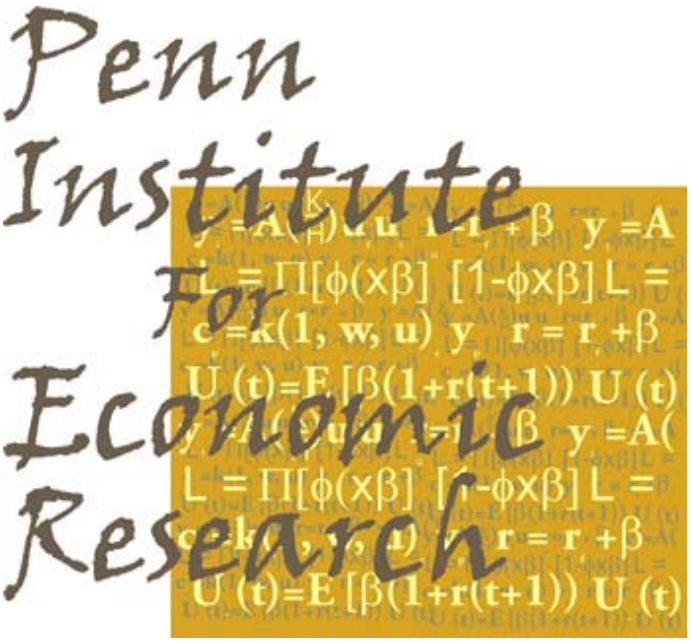

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"Value Added Taxes, Chain Effects and Informality"

by

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Value Added Taxes, Chain Effects and Informality*

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Value Added Taxes, Chain Effects and Informality

Abstract

This paper investigates determinants of informal economic activity. We present an equilibrium model of informality and test its implications using a survey of 48,000+ small firms in Brazil. We define informality as tax avoidance; firms in the informal sector avoid tax payments but suffer other limitations. A novel theoretical contribution in this model is the role of value added taxes in transmitting informality. It predicts that the informality of a firm is correlated to the informality of firms from which it buys or sells. The model also implies that higher tolerance for informal firms in one production stage increases tax avoidance in downstream and upstream stages. Empirical analysis shows that, in fact, various measures of formality of suppliers and purchasers (and its enforcement) are correlated with the formality of a firm. Even more interestingly, when we look at sectors where Brazilian firms are not subject to the credit system of value added tax, but instead the value added tax is applied at some stage of production at a rate that is estimated by the tax authorities, this chain effect vanishes.

JEL Codes: H2, H3, K4.

1 Introduction

In this paper we investigate determinants of informality. It is difficult to define informal activities unambiguously, but estimates indicate that in 1990-1993 approximately 10% of GDP in the United States was produced by individuals or firms that evaded taxes or engaged in illegal pursuits. These activities may produce 25 to 35% of aggregate output in Latin America, between 13 to 70% in Asian countries, around 15% in O.E.C.D. countries (see Table 2 in Schneider and Enste [23]). The underground economy is also estimated to have comprised between 6 and 63.5% in countries in the former Soviet Union and other Eastern and Central European nations during the first half of the 90's. (see Table 1 in Johnson *et al* [13]).

Informality creates a fiscal problem, but there is also growing evidence that informal firms are less efficient, perhaps because of their necessarily small scale, perhaps because of their lack of access to credit or access to the infrastructure of legal protection provided by the State. In many less developed countries, creating incentives for formalization is viewed as an important step to increase aggregate productivity.

We present an equilibrium model of informality and test its implications using a survey of 48,000+ small firms in Brazil. In our model informality is defined as tax avoidance. Firms in the informal sector avoid paying taxes but suffer from other limitations.

The model can be seen as a development of Rausch [19], who relied in the modeling strategy of Lucas [16] in which managerial ability differs across agents in the economy, and assumed a limitation on the size of informal firms. As in that paper, agents with low managerial ability become informal managers and those with highest ability become formal managers.[1] Managers with more ability would naturally run larger firms; for this reason they choose to join the formal sector, where they do not face size limitation. The marginal firm trades off the cost of paying taxes versus the scale limitations of informal firms. As a result, the marginal firm is smaller in the informal sector than it would be if it joined the formal

[1] In the working paper version of this article we provide an generalization of Rausch's model in which capital and labor are chosen and the group of lowest ability individuals becomes workers. See de Paula and Scheinkman [18].

sector. Thus, as in Rausch [19], Fortin *et al.* [9] or Dabla-Norris *et al.* [5], a size gap develops. Managers that are slightly more efficient than the manager of the marginal informal firm run discretely larger firms. In this class of models, entrepreneurs that operate in the informal sector are too inefficient to benefit from the lower capital costs and scale economies afforded to formal entrepreneurs. In this sense these models agree with the results from a survey of informal Mexican firms conducted by Mckenzie and Woodruff that is reported in Fajnzylber *et al* [8], where 75% of the respondents reported that they were too small to make it worth their while to become formal.[2]

The main innovation of our theoretical analysis though is a focus on the role of value added taxes (VAT) in transmitting informality. It exploits the idea that collecting value added taxes according to a credit scheme sets in motion a mechanism for the transmission of informality. The VAT is a prevalent form of indirect taxation: more than 120 nations had adopted it by 2000.[3] In the credit or invoice method that is often used, the value added tax applies to each sale and each establishment receives a credit for the amount of tax paid in the previous stages of the production chain. This credit is then used by the taxpayer against future liabilities with the tax authorities. Since purchases from informal suppliers do not generate tax credits and informal buyers cannot use tax payment from formal suppliers, there is an incentive for informal (formal) firms to deal with other informal (formal) firms.[4] Our empirical analysis shows that, in fact, various measures of formality of suppliers and purchasers (and its enforcement) are correlated with the formality of a firm. These findings survive when we use instrumental variables to control for possible simultaneity. Even more interestingly, when we look at sectors where Brazilian firms are not subject to the credit system of value added tax, but instead the VAT is applied at some stage of production at a rate that is estimated by the tax authorities, this chain effect vanishes. We should also mention that we have a sample of small firms and the chain effects obtained in the model

[2]...presumably relative to cost.

[3]See Appendix 4 in Schenk and Oldman [22].

[4]To our knowledge, the only other study to investigate the informal sector in conjunction with a VAT structure is Emran and Stiglitz [7]. Their focus is on the consequences of informality for a revenue neutral tax reform involving value added and trade taxes.

may be even more important for larger firms with lower, more involved production processes. In this sense, our estimates may be seen as a "lower bound".

Using data from the Brazilian Ministry of Labor, we also construct measures of enforcement of formality in the labor market. Our estimates show that enforcement in the clients' or suppliers' sectors has a positive and significant effect on the probability of formalization of an economic unit.

Since the mid 90's, following the lead of the Federal government, several Brazilian states introduced SIMPLES programs that simplified and lowered the VAT rates for small firms. The State of São Paulo, the largest and richest state in Brazil, for example introduced its SIMPLES in 1998. Rio Grande do Sul, another large and relatively rich southern state, started its own program only in 2005. We use data on the Brazilian states and two rounds of the Brazilian survey of small firms to evaluate the impact of the introduction of these state programs. Our results point to a significantly positive impact of the program introduction, increasing the probability of formalization of small firms and the formalization chain across firms.

The model in this paper ignores several alternative reasons for informality, such as the fixed cost of complying with regulations, labor taxes or the existence of a minimum-wage. It also ignores benefits of formality that have been highlighted in the literature — such as access to participation in the legal system and other civic institutions. Considering these omitted costs and benefits should not change the qualitative implications of our models, which do not aim at providing an exhaustive explanation for informality. In addition, we focus on informality from the viewpoint of firms, not workers. There is a vast literature on labor informality, which is not addressed in this paper. Finally, our model ignores partial compliance: firms either pay their taxes in full or not at all. This is a simplification that is also present in our data, which only provides us with binary information on formalization.

Other papers that investigate causes and determinants of informality include Loayza [15], Johnson *et al.* [13] and Friedman *et al.* [10] which provide evidence of an association between the size of the underground economy and higher taxes, more labor market restrictions, and poorer institutions (bureaucracy, corruption and legal environment). Junqueira and Mon-

teiro [14] and Fajnzylber *et al.* [8] are recent papers that use an earlier (1997) wave of the the survey that we employ in this paper. They both explore the institution of the federal SIMPLES, which simplified and reduced rates for tax compliance for small firms in Brazil, to make inferences on the relation of taxes and informality. Although our empirical results speak to a somewhat different set of questions (for instance, the multi-stage transmission of informality captured by our second model), use data from a different year (2003 versus 1997) and refer to a different definition for formalization,[5] their empirical results are broadly in line with the implications of our models. Both papers find that the enactment of SIMPLES has increased formality through a smaller tax burden and cheaper formalization costs.

The remainder of this paper is organized as follows. In the next Section we develop a model with two stages of production and VAT. Section 3 contains the empirical results obtained using data on informal firms in Brazil and Section 4 concludes.

2 A Model for informality

In this section we present a very simple model that illustrates the transmission of informality across sectors as a result of taxes on value added. In Section 3 we document that this mechanism is relevant for informality in Brazil.

There are two stages of production: "upstream" and "downstream". All individuals in this model are entrepreneurs and, for simplicity, we assume that they are specialized in one of the stages. Each entrepreneur in the upstream sector is characterized by his ability $\theta_u > 0$. The density of θ_u is $g_u(\cdot)$. An entrepreneur of ability θ_u can produce θ_u units of the intermediate good in the formal sector. An informal entrepreneur, if detected by the authorities, looses all profit. The probability of being detected increases with the size of the

[5] Junqueira and Monteiro [14] and Fajnzylber *et al.* [8] use municipal licensing as proxy for formalization instead tax registration, the measure we use. Junqueira and Monteiro recognize that tax registration would be a more appropriate indication of formalization, but opt for licensing because the question on tax registration was only asked for those who indicated that their firm had been "legally constituted" — that is, a contract had been registered with the proper authorities. We do not view this as a problem, since according to Brazilian law only legally constituted firms are eligible for tax registration.

firm. While a more general form for this probability of detection can be adopted, we assume here, for simplicity that there exists an output level $\overline{y} > 0$, such that entrepreneurs that produce no more than $\overline{y} > 0$ are never detected but those who produce more than $\overline{y} > 0$ are detected with probability 1. Thus the production function in the informal sector for an entrepreneur of ability θ_u is $\min(\overline{y}, \theta_u)$.

Downstream entrepreneurs are characterized by an ability parameter θ_d with density $g_d(\cdot)$. An agent with ability θ_d, if in the formal sector, produces $\theta_d x^\alpha$ units of the formal good using x units of the intermediate good. In analogy to what we assumed for upstream entrepreneurs, we assume that in the informal sector, only a limited amount of input can be used and the production function becomes $\theta_d \min(\overline{x}, x)^\alpha$, where $\overline{x}0$.

Both g_u and g_d are continuous and that there exists $\theta_u < \overline{y}$ for which $g_u(\theta_u) > 0$, and that $g_d(\theta_d) > 0$ for $\theta_d > 0$.

The final good is tradeable and has an exogenous price which we use as numeráire. Firms in the formal sector pay an ad-valorem tax rate of τ and we write $\pi = 1 - \tau$. The value added tax is levied by the credit method: the tax rate applies to each sale and each establishment receives a credit for the amount of tax paid in the previous stages of production. Because of the tax credit, the prices paid for informal and formal goods may be distinct and we let p_f be the price of the intermediate good in the formal sector and p_i in the informal sector.

We write

$$\Pi_f^u(\theta_u) = \pi p_f \theta_u \tag{1}$$

$$\Pi_i^u(\theta_u) = p_i \min\{\theta_u, \overline{y}\} \tag{2}$$

for the profit of an upstream firm with manager of quality θ_u if it produces in the formal (informal) sector. Downstream firms face a slightly more complicated problem, since they must also choose which intermediate good (formal or informal) to purchase.

Write

$$\Pi_f^d(\theta_d) = \max\{\max_x[\pi(\theta_d x^\alpha - p_f x)], \max_x[\pi\theta_d x^\alpha - p_i x]\}, \tag{3}$$

for the profit of a downstream firm with a manager with ability θ_d that chooses to operate in the formal sector. In an analogous manner, write

$$\Pi_i^d(\theta_d) = \max\{\max_x[\theta_d \min(x, \overline{x})^\alpha - p_f x], \max_x[\theta_d \min(x, \overline{x})^\alpha - p_i x]\}, \tag{4}$$

for the profit of a downstream firm with a manager of ability θ_d that chooses to operate in the informal sector.

The demand of an informal entrepreneur of ability θ_d facing an input price p is:

$$x_i(\theta_d, p) = \min\left(\overline{x}, \left(\frac{\alpha\theta_d}{p}\right)^{1/(1-\alpha)}\right). \tag{5}$$

In turn, a formal entrepreneur demands, if he buys from the formal sector at a unit price p:

$$x_f(\theta_d, p) = \left(\frac{\alpha\theta_d}{p}\right)^{1/(1-\alpha)}, \tag{6}$$

while if he buys from the informal sector he demands $x_f(\theta_d, \frac{p}{\pi})$, since the tax credit does not apply.

We now derive aggregate demand and supply of the intermediate good in the formal and informal sectors as a function of prevailing prices. Since we are interested in equilibrium prices we may restrict the range of prices to $0 < \pi p_f \leq p_i \leq p_f$. In fact, if $\pi p_f > p_i$ profit maximization and equations (3) and (4) imply that both formal and informal entrepreneurs downstream would buy from informal upstream firms. However, every upstream entrepreneur will prefer to produce in the formal sector. Similarly, if $p_i p_f$ every downstream entrepreneur would prefer to buy from formal firms. However, small θ_u agents would prefer to produce informally. Furthermore when $\pi p_f \leq p_i \leq p_f$ downstream informal (formal) entrepreneurs weakly prefer to buy from informal (formal) producers. If these inequalities are strict, preferences are also strict.

The following proposition shows the existence of cutoff points for each stage, $\overline{\theta}_u(p_i, p_f)$ and $\overline{\theta}_d(p_i, p_f)$ such that all managers with ability below the cutoff (weakly) prefer informality and all those with ability above the cut-off points prefer to join the formal sector.

Proposition 1 *(i) If* $\theta_u < \overline{\theta}_u(p_i, p_f) = \frac{p_i \overline{y}}{\pi p_f} \geq \overline{y}$ *then* $\Pi_i^u(\theta_u) \geq \Pi_f^u(\theta_u)$, *and if* $\theta_u > \overline{\theta}_u(p_i, p_f) = \frac{p_i \overline{y}}{\pi p_f}$ *then* $\Pi_i^u(\theta_u) < \Pi_f^u(\theta_u)$.

(ii) There exists a $\overline{\theta}_d(p_i, p_f)$ *such that if* $\theta_d < \overline{\theta}_d(p_i, p_f)$ *then* $\Pi_i^d(\theta_d) \geq \Pi_f^d(\theta_d)$ *and if* $\theta_d > \overline{\theta}_d(p_i, p_f)$ *then* $\Pi_i^d(\theta_d) < \Pi_f^d(\theta_d)$.

At the cutoff points the size and input demand of firms are discontinuous with respect to the quality of the entrepreneur: the marginal informal managers in both sectors are always constrained.

Proposition 2 *(i) If* $p_i > \pi p_f$ *the output of the smallest upstream formal firm is* $\frac{p_i \overline{y}}{\pi p_f} \overline{y}$.

(ii) If $\Pi_f^d(\theta_d) > \Pi_i^d(\theta_d)$ *then the optimal choice of the firm with manager of quality* θ_d, $x_f(\theta_d, p)$, *where* $p = p_f$ *if the firm's optimal choice is to buy the formal good and* $p = \frac{p_i}{\pi}$ *if the firm's optimal choice is to buy the informal good, satisfies*

$$x_f(\theta_d, p) \geq \frac{\overline{x}}{\pi} > \overline{x} \geq x_i(\theta_d, p),$$

for any θ. *In particular, the output of the smallest downstream formal firm is strictly bigger than the output of the largest informal firm.*

The proofs for these results and the existence of an equilibrium price vector (p_i, p_f) are established in the Appendix.

2.1 Comparative statics

In the model all inefficiencies arise from firms that choose to be informal and too small. As exposited in the previous Section, these choices depend on the values of parameters such as the tax rate $(1 - \pi)$ and the level of tolerance in the downstream stage $\bar{x}$ and in the upstream stage $\bar{y}$. Although these parameters can in principle be optimally chosen by a government,subject to the government's enforcement technology and budget constraint, we

will not consider here the optimal tax and enforcement problems.[6]. Nonetheless it is useful to consider the effect of these parameters on equilibrium outcomes.

The effect of tax rate changes on the composition of the economy can be disentangled into a direct effect, holding prices constant, and an indirect effect, which operates through the adjustment of equilibrium prices. The direct effect of taxes on $\overline{\theta}_u$ and $\overline{\theta}_d$ is easily seen to be negative:

$$\frac{\partial \overline{\theta}_u}{\partial \pi} < 0 \qquad \frac{\partial \overline{\theta}_d}{\partial \pi} < 0$$

In other words, for given prices, an increase in the tax rate (decrease in π) leads to an increase in the informal sector in both stages. The first inequality follows because $\overline{\theta}_u = p_i \overline{y}/\pi p_f$. The effect on $\overline{\theta}_d$ is established by noticing that $\overline{\theta}_d$ is defined at the intersection of $\Pi_i^d(\cdot)$ and $\Pi_f^d(\cdot)$. Since $\Pi_i^d(\cdot)$ does not depend on π directly (equation (4)) and $\Pi_f^d(\cdot)$ increases in π (equation (3)), $\overline{\theta}_d$ increases with the tax rate (decreases with π).

Since a change in taxes also affects prices and these in turn affect the cutoffs, the total effect must account for the equilibrium adjustment of prices. The next proposition establishes that increases in taxes increase informality in both stages. In addition, it states comparative statics results for prices and cutoffs in one stage when tolerance to informality in the other stage changes.

Proposition 3 *(i)*

$$\frac{dp_i}{d\overline{y}} < 0 \qquad \frac{d\overline{\theta}_d}{d\overline{y}} > 0$$

(ii)

$$\frac{dp_i}{d\overline{x}} > 0 \qquad \frac{d\overline{\theta}_u}{d\overline{x}} > 0$$

(iii)

$$\frac{d\overline{\theta}_u}{d\pi} < 0 \qquad \frac{d\overline{\theta}_d}{d\pi} < 0$$

The derivation of these results is presented in the Appendix. The total effect of taxes is still in the same direction as the direct effect. Also, as expected, higher tolerance in one stage implies a larger informal sector in the other stage. The impact of an increase in

[6] An exception is a numerical example involving tax substitution that is discussed in subsection 3.6

tolerance on the informality at the same stage of are less clear. In the case of the upstream stage, for example, the direct effect ($\partial\overline{\theta}_u/\partial\overline{y}$) is positive, but the indirect effect (change in prices times reaction of $\overline{\theta}_u$ to prices) is negative (see Appendix). We nevertheless conjecture that the total effect is positive. Numerical simulations support our conjecture: an increase in tolerance in the upstream sector increases the proportion of informal firms upstream and downstream. For $\alpha = 0.7, \tau = 0.15, \overline{x} = 0.1, \theta_d, \theta_u \sim U[0,1]$, Figure 1 shows that as $\overline{y}$ increases, the proportion of upstream firms that are informal increases. As a result the price of the informal intermediate good p_i decreases and some of the downstream formal firms opt for informality. The fall in demand for the formal intermediate good causes a fall in its price p_f. These qualitative features are unchanged for other parameter choices. A symmetric picture arises when we consider changes in the tolerance for informality in the downstream stage, $\overline{x}$.

[Figure 1 here]

3 Empirical Application

In this section we explore implications of our theoretical framework using a dataset on informal firms in Brazil. Tax noncompliance is an important phenomenon in Brazil. Schneider and Enste [23] estimate that informality represents more than one-quarter of the Brazilian economy. Its value added tax system was established in the sixties and value added taxes represent approximately 35% of total tax collection.

3.1 Data

Our principal data source is the ECINF survey (Pesquisa de Economia Informal Urbana) on informal firms realized by the Brazilian Statistics Bureau (IBGE). We used the 2003 edition of that survey, collected in October 2003, which contains information on 48,701 entrepreneurs in urban regions from all states in the Brazilian federation. We also used the 1997 edition for the analysis present in subsection 3.5. The survey focused on units with five

or less employees.[7] The sampling strategy uses the demographic census as a frame. First, preliminary interviews screened households for the presence of at least one entrepreneur with a business employing five or less people, for possible inclusion in the survey. The sampling was done in two stages: in each state (of a total of 27) the primary sampling units (census tracts) were stratified geographically in three strata (state capital, other urban sectors in the capital's metropolitan area and remaining urban sectors). In a second step, the primary sampling units were stratified according to levels of income within the geographical stratum. Census tracts were randomly selected with a probability proportional to the number of households in the sector. From each selected urban sector a total of 16 households was randomly selected for interviews.[8]. Interviewees were told that the information collected for the survey was confidential and would only be utilized for statistical purposes and, in fact, a vast majority declared that their firm was informal.

An ideal dataset for testing our second model would contain information on the production chain associated with each firm. Although the ECINF contains certain characteristics of a firm's clientele (whether they were predominantly large or small companies, persons or governmental institutions), this information is quite limited. To complement these data we used the input-output matrix information available from the Brazilian Statistics Bureau (IBGE). We computed inter-sectoral technical input coefficients and measures of output sectoral destination using the 2003 Brazilian national accounts.[9]

[7]The Brazilian small business administration SEBRAE defines small businesses as those with less than 10 workers in commerce or services or less than 20 workers in all other sectors. According to SEBRAE's Boletim Estatístico de Micros e Pequenas Empresas: Primeiro Semestre de 2005 (http://www.sebrae.com.br), in 2002 small businesses accounted for 93.6% of the total number of firms, employed 36.2% of the workers and responded for 10.3% of wages in 2002.

[8]The census tract is a sub-municipal geographical partition. For the 1997 edition of the survey there were on average 3.1 such census tracts per municipality covered by the survey, with as many as 62 tracts in the municipality of Goiânia and as few as 1.7 tracts per municipality in the State of Maranhão. For more information on the sampling strategies employed, see Almeida and Bianchini [1].

[9]Tables 1 and 2 under "Tabelas de Recursos e Usos" available under National Accounts on http://www.ibge.gov.br for 2003. The construction of technical coefficients follows the European System of Integrated Economic Accounts (ESA) specifications (see ten Raa [26]).

3.2 Description of Variables

We eliminated firms with owners who were less than 15 years old and the observations lacking education or gender information. Entrepreneurs who claimed that their main client was a governmental institution, which comprised less than 1% of the original data, were also discarded. This restricted our sample to around 48,000 observations.

Table 1 summarizes the main variables used in this paper. The first variable indicates formalization; it is a dummy variable that equals one if the firm is registered with the Brazilian tax authorities.[10] For firms in economic sectors that qualify for forward tax substitution (see subsection 3.6 for an explanation), *tax sub* takes the value one. The next two variables are dummies for firms that sell their products mainly to large firms (*large cl*) or small firms (*small cl*) (where large firms are those with more than five employees). Other alternatives are persons or ignored. *Outsidehouse* is a dummy that equals one when the activity is performed outside the home. The number of employees (*# employees*) includes the owner. Even though the survey focused on firms with five or less employees, a few units (less than 0.1%) employ more than five people due to the lag between screening and interviewing and the fact that firms may have multiple partners which are also counted as employees. The variables *revenue*, *other job* and *bank loan* are self-explanatory. *Education* is a categorical variable with values depicted in Table 2. *Age* of the owner is in years and *gender* equals 1 for male. The variable *homeowner* $\times$ *# rooms* is a measure of wealth and is zero for non-homeowners and otherwise displays the number of rooms in the house. The variables *log inv* and *log inst* measure the logarithm of investments and capital installations in October/2003 (R$ 1,000).[11] *Profit* equals revenue minus expenses in October/2003 (also in R$ 1,000). *Logwage* denotes the logarithm of the total expenditures in wages (in R$1,000)

[10]The tax registry is the Cadastro Nacional de Pessoas Jurídicas, which replaced the previous system, the Cadastro Geral de Contribuintes (CGC), used in the 1997 survey. This variable is the most representative of formalization for our purposes, but we have nonetheless experimented with using "legally constituted firms" and obtained virtually identical results. This is not surprising, since, as we already mentioned, the latter is a prerequisite for tax registration and the correlation between the two measures of informality is 0.98.

[11]The value of installations refers to owned installations. Rented equipment is not included. Only 7% of formal firms and 7% of informal firms reported any rented equipment

divided by the number of employees in the firm.[12] The variables *cl form* and *sup form* measure formalization among customers and suppliers of a firm (see subsection 3.3 for the construction of these variables).

[Tables 1 and 2 here]

Each firm in the sample is classified into economic activities following the CNAE (Classificação Nacional de Atividades Econômicas) classification.[13] Using technical coefficients as well as sectoral output allocation coefficients from the National Accounts System (NAS) (using NAS sector classification) we are able to assign to each activity in the survey a vector with these coefficients. Typically a CNAE activity corresponds to a single NAS sector, but there are a few exceptions. Whenever such a multiple match occurred, we assigned to a CNAE sector the weighted averages (using NAS sector production value) of the coefficients in the corresponding NAS sectors. The ECINF survey also has its own aggregate sectoral characterization, displayed in Table 3.

We use these coefficients as a vector measure of sectoral allocation of output and sectoral input assignment by a firm. The last two variables in Table 1 are measures of formalization enforcement for suppliers and customers and were constructed as follows. We used information available from the Brazilian Ministry of Labor on the number of firms visited in a given economic sector and state during 2002 to monitor labor regulation compliance.[14] We normalized the number of visits in each state and sector by the number of persons employed in that state and sector provided by the Brazilian Statistics Bureau (IBGE) (through the Cadastro Central de Empresas).[15] Assuming that a firm's clients were in the same state,[16] we generated an index of client formalization enforcement as a weighted average of these

[12]For comparison, annual GDP per capita in Brazil in 2003 was R$ 8,694.47.($\log(8.69447/12) = \log(0.72454) = -0.13$).

[13]The Brazilian Bureau of Statistics website (http://www.ibge.gov.br) provides a description of this classification as well as various matching tables to other classification schemes.

[14]The enforcement information was also used by Almeida and Carneiro [2].

[15]Similar calculations were also performed using as normalizing variable the number of firms in the state-sector (also obtained through the Cadastro Central de Empresas).

[16]Only 5.23% of the firms in our sample sell in other states

variables, where the weights were the sectoral output allocation coefficients. We used sectoral input demand coefficients to obtain an analogous measure of supplier formalization enforcement.

The correlation matrix for our variables is in Table 4.

[Tables 3 and 4 here]

Table 5 contains probit estimates for the formalization variable *tax reg* using two different sets of controls. The signs obtained for each one of the regressors are as expected. The coefficient of the variable "working outside the home" is positive. The coefficients are also positive for variables related to the size of the firm (number of employees and revenue), credit (bank loan), or the quality of the entrepreneurial input (education, age or having no additional job). Since women in Brazil are likely to have substantial household duties, the sign on the gender variable is probably related to entrepreneurial input. These variables may also partially control for other determinants of informality, such as opportunities in the labor market.[17] The coefficients on all these variables are statistically significant.

[Table 5 here]

The two sets of estimates use different sectoral controls. In the first set we used dummies for state and sector (according to the specification in Table 3). In the second set of results we used the derived output coefficients obtained from the Brazilian National Accounts (similar results obtain using input coefficients). The National Accounts System in Brazil categorizes economic activity into forty-two sectors. The "use table" in the NAS allows one to obtain how much in a given year a sector required in terms of input from another sector in the economy. This can be used to derive technical coefficients for each NAS sector (see footnote 9). We were able to identify the NAS sector for each firm in the ECINF survey using equivalence tables among the different classification schemes that are available from the Brazilian Statistics Bureau. The "make table" in the National Accounts

[17]With the available data it is not possible to control for business cycle variations another potentially relevant determinant of informality that is not captured in our model.

provides the quantity of output destined to each sector of the economy and final demand. We used this information to assemble a vector of sectoral allocation for each monetary unit of output generated for each activity in our sample (and hence each observation in our sample): $(oa_j)_{j=1,...,42}$. These controls, in additional to state dummies, were used in the second set of estimates presented in the table.

3.3 Chain Effects on Formalization

One initial approach to investigate the existence of cross-firms effects of formalization was to employ a characterization of a firm's clientele in the ECINF survey. Interviewees were asked to declare whether sales were principally to large firms (more than five employees), small firms, or persons. Sales to firms tend to increase the probability of formalization with a bigger effect being associated with large firms as depicted in Table 7. These correlations are supportive of the existence of a chain effect in formalization.

We also used a composite measure of formalization among a firm's suppliers to examine this chain effect. This measure consists of a weighted average of the formalization variable (*tax reg*) across supplying sectors using as weights the technical coefficients for input utilization from each sector. More precisely, the formality measure for the suppliers of firm i is given by

$$supplierformal_i = \frac{\sum_j tc_{ij} \times \text{formality}_j}{\sum_j tc_{ij}} \tag{7}$$

where formality_j is the percentage of firms in sector j that display tax registration[18] and tc_{ij} is the required amount of input from sector j per monetary unit of output produced by firm i (obtained from the technical coefficients for that firm's sector). Some caveats apply. This measure of supplier's formality only accounts for potential suppliers that are present in the survey and, in particular, ignores all suppliers that are large firms. On the other hand, the technical coefficients are obtained from a sample of formal firms. The net impact on

[18]Four NAS sectors were excluded since they are not sampled in the ECINF survey: agriculture, mineral extraction, the sugar industry and other food products.

the measure of formalization is unclear. Nevertheless, the results of our analysis favor the model: the coefficients attached to this variable are positive and statistically significant. The estimation results are displayed in Table 7. The marginal impact of supplier formalization on the probability of being formal is 0.358.

A similar strategy uses the sales of each firm, where a sectors' formalization is now weighted according to the output break up by sector:

$$clientformal_i = \frac{\sum_j oa_{ij} \times \text{formality}_j}{\sum_j oa_{ij}} \tag{8}$$

The results are depicted in Table 7. The coefficient on this composite measure of client formalization is positive and statistically significant, with a marginal impact of 0.618.

[Table 7 here]

To address the likely correlation of `large cl` with observable determinants of formality for a given firm, the regressions in Table 7 include sector and state controls and other covariates which we viewed as the most natural confounding variables. We have nonetheless tried additional specifications. First, we experimented with a narrower sector classification (the CNAE numerical activity designation) for controls instead of that displayed in Table 3. The results are unchanged. We also estimated the regressions of subsamples with different sectors (manufacturing and services) and the conclusions are basically unchanged. To handle potential correlation of capital intensity of production across various stages of a given production process we included a quadratic polynomial on capital (installations) in our regressions and the marginal effects are essentially unchanged for the main variables (`large cl` and `small cl`). We also ran regressions including `large cl`, `small cl` and the client and supplier formality proxies from regressions 2 and 3 together. The coefficients are all positive and only the supplier-formality coefficient is no longer significant.

While the degree of tax compliance among a firm's suppliers and customers seems to affect formalization, an endogeneity problem may arise since suppliers and customers of a firm respond to the degree of tax compliance of that firm. This would tend to bias the estimator upwards. Since the variable we use as a proxy for formalization among clients is

an imperfect measure of tax compliance, one cannot rule out the possibility of attenuation bias in the opposite direction of the simultaneity bias as would be the case in some models with misclassified categorical regressors (see Bound *et al.* [3]). To address this potential endogeneity problem we ran instrumental variable versions for the estimation results displayed in Table 7 using the average education level in an entrepreneur's urban sector as an instrument for the formalization of his clients. For the average education level in the urban sector to be a valid instrument, it must only affects one's propensity to be formal through formalization of his or her clients and not respond to the formalization of a single individual.[19] Since we use a single instrumental variable (and hence can only handle one endogenous variable), we consolidate the dummy variables indicating large firms and small firms as a single variable (`lscl` ≡ `large cl` + `small cl`). Table 8 displays the results for the first set of estimates in Table 7 using the aggregate variable in place of `large cl` and `small cl` and its IV version.[20] The coefficient on the consolidated variable, `lscl`, is positive and remains so in the IV version. In fact, the IV version displays an even larger coefficient, which we ascribe to the attenuation effect of imperfect measurement of supplier formalization in the non-instrumented estimation.[21]

[Table 8 here]

We also ran instrumented and non-instrumented probit regressions using a subsample of firms having only large and small firm clients and using the latter as baseline. The coefficient for the large client dummy is again positive in the non-instrumented version of this estimation and it also increases when we use the instrumental variable.

[19] A similar strategy was used in DiPasquale and Glaeser [6].

[20] Since both our outcome of interest (`tax reg`) and endogenous variable (`lscl` ≡ `large cl` + `small cl`) are dichotomous, standard procedures such as TSLS or Rivers and Vuong [21] are inadequate and we used a bivariate probit to generate our IV estimates as suggested in Heckman [11]. To achieve numerical convergence to a maximum, we had to drop `revenue`. We repeated the estimation using linearized TSLS and the Rivers-Vuong approach including `revenue` and the results are qualitatively unchanged — the coefficient on `lscl` is even larger.

[21] A similar phenomenon is observed in Card [4], where IV estimates for return to schooling are about twice as large as OLS estimates.

We also obtained results for a model where the presence of a large client (`large cl`) depends on whether the entrepreneur is formal (`tax reg`=1) and other covariates. We allow for endogeneity of `tax reg` and instrument it using the time it takes for the entrepreneur to reach the nearest bank. The assumption here is that the distance to a bank branch will affect the likelihood that an individual decides to become formal but will only affect whether the entrepreneur has a formal client (proxied by a large customer) via his own status as a formal or informal firm. The results once again point to a chain effect of formalization: an individual who obtains a tax registration is significantly more likely to transact with formal entrepreneurs. The coefficient on `tax reg` is 2.62, with a p-value of 0.013. Theses estimates are presented on Table 9. Results remain unchanged if we restrict ourselves to those who only sell to large and small companies (coefficient=2.00, p-value=0.007).

[Table 9 here]

3.4 The Effect of Enforcement

The previous results show evidence of correlation in the degree of informality across stages of production. Our model also suggests that increased tolerance towards informality in the upstream stage leads to a reduction in formalization in the downstream stage. Similarly, higher tolerance for informality among downstream firms should be accompanied by higher degree of tax avoidance in the upstream stage. We use the measures of formalization enforcement in the labor market described in subsection 3.2 as an indicator for monitoring within each state-economic sector from which a firm buys (using the technical coefficients as weights) and to which a firm sells (using the output allocations as weights). Our estimates in Table 10 show that enforcement in upstream or downstream stages has a positive and significant effect on the probability of formalization.

[Table 10 here]

One potential concern is whether the above results are produced by a higher number of inspections in response to a higher perceived level of informality in a given state/sector.

We believe this reverse causality concern is attenuated because the enforcement measures refer to 2002 and hence predate the ECINF survey. As long as enforcement is predetermined (in the econometric sense) with respect to the formalization decision, the issue is not present. In this spirit, we have obtained results for the regressions presented on Table 10 only for those individuals who have become owners within the last year (between October/2002 and October/2003). The results remain highly significant: the marginal effect of `sup enf` in the first regression is 0.589 (std.error = 0.283,p-value = 0.038) and the marginal effect of `cl enf` is still positive at 1.12 (std.error = 0.301, p-value=0.000).

3.5 The SIMPLES Tax Program

In 1996 the Brazilian federal government established the SIMPLES tax program, targeted at small firms – those with roughly less than R$1,000,000 in annual revenues. It consolidated taxes and social security contributions in a single payment and simplified verification and remittance procedures for tax collection. Although states and municipalities were allowed to join the system for the collection of value added taxes, very few did. More than 20 states eventually established instead their own state-level versions of the SIMPLES system for the collection of VAT and other state taxes. In 1998, for example, the state of São Paulo established a local version of the SIMPLES program. The system exempted firms with less than R$ 120,000 annual revenues from the collection of state VAT and offered reduced rates to firms with at most R$1.2 million in annual revenues. The program provided firms with a significant reduction in VAT. For example, a firm with monthly sales of R$60,000 with input costs of R$20,000 would owe R$7,200 in VAT before the SIMPLES. Under the new program the VAT would total less than R$1,300.

We used the first round of the ECINF survey, collected in 1997, and its 2003 edition to measure the effect of this reduction in VAT on formalization. For comparison we obtained data for states that established their state SIMPLES before and after 2003. Table 11 displays the state laws and their publication dates for those states that established SIMPLES programs before 2003. Two states, Rio Grande do Sul (RS) and Minas Gerais (MG), implemented SIMPLES after 2003, and others (Amazonas (AM), Piauí (PI), Mato Grosso (MT),

Tocantins (TO) and Roraima (RR)) have not established a program.

[Table 11 here]

Table 12 displays results from a probit model where dummy variables for the state and pre- and post-introduction of the state SIMPLES are used to assess the variation in the formalization. We applied the same controls we used in our previous formalization regressions.[22] The results point to a positive impact of the program's introduction with a marginal effect of 1.4 percentage points on formalization, an increase in the probability of formalization of approximately one-tenth.

[Table 12 here]

We could not obtain data on economic activity and State specific tax rates as each state has a different tax code with a myriad of case-specific rules and contingencies. Instead, we investigated the impact of the State SIMPLES on the coefficients of Table 7. Table 13 displays the results distinguishing firms in sectors eligible and not eligible to participate in the Federal version of SIMPLES according to Assuncão and Monteiro [14] (see their Appendix for a list of eligible and ineligible sectors). We take their eligibility criterion as a proxy for the eligibility to participate in the State SIMPLES programs in our sample. For the eligible firms in states where the VAT SIMPLES was introduced by 2003, the marginal effect of selling mostly to large or small clients on the likelihood of having a tax registration, a measure of the chain effect on formalization, is positive for both and significant for the former. The marginal effect associated with having a large client increased by 5.6% for firms in states that instituted the SIMPLES between 1997 and 2003. The equivalent measure associated to having a small client was 1.4%. The marginal effect for those firms that were not eligible was nonetheless not significant. The null hypothesis that $H_0 : \text{SIMPLES} \times 2003 \times \text{Large Client} + \text{Ineligible} \times \text{SIMPLES} \times 2003 \times \text{Large Client} = 0$ has a p-value of 0.67. The p-value for the null hypothesis that SIMPLES $\times$ 2003 $\times$ Small Client + Ineligible $\times$ SIMPLES $\times$ 2003 $\times$ Small Client $= 0$ is also high: 0.4577. It is consequently not possible to reject the hypothesis that the State

[22] Standard errors are not clustered by urban sector since their definition varied between 1997 and 2003.

SIMPLES had no impact on this measure of the chain effect on formalization for ineligible entrepreneurs as one would expect.

[Table 13 here]

3.6 Robustness: Tax Substitution

Brazilian tax law imposes *forward tax substitution* ("substituição tributária para frente") in certain sectors.[23] Under this tax collection system, the value added tax is charged at some stage in the production chain at a rate estimated by the State. This method tends to be adopted for activities with a reduced set of initial producers and many smaller units at the subsequent stages of production. Since no extra value added tax is imposed one should not expect a chain effect in these sectors.

We ran probit estimates on activities where tax substitution is imposed. These activities (and their CNAE numerical activity designation) are automobile and auto-parts manufacturing (34001, 34002, 35010, 35020, 35030, 35090), production of tires (25010), production and distribution of liquor (15050 and 53030), cigarettes (16000), commercialization of automobiles and tires (50010, 50020, 50030 and 54040), distribution of fuel (50050 and 53065), bars and similar establishments (55030) and oil refining (23010 and 23020).

The results concerning investment and installations, number of employees, and the entrepreneur's education level remain qualitatively as before. In Table 14 we interact tax-substitution with our measure of formality of the clients. To facilitate comparisons with the results in Table 8 we again consolidate the dummy variables indicating large firm and small firm clients as a single variable. The coefficient of the interaction term is negative and significant. This is implied by the regression run on the subsample of firms eligible for tax substitution displayed in Table 14. The coefficient on the client formalization ceases to be positive. If anything, there is evidence for a *negative* rather than positive coefficient.

[Table 14 here]

[23] Tax substitution is not peculiar to Brazil. See [20].

Tax substitution raises a natural question: When is it advantageous to replace the VAT by a tax on a single stage? We now discuss a numerical example that illustrates the economic forces that determine the optimal choice. Fix the level $\overline{x}$ of tolerance in the downstream stage and vary the level of tolerance $\overline{y}$ in the upstream stage. We will consider two scenarios. In the first scenario a sales tax is applied only in the downstream sector; producers of the intermediary good are tax exempt. In this case all upstream firms choose to be formal and a single price prevails in the market for the intermediate good. The upstream tolerance level $\overline{y}$ has no effect on equilibrium quantities and, in particular, on the total tax collection T or on the output net of taxes of the final good. Now consider a value added tax that affects both sectors. To insure that the government's budget constraint is satisfied, for each level of $\overline{y}$ find a value added tax rate $\tau(\overline{y})$ such that the total tax collection equals T. It is intuitive that as $\overline{y}$ becomes larger, making informality easier in the upstream sector, the tax rate must be increased to maintain total tax collection. This intuition is verified in this example as illustrated in Figure 3. The tax rate is relatively low when $\overline{y}$ is small, but increases as $\overline{y}$ grows. Furthermore, as shown in Figure 2, for small values of $\overline{y}$, the net output of the final good is larger than it would be if we only taxed the final good. At the other extreme, when $\overline{y}$ is very large, the output of the final good is smaller under a VAT system. Again this result is quite intuitive - if it is very difficult to impose formality in one stage relative to imposing on the other stage, it may be preferable to only tax the latter. When the stage where enforcement is easiest is downstream, the optimal choice is a sales tax, otherwise forward tax substitution may be desirable.

[Figures 2 and 3 here]

4 Conclusion

An implication of our model is that informal firms are smaller, less productive and with less inputs. In addition, informality is transmitted through vertical relationships when value added taxes are levied through the credit method. Using microdata from surveys conducted in Brazil, we confirmed implications of the model.

In the model, informal firms are less efficient than formal ones, but our analysis is silent concerning the optimal amount of informality. To discuss this question one must also model the cost of enforcement, presumably as a function of a firm's size, and the value of tax revenues.

Tax authorities in Brazil impose tax substitution hoping to increase compliance. Our model predicts a decrease in the interaction effect when tax substitution is imposed, but does not make any prediction concerning the effect of tax substitution on the level of informality. The firms in our sample that belong to tax substitution sectors tend to have more individuals as main clients and to be owned by less educated entrepreneurs; factors associated with less formality. Nonetheless, the difference in the rate of formalization between firms in tax substitution sectors and the other firms is 7.8 percentage points (with a standard error of .4), a very large difference when compared with the average level of 13.2% in our sample. This increased formalization probably reflects the criterium used by Brazilian tax authorities. Tax substitution is imposed when at some level in the chain the typical producer is a large firm which is then charged the estimated value added tax. If these large firms cannot afford to become informal and pay the estimated value added tax, smaller firms in the same production chain will face lower costs to formality than the typical small firm in Brazil and, for this reason, may often opt to become formal more often.

Appendix A: Proofs

Proof of Proposition 1

(i) is immediate from equations (1) and (2). To show that (ii) holds note that θ_d enters the definition of the profit function of formal firms exactly as an output price and hence, from the properties of profit functions with respect to output prices, we know that its derivative with respect to θ_d is proportional to $x_f(\theta_d, p)$ which goes to infinity as $\theta_d \to \infty$. Furthermore, the function $\Pi_i^d(\theta_d)$ is convex and, since supply functions of firms must slope up, if the choice, conditional on informality, of a firm of ability θ satisfies $x_i(\theta) = \overline{x}$ then the optimal choice conditional on informality, $x_i(\theta_d) = \overline{x}$ for $\theta_d \geq \theta$, and as a consequence, $\Pi_i^d(\theta_d)$ is linear for

$\theta_d \geq \theta$. In addition, whenever $x_i(\theta_d) < \overline{x}$, the informal firm's constraint is not binding. In this case, since $p_f \geq p_i$

$$\Pi_i^d = \varphi(p_i) > \varphi(p_f)$$

where $\varphi(p) = [\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}]\left(\frac{q\theta_d}{p^\alpha}\right)^{1/(1-\alpha)}$. Since

$$\Pi_f^d = \max\{\pi\varphi(p_f), \pi^{1/(1-\alpha)}\varphi(p_i)\}$$

then $\Pi_i^d(\theta_d) > \Pi_f^d(\theta_d)$, provided $\theta_d > 0$. ■

Proof of Proposition 2

(i) is obvious. For (ii), suppose first that it is optimal for the firm with manager of quality θ_d to buy the formal good. If $\pi x_f(\theta_d, p_f) < \overline{x}$, since

$$\theta_d(\pi x_f(\theta_d, p_f))^\alpha - \pi p_f x_f(\theta_d, p_f) \geq \pi(\theta_d x_f^\alpha(\theta_d, p_f) - p_f x_f(\theta_d, p_f)),$$

the firm would prefer to be in the informal sector and buy $\pi x_f(\theta_d, p_f)$ of formal inputs. If the firm bought the informal good and $\pi x_f(\theta_d, \frac{p_i}{\pi}) < \overline{x}$, since

$$q\theta_d(\pi x_f(\theta_d, \frac{p_i}{\pi}))^\alpha - \pi p_i x_f(\theta_d, \frac{p_i}{\pi}) \geq \pi q\theta_d x_f^\alpha(\theta_d, \frac{p_i}{\pi}) - \pi p_i x_f(\theta_d, \frac{p_i}{\pi}),$$

the firm would prefer to be in the informal sector and buy $\pi x_f(\theta_d, \frac{p_i}{\pi})$ of informal inputs. Consequently, $\pi x_f(\theta_d, p_f) \geq \overline{x}$ and $\overline{x} \geq x_i(\theta_d, p)$ for any θ_d. Furthermore, the entrepreneur $\overline{\theta}_d(p_i, p_f)$ must be indifferent between being formal or informal. Since informal (formal) entrepreneurs weakly prefer to buy from informal (formal) suppliers, we must have:

$$\overline{\theta}_d(p_i, p_f)\overline{x}^\alpha - p_i\overline{x} = \pi\left[\overline{\theta}_d(p_i, p_f)x_f^\alpha(\overline{\theta}_d(p_i, p_f)) - p_f x_f(\overline{\theta}_d(p_i, p_f))\right]. \tag{9}$$

In particular, choosing $\theta_d = \overline{\theta}_d(p_i, p_f)$ and $p = p_f$ shows the discontinuity at the marginal firms. ■

Existence of Equilibrium

Because of the possibility of indifference, we have supply and demand correspondences instead of functions. We will write $S(p_i, p_f)$ for the set of possible aggregate supply vectors

$(s_i(p_i, p_f), s_f(p_i, p_f))$ obtained from the choices of profit maximizing entrepreneurs in the upstream stage. If $p_i \neq \pi p_f$ the set $S(p_i, p_f)$ contains a single vector (s_i, s_f) given by

$$s_i = \int_0^{\frac{p_i \overline{y}}{\pi p_f}} \min\{\theta, \overline{y}\} g_u(\theta) d\theta \tag{10}$$

$$s_f = \int_{\frac{p_i \overline{y}}{\pi p_f}}^{\infty} \theta g_u(\theta) d\theta \tag{11}$$

If $\pi p_f = p_i = 0$ then $S(p_i, p_f) = \{0\}$. Finally when $\pi p_f = p_i \neq 0$ a point $(s_i, s_f) \in S(p_i, p_f)$ if there exists a $\overline{\theta}_u \leq \overline{y}$ such that:[24]

$$s_i = \int_0^{\overline{\theta}_u} \theta g_u(\theta) d\theta \tag{12}$$

$$s_f = \int_{\overline{\theta}_u}^{\infty} \theta g_u(\theta) d\theta \tag{13}$$

We write $X(p_i, p_f)$ for the set of possible aggregate demand vectors $(x_i(p_i, p_f), x_f(p_i, p_f))$ obtained from the choices of profit maximizing entrepreneurs in the downstream stage.

When $\pi p_f = p_i$ formal firms are indifferent between buying the formal or informal input, but informal firms prefer buying from informal firms. Hence we can allocate all formal firms with managers below a certain threshold to buying in the informal sector with the complement interval assigned to purchase in the formal sector.[25] In this case, a point $(x_i, x_f) \in X(p_i, p_f)$ if there exists a $\gamma \geq \overline{\theta}_d(p_i, p_f)$ such that:

$$x_i = \int_0^{\overline{\theta}_d(p_i, p_f)} x_i(\theta, p_i) g_d(\theta) d\theta + \int_{\overline{\theta}_d(p_i, p_f)}^{\gamma} x_f(\theta, \frac{p_i}{\pi}) g_d(\theta) d\theta \tag{14}$$

$$x_f = \int_{\gamma}^{\infty} x_f(\theta, p_f) g_d(\theta) d\theta \tag{15}$$

If $\pi p_f < p_i < p_f$ formal (informal) firms prefer to buy from formal (informal) firms.

[24] In principle we could assign any subset of entrepreneurs with productivity below $\overline{y}$ to the informal sector, but there is always an interval containing the origin that would produce exactly the same aggregate output.

[25] As before, these assignments can reproduce the demands realized by any arbitrary assignment of firms to each sector.

In this case, a point $(x_i, x_f) \in X(p_i, p_f)$ if :

$$x_i = \int_0^{\overline{\theta}_d(p_i,p_f)} x_i(\theta, p_i) g_d(\theta) d\theta \tag{16}$$

$$x_f = \int_{\overline{\theta}_d(p_i,p_f)}^{\infty} x_f(\theta, p_f) g_d(\theta) d\theta \tag{17}$$

If $p_f = p_i$ informal firms are indifferent, but formal firms prefer buying from formal firms. Hence we may assign informal firms arbitrarily to buying formal or informal inputs. In this case, a point $(x_i, x_f) \in X(p_i, p_f)$ if there exists $\gamma \leq \overline{\theta}_d(p_i, p_f)$ such that:

$$x_i = \int_0^{\gamma} x_i(\theta_d, p_i) g_d(\theta) d\theta \tag{18}$$

$$x_f = \int_{\gamma}^{\overline{\theta}_d(p_i,p_i)} x_i(\theta_d, p_i) g_d(\theta) d\theta + \int_{\overline{\theta}_d(p_i,p_i)}^{\infty} x_f(\theta_d, p_i) g_d(\theta) d\theta \tag{19}$$

An equilibrium is a vector (p_i, p_f) such that $\exists z \in X(p_i, p_f) \bigcap S(p_i, p_f)$. We will present the proof of existence of an equilibrium price in two steps. First we will set $p_i = \mu p_f$ with $\pi \leq \mu \leq 1$. For each μ we will show that there exists a unique $p_i(\mu)$ such that if $(p_i, p_f) = (p_i(\mu), \frac{p_i(\mu)}{\mu})$ then the sum of aggregate supply of the formal and informal intermediate goods equals the sum of aggregate demands. We then show that there exits a unique μ^* such that $(p_i(\mu^*), \frac{p_i(\mu^*)}{\mu^*})$ is an equilibrium. We will use the following preliminary result:

Lemma 1 *If $\pi p_f < p_i < p_f$ then $\overline{\theta}_d(p_i, p_f)$ decreases with p_i and it increases with p_f. Further, if $\pi \leq \mu \leq 1$ then, $\overline{\theta}_d(p_i, \frac{p_i}{\mu})$ increases with p_i.*

Proof: If $\pi p_f < p_i \leq p_f$ formal firms prefer to buy the formal good. Hence

$$\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f} = -\pi x_f(\theta_d, p_f) \tag{20}$$

Similarly, if $\pi p_f \leq p_i < p_f$, informal firms prefer to buy the informal good, and in an analogous fashion

$$\frac{\partial \Pi_i^d(\theta_d)}{\partial p_i} = -x_i(\theta_d, p_i) \tag{21}$$

This establishes the first part of the lemma, since increasing p_i reduces profits for informal firms and increasing p_f reduces profits for formal firms.

In order to sign the change in $\overline{\theta}_d(p_i, \frac{p_i}{\mu})$ we must establish the sign of:

$$\frac{1}{\mu}\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f} - \frac{\partial \Pi_i^d(\theta_d)}{\partial p_i}. \tag{22}$$

for the marginal firm. If this is negative, the difference in profits in the formal and informal sectors for the marginal firm decreases and more firms will become informal. If $\pi p_i < p_f < p_i$,

$$\frac{1}{\mu}\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f} - \frac{\partial \Pi_i^d(\theta_d)}{\partial p_i} = -\frac{\pi}{\mu}x_f(\theta_d, \frac{p_i}{\mu}) + x_i(\theta_d, p_i). \tag{23}$$

The marginal informal firm buys exactly $\overline{x}$. Hence, from Proposition 2

$$-\frac{\pi}{\mu}x_f(\overline{\theta}_d, \frac{p_i}{\mu}) + x_i(\overline{\theta}_d, p_i) \leq -\frac{\overline{x}}{\mu} + \overline{x} \leq 0$$

since we assume that $\mu \leq 1$ and the second part of the lemma follows.

The derivative $\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f}$ ($\frac{\partial \Pi_i^d(\theta_d)}{\partial p_i}$) is not well defined when $p_i = \pi p_f$ (resp. $p_i = p_f$), but it is easy to see that, in this case, the change in profit difference between formality and informality for the marginal firm still equals $-\frac{\pi}{\mu}x_f(\overline{\theta}_d, \frac{p_i}{\mu}) + x_i(\overline{\theta}_d, p_i)$. ■

We now return to the equilibrium analysis. For $\mu = \pi$ ($p_i = \pi p_f$) the sum of the aggregate supply always equals

$$\int_0^\infty \theta g_u(\theta)d\theta. \tag{24}$$

On the other hand, the sum of aggregate demands always equals

$$\int_0^{\overline{\theta}_d(p_i, \frac{p_i}{\pi})} x_i(\theta, p_i)g_d(\theta)d\theta + \int_{\overline{\theta}_d(p_i, \frac{p_i}{\pi})}^\infty x_f(\theta, \frac{p_i}{\pi})g_d(\theta)d\theta \tag{25}$$

It is easy to check that this last expression goes to zero as $p_i \to \infty$ and to ∞ as $p_i \to 0$. Furthermore, since demand of any type decreases with the price of the input, and, from Proposition 2 $x_f(\overline{\theta}_d, p_i/\pi)\overline{x}$, using the Lemma above it is immediate that aggregate demand is monotonically decreasing with p_i. Hence there exists a unique $p_i(\pi)$ for which the sum of supplies equal the sum of demands.

For $\pi < \mu \leq 1$, using expressions (10) and (11) we obtain that the sum of the aggregate supplies is:

$$\int_0^{\frac{\mu\overline{y}}{\pi}} \max\{\theta, \overline{y}\}g_u(\theta)d\theta + \int_{\frac{\mu\overline{y}}{\pi}}^\infty \theta g_u(\theta)d\theta. \tag{26}$$

On the other hand, using equations (16) and (17), the sum of the aggregate demands equals:

$$\int_0^{\overline{\theta}_d(p_i,\frac{p_i}{\mu})} x_i(\theta_d,p_i)g_d(\theta)d\theta + \int_{\overline{\theta}_d(p_i,p_i/\mu)}^{\infty} x_f(\theta_d,\frac{p_i}{\mu})g_d(\theta)d\theta. \tag{27}$$

Just as before, the result in the Lemma insures the monotonicity properties that yield the existence of a unique $p_i(\mu)$ that equates the sum of aggregate demands with that of aggregate supplies.

An increase in μ always decreases aggregate supply since it causes some firms in the upstream sector to switch from formal to informal. In addition, an increase in μ increases the demand by formal firms at each p_i and causes some firms to switch from informal to formal in the downstream sector. Thus, at each p_i, aggregate demand goes up. Hence $p_i(\mu)$ increases with μ.

The supply of the informal sector when $p_i = \pi p_f$ is some amount in the interval $[0, \int_0^{\overline{y}} \theta g_u(\theta)d\theta]$. The demand is some number in the interval $[\int_0^{\overline{\theta}_d(p_i,p_i/\pi)} x_i(\theta_d,p_i)g_d(\theta)d\theta, \int_0^{\overline{\theta}_d(p_i,p_i/\pi)} x_i(\theta_d,p_i)g_d(\theta)d\theta + \int_{\overline{\theta}_d(p_i,p_i/\pi)}^{\infty} x_f(\theta_d,p_i/\pi)g_d(\theta)d\theta]$. If these intervals overlap, at $p_i = p_i(\pi)/\pi$ then $(p_i(\pi), p_i(\pi)/\pi)$ is an equilibrium. This will happen whenever the tolerance for informality in the upstream sector ($\overline{y}$) is high enough.

If these intervals do not overlap the informal supply of the intermediate good must necessarily go up with μ. On the other hand, the informal demand at $(p_i(\mu), \frac{p_i(\mu)}{\mu})$ will go down since $p_i(\mu)$ goes up and the relative price of the formal good goes down. At $\mu = 1$, the supply of the informal good is $\int_0^{\frac{\overline{y}}{\pi}} \max\{\theta, \overline{y}\}g_u(\theta)d\theta$ whereas the demand is any number in the interval $[0, \int_0^{\overline{\theta}_d(p_i,p_f)} x_i(\theta_d,p_f)g_d(\theta)d\theta]$. Hence there always exists a unique μ^* such that $(p_i(\mu^*), p_i(\mu^*)/\mu^*)$ is an equilibrium.

Comparative Statics

Market equilibrium yields:

$$I(p_i, p_f, \bar{x}, \bar{y}) = \underbrace{\int_0^{\overline{\theta}_u(p_i,p_f,\bar{x},\bar{y})} \theta g_u(\theta) d\theta}_{\text{informal supply}} - \underbrace{\int_0^{\overline{\theta}_d(p_i,p_f,\bar{x},\bar{y})} x_i(\theta, p_i) g_d(\theta) d\theta}_{\text{informal demand}} = 0 \tag{28}$$

$$F(p_i, p_f, \bar{x}, \bar{y}) = \underbrace{\int_{\overline{\theta}_u(p_i,p_f,\bar{x},\bar{y})}^{\infty} \theta g_u(\theta) d\theta}_{\text{formal supply}} - \underbrace{\int_{\overline{\theta}_d(p_i,p_f,\bar{x},\bar{y})}^{\infty} x_f(\theta, p_f) g_d(\theta) d\theta}_{\text{formal demand}} = 0 \tag{29}$$

The matrix of partial derivatives with respect to (p_i, p_f) is:

$$\begin{aligned} A &= \begin{bmatrix} \frac{\partial I}{\partial p_i} & \frac{\partial I}{\partial p_f} \\ \frac{\partial F}{\partial p_i} & \frac{\partial F}{\partial p_f} \end{bmatrix} = \\ &= \begin{bmatrix} \underbrace{\begin{array}{c} \overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_i} - \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_i} \\ - \int_0^{\overline{\theta}_d(p_i,p_f)} \frac{\partial x_i(\theta,p_i)}{\partial p_i} g_d(\theta) d\theta \end{array}}_{>0} & \underbrace{\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_f} - \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_f}}_{<0} \\ \underbrace{-\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_i} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_i}}_{<0} & \underbrace{\begin{array}{c} -\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_f} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_f} \\ - \int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta,p_f)}{\partial p_f} g_d(\theta) d\theta \end{array}}_{>0} \end{bmatrix} \end{aligned}$$

where the signs are obtained from noticing first that

$$\frac{\partial \overline{\theta}_u}{\partial p_i} = \frac{\overline{y}}{\pi p_f} > 0 \text{ and } \frac{\partial \overline{\theta}_u}{\partial p_f} = -\frac{p_i \overline{y}}{\pi p_f^2} < 0.$$

In addition, since $\pi x_f(\overline{\theta}_d) \geq \overline{x}$ and $p_f > p_i$, $\pi x_f^{\alpha}(\overline{\theta}_d) > \overline{x}^{\alpha}$. Hence,

$$\frac{\partial \overline{\theta}_d}{\partial p_i} = \frac{\overline{x}}{\overline{x}^{\alpha} - \pi x_f(\overline{\theta}_d)} < 0,$$

$$\frac{\partial \overline{\theta}_d}{\partial p_f} = \frac{-\pi x_f(\overline{\theta}_d)}{\overline{x}^{\alpha} - \pi x_f(\overline{\theta}_d)} < 0$$

and

$$\left| \frac{\partial \overline{\theta}_d}{\partial p_f} \right| \geq \left| \frac{\partial \overline{\theta}_d}{\partial p_i} \right|.$$

The determinant of the above matrix is:

$$\begin{aligned} |A| &= \begin{pmatrix} \overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_i} - \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_i} \\ - \int_0^{\overline{\theta}_d(p_i,p_f)} \frac{\partial x_i(\theta,p_i)}{\partial p_i} g_d(\theta) d\theta \end{pmatrix} \times \begin{pmatrix} -\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_f} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_f} \\ - \int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta,p_f)}{\partial p_f} g_d(\theta) d\theta \end{pmatrix} \\ &\quad - \left(\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_f} - \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_f} \right) \times \left(-\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_i} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_i} \right) \end{aligned}$$

After cancelations and *ignoring the terms involving the integrals (which are positive)*, this expression becomes

$$\overline{\theta}_u g_u(\overline{\theta}_u) g_d(\overline{\theta}_d) \left[\underbrace{\left(x_f(\overline{\theta}_d) \frac{\partial \overline{\theta}_u}{\partial p_i} \frac{\partial \overline{\theta}_d}{\partial p_f} + \overline{x} \frac{\partial \overline{\theta}_u}{\partial p_f} \frac{\partial \overline{\theta}_d}{\partial p_i} \right)}_{\text{from } a_{11} \times a_{22}} - \underbrace{\left(x_f(\overline{\theta}_d) \frac{\partial \overline{\theta}_u}{\partial p_f} \frac{\partial \overline{\theta}_d}{\partial p_i} + \overline{x} \frac{\partial \overline{\theta}_u}{\partial p_i} \frac{\partial \overline{\theta}_d}{\partial p_f} \right)}_{\text{from } a_{12} \times a_{21}} \right] =$$

$$= \underbrace{\overline{\theta}_u g_u(\overline{\theta}_u) g_d(\overline{\theta}_d)}_{>0} \left[\underbrace{\left(x_f(\overline{\theta}_d) - \overline{x} \right)}_{>0} \left(\frac{\partial \overline{\theta}_u}{\partial p_i} \frac{\partial \overline{\theta}_d}{\partial p_f} - \frac{\partial \overline{\theta}_u}{\partial p_f} \frac{\partial \overline{\theta}_d}{\partial p_i} \right) \right]$$

Hence, as long as

$$\left(\frac{\partial \overline{\theta}_u}{\partial p_i} \frac{\partial \overline{\theta}_d}{\partial p_f} - \frac{\partial \overline{\theta}_u}{\partial p_f} \frac{\partial \overline{\theta}_d}{\partial p_i} \right) > 0$$

the determinant is positive. Also,

$$\frac{\partial \overline{\theta}_u}{\partial p_i} p_i + \frac{\partial \overline{\theta}_u}{\partial p_f} p_f = 0$$

and since $p_f > p_i$,

$$\left| \frac{\partial \overline{\theta}_u}{\partial p_i} \right| > \left| \frac{\partial \overline{\theta}_u}{\partial p_f} \right|.$$

Hence, since we showed that

$$\left| \frac{\partial \overline{\theta}_d}{\partial p_f} \right| \geq \left| \frac{\partial \overline{\theta}_d}{\partial p_i} \right|$$

we may conclude that the determinant is positive.

By definition,

$$\Delta \Pi^d = \Pi_i^d(\overline{\theta}_d, p_i, p_f) - \Pi_f^d(\overline{\theta}_d, p_i, p_f) = 0. \tag{30}$$

We can then apply the Implicit Function Theorem to get:

$$\frac{\partial \overline{\theta}_d / \partial p_i}{\partial \overline{\theta}_d / \partial p_f} = \frac{\partial \Pi_i / \partial p_i - \partial \Pi_f / \partial p_i}{\partial \Pi_i / \partial p_f - \partial \Pi_f / \partial p_f} = \frac{\partial \Pi_i / \partial p_i}{-\partial \Pi_f / \partial p_f}$$

because equation (3) is active in the first argument and equation (4), in the second as previously noted. We can now appeal to the Envelope Theorem, which delivers

$$\frac{\partial \Pi_i^d}{\partial p_i} = -\overline{x}$$

and

$$\frac{\partial \Pi_f^d}{\partial p_f} = -x_f(\overline{\theta}_d)\pi.$$

By Proposition 2, $x_f(\overline{\theta}_d)\pi \geq \overline{x}$ and

$$\left|\frac{\partial \overline{\theta}_d}{\partial p_f}\right| \geq \left|\frac{\partial \overline{\theta}_d}{\partial p_i}\right|.$$

This allows us to compute the inverse of A as

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

where the adjoint matrix is given by

$$\begin{aligned}
\text{adj}(A) &= \begin{bmatrix} \frac{\partial F}{\partial p_f} & -\frac{\partial I}{\partial p_f} \\ -\frac{\partial F}{\partial p_i} & \frac{\partial I}{\partial p_i} \end{bmatrix} = \\
&= \begin{bmatrix} \underbrace{\begin{array}{c} -\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_f} + x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_f} \\ -\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta \end{array}}_{>0} & \underbrace{-\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_f} + \overline{x}g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_f}}_{>0} \\ \underbrace{\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_i} - x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_i}}_{>0} & \underbrace{\begin{array}{c} \overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_i} - \overline{x}g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_i} \\ -\int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta \end{array}}_{>0} \end{bmatrix}
\end{aligned}$$

Comparative Statics with respect to $\overline{y}$. To study the sensitivity of the equilibrium to $\overline{y}$, let $\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial \overline{y}} = c > 0$ and notice that

$$\begin{bmatrix} \frac{\partial I}{\partial \overline{y}} \\ \frac{\partial F}{\partial \overline{y}} \end{bmatrix} = \begin{bmatrix} c \\ -c \end{bmatrix}$$

Consequently,

$$\begin{bmatrix} \frac{dp_i}{d\overline{y}} \\ \frac{dp_f}{d\overline{y}} \end{bmatrix} = -\frac{1}{|A|}\text{adj}(A)\begin{bmatrix} c \\ -c \end{bmatrix} = \frac{1}{|A|}\text{adj}(A)\begin{bmatrix} -c \\ c \end{bmatrix}$$

Hence

$$\frac{dp_i}{d\overline{y}} = -c\frac{(a_{22}+a_{12})}{|A|} = -\frac{c}{|A|}\left((x_f(\overline{\theta}_d)-\overline{x})g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_f} - \int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta\right) < 0$$

and

$$\frac{dp_f}{d\overline{y}} = c\frac{(a_{21}+a_{11})}{|A|} = \frac{c}{|A|}\left((x_f(\overline{\theta}_d)-\overline{x})g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_i} - \int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta\right)$$

Hence:

$$\begin{aligned}\frac{d\overline{\theta}_d}{d\overline{y}} &= \frac{\partial\overline{\theta}_d}{\partial p_i}\frac{dp_i}{d\overline{y}} + \frac{\partial\overline{\theta}_d}{\partial p_f}\frac{dp_f}{d\overline{y}} = \\ &= \frac{c}{|A|}\left(\frac{\partial\overline{\theta}_d}{\partial p_i}\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta - \frac{\partial\overline{\theta}_d}{\partial p_f}\int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta\right) > 0.\end{aligned}$$

For the threshold in the upstream stage,

$$\frac{d\overline{\theta}_u}{d\overline{y}} = \underbrace{\underbrace{\frac{\partial\overline{\theta}_u}{\partial p_i}}_{>0}\underbrace{\frac{dp_i}{d\overline{y}}}_{<0} + \frac{\partial\overline{\theta}_u}{\partial p_f}\frac{dp_f}{d\overline{y}}}_{<0\ (indirect)} + \underbrace{\frac{\partial\overline{\theta}_u}{\partial\overline{y}}}_{>0\ (direct)}$$

In fact,

$$\begin{aligned}\frac{\partial\overline{\theta}_u}{\partial p_i}\frac{dp_i}{d\overline{y}} + \frac{\partial\overline{\theta}_u}{\partial p_f}\frac{dp_f}{d\overline{y}} &= \frac{c}{|A|}\Bigg(\underbrace{\left(\frac{\partial\overline{\theta}_u}{\partial p_f}\frac{\partial\overline{\theta}_d}{\partial p_i} - \frac{\partial\overline{\theta}_u}{\partial p_i}\frac{\partial\overline{\theta}_d}{\partial p_f}\right)(x_f(\overline{\theta}_d)-\overline{x})g_d(\overline{\theta}_d)}_{<0} \\ &\quad + \underbrace{\frac{\partial\theta_u}{\partial p_i}\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta}_{<0} - \underbrace{\frac{\partial\overline{\theta}_u}{\partial p_f}\int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta}_{>0}\Bigg) < 0\end{aligned}$$

where the first term above is negative since $\frac{\partial\overline{\theta}_u}{\partial p_f}\frac{\partial\overline{\theta}_d}{\partial p_i} = \left|\frac{\partial\overline{\theta}_u}{\partial p_f}\right|\left|\frac{\partial\overline{\theta}_d}{\partial p_i}\right| < \left|\frac{\partial\overline{\theta}_u}{\partial p_i}\right|\left|\frac{\partial\overline{\theta}_d}{\partial p_f}\right| = \frac{\partial\overline{\theta}_u}{\partial p_i}\frac{\partial\overline{\theta}_d}{\partial p_f}$. It is unclear how this expression compares to $\frac{\partial\overline{\theta}_u}{\partial\overline{y}} = \frac{p_i}{\pi p_f}$ or what is the sign of $\frac{d\overline{\theta}_u}{d\overline{y}}$.

Comparative Statics with respect to $\overline{x}$. Let θ^* denote the θ at which an informal entrepreneur in the downstream sector becomes constrained by $\overline{x}$. Notice that

$$\begin{aligned}I(p_i,p_f) &= \underbrace{\int_0^{\overline{\theta}_u}\theta g_u(\theta)d\theta}_{\text{informal supply}} - \underbrace{\int_0^{\overline{\theta}_d(p_i,p_f)}x_i(\theta,p_i)g_d(\theta)d\theta}_{\text{informal demand}} = \\ &= \int_0^{\overline{\theta}_u}\theta g_u(\theta)d\theta - \int_0^{\theta^*}x_i(\theta,p_i)g_d(\theta)d\theta - \overline{x}(G(\overline{\theta}_d)-G(\theta^*))\end{aligned}$$

Hence,

$$\begin{bmatrix} \frac{\partial I}{\partial \overline{x}} \\ \frac{\partial F}{\partial \overline{x}} \end{bmatrix} = \begin{bmatrix} -(G(\overline{\theta}_d) - G(\theta^*)) - \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial \overline{x}} \\ x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial \overline{x}} \end{bmatrix}$$

As was the case before, we need to compute

$$\begin{aligned} \frac{dp_i}{d\overline{x}} &= \frac{1}{|A|} \Bigg[\underbrace{\left(G(\overline{\theta}_d) - G(\theta^*)\right)}_{>0} \underbrace{\left(- \overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_f} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_f} - \int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta, p_f)}{\partial p_f} g_d(\theta) d\theta \right)}_{>0} \\ &\quad + \underbrace{g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial \overline{x}} \frac{\partial \overline{\theta}_d}{\partial p_f} \overline{\theta}_u g_u(\overline{\theta}_u)(x_f(\overline{\theta}_d) - \overline{x}) - \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial \overline{x}} \int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta, p_f)}{\partial p_f} g_d(\theta) d\theta}_{>0} \Bigg] > 0 \end{aligned}$$

and

$$\begin{aligned} \frac{dp_f}{d\overline{x}} &= \frac{1}{|A|} \Bigg[\left(G(\overline{\theta}_d) - G(\theta^*) + \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial \overline{x}} \right) \left(\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_i} - x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_i} \right) \\ &\quad - \left(\overline{\theta}_u g_u(\overline{\theta}_u) \frac{\partial \overline{\theta}_u}{\partial p_i} - \overline{x} g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial p_i} - \int_0^{\overline{\theta}_d(p_i,p_f)} \frac{\partial x_i(\theta, p_i)}{\partial p_i} g_d(\theta) d\theta \right) x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial \overline{x}} \Bigg] \end{aligned}$$

Consequently,

$$\begin{aligned} \frac{d\overline{\theta}_u}{d\overline{x}} &= \frac{\partial \overline{\theta}_u}{\partial p_i} \frac{dp_i}{d\overline{x}} + \frac{\partial \overline{\theta}_u}{\partial p_f} \frac{dp_f}{d\overline{x}} = \\ &= \frac{1}{|A|} \Bigg[\left(G(\overline{\theta}_d) - G(\theta^*)\right) x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \left(\frac{\partial \overline{\theta}_d}{\partial p_f} \frac{\partial \overline{\theta}_u}{\partial p_i} - \frac{\partial \overline{\theta}_d}{\partial p_i} \frac{\partial \overline{\theta}_u}{\partial p_f} \right) \\ &\quad - \frac{\partial \overline{\theta}_u}{\partial p_i} \int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta, p_f)}{\partial p_f} g_d(\theta) d\theta \left(G(\overline{\theta}_d) - G(\theta^*) + \overline{x} g_d(\overline{\theta}_d) \right) \\ &\quad + \frac{\partial \overline{\theta}_u}{\partial p_f} x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \frac{\partial \overline{\theta}_d}{\partial \overline{x}} \int_0^{\overline{\theta}_d(p_i,p_f)} \frac{\partial x_i(\theta, p_i)}{\partial p_i} g_d(\theta) d\theta \Bigg] \\ &> 0 \end{aligned}$$

Comparative Statics with respect to π. Notice that

$$\begin{bmatrix} \frac{\partial I}{\partial \pi} \\ \frac{\partial F}{\partial \pi} \end{bmatrix} = \begin{bmatrix} k \\ -k + \frac{\partial \overline{\theta}_d}{\partial \pi} (x_f(\overline{\theta}_d) - \overline{x}) g_d(\overline{\theta}_d) \end{bmatrix}$$

where $k = \frac{\partial \overline{\theta}_u}{\partial \pi}\overline{\theta}_u g_u(\overline{\theta}_u) - \frac{\partial \overline{\theta}_d}{\partial \pi}\overline{x} g_d(\overline{\theta}_d)$. Then,

$$\begin{aligned}
\begin{bmatrix} \frac{dp_i}{d\pi} \\ \frac{dp_f}{d\pi} \end{bmatrix} &= -\frac{1}{|A|}\text{adj}(A)\begin{bmatrix} k \\ -k + \frac{\partial \overline{\theta}_d}{\partial \pi}(x_f(\overline{\theta}_d) - \overline{x})g_d(\overline{\theta}_d) \end{bmatrix} = \\
&= \frac{1}{|A|}\text{adj}(A)\begin{bmatrix} -k \\ k - \frac{\partial \overline{\theta}_d}{\partial \pi}(x_f(\overline{\theta}_d) - \overline{x})g_d(\overline{\theta}_d) \end{bmatrix}
\end{aligned}$$

and consequently,

$$\frac{dp_i}{d\pi} = -k\frac{(a_{22} + a_{12})}{|A|} + \frac{a_{12}\frac{\partial \overline{\theta}_d}{\partial \pi}(x_f(\overline{\theta}_d) - \overline{x})g_d(\overline{\theta}_d)}{|A|}$$

and

$$\frac{dp_f}{d\pi} = k\frac{(a_{21} + a_{11})}{|A|} - \frac{a_{11}\frac{\partial \overline{\theta}_d}{\partial \pi}(x_f(\overline{\theta}_d) - \overline{x})g_d(\overline{\theta}_d)}{|A|}.$$

Let $\Gamma_1 = \int_0^{\overline{\theta}_d(p_i,p_f)} \frac{\partial x_i(\theta,p_i)}{\partial p_i} g_d(\theta)d\theta$ and $\Gamma_2 = \int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta,p_f)}{\partial p_f} g_d(\theta)d\theta$. The total derivative (multiplied by $|A|$) is then

$$\begin{aligned}
|A|\frac{d\overline{\theta}_d}{d\pi} &= |A|\left[\frac{\partial \overline{\theta}_d}{\partial p_i}\frac{dp_i}{d\pi} + \frac{\partial \overline{\theta}_d}{\partial p_f}\frac{dp_f}{d\pi} + \frac{\partial \overline{\theta}_d}{\partial \pi}\right] = \\
&= \left(\underbrace{\frac{\partial \overline{\theta}_d}{\partial p_i}\Gamma_2 - \frac{\partial \overline{\theta}_d}{\partial p_f}\Gamma_1}_{>0}\right)\underbrace{\frac{\partial \overline{\theta}_u}{\partial \pi}\overline{\theta}_u g_u(\overline{\theta}_u)}_{<0} \\
&\quad + \underbrace{\frac{\partial \overline{\theta}_d}{\partial \pi}}_{<0}\left(\underbrace{\Gamma_1\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_f} + \Gamma_1\Gamma_2 - \Gamma_2\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_i}}_{>0}\right) < 0
\end{aligned}$$

Hence, both the direct and total effect of taxes on the cutoff are negative. Similarly

$$\begin{aligned}
|A|\frac{d\overline{\theta}_u}{d\pi} &= |A|\left[\frac{\partial \overline{\theta}_u}{\partial p_i}\frac{dp_i}{d\pi} + \frac{\partial \overline{\theta}_u}{\partial p_f}\frac{dp_f}{d\pi} + \frac{\partial \overline{\theta}_u}{\partial \pi}\right] = \\
&= \underbrace{\Gamma_1 x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)}_{<0}\left(\underbrace{\frac{\partial \overline{\theta}_d}{\partial \pi}\frac{\partial \overline{\theta}_u}{\partial p_f} - \frac{\partial \overline{\theta}_u}{\partial \pi}\frac{\partial \overline{\theta}_d}{\partial p_f}}_{>0}\right) \\
&\quad + \underbrace{\Gamma_2\overline{x}g_d(\overline{\theta}_d)}_{<0}\left(\underbrace{\frac{\partial \overline{\theta}_d}{\partial p_i}\frac{\partial \overline{\theta}_u}{\partial \pi} - \frac{\partial \overline{\theta}_d}{\partial \pi}\frac{\partial \overline{\theta}_u}{\partial p_i}}_{>0}\right) \\
&\quad + \underbrace{\frac{\partial \overline{\theta}_u}{\partial \pi}\Gamma_1\Gamma_2}_{<0} < 0
\end{aligned}$$

and again both the direct and the total effect of taxes on the cutoff are negative.

References

[1] Almeida, R. and Z. Bianchini, 1998, "Aspectos de Amostragem da Pesquisa de Economia Informal Urbana", Texto para Discusso No.89, IBGE.

[2] Almeida, R. and P. Carneiro, 2005, "Enforcement of Regulation, Informal Labor and Firm Performance", IZA Discussion Paper No.1759.

[3] Bound, J., C. Brown and N. Mathiowetz, 2005, "Measurement Error in Survey Data" **in** *Handbook of Econometrics*, V.5., J. Heckman and E. Leamer (eds.), Amsterdam: North-Holland.

[4] Card, D. (1995) "Using Geographical Variation in College Proximity to Estimate the Return to Schooling" **in** *Aspects of Labour Market Behavior: Essays in Honour of John Vanderkamp*, ed. L.N.Christophides, E.K. Grant and R. Swedinsky, Toronto: University of Toronto Press, pp.201-222.

[5] Dabla-Norris, E., M. Gradstein and G. Inchauste, 2008, "What Causes Firms to Hide Output? The Determinants of Informality", *Journal of Development Economics*.

[6] DiPasquale, D. and E. Glaeser, 1999, "Incentives and Social Capital: Are Homeowners Better Citizens?", *Journal of Urban Economics*.

[7] Emran, M. and J. Stiglitz, 2005, "On Selective Indirect Tax Reform in Developin Countries", *Journal of Public Economics*, **89**, pp.599-623.

[8] Fajnzylber, P., W. Maloney and G. Montes Rojas, 2007, "Does Formality Improve Micro-Firm Performance? Quasi-Experimental Evidence from the Brazilian SIMPLES Program", World Bank Working Paper.

[9] Fortin, B., N. Marceau and L. Savard, 1997, "Taxation, Wage Controls and the Informal Sector", *Jounal of Public Economics*, **66**, pp.293-312.

[10] Friedman, E., S. Johnson, D. Kaufmann and P. Zoido-Lobaton, 2000, "Dodging the grabbing hand: the determinants of unofficial activity in 69 countries", *Journal of Public Economics*, **76**, pp.459-493.

[11] Heckman, J., 1978, "Dummy Endogenous Variable in a Simultaneous Equation System", *Econometrica*, **46**, pp.931-959.

[12] Intituto Brasileiro de Geografia e Estatística (IBGE), 2003, *Sistema de Contas Nacionais - Brasil*, Série Relatórios Metodológicos, V.24.

[13] Johnson, S., D. Kaufmann and A. Shleifer, 1997, "The Unofficial Economy in Transition", *Brookings Papers on Economic Activity*, No.2.

[14] Junqueira, J. and J. Monteiro, 2005, "Outgoing the shadows: estimating the impact of bureaucracy and tax reduction on formality", Working Paper, PUC-Rio.

[15] Loayza, N., 1996, "The Economics of the Informal Sector: A Simple Model and Some Empirical Evidence from Latin America", *Carnegie-Rochester Conference Series on Public Policy*, **45**, pp.129-162.

[16] Lucas, R., 1978, "On the Size Distribution of Business Firms", *Bell Journal of Economics*, **9**, pp.508-523.

[17] Maloney, W. and J.N. Mendez, 2004, "Measuring the Impact of Minimum Wages: Evidence from Latin America", **in** J. Heckman and C. Pagés (eds.), *Law and Employment: Lessons from Latin America and the Caribbean*, Chicago: The University of Chicago Press.

[18] de Paula, A. and J. Scheinkman, 2008, "The Informal Sector", NBER Working Paper 13486.

[19] Rausch, J.E., 1991, "Modelling the Informal Sector Formally", *Journal of Development Economics*, **35**, pp.33-47.

[20] Ribeiro, A.P., 1997, "Substituição Tributária para Frente", *Revista CEJ*, Centro de Estudos Judiciários (Conselho de Justiça Federal), No.3, Brasília: CEJ.

[21] Rivers, D. and Q. Vuong, 1988, "Limited Information Estimators and Exogeneity Tests for Simultaneous Probit Models", *Journal of Econometrics*, **39**, pp.347-366.

[22] Schenk, A. and O. Oldman, 2001, *Value Added Taxes: A Comparative Approach in Theory and Practive*, Ardsley: Transnational Publishers.

[23] Schneider, F. and D. Enste, 2000, "Shadow Economies: Size, Causes and Consequences", *Journal of Economic Literature*, **38**, pp.77-114.

[24] Straub, S., 2005, "Informal Sector: The Credit Market Channel", *Journal of Development Economics*, **78**, pp.299-321.

[25] Tait, A., 1991, "Value-Added Tax: Administrative and Policy Issues", Occasional Paper 88, IMF.

[26] ten Raa, T., 2005, *The Economics of Input-Output Analysis*, Cambridge: Cambridge University Press.

Table 1: Variable Description

Variable	Description	Obs	Mean	Std. Dev.
tax reg	1 = Tax Registration	48308	0.130	0.337
tax sub	1 = Tax Substitution	48314	0.179	0.384
large cl	1 = Large Client	48306	0.041	0.199
small cl	1 = Small Client	48306	0.070	0.256
outside hh	1 = Outside Household	48310	0.640	0.480
# employees	Number of Employees	48314	1.473	1.044
revenue	Revenue in Oct/2003 (R$ 1,000)	47570	2.077	6.276
other job	1 = Owner has Other Job	48288	0.125	0.330
bank loan	1 = Bank Loan	48292	0.062	0.241
education	Education Level (Owner)	48253	4.367	1.884
age	Age (Owner)	48314	41.026	12.313
gender	Gender (Owner)	48312	0.644	0.479
homeowner × # rooms	Homeowner × Number of Rooms	48040	4.889	3.316
log inst	Log of Installations (R$)	39818	5.830	1.764
log inv	Log of Investments (R$)	8119	6.504	2.161
profit	Profit in Oct/2003 (R$ 1,000)	44707	0.771	4.514
sup enf	Supplier Enforcement	47846	0.012	0.010
cl enf	Client Enforcement	47846	0.010	0.010
logwage	Log of Mean Wage (R$ 1,000)	6491	-1.831	0.855
supplierformal	Formalization among Suppliers	47749	0.159	0.035
clientformal	Formalization among Clients	47846	0.127	0.039

Table 2: Education

1 =	No education
2 =	Reads and writes
3 =	Some primary education
4 =	Graduated primary school
5 =	Some secondary education
6 =	Graduated secondary school
7 =	Some College education
8 =	Graduated College

Table 3: Economic Sector

	Freq.	%	Description
1	5,130	10.62	Transformation and Mineral Extraction Industry
2	7,000	14.49	Construction
3	14,675	30.37	Retail and Repair Services
4	4,104	8.49	Lodging and Food Services
5	4,451	9.21	Transportation and Communications
6	3,125	6.47	Real Estate and Services
7	2,937	6.08	Education, Health and Social Services
8	4,693	9.71	Other Collective, Social and Personal Services
9	2,199	4.55	Other Activities

Table 4: Correlation Matrix

	tax reg	tax sub	large cl	small cl	outsd hh	# empl	rev	other job	bk loan
tax sub	0.01	1.00							
large cl	0.12	-0.06	1.00						
small cl	0.06	-0.16	-0.10	1.00					
outside hh	0.09	-0.05	0.01	-0.03	1.00				
# employees	0.35	0.02	0.08	0.10	0.08	1.00			
revenue	0.29	0.05	0.17	0.03	0.09	0.30	1.00		
other job	-0.01	-0.06	-0.02	-0.06	0.01	0.04	-0.01	1.00	
bank loan	0.11	-0.02	0.01	0.00	-0.03	0.04	0.07	0.01	1.00
education	0.30	-0.16	0.08	0.10	0.06	0.17	0.13	0.24	0.06
age	0.03	0.02	0.01	0.01	0.00	0.04	0.05	0.01	0.00
gender	-0.07	0.05	0.06	0.08	0.10	-0.03	0.03	0.01	-0.06
h owner × # rooms	0.15	-0.06	0.05	0.03	0.05	0.08	0.10	0.06	-0.02
log inst	0.55	0.16	0.16	0.06	0.10	0.50	0.52	-0.05	0.15
log inv	0.38	0.01	0.12	0.09	0.07	0.29	0.27	0.06	0.16
profit	0.04	-0.08	0.07	0.04	0.05	0.06	0.37	-0.02	0.02
sup enf	-0.02	0.03	-0.04	-0.03	-0.11	0.03	0.01	0.02	0.04
cl enf	0.09	-0.06	0.03	0.16	-0.06	0.02	0.09	-0.01	0.04
log wage	0.33	-0.08	0.19	0.13	0.12	0.24	0.30	-0.08	0.05
cl form	0.31	-0.04	0.10	0.15	0.05	0.08	0.20	-0.07	0.05
sup form	0.04	-0.10	0.09	0.19	0.04	-0.01	0.03	-0.03	-0.05

Correlation Matrix (cont'd)

	education	age	gender	h owner × # rooms	log inst	log inv	profit	sup enf	cl enf
age	-0.12	1.00							
gender	-0.20	0.05	1.00						
h owner × # rooms	0.23	0.17	-0.06	1.00					
log inst	0.25	0.05	-0.02	0.12	1.00				
log inv	0.34	-0.02	-0.07	0.13	0.55	1.00			
profit	0.07	0.05	0.00	0.04	-0.04	0.07	1.00		
sup enf	-0.11	-0.04	-0.03	-0.01	0.05	0.01	-0.06	1.00	
cl enf	0.02	0.00	0.05	0.04	0.15	0.12	-0.02	0.63	1.00
log wage	0.21	0.09	0.08	0.09	0.55	0.37	0.05	-0.15	-0.03
cl form	0.11	0.03	0.07	0.08	0.35	0.21	0.01	0.10	0.47
sup form	0.10	0.01	0.13	0.07	0.01	0.04	0.05	-0.11	0.23

Correlation Matrix (cont'd)

	log wage	cl form
cl form	0.09	1.00
sup form	0.08	0.54

Table 5: Probit Estimates

Dep. Var. = tax reg	**Coeff.** (Std. Err.)	**Marg. Eff.**	**Coeff.** (Std. Err.)	**Marg. Eff.**
outside hh	0.174**	0.021	0.178**	0.020
	(0.024)		(0.024)	
# employees	0.407**	0.052	0.419**	0.050
	(0.012)		(0.012)	
revenue	0.051**	0.006	0.044**	0.005
	(0.005)		(0.004)	
bank loan	0.379**	0.062	0.348**	0.052
	(0.033)		(0.034)	
other job	-0.242**	-0.027	-0.275**	-0.028
	(0.033)		(0.033)	
education	0.192**	0.0246	0.175**	0.021
	(0.006)		(0.006)	
age	0.036**	0.005	0.041**	0.005
	(0.004)		(0.005)	
age^2	0.000**	0.000	0.000**	0.000
	(0.000)		(0.000)	
gender	0.148**	0.018	0.215**	0.025
	(0.020)		(0.021)	
homeowner × # rooms	0.030**	0.004	0.028**	0.003
	(0.003)		(0.003)	
Sector Dummies	Yes		No	
Output Coeff.	No		Yes	
State Dummies	Yes		Yes	
N	47201		46749	
Pseudo-R^2	0.3634		0.3767	
$\chi^2_{(44)}$	5435.96			

1. Significance levels : † : 10% ∗ : 5% ∗∗ : 1%
2. Standard errors clustered by urban sector.

Table 6: Investment, Installations and Profits

Dep. Var. =	log inv per worker	log inst per worker	profit
	Coefficient	**Coefficient**	**Coefficient**
	(Std. Err.)	(Std. Err.)	(Std. Err.)
tax reg	0.649**	0.800**	0.680**
	(0.062)	(0.033)	(0.138)
outside hh	0.204**	0.289**	0.186**
	(0.045)	(0.017)	(0.053)
bank loan	0.737**	0.626**	0.061
	(0.059)	(0.026)	(0.116)
other job	-0.276**	-0.257**	-0.180†
	(0.058)	(0.022)	(0.099)
education	0.240**	0.127**	0.178**
	(0.013)	(0.005)	(0.016)
age	0.031**	0.067**	0.029**
	(0.010)	(0.003)	(0.008)
age^2	0.000**	-0.001**	0.000*
	(0.000)	(0.000)	(0.000)
gender	0.509**	0.355**	0.264**
	(0.044)	(0.015)	(0.044)
homeowner × # rooms	0.030**	0.020**	0.017*
	(0.006)	(0.002)	(0.008)
revenue	0.018**	0.062**	
	(0.005)	(0.006)	
# employees			0.408**
			(0.054)
N	7954	39176	44368
R^2	0.330	0.356	0.038
$F_{(44,\cdot)}$	68.51	300.16	20.82

1. Significance levels : † : 10% * : 5% ** : 1%
2. The regressions also control for state and sector.
3. Standard errors are clustered by urban sector.

Table 7: Probit Estimates (Chain Effects)

Dep. Var. =	**Coeff.**	**Marg. Eff.**	**Coeff.**	**Marg. Eff.**	**Coeff.**	**Marg. Eff.**
tax reg	(Std. Err.)		(Std. Err.)		(Std. Err.)	
large cl	0.373**	0.061				
	(0.049)					
small cl	0.168**	0.024				
	(0.035)					
supplier formal			2.803**	0.358		
			(0.294)			
client formal					4.976**	0.618
					(0.296)	
outside hh	0.179**	0.022	0.167**	0.021	0.161**	0.02
	(0.024)		(0.024)		(0.024)	
# employees	0.407**	0.052	0.407**	0.052	0.421**	0.052
	(0.012)		(0.012)		(0.012)	
revenue	0.049**	0.006	0.050**	0.006	0.046**	0.006
	(0.005)		(0.005)		(0.004)	
bank loan	0.381**	0.062	0.382**	0.062	0.361**	0.057
	(0.033)		(0.034)		(0.034)	
other job	-0.229**	-0.026	-0.238**	-0.026	-0.234**	-0.025
	(0.033)		(0.033)		(0.033)	
education	0.186**	0.024	0.184**	0.024	0.186**	0.023
	(0.006)		(0.006)		(0.006)	
age	0.035**	0.005	0.035**	0.005	0.037**	0.005
	(0.005)		(0.005)		(0.005)	
age^2	0.000**	0.000	0.000**	0.000	0.000**	0.000
	(0.000)		(0.000)		(0.000)	
gender	0.125**	0.015	0.114**	0.014	0.134**	0.017
	(0.020)		(0.021)		(0.020)	
homeowner × # rooms	0.030**	0.004	0.029**	0.004	0.029**	0.004
	(0.003)		(0.003)		(0.003)	
N	47196		46654		46749	
Pseudo-R^2	0.3664		0.3657		0.3722	
χ^2	5491.36		5469.05		5597.23	

1. Significance levels : † : 10% ∗ : 5% ∗∗ : 1%
2. Standard errors clustered by urban sector.
3. The regressions also control for state and sector.

Table 8: IV Probit Estimates (Chain Effects)

Dep. Var. = tax reg	**Non-IV Coeff.** (Std. Err.)	**IV Coeff.** (Std. Err.)	**First Stage (IV)** Dep. Var. = lscl	**Coeff.** (Std. Err.)
lscl	0.296**	0.447**	educurbsec	0.096**
	(0.029)	(0.088)		(0.012)
outside hh	0.213**	0.213**	outside hh	0.007
	(0.024)	(0.024)		(0.021)
# employees	0.482**	0.478**	# employees	0.076**
	(0.010)	(0.010)		(0.012)
bank loan	0.423**	0.405**	bank loan	0.074*
	(0.033)	(0.033)		(0.034)
other job	-0.252**	-0.246*	other job	-0.174**
	(0.033)	(0.033)		(0.029)
education	0.204**	0.200**	education	0.089**
	(0.006)	(0.006)		(0.006)
age	0.038**	0.038**	age	0.006†
	(0.004)	(0.004)		(0.004)
age^2	-0.000**	-0.000**	age^2	0.000*
	(0.000)	0.000		(0.000)
gender	0.164**	0.150**	gender	0.498**
	(0.020)	(0.021)		(0.003)
homeowner × # rooms	0.033**	0.033**	homeowner × # rooms	-0.005†
	(0.003)	(0.003)		(0.003)
N	47,930	47,196		

1. Significance levels : † : 10% * : 5% ** : 1%
2. The regressions also control for state and sector.
3. The second regression uses the average level of education in the urban sector as an instrument.
4. IV results obtained as bivariate probit.
5. Standard errors clustered by urban sector.

Table 9: IV Probit Estimates (Chain Effects)

Dep. Var. = large cl	**Non-IV Coeff.** (Std. Err.)	**IV Coeff.** (Std. Err.)	**First Stage (IV)** Dep. Var. = tax reg	**Coeff.** (Std. Err.)
tax reg	0.380**	2.620*	nearestbank	-0.050 $\times 10^{-3\dagger}$
	(0.032)	(1.056)		(0.027 $\times 10^{-3}$)
outside hh	0.067*	-0.063	outside hh	0.231**
	(0.028)	(0.081)		(0.022)
# employees	0.038**	-0.283†	# employees	0.475**
	(0.010)	(0.171)		(0.008)
bank loan	0.030	-0.180	bank loan	0.398**
	(0.042)	(0.126)		(0.032)
other job	-0.206**	-0.046	other job	-0.247**
	(0.039)	(0.132)		(0.033)
education	0.093**	-0.022	education	0.197**
	(0.007)	(0.076)		(0.006)
age	0.010†	-0.009	age	0.039**
	(0.006)	(0.010)		(0.005)
age^2	-0.000*	0.000	age^2	-0.000**
	(0.000)	0.000		(0.000)
gender	0.376**	0.189	gender	0.187**
	(0.029)	(0.170)		(0.022)
homeowner $\times$ # rooms	0.005	-0.010	homeowner $\times$ # rooms	0.031**
	(0.003)	(0.011)		(0.003)
N	47,550	34,284		

1. Significance levels : $\dagger$: 10% $*$: 5% $**$: 1%
2. The regressions also control for state and sector.
3. The second regression uses the distance to the nearest bank as an instrument for firm formalization.
4. IV results obtained as bivariate probit.
5. Standard errors clustered by urban sector.

Table 10: Probit Estimates (Enforcement)

Dep. Var. = tax reg	**Coeff.** (Std. Err.)	**Marg. Eff.**	**Coeff.** (Std. Err.)	**Marg. Eff.**
sup enf	5.607**	0.724		
	(1.463)			
cl enf			11.817**	1.510
			(1.294)	
outside hh	0.178**	0.022	0.177**	0.022
	(0.024)		(0.024)	
# employees	0.407**	0.053	0.412**	0.053
	(0.012)		(0.012)	
revenue	0.051**	0.006	0.049**	0.006
	(0.005)		(0.004)	
bank loan	0.377**	0.062	0.373**	0.062
	(0.033)		(0.033)	
other job	-0.243**	-0.027	-0.238**	-0.027
	(0.033)		(0.033)	
education	0.192**	0.025	0.186**	0.024
	(0.006)		(0.006)	
age	0.035**	0.005	0.035**	0.004
	(0.004)		(0.004)	
age^2	0.000**	0.000	0.000**	0.000
	(0.000)		(0.000)	
gender	0.152**	0.019	0.141**	0.018
	(0.020)		(0.020)	
homeowner × # rooms	0.030**	0.004	0.029**	0.004
	(0.003)		(0.003)	

N	46749	46749
Pseudo-R^2	0.3628	0.3649
$\chi^2_{(45)}$	5410.44	5482.02

1. Significance levels : † : 10% ∗ : 5% ∗∗ : 1%
2. Standard errors clustered by urban sector.
3. The regressions also control for state and sector.

Table 11: State SIMPLES Programs (Before 2003)

State	State Law #	Date		State Law #	Date
Mato Grosso do Sul[1]	1866	7/8/1998	Goiás[1]	13270	6/4/1998
São Paulo[2]	10086	11/19/1998	Paraíba	7332	4/28/2003
Rio de Janeiro[2]	3342	12/29/1999	Espírito Santo[1]	5389	4/24/1997
Bahia[2]	7357	11/4/1998	Alagoas[2]	6271	10/3/2001
Paraná[1]	246	1/29/2003	Rio Grande do Norte[3]	8296	1/28/2003
Pernambuco[2]	11157	12/29/1997	Distrito Federal[4]	2510	12/29/1999
Ceará[2]	13298	4/2/2003	Sergipe[1]	4185	12/22/1999
Pará[3]	6616	1/7/2003	Rondônia	8945	12/30/1999
Maranhão[1]	6904	3/24/1997	Acre[1]	1340	7/19/2000
Santa Catarina[2]	11398	5/8/2000	Amapá	1933	6/17/1998

1. `http://www.telecentros.desenvolvimento.gov.br`
2. Cartilha da Lei Geral (SEBRAE)
3. Secretaria de Fazenda Estadual

Table 12: Probit Estimates (SIMPLES)

Dep. Var. = tax reg	**Coefficient** (Std. Err.)	**Marg. Eff.**
SIMPLES × 2003	0.101**	0.014
	(0.030)	
SIMPLES	-0.241**	-0.036
	(0.021)	
2003	-0.754**	-0.11
	(0.027)	
N	90224	
Pseudo-R^2	0.3215	
$\chi^2_{(21)}$	21951.11	

1. Significance levels : † : 10% ∗ : 5% ∗∗ : 1%
2. SIMPLES stands for states that implemented a version of SIMPLES for the VAT.
3. Controls include `outside hh`, # `employees`, `revenue`, `bank loan`, `other job`, `education`, `age`, `age`^{2}, `gender`, `homeown_numroom` and sector of activity.

Table 13: Probit Estimates (SIMPLES, Ineligible)

Dep. Var. = tax reg	**Coefficient**	**(Std. Err.)**	**Marg. Eff.**
ineligible × SIMPLES × 2003 × large cl	-0.253	(0.256)	-0.028
SIMPLES × 2003 × large cl	0.332†	(0.178)	0.056
ineligible × SIMPLES × 2003 × small cl	-0.198	(0.195)	-0.023
SIMPLES × 2003 × small cl	0.089	(0.129)	0.013
ineligible × 2003 × large cl	-0.069*	(0.221)	-0.008
2003 × large cl	0.023	(0.153)	0.003
ineligible × 2003 × small cl	0.051	(0.171)	0.007
2003 × small cl	0.005	(0.110)	0.001
ineligible × SIMPLES × large cl	0.237	(0.202)	0.037
SIMPLES × large cl	-0.214	(0.143)	-0.025
ineligible × SIMPLES × small cl	0.327*	(0.081)	0.055
SIMPLES × small cl	-0.222*	(0.094)	-0.026
ineligible × large cl	0.378*	(0.171)	0.065
large cl	0.254*	(0.122)	0.040
ineligible × small cl	0.050	(0.127)	0.007
small cl	0.244**	(0.080)	0.038
ineligible × SIMPLES	0.046	(0.029)	0.006
SIMPLES	-0.193*	(0.018)	-0.028
N		22312	
Pseudo-R^2		0.3268	
$\chi^2_{(28)}$		22312	

1. Significance levels : $\dagger$: 10% $*$: 5% $**$: 1%
2. SIMPLES stands for states that implemented a version of SIMPLES for the VAT.
3. Controls include `outside hh`, # `employees`, `revenue`, `bank loan`, `other job`, `education`,`age`, `age`^{2}, `gender`,`homeown_numroom` and sector of activity.

Table 14: Probit Estimates (Tax Substitution)

Variable	Full Sample		Tax Sub = 1	
	Coefficient	**Coefficient**	**Coefficient**	**Coefficient**
	(Std. Err.)	(Std. Err.)	(Std. Err.)	(Std. Err.)
large cl	0.446**		0.01	
	(0.049)		(0.190)	
small cl	0.265**		-0.425**	
	(0.036)		(0.121)	
lscl		0.336**		-0.282**
		(0.031)		(0.108)
tax sub × large cl	-0.385*			
	(0.187)			
tax sub × small cl	-0.617**			
	(0.119)			
tax sub × lscl		-0.555**		
		(0.104)		
tax sub	0.345**	0.346**		
	(0.027)	(0.027)		
outside hh	0.202**	0.204**	0.215**	0.217**
	(0.024)	(0.024)	(0.046)	(0.046)
# employees	0.398**	0.398**	0.336**	0.337**
	(0.012)	(0.012)	(0.022)	(0.022)
revenue	0.047**	0.048**	0.049**	0.049**
	(0.004)	(0.004)	(0.009)	(0.009)
bank loan	0.380**	0.378**	0.401**	0.397**
	(0.033)	(0.033)	(0.064)	(0.064)
other job	-0.223**	-0.224**	-0.248**	-0.248**
	(0.033)	(0.033)	(0.068)	(0.068)
education	0.196**	0.196**	0.174**	0.173**
	(0.006)	(0.006)	(0.013)	(0.013)
age	0.034**	0.034**	0.054**	0.054**
	(0.005)	(0.005)	(0.010)	(0.010)
age^2	0.000**	0.000**	0.000**	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)
gender	0.099**	0.097**	0.096*	0.094*
	(0.021)	(0.021)	(0.042)	(0.042)
homeowner × # rooms	0.030**	0.030**	0.046**	0.047**
	(0.003)	(0.003)	(0.007)	(0.007)
N	47196	47196	8440	8440
Pseudo-R^2	0.3717	0.3712	0.3261	0.3255
$\chi^2_{(47)}$	5800.83	5791.56	1420.92	1407.81

1. Significance levels : † : 10% ∗ : 5% ∗∗ : 1%
2. Standard errors clustered by urban sector.
3. The regressions also control for state and sector.

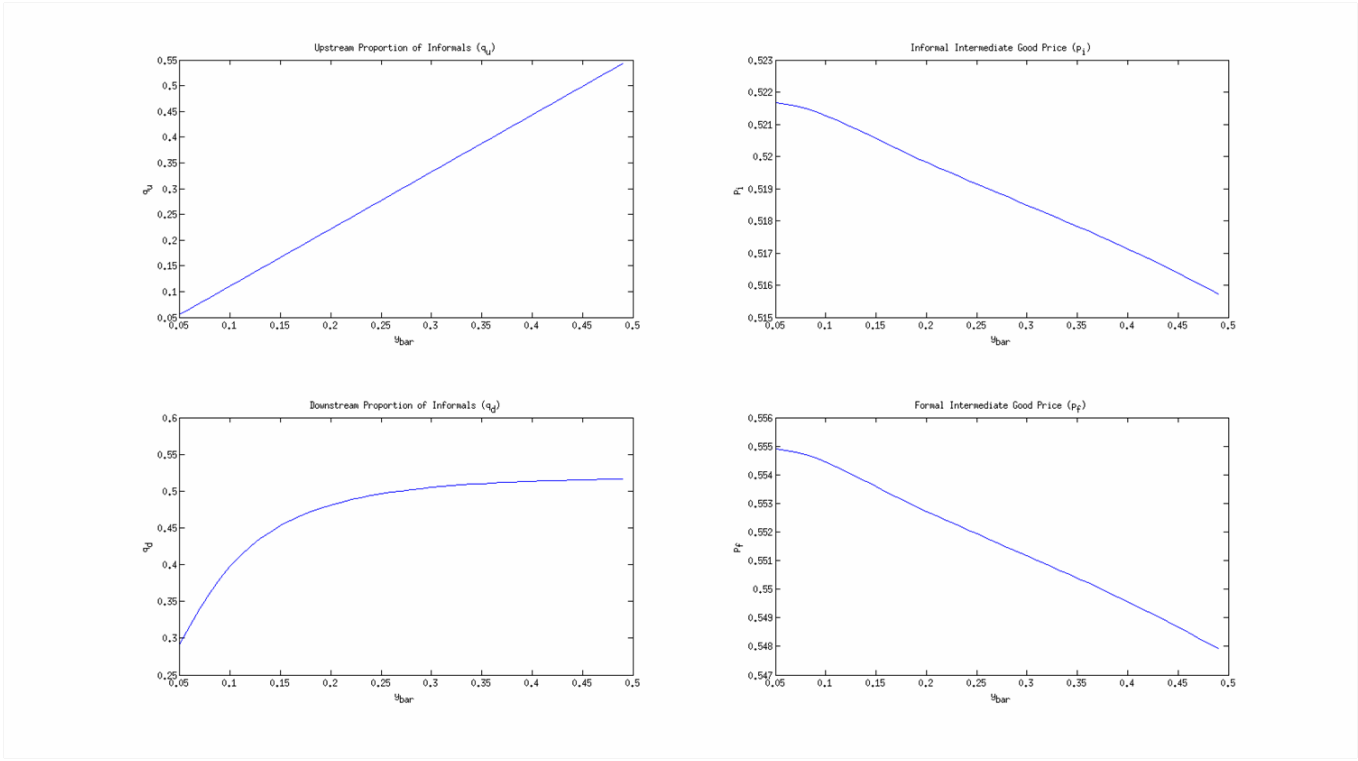

Figure 1: $\alpha = 0.7, \tau = 0.15, \overline{x} = 0.05, \theta_d, \theta_u \sim U[0,1]$

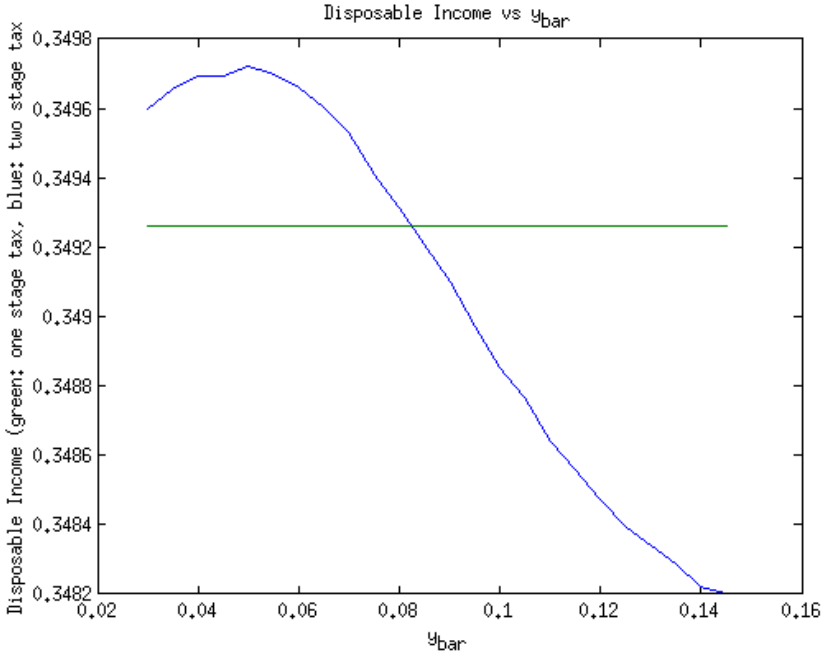

Figure 2: $\alpha = 0.4, \tau = 0.17$ (sales tax)), $\overline{x} = 0.05, \theta_d, \theta_u \sim U[0,1]$

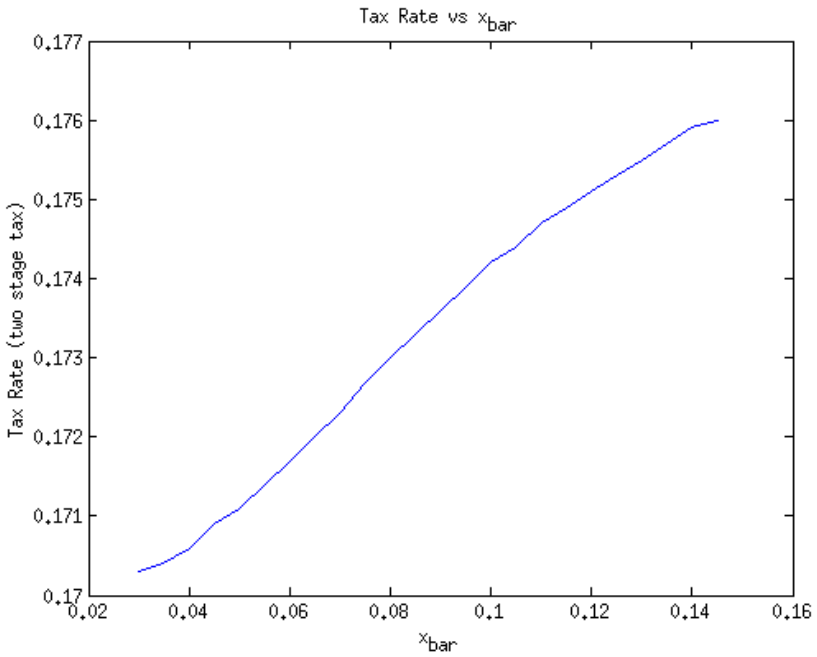

Figure 3: $\alpha = 0.4, \tau = 0.17$ (sales tax)), $\overline{x} = 0.05, \theta_d, \theta_u \sim U[0,1]$