m), x_m \rangle$.

1. *If $x_1 \notin d(p)$ and $x_1' > x_1$ then $x_1' \in e(p')$.*

2. *If $x_1 \notin e(p)$ and $x_1' < x_1$ then $x_1' \in d(p')$.*

Proof. We prove statement (1), since the proof of (2) is analogous. If if $EDD(p) = EDD(p')$ then

$$u(x_1') - u(x_1) > V\left(p'\right) - V(p) = \frac{p(x_j)}{1+\beta \sum_{x_j \notin e(p)} p(x_j)} \left(u(x_1') - u(x_1)\right).$$

So, suppose that $EDD(p) \neq EDD(p')$. Note that by monotonicity with respect to first-order stochastic dominance, $x \in d(p) \Rightarrow x \in d(p')$. So suppose there exists $x \in e(p)$ such that $x \in d(p')$. Then $V(p)$ equals

$$\frac{\sum\limits_{x_j \in e(p'), j \neq 1} p(x_j)u(x_j) + \sum\limits_{x_j \in e(p), x_j \notin e(p')} p(x_j)u(x_j) + (1+\beta) \sum\limits_{x_j \notin e(p)} p(x_j)u(x_j) + p(x_1)u(x_1)}{1+\beta \sum\limits_{x_j \notin e(p)} p(x_j)}$$

and $V(p')$ equals

$$\frac{\sum\limits_{x_j \in e(p'), j \neq 1} p(x_j)u(x_j) + (1+\beta)\left(\sum\limits_{x_j \in e(p), x_j \notin e(p'), j \neq 1} p(x_j)u(x_j) + \sum\limits_{x_j \notin e(p)} p(x_j)u(x_j)\right) + (1+I_{x_1' \notin e(p')}\beta)p(x_1)u(x_1')}{1+\beta \sum\limits_{x_j \notin e(p)} p(x_j) + \beta\left(\sum\limits_{x_j \in e(p), x_j \notin e(p'), j \neq 1} p(x_j) + I_{x_1' \notin e(p')} p(x_1)\right)}$$

So $V(p')$ equals

$$\frac{V(p)\left(1+\beta \sum\limits_{x_j \notin e(p)} p(x_j)\right) + \beta\left(\sum\limits_{x_j \in e(p), x_j \notin e(p'), j \neq 1} p(x_j)x_j + 1_{x_1' \notin e(p')} p(x_1)x_1'\right) + p(x_1)\left(u(x_1') - u(x_1)\right)}{1+\beta \sum\limits_{x_j \notin e(p)} p(x_j) + \beta\left(\sum\limits_{x_j \in e(p), x_j \notin e(p'), j \neq 1} p(x_j) + 1_{x_1' \notin e(p')} p(x_1)\right)}$$

Multiply both sides by the denominator and rearrange to get:

$$
\begin{aligned}
&\left(1+\beta \sum_{x_j \notin e(p)} p(x_j)\right)\left(V\left(p'\right)-V\left(p\right)\right) \\
&+\beta\left(\sum_{x_j \in e(p), x_j \notin e(p'), j \neq 1} p(x_j)\left(V\left(p\right)-u(x_j)\right)+1_{x_1' \notin e(p')} p(x_1)\left(V\left(p\right)-u(x_1')\right)\right) \\
&= p(x_1)\left(u(x_1')-u(x_1)\right)
\end{aligned}
$$

Note that $u(x_1')-u(x_1)$ must be greater than $(V\left(p'\right)-V\left(p\right))$ to avoid a contradiction (since the coefficient of $(V\left(p'\right)-V\left(p\right))$ is greater than $p(x_1)$. Since by assumption $u(x_1)>V\left(p\right)$, we must have $u(x_1')>V\left(p'\right)$, that is, $x_1' \in e\left(p'\right)$. ■

Lemma 3. *Suppose that for any nondegenerate $p \in \mathscr{L}^1$, $CE_e(p) > CE_d(p)$. Then for any nondegenerate $P \in \mathscr{L}^2$, a consistent decomposition (using only strict elation and disappointment for nondegenerate lotteries in its support) exists.*

Proof. Consider $P = \langle \alpha_1, p_1; \ldots; \alpha_m, p_m \rangle$. Suppose for simplicity that all p_i are nondegenerate (if $p_i = \delta_x$ is degenerate, then $CE_e(p_i) = CE_d(p_i) = u(x)$, so the algorithm can be run on the nondegenerate sublotteries, with the degenerate ones labeled ex-post according to internal consistency). Without loss of generality, suppose that the indexing in P is such that $p_1 \in \arg\max_{i=1,\ldots,m} CE_e(p_i)$, $p_m \in \arg\min_{i=2,\ldots,m} CE_d(p_i)$, and $CE_e(p_2) \geq CE_e(p_3) \geq \cdots \geq CE_e(p_{m-1})$. A consistent decomposition is constructed by the following algorithm. Set $h^1(p_1) = e$ and $h^1(p_j) = d$ for all $i > 1$. Let V^1 be the folded back value under h^1; if V^1 is consistent with h^1, the algorithm and proof are complete. If not, consider $i = 2$. If $u(CE_d(p_2)) \geq V^1$, then set $h^2(p_2) = e$ and $h^2(p_i) = h^1(p_i)$ for all $i \neq 2$ (if $u(CE_d(p_2)) < V^1$, let $h^2(p_i) = h^1(p_i)$ for all i). Let V^2 be the resulting folded back value. If V^2 is consistent with h^2, the algorithm and proof are complete. If not, move to $i = 3$, and so on and so forth, so long as $i \leq m-1$. Notice from Lemma 2 that if $u(CE_d(p_i)) \geq V^{i-1}$, then $u(CE_e(p_i)) > V^i$. Moreover, notice that if $u(CE_e(p_i)) > V^i$, then for any $j < i$, $u(CE_e(p_j)) \geq u(CE_e(p_i)) > V^i$, so previously switched labels remain strict elations; also, because $V^i \geq V^{i-1}$ for all i, previous disappointments remain disappointments. If the final step of the algorithm reaches $i = m-1$, notice that $CE_d(p_m)$ is the lowest disappointment value, therefore the lowest value among $\{CE_{h^{m-1}(p_j)}(p_j)\}_{j=1,\ldots,m}$. Hence, the final constructed decomposition h^{m-1} is consistent with V^{m-1}. ■

Proof of Theorem 2. By Lemma 3 and overreaction to news, an internally consistent (strict) elation-disappointment decomposition exists for any nondegenerate $P \in \mathscr{L}^2$, using any initial β. By induction, suppose that for any $(t-1)$-stage lottery an internally consistent history assignment exists, using any initial β. Consider a t-stage nondegenerate lottery $P^t = \langle \alpha_1, P_1^{t-1}; \ldots; \alpha_m, P_m^{t-1} \rangle$. Notice that the algorithm in Lemma 3 for $\mathscr{L}^2$ only uses the fact that $CE_e(p) > CE_d(p)$ for any nondegenerate $p \in \mathscr{L}^1$. But the same algorithm can be used to construct an internally consistent history assignment for P^t if for any $P^{t-1} \in \mathscr{L}^{t-1}$, $CE_e(P^{t-1}) > CE_d(P^{t-1})$. While there may be multiple consistent decompositions of P^{t-1} using each of β_e and β_d, the strong primacy effect (with strict inequality) ensures this strict inequality regardless of the chosen decomposition. Indeed, starting with β_h, the tree is folded back using higher certainty equivalents sublottery by sublottery, and evaluated using lower β's, as compared to starting with the strictly higher β_d. As in Lemma 3, the history for any degenerate sublottery can be assigned ex-post according to what is consistent; its value is not affected by the assignment of e or d. ■

Here we study the possibility of a third assignment of neutrality (n) for the case of equality in value. For simplicity the characterization of β_{hn} is given for $h = 0$; the generalization is immediate.

Lemma 4. *Suppose there is a nondegenerate r that is neutral in $P = \langle \alpha_1, r; \ldots; \alpha_j, \delta_{x_j}; \ldots \alpha_m, \delta_{x_m} \rangle$ in $\mathscr{L}^2$. For any r', define $P(r') = \langle \alpha_1, r'; \ldots; \alpha_j, \delta_{x_j}; \ldots \alpha_m, \delta_{x_m} \rangle$. There is an open neighborhood N_r of r such that if there is (1) nondegenerate $r' \in N_r$ strictly elating in $P(r')$, then $\beta_e \leq \beta_n$; and (2) nondegenerate $r'' \in N_r$ disappointing in $P(r'')$, then $\beta_n \leq \beta_d$. Moreover, at least one of (1) or (2) holds.*

Proof. If $\beta_n = \beta_e$ or $\beta_n = \beta_d$ we are done. Suppose that for some nondegenerate r,

$$P = \langle p_1, r; p_2, \delta_x; p_3, \delta_y \rangle \sim \langle p_1, \delta_{CE_n(r)}; p_2, \delta_x; p_3, \delta_y \rangle,$$

where $CE_n(r) = u^{-1}(V(r; u, \beta_n))$. Let $\gamma = \min_{h \in \{e,d\}} |CE_h(r) - CE_n(r)|$ ($\neq 0$ if $\beta_n \neq \beta_d$ and $\beta_n \neq \beta_e$). Pick r^ε such that

$$\max\{|CE_e(r^\varepsilon) - CE_e(r)|, |CE_d(r^\varepsilon) - CE_d(r)|, |CE_n(r^\varepsilon) - CE_n(r)|\} < \frac{\gamma}{6}.$$

Suppose first that $r^\varepsilon >_1 r$. Then it cannot be that $\langle p_1, r^\varepsilon; p_2, \delta_x; p_3, \delta_y \rangle \sim \langle p_1, \delta_{CE_n(r)}; p_2, \delta_x; p_3, \delta_y \rangle$. To see this, first note that the RHS is indifferent to $\langle 1, \delta_{CE_n(r)} \rangle$. If r^ε is neutral then the LHS is indifferent to $\langle 1, \delta_{CE_n(r^\varepsilon)} \rangle$, but indifference then contradicts monotonicity and $CE_n(r^\varepsilon) > CE_n(r)$. So by construction we know that $CE_n(r) \notin \{CE_e(r^\varepsilon), CE_n(r^\varepsilon), CE_d(r^\varepsilon)\}$.

Suppose that r^{ε} is a strict elation. We claim $CE_e(r^{\varepsilon}) > CE_n(r)$. Suppose otherwise. We know

$$\langle p_1, \delta_{CE_n(r)}; p_2, \delta_x; p_3, \delta_y \rangle \sim \delta_{CE_n(r)} \succeq \delta_{CE_e(r^{\varepsilon})} \succ \langle p_1, \delta_{CE_e(r^{\varepsilon})}; p_2, \delta_x; p_3, \delta_y \rangle.$$

But if the prize $CE_e(r^{\varepsilon})$ is elating in the single-stage lottery, and it is improved to $CE_n(r)$, then as shown in Lemma 2, it must remain elating, a contradiction to being neutral. The same argument says that if r^{ε} is a disappointment then $CE_d(r^{\varepsilon}) < CE_n(r)$. Given the choice of r^{ε} in the γ-neighborhood above, this implies the desired conclusion. ■

A.2. Proof of Theorem 3

The proof of necessity is analogous to that of Theorem 1. The proof of sufficiency is analogous to that of Theorem 2, with two additions of note. First, since overreaction to news and the strong primacy effect (with strict inequalities) imply the value of each stochastic decision tree in a choice set increases when evaluated as an elation, the value of the choice set (the maximum of those values) also increases when viewed as an elation (relative to being viewed as a disappointment). Second, if the value of a choice set is the same when viewed as an elation and as a disappointment, the best option in both choice sets must be a degenerate continuation. Then its history assignment may be made ex-post according to internal consistency. ■

A.3. Proof of Theorem 4

In proving sufficiency, we weaken the assumption in DA that u under each history is the same and replace it with Axiom MRS below. Wherever it exists, the MRS between y and x in $\langle \alpha, x; 1-\alpha, y \rangle$ given β is

$$MRS(x,y;\beta) := \frac{\frac{\partial V(\langle \alpha,x;1-\alpha,y\rangle;u,\beta)}{\partial x}}{\frac{\partial V(\langle \alpha,x;1-\alpha,y\rangle;u,\beta)}{\partial y}} = \begin{cases} \frac{u'(x)\alpha}{(1+\beta)u'(y)(1-\alpha)} & x > y \\ \frac{(1+\beta)u'(x)\alpha}{u'(y)(1-\alpha)} & x < y \end{cases}$$

Axiom MRS (Constant relative MRS). For all $x, x' > y, y'$, and β, β', $\frac{MRS(x,y;\beta)}{MRS(x,y;\beta')} = \frac{MRS(x',y';\beta)}{MRS(x',y';\beta')}$.

Step 1: Evident elation or disappointment. For any $\alpha \in (0,1)$, define the sets

$$\mathscr{L}_{d,\alpha} := \{\langle \alpha, \delta_b; 1-\alpha, p \rangle \mid p \in \mathscr{L}^1\},$$
$$\mathscr{L}_{e,\alpha} := \{\langle \alpha, \delta_w; 1-\alpha, p \rangle \mid p \in \mathscr{L}^1\}, \text{ and}$$
$$\mathscr{L}_0 := \{\langle 1, p \rangle \mid p \in \mathscr{L}^1\}$$

which consist of all the lotteries of the form in Figure 5 in the main text. Without loss of generality, consider the restriction of $\succeq$ to $\mathscr{L}_{e,\alpha}$." This induces $\succeq_{e,\alpha}$. The case of $\succeq$ restricted to $\mathscr{L}_{d,\alpha}$ is analogous.

Note that by Axiom CE and the definition of $CE_{e,\succeq}(\cdot)$, for any p and $\alpha \in (0,1)$,

$$\langle \alpha, \delta_w; 1-\alpha, p \rangle \sim \langle \alpha, \delta_w; 1-\alpha, \delta_{CE_{e,\succeq}(p)} \rangle. \tag{4}$$

Let $V_{e,\alpha} : \mathscr{L}_{e,\alpha} \to \mathbb{R}$ be a utility representation of $\succeq$ restricted to $\mathscr{L}_{e,\alpha}$. Define

$$\Gamma := \{ \langle \alpha_1, \delta_{x_1}; \alpha_2, \delta_{x_2}; ...; \alpha_m, \delta_{x_n} \rangle | \alpha_j > 0 \text{ for all } j, \ \sum_{j=1}^{m} \alpha_j = 1, \text{ and } x_1, \ldots, x_m \in X \ \}.$$

By (4),

$$V_{e,\alpha}(\mathscr{L}_{e,\alpha}) = V_{e,\alpha}(\mathscr{L}_{e,\alpha} \cap \Gamma),$$

or the range of $V_{e,\alpha}$ is the same on $\mathscr{L}_{e,\alpha}$ and on $\mathscr{L}_{e,\alpha} \cap \Gamma$. Let $V_0 : \mathscr{L}_0 \to \mathbb{R}$ be a utility representation of $\succeq$ restricted to $\mathscr{L}_0$, which has a disappointment aversion representation by Axiom DA. By uniqueness of V_0 up to increasing transformation, assume that V_0 itself is a disappointment aversion representation. Define $V : \mathscr{L}^1 \to \mathbb{R}$ by $V(p) = V_0(\langle 1, p \rangle)$. By Axiom TN, this represents lotteries in Γ. By uniqueness of $V_{e,\alpha}$ up to increasing transformation, pick a transformation such that for any $x \in X$,

$$V_{e,\alpha}(\langle \alpha, \delta_w; 1-\alpha, \delta_x \rangle) = V(\langle \alpha, \delta_w; 1-\alpha, \delta_x \rangle). \tag{5}$$

Call u_0 and β_0 the Bernoulli utility and disappointment coefficient corresponding to V_0 (and therefore V), respectively. Then (4) and (5) imply

$$V_{e,\alpha}\left(\langle \alpha, \delta_w; 1-\alpha, p \rangle\right) = \frac{(1-\alpha)u_0(CE_{e,\succeq}(p)) + \alpha(1+\beta_0)u_0(w)}{1+\beta_0\alpha}. \tag{6}$$

Because it is the only part on the RHS of (6) depending on p, $\succeq$ restricted to $\mathscr{L}_{e,\alpha}$ is identical to $\succeq$ restricted to $\mathscr{L}_{e,\alpha'}$ for $\alpha \neq \alpha' \in (0,1)$, so we drop the subscript α from $V_{e,\alpha}$.

Since $V_e(\langle \alpha, \delta_w; 1-\alpha, \cdot \rangle)$ represents the restriction of $\succeq$ to $\mathscr{L}_{e,\alpha}$, by Axiom DA it is an increasing transformation of a Gul form $G_e(\cdot)$ (which has Bernoulli utility $u_e(\cdot)$ and disappointment coefficient β_e). Because it is the only part on the RHS of (6) depending on p, $u_0(CE_{e,\succeq}(\cdot))$ itself must be an increasing transformation $f(\cdot)$ of $G_e(\cdot)$ over $\mathscr{L}^1$: that is,

$$u_0(CE_{e,\succeq}(p)) = f(G_e(p)) \text{ for all } p. \tag{7}$$

By definition, $y = CE_{e,\succeq}(\delta_y)$. Plugging $p = \delta_y$ into (7) means

$$u_0(y) = u_0(CE_{e,\succeq}(\delta_y)) = f(G_e(\delta_y)) = f(u_e(y)). \tag{8}$$

Now use constant relative MRS. Let $q = \langle \alpha, \delta_x; 1-\alpha, \delta_y \rangle$, where $x > y$. Then,

$$V(q) = \frac{\alpha u_0(x) + (1+\beta_0)(1-\alpha)u_0(y)}{1+\beta_0(1-\alpha)}, \text{ and } G_e(q) = \frac{\alpha u_e(x) + (1+\beta_e)(1-\alpha)u_e(y)}{1+\beta_e(1-\alpha)}.$$

The MRS corresponding to β_0 when $x > y$ is $-\frac{u_0'(x)}{u_0'(y)} \cdot \frac{1}{1+\beta_0}$, and because G_e represents $\succeq$ on $\mathscr{L}_{e,\alpha}$, the elation MRS when $x > y$ is $-\frac{u_e'(x)}{u_e'(y)} \cdot \frac{1}{1+\beta_e}$. Relative MRS is then

$$\frac{u_e'(x)}{u_0'(x)} \cdot \frac{u_0'(y)}{u_e'(y)} \cdot \frac{1+\beta_0}{1+\beta_e}.$$

(By Gul monotonicity, all u's are strictly positive functions). Constant relative MRS says this is independent of x, y wherever it exists. For simplicity assume u is differentiable (otherwise use continuity to piece together intervals). Then $\frac{u_e'(x)}{u_0'(x)} \cdot \frac{u_0'(y)}{u_e'(y)}$ is constant in x, y. Pick $w < z_l^* < z_h^* < b$. In particular, setting $y = z_l^*$, $\frac{u_e'(x)}{u_0'(x)}$ is constant for $x \in (z_l^*, b]$, or there exists κ_1 such that $u_e'(x) = \kappa_1 u_0'(x)$ for all $x \in (z_l^*, b]$. Similarly, setting $x = z_h^*$, there exists κ_2 such that $u_e'(y) = \kappa_2 u_0'(y)$ for all $y \in [w, z_h^*)$. Because $z_l^* < z_h^*$, the intersection is nonempty and $\kappa_1 = \kappa_2 = \kappa$. Hence u_e and u_0 are affine transformations of each other. Moreover, $\kappa > 0$.

Now recall (8), which implies that f must be the inverse affine transformation mapping u_e back to u_0. But if u_e is a linear transformation of u_0, then for any β, the Gul value of a lottery p using u_e is a linear transformation of the corresponding Gul value of p using u_0; and f undoes this transformation. Denote by $G_{0,e}(\cdot)$ the value of a Gul functional calculated using u_0 and β_e. We have $G_{0,e}(p) = f(G_e(p))$. Therefore $u(CE_{e,\succeq}(p)) = G_{0,e}(p)$, indicating that $CE_{e,\succeq}(p)$ is indeed the certainty equivalent calculated according to u_0 and β_e. A similar argument works for $CE_{d,\succeq}$. Since u_0 is used after all histories we refer to it simply as u.

To summarize, in this step we established that all the lotteries in $\mathscr{L}_{e,\alpha}$, $\mathscr{L}_{d,\alpha}$, and $\mathscr{L}_0$ are evaluated by folding back, using disappontment aversion functionals with the same u but potentially different β's.

Step 2: Endogenous neutrality, elation or disappointment. Consider any nondegenerate $P = \langle \alpha_1, p_1; \cdots; \alpha_m, p_m \rangle$. By Axiom CAT, for every $j = 1, 2, \ldots, m$, p_j is elating or disappointing in P.

Beginning with $j = 1$, this implies that

$$P \sim P^{(1)} = \langle \alpha_1, \delta_{CE_{a(1),\succeq}(p_1)}; \alpha_2, p_2 \cdots ; \alpha_m, p_m \rangle \text{ for some } a(1) \in \{e, d\}.$$

Now, notice by Axiom CAT that p_2 is elating or disappointing in $P^{(1)}$. Hence

$$P \sim P^{(1)} \sim P^{(2)} = \langle \alpha_1, \delta_{CE_{a(1),\succeq}(p_1)}; \alpha_2, \delta_{CE_{a(2),\succeq}(p_2)} \cdots ; \alpha_m, p_m \rangle \text{ for some } a(2) \in \{e, d\}.$$

By repeatedly applying categorization in this manner,

$$P \sim P^{(1)} \sim P^{(2)} \sim \cdots \sim P^{(m-1)} \sim P^{(m)} = \langle \alpha_1, \delta_{CE_{a(1),\succeq}(p_1)}; \alpha_2, \delta_{CE_{a(2),\succeq}(p_2)} \cdots ; \alpha_m, \delta_{CE_{a(m),\succeq}(p_m)} \rangle,$$

where each $a(j) \in \{e, d\}$. Moreover, by Axiom CAT and use of transitivity, if $a(j) = e$ then $\delta_{CE_{e,\succeq}(p_j)} \succeq P^{(m)}$, and if $a(j) = d$ then $P^{(m)} \succ \delta_{CE_{d,\succeq}(p_j)}$.[13] ■

[13] Notice that the construction of $P^{(m)}$ may have been path-dependent (potentially more than one of the relations holds). But since $P^{(m)}$ is evaluated using a disappointment aversion representation (by Axioms TN and DA), for any path of construction, either (i) $P^{(m)}$ has at least one $CE_{d,\succeq}$ and one $CE_{e,\succeq}$ or (ii) $P^{(m)}$ consists entirely of $CE_{e,\succeq}$'s, all of which are indifferent to $P^{(m)}$.

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