THE EFFECT OF INFLATION ON GROWTH INVESTMENTS: A NOTE

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Resumen: A partir de una nota previa enfocada hacia el efecto aislado de los impuestos sobre la duración óptima de una inversión en un entorno sin inflación (de Faro, 1996), motivada por el trabajo pionero de Brenner y Venetia (1983), este trabajo complementa dicho análisis al investigar los efectos aislados de inflación. Específicamente, al tomar en cuenta sólo el caso de no-reinversión, nuestro objetivo es el de incluir en el análisis una ilustración numérica de los resultados característicos de BV, y algunos procesos de indexación que se han usado en algunas economías con alta inflación.

Abstract: Motivated by the pioneering work of Brenner and Venetia (1983), henceforth designated as BV, a previous note that focused attention on the effect of taxation on the optimal duration of investments in a non-inflationary evironment (de Faro, 1996), the present note is aimed at investigating the effects of inflation on the optimal duration of investments. Specifically, considering the no-reinvestment case only, our objective is to include in the analysis both a numerical illustration of the peculiar results of BV as well as indexation procedures that have been used in some high inflation economies.

1. Introduction

Motivated by the pioneering work of Brenner and Venetia (1983), henceforth designated as BV, who seem to have been the first authors to include taxes and inflation in the analysis of the classical growth investment problem; and by a previous note that focused attention on the effect of taxation on the optimal duration of investments in a non-inflationary evironment (de Faro, 1996), the present note is aimed at investigating the effects of inflation on the optimal duration of investments. Specifically, considering the no-reinvestment case only, our objective is to include in the analysis both a numerical illustration of the peculiar results of BV as well as indexation procedures that have been used in some high inflation economies.

2. Basic Model

Adapting the notation in BV, let

$F(T)$ = Net present value of the investment cash-flow, as measured in the monetary units in use at the date of the investment;
$f(T)$ = Pre-tax net receipts, as measured in the monetary units in use at the date of the investment, derived from termination at time T;
$h(t)$ = Pre-tax net receipts, as measured in nominal terms, derived from termination at time T;
R = Real rate of interest (measured instantaneously and assumed to be positive);
C = Cost of investment, as measured in the monetary units in use at the date of the investment;
T = Time to termination of investment;
θ = Constant rate of inflation (as measured instantaneously and assumed to be positive);
φ = Tax rate.

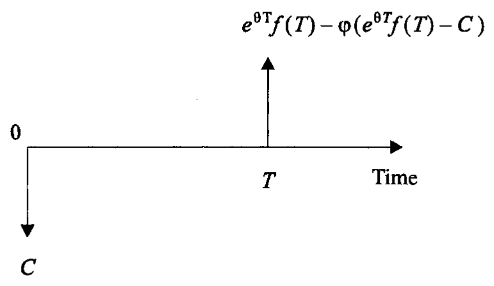
Assuming that the asset can be depreciated, for tax purposes, in lump sum form at the time of termination, with no indexation of the investment cost, and that pre-tax net receipts is fully indexed (i.e. $h(T) = e^{\theta T} f(T)$), the cash flow associated with the investment, as expressed in current prices (i.e. nominal terms), is as depicted in figure 1.

The above cash-flow corresponds to what will be called the basic model, and is the one that was addressed by BV.

Working with constant prices, calculated at date of the investment, it follows that the function whose maximization determines the optimal duration is:[1]

[1] Observe that, if we denote by $i = R + \theta$ the corresponding nominal rate of interest, if $\varphi = 0$ and if we work directly with $h(T)$, equation (1) corresponds to the classical

Figure 1
Cash-Flow for the Basic Model



$$F(T) = -C + e^{-(R+\theta)T}\{(1-\varphi)e^{\theta T}f(T) + \varphi C\} \tag{1}$$

Repeating here, for completeness, the presentation of BV, we have:

$$\frac{dF(T)}{dT} = \{(1-\varphi)(f'(T) - Rf(T))e^{\theta t} - (R+\theta)\varphi C\}e^{-(R+\theta)T} \tag{2}$$

Thus

$$\frac{dF(T)}{dT} = 0 \Rightarrow (1-\varphi)\,(f'(T) - Rf(T))e^{\theta T} - (R+\theta)\varphi C = 0 \tag{3}$$

On the other hand

$$\frac{d^2F(T)}{dT^2} = \{(1-\varphi)(f''(T) - Rf'(T))e^{\theta T} + \theta(1-\varphi)(f'(T) - Rf(T))e^{\theta T}\}e^{-(R+\theta)T} - (R+\theta)\frac{dF(T)}{dT} \tag{4}$$

Therefore, denoting by T^* the solution of (3), we have:

$$\left.\frac{d^2F(T)}{dT^2}\right]_{T*} = (1-\varphi)\{f''(T^*) - Rf'(T^*) + \theta(f'(T^*) - (Rf(T^*))\}e^{-RT^*} \tag{5}$$

text-book presentation of the problem (cf. Hirshleifer, 1970, Simon and Blume, 1994 and Varian, 1990).

Thus, as $1 - \varphi > 0$ and $e^{-RT} > 0$, it follows that T^* will be the optimal duration if

$$f''(T^*) - Rf'(T^*) + \theta(f'(T^*) - (Rf(T^*)) < 0 \tag{6}$$

To investigate the effect of changes in the inflation rate θ on the optimal duration T^*, differentiate (3) holding φ and R as constant. It follows that:

$$\frac{dT}{d\theta} = \frac{\varphi C - T(1-\varphi)(f'(T) - Rf(T))e^{\theta T}}{(1-\varphi)\{f''(T) - Rf'(T) + \theta(f'(T) - Rf(T))\}e^{\theta T}} \tag{7}$$

Taking into account (6), we know that, at T^*, the denominator of (7) is negative. Accordingly, the sign of $dT^*/d\theta$ is the opposite of the sign of the numerator of (7), which depends on T^*.

Rather than trying to determine the sign of the numerator of (7), a cumbersome task indeed, BV turned their attention to the investigation of the sign of $\partial^2 F(T)/(\partial T\, \partial\theta)$, at T^*, as it is the same as the sign of $dT/d\theta$, also at T^*.[2]

From (2), it follows that

$$\frac{\partial^2 F(T)}{\partial T\, \partial \theta} = \{T(1-\varphi)(f'(T) - Rf(T))e^{\theta T} - \varphi C\}e^{-(R+\theta)T}$$
$$- T\{(1-\varphi)(f'(T) - Rf(T))e^{\theta T} - (R+\theta)\varphi C\}e^{-(R+\theta)T}$$

Thus, taking into account (3), we have:

$$\left.\frac{\partial^2 F(T)}{\partial T\, \partial \theta}\right]_{T^*} = \{T^*(1-\varphi)(f'(T^*) - Rf(T^*))e^{\theta T^*} - \varphi C\}e^{-(R+\theta)T^*}$$

or, given that $(1-\varphi)(f'(T^*) - Rf(T^*))e^{-\theta T^*} = (R+\theta)\varphi C$

$$\left.\frac{\partial^2 F(T)}{\partial T\, \partial \theta}\right]_{T^*} = \{T^*(R+\theta)\varphi C - \varphi C\}e^{-(R+\theta)T^*}$$

$$= \varphi C\{T^*(R+\theta) - 1\}e^{-(R+\theta)T^*} \tag{8}$$

[2] This result follows from the so-called envelope theorem (cf. Varian, 1992).

Therefore, the sign of $dT/d\theta$ at T^* is the same as the sign of $T^*(R + 0) - 1$. Thus, as $R + \theta$ was assumed to be positive, it follows that:

$$\frac{dT^*}{d\theta} \gtrless 0 \text{ if } T^* \gtrless \frac{1}{R+\theta} \tag{9}$$

or

$$\frac{dT^*}{d\theta} \gtrless 0 \text{ if } \theta \gtrless \frac{1}{T^*} - R \tag{10}$$

Quoting BV, we can say that "for investments increased inflation increases the duration and the opposite holds for short durations. Alternatively, as expressed by (10), at high levels of inflation, increased inflation calls for longer duration and the opposite holds at low levels of inflation. In particular, a change from no inflation to some low level of inflation should decrease optimal duration".

With the sole purpose of giving at least some indication of the magnitude of an example that was presented in Hirshleifer (1970), we have:

$$f(T) = 90 \log(1 + T) + 120$$

If $C = 100$, $\varphi = 20\%$, $R = 10\%$, and if we have no inflation ($\theta = 0$), it is easily verified that the optimal duration is $T^* \cong 2.493902$. On the other hand, if we move to the situation where we start to have inflation, at the very moderate rate $\theta = 1\%$, the optimal duration will be slightly reduced to $T^* \cong 2.475614$.

Supposing now that $C = 56$, and mantaining the values of φ and R, consider the situation where we have rampant inflation, with $\theta = 50\%$. In this state of affairs we will have $T^* \cong 2.503765$. On the other hand, if inflation increases even further, say $\theta = 60\%$, we will also have an increase (though small) in the optimal duration, as $T^* \cong 2.529576$.

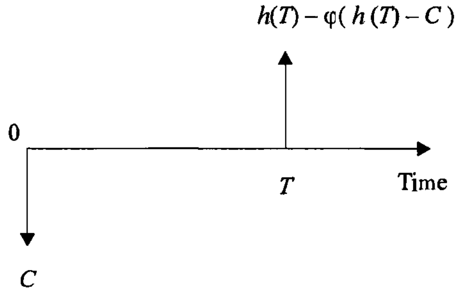
3. Variations of the Basic Model

The persistent presence of high inflation in some countries has led to the use of indexation. In particular, Brazil's monetary correction scheme,[3] which was made official in 1964 and remains in use, makes indexation a crucial factor for tax purposes. Accordingly, based primarily on the Brazilian experience, we are going to consider the following variations of the basic model.

3.1. *Absence of Indexation*

As the first variation of the basic model, one which is more appropriate for low inflation economies, let us consider the case where neither the pre-tax net receipts nor the investment, are indexed to the inflation rate. In this situation, the cash flow associated with the investment, as expressed in current prices, is as depicted in figure 2.

Figure 2
Cash-Flow in the Absence of Indexation



Thus, at constant prices, as referred to the date of the investment, we have:

$$F(T) = -C + e^{-(R+\theta)T}\{(1-\varphi)h(T) + \varphi C\} \qquad (11)$$

or, working with the nominal interest rate $i = R + \theta$

$$F(T) = -C + e^{-iT}\{(1-\varphi)h(T) + \varphi C\} \qquad (11')$$

[3] For assessments of the monetary correction scheme see, besides the early work of Fishlow (1974), Holanda (1993) and the very comprehensive surveys of Simonsen (1983, 1995).

That is, if the classical Fisher equation relating i, R and θ holds, and if we do not have indexation, the optimal duration is a function of the nominal interest rate i. In other words, with $h(T)$ playing the role of $f(T)$, we have, formally, exactly the case treated in equation (2) in BV. The difference is that, rather than analyzing the effects of changes in φ on the optimal duration T^*, we are interested now in investigating the effects of changes in the inflation rate θ.

Proceeding with the maximization of $F(T)$, we have:

$$\frac{dF(T)}{dT} = \{(1-\varphi)(h'(T) - ih(T)) - i\varphi C\}e^{-iT^*} \tag{12}$$

Thus

$$\frac{dF(T)}{dT} = 0 \Rightarrow i = \frac{(1-\varphi)h'(T)}{(1-\varphi)h(T) + \varphi C} \tag{13}$$

That is, the optimal duration T^* has to satisfy the extended version of the classical Jevon's formula, as given by (13).

With regard to the second order condition, we have:

$$\frac{d^2F(T)}{dT^2}\Big]_{T^*} = \{(1-\varphi)(h''(T^*) - ih'(T*))\}e^{-iT^*} \tag{14}$$

Thus, if nominal pre-tax net receipts $h(T)$ is a well behaved function of T, i.e. increasing and concave, the considered solution T^* is indeed optimal.

In order to investigate the effects of changes in the inflation rate θ on the optimal duration T^*, let us consider the effects of changes in the nominal interest rate i. To do this, differentiating (13), holding the tax rate φ constant, we have:

$$\frac{dT}{di} = \frac{(1-\varphi)h(T) + \varphi C}{(1-\varphi)(h''(T) - ih'(T))} \tag{15}$$

Therefore, given the assumptions on the behavior of $h(T)$, it follows that increases in the nominal rate of interest i shortens the optimal duration T^*. Making use of the chain rule of differentiation, and taking into account that $di/d\theta > 0$, it follows then that:

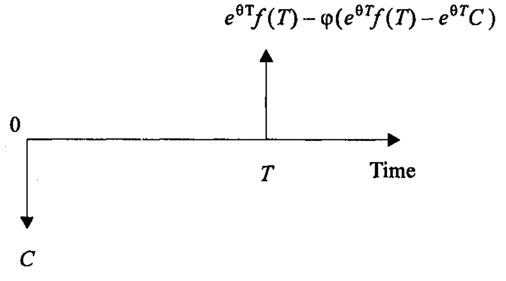
$$\frac{dT}{d\theta} = \left(\frac{dT}{di}\right)\left(\frac{di}{d\theta}\right) < 0 \tag{16}$$

Thus, if there is no indexation at all, an increase in the rate of inflation shortens the optimal duration.

3.2. *Full Indexation*

In the Brazilian case, the usual procedure with regard to taxation, at least until the implementation of the Real Plan in 1994, was to index value of the investments to the rate of inflation. Therefore, if the nominal value of pre-tax net receipts $h(T)$ is also indexed, i.e. $h(T) = e^{\theta T} f(T)$, we are fully indexed, and the cash flow associated with the investment, as expressed in current prices, is as depicted in figure 3.

Figure 3
Cash-Flow in the Case of Full Indexation



Thus, as measured in constant prices, calculated at the date of the investment, the present value function to be maximized is:

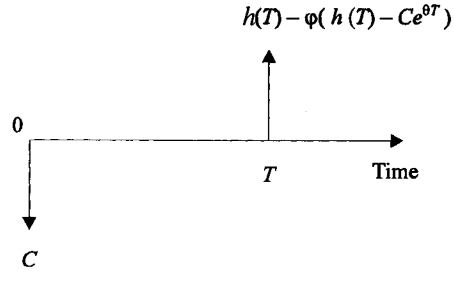
$$F(T) = -C + e^{-RT}\{(1-\varphi)f(T) + \varphi C\} \tag{17}$$

It is then obvious that the optimal duration does not depend on the inflation rate θ.

3.3. *Partial Indexation*

Concluding the analysis, let us now consider the case where the investment value is indexed for tax purpose, but the nominal value of pre-tax net receipts $h(T)$ is not. In this situation, assuming that $h(T) > Ce^{\theta T}$, for $T > 0$, the cash flow associated with the investment, as expressed in current prices, is as depicted figure **4**.[4]

Figure 4
Cash-Flow in the Case of Partial Indexation



In terms of constant prices, calculated at the date of the investment, the correspondent present value is:

$$F(T) = -C + e^{-(R+\theta)T}\{(1 - \varphi)h(T) + \varphi Ce^{\theta T}\} \quad (18)$$

with

$$\frac{dF(T)}{dT} = \{(1 - \varphi)(h'(T) - (R + \theta)h(T)) - R\varphi Ce^{\theta T}\}e^{-(R+\theta)T} \quad (19)$$

Therefore, the optimal duration T^* has to satisfy the following equation:

$$(1 - \varphi)(h'(T) - (R + \theta)h(T)) - R\varphi Ce^{\theta T} = 0 \quad (20)$$

On the other hand, as

[4] Note that, more generally, the value of the tax is given by $\max\{0; \varphi(h(T) - Ce^{\theta T})\}$.

$$\left.\frac{d^2F(T)}{dT^2}\right]_{T^*} = \{(1-\varphi)(h''(T^*) - (R+\theta)h'(T^*)) - \theta R\varphi C e^{\theta T^*}\} e^{-(R+\theta)T^*} \tag{21}$$

T^* will be indeed the optimal duration if

$$(1-\varphi)(h''(T^*) - (R+\theta)h'(T^*)) - \theta R\varphi C e^{\theta T^*} < 0 \tag{22}$$

Also, differentiating (20), holding constant R and φ, we have that:

$$\frac{dT^*}{d\theta} = \frac{(1-\varphi)h(T^*) + T^* R\varphi C e^{\theta T^*}}{(1-\varphi)(h''(T^*) - (R+\theta)h'(T^*)) - \theta R\varphi C e^{\theta T^*}} \tag{23}$$

Thus, as the numerator of (23) is obviously positive, it follows from (22) that an increase in the rate of inflation decreases the optimal duration.

4. Conclusion

Taking into account indexation procedures that have been used in some high inflation economies, particularly in Brazil, we have extended the basic model presented by BV in order to cover three additional situations. The fundamental conclusion is that while the effect of an increase in the inflation rate is somewhat ambiguous in the basic situation, since it depends on the relative size of the optimal duration itself, the effect is quite predictable in the other three situations. Thus, while the optimal duration is independent of the inflation rate in the case of full indexation, an increase in the rate of inflation always shortens the optimal duration, both in the case of partial indexation and in the absence of indexation.

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