

# SAVING AND INVESTMENT IN A TWO-SECTOR MODEL OF ENDOGENOUS GROWTH OF A SMALL OPEN ECONOMY

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**Resumen:** Se desarrolla un modelo de dos sectores con crecimiento endógeno. El sector exportador es el único sector que genera progreso técnico. El conocimiento tecnológico puede ser usado por el sector importador. Las empresas enfrentan costos de ajuste para la inversión. El modelo tiene ecuaciones bien definidas para la tasa de crecimiento del capital y para la tasa de crecimiento del consumo. El modelo tiene una solución de estado estacionario. Se estudia la relación entre la tasa de interés y la tasa de crecimiento. La tasa de crecimiento óptima es mayor que la lograda en la economía de mercado sin gobierno. La política óptima es un subsidio a la inversión en el sector exportador.

**Abstract:** We develop a two-sector model of endogenous growth. The export sector is the only sector that generates technological progress. Technological knowledge can be used by the import sector. Firms face adjustment costs for investment. The model has well-defined equations for the growth rate of capital, and for the growth rate of consumption. The model has a steady-state solution. We study the relation between the interest rate and the growth rate. The optimal growth rate is higher than that achieved in the market economy without government. The optimal policy is an investment subsidy in the export sector.

## 1. Introduction

In the one-sector, AK, model of endogenous growth, Rebelo (1991), the accumulation of capital is determined residually. In this model, the steady-state growth rate is determined by the usual Euler equation. When we

suppose that the firms face adjustment costs for investment, the AK model has a well-defined equation for the accumulation of capital, a different dynamic behaviour, and so this model gives interesting economic findings (see Barro and Sala-i-Martin, 1992). Likewise, it is possible to introduce adjustment costs in multisector models (see Mercenier and Sampaio de Souza, 1994).

In this paper we develop a two-sector endogenous growth model with adjustment costs for investment. Casares (1996) studies outward-oriented economies. He argues that, in outward-oriented economies, the export sector firms are the ones that produce more technological change. Consequently, the export sector firms are technologically more advanced. In order to model outward-oriented economies in a two sector model, he assumes that the export sector firms are the firms with the fastest learning rate in the economy. Thus, he develops an export-sector-led endogenous growth model where the accumulation of capital is determined residually. We extend the model to allow adjustment costs for investment. Thus, we construct an export-sector-led endogenous growth model where there are well-defined equations for the growth rate of capital.

The economy, in this paper, consists of an export and import sector. The model has two key features on the production side. First, we assume that the export sector firms are the only firms that generate technological knowledge through learning by doing. Since knowledge spills over from one sector to another technological knowledge generated in the export sector is used by the import sector firms. Thus, the export sector leads technological advances.<sup>1</sup> The second feature is that the firms in both sectors face adjustment costs for investment. The adjustment cost per unit of investment is proportional to the ratio of investment to installed capital (as in Hayashi, 1982 and Abel and Blanchard, 1983). So in both sectors the model has well-defined equations for the steady-state growth rate of capital (Tobin's theory of investment in a context of two sectors, and of endogenous growth). On the consumption side, the model also has a well-defined equation for the growth rate of consumption (the usual Euler equation). The combination of these features results in a new model that is novel in the endogenous growth literature.<sup>2</sup>

<sup>1</sup> Feder (1983) argues that the export sector generates positive externalities on export sectors (see also Bardhan, 1993 and Edwards, 1993). Young (1991) discusses substantial spillover effects of knowledge across industries.

<sup>2</sup> Endogenous growth models with two production sectors are studied, among others, in Rebelo (1991), Mulligan and Sala-i-Martin (1993) and Barro and Sala-i-Martin (1995). Aghion and Howitt (1998) expound the main empirical criticisms of endogenous growth theory.

This model can be used to study how the endogenous variables respond to shocks. Barro and Sala-i-Martin (1992) study the AK model of endogenous growth without adjustment costs (Rebelo, 1991). In general, they conclude that shifts in the different parameters of the model produce movements in the steady-state interest rate and the steady-state growth rate that are positively correlated. Nevertheless, empirically this positive correlation is debatable. The authors cited above argue that there is no correlation between the interest rate and the growth rate. In order to eliminate the positive correlation, they introduce adjustment costs into the AK model. The AK model with adjustment costs predicts that shifts in the different parameters generate positively, or negatively correlated movements between the steady-state interest rate and the steady-state growth rate.<sup>3</sup> We will generalize these results with our two-sector model and will present new findings in this theme.

The objectives of this paper are: (i) to deduce, in the market economy, the steady-state growth rate, (ii) to show how the steady-state interest rate and the steady-state growth rate respond to shifts in the cost-of-adjustment parameter, the fixed-technology parameters, the terms of trade, and the preference parameters, (iii) to obtain, in the steady-state, the optimal growth rate, and finally, (iv) to deduce the optimal investment subsidy for the export sector.

First, we find the steady-state solution of the market economy. The steady-state growth rate exists and is unique. This is an important contribution to the understanding of the two-sector models of endogenous growth with externalities and with adjustment costs. Second, we find that shifts in the cost-of-adjustment parameter, in the fixed-technology parameters, or in the terms of trade, produce movements in the steady-state interest rate and the steady-state growth rate which are positively correlated. Shifts in preference parameters generate, in the steady-state, negatively correlated movements between the interest rate and the growth rate. The general correlation between the steady-state interest rate and the steady-state growth rate depends on the relative importance of shifts in the different parameters. Thus, we generalize the results obtained in Barro and Sala-i-Martin (1992). Fry (1997) shows that empirically the relation between the interest rate and the growth rate is an inverted U curve. Given that we use a

<sup>3</sup> Ploeg and Alogoskoufis (1994) explain that changes in budgetary policies produce a negative correlation between the steady-state interest rate and the steady-state growth rate.

two-sector model of a small open economy, we can study how changes in the terms of trade affect the steady-state interest rate and the steady-state growth rate. We define the terms of trade as the price of exportable good relative to the price of importable good. Third, we deduce the optimal growth rate and we also show that the optimal growth rate is higher than that achieved in the market economy without government. Finally, the government, in the market economy, can achieve the optimal growth rate through an investment subsidy in the export sector. Thus, the optimal policy is to promote the learning process (see Rauch, 1992). All these findings, in an export-sector-led endogenous growth model with adjustment costs for investment, are not present in the endogenous growth literature and are contributions to the economic theory.

The paper is organized as follows. In section 2, we develop a competitive market economy where firms maximize the present value of their cash flow subject to their constraints, and households maximize the present value of their utility subject to their constraints. In order to solve the model, we transform the model in terms of control-like and state-like variables (these variables are constants in the steady-state). In section 3, we obtain the steady-state solution of the market economy. In section 4, we solve the social planner's problem. That is, we obtain the optimal growth rate in the steady-state. In section 5, we deduce the optimal investment subsidy to the export sector. In section 6, we present our conclusions.

## 2. The Competitive Market Economy

The economy is small and open. Thus, the economy is small relative to world goods market, and it therefore takes the terms of trade as given. We define the terms of trade as the price of exportable good relative to the price of importable good. There are two production sectors, one producing a good that is exported, and one producing a good that is also imported. There is international trade in goods, but, for simplicity, there is no international trade in assets (see Grossman and Helpman, 1991). The value of exports is equal to the value of imports. All domestic financial assets are perfect substitutes, so rates of return on different assets are equalized to the domestic interest rate. The output in each sector is produced with physical capital, labour and technological knowledge. The output in each sector can be consumed or invested. Firms own capital stock and they have perfect foresight. Firms face

adjustment costs for investment. The total labour supply is constant. Labour is freely mobile between the sectors. Firms maximize the present value of their cash flow subject to the capital accumulation constraint. Households maximize the present value of an instantaneous utility function subject to the flow budget constraint. The consumption basket consists of exportable and importable goods.

### 2.1. Production Functions

There are a large number,  $N_X$ , of competitive export firms and  $N_M$  competitive import firms. We drop the time index for legibility. Thus, we assume that the production functions for the  $i$ th export firm and the  $i$ th import firm are Cobb-Douglas:

$$Y_{Xi} = A_X K_{Xi}^\alpha L_{Xi}^{1-\alpha} T_1 \text{ where } T_1 = K_X^{1-\alpha} \quad (1)$$

$$Y_{Mi} = A_M K_{Mi}^\beta L_{Mi}^{1-\beta} T_2 \text{ where } T_2 = K_X^{1-\beta} \quad (2)$$

where  $Y_{Xi}$  is the output of the  $i$ -th export firm,  $K_{Xi}$  the stock of physical capital accumulated from the exportable good of the  $i$ -th export firm,  $L_{Xi}$  the quantity of labour employed by the  $i$ -th export firm,  $\alpha$  and  $1 - \alpha$  the shares of  $K_{Xi}$  and  $L_{Xi}$  respectively, and  $A_X$  is a fixed technology parameter. It is assumed that the  $i$ -th export firm uses only  $K_{Xi}$ . Technological knowledge is created through learning by doing in the export sector, thus knowledge is a by-product of investment. Then, knowledge of the  $i$ -th export firm increases with  $K_{Xi}$ . We denote  $K_X$  as the aggregate stock of physical capital accumulated by the exportable good in the export sector. Since knowledge is a public good, there are intra-industry spillover effects, that is, the knowledge of the  $i$ -th export firm is available to all the export firms. Thus,  $K_X$  is the index of the aggregate stock of knowledge in the export sector. Therefore,  $T_1$  is the contribution of technological knowledge to  $Y_{Xi}$ . In this way,  $T_1$  is an external variable which affects the  $i$ -th export production function. This specification follows Arrow (1962) and Sheshinski (1967).

In equation (2),  $Y_{Mi}$  is the output of the  $i$ -th import firm,  $K_{Mi}$  the stock of physical capital accumulated by the importable good of the  $i$ -th import firm,  $L_{Mi}$  the quantity of labour employed by the  $i$ -th import firm,  $\beta$  and  $1 - \beta$  the shares of  $K_{Mi}$  and  $L_{Mi}$  respectively, and  $A_M$  is a fixed-technology parameter. Since there are spillover effects of knowledge between the two

production sectors (inter-industry benefits of knowledge), technological knowledge generated in the export sector can be used in the import sector. Thus,  $T_2$  is the contribution of technological knowledge to  $Y_{Mi}$ . Therefore,  $T_2$  is an external variable which affects the  $i$ -th import production function.

Since in equation (1) the exponent on  $K_X$  in the externality  $\Gamma_1$  is  $1 - \alpha$ , then the production function of the  $i$ -th export firm has constant returns with respect to  $K_{Xi}$  and  $K_X$  (for a broad measure of capital). Since we are assuming in equation (2) that the exponent on  $K_X$  in the externality  $T_2$  is  $1 - \beta$ , then the production function of the  $i$ -th import firm also has constant returns with respect to  $K_{Mi}$  and  $K_X$  (for a broad measure of capital). Thus, with constant returns to a broad measure of capital in both sectors, the model exhibits endogenous growth and has a solution. This specification follows Romer (1986, 1989) with respect to the one-sector endogenous-growth model (see also Jones and Manuelli, 1990). Mulligan and Sala-i-Martin (1993) deduce necessary conditions so that the models with two types of capital and two sectors display endogenous growth (they consider the more general case where capital goods are used in both sectors and non-reproducible inputs are not allowed). We assume zero rate of population growth.

## 2.2. The Export Sector

The  $i$ -th export firm owns capital stock. The total investment spending of the  $i$ -th export firm is:

$$P_X^* I_{Xi}(1 + \phi_{Xi}) = P_X^* I_{Xi} + P_X^* I_{Xi}\phi_{Xi} \quad (3)$$

where  $P_X^*$  is the world price of the exportable good,  $I_{Xi}$  is the investment to increase  $K_{Xi}$  and  $\phi_{Xi}$  is the adjustment cost per unit of  $I_{Xi}$ , which is defined as:

$$\phi_{Xi} = \frac{b}{2} \frac{I_{Xi}}{K_{Xi}} \quad (4)$$

The total installation cost is  $I_{Xi}\phi_{Xi}$  which is linearly homogeneous in  $I_{Xi}$  and  $K_{Xi}$ . This formulation is consistent with maintaining a steady-state growth path (see Turnovsky, 1996). Note that adjustment cost per unit of  $I_{Xi}$  is proportional to the ratio of  $I_{Xi}$  to  $K_{Xi}$  as in Hayashi (1982), Abel and Blanchard (1983) and Blanchard and Fischer (1989). We define the parameter  $b$  as the cost-of-adjustment parameter, so  $b$  is the sensitivity of the

adjustment costs to the total amount invested ( $b > 0$ ). We assume that  $b$  has the same value in the two sectors.

The  $i$ -th export firm takes into account the expected future price of the exportable good (that remains constant all the time), and the time paths for wage rate,  $w$ , and interest rate,  $r$ , so that  $\{w(t), r(t)\}_{t=[0, \infty)}$  is given. The decision problem of the  $i$ -th export firm is to select the time paths of investment and employment that maximize the present discounted value of its cash flow:

$$\max V_{Xi} = \int_0^\infty (P_X^* A_X K_{Xi}^\alpha L_{Xi}^{1-\alpha} T_1 - wL_{Xi} - P_X^* I_{Xi} - P_X^* \frac{b}{2} \frac{I_{Xi}^2}{K_{Xi}}) e^{-\int_0^t r(v)dv} dt \quad (5)$$

subject to the capital accumulation constraint,  $\dot{K}_{Xi} = I_{Xi}$ . The Hamiltonian is:

$$H = (P_X^* A_X K_{Xi}^\alpha L_{Xi}^{1-\alpha} T_1 - wL_{Xi} - P_X^* I_{Xi} - P_X^* \frac{b}{2} \frac{I_{Xi}^2}{K_{Xi}} + q_X I_{Xi}) e^{-\int_0^t r(v)dv} \quad (6)$$

where  $q_X$  is the current-value shadow price, in units of contemporaneous output of  $K_{Xi}$ , that is,  $q_X = \Gamma_X e^{\int_0^t r(v)dv}$ , where  $\Gamma_X$  is the present-value shadow price. The control variables are  $L_{Xi}$  and  $I_{Xi}$  and the state variable is  $K_{Xi}$ . The  $i$ -th export firm takes the externality  $T_1$  as given. Supposing identical firms and that in equilibrium all the export firms make the same choices, we have  $Y_X = N_X Y_{Xi}$ ,  $K_X = N_X K_{Xi}$ ,  $L_X = N_X L_{Xi}$  and  $I_X = N_X I_{Xi}$ , where  $Y_X$  is the aggregate output in the export sector,  $K_X$  is the aggregate stock of physical capital accumulated from the exportable good,  $L_X$  is the aggregate labour employed in the export sector and  $I_X$  is the aggregate investment in the export sector. The first order conditions are:

$$w = P_X^* A_X K_X^\alpha (1 - \alpha) L_X^{-\alpha} \quad (7)$$

$$q_X = P_X^* (1 + b \frac{I_X}{K_X}) \quad (8)$$

$$r = \frac{P_X^* A_X \alpha L_X^{1-\alpha} + P_X^* (b/2) (I_X/K_X)^2}{q_X} + \frac{\dot{q}_X}{q_X} \quad (9)$$

$$\lim_{t \rightarrow \infty} e^{-\int_0^t r(v)dv} q_X K_X = 0 \quad (10)$$

Equation (7) indicates that the wage rate is equal to the value of marginal product of labour in the export sector. Notice that we have explicitly used the value of the externality.<sup>4</sup> Equation (8) states that the shadow price of  $K_X$ ,  $q_X$ , is equal to the purchase price of  $K_X$ ,  $P_X^*$ , plus the marginal adjustment cost multiplied by  $P_X^*$ . Equation (9) indicates that the expected return on domestic bonds is equal to the total private return of holding  $K_X$ , that is, interest rate is equal to the private marginal product of  $K_X$  plus the marginal reduction in adjustment costs arising from an additional unit of  $K_X$ , all multiplied by  $P_X^*$  and all deflated by  $q_X$ , plus the rate of capital gains. Equation (10) is the transversality condition.

The average  $q_X$  is the ratio  $V_{X_i}/K_{X_i} = V_X/K_X$ , where  $V_X$  is the stock-market value of the export firms. Thus the average  $q_X$  is the market value of a unit of  $K_X$ . Since the production functions and the adjustment cost functions of the export firms are linearly homogeneous in investment, physical capital and labour, and firms operate in competitive markets, it is easy to show that the shadow price  $q_X$  is equal to the average  $q_X$ . Using equation (8) and the aggregate capital accumulation constraint,  $\dot{K}_X = I_X$ , we obtain the following equation for the growth rate of  $K_X$ :

$$\frac{\dot{K}_X}{K_X} = \frac{q_X - P_X^*}{P_X^* b} \quad (11)$$

then the growth rate of  $K_X$  is positive (negative) when the market value of a unit of installed capital,  $q_X$ , is more than (less than) the price of a unit of uninstalled capital,  $P_X^*$ . This is an application of Tobin's theory of investment in a context of two sectors, and of endogenous growth.

It is assumed that the  $i$ th export firm finances investment, including adjustment costs, by issuing new bonds.<sup>5</sup> This implies that the interest rate is directly observable. The aggregate bonds of the export sector firms are denoted by  $B_X$ , so the aggregate issue of bonds is:

$$\dot{B}_X = P_X^* I_X + P_X^* \frac{b}{2} \frac{I_X^2}{K_X} \quad (12)$$

The  $i$ -th export firm distributes dividends to the households. The aggregate dividends of the export sector firms,  $\pi_X$ , are:

<sup>4</sup> When the firm makes its choice, it does not perceive the externality, but the externality nevertheless affects the production function and the first order conditions.

<sup>5</sup> The amount of equity shares remains unchanged.



$$\pi_X = P_X^* Y_X - wL_X - rB_X \tag{13}$$

where the aggregate production function of the export sector,  $Y_X$ , is given by:

$$Y_X = A_X K_X^\alpha L_X^{1-\alpha} T_1 = A_X K_X L_X^{1-\alpha} \tag{14}$$

Note that the aggregate production function of the export sector has increasing returns to scale (on the right-hand side of the second equality, we have explicitly used the value of the externality).

### 2.3. The Import Sector

The  $i$ -th import firm owns capital stock. The total investment spending is:

$$P_M^* I_{Mi} (1 + \phi_{Mi}) = P_M^* I_{Mi} + P_M^* I_{Mi} \phi_{Mi} \tag{15}$$

where  $P_M^*$  is the world price of the importable good,  $I_{Mi}$  is the investment to increase  $K_{Mi}$  and  $\phi_{Mi}$  is the adjustment cost per unit of  $I_{Mi}$  which is defined as:

$$\phi_{Mi} = \frac{b}{2} \frac{I_{Mi}}{K_{Mi}} \tag{16}$$

The total installation cost is  $I_{Mi} \phi_{Mi}$ . The decision problem of the  $i$ -th import firm is to select the time paths of  $L_{Mi}$  and  $I_{Mi}$  (given  $P_M^*$  and  $\{w(t), r(t)\}_{t=[0, \infty)}$ ), that maximize the present discounted value of its cash flow:

$$\max V_{Mi} = \int_0^\infty (P_M^* A_M K_{Mi}^\beta L_{Mi}^{1-\beta} T_2 - wL_{Mi} - P_M^* I_{Mi} - P_M^* \frac{b}{2} \frac{I_{Mi}^2}{K_{Mi}}) e^{-\int_0^t r(v)dv} dt \tag{17}$$

subject to the capital accumulation constraint,  $\dot{K}_M = I_{Mi}$ . The Hamiltonian is:

$$H = (P_M^* A_M K_{Mi}^\beta L_{Mi}^{1-\beta} T_2 - wL_{Mi} - P_M^* I_{Mi} - P_M^* \frac{b}{2} \frac{I_{Mi}^2}{K_{Mi}} + q_M I_{Mi}) e^{-\int_0^t r(v)dv} \tag{18}$$

where  $q_M$  is the current-value shadow price, in units of contemporaneous output, of  $K_{Mi}$ , that is,  $q_M = \Gamma_M e^{\int_0^t r(v)dv}$ , where  $\Gamma_M$  is the present-value shadow price. The control variables are  $L_{Mi}$  and  $I_{Mi}$  and the state variable is  $K_{Xi}$ . The  $i$ -th import firm takes the externality  $T_2$  as given. Supposing identical firms and that in equilibrium all the import firms make the same choices, we have  $Y_M = N_M Y_{Mi}$ ,  $K_M = N_M K_{Mi}$ ,  $L_M = N_M L_{Mi}$  and  $I_M = N_M I_{Mi}$ , where  $Y_M$  is the aggregate output in the import sector,  $K_M$  the aggregate stock of physical capital accumulated from the importable good,  $L_M$  is the aggregate labour employed in the import sector and  $I_M$  is the aggregate investment in the import sector. The first order conditions are:

$$w = P_M^* A_M K_M^\beta K_X^{1-\beta} (1-\beta) L_M^{-\beta} \tag{19}$$

$$q_M = P_M^* (1 + b \frac{I_M}{K_M}) \tag{20}$$

$$r = \frac{P_M^* A_M \beta K_M^{\beta-1} K_X^{1-\beta} L_M^{1-\beta} + P_M^* (b/2) (I_M/K_M)^2}{q_M} + \frac{q_M}{q_M} \tag{21}$$

and the transversality condition:

$$\lim_{t \rightarrow \infty} e^{-\int_0^t r(v)dv} q_M K_M = 0 \tag{22}$$

Equation (19) equates the wage rate to the value of the marginal product of labour in the import sector. Equation (20) indicates that the market value of a unit of  $K_M$ ,  $q_M$ , is equal to the purchase price of  $K_M$ ,  $P_M^*$ , plus the marginal adjustment cost multiplied by  $P_M^*$ . Equation (21) states that domestic interest rate is equal to the private marginal product of  $K_M$  plus the marginal reduction in adjustment costs arising from an additional unit of  $K_M$ , all multiplied by  $P_M^*$  and all deflated by  $q_M$ , plus the rate of capital gains. Using equation (20) and the aggregate capital accumulation constraint,  $\dot{K}_M = I_M$ , we obtain the equation for the growth rate of  $K_M$ :

$$\frac{\dot{K}_M}{K_M} = \frac{q_M - P_M^*}{P_M^* b} \tag{23}$$

thus the import firms have positive (negative) rates of capital accumulation if  $q_M > P_M^*$  ( $q_M < P_M^*$ ).

The  $i$ -th import firm finances investment, including installation costs, by issuing new bonds. The aggregate bonds of the import sector firms are denoted by  $B_M$ , so the aggregate issue of bonds is:

$$\dot{B}_M = P_M^* I_M + P_M^* \frac{b}{2} \frac{I_M^2}{K_M} \quad (24)$$

The  $i$ -th import firm distributes dividends to the households. The aggregate dividends of the import sector firms,  $\pi_M$ , are:

$$\pi_M = P_M Y_M - wL_M - rB_M \quad (25)$$

where the aggregate production function of the import sector,  $Y_M$ , is given by:

$$Y_M = A_M K_M^\beta L_M^{1-\beta} T_2 = A_M K_M^\beta K_X^{1-\beta} L_M^{1-\beta} \quad (26)$$

Note that the aggregate production function of the import sector has increasing returns to scale.

#### 2.4. The Representative Household

We assume households with perfect foresight, thus they take

$$\{w(t), r(t)\}_{t=[0, \infty)}$$

as given. The representative household can not lend or borrow in the international financial market. It views  $B_X$  and  $B_M$  as perfect substitutes, thus rates of return on  $B_X$  and  $B_M$  are equalized to the domestic interest rate. The decision problem of the representative household is to choose a time path of aggregate consumption that maximizes the present discounted value of a utility function with a constant elasticity of intertemporal substitution:

$$\max U(0) = \int_0^\infty \frac{C^{1-1/\sigma}}{1-1/\sigma} e^{-\rho t} dt \quad (27)$$

subject to the household's flow budget constraint:

$$\dot{B} = rB + w(L_X + L_M) + \pi_X + \pi_M - P_c C \quad (28)$$

and the solvency condition:

$$\lim_{t \rightarrow \infty} e^{-\int_0^t r(v)dv} B = 0 \tag{29}$$

In equation (27),  $C$  is aggregate real consumption,  $\sigma$  is the elasticity of intertemporal substitution and  $\rho$  is the subjective discount factor. In equation (28),  $B$  (assets) is  $B = B_X + B_M$ , thus the representative household receives interest income,  $rB$ , labour income and dividends from the firms. The representative household allocates this income between consumption and saving. The total expenditure on consumption,  $P_C C$ , in equation (28), is given by:

$$P_C C = P_X^* C_X + P_M^* C_M \tag{30}$$

where  $P_C$  is the consumer price index and  $C_X$  and  $C_M$  are the consumption of the exportable and the importable good, respectively. Savings are the demand for new bonds,  $\dot{B} = \dot{B}_X + \dot{B}_M$

We notice that  $C$  is a homothetic index of  $C_X$  and  $C_M$ , that is,  $C = DC_X^\gamma C_M^{1-\gamma}$ , where  $D = 1/[\gamma^\gamma(1-\gamma)^{1-\gamma}]$  is a parameter, and  $\gamma$  and  $(1-\gamma)$  are the shares of  $C_X$  and  $C_M$  with respect to total consumption expenditure. We also notice that  $P_C$  is given by  $P_C = P_X^\gamma P_M^{1-\gamma}$ , with the characteristic that when the total expenditure on consumption, equation (30), is divided by  $P_C$ , the resultant real consumption  $C$  is a measure of the level of utility (see Gavin, 1991).

The Hamiltonian is:

$$H = \left[ \frac{C^{1-1/\sigma}}{1-1/\sigma} + \lambda_C [rB + w(L_X + L_M) + n_X + \pi_M - P_C C] \right] e^{-\rho t} \tag{31}$$

where  $\lambda_C$  is the current-value shadow price of  $B$ , that is,  $\lambda_C = \Gamma_C e^{\rho t}$ , where  $\Gamma_C$  is the present-value shadow price of  $B$ .

The first order conditions are:

$$\frac{\dot{\lambda}_C}{\lambda_C} = (\rho - r) \tag{32}$$

$$\lambda_C = \frac{C^{-1/\sigma}}{P_C} \tag{33}$$

and the boundary condition  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_C B = 0$ .

Taking logs and derivatives of equation (33) with respect to time, and substituting the result in (32), we obtain the dynamic allocation condition for aggregate consumption among different time periods:

$$\frac{\dot{C}}{C} = \sigma(r - p) \quad (34)$$

In order to select the optimal consumption basket of  $C_X$  and  $C_M$ , the representative household maximizes the utility  $u = C_X^\gamma C_M^{1-\gamma}$  subject to (30), where  $P_C C$  is determined by the household's flow budget constraint, equation (28). We now get the static first order condition:

$$\frac{U_{C_M}}{U_{C_X}} = \frac{(1-\gamma)C_X}{\gamma C_M} = \frac{P_M^*}{P_X^*} \quad (35)$$

where  $U_{C_M}$  and  $U_{C_X}$  are the marginal utilities of the importable good and the exportable good respectively. Equation (35) states that the marginal rate of substitution of  $C_M$  for  $C_X$  is equal to relative world prices. Using equations (30) and (35), we obtain the levels of  $C_X$  and  $C_M$ .  $C_X = (\gamma P_C C) / P_X^*$  and  $C_M = [(1-\gamma)P_C C] / P_M^*$ .

### 2.5. Equilibrium in Goods and Labour Markets

Substituting equations (12), (13), (24), (25), (30) and  $\dot{B} = \dot{B}_X + \dot{B}_M$  in the household's flow budget constraint, equation (28), we obtain the aggregate equilibrium condition for the goods market at world prices:

$$P_X^* Y_X + P_M^* Y_M = P_X^* C_X + P_M^* C_M + P_X^* L_X + P_X^* \frac{b}{2} \frac{I_X^2}{K_X} + P_M^* I_M + P_M^* \frac{b}{2} \frac{I_M^2}{K_M} \quad (36)$$

where the value of total output at world prices is  $Y = P_X^* Y_X + P_M^* Y_M$ .

Since the size of total population is constant and equal to the total labour supply,  $L$ , then the total labour supply is also constant and normalized to one. Thus, the equilibrium condition for the labour market is:

$$L_X + L_M = L = n + (1 - n) = 1 \quad (37)$$

where  $n$  is the fraction of labour employed in the export sector and  $(1 - n)$  the fraction of labour employed in the import sector.

### 2.6. Transformation of Variables

If we consider that the economy has a positive growth rate, the variables  $K_X$ ,  $K_M$ ,  $Y_X$ ,  $Y_M$ ,  $Y$  and  $C$  grow forever. In order to study the model, it is convenient to transform the variables of the model to variables that are constants in the steady-state. Thus, we reformulate the model in terms of state-like and control-like variables. These variables are constants in the steady-state (see Mulligan and Sala-i-Martin, 1991; 1993). Let  $z = K_M/K_X$  be the state-like variable and let  $a = C/K_M$  be the first control-like variable. Given that  $n$ , the fraction of labour employed in the export sector, is constant in the steady-state,  $n$  is the second control-like variable.

The aggregate production functions become:

$$Y_X = A_X K_X n^{1-\alpha} \quad (38)$$

$$Y_M = A_M Z^\beta K_X (1-n)^{1-\beta}. \quad (39)$$

Next, using the equilibrium condition for the labour market, the first order conditions (7),(8),(9),(19),(20) and (21) can be rewritten as:

$$w = P_X^* A_X K_X (1-\alpha)n^{-\alpha} \quad (40)$$

$$q_X = P_X^* (1 + b g_{K_X}) \quad (41)$$

$$r = \frac{P_X^* A_X \alpha n^{1-\alpha} + P_X^* (b/2) g_{K_X}^2}{q_X} + \frac{\dot{q}_X}{q_X} \quad (42)$$

$$w = P_M^* A_M z^\beta K_X (1-\beta)(1-n)^{-\beta} \quad (43)$$

$$q_M = P_M^* (1 + b g_{K_M}) \quad (44)$$

$$r = \frac{P_M^* A_M \beta z^{\beta-1} (1-n)^{1-\beta} + P_M^* (b/2) g_{K_M}^2}{q_M} + \frac{\dot{q}_M}{q_M} \quad (45)$$

where  $g_{K_X} = I_X/K_X = \dot{K}_X/K_X$  and  $g_{K_M} = I_M/K_M = \dot{K}_M/K_M$  are the growth rates of  $K_X$  and  $K_M$  respectively. The growth rate of consumption,  $g_C$ , remains similar to equation (34), where the interest rate now is given by (42) or (45).

Finally, using the aggregate production functions, (38) and (39), the equilibrium condition for the goods market at world prices can be rewritten as:

$$\begin{aligned}
 P_X^* A_X K_X n^{1-\alpha} + P_M^* A_M z^\beta K_X (1-n)^{1-\beta} = P_C C + P_X^* K_X (g_{K_X} + \frac{b}{2} g_{K_X}^2) \\
 + P_M^* K_M (g_{K_M} + \frac{b}{2} g_{K_M}^2)
 \end{aligned}
 \quad (46)$$

### 3. The Steady-State Solution

In this section we solve the model in the steady-state. The variables  $z$ ,  $n$ , and  $a = C/K_M$  are constants in the steady-state. Then, using equations (38), (39) and  $Y = P_X^* Y_X + P_M^* Y_M$ , it is easy to show that  $\dot{Y}_X/Y_X = g_{Y_X} = g_{K_X}$ ,  $\dot{Y}_M/Y_M = g_{Y_M} = g_{K_X}$  and that  $\dot{Y}/Y = g_Y = g_{K_X}$ , where  $g_{Y_X}$ ,  $g_{Y_M}$  and  $g_Y$  are the growth rates of  $Y_X$ ,  $Y_M$  and  $Y$  respectively. Furthermore, in the steady-state,  $g_{K_X} = g_{K_M} = g_C = g_Y = g$  and  $\dot{q}_X = \dot{q}_M = 0$ , where  $g$  is the steady-state growth rate. Using these relations, we proceed to solve the model.

Equating (40) and (43), we obtain:

$$P_X^* A_X (1-\alpha)n^{-\alpha} = P_M^* A_M z^\beta (1-\beta)(1-n)^{-\beta} \quad (47)$$

this equation is the static efficient allocation condition for labour between the sectors, where the value of marginal product of labour in both sectors is equal.

Using (41) and (42), we obtain the steady-state interest rate:

$$r = \frac{A_X \alpha n^{1-\alpha} + (b/2)g^2}{(1+bg)} \quad (48)$$

or alternatively with (44) and (45), we get:

$$r = \frac{A_M \beta z^{\beta-1} (1-n)^{1-\beta} + (b/2)g^2}{(1+bg)} \quad (49)$$

Equating (48) and (49), we obtain the dynamic arbitrage condition for the two capital goods:

$$A_X \alpha n^{1-\alpha} = A_M \beta z^{\beta-1} (1-n)^{1-\beta}. \quad (50)$$

This condition states that, in the steady state, the private marginal product of  $K_X$  is equal to the private marginal product of  $K_M$ . The condition (50) constitutes the basis for the decision as to whether to accumulate  $K_X$  or  $K_M$ . Since  $K_X$  and that  $K_M$  are only used in their respective sectors, our model does not include static allocation conditions for the existing capitals (see Rebelo, 1991).

The model is recursive, that is, with equations (47) and (50), we can determine the values of  $n$  and  $z$ . These variables determine the values of remaining variables of the model, that is, they determine the growth rate of the economy and the value of  $a = C/K_M$ .

With equations (47) and (50), we obtain the level of  $n$ :

$$n = \frac{1}{\left[ \frac{(1-\beta)}{(1-\alpha)} \left( \frac{\beta}{\alpha} \right)^{\frac{\beta}{1-\beta}} \right]^{\frac{(1-\beta)}{\alpha-\beta}} \left( \frac{P_M^*}{P_X^*} \right)^{\frac{(1-\beta)}{\alpha-\beta}} \left( \frac{A_M}{A_X} \right)^{\frac{1}{\alpha-\beta}}} \quad (51)$$

we can see that  $n$  depends only on parameters. Since the value of  $n$  must be less than 1, we impose the following two numerical-simulation conditions that ensure that the denominator of (51) is bigger than 1, and so  $n < 1$ : (i)  $\alpha > \beta$  and (ii):

$$\frac{P_M^*}{P_X^*} > \frac{1}{\left[ \frac{(1-\beta)}{(1-\alpha)} \left( \frac{\beta}{\alpha} \right)^{\frac{\beta}{1-\beta}} \right]^{\frac{1}{\alpha-\beta}} \left( \frac{A_M}{A_X} \right)^{\frac{1}{\alpha-\beta}}} \quad (52)$$

These conditions are also the only way to ensure that we can find feasible solutions for the private and social levels of  $n$  ( $n < 1$ ) at the same time (see section 4). The quantity of  $z$  is deduced through the efficient allocation condition for labour, (47):

$$z = \left[ \frac{P_X^* A_X (1-\alpha) (1-n)^\beta}{P_M^* A_M (1-\beta) n^\alpha} \right]^{1/\beta} \quad (53)$$

We can see that  $z$  depends only on parameters, given that  $n$  depends only on parameters. Next, dividing equation (46) by  $K_M$ , we get the equation for  $a = C/K_M$ :



$$a = \frac{1}{P_C} \left[ P_X^* A_X \frac{1}{z} n^{1-\alpha} + P_M^* A_M \frac{1}{z^{1-\beta}} (1-n)^{1-\beta} - P_X^* \frac{1}{z} \left( g + \frac{b}{2} g^2 \right) - P_M^* \left( g + \frac{b}{2} g^2 \right) \right] \quad (54)$$

We now proceed to find the steady-state growth rate. Using equation (34), we can define the steady-state growth rate as:

$$g = \sigma(r - p) \quad (55)$$

where  $r$  is given by equations (48) or (49). Given that the steady-state growth rate will be positive, we have  $r > p$ , in equation (55), and as  $\sigma < 1$ , we also have  $r > g$ .

Using (55) and (48), we obtain:

$$g = \sigma \left[ \frac{A_X \alpha n^{1-\alpha} + (b/2)g^2}{(1 + b g)} - \rho \right]. \quad (56)$$

With the previous equation, we obtain:

$$b(1 - \frac{\sigma}{2})g^2 + (\sigma p b + 1)g - \sigma(A_X \alpha n^{1-\alpha} - \rho) = 0. \quad (57)$$

We can see that this equation depends only on parameters, given that  $n$  and  $z$  depend only on technological and preference parameters. Thus, equation (57) is a quadratic equation of the variable  $g$ . Resolving equation (57) for  $g$  we obtain the steady-state growth rate.

In order to have a clear solution, we show a numerical solution of the model in the steady-state. We use the following parameter values:  $\alpha = 0.6$ ,  $\beta = 0.3$  (in a more general definition, the shares of physical capital can be defined as the shares of a broad measure of capital that can include human capital),  $P_X^* = 1$ ,  $P_M^* = 1.2$ ,  $\sigma = 0.10$ ,  $p = 0.03$ ,  $\gamma = 0.3$ ,  $(1 - \gamma) = 0.7$ ,  $b = 6$ ,  $K_X(0) = 1$ ,  $K_M(0) = 1$ ,  $A_X = 1$ ,  $A_M = 1$ . The parameter values are only for illustrative purposes. With these values, the import sector is an excess demand sector.

First, using equations (51) and (53), we obtain the steady-state quantities of  $n$  and  $z$ :  $n = 0.3541$  and  $z = 0.4342$ . Next, solving equation (57), we get a positive root (positive  $g$ ) and a negative root (negative  $g$ ). The positive solution is:  $g = 0.0306$  (3.06 % per year). The corresponding values of  $q_X$

and  $q_M$ , equations (41) and (44), are:  $q_X = 1.1836$  and  $q_M = 1.4203$ . Solving equations (11) and (23):

$$K_X(t) = K_X(0)e^{[(q_X - P_X^*)t/(P_X^*b)]} \quad (58)$$

$$K_M(t) = K_M(0)e^{[(q_M - P_M^*)t/(P_M^*b)]} \quad (59)$$

where  $[(q_X - P_X^*)/(P_X^*b)] = [(q_M - P_M^*)/(P_M^*b)] = g$ . Substituting (58) in the transversality condition on  $K_X$ , (10), and substituting (59) in the transversality condition on  $K_M$ , (22), it is easy to show, with the previous values of  $g$ ,  $q_X$ , and  $q_M$ , that the transversality conditions are not violated (given that  $r > g$ ). If we use the negative value of  $g$  and the corresponding negative values of  $q_X$  and  $q_M$ , the transversality conditions are violated (see Turnovsky, 1996). Then, we can eliminate the negative value of  $g$ . We need to assume that the steady-state growth rate of this small open economy must be equal to or less than the world rate, otherwise it becomes a large economy. With equation (54), we obtain that  $a = 2.6299$ .

In short, for the market economy without government:

$$n = 0.3541, z = 0.4342, a = 2.6299, g = 0.0306$$

Thus, we have found the steady-state solution of the market economy.

Note that the model defines two relations between  $r$  and  $g$ . Equation (48), or alternatively (49) defines a negative relationship between the interest rate and the growth rate. This equation states that when the interest rate increases,  $q_X$  decreases, and so investment declines and the growth rate decreases. Equation (55) defines a positive relationship between the interest rate and the growth rate. Thus, when the interest rate increases, saving increases and the growth rate is stimulated.

Next, we generalize the result, presented in Barro and Sala-i-Martin (1992), that in the  $AK$  model with adjustment costs, shifts in the different parameters, generate positively, or negatively correlated movements between the steady-state interest rate and the steady state growth rate. Using our two-sector model, it is easy to show numerically that when  $b$  increases, the steady-state interest rate decreases and the steady-state growth rate decreases. Thus, shifts in the cost-of-adjustment parameter produce movements in  $r$  and  $g$  that are positively correlated. Another conclusion is that a reduction in the costs of adjustment for investment improves economic

growth. When  $A_X$  increases, or  $A_M$  decreases, the steady-state interest rate increases and the steady-state growth rate increases. Thus, shifts in the fixed technology parameters ( $A_X$  or  $A_M$ ) produce movements in  $r$  and  $g$  that are positively correlated. Next, since our economy is a two-sector model of a small open economy, we study how changes in the terms of trade affect  $r$  and  $g$ . We define the terms of trade as the price of the exportable good relative to the price of the importable good ( $P_X^*/P_M^*$ ). When the terms of trade increase,  $P_X^*$  increases or  $P_M^*$  decreases, the steady-state interest rate increases and the steady-state growth rate increases. Therefore shifts in the terms of trade generate movements in  $r$  and  $g$  that are positively correlated. This is another contribution to the literature.

Next, when  $\rho$  increases, the steady-state growth rate decreases and the interest rate increases. Also, when  $\sigma$  increases, the steady-state growth rate increases and the interest rate decreases. Thus, shifts in the preference parameters ( $\rho$  and  $\sigma$ ) generate negatively correlated movements between  $r$  and  $g$ . In short, the general correlation between  $r$  and  $g$ , in our model, depends on the relative importance of shifts in the different parameters (see also Ploeg and Alogoskoufis, 1994). In general, the correlation between  $r$  and  $g$  is debatable empirically. Fry (1997) shows that empirically the relationship between  $r$  and  $g$  is an inverted  $U$  curve.

#### 4. The Optimal Economic Growth Rate

In order to obtain the (Pareto) optimal solution, we need to obtain the social planner's solution, where the planner internalizes the two externalities. Thus the planner's problem is to maximize the present discounted value of the instantaneous utility function, equation (27), subject to the aggregate equilibrium condition for the goods market at world prices:

$$\begin{aligned}
 P_X A_X^* K_X n^{1-\alpha} + P_M^* A_M K_M^\beta K_X^{1-\beta} (1-n)^{1-\beta} = P_C C + P_X^* I_X \\
 + P_X^* \frac{b}{2} \frac{I_X^2}{K_X} + P_M^* I_M + P_M^* \frac{b}{2} \frac{I_M^2}{K_M}
 \end{aligned}
 \tag{60}$$

and to the capital accumulation constraints:  $\dot{K}_X = I_X$  and  $\dot{K}_M = I_M$ .

Note in equation (60) that the aggregate production functions are  $Y_X = A_X K_X n^{1-\alpha}$  and  $Y_M = A_M K_M^\beta K_X^{1-\beta} (1-n)^{1-\beta}$ , where the externalities

have been internalized and the equilibrium condition for the labour market, equation (37), has been taken into account. Likewise, in equation (60),  $P_C C$  is given by equation (30) and the consumer price index remains similar to that in section 2.4. Rewriting equation (60), we obtain the equation for  $C$ :

$$C = \frac{P_X^*}{P_C} A_X K_X n^{1-\alpha} + \frac{P_M^*}{P_C} A_M K_M^\beta K_X^{1-\beta} (1-n)^{1-\beta} - \frac{P_X^*}{P_C} I_X - \frac{P_X^* b}{P_C} \frac{I_X^2}{2 K_X} - \frac{P_M^*}{P_C} I_M - \frac{P_M^* b}{P_C} \frac{I_M^2}{2 K_M} \quad (61)$$

The Hamiltonian is:

$$H = \left( \frac{C^{1-1/\sigma}}{1-1/\sigma} + \lambda_X I_X + \lambda_M I_M \right) e^{-\rho t} \quad (62)$$

where  $C$  is given by equation (61). The control variables are  $I_X$  and  $I_M$ , the state variables are  $K_X$  and  $K_M$ , and  $\lambda_X$  and  $\lambda_M$  are the current-value co-state variables of  $K_X$  and  $K_M$  respectively. The first order conditions are:

$$P_X^* A_X K_X (1-\alpha) n^{-\alpha} = P_M^* A_M K_M^\beta K_X^{1-\beta} (1-\beta) (1-n)^{-\beta} \quad (63)$$

$$\lambda_X = \frac{P_X^*}{P_C} \left( 1 + b \frac{I_X}{K_X} \right) C^{-1/\sigma} \quad (64)$$

$$\lambda_M = \frac{P_M^*}{P_C} \left( 1 + b \frac{I_M}{K_M} \right) C^{-1/\sigma} \quad (65)$$

$$\frac{C^{-1/\sigma}}{\lambda_X} \left[ \frac{P_X^*}{P_C} A_X n^{1-\alpha} + \frac{P_M^*}{P_C} A_M K_M^\beta (1-\beta) K_X^{-\beta} (1-n)^{1-\beta} + \frac{P_X^* b}{P_C} \frac{I_X}{K_X} \right] + \frac{\dot{\lambda}_X}{\lambda_X} = \rho \quad (66)$$

$$\frac{C^{-1/\sigma}}{\lambda_M} \left[ \frac{P_M^*}{P_C} A_M \beta K_M^{\beta-1} K_X^{1-\beta} (1-n)^{1-\beta} + \frac{P_M^* b}{P_C} \frac{I_M}{K_M} \right] + \frac{\dot{\lambda}_M}{\lambda_M} = \rho \quad (67)$$

and the transversality conditions:  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_X K_X = 0$  and  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_M K_M = 0$ .

We now proceed to find the steady-state solution of the planner's problem in terms of the state-like and control-like variables. Equation (63) is the static efficient allocation condition for labour. Rewriting this condition, we get:

$$P_X^* A_X (1 - \alpha) n^{-\alpha} = P_M^* A_M z^\beta (1 - \beta) (1 - n)^{-\beta}. \quad (68)$$

Thus the value of the marginal product of labour is equal in both sectors. Using equations (64) and (65), we obtain the following relationship:  $\lambda_X/\lambda_M = P_X^*/P_M^*$ ; taking logs and derivatives of this relationship with respect to time, we get:

$$\frac{\dot{\lambda}_X}{\lambda_X} = \frac{\dot{\lambda}_M}{\lambda_M} \quad (69)$$

thus, the rate of growth of  $\lambda_X$  is always equal to the rate of growth of  $\lambda_M$ .

Equating equations (66) and (67), and using equations (64) and (65), we obtain the dynamic arbitrage condition for the two capital goods in the steady-state:

$$A_X n^{1-\alpha} + (P_M^*/P_X^*) A_M z^\beta (1 - \beta) (1 - n)^{1-\beta} = A_M \beta z^{\beta-1} (1 - n)^{1-\beta} \quad (70)$$

The left-hand side of this equation is the total social marginal product of  $K_X$ , which is formed by the social marginal product of  $K_X$  in the export sector, plus the social marginal product of  $K_X$  in the import sector (expressed relative to the price of the exportable good). Equation (70) states that the total social marginal product of  $K_X$  is equal to the social marginal product of  $K_M$  in the import sector. We can see that the total social marginal product of  $K_X$  is higher than the private marginal product of  $K_X$ . Thus the optimal growth rate will be higher than the growth rate of the market economy without government.

From equations (68) and (70), we can obtain the equations for  $n$  and  $z$  in the steady-state. Using equation (68), we get the equation for  $z$ :

$$z = \left( \frac{P_X^* A_X (1 - \alpha) (1 - n)^\beta}{P_M^* A_M (1 - \beta) n^\alpha} \right)^{1/\beta} \quad (71)$$

Substituting equation (71) in (70), we get:

$$\begin{aligned}
 & A_X n^{1-\alpha} + \frac{A_X(1-\alpha)(1-n)}{n^\alpha} \\
 &= \frac{P_X^* A_X \beta(1-\alpha)(1-n)}{P_M^*(1-\beta)n^\alpha} \left[ \frac{P_M^* A_M(1-\beta)n^\alpha}{P_X^* A_X(1-\alpha)(1-n)^\beta} \right]^{1/\beta} \quad (72)
 \end{aligned}$$

If we multiply the previous equation by  $n^\alpha$ , we obtain the following equation:

$$\left( \frac{A_M}{A_X} \right)^{1/\beta} \beta \left[ \frac{(1-\alpha)}{(1-\beta)} \right]^{1-1/\beta} \left( \frac{P_X^*}{P_M^*} \right)^{(1-1/\beta)} n^{\alpha/\beta} - \alpha n + (\alpha - 1) = 0 \quad (73)$$

Equation (73) is a nonlinear equation. When  $\alpha/\beta = 2$ , the equation is a quadratic equation of variable  $n$ , with feasible solutions ( $0 < n < 1$ ) and non-feasible solutions ( $n > 1$ ), depending upon the parameter values. We can resolve the quadratic equation to obtain the value of  $n$ . We do not have the set of all feasible solutions for all parameter values.

Dividing equation (60) by  $K_M$ , we find the value of  $a = C/K_M$ :

$$a = \frac{1}{P_C} \left[ \frac{P_X^* A_X n^{1-\alpha}}{z} + \frac{P_M^* A_M (1-n)^{1-\beta}}{z^{1-\beta}} - P_X^* \frac{1}{z} \left( g + \frac{b}{2} g^2 \right) - P_M^* \left( g + \frac{b}{2} g^2 \right) \right] \quad (74)$$

where  $g$  is the steady-state growth rate (remember that all the relevant variables grow at the same rate). We will now solve for the optimum steady-state growth rate. First, taking logs and derivatives of (64) with respect to time, we get:

$$\dot{C}/C = g = -\sigma(\lambda_X/\lambda_X) \quad (75)$$

Using the previous equation (75), and (66) and (64), we obtain:

$$g = \sigma \left[ \frac{A_X n^{1-\alpha} + (P_M^*/P_X^*) A_M z^\beta (1-\beta)(1-n)^{1-\beta} + (b/2)g^2}{(1+bg)} - \rho \right] \quad (76)$$

or alternatively, with equations (75), (69), (67) and (65), we get:

$$g = \sigma \left[ \frac{A_M \beta z^{\beta-1} (1-n)^{1-\beta} + (b/2)g^2}{(1+bg)} - \rho \right] \quad (77)$$

We can obtain the steady-state growth rate through equations (76) or (77). Using equation (77), we obtain a quadratic equation of variable  $g$ :

$$b(1 - \frac{\sigma}{2})g^2 + (1 + \sigma pb)g - \sigma[A_M \beta z^{\beta-1} (1-n)^{1-\beta} - \rho] = 0 \quad (78)$$

Here, we present the steady-state numerical solution of the planner's problem. Using the same parameter values, and equation (73), we find that the feasible solution of the quantity of  $n$  is  $n = 0.6942$ , and with equation (71) we get  $z = 0.0535$ . Solving equation (78), we obtain a positive root (positive  $g$ ) and a negative root (negative  $g$ ). The positive solution is  $g = 0.0696$  (6.96 % per year). With equation (74) we get  $a = 16.3227$ . We have experimented with different parameter values and the results hold.

In short for the planned economy, the optimal values are:

$$n = 0.6942, z = 0.0535, a = 16.3227, g = 0.0696.$$

We can see that the optimal growth rate is higher than that achieved in the market economy without government intervention.

In the next section, we show that with an optimal subsidy on investment in the export sector the market economy can reach the optimal growth rate.

### 5. The Optimal Investment Subsidy

The aim of government intervention is to allow the market economy to reach the optimal growth rate. The optimal policy is subsidy on investment in the export sector. This increases the growth rate by stimulating the source of the learning process. The problem of the  $i$ -th export firm is to maximize:

$$V_{X_i} = \int_0^{\infty} [P_X^* A_X K_{X_i}^{\alpha} L_{X_i}^{1-\alpha} T_1 - wL_{X_i} - (1-\mu)(P_X^* I_{X_i} + P_X^* \frac{b}{2} \frac{I_{X_i}^2}{K_{X_i}})] e^{-\int_0^t r(v)dv} dt \quad (79)$$

subject to  $\dot{K}_X = I_X$ , where  $\mu$  is the rate of the investment subsidy. The Hamiltonian is standard. Supposing identical firms and that in equilibrium all the export firms make the same choices, the first order conditions, in terms of control-like and state-like variables, are:

$$w = P_X^* A_X K_X (1 - \alpha) n^{-\alpha} \tag{80}$$

$$q_X = (1 - \mu) P_X^* \left( 1 + b \frac{I_X}{K_X} \right) \tag{81}$$

$$r = \frac{P_X^* A_X \alpha n^{1-\alpha} + (1 - \mu) P_X^* (b/2) (I_X/K_X)^2}{q_X} + \frac{\dot{q}_X}{q_X} \tag{82}$$

and the transversality condition  $\lim_{t \rightarrow \infty} e^{-\int_0^t r(v) dv} q_X K_X = 0$ . The aggregate rate at which bonds are issued is:

$$\dot{B}_X = (1 - \mu) \left( P_X^* I_X + P_X^* \frac{b}{2} \frac{I_X^2}{K_X} \right). \tag{83}$$

The distribution of aggregate dividends of the export sector firms is given by equation (13). The  $i$ -th import firm problem is similar to that in section 2.3. The representative household now faces the following flow budget constraint:

$$\dot{B} = rB + w(L_X + L_M) + \pi_X + \pi_M - R - P_C C \tag{84}$$

where the investment subsidy is financed through lump-sum taxes,  $R$ , to the households. The total expenditure on consumption and the consumer price index remain unchanged. Since the subsidy is financed in a non-distorting lump-sum fashion, the dynamic allocation condition for aggregate consumption among different time periods, equation (34), remains valid. The static first order condition of the representative household is similar to that in section 2.4, equation (35).

The government collects taxes,  $R$ , and subsidizes investment in the export sector to the amount of  $\mu P_X^* I_X [1 + (b/2)(I_X/K_X)]$ . Thus the balanced government budget constraint is:

$$R = \mu \left( P_X^* I_X + P_X^* \frac{b}{2} \frac{I_X^2}{K_X} \right). \tag{85}$$



Consolidating, we obtain the aggregate equilibrium condition for the goods market at world prices, equation (36).

We will now deduce the steady-state solution. With the first order conditions (80) and (43), we obtain the static efficient allocation condition for labour, equation (47). With (82), the steady state interest rate is:

$$r = \frac{P_x^* A_x \alpha n^{1-\alpha}}{q_x} + \frac{(1-\mu)P_x^*(b/2)g^2}{q_x} \quad (86)$$

Equating (86) with (49), and using (81), we obtain the dynamic arbitrage condition for the two capital goods:

$$\frac{A_x \alpha n^{1-\alpha}}{(1-\mu)} = A_M \beta z^{\beta-1} (1-n)^{1-\beta} \quad (87)$$

With equations (47) and (87), we can again obtain the equation for  $n$ :

$$n = \frac{1}{\left[ \frac{(1-\beta)}{(1-\alpha)} \left( \frac{\beta}{\alpha} \right)^{\frac{\beta}{1-\beta}} \right]^{\frac{(1-\beta)}{\alpha-\beta}} \left( \frac{P_M^*}{P_x^*} \right)^{\frac{(1-\beta)}{\alpha-\beta}} \left( \frac{A_M}{A_x} \right)^{\frac{1}{\alpha-\beta}} (1-\mu)^{\frac{\beta}{\alpha-\beta}}} \quad (88)$$

The quantity of  $z$  is given by equation (53), and the quantity of  $a$  is given by equation (54). With equations (55), (86) and (81), we can define the steady-state growth rate,  $g$ , as:

$$g = \sigma \left[ \frac{A_x \alpha n^{1-\alpha}}{(1-\mu)(1+bg)} + \frac{(b/2)g^2}{(1+bg)} - \rho \right] \quad (89)$$

With the previous equation, we get a quadratic equation of variable  $g$ :

$$b(1 - \frac{\sigma}{2})g^2 + (1 + \sigma pb)g - \sigma \left[ \frac{A_x \alpha n^{1-\alpha}}{(1-\mu)} - \rho \right] = 0 \quad (90)$$

Using the same parameter values, we deduce that the optimal subsidy is  $\mu = 0.4898$ . Using this subsidy rate and equations (88) and (53), we deduce the optimal quantities of  $n = 0.6942$  and  $z = 0.0535$ . Solving equation (90), we get the optimal steady-state growth rate,  $g = 0.0696$  (6.9

% per year). Finally, with equation (54), we get the optimal quantity of  $a = 16.3227$ .

Therefore, when the government imposes the optimal investment subsidy, the marginal product of  $K_X$  increases (left-hand side of equation (87)), and so the export firms want to invest in  $K_X$  and the import firms disinvest in  $K_M$ , and the quantity of  $z$  decreases. Since  $z$  decreases, the value of the marginal product of labour in the import sector decreases (right-hand side of equation (47)), labour flows to the export sector and the level of  $n$  increases, so the steady state growth rate increases, and the quantity of  $a = C/K_M$  increases.

## 6. Conclusions

We have developed an export-sector-led endogenous growth model with adjustment costs for investment. We have found the steady-state solution of the market economy without government intervention. The steady-state growth rate exists and is unique. We have shown that when the cost-of-adjustment parameter increases, that is, when adjustment costs are higher, the steady-state interest rate and the steady-state growth rate decrease. Thus, the adjustment costs damage economic growth. We have shown that shifts in the fixed technological parameters generate positively correlated movements between  $r$  and  $g$ . We have found that shifts in the preference parameters produce negatively correlated movements between the steady state interest rate and the steady-state growth rate. The correlation between  $r$  and  $g$  can be obtained in some one-sector endogenous growth models with adjustment costs (see Barro and Sala-i-Martin, 1992; and Ploeg and Alogoskoufis, 1994). Given that our model is a two-sector model of endogenous growth of a small open economy, we have also been able to study how changes in the terms of trade affect the steady-state interest rate and the steady-state growth rate. We found that shifts in the terms of trade generate positively correlated movements between  $r$  and  $g$ . Thus, in our export-sector-led endogenous growth model with adjustment costs, the correlation between  $r$  and  $g$  depends on the relative importance of shifts in the cost-of-adjustment parameter, in the fixed technological parameters, in the preference parameters, and in the terms of trade. Since there are two externalities, the competitive market economy without government intervention is inefficient: the optimal steady-state growth rate is higher than that achieved in the competitive

market economy without government intervention. Since the export sector is the only sector in the economy that learns, the optimal subsidy should be assigned to investments in the export sector. With the optimal investment subsidy, the competitive market economy reaches the optimal growth rate. Thus, the timal public policy is to stimulate the sources of technological change. All these findings, in an export-sector-led endogenous growth model with adjustment costs for investment, are not present in the literature and are contributions to the endogenous growth theory.

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