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"Pro-cyclical Unemployment Benefits?
Optimal Policy in an Equilibrium Business Cycle Model"

by

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Pro-cyclical Unemployment Benefits? Optimal Policy in an Equilibrium Business Cycle Model

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Abstract

We study the optimal provision of unemployment insurance (UI) over the business cycle. We use an equilibrium search and matching model with aggregate shocks to labor productivity, incorporating risk-averse workers, endogenous worker search effort decisions, and unemployment benefit expiration. We characterize the optimal UI policy, allowing both the benefit level and benefit duration to depend on the history of past aggregate shocks. We find that the optimal benefit is decreasing in current productivity and decreasing in current unemployment. Following a drop in productivity, benefits initially rise in order to provide short-run relief to the unemployed and stabilize wages, but then fall significantly below their pre-recession level, in order to speed up the subsequent recovery. Under the optimal policy, the path of benefits is pro-cyclical overall. As compared to the existing US UI system, the optimal history-dependent benefits smooth cyclical fluctuations in unemployment and deliver substantial welfare gains.

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1 Introduction

The unemployment insurance (UI) systems in the US and many other countries contain provisions for extending unemployment benefits in response to economic downturns. The optimality of such extensions is the subject of an ongoing debate. The recent recession has further intensified the discussion of this important policy issue. Unemployment benefits provide insurance to workers against heightened unemployment risk. However, they may distort worker search decisions as well as firms' hiring decisions, possibly exacerbating the negative effects of an adverse economic shock. In this paper, we use a general equilibrium search model to characterize optimal UI policy over the business cycle.

We study UI provision in a Pissarides model with risk-averse workers and aggregate shocks to labor productivity. Our approach has three key features. First, we use a general equilibrium model, which enables us to capture the effects of policy changes on both firms' vacancy creation and worker search behavior. Second, we allow unemployment benefits to expire. This enables us to study the optimal choice of benefit duration as well as benefit level and to characterize the optimal behavior of both policy dimensions over the business cycle. Third, we allow the benefit policy to depend not only on the current aggregate state but also on its past history. This is important, since the social costs and benefits of providing job creation incentives in the current period depend on both current and past economic conditions.

Formally, we consider the optimal policy choice of a benevolent, utilitarian government that can choose both the level and the duration of unemployment benefits. The government can change the benefit level and duration in response to aggregate conditions and run deficits in some states of nature, as long as it balances its budget on average. We solve for the optimal state-contingent UI policy and find that it prescribes for the benefit level and duration to *rise* immediately following a drop in productivity. Subsequently, however, it prescribes a persistent *decline* in benefit levels and duration below their pre-recession values. The optimal response of benefits to a negative shock is thus non-monotonic. Right after a negative productivity shock hits, the social returns to job creation are low, so the government is more concerned with providing short-term relief for the unemployed and slowing the decline of wages than with inducing high job finding. Therefore, it temporarily raises the generosity of benefits - both level and duration - triggering a decrease in both vacancy creation and worker search effort. However, since the shock is mean-reverting, the government expects an economic recovery and subsequently lowers benefits and shortens their duration to stimulate job finding.

Central to this result is our finding that, all else equal, the optimal benefit level and duration are decreasing in current productivity and decreasing in current unemployment. In low-productivity states, the social benefits of creating additional worker-firm matches are relatively low, and so the government optimally raises the generosity of UI benefits. In high-unemployment states, however, the social benefits of raising employment are relatively high, and so the government optimally lowers the generosity of UI benefits. This suggests that, in a recession, there are two opposing forces - low productivity and high unemployment - which give opposite prescriptions for the behavior of optimal benefits. We find that the first effect is stronger at the very beginning of a recession, but the second effect dominates as the recession progresses, and inducing a recovery becomes desirable. As a consequence, we find that the time path of optimal benefit levels and benefit duration is pro-cyclical overall.

1.1 Relationship to the previous literature on unemployment insurance

The literature on the design of optimal UI policy has emphasized two key tradeoffs. The first is the tradeoff between providing insurance against unemployment risk and providing job search incentives to unemployed workers. This tradeoff has been extensively analyzed in principal-agent models of optimal UI, starting with Baily (1978), Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), and Shimer and Werning (2008). The second tradeoff is the tradeoff between insurance and providing firms with incentives for vacancy creation. This tradeoff has been emphasized by Fredriksson and Holmlund (2001), Cahuc and Lehmann (2000), Coles and Masters (2006), and Lehmann and van der Linden (2007), who study optimal UI design in equilibrium models with endogenous job creation and wage bargaining. Our framework incorporates the tradeoffs from both literatures. Moreover, it introduces aggregate shocks into such optimal policy analysis and quantitatively characterizes the optimal policy.

The paper closest to ours is Landais, Michaillat, and Saez (2010), who also examine optimal UI policy over the business cycle. Unlike our paper, they find that optimal UI benefits should be countercyclical. There are important differences between the assumptions of their paper and ours. The key difference is that, while we assume that wages are determined by bargaining, Landais, Michaillat, and Saez (2010) assume an extreme form of wage rigidity, namely that wages are a reduced-form function of labor productivity. In section 6.1 we elaborate on this difference in assumptions and discuss its importance for the difference in results.

Several other recent studies (Kiley (2003) , Sanchez (2008), Andersen and Svarer (2010, 2011), Kroft and Notowidigdo (2010)) have examined the optimal design of a state-contingent policy.[1] Our results that optimal benefits respond non-monotonically to a productivity shock, and that the optimal path of benefits is pro-cyclical, are new to this literature. Furthermore, introducing optimal benefit duration into this literature is particularly important, since the current debate on the optimality of UI benefit extensions has focused almost entirely on the duration of benefits. To our knowledge, our paper is the first to incorporate both policy dimensions in the context of optimal UI provision over the business cycle.

The paper is organized as follows. We present the model in section 2. Section 3 describes the optimal policy. We describe how we calibrate the model to US data in section 4. We report our results in section 5. In section 6, we discuss our results and conduct sensitivity analysis. Finally, we conclude in section 7.

2 Model Description

2.1 Economic Environment

We consider an infinite-horizon discrete-time model. The economy is populated by a unit measure of workers and a larger continuum of firms.

[1] Another strand of the recent literature examines the effect of the recent unemployment benefit extensions on the unemployment rate. See e.g. Fujita (2010), Nakajima (2011), Valletta and Kuang (2010).

Agents. In any given period, a worker can be either employed (matched with a firm) or unemployed. Workers are risk-averse expected utility maximizers and have expected lifetime utility

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(x_t\right) - c\left(s_t\right) \right],$$

where $\mathbb{E}_0$ is the period-0 expectation operator, $\beta \in (0,1)$ is the discount factor, x_t denotes consumption in period t, and s_t denotes search effort exerted in period t if unemployed. Only unemployed workers can supply search effort: there is no on-the-job search. The within-period utility of consumption $u : \mathbb{R}_+ \to \mathbb{R}$ is twice differentiable, strictly increasing, strictly concave, and satisfies $u'(0) = \infty$. The cost of search effort for unemployed workers $c : [0,1] \to \mathbb{R}$ is twice differentiable, strictly increasing, strictly convex, and satisfies $c'(0) = 0$, $c'(1) = \infty$. An unemployed worker produces h units of the consumption good via home production. There do not exist private insurance markets and workers cannot save or borrow.

Firms are risk-neutral and maximize profits. Workers and firms have the same discount factor β. A firm can be either matched to a worker or vacant. A firm posting a vacancy incurs a flow cost k.

Production. The economy is subject to aggregate shocks to labor productivity. Specifically, a matched worker-firm pair produces output z_t, where z_t is stochastic. We assume that $\ln z_t$ follows an AR(1) process

$$\ln z_t = \rho \ln z_{t-1} + \sigma_\varepsilon \varepsilon_t,$$

where $0 \leq \rho < 1$, $\sigma_\varepsilon > 0$, and ε_t are independent and identically distributed standard normal random variables. We will write $z^t = \{z_0, z_1, ..., z_t\}$ to denote the history of shocks up to period t.

Matching. Job creation occurs through a matching function. The number of new matches in period t equals

$$M\left(S_t\left(1 - L_{t-1}\right), v_t\right),$$

where $1 - L_{t-1}$ is the unemployment level in period $t-1$, S_t is the average search effort exerted by unemployed workers in period t, and v_t is the measure of vacancies posted in period t. The quantity

$\mathcal{N}_t = S_t(1 - L_{t-1})$ represents the measure of efficiency units of worker search.

The matching function M exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and has the property that the number of new matches cannot exceed the number of potential matches: $M(\mathcal{N}, v) \leq \min\{\mathcal{N}, v\}\ \forall \mathcal{N}, v$. We define

$$\theta_t = \frac{v_t}{S_t(1 - L_{t-1})}$$

to be the market tightness in period t. We define the functions

$$f(\theta) = \frac{M(S(1-L), v)}{S(1-L)} = M(1, \theta) \qquad \text{and}$$
$$q(\theta) = \frac{M(S(1-L), v)}{v} = M\left(\frac{1}{\theta}, 1\right)$$

where $f(\theta)$ is the job-finding probability per efficiency unit of search and $q(\theta)$ is the probability of filling a vacancy. By the assumptions on M made above, the function $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ. For an individual worker exerting search effort s, the probability of finding a job is $sf(\theta)$. When workers choose the amount of search effort s, they take as given the aggregate job-finding probability $f(\theta)$.

Existing matches are exogenously destroyed with a constant job separation probability δ. Thus, any of the L_{t-1} workers employed in period $t-1$ has a probability δ of becoming unemployed.

2.2 Government Policy

The US UI system is financed by payroll taxes on firms and is administered at the state level. However, under the provisions of the Social Security Act, each state can borrow from a federal unemployment insurance trust fund, provided it meets certain federal requirements. Motivated by these features of the UI system, we assume that the government in the model economy can insure against aggregate shocks by buying and selling claims contingent on the aggregate state and is required to balance its budget only in expectation. Further, we assume that the price of a claim to one unit of consumption in state z_{t+1} after a history z^t is equal to the probability of z_{t+1} conditional on z^t; this would be the case, e.g., in the presence of a large number of out-of state risk-neutral investors with the same discount factor.

Government policies are restricted to take the following form. The government levies a constant lump sum tax τ on firm profits and uses its tax revenues to finance unemployment benefits. The government is allowed to choose both the level of benefits and the rate at which they expire. We assume stochastic benefit expiration. A benefit policy at time t thus consists of a pair (b_t, e_t), where $b_t \geq 0$ is the level of benefits provided to those workers who are eligible for benefits at time t, and $e_t \in [0, 1]$ is the probability that an unemployed worker eligible for benefits becomes ineligible the following period. The eligibility status of a worker evolves as follows. A worker employed in period t is automatically eligible for benefits in case of job separation. An unemployed worker eligible for benefits in period t becomes ineligible the following period with probability e_t, and an ineligible worker does not regain eligibility until he finds a job. All eligible workers receive the same benefits b_t; ineligible workers receive no unemployment benefits, but instead receive an exogenously given welfare payment p.

We allow the benefit policy to depend on the entire history of past aggregate shocks; thus the policy $b_t = b_t\left(z^t\right), e_t = e_t\left(z^t\right)$ must be measurable with respect to z^t. Benefits are constrained to be non-negative: the government cannot tax home production.

2.3 Timing

The government commits to a policy $(\tau, b_t(\cdot), e_t(\cdot))$ once and for all before the period-0 shock realizes. Within each period t, the timing is as follows.

1. The economy enters period t with a level of employment L_{t-1}. Of the $1 - L_{t-1}$ unemployed workers, a measure $D_{t-1} \leq 1 - L_{t-1}$ are eligible for benefits, i.e. will receive benefits in period t if they do not find a job.

2. The aggregate shock z_t then realizes. Firms observe the aggregate shock and decide how many vacancies to post, at cost k per vacancy. At the same time, workers choose their search effort s_t at the cost of $c(s_t)$. Letting S_t^E and S_t^I be the search effort exerted by an eligible unemployed worker and an ineligible unemployed worker, respectively, the aggregate search effort is then equal

to $S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})$, and the market tightness is therefore equal to

$$\theta_t = \frac{v_t}{S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})} \tag{1}$$

3. $f(\theta)\left(S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})\right)$ unemployed workers find jobs. At the same time, a fraction δ of the existing L_{t-1} matches are exogenously destroyed.

4. All the workers who are now employed produce z_t and receive a bargained wage w_t (below we describe wage determination in detail). Workers who (i) were employed and lost a job, or (ii) were eligible unemployed workers and did not find a job, consume home production plus unemployment benefits, $h + b_t$ and lose their eligibility for the next period with probability e_t. Ineligible unemployed workers who have not found a job consume home production plus public assistance, $h + p$, and remain ineligible for the following period.

This determines the law of motion for employment

$$\begin{aligned} L_t\left(z^t\right) = &(1-\delta) L_{t-1}\left(z^{t-1}\right) \\ &+ f\left(\theta_t\left(z^t\right)\right)\left[S_t^E\left(z^t\right) D_{t-1}\left(z^{t-1}\right) + S_t^I\left(z^t\right)\left(1 - L_{t-1}\left(z^{t-1}\right) - D_{t-1}\left(z^{t-1}\right)\right)\right] \end{aligned} \tag{2}$$

and the law of motion for the measure of eligible unemployed workers:

$$D_t\left(z^t\right) = \left(1 - e_t\left(z^t\right)\right)\left[\delta L_{t-1}\left(z^{t-1}\right) + \left(1 - s_t\left(z^t\right) f\left(\theta_t\left(z^t\right)\right)\right) D_{t-1}\left(z^{t-1}\right)\right] \tag{3}$$

Thus, the measure of workers receiving benefits in period t is $\delta L_{t-1} + (1 - s_t f(\theta_t)) D_{t-1} = \frac{D_t}{1-e_t}$.

Since we assume that the government has access to financial markets in which a full set of state-contingent claims is traded, its budget constraint is a present-value budget constraint

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t\left(z^t\right)\tau - \left(\frac{D_t\left(z^t\right)}{1 - e_t\left(z^t\right)}\right) b_t\left(z^t\right) \right\} \geq 0 \tag{4}$$

2.4 Worker Value Functions

A worker entering period t employed retains his job with probability $1-\delta$ and loses it with probability δ. If he retains his job, he consumes his wage $w_t\left(z^t\right)$ and proceeds as employed to period $t+1$. If he loses his job, he consumes his home production plus benefits, $h+b_t\left(z^t\right)$ and proceeds as unemployed to period $t+1$. With probability $1-e_t\left(z^t\right)$ he then retains his eligibility for benefits in period $t+1$, and with probability $e_t\left(z^t\right)$ he loses his eligibility. Denote by $W_t\left(z^t\right)$ the value after a history z^t for a worker who enters period t employed.

A worker entering period t unemployed and eligible for benefits chooses search effort s_t^E and suffers the disutility $c\left(s_t^E\right)$. He finds a job with probability $s_t^E f\left(\theta_t\left(z^t\right)\right)$ and remains unemployed with the complementary probability. If he finds a job, he earns the wage $w_t\left(z^t\right)$ and proceeds as employed to period $t+1$. If he remains unemployed, he consumes his home production plus benefits, $h+b_t\left(z^t\right)$, and proceeds as unemployed to the next period. With probability $1-e_t\left(z^t\right)$ he retains his eligibility for benefits in period $t+1$, and with probability $e_t\left(z^t\right)$ he loses his eligibility. Denote by $U_t^E\left(z^t\right)$ the value after a history z^t for a worker who enters period t as eligible unemployed.

Finally, a worker entering period t unemployed and ineligible for benefits chooses search effort s_t^I and suffers the disutility $c\left(s_t^I\right)$. He finds a job with probability $s_t^I f\left(\theta_t\left(z^t\right)\right)$ and remains unemployed with the complementary probability. If he finds a job, he earns the wage $w_t\left(z^t\right)$ and proceeds as employed to period $t+1$. If he remains unemployed, he consumes his home production plus welfare payments, $h+p$, and proceeds as ineligible unemployed to the next period. Denote by $U_t^I\left(z^t\right)$ the value after a history z^t for a worker who enters period t as ineligible unemployed.

The Bellman equations for the three types of workers are then:

$$\begin{aligned} W_t\left(z^t\right) = &(1-\delta)\left[u\left(w_t\left(z^t\right)\right)+\beta \mathbb{E} W_{t+1}\left(z^{t+1}\right)\right] \\ &+\delta\left[u\left(h+b_t\left(z^t\right)\right)+\beta\left(1-e_t\right) \mathbb{E} U_{t+1}^E\left(z^{t+1}\right)+\beta e_t \mathbb{E} U_{t+1}^I\left(z^{t+1}\right)\right] \end{aligned} \tag{5}$$

$$\begin{aligned} U_t^E\left(z^t\right) = &\max_{s_t^E} -c\left(s_t^E\right)+s_t^E f\left(\theta_t\left(z^t\right)\right)\left[u\left(w_t\left(z^t\right)\right)+\beta \mathbb{E} W_{t+1}\left(z^{t+1}\right)\right] \\ &+\left(1-s_t^E f\left(\theta_t\left(z^t\right)\right)\right)\left[u\left(h+b_t\left(z^t\right)\right)+\beta\left(1-e_t\left(z^t\right)\right) \mathbb{E} U_{t+1}^E\left(z^{t+1}\right)+\beta e_t \mathbb{E} U_{t+1}^I\left(z^{t+1}\right)\right] \end{aligned} \tag{6}$$

$$\begin{aligned} U_t^I\left(z^t\right) = &\max_{s_t^I} -c\left(s_t^I\right)+s_t^I f\left(\theta_t\left(z^t\right)\right)\left[u\left(w_t\left(z^t\right)\right)+\beta \mathbb{E} W_{t+1}\left(z^{t+1}\right)\right] \\ &+\left(1-s_t^I f\left(\theta_t\left(z^t\right)\right)\right)\left[u(h+p)+\beta \mathbb{E} U_{t+1}^I\left(z^{t+1}\right)\right] \end{aligned} \tag{7}$$

It will be useful to define the worker's surplus from being employed. The surplus utility from being employed, as compared to eligible unemployed, in period t is

$$\Delta_t\left(z^t\right)=\left[u\left(w_t\left(z^t\right)\right)+\beta \mathbb{E}_t W_{t+1}\left(z^{t+1}\right)\right]-\left[u\left(h+b_t\left(z^t\right)\right)+\beta\left(1-e_t\right) \mathbb{E} U_{t+1}^E\left(z^{t+1}\right)+\beta e_t \mathbb{E} U_{t+1}^I\left(z^{t+1}\right)\right] \tag{8}$$

Similarly, we define the surplus utility from being employed as compared to being unemployed and ineligible for benefits:

$$\Xi_t\left(z^t\right)=\left[u\left(w_t\left(z^t\right)\right)+\beta \mathbb{E}_t W_{t+1}\left(z^{t+1}\right)\right]-\left[u(h+p))+\beta \mathbb{E} U_{t+1}^I\left(z^{t+1}\right)\right] \tag{9}$$

2.5 Firm Value Functions

A matched firm retains its worker with probability $1-\delta$. In this case, the firm receives the output net of wages and taxes, $z_t - w_t\left(z^t\right) - \tau$, and then proceeds into the next period as a matched firm. If the firm loses its worker, it gains nothing in the current period and proceeds into the next period unmatched. A firm that posts a vacancy incurs a flow cost k and finds a worker with probability $q\left(\theta_t\left(z^t\right)\right)$. If the firm finds a worker, it gets flow profits $z_t - w_t\left(z^t\right) - \tau$ and proceeds into the next period as a matched firm. Otherwise, it proceeds unmatched into the next period.

Denote by $J_t\left(z^t\right)$ the value of a firm that enters period t matched to a worker, and denote by $V_t\left(z^t\right)$

the value of an unmatched firm posting a vacancy. These value functions satisfy the following Bellman equations:

$$J_t\left(z^t\right) = (1-\delta)\left[z_t - w_t\left(z^t\right) - \tau + \beta\mathbb{E}_t J_{t+1}\left(z^{t+1}\right)\right] + \delta\beta\mathbb{E}_t V_{t+1}\left(z^{t+1}\right) \tag{10}$$

$$V_t\left(z^t\right) = -k + q\left(\theta_t\left(z^t\right)\right)\left[z_t - w_t\left(z^t\right) - \tau + \beta\mathbb{E}_t J_{t+1}\left(z^{t+1}\right)\right] + \left(1 - q\left(\theta_t\left(z^t\right)\right)\right)\beta\mathbb{E}_t V_{t+1}\left(z^{t+1}\right) \tag{11}$$

The firm's surplus from employing a worker in period t is denoted

$$\Gamma_t\left(z^t\right) = z_t - w_t\left(z^t\right) - \tau + \beta\mathbb{E}_t J_{t+1}\left(z^{t+1}\right) - \beta\mathbb{E}_t V_{t+1}\left(z^{t+1}\right) \tag{12}$$

2.6 Wage Bargaining

We assume that wages are determined according to Nash bargaining: the wage is chosen to maximize a weighted product of the worker's surplus and the firm's surplus. Further, the worker's outside option is being unemployed and eligible for benefits, since he becomes eligible upon locating an employer and retains eligibility if negotiations with the employer break down. The worker-firm pair therefore chooses the wage $w_t\left(z^t\right)$ to maximize

$$\Delta_t\left(z^t\right)^{\xi}\Gamma_t\left(z^t\right)^{1-\xi}, \tag{13}$$

where $\xi \in (0,1)$ is the worker's bargaining weight.

2.7 Equilibrium Given Policy

In this section, we define the equilibrium of the model, taking as given a government policy $(\tau, b_t(\cdot), e_t(\cdot))$ and characterize it.

2.7.1 Equilibrium Definition

Taking as given an initial condition (z_{-1}, L_{-1}), we define an equilibrium given policy:

Definition 1 *Given a policy* $(\tau, b_t(\cdot), e_t(\cdot))$ *and an initial condition* (z_{-1}, L_{-1}) *an equilibrium is a sequence of* z^t*-measurable functions for wages* $w_t\left(z^t\right)$*, search effort* $S_t^E\left(z^t\right)$*,* $S_t^I\left(z^t\right)$*, market tightness*

$\theta_t(z^t)$, *employment* $L_t(z^t)$, *measures of eligible workers* $D_t(z^t)$, *and value functions*

$$\left\{W_t(z^t), U_t^E(z^t), U_t^I(z^t), J_t(z^t), V_t(z^t), \Delta_t(z^t), \Xi_t(z^t), \Gamma_t(z^t)\right\}$$

such that:

1. *The value functions satisfy the worker and firm Bellman equations (5), (6), (7), (8), (9), (10), (11), (12)*
2. *Optimal search: The search effort* S_t^E *solves the maximization problem in (6) for* s_t^E*, and the search effort* S_t^I *solves the maximization problem in (7) for* s_t^I
3. *Free entry: The value* $V_t(z^t)$ *of a vacant firm is zero for all* z^t
4. *Nash bargaining: The wage maximizes equation (13)*
5. *Law of motion for employment and eligibility status: Employment and the measure of eligible unemployed workers satisfy (2) , (3)*
6. *Budget balance: Tax revenue and benefits satisfy (4)*

2.7.2 Characterization of Equilibrium

We characterize the equilibrium given policy via a system of equations that involves allocations only, and does not involve the value functions. This will be helpful in computing the optimal policy.

Lemma 1 *Fix an initial condition and a policy* $(\tau, b_t(\cdot), e_t(\cdot))$*. Suppose that the sequence*

$$\Upsilon_t(z^t) = \{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t), \\ W_t(z^t), U_t^E(z^t), U_t^I(z^t), J_t(z^t), V_t(z^t), \Delta_t(z^t), \Xi_t(z^t), \Gamma_t(z^t)\}$$

is an equilibrium. Then the sequences $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$ *satisfy:*

1. *The laws of motion (2) , (3)*

2. The budget equation (4)

3. Modified worker Bellman equations (dependence on z^t is understood throughout)

$$\frac{c'\left(S_t^E\right)}{f\left(\theta_t\right)} = u\left(w_t\right) - u\left(h+b_t\right) + \left(1-e_t\right)\beta\mathbb{E}_t\left(c\left(S_{t+1}^E\right) + \left(1-\delta-S_{t+1}^E f\left(\theta_{t+1}\right)\right)\frac{c'\left(S_{t+1}^E\right)}{f\left(\theta_{t+1}\right)}\right)$$
$$+ e_t\beta\mathbb{E}_t\left(c\left(S_{t+1}^I\right) + \left(1-S_{t+1}^I f\left(\theta_{t+1}\right)\right)\frac{c'\left(S_{t+1}^I\right)}{f\left(\theta_{t+1}\right)} - \delta\frac{c'\left(S_{t+1}^E\right)}{f\left(\theta_{t+1}\right)}\right) \tag{14}$$

$$\frac{c'\left(S_t^I\right)}{f\left(\theta_t\right)} = u\left(w_t\right) - u\left(h+p\right) + \beta\mathbb{E}_t\left(c\left(S_{t+1}^I\right) + \left(1-S_{t+1}^I f\left(\theta_{t+1}\right)\right)\frac{c'\left(S_{t+1}^I\right)}{f\left(\theta_{t+1}\right)} - \delta\frac{c'\left(S_{t+1}^E\right)}{f\left(\theta_{t+1}\right)}\right) \tag{15}$$

4. Modified firm Bellman equation

$$\frac{k}{q\left(\theta_t\right)} = z_t - w_t - \tau + \beta\left(1-\delta\right)\mathbb{E}_t\frac{k}{q\left(\theta_{t+1}\right)} \tag{16}$$

5. Nash bargaining condition

$$\xi u'\left(w_t\right)k\theta_t = \left(1-\xi\right)c'\left(S_t^E\right) \tag{17}$$

Conversely, if $\left\{w_t\left(z^t\right), S_t^E\left(z^t\right), S_t^I\left(z^t\right), \theta_t\left(z^t\right), L_t\left(z^t\right), D_t\left(z^t\right)\right\}$ satisfy (2)-(4) and (14)-(17), then there exist value functions such that $\Upsilon_t\left(z^t\right)$ is an equilibrium.

Proof. First, observe that the necessary first-order conditions for optimal search effort are

$$\Delta_t = \frac{c'\left(S_t^E\right)}{f\left(\theta_t\right)} \tag{18}$$
$$\Xi_t = \frac{c'\left(S_t^I\right)}{f\left(\theta_t\right)} \tag{19}$$

Next, taking the differences of the workers' value functions from equations (5), (6), (7), we have

$$\begin{aligned} W_t - U_t^E &= c\left(S_t^E\right) + \left(1 - \delta - S_t^E f\left(\theta_t\right)\right)\Delta_t \\ &= c\left(S_t^E\right) + \left(1 - \delta - S_t^E f\left(\theta_t\right)\right)\frac{c'\left(S_t^E\right)}{f\left(\theta_t\right)} \end{aligned} \tag{20}$$

$$\begin{aligned} W_t - U_t^I &= c\left(S_t^I\right) + \left(1 - S_t^I f\left(\theta_t\right)\right)\Xi_t\left(z^t\right) - \delta\Delta_t \\ &= c\left(S_t^I\right) + \left(1 - S_t^I\left(z^t\right) f\left(\theta_t\right)\right)\frac{c'\left(S_t^I\right)}{f\left(\theta_t\right)} - \delta\frac{c'\left(S_t^E\right)}{f\left(\theta_t\left(z^t\right)\right)} \end{aligned} \tag{21}$$

Next, we rearrange the expressions for worker surpluses (8), (9) to get

$$\begin{aligned} \Delta_t =& u\left(w_t\right) - u\left(h + b_t\right) \\ &+ \beta\left(1 - e_t\right)\mathbb{E}_t\left(W_{t+1} - U_{t+1}^E\right) + \beta e_t \mathbb{E}_t\left(W_{t+1} - U_{t+1}^I\right) \end{aligned} \tag{22}$$

$$\Xi_t = u\left(w_t\right) - u\left(h + p\right) + \beta\mathbb{E}_t\left(W_{t+1} - U_{t+1}^I\right) \tag{23}$$

Now, substituting (18) and (20) into the left and right hand sides of (22) gives (14); similarly, substituting (19) and (21) into the left and right hand sides of (23) gives (15).

Next, we derive the law of motion for the firm's surplus from hiring. By the free-entry condition, the value $V_t\left(z^t\right)$ of a firm posting a vacancy must be zero. Equations (10) and (11) then simplify to:

$$J_t = (1 - \delta)\left[z_t - w_t - \tau + \beta\mathbb{E}_t J_{t+1}\right] \tag{24}$$

$$0 = -k + q\left(\theta_t\right)\left[z_t - w_t - \tau + \beta\mathbb{E}_t J_{t+1}\right] \tag{25}$$

which together imply

$$J_t = (1 - \delta)\frac{k}{q\left(\theta_t\right)} \tag{26}$$

$$\Gamma_t = \frac{k}{q\left(\theta_t\right)} \tag{27}$$

Equations (24) and (26) imply that Γ_t follows the law of motion $\Gamma_t = z_t - w_t - \tau + \beta(1 - \delta)\mathbb{E}_t\Gamma_{t+1}$, which, by (27), is precisely (16).

Finally, the first-order condition with respect to w_t for the Nash bargaining problem (13) is

$$\xi u'(w_t)\Gamma_t = (1-\xi)\Delta_t \tag{28}$$

Substituting (27) and (18) into (28) and using the fact that $f(\theta) = \theta q(\theta)$ yields (17).

The converse of the result holds since the value functions can be recovered via the corresponding Bellman equations. ■

The conditions (14)-(17) are straightforward to interpret. Equations (14) and (15) state that the marginal cost of increasing the job finding probability for the eligible and ineligible workers, respectively, equals the marginal benefit. The marginal cost (left-hand side of each equation) of increasing the job finding probability is the marginal disutility of search for that worker weighted by the aggregate job finding rate. The marginal benefit (right-hand side of each equation) equals the current consumption gain from becoming employed plus the benefit of economizing on search costs in the future. Equation (16) gives a similar optimality condition for firms: it equates the marginal cost of creating a vacancy, weighted by the probability of filling that vacancy, to the benefit of employing a worker. Finally, (17) is a restatement of the first-order condition of the bargaining problem. It will be clear in section 3 that the conditions (14)-(17) will play the role of incentive constraints in the optimal policy problem, analogous to incentive constraints in principal-agent models of unemployment insurance, e.g. Hopenhayn and Nicolini (1997).

3 Optimal Policy

We assume that the government is utilitarian: it chooses a policy to maximize the period-0 expected value of worker utility, taking the equilibrium conditions as constraints.

Definition 2 *A policy $\tau, b_t(z^t), e_t(z^t)$ is feasible if there exists a sequence of z^t-measurable functions $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$ such that (2), (3), (14)-(17) hold for all z^t, and the government budget constraint (4) is satisfied.*

Definition 3 *The optimal policy is a policy* $\tau, b_t(z^t), e_t(z^t)$ *that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} L_t(z^t)\, u(w_t(z^t)) + \left(\frac{D_t(z^t)}{1-e_t(z^t)}\right) u(h + b_t(z^t)) + \left(1 - L_t(z^t) - \frac{D_t(z^t)}{1-e_t(z^t)}\right) u(h+p) \\ -D_{t-1}(z^{t-1})\, c(S_t^E(z^t)) - (1 - L_{t-1}(z^{t-1}) - D_{t-1}(z^t-1))\, c(S_t^I(z^t)) \end{array} \right\} \tag{29}$$

over the set of all feasible policies.

The government's problem can be written as one of choosing a policy $\tau, b_t(z^t), e_t(z^t)$ together with functions $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$ to maximize (29) subject to (2), (3), (14)-(17) holding for all z^t, and subject to the government budget constraint (4). We find the optimal policy by solving the system of necessary first-order conditions for this problem. The period-t solution will naturally be state-dependent: in particular, it will depend on the current productivity z_t, as well as the current unemployment level $1 - L_{t-1}$, and current measure of benefit-eligible workers D_{t-1} with which the economy has entered period t. However, in general the triple $(z_t, 1 - L_{t-1}, D_{t-1})$ is not a sufficient state variable for pinning down the optimal policy, which may depend on the entire past history of aggregate shocks. In the appendix, we show that the optimal period t solution is a function of $(z_t, 1 - L_{t-1}, D_{t-1})$ as well as $(e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$, where e_{t-1} is the previous period's benefit expiration rate and $\mu_{t-1}, \nu_{t-1}, \gamma_{t-1}$ are Lagrange multipliers on the constraints (14),(15),(16), respectively, in the maximization problem (29). The tuple $(z_t, 1 - L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$ captures the dependence of the optimal b_t, e_t on the history z^t. The fact that the z_t, $1 - L_{t-1}$ and D_{t-1} are not sufficient reflects the fact that the optimal policy is time-inconsistent: for example, the optimal benefits after two different histories of shocks may differ even though the two histories result in the same current productivity and the same current unemployment level. Intuitively, the government might want to induce firms to post vacancies - and workers to search - by promising low unemployment benefits, but has an ex post incentive to provide higher benefits, so as to smooth worker consumption, after employment outcomes have realized. Including the variables $e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1}$ as state variables in the optimal policy captures exactly this trade-off. Note that we assume throughout the paper that the government can fully commit to its policy. In the appendix we explain the method used to solve for the optimal policy.

4 Calibration

We calibrate the model to verify that it captures salient features of the US labor market, and is thus a useful one for studying optimal policy design. Unlike previous versions of the Pissarides model calibrated in the literature, e.g. Shimer (2005) and Hagedorn and Manovskii (2008), our model incorporates endogenous search intensity choices and stochastic benefit expiration. Moreover, the market tightness in our model is not equal to the vacancy-unemployment ratio; rather, it is the object defined in (1), which we do not directly observe in the data. Our calibration strategy will be correspondingly modified relative to the previous literature. As explained below, we will calibrate the model to ensure that it is consistent both with aggregate US labor market data and with results from micro studies on the responsiveness of unemployment duration to benefit generosity.

We normalize mean productivity to one. We assume a benefit scheme that mimics the benefit extension provisions currently in place within the US policy. The standard benefit duration is 26 weeks; local and federal employment conditions trigger automatic 20-week and 33-week extensions. In the model we assume that $e_t = 1/59$ when productivity is below two standard deviations below the mean, $e_t = 1/46$ when productivity is between one and two standard deviations below the mean, and $e_t = 1/26$ otherwise. We set the welfare payment $p = 0.05$ to match the amount of Food Stamp payments as a fraction of average weekly earnings.[2] We pick the tax rate $\tau = 0.023$ so that the government balances its budget if the unemployment rate is 5.5%.

We assume log utility: $u(x) = \ln x$. For the cost of search, we assume the functional form

$$c(s) = \frac{A}{1+\psi}\left[(1-s)^{-(1+\psi)} - 1\right] - As \tag{30}$$

This functional form satisfies all the assumptions made on the search cost function; in particular, it implies that the optimal search effort will always be between 0 and 1 for any $A > 0$.

[2] See the US Department of Health and Human Services (2008) Annual Report.

For the matching function, we follow den Haan, Ramey, and Watson (2000) and pick

$$M(\mathcal{N}, v) = \frac{\mathcal{N}v}{[\mathcal{N}^{\chi} + v^{\chi}]^{1/\chi}}$$

This matching technology satisfies all the assumptions made earlier, in particular the assumption that the implied job-finding rate is always less than one. We have:

$$f(\theta) = \frac{\theta}{(1+\theta^{\chi})^{1/\chi}}$$

$$q(\theta) = \frac{1}{(1+\theta^{\chi})^{1/\chi}}$$

The model period is taken to be 1 week. We set the discount factor $\beta = 0.99^{1/12}$, implying a yearly discount rate of 4%. Following Shimer (2005), labor productivity z_t is taken to mean real output per person in the non-farm business sector. This measure of productivity is taken from the data constructed by the BLS and the parameters for the shock process are estimated, at the weekly level, to be $\rho = 0.9895$ and $\sigma_{\varepsilon} = 0.0034$. The job separation parameter δ is set to 0.0081 to match the average weekly job separation rate.[3] We set $k = 0.58$ following Hagedorn and Manovskii (2008), who estimate the costs of vacancy creation to be 58% of weekly labor productivity.

This leaves five parameters to be calibrated: (1) the value h of home production; (2) the worker bargaining weight ξ; (3) the matching function parameter χ; (4) the level coefficient of the search cost function A; and (5) the curvature parameter of the search cost function ψ. We jointly calibrate these five parameters to simultaneously match five data targets: (1) the average vacancy-unemployment ratio; (2) the standard deviation of vacancy-unemployment ratio; (3) the average weekly job-finding rate; (4) the average duration of unemployment; and (5) the elasticity of unemployment duration with respect to benefits. The first four of these targets are directly measured in the data. For the elasticity of unemployment duration with respect to benefits, $\mathcal{E}_{d,b}$, we use micro estimates reported by Meyer (1990) and target an elasticity of 0.9. Intuitively, given the first three parameters, the average

[3]See Hagedorn and Manovskii (2008) on how to obtain the weekly estimates for the job finding rate and the job separation rate from monthly data.

unemployment duration and its elasticity with respect to benefits identify the parameters A and ψ, since these parameters govern the distortions in search behavior induced by benefits. Table 1 below reports the calibrated parameters. Our calibrated model is also consistent with non-targeted observations in the data: for example, the elasticity of unemployment duration with respect to the potential duration of benefits is 0.167 in the model, consistent with the estimates reported in Moffitt (1985) and close to other estimates in the literature.

Table 1: Internally Calibrated Parameters

	Parameter	Value	Target	Data	Model
h	Home production	0.580	Mean $v/(1-L)$	0.634	0.634
ξ	Bargaining power	0.114	St. dev of $\ln(v/(1-L))$	0.259	0.259
χ	Matching parameter	0.492	Mean job finding rate	0.139	0.139
A	Disutility of search	0.0015	Unemployment duration	13.2	13.2
ψ	Search cost curvature	3.786	$\mathcal{E}_{d,b}$	0.9	0.9

5 Results

In order to illustrate the mechanism behind the optimal policy, in Figure 1 we plot the optimal benefit policy function $b_t\left(z, 1-L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1}\right)$ as a function of current z and last period's $1-L$ only, keeping D_{t-1}, e_{t-1}, μ_{t-1}, ν_{t-1} and γ_{t-1} fixed at their average values. The optimal benefit level is decreasing in current productivity z and decreasing in unemployment $1-L$. The intuition for this result is that the optimal benefit is lower in states of the world when the marginal social benefit of job creation is higher, because lower benefits are used to encourage search effort by workers and vacancy creation by firms. The marginal social benefit of job creation is higher when z is higher, since the output of an additional worker-firm pair is then higher. The marginal social benefit is also higher when current employment is lower. As a consequence, optimal benefits are lowest, all else equal, when current productivity is high and current employment is low, i.e. at the beginning of an economic *recovery*. Figure 2 illustrates the same result for the optimal duration of benefits: optimal benefit duration is lowest at times of high productivity and high unemployment. This shape of the policy function also implies that during a recession, there are two opposing forces at work - low productivity and high unemployment - which give opposite prescriptions for the response of optimal benefits. This gives an ambiguous prediction for the overall cyclicality of benefit levels and benefit duration.

In order to understand the overall behavior of the optimal policy, in Figures 3 and 4 we analyze the response of the economy to a negative productivity shock under the optimal policy and compare it to the response under the current policy. In Figure 3 we plot the response of the optimal policy when productivity drops by 2.3% after a long sequence of productivity held at 1. The optimal benefit level initially jumps up, but then falls for about two quarters following the shock, and slowly reverts to its pre-shock level. The same is true of optimal benefit duration. Unemployment rises in response to the drop in productivity and continues rising for about one quarter before it starts to return to its pre-shock level. Note that the rise in unemployment is significantly lower than under the current benefit policy.

In Figure 4 we plot the response of other key labor market variables. As compared to the current benefit policy, the optimal policy results in a faster recovery of the vacancy-unemployment ratio, the search intensity of unemployed workers eligible for benefits, and the job finding rate. Wages also fall more gradually under the optimal policy than they do under the current policy.

The intuition for this optimal policy response is that the government would like to provide immediate insurance against the negative shock and, expecting future productivity to rise, would like to induce a recovery in vacancy creation and search effort. Thus, benefit generosity responds positively to the initial drop in productivity but negatively to the subsequent rise in unemployment, precisely as implied by Figures 1 and 2. The initial rise in benefits smooths the fall in wages through an increase in the worker outside option. The subsequent benefit decline, as well as the increase in the rate of benefit expiration, ameliorates the rise in unemployment. The government optimally uses a combination of both available policy instruments - benefit level and benefit duration - to achieve this effect.

We next investigate how the economy behaves over time under the optimal policy. To this end, we simulated the model both under the current benefit policy and under the optimal policy. Table 2 reports the summary statistics, under the optimal policy, for the behavior of unemployment benefit levels b and potential benefit duration $1/e$. Benefits are higher and expire faster under the optimal policy than under the current policy. The optimal tax rate under the optimal policy is $\tau = 0.018$, lower than under the current policy.

The key observation is that, over a long period of time, the correlation of optimal benefits with productivity is positive: both benefit levels and potential benfit duration are pro-cyclical in the long run and, in particular, negatively correlated with the unemployment rate. Moreover, this result is not driven by any balanced budget requirement, since we allow the government to run deficits in recessions.

Tables 3 and 4 report the moments of key labor market variables when the model is simulated under the current policy and the optimal policy, respectively. As compared to the optimal policy, the optimal policy results in lower average unemployment and lower unemployment volatility. These results corroborate our earlier intuition that the benefit policy serves to smooth the cyclical fluctuations in unemployment.

Finally, we compute the expected welfare gain from switching from the current policy to the optimal policy. We find that implementing the optimal policy results in a significant welfare gain: 0.67% as measured in consumption equivalent variation terms.

6 Discussion

6.1 The importance of the wage setting mechanism

The assumption of Nash bargaining in our model is an important modeling choice. Our wage-setting mechanism is flexible enough to allow wages to respond to both economic conditions - including productivity, labor market tightness, and the worker value of home production - and government policy, such as benefits and taxes. Empirical evidence indicates that increases in unemployment benefits do not leave wages unaffected, since they raise workers' reservation wages (see e.g. Fishe (1982) and Feldstein and Poterba (1984)). This highlights the importance of a wage setting mechanism in which wages do react to the worker outside option.

This wage setting mechanism distinguishes our paper from the concurrent work by Landais, Michaillat, and Saez (2010), who assume that wages are a reduced-form function of productivity only, and thus completely invariant to all policy changes and labor market tightness. As a result of their assumption, the main driver of unemployment in their model is job rationing, which does not respond to changes in policy. Consequently, in the model of Landais, Michaillat, and Saez (2010), in recessions the unemployment rate is high but cannot be affected through UI benefit policy. Not surprisingly, they find that optimal UI benefits should be countercyclical: in recessions, unemployment benefits do provide insurance to agents against heightened unemployment risk, but do not have a substantial effect on the unemployment rate. By contrast, in our paper, the UI benefit policy affects the unemployment rate significantly and can be used to manipulate both worker search behavior and firm vacancy posting behavior; in particular, it can be used to stimulate a recovery of employment during recessions. We find that this use of the UI benefits is an integral part of the optimal policy, and optimal benefits should therefore be pro-cyclical.

6.2 The Hosios condition and its relationship to our model

An important concern in the Pissarides model with Nash bargaining is that the laissez-faire equilibrium is not constrained efficient. Even with risk-neutral workers, the Hosios (1990) condition requires that the worker bargaining weight be equal to the elasticity of the matching function in order to attain efficiency. If the Hosios condition is violated, there is a role for government intervention - such as unemployment benefits - even in the absence of insurance considerations. This raises the question to what extent our optimal policy results are driven by violations of the Hosios condition, as opposed to insurance-incentives tradeoffs considered in the optimal UI literature. To investigate this question, we have solved for the optimal policy in a version of the model with *risk-neutral* workers, in which the violation of the Hosios condition is the only reason for government intervention. We find that the cyclicality of optimal benefits depends crucially on whether the worker bargaining weight is too high or too low: optimal benefits should be pro-cyclical for small values of worker bargaining power, but countercyclical for large values. However, with risk-averse workers, we find that optimal benefits should be pro-cyclical both for extremely low and for extremely high values of worker bargaining power. This implies that the violation of the Hosios condition is not the driving force of our results.[4]

6.3 The complementarity of benefit level and benefit duration

An important aspect of our analysis is the simultaneous treatment of optimal benefit level and optimal benefit duration. Our results indicate that optimal benefit levels and optimal benefit duration move in the same direction in response to a productivity shock, and therefore operate as complements over the business cycle. To further emphasize this complementarity, we illustrate how the optimal policy would change if the government were restricted to change only one of these two policy dimensions. This may be relevant, for example, because benefit duration may be more flexible in practice than the benefit level. This also facilitates comparison to the existing policy, in which mostly the duration of benefits, rather than the level, changes over the business cycle. We conduct three alternative policy experiments. In the first, we fix the benefit level at its current level: $b = 0.4$, and allow only the duration to change over the business cycle. The results, reported in Figure 5, show that the optimal policy response is similar qualitatively to our benchmark: in response to a negative productivity shock, potential duration of benefits should initially rise, and then fall considerably below its initial level. However, both the initial rise in the potential duration and its subsequent decline are greater than in the benchmark optimal policy result. In the second experiment, we fix the benefit expiration rate at its current level of $e = 1/26$ and compute the optimal benefit policy. Finally, in the third experiment, we ask how the benefit level should vary if benefits are not allowed to expire at all, i.e. if we fix $e = 0$. The results are shown in Figures 6 and 7 We find that the shape of the policy response is once again similar to the benchmark: benefits initially rise and then fall. However, both the initial rise and the subsequent decline are greater in magnitude than in the benchmark optimal policy experiment. In each of these cases, the government has one policy instrument at its disposal rather than two, and the optimal cyclical

[4] As an extension, it would be interesting to investigate optimal policy under alternative modeling of the wage process, in particular directed search. It is well known that, while in a directed search model with risk neutral workers, the laissez-faire equilibrium is constrained efficient, this is no longer the case in a model with risk-averse workers (see e.g. Acemoglu and Shimer (1999)) and a role for unemployment benefits therefore exists. We conjecture that our results would still be valid in a directed search model.

response of this policy instrument becomes stronger as a result.

6.4 Sensitivity analysis

We examine the robustness of our results to the parameterization of the model. We have calibrated the model parameters - in particular, the value of home production and the worker bargaining power - to make the model's behavior consistent with US labor market volatility data. However, since several alternative calibrations exist in the literature (see e.g. Shimer (2005)), we conduct sensitivity analysis to determine whether our optimal policy results remain valid under these alternative calibrations. Figure 8 displays the optimal policy results when home production is set to 0. Because the value of unemployment is considerably lower under this calibration, the optimal policy prescribes for benefits not to expire at all, but the optimal response of the benefit level is similar to our benchmark. Figure 9 displays the results when worker bargaining power is increased to 0.5. Next, we adopt the Shimer (2005) calibration, in which we set home production to 0 and the bargaining power of the workers to 0.72. The result is displayed in Figure 10; once again, optimal benefits do not expire, but the optimal response of the benefit level is the same as in our benchmark. The main qualitative features of our results, including the result that the optimal benefit scheme is pro-cyclical, do not depend on which calibration is used.
In addition, we have computed the optimal policy for different values of worker risk aversion: specifically, we have computed it for constant relative risk aversion utility, for values of relative risk aversion equal to 1/2 and 2. The results are displayed in Figures 11 and 12. Once again, the qualitative features of our results remain intact.

7 Conclusion

We analyzed the design of an optimal UI system in the presence of aggregate shocks in an equilibrium search and matching model. Optimal benefits respond non-monotonically to productivity shocks: while raising benefit generosity may be optimal at the onset of a recession, it becomes suboptimal as the recession progresses and inducing a recovery is desirable. We find that optimal benefits are pro-cyclical overall, counter to previous results in the literature. Adopting the optimal policy would yield significant welfare gains. Furthermore, we find that the optimal benefit policy, in addition to providing insurance to workers, results in the smoothing of unemployment over the business cycle.

An important extension for future research is investigating the role of government commitment. The ability of the government to commit matters because the behavior of agents in our model depends not only on the current policy, but also on their expectations about future policy. Throughout the paper, we have assumed that the government can fully commit to its policy. A government without commitment power might be tempted not to lower benefits when there are a lot of unemployed workers. It would therefore be interesting to characterize the time-consistent policy and compare it to the optimal policy in the presence of aggregate shocks.

Our paper has focused on the optimal cyclical behavior of UI benefits and thus serves to inform the ongoing policy debate on the desirability of benefit extensions in recessions. UI benefits are a worker-side intervention, as they affect the economy by changing the workers' value of being unemployed. An interesting extension would be to consider the optimal behavior of UI benefits in conjunction with firm-side interventions, such as hiring subsidies. Increasing hiring subsidies in recessions may be desirable as another instrument for stimulating an employment recovery. A potential concern with hiring subsidies, frequently articulated in policy debates, is the firm-side moral hazard they generate: firms could, for example, fire existing employees only to hire them again in order to receive hiring subsidies. A thorough investigation of the tradeoffs involved with such policies seems a fruitful direction for future research.

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A Solving for the Optimal Policy

The government is maximizing

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} L_t\left(z^t\right) u\left(w_t\left(z^t\right)\right) + \left(\frac{D_t\left(z^t\right)}{1-e_t\left(z^t\right)}\right) u\left(h+b_t\left(z^t\right)\right) + \left(1 - L_t\left(z^t\right) - \frac{D_t\left(z^t\right)}{1-e_t\left(z^t\right)}\right) u\left(h+p\right) \\ -D_{t-1}\left(z^{t-1}\right) c\left(S_t^E\left(z^t\right)\right) - \left(1 - L_{t-1}\left(z^{t-1}\right) - D_{t-1}\left(z^t - 1\right)\right) c\left(S_t^I\left(z^t\right)\right) \end{array} \right\} \tag{31}$$

subject to the conditions (2), (3), (14). (15),(16),(17) holding for all z^t, and subject to the government budget constraint (4).

Let $\pi\left(z^t\right)$ be the probability of history $z^t = \{z_0, z_1, ..., z_t\}$ given the initial condition z_{-1}. Denote by η the Lagrange multiplier on (4), and denote the Lagrange multipliers on (2), (3), (14). (15),(16),(17) by

$$\beta^t \pi\left(z^t\right) \lambda_t\left(z^t\right), \beta^t \pi\left(z^t\right) \alpha_t\left(z^t\right), \beta^t \pi\left(z^t\right) \mu_t\left(z^t\right), \beta^t \pi\left(z^t\right) \nu_t\left(z^t\right), \beta^t \pi\left(z^t\right) \gamma_t\left(z^t\right), \beta^t \pi\left(z^t\right) \phi_t\left(z^t\right),$$

respectively. In what follows, we suppress the dependence on z^t for notational simplicity. The first order necessary conditions with respect to $b_t, e_t, w_t, S_t^E, S_t^I, L_t, D_t, \theta_t$, respectively, are:

$$\left(D_t - \left(1 - e_t\right) \mu_t\right) u'\left(h + b_t\right) = \eta D_t \tag{32}$$

$$D_t\left[u\left(h+b_t\right) - u\left(h+p\right) - \eta b_t - \alpha_t\right] = \mu_t\left(1 - e_t\right)\left[u\left(h+b_t\right) - u\left(h+p\right) - \frac{c'\left(S_t^I\right) - c'\left(S_t^E\right)}{f\left(\theta_t\right)}\right] \tag{33}$$

$$\gamma_t = \left(L_t + \mu_t + \nu_t\right) u'\left(w_t\right) - \phi_t \xi u''\left(w_t\right) k \theta_t \tag{34}$$

$$\begin{aligned} \phi_t\left(\xi - 1\right) c''\left(S_t^E\right) =& D_{t-1}\left[\left(\lambda_t - \alpha_t\right) f\left(\theta_t\right) - c'\left(S_t^E\right)\right] \\ &+ \frac{c''\left(S_t^E\right)}{f\left(\theta_t\right)}\left[\mu_{t-1}\left(\left(1 - e_{t-1}\right)\left(1 - S_t^I f\left(\theta_t\right)\right) - \delta\right) - \mu_t - \delta \nu_{t-1}\right] \end{aligned} \tag{35}$$

$$\left(1 - L_{t-1} - D_{t-1}\right)\left[c' - \lambda_t f\left(\theta_t\right)\left(S_t^I\right)\right] = \frac{c''\left(S_t^I\right)}{f\left(\theta_t\right)}\left[\left(\mu_{t-1} e_{t-1} + \nu_{t-1}\right)\left(1 - S_t^I f\left(\theta_t\right)\right) - \nu_t\right] \tag{36}$$

$$\lambda_t = u\left(w_t\right) - u\left(h+p\right) + \eta \tau + \beta \mathbb{E}_t\left\{c\left(S_{t+1}^I\right) + \lambda_{t+1}\left(1 - \delta - S_{t+1}^I f\left(\theta_{t+1}\right)\right) + \alpha_{t+1} \delta\right\} \tag{37}$$

$$\begin{aligned} &\alpha_t = u\left(h+b_t\right) - u\left(h+p\right) - \eta b_t \\ &+ \beta\left(1 - e_t\right) \mathbb{E}_t\left\{c\left(S_{t+1}^I\right) - c\left(S_{t+1}^E\right) + \lambda_{t+1} f\left(\theta_{t+1}\right)\left(S_{t+1}^E - S_{t+1}^I\right) + \alpha_{t+1}\left(1 - S_{t+1}^E f\left(\theta_{t+1}\right)\right)\right\} \end{aligned} \tag{38}$$

$$\phi_t \xi u'(w_t) k - f'(\theta_t) \left\{ \lambda_t \left[S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1}) \right] - \alpha_t S_t^E D_{t-1} \right\} - [\gamma_t - (1-\delta)\gamma_{t-1}] \frac{k q'(\theta_t)}{(q(\theta_t))^2}$$

$$= [\mu_t - \mu_{t-1}(1 - e_{t-1} - \delta) + \nu_{t-1}\delta] \frac{c'(S_t^E) f'(\theta_t)}{(f(\theta_t))^2} + [\nu_t - \nu_{t-1} - \mu_{t-1} e_{t-1}] \frac{c'(S_t^I) f'(\theta_t)}{(f(\theta_t))^2} \tag{39}$$

The first-order necessary condition for the optimal tax rate τ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \eta L_t(z^t) - \gamma_t(z^t) \} = 0 \tag{40}$$

To find the optimal policy given η and τ, we solve the above system of difference equations (32)-(39) and (2), (3), (14). (15),(16),(17) for the optimal policy vector

$$\Omega(z^t) = \{ b_t(z^t), e_t(z^t), w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t),$$
$$\lambda_t(z^t), \alpha_t(z^t), \mu_t(z^t), \nu_t(z^t), \gamma_t(z^t), \phi_t(z^t) \}$$

We then pick η and τ so that (4) and (40) are satisfied.

Observe that the only period-$t-1$ variables that enter the period-t first-order conditions are

$$L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1},$$

and no variables from periods prior to $t-1$ enter the period-t first-order conditions. This implies that $(z_t, L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$ is a sufficient state variable for the history of shocks z^t up to and including period t. Specifically, fix η, τ, and let $(-)$ and $(+)$ denote the previous period's variable and the next period's variable, respectively. Let

$$\Psi : (z, L_-, D_-, e_-, \mu_-, \nu_- \gamma_-) \mapsto \left(b, e, w, S^E, S^I, L, D, \theta, \lambda, \alpha, \mu, \nu, \gamma, \phi \right)$$

be a function that satisfies

$$(D - (1-e)\mu) u'(h+b) = \eta D \tag{41}$$

$$D[u(h+b) - u(h+p) - \eta b - \alpha] = \mu(1-e) \left[u(h+b) - u(h+p) - \frac{c'(S^I) - c'(S^E)}{f(\theta)} \right] \tag{42}$$

$$\gamma = (L + \mu + \nu) u'(w) - \phi \xi u''(w) k \theta \tag{43}$$

$$\phi(\xi - 1) c''(S^E) = D_- \left[(\lambda - \alpha) f(\theta) - c'(S^E) \right]$$
$$+ \frac{c''(S^E)}{f(\theta)} \left[\mu_- \left((1 - e_-) \left(1 - S^I f(\theta) \right) - \delta \right) - \mu - \delta \nu_- \right] \tag{44}$$

$$(1-L_{-}-D_{-})\left[c'-\lambda f(\theta)\left(S^{I}\right)\right]=\frac{c''\left(S^{I}\right)}{f(\theta)}\left[(\mu_{-}e_{-}+\nu_{-})\left(1-S^{I}f(\theta)\right)-\nu\right] \tag{45}$$

$$\lambda=u(w)-u(h+p)+\eta\tau+\beta\mathbb{E}\left\{c\left(S_{+}^{I}\right)+\lambda_{+}\left(1-\delta-S_{+}^{I}f(\theta_{+})\right)+\alpha_{+}\delta\right\} \tag{46}$$

$$\begin{aligned}\alpha&=u(h+b)-u(h+p)-\eta b\\&+\beta(1-e)\mathbb{E}\left\{c\left(S_{+}^{I}\right)-c\left(S_{+}^{E}\right)+\lambda_{+}f(\theta_{+})\left(S_{+}^{E}-S_{+}^{I}\right)+\alpha_{+}\left(1-S_{+}^{E}f(\theta_{+})\right)\right\}\end{aligned} \tag{47}$$

$$\begin{aligned}&\phi\xi u'(w)k-f'(\theta)\left\{\lambda\left[S^{E}D_{-}+S^{I}(1-L_{-}-D_{-})\right]-\alpha S^{E}D_{-}\right\}-\left[\gamma-(1-\delta)\gamma_{-}\right]\frac{kq'(\theta)}{(q(\theta))^{2}}\\&=\left[\mu-\mu_{-}(1-e_{-}-\delta)+\nu_{-}\delta\right]\frac{c'\left(S^{E}\right)f'(\theta)}{(f(\theta))^{2}}+\left[\nu-\nu_{-}-\mu_{-}e_{-}\right]\frac{c'\left(S^{I}\right)f'(\theta)}{(f(\theta))^{2}}\end{aligned} \tag{48}$$

as well as

$$\begin{aligned}L&=(1-\delta)L_{-}+f(\theta)\left[S^{E}D_{-}+S^{I}(1-L_{-}-D_{-})\right]\\D&=(1-e)\left[\delta L_{-}+(1-sf(\theta))D_{-}\right]\end{aligned} \tag{49}$$

$$\begin{aligned}\frac{c'\left(S^{E}\right)}{f(\theta)}=&u(w)-u(h+b)+(1-e)\beta\mathbb{E}\left(c\left(S_{+}^{E}\right)+\left(1-\delta-S_{+}^{E}f(\theta_{+})\right)\frac{c'\left(S_{+}^{E}\right)}{f(\theta_{+})}\right)\\&+e\beta\mathbb{E}\left(c\left(S_{+}^{I}\right)+\left(1-S_{+}^{I}f(\theta_{+})\right)\frac{c'\left(S_{+}^{I}\right)}{f(\theta_{+})}-\delta\frac{c'\left(S_{+}^{I}\right)}{f(\theta_{+})}\right)\end{aligned} \tag{50}$$

$$\frac{c'\left(S^{I}\right)}{f(\theta)}=u(w)-u(h+p)+\beta\mathbb{E}\left(c\left(S_{+}^{I}\right)+\left(1-S_{+}^{I}f(\theta_{+})\right)\frac{c'\left(S_{+}^{I}\right)}{f(\theta)}-\delta\frac{c'\left(S_{+}^{E}\right)}{f(\theta_{+})}\right) \tag{51}$$

$$\frac{k}{q(\theta)}=z-w-\tau+\beta(1-\delta)\mathbb{E}\frac{k}{q(\theta_{+})} \tag{52}$$

$$\xi u'(w)k\theta=(1-\xi)c'\left(S^{E}\right) \tag{53}$$

Then the sequence defined by

$$\Omega\left(z^{t}\right)=\Psi\left(z_{t},L_{t-1}\left(z^{t-1}\right),D_{t-1}\left(z^{t-1}\right),e_{t-1}\left(z^{t-1}\right),\mu_{t-1}\left(z^{t-1}\right),\nu_{t-1}\left(z^{t-1}\right),\gamma_{t-1}\left(z^{t-1}\right)\right)$$

satisfies the system (32)-(39) and (2), (3), (14). (15),(16),(17).

To find the optimal policy given η, we therefore solve the system of functional equations (41)-(53).

B Tables and Figures

Figure 1: Optimal policy: benefit level

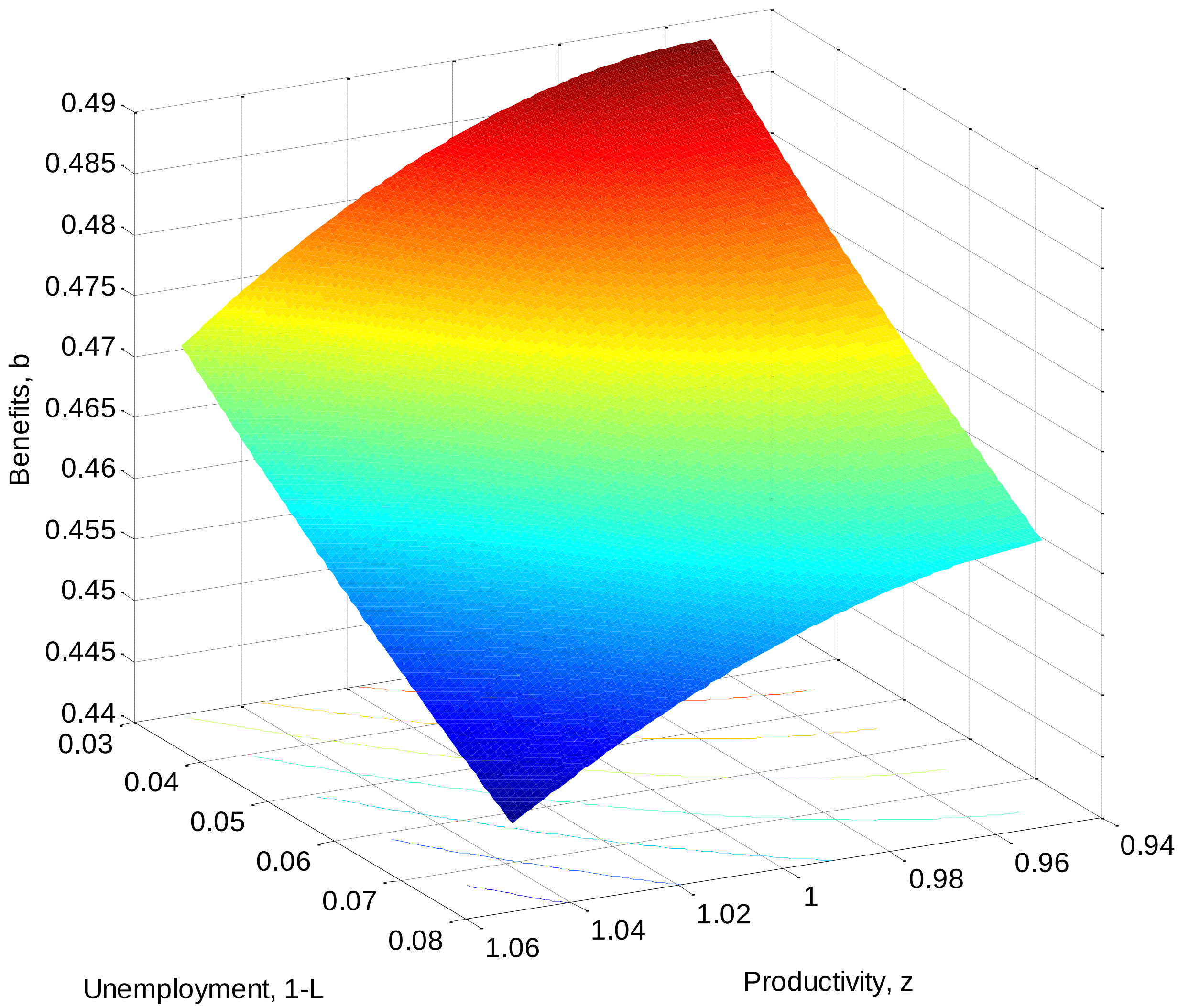

Figure 2: Optimal policy: benefit duration

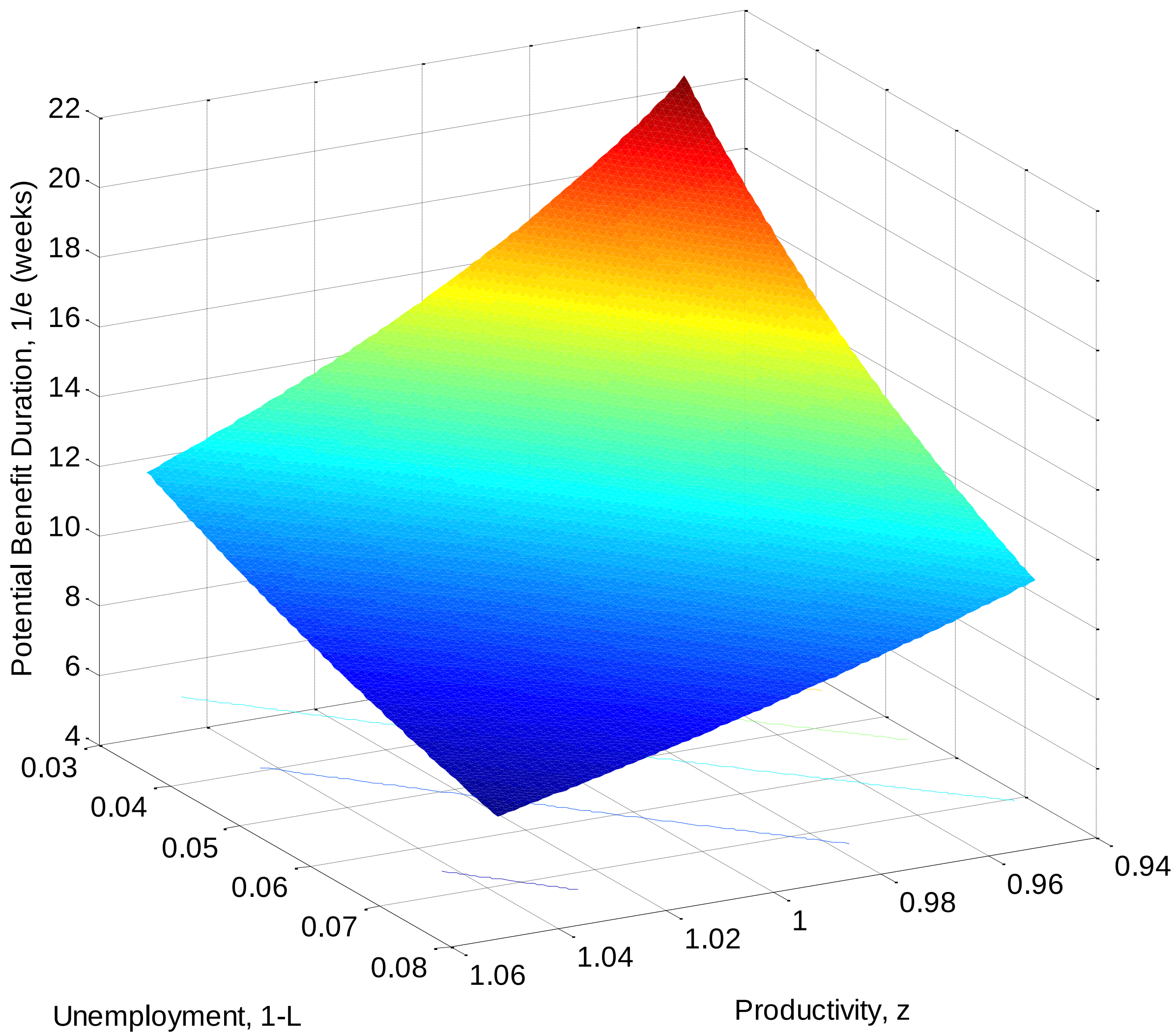

Table 2: Optimal benefit behavior

	Benefit level b	Potential duration $1/e$
Mean	0.472	12.5
Standard deviation	0.010	0.059
Correlation with z	0.758	0.520
Correlation with $1-L$	-0.420	-0.136
Correlation with b	1	0.950

Table 3: Model statistics simulated under the current US policy

		z	$1-L$	$v/(1-L)$	$\hat{f}$	w	S^E	S^I
Mean		1	0.059	0.634	0.139	0.954	0.505	0.655
Standard Deviation		0.013	0.128	0.259	0.150	0.010	0.040	0.003
	z	1	-0.849	0.907	0.945	0.883	0.873	0.943
	$1-L$	-	1	-0.902	-0.723	-0.908	-0.916	-0.891
	$v/(1-L)$	-	-	1	0.775	0.996	0.987	0.959
Correlation	$\hat{f}$	-	-	-	1	0.742	0.731	0.861
Matrix	w	-	-	-	-	1	0.997	0.958
	S^E	-	-	-	-	-	1	0.960
	S^I	-	-	-	-	-	-	1

Note: Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600. $\hat{f}$ denotes the weekly job finding rate.

Table 4: Model statistics simulated under the optimal US policy

		z	$1-L$	$v/(1-L)$	$\hat{f}$	w	S^E	S^I
Mean		1	0.049	0.742	0.157	0.956	0.520	0.655
Standard Deviation		0.13	0.027	0.062	0.032	0.011	0.008	0.003
	z	1	-0.877	0.814	0.775	0.917	0.743	0.995
	$1-L$	-	1	-0.934	-0.920	-0.656	-0.904	-0.847
	$v/(1-L)$	-	-	1	0.998	0.515	0.993	0.768
Correlation	$\hat{f}$	-	-	-	1	0.459	0.999	0.726
Matrix	w	-	-	-	-	1	0.416	0.942
	S^E	-	-	-	-	-	1	0.692
	S^I	-	-	-	-	-	-	1

Note: Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600. $\hat{f}$ denotes the weekly job finding rate.

Figure 3: Responses to 2.3% drop in productivity

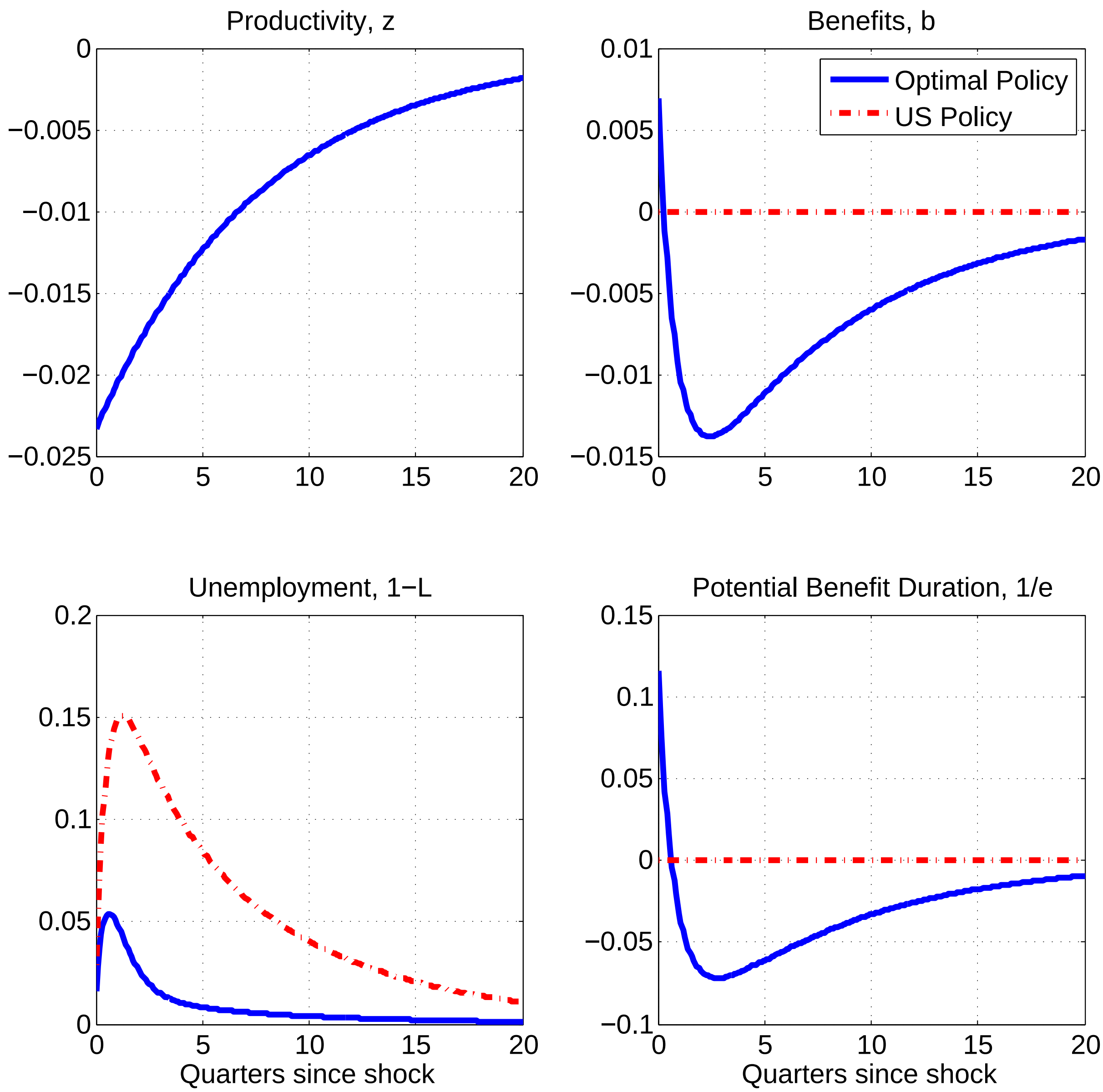

Figure 4: Responses to 2.3% drop in productivity

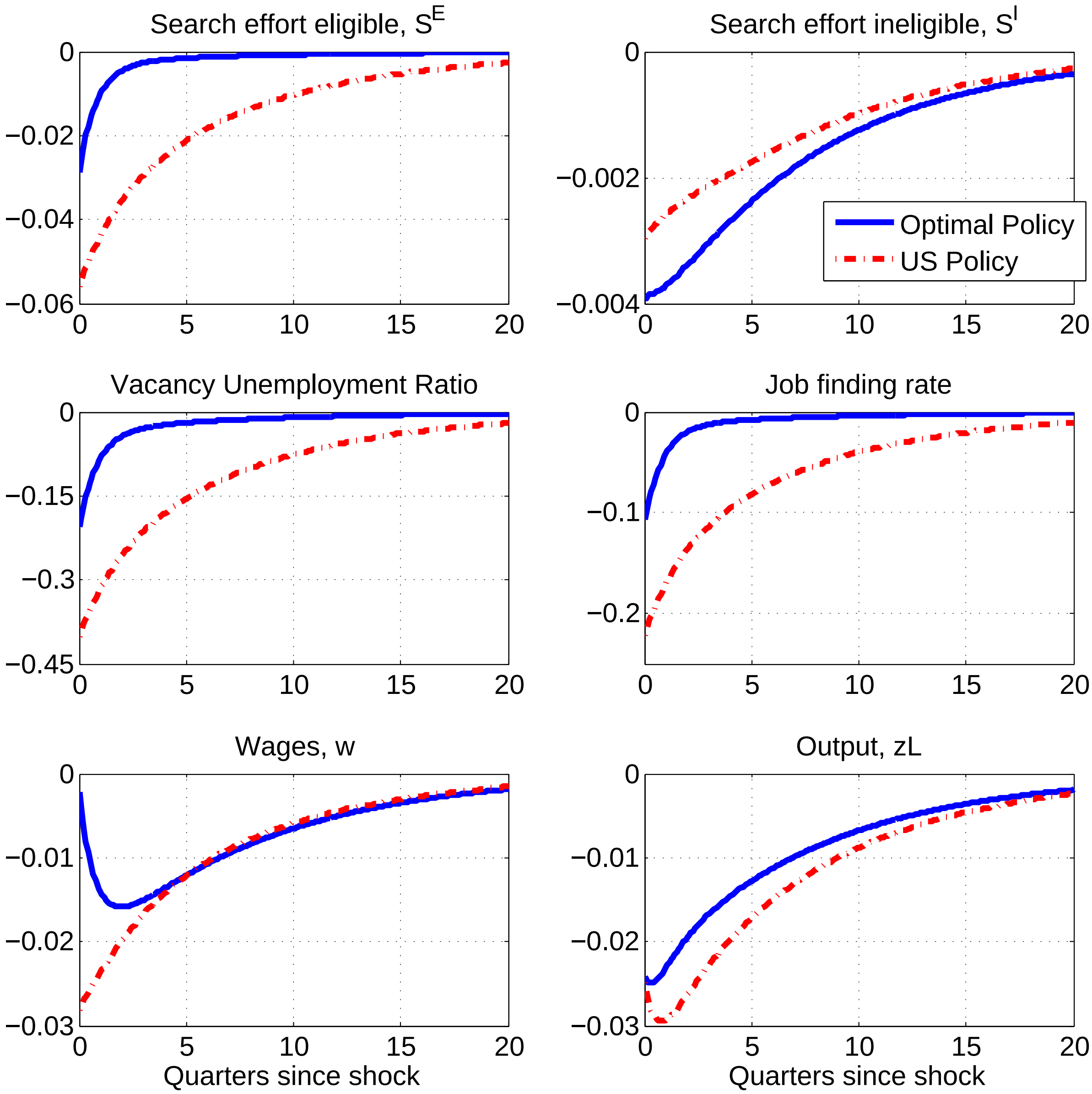

Figure 5: Response of duration to a 2.3% shock, fixing benefit level at $b = 0.4$

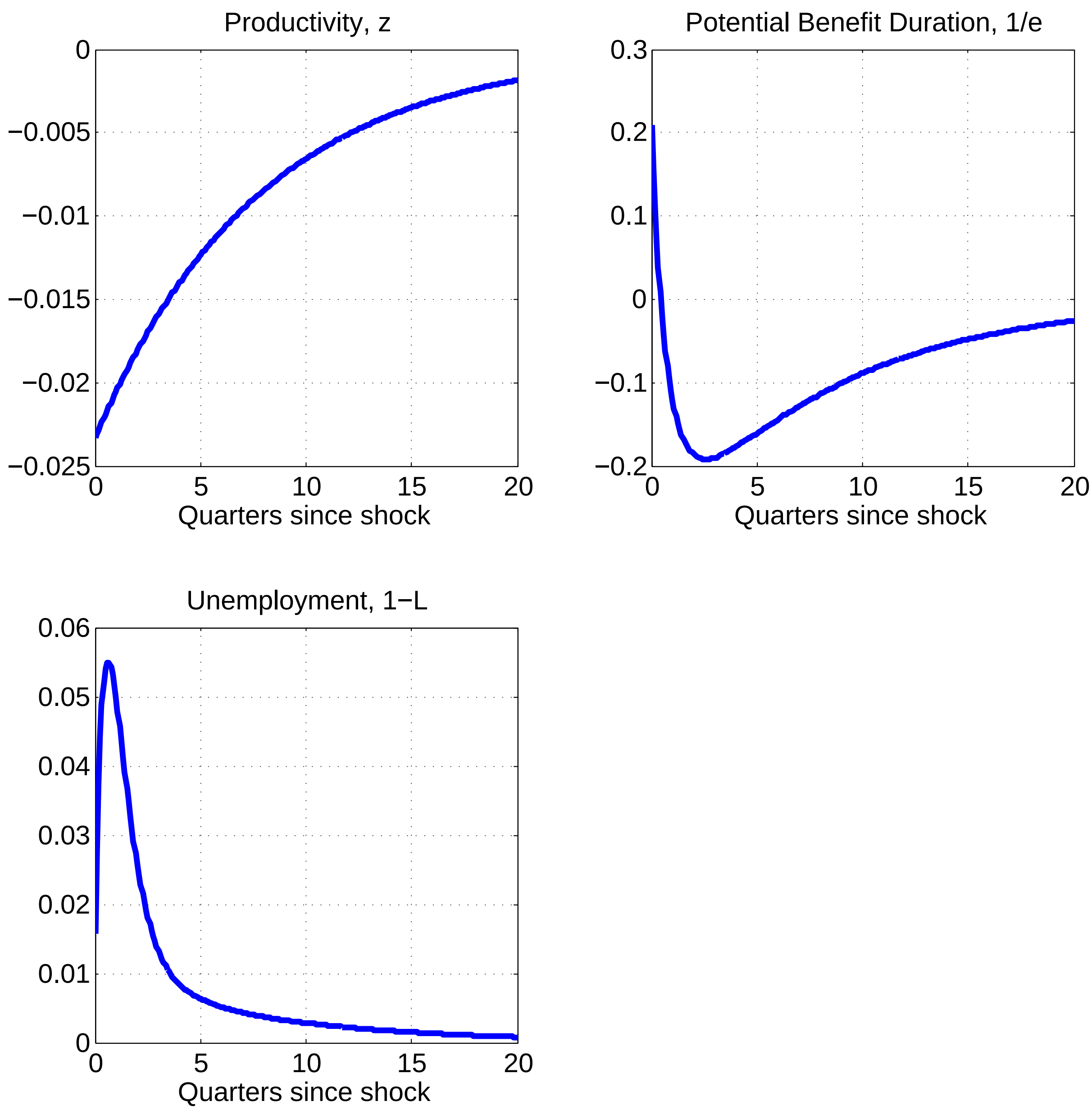

Figure 6: Response of benefit level to a 2.3% shock, fixing expected duration at 26 weeks

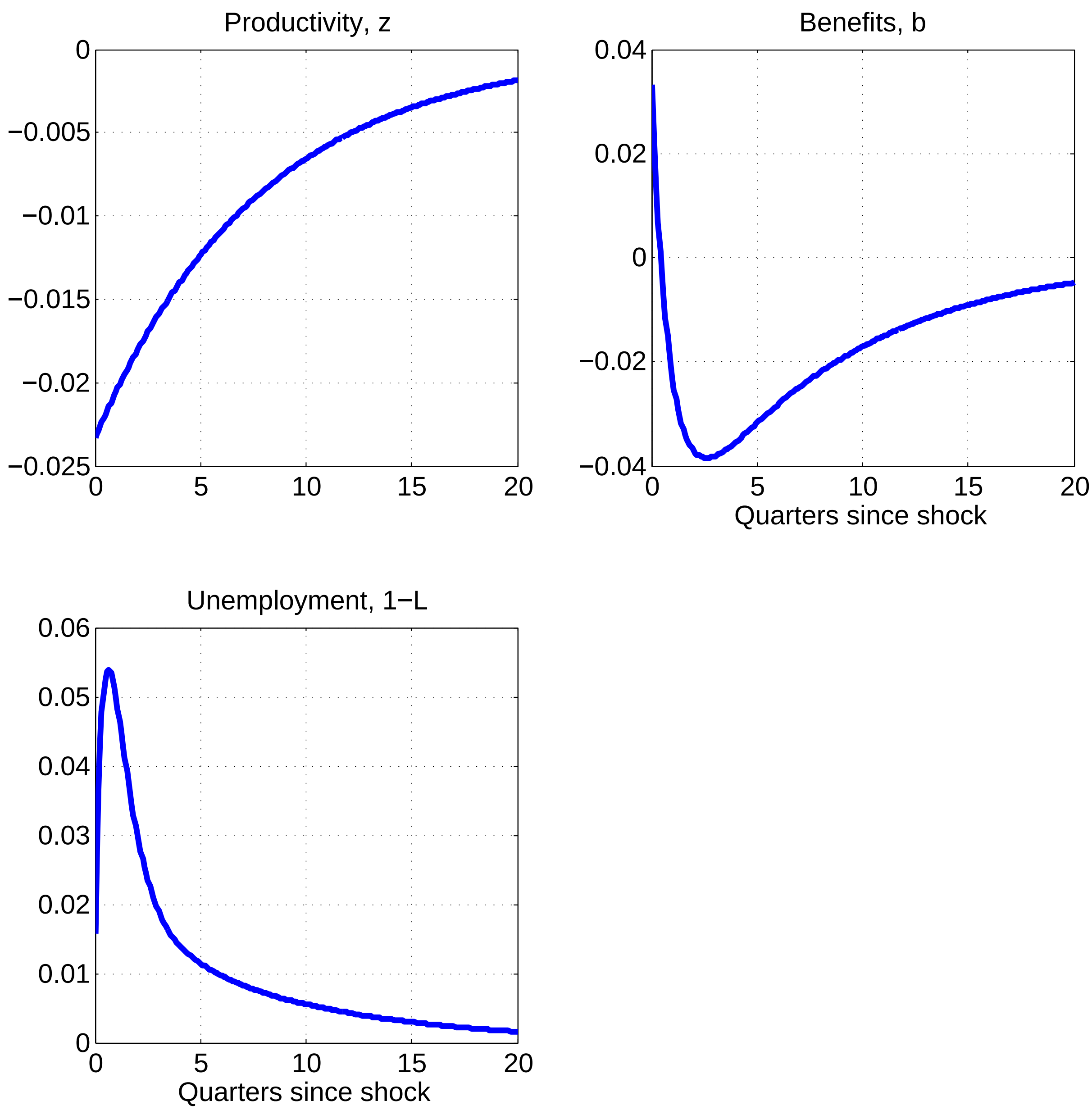

Figure 7: Response of benefit level to a 2.3% shock with no benefit expiration

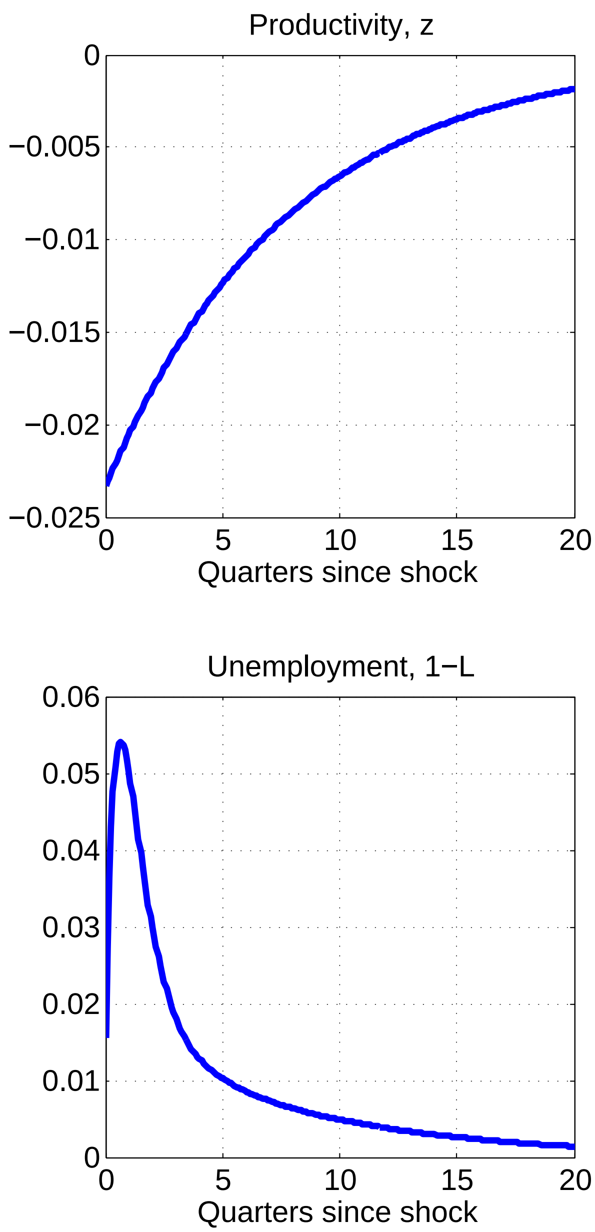

Figure 8: Response to a 2.3% shock with $h = 0$

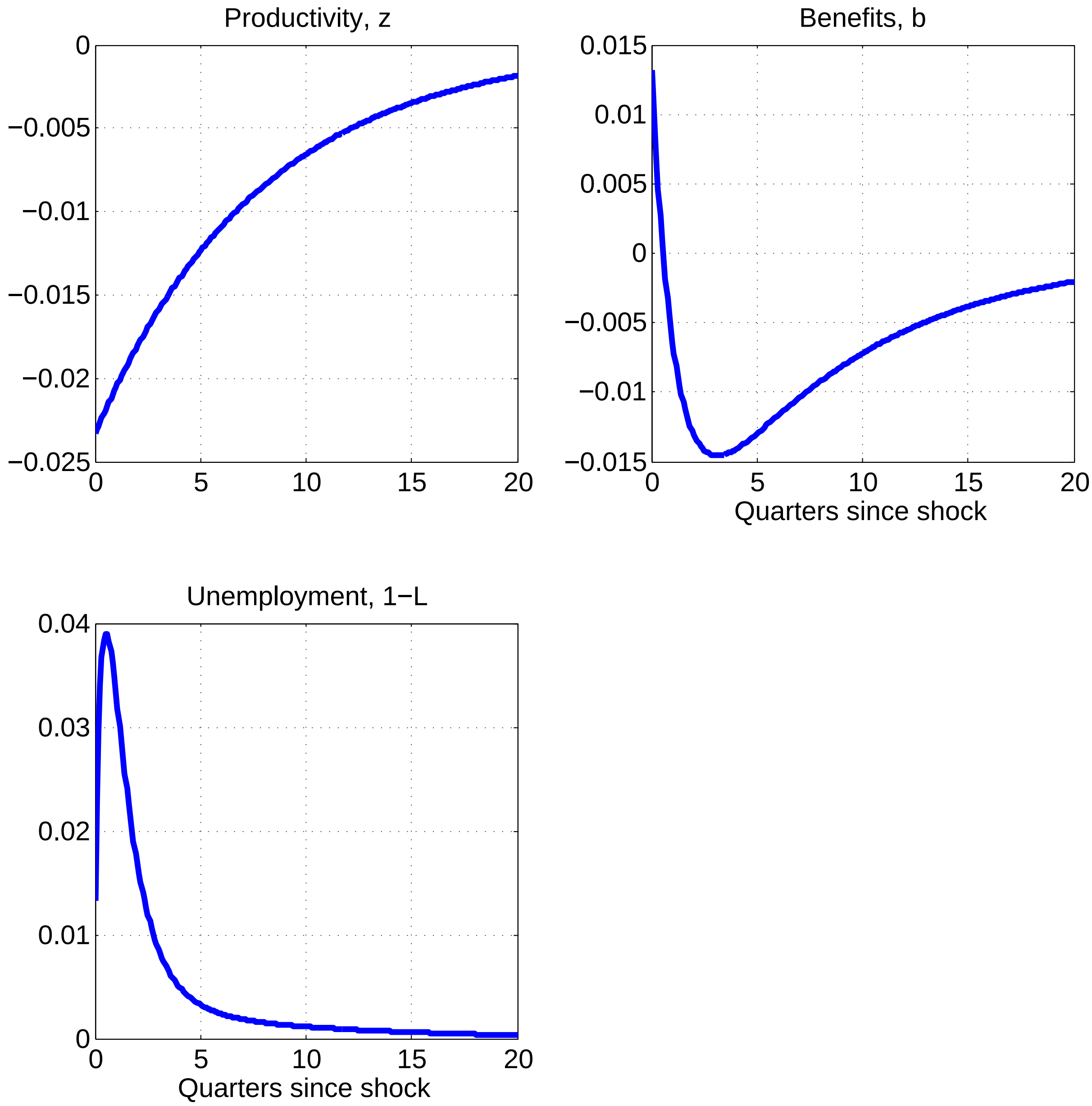

Figure 9: Response to a 2.3% shock with $\xi = 0.5$

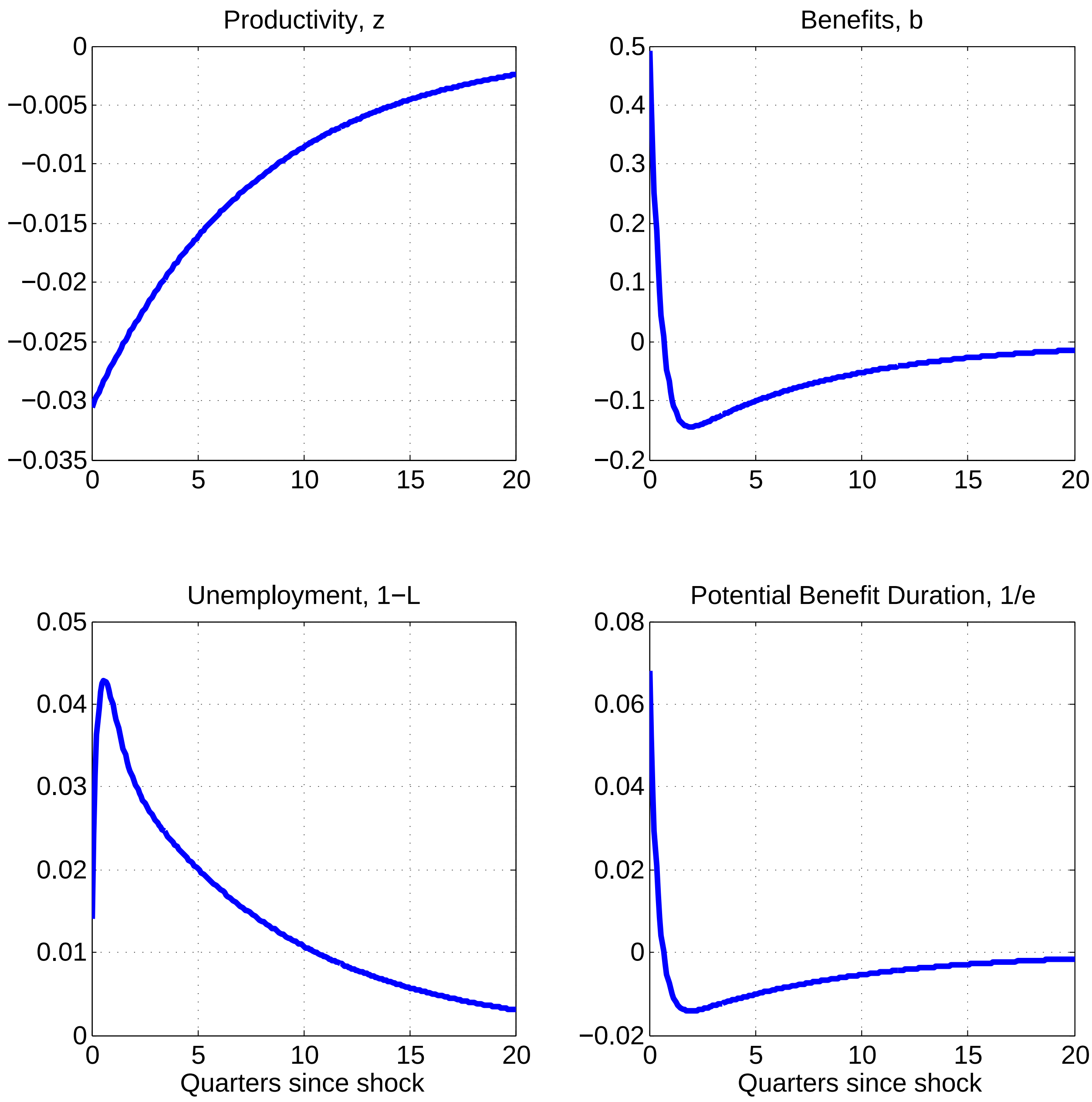

Figure 10: Response to a 2.3% shock under Shimer (2005) calibration

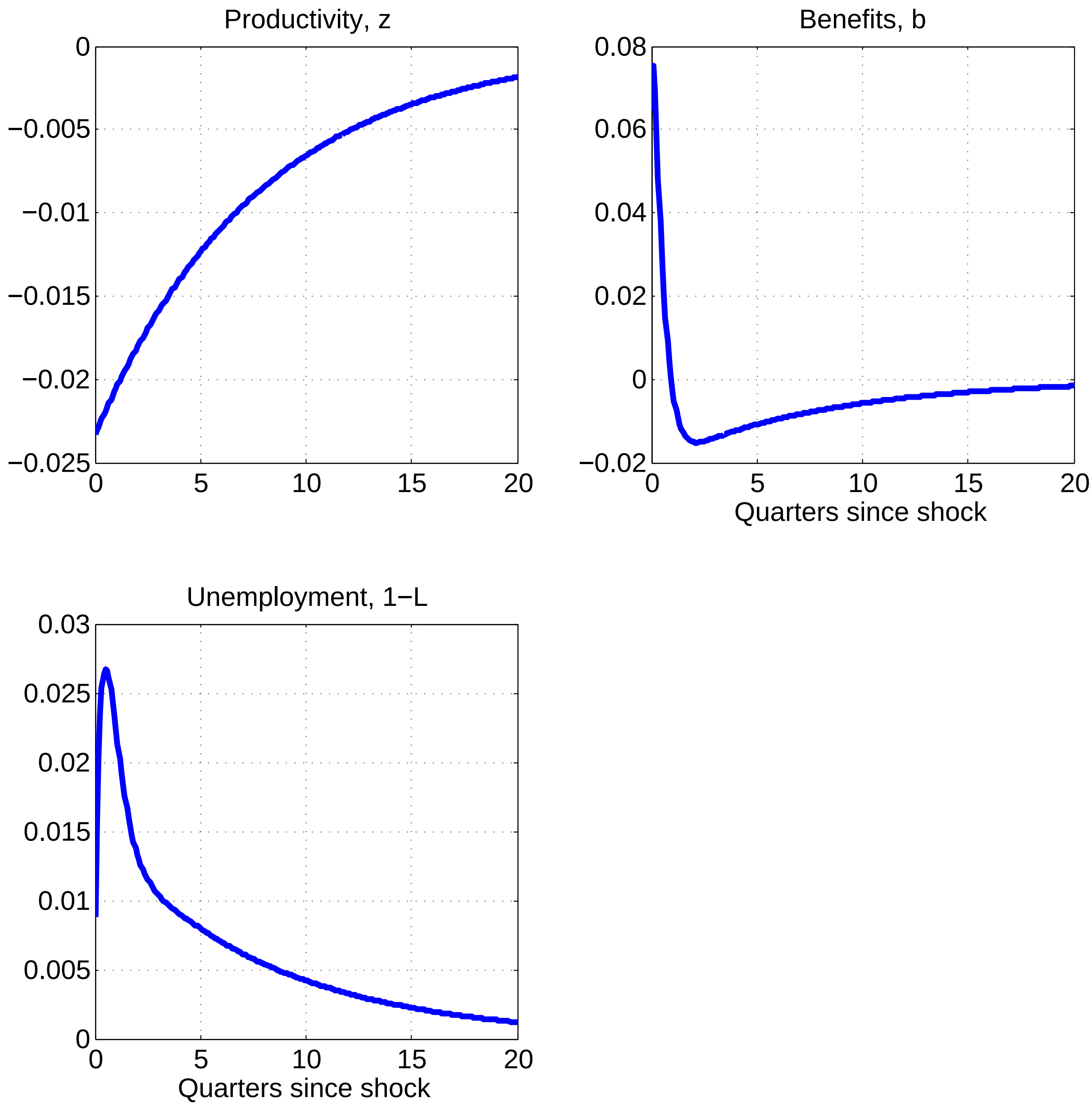

Figure 11: Response to a 2.3% shock under risk aversion of $\sigma = 1/2$

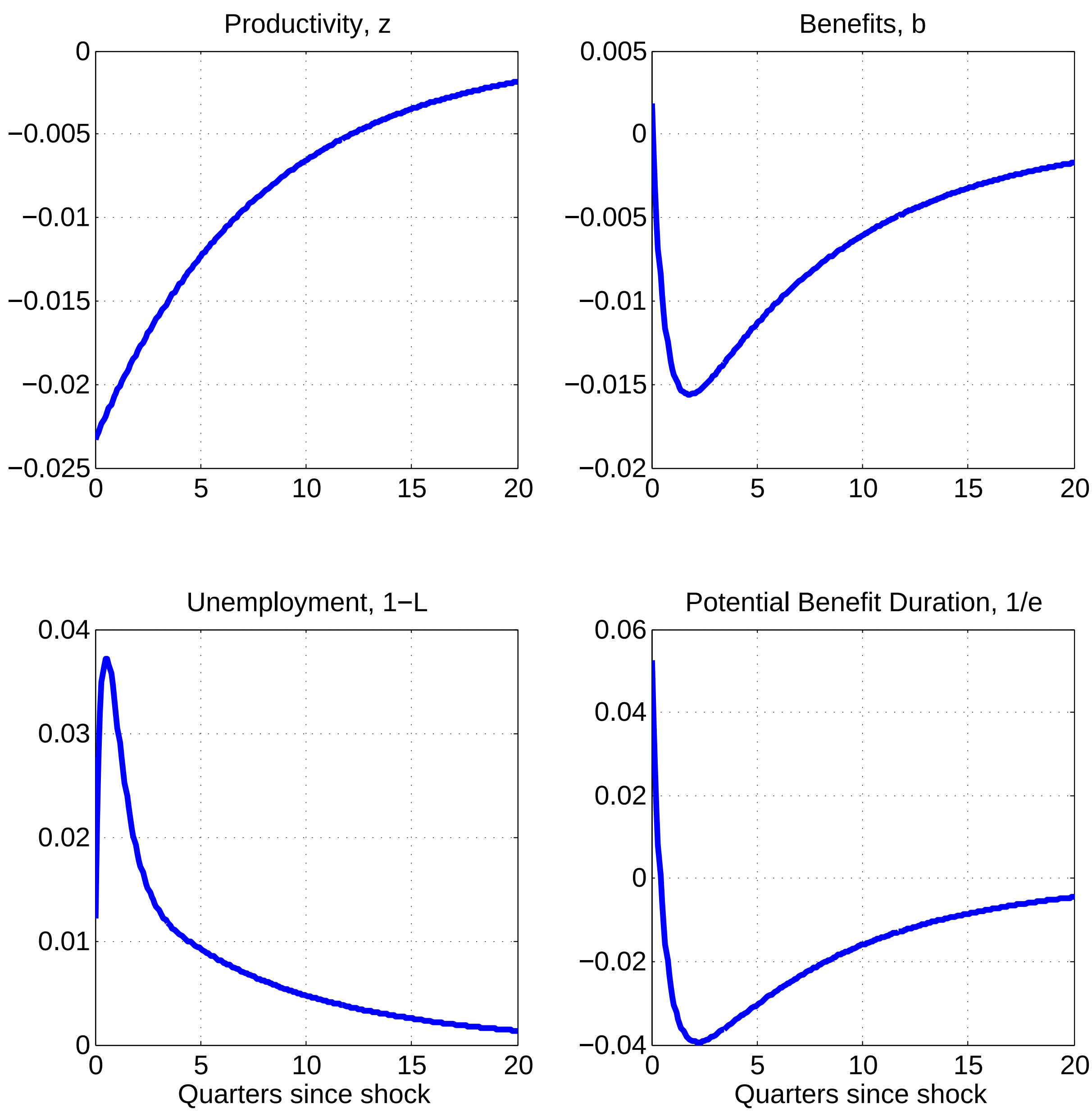

Figure 12: Response to a 2.3% shock under risk aversion of $\sigma = 2$