Effectively Hedging the Interest Rate Risk of Wide Floating Rate Coupon Spreads

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ABSTRACT

Bond issuers frequently immunize/hedge their interest rate exposure by means of interest rate swaps (IRS). The receiving leg matches all bond cash-flows, while the pay leg requires floating rate coupon payments of form LIBOR + a spread. The goal of hedging against interest rate risk is only achieved in full if the present value of this spread is zero. Using market data we show that under a traditional IRS hedging strategy an investor could still experience significant cash flow losses given a 1% shift in the underlying benchmark yield curve.

We consider the instantaneous interest-rate risk of a bond portfolio that allows for general changes in interest rates. We make two contributions. The paper analyzes the size of hedging imperfections arising from the widening of the floating rate spread in a traditional swap contract and subsequently proposes two new practical, effective and analytically tractable swap structures; Structure 1: An Improved Parallel Hedge Swap, hedges against parallel shifts of the yield curve and Structure 2: An Improved Non-Parallel Hedge Swap, hedges against any movement of the swap curve. Analytical representations of these swaps are provided such that spreadsheet implementations are easily attainable.

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1.0   Introduction

Interest-rate swaps (IRS), which involve the exchange of a fixed-rate for a floating-rate interest payment, have grown rapidly over the last decade. In fact, IRS which first appeared in 1981 currently makes up a significant portion of the Over-the-Counter derivatives' market. As of December 2008 the notional amount of interest rate swaps outstanding was over \$328.1 trillion, up \$158.4 trillion from December 2005.[3] With this development, there has been a corresponding need for investors to better understand the potential cash-flow losses that might occur from unfavorable shifts in the interest rate swaps' underlying benchmark index. Such as the risk of a loss arising from a widening in the floating rate spread embedded in the coupon structure of the "*pay* leg" of the swap. This article examines and proposes two improved IRS immunization[4] structures that are designed to more adequately address yield curve shifts that would traditionally result in significant cash flow losses.

Bond issuers frequently immunize/hedge their perceived interest rate exposure by means of interest rate swaps. However as pointed out in Wall and Pringle (1993) these derivative financial instruments are not without risks because the counterparty in the interest rate swap that is paying a floating rate is exposed to interest rate risk. In fact, Stewart *et al* (2006) also points out that in the event that the underlying interest rate index

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[3] Data was obtained from International Swaps and Derivatives Association and Bank for International Settlements.

[4] Interest rate immunization is an investment strategy used to minimize the effects of interest rate risk to the value of a portfolio.

shifts rapidly upward then the interest rate swap will expose the floating ratepayer to negative cash flow consequences.

In the financial literature, IRS hedging is typically motivated by counterparties with a menu of funding opportunities who discover that their funding costs can be reduced by engaging in an interest rate swap with each other. To do this they each issue liabilities to the market on a basis that the other counterparty wants and agrees to exchange payments on these liabilities in order to obtain net financing on the basis that they initially desire (see Minton (1997), Bicksler and Chen (1986), and Reitano (1992)). However, given the recent economic reality of zero percent interest rates, the bench-mark yield curve could shift unfavourable or flatten, thus requiring the consideration of improved IRS pricing dynamics to avoid significant cash flow losses.

To simulate these improved dynamics this paper first considers a vanilla IRS in which the "*receive*" leg of the swap[5] generally matches all bond cash-flows, while the "*pay*" leg is usually satisfied by floating rate coupon payments of LIBOR[6] + a spread (s). See Figure 1 illustrating a typical interest rate swap transaction. For the practitioner, the generally stated goal of fully hedging/immunizing against interest rate risk is only achieved if there is no such spread. However, in almost all hedging operations of new bond issues this spread differs from zero because different issuers have different funding cost, largely due to differentials in credit quality.

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[5] Interest rate swaps do not generate new sources of funding themselves; rather, they convert one interest rate basis to a different rate basis (e.g., from a fixed interest rate basis to a floating or variable interest rate basis, or vice versa).

[6] Typically, floating rate interest payments may be based on the London Inter Bank Offered Rate (LIBOR), EURIBOR or the Securities Industry and Financial Markets Association (SIFMA) Municipal Swap index while payments made by the other counterparty are based on a fixed rate of interest.

Figure 1: Illustration showing a typical Interest Rate Swap with a "pay" leg of 5.625% and a
floating leg of Libor + a spread.
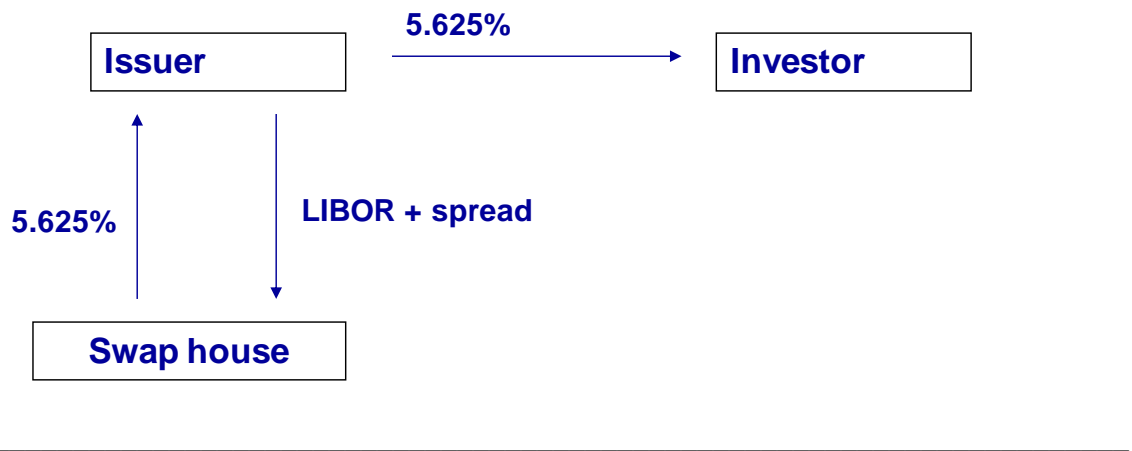
The paper analyzes the size of such hedging imperfections arising from the widening of this spread in a traditional swap contract and subsequently proposes two new practical and analytically tractable swap structures: (a) *Structure 1*: An Improved Parallel Hedge Swap, which hedges against parallel shifts of the yield curve and (b) *Structure 2*: An Improved Non-Parallel Hedge Swap, which hedges against any movement of the swap curve. Table 1 lists the differences in the traditional and proposed IRS structures. The proposed IRS coupon structures include 2 new transaction parameters (α, λ) for the Improved Parallel Hedge Swap and $2T$ new transaction parameters α_t, λ_t $(t = 1,...,T)$ for the Improved Non-Parallel Hedge Swap, which are discussed later in section 2.1. The swap structures we develop also have another important virtue; at inception there is absolutely no lending bias in the swap transaction, which is not the case in the traditional market quoted interest rate swap.

Table 1: Comparative differences between the Traditional and Improved IRS Hedge Swaps

	Traditional IRS	Improved IRS under Parallel Yield Curve Shifts	Improved IRS under Non-Parallel Yield Curve Shifts
(a) Coupon Form	$(L_t + s)$	$(\alpha * L_t + \lambda)$	$(\alpha_t * L_t + \lambda_t)$
(b) Hedge performance	Generally hedges most interest rate risk	Effectively hedges against parallel shifts in the Yield Curve	Effectively hedges against all shifts in the Yield Curve
(c) Complexity	Easy, Straightforward	Easy, Straightforward	Straightforward but has 2T parameters for T-years
(d) Credit exposure	Loan embedded due to upfront payment and slope of the yield curve	Loan embedded due to upfront payment and slope of the yield curve	No loan embedded as no upfront payment needed

For some market practitioners such as bond and derivatives traders in the large investment banks, continuous rebalancing of their investment portfolio might be a realistic option when there are shifts in the underlying benchmark yield curve. However, for others, such as treasurers in multinational corporations, public agencies and supranational financial institutions, bond issues are usually hedged at the launch of a funding transaction, and as such rebalancing is not necessary during the life of the bond issue.

This latter approach requires a high degree of customization of hedge swaps. Usually, the issuer receives the cash flows of the bond on payment dates, while paying some floating rate plus a fixed spread (s). The latter is fixed by the requirement that the PV of the swap shall be zero at time of execution (PV of the spread $s = 0$). However, as we show later in this paper, such an approach only approximately hedges against possible interest rate risk and a bit more sophistication in the structuring of the floating leg of the swap could result in a more effective swap. By *more effective* we mean that changes beyond a marginal parallel shift in the benchmark curve will not lead to cash flow losses or the necessity of re-hedging of the bond portfolio.

1.1. Prior research

The fixed income literature suggests that interest rate risk can seriously affect any fixed-income portfolio. This is the case even if the portfolio consists of all default-free fixed-income securities. Until recently, most portfolio immunization strategies focused on bond portfolios, and were generally accomplished by methods such as, cash flow matching, duration matching, volatility and convexity matching (Nawalkha and Soto (2009), Nawalkha and Latif (2004), Lacey and Nawalkha (1993), Reitano (1992)). Currently it has been shown that interest rate futures or options can be used in conjunction with bond portfolios to provide the same kind of immunization (Stulz (2003) and Kolb (2007)). For example, corporate pension funds use interest rate swaps to ease the burden of long-term liabilities, because swaps enable a pension fund to synthetically increase the duration of its portfolio to match liabilities.

To demonstrate the effects of portfolio immunization consider a portfolio $g(x)$

$$g(x+\Delta x) \approx g(x) + \frac{\partial g}{\partial x} \Delta x$$

If $\frac{\partial g}{\partial x} = 0$ then $g(x+\Delta x) \approx g(x)$ and the portfolio is said to be immunized against interest rate risk. Immunization can also be achieved on a net worth basis (e.g. banks try to mitigate the effects of interest on their worth) or in terms of a target date (e.g. pension funds with a fixed future obligation).

However there are a number of potential problems with this approach. The strategy assumes there is no default risk or call risk for bonds in the portfolio. Secondly, it is well known that the traditional duration and convexity risk measures (see Lacey and Nawalkha (1993), Nawalkha and Latif (2004) and Reitano (1992)) are only valid when

the whole yield curve moves in a parallel fashion. Thirdly, as duration changes over time the portfolio will have to be rebalanced. There are extensions available to handle non parallel shifts in the term structure but most of these are cumbersome at best (see Nawalkha and Soto for a survey of a number of models in the fixed income literature that deal with hedging the risk of large, non-parallel yield curve shifts).

The subsequent rapid growth in the use of interest rate swaps for portfolio immunization and other speculative purposes led in part to the well-publicized derivative driven losses in the mid-1990s which motivated a significant body of research[7] (Bodnar, et al (1995, 1996), Howton and Perfect (1998), Phillips (1995), Saunders (1999), Li and Mao (2003) and Balsam and Kim (2001)) on the value of interest rate swaps to a firm's assets.

In fact, in trying to better understand the potential effects of these complex instruments on the firm's assets (see Balsam and Kim (2001)), one strand of literature shows researchers such as Nance et al. (1993) investigating the determinants of firm hedging. Their analysis found that firms who chose to hedge are larger, face more convex tax functions, lower interest coverage and have more growth opportunities (Geczy et al. (1997). Several researchers have put forward another argument on the benefits of swaps to firms by explaining that swaps enable firms to exploit financing comparative advantage (Stulz (2003), Balsam and Kim (2001), Bicksler and Chen (1986), Turnbull (1986) and Smith, Smithson and Wakeman (1986)). However Shultz (2003) argues that the benefits highlighted by the abundance of these models are often illusionary at best.

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[7] The use of derivative interest rate swaps resulted in large losses for a number of public companies such as Procter and Gamble & Gamble Greetings in the United States.

In another strand of the literature we find an abundant body of scholarship that investigates change in swap spreads. See for instance Sun et al. (1993), Brown et al. (1994), Duffie and Huang (1996), Cossin and Pirotte (1997), Minton (1997), Lang et al. (1998), Lekkos and Milas (2001), Fehle (2003) and Huang and Chen (2007). We identify three issues central to this body of swap spread analysis. The first issue addresses the factors that contribute to the dynamics of swap spreads. In fact, this question is perhaps one of the most researched swap spread topics so far, yet empirical findings appear inconclusive. The second contention is the differential impacts of these factors on swap spreads across a spectrum of maturities. For example, Lekkos and Milas (2001) find that the impact from changes in the term structure on swap spreads is not uniform across swap maturities. They identify that an increasing yield curve slope is positively related to short-term swap spreads, but negatively related to the long-term swap spreads.

The last issue deals with the underlying effect of economic/market conditions on swap spreads. Work by Lang et al. (1998) report that swap spreads are pro-cyclical, suggesting that while the fixed ratepayer in a swap transaction gains value when the yield curve becomes steeper, the increasing default risk may offset this value enhancement[8]. However, as we examine the issue of derivative losses by market practitioners a more interesting question that begs to be answered then is, whether the swap's coupon structure can be adjusted to be more effective to changes in economic or financial market conditions that alter the swaps' benchmark yield curve. Unlike the prior research motivations this paper seeks to investigate the coupon structure of the "*pay* leg" with a

[8] However more recently, swap contracts are largely collateralized between counterparties or even traded via a high-grade central counterpart; thus eliminating default risk to a large extend.

view of proposing a modified IRS structure that could better withstand unexpected swings in the benchmark rate.

The remainder of the paper is organized into 3 sections. In section 2 we formulate the general model framework, which lays out the basic setup of the IRS model presented in this paper. This section develops an analytical implementation of the model using recent market data; estimates the closed form solution of the improved IRS models and tests the model against a 1% shift in the yield curve. Section 3 presents a tractable methodology for simulating and implementing the improved swap models. Section 4 summarizes the study's finding and proposes areas of future research.

2.0   Implementation and Evaluation: Traditional and Improved IRS Structures

We begin by considering a fixed rate bond issue of nominal 1 to be hedged with an IRS.[9] We allow for annual coupons that may be non-identical but fixed. Consequently, the bond's cash flow stream looks like Table 2:

Table 2: Bond Cashflows

t	Bond
0	P
1	$-b_1$
2	$-b_2$
...	...
T-1	$-b_{T-1}$
T	$-1-b_T$

with T being the maturity, P the net bond price[10] and b_t $(t=1,...,T)$ the annual bond coupons. In order to hedge against interest rate risk, an IRS is agreed. The usual interest rate swap[11] has the form:

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[9] The reader is referred to Stulz (2003) pp 550 for a discussion on IRS structure and valuation.

Table 3: Interest Rate Swap Structure

t	Receive Leg	Pay Leg
0	1-P	
1	b_1	$-(L_1+s)$
2	b_2	$-(L_2+s)$
...	...	...
T-1	b_{T-1}	$-(L_{T-1}+s)$
T	b_T	$-(L_T+s)$

where $1-P$ is the traditional upfront payment to bring proceeds to par. L_t (t=1,…,T) are the LIBOR floating-rate amounts, which are fixed at the beginning of period t and paid at the end of the same period. The spread (s) can be determined by the requirement that the PV of the swap must be zero at inception. We introduce the discount factors DF_t (t=1,…,T) and determine the PVs of both legs of the swap agreement as

$$PV_{rec} = 1 - P + \sum_{t=1}^{T} b_t DF_t \tag{1}$$

$$PV_{pay} = -\sum_{t=1}^{T} (f_t + s) \cdot DF_t \tag{2}$$

where $f_t(t=1,...,T)$ denotes the forward rate, the estimator of future LIBOR rates. At inception the PV of the swap must be zero. Therefore

$$0 = 1 - P + \sum_{t=1}^{T} b_t DF_t - \sum_{t=1}^{T} (f_t + s) \cdot DF_t \tag{3}$$

$$s = \frac{1 - P + \sum_{t=1}^{T} b_t DF_t - \sum_{t=1}^{T} f_t DF_t}{\sum_{t=1}^{T} DF_t} \tag{4}$$

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[10] In this paper we make some simplifying assumptions, in order to show clearly the concept of a better micro hedge of new issues.
- Fixed rate bond with annual coupons, no broken period, no accrued interest, clean price = dirty price, no fees. Annual coupons might be irregular.
- Swap into 1Y LIBOR (or EURIBOR). An extension of the model to the realistic case of quarterly or semi-annual rolls, and with short or long first and final stub is possible.

[11] An IRS is assumed, but a Cross Currency Swap would work similarly.

As the PV of a pure FRN is par the following holds:

$$PV(FRN) = 1 = DF_T + \sum_{t=1}^{T} f_t DF_t \qquad (5)$$

$$DF_T = 1 - \sum_{t=1}^{T} f_t DF_t$$

Therefore:

$$s = \frac{DF_T - P + \sum_{t=1}^{T} b_t DF_t}{\sum_{t=1}^{T} DF_t} \qquad (6)$$

This package of bond funding and hedge swap is equivalent to funding via a floating rate note (FRN) in LIBOR plus a spread (see Klein (2004)). The resulting net cash flows of the hedged funding transaction are of the form in Table 4

Table 4: Net Cashflows from Hedged Funding Transaction

t	Pay Leg
0	+1
1	$-(L_1 + s)$
2	$-(L_2 + s)$
...	...
T	$-(1 + L_T + s)$

In the special case where the spread s is zero, we have a "*pure*" FRN and the *PV* of the associated cash flow stream will be exactly zero, if discount factors and forward LIBOR rates are derived from the same yield curve[12]. This means that the *PV* of the micro-hedged package of bonds and IRS would be unaffected by movements in the underlying yield curve, so that in this case an effective hedge would be in place. However, consistent with observed market data, the spread s is normally far away from zero and more importantly strongly dependent on interest rate levels. The available empirical data show that the spread s impairs the typically effective hedge strategy

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[12] The important question whether the (non-cash) swap curve is suitable to derive (cash) EURIBOR forwards is left out of consideration here. This problem is not specific to the problem discussed in this paper, i.e. interest rate risk arising from wide funding spreads.

because as interest rates fall, discount factors rise and the PV of the funding spread's cash flow stream increases. The question therefore arises: *Is the issuer adequately hedged against interest rate risk by following the routine of swapping into the LIBOR + spread ($L_t + s$) format?*

In 2007, 3-month USD LIBOR rates and the associated forward rate were typically in the area of 5.5% with funding spread s for AA rated issuers in the area of 0.25%. This means that the floating part of the funding interest was 96% and the fixed spread component (s) only 4%. Recently however, 3-month USD LIBOR was in the region of 0.25% [13], with the accompanying funding spread in the area of 1 to 2%, so the relative importance of the fixed-rate part increased sharply, rising from 4% of the interest to be paid to between 80-90%. This is particularly pronounced for long maturities, where interest rate risk is enhanced by higher duration. Table 5, which assumes a traditional IRS hedge ($L_t + s$) shows an issuer's loss in the event of an instantaneous 1% downward shift of the yield curve (in percent of the nominal issued), for various maturities (3-50Y) and funding spreads (0-350 bps). From the table, a 10Y fixed rate bond hedged at LIBOR + 350 bps would experience a loss of 158 bps of the nominal if there was a 1% shift in the yield curve.

Table 5: Relative cash flow loss caused by a 1% downward shift of the yield curve.

Maturity	0.00%	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%	3.50%
3 Y	0.00%	0.03%	0.06%	0.09%	0.12%	0.15%	0.18%	0.20%
5 Y	0.00%	0.07%	0.14%	0.21%	0.28%	0.35%	0.42%	0.49%
10 Y	0.00%	0.23%	0.45%	0.68%	0.90%	1.13%	1.35%	1.58%
20 Y	0.00%	0.65%	1.31%	1.96%	2.62%	3.27%	3.93%	4.58%
30 Y	0.00%	1.12%	2.23%	3.35%	4.46%	5.58%	6.69%	7.81%
50 Y	0.00%	1.92%	3.83%	5.75%	7.67%	9.58%	11.50%	13.42%

Source: Computed from Bloomberg Data for the period January 12th 2010

[13] 0.25219% on Feb 22nd 2010 (source: Bloomberg).

Now consider the following Bloomberg market data example in Table 6: On January 10th 2010 Banco do Brazil launched a USD 500M 10-year fixed-rate bond. If hedged with an IRS the all-in cost would have been approximately LIBOR + 2.275%. If left completely unhedged *a drop of the USD swap curve by 1% would have resulted in a loss of USD 49.3m, which is almost 10% of the nominal.* If swapped with a traditional interest rate swap at LIBOR + 2.275%, interest rate risk is only hedged partially so *a drop of the swap curve by 1% would have resulted in a loss of USD 5.1m, which is still more than 10% of the loss in case of no hedging at all* (Table 6). Note that such a drop of 1% is not unrealistic, as the 10-year USD swap rate fell from 4.5% to 2.4% in the period August - December 2008 (see Figure 2 below).

Table 6: Summary of results of IRS Hedge Strategy on USD 500m bond issue by Banco do Brazil

Bond: Banco do Brazil 10Y launched January 10th 2010 (Millions)			
Nominal	500	USD	
Maturity	10	years	
Coupon	6.000%	p.a.	
SPREAD	2.275%	The correct spread taking into account the whole swap curve.	
Price	99.395%		
Yield	6.083%	p.a. (annual compounding)	
Duration	7.794	years	
Mod. Duration	7.347	years	
s_20Y	3.797%	20Y swap rate	
DF_20Y	0.6786	20Y discount factor	
Q_20Y	8.466124	Sum of all discount factors up to 20Y (i.e. BPV)	
Profit/Loss	9.912%	49.562 M USD	if bond left unhedged
Profit/Loss	1.027%	5.136 M USD	for traditional swap
Profit/Loss	0.000%	0.000 M USD	for Improved Hedge Swap Structure

Source: Bloomberg Data and our calculations in Appendices D and E.

Figure 2: USD swap rates (3Y, 5Y, 10Y, 30Y) for the period Jan 3rd 2000 – February 22nd 2010
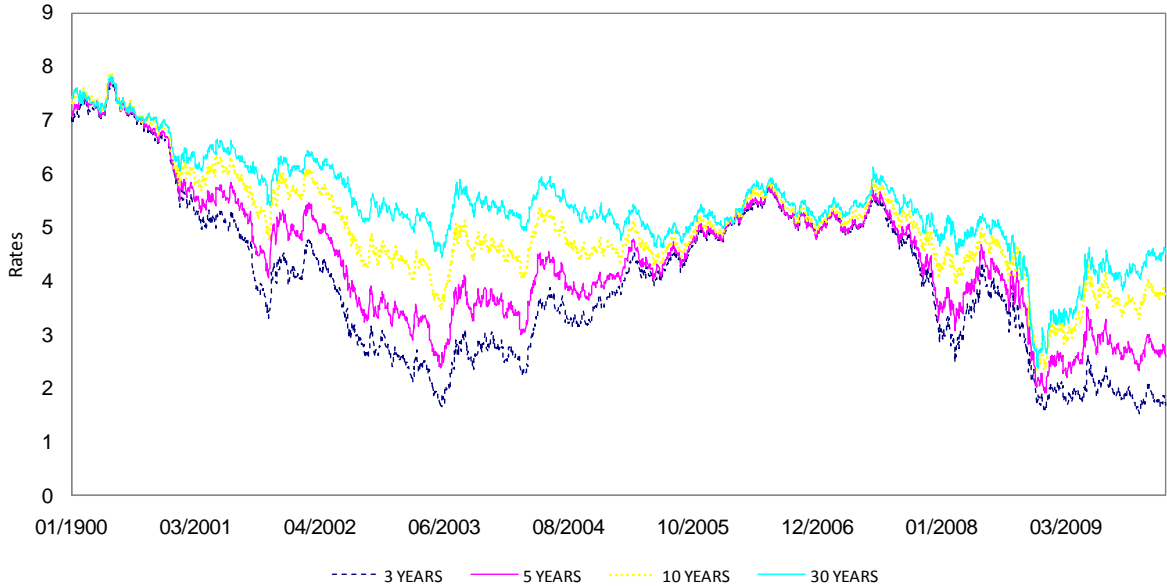
Source: Bloomberg

2.1 Structure 1: *Hedging against Parallel Shifts of the Yield Curve*

In order to mitigate the interest rate risk discussed in the preceding section, part of the fixed spread *s* in the swap structure can be replaced by some additional floating-rate amount in the *pay* leg. This can be done by introducing a factor α to the LIBOR rate and estimating it appropriately. Since the proposed spread component will be different from that of the traditional IRS model in Tables 3 and 4, we denote the new "*pay*" leg coupon spread by λ. The swap would now have the following characteristic form in Table 7:

Table 7: Modified Interest Rate Swap Structure

t	Receive Leg	Pay Leg
0	1-P	
1	b_1	$-(\alpha*L_1 + \lambda)$
2	b_2	$-(\alpha*L_2 + \lambda)$
...	...	...
T-1	b_{T-1}	$-(\alpha*L_{T-1} + \lambda)$
T	b_T	$-(\alpha*L_T + \lambda)$

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The parameters α and λ are fully determined by the requirements that (1) the PV of the swap is zero and (2) that an infinitesimal parallel shift of the yield curve

$$s_t \longrightarrow s_t{'} = s_t + \delta s_t$$

leaves the PV of the package of bond and swap unchanged to the first order in the infinitesimal shift parameter $\delta s_t = \varepsilon$ (see Appendix A.1).

The two determining equations are:

$$[PV] \qquad\qquad 1 - P + \sum_{i=1}^{T} DF_i b_i - \sum_{i=1}^{T} DF_i(\alpha f_i + \lambda) = 0 \qquad\qquad (7)$$

$$[HEDGE] \qquad\qquad \sum_{i=1}^{T} DF_i(\alpha f_i + \lambda) + DF_T = \sum_{i=1}^{T} DF_i{'}(\alpha f_i{'} + \lambda) + DF_T{'} \qquad (8)$$

Here DF_i are discount factors and forward rates of the actual yield curve and $DF_i{'}$ are the discount factors and forward rates of the infinitesimally parallel shifted yield curve. We give an iterative formula for the shifted discount factors and shifted forward rates in Appendix A.2.

These two equations are linear in the transaction parameters α and λ and can be rewritten in matrix form as:

$$\begin{pmatrix} \sum_{i=1}^{T} DF_i f_i & \sum_{i=1}^{T} DF_i \\[2em] \sum_{i=1}^{T} DF_i f_i - DF_i{'} f_i{'} & \sum_{i=1}^{T} DF_i - DF_i{'} \end{pmatrix} \times \begin{pmatrix} \alpha \\ \lambda \end{pmatrix} = \begin{pmatrix} 1 - P + \sum_{i=1}^{T} DF_i b_i \\[2em] DF_T{'} - DF_T \end{pmatrix} \qquad (9)$$

We get the solution of the parameters by inverting the matrix:

$$\begin{pmatrix} \alpha \\ \lambda \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{T} DF_i f_i & \sum_{i=1}^{T} DF_i \\[2em] \sum_{i=1}^{T} DF_i f_i - DF_i{'} f_i{'} & \sum_{i=1}^{T} DF_i - DF_i{'} \end{pmatrix}^{-1} \times \begin{pmatrix} 1 - P + \sum_{i=1}^{T} DF_i b_i \\[2em] DF_T{'} - DF_T \end{pmatrix} \qquad (10)$$

All parameters α and λ are expressed in terms of the bond coupon rates b_t, the discount factors DF_t and the forward rates f_t derived from the swap curve. In case $s>0$, we expect $\alpha>1$ and $\lambda<s$, as we expect that the fixed part of the coupon expression should be reduced and the floating part increased to yield a better hedge performance. Table 8 calculates and presents estimates of the closed form solution for α and the new spread λ, where the solutions are given as $\alpha=1.115>1$ and $\lambda=1.83\%<s$. The results in Table 8 illustrates that in order to hedge against a parallel shift of the swap curve, the interest paid by the issuer under *the swap should be modified from 1 × LIBOR + 2.275% to 1.115 × LIBOR + 1.838%.* This modification would not change the PV of the swap but would ensure a first-order hedge against interest rate movements from parallel shifts of the USD swap curve. Such hedge is a delta hedge only, but remains very efficient for curve shifts of 1-2%.

Table 8: The solutions (α, λ) for maturities T=1...50 and spreads s= 0%...+3.5%

T	0%	0.5%	1%	1.5%	2%	2.275%	2.5%	3.0%	3.5%
1	(1, 0%)	(1.005, 0.497%)	(1.01, 0	(1.015, 1.492%)	(1.02, 1.99%)	(1.023, 2.264%)	(1.025, 2.487%)	(1.03, 2.985%)	(1.035, 3.482%)
2	(1, 0%)	(1.007, 0.491%)	(1.015,	(1.022, 1.474%)	(1.03, 1.965%)	(1.034, 2.236%)	(1.037, 2.457%)	(1.044, 2.948%)	(1.052, 3.439%)
3	(1, 0%)	(1.01, 0.482%)	(1.02, 0	(1.029, 1.447%)	(1.039, 1.929%)	(1.045, 2.194%)	(1.049, 2.411%)	(1.059, 2.894%)	(1.069, 3.376%)
4	(1, 0%)	(1.012, 0.472%)	(1.024,	(1.036, 1.415%)	(1.049, 1.887%)	(1.055, 2.146%)	(1.061, 2.358%)	(1.073, 2.83%)	(1.085, 3.302%)
5	(1, 0%)	(1.014, 0.46%)	(1.029,	(1.043, 1.381%)	(1.058, 1.841%)	(1.066, 2.094%)	(1.072, 2.301%)	(1.087, 2.762%)	(1.101, 3.222%)
6	(1, 0%)	(1.017, 0.449%)	(1.033,	(1.05, 1.346%)	(1.067, 1.795%)	(1.076, 2.041%)	(1.084, 2.243%)	(1.1, 2.692%)	(1.117, 3.141%)
7	(1, 0%)	(1.019, 0.437%)	(1.038,	(1.057, 1.312%)	(1.076, 1.749%)	(1.086, 1.989%)	(1.095, 2.186%)	(1.114, 2.623%)	(1.133, 3.06%)
8	(1, 0%)	(1.021, 0.426%)	(1.042,	(1.063, 1.278%)	(1.084, 1.704%)	(1.096, 1.938%)	(1.106, 2.13%)	(1.127, 2.555%)	(1.148, 2.981%)
9	(1, 0%)	(1.023, 0.415%)	(1.046,	(1.07, 1.245%)	(1.093, 1.659%)	(1.106, 1.888%)	(1.116, 2.074%)	(1.139, 2.489%)	(1.163, 2.904%)
10	(1, 0%)	(1.025, 0.404%)	(1.051,	(1.076, 1.212%)	(1.101, 1.616%)	**(1.115, 1.838%)**	(1.127, 2.019%)	(1.152, 2.423%)	(1.177, 2.827%)
11	(1, 0%)	(1.027, 0.393%)	(1.055,	(1.082, 1.179%)	(1.109, 1.572%)	(1.124, 1.789%)	(1.137, 1.965%)	(1.164, 2.358%)	(1.191, 2.752%)
12	(1, 0%)	(1.029, 0.382%)	(1.059,	(1.088, 1.146%)	(1.117, 1.528%)	(1.134, 1.739%)	(1.147, 1.911%)	(1.176, 2.293%)	(1.205, 2.675%)
13	(1, 0%)	(1.031, 0.372%)	(1.063,	(1.094, 1.116%)	(1.125, 1.488%)	(1.142, 1.693%)	(1.157, 1.86%)	(1.188, 2.232%)	(1.219, 2.604%)
14	(1, 0%)	(1.033, 0.362%)	(1.066,	(1.1, 1.085%)	(1.133, 1.447%)	(1.151, 1.646%)	(1.166, 1.809%)	(1.199, 2.171%)	(1.233, 2.533%)
15	(1, 0%)	(1.035, 0.351%)	(1.07, 0	(1.105, 1.054%)	(1.14, 1.406%)	(1.16, 1.599%)	(1.175, 1.757%)	(1.211, 2.109%)	(1.246, 2.46%)
16	(1, 0%)	(1.037, 0.343%)	(1.074,	(1.111, 1.028%)	(1.148, 1.371%)	(1.168, 1.559%)	(1.185, 1.714%)	(1.222, 2.056%)	(1.258, 2.399%)
17	(1, 0%)	(1.039, 0.334%)	(1.077,	(1.116, 1.002%)	(1.155, 1.336%)	(1.176, 1.52%)	(1.194, 1.67%)	(1.232, 2.004%)	(1.271, 2.338%)
18	(1, 0%)	(1.04, 0.325%)	(1.081,	(1.121, 0.976%)	(1.162, 1.301%)	(1.184, 1.48%)	(1.202, 1.627%)	(1.243, 1.952%)	(1.283, 2.277%)
19	(1, 0%)	(1.042, 0.317%)	(1.084,	(1.127, 0.95%)	(1.169, 1.267%)	(1.192, 1.441%)	(1.211, 1.584%)	(1.253, 1.901%)	(1.295, 2.217%)
20	(1, 0%)	(1.044, 0.308%)	(1.088,	(1.132, 0.925%)	(1.175, 1.233%)	(1.2, 1.403%)	(1.219, 1.541%)	(1.263, 1.85%)	(1.307, 2.158%)
...									
45	(1, 0%)	(1.076, 0.164%)	(1.151,	(1.227, 0.492%)	(1.303, 0.656%)	(1.344, 0.746%)	(1.378, 0.82%)	(1.454, 0.984%)	(1.53, 1.148%)
46	(1, 0%)	(1.077, 0.16%)	(1.153,	(1.23, 0.48%)	(1.306, 0.641%)	(1.348, 0.729%)	(1.383, 0.801%)	(1.46, 0.961%)	(1.536, 1.121%)
47	(1, 0%)	(1.078, 0.156%)	(1.155,	(1.233, 0.469%)	(1.31, 0.625%)	(1.353, 0.711%)	(1.388, 0.782%)	(1.465, 0.938%)	(1.543, 1.094%)
48	(1, 0%)	(1.078, 0.153%)	(1.157,	(1.235, 0.458%)	(1.314, 0.611%)	(1.357, 0.695%)	(1.392, 0.763%)	(1.47, 0.916%)	(1.549, 1.069%)
49	(1, 0%)	(1.079, 0.149%)	(1.159,	(1.238, 0.447%)	(1.317, 0.596%)	(1.361, 0.678%)	(1.396, 0.745%)	(1.476, 0.894%)	(1.555, 1.043%)
50	(1, 0%)	(1.08, 0.146%)	(1.16, 0.	(1.24, 0.437%)	(1.321, 0.582%)	(1.365, 0.662%)	(1.401, 0.728%)	(1.481, 0.873%)	(1.561, 1.019%)

2.2   Structure 2: *Hedging against Non-Parallel Shifts of the Yield Curve*

As parallel shifts represent the dominant movement in yield curve dynamics, the swap structure proposed in Section 2.1 is a substantial improvement to the hedge performance of the traditional hedge swap. However, since recent central bank policy and market activity has also resulted in substantial non-parallel movements in the yield curve as evidenced by the consequence of coordinated central banks effort to lower short-term reference rates to near zero percent[14], and where a normalization of the yield curve would require a sizable non-parallel shift. In this section we therefore want to present a way to appropriately hedge against such non-parallel movements in the underlying benchmark rate.

In order to hedge against possible non-parallel movements of the yield curve, additional parameters are added to the structure of the swap in *Structure* 1. For this purpose a larger number of parameters α_t, λ_t, (for $t = 1,...,T$) are introduced and the IRS hedge swap is of the following form in Table 9:

Table 9: IRS for Parallel and Non-Parallel Yield Curve Movements

t	*Receive Leg*	*Pay Leg*
0		
1	b_1	$-(\alpha_1 *L_1 + \lambda_1)$
2	b_2	$-(\alpha_2 *L_2 + \lambda_2)$
...	*...*	*...*
T-1	b_{T-1}	$-(\alpha_{T-1} *L_{T-1} + \lambda_{T-1})$
T	b_T	$-(\alpha_T *L_T + \lambda_T)$

Note that, unlike in the case of the parallel shift in section 2.1 there are no upfront payments (i.e. cash-flow at time zero) in this swap structure such as that in the traditional

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[14] During the 2007/2008 global credit market crisis which resulted in significant central bank involvement in credit markets.

swap needed to bring proceeds to par $(1-P)$. We show below in section 3 that this is possible and, together with a careful choice of the parameters α_t and λ_t any loan bias in the swap structure can be avoided at inception. The swap can also be accreting or decreting, depending on whether the bond being issued is at a premium or at a discount. The number of free parameters that are chosen $(\alpha_t,...,\alpha_n)$ and $(\lambda_t,...,\lambda_n)$ allow us to achieve the goals of:

1. A more effective hedge against any movement of the yield curve;

2. No loan bias; *Ex-ante* no such loan would ever be embedded in the swap, if market developed in line with forward rate expectations.

   The closed form solutions for the $2T$ parameters α_t and λ_t (for $t=1,...,T$) are derived in Appendix B and presented in expressions 11 and 12.

$$\alpha_t = \frac{DF_T + \sum_{i=t}^{T} b_i DF_i}{DF_{t-1}} \tag{11}$$

$$\lambda_t = b_t - \alpha_t f_t = \frac{b_t DF_{t-1} - f_t \left((1+b_T)DF_T + \sum_{i=t}^{T-1} b_i DF_i \right)}{DF_{t-1}} \tag{12}$$

3.0   Simulating the proposed IRS hedging Structures

We start with the usual bootstrapping procedure; in Appendix A.2. For the exercise a USD swap curve up to 50 years with annual time steps is used. Forward rates (f_t) and discount factors (DF_t) are extracted from this swap curve via the usual bootstrapping process.

Appendix C presents the result of the bootstrapping exercise on January 10th 2010 swap curve data where Q_t is the sum of all discount factors up to t (referred to as the Basis Point Value)[15]. Appendices D - E provides a simulation of the swap, providing a bootstrapping of the original curve, the improved IRS hedging structure and the shifted curves[16]. Appendix E provides the estimates of the 2T new transaction parameters α_t, λ_t $(t = 1,...,T)$ for the Improved Non-Parallel Hedge Swap, which is presented in Table 10. The simulation exercises in Appendices B – D shows that for a 1 percent swing in the yield curve the traditional swap would experience a cash flow loss of USD 5.1M. In the case of our improved/more effective swap structure this move in the yield curve is immunized resulting in a positive outcome for the investor as the potential cash flow loss is minimized. The following table shows the parameters of the Improved Hedge Swap in the case of a 10Y bond issue with funding spread of 2.275%.

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[15] BPV tells you how much money your positions will gain or lose in relative terms for a parallel movement of 1 bps in the yield curve. It therefore quantifies your interest rate risk for small changes in interest rates.

[16] **Traditional hedge swap**
    (a) There is an upfront payment of PRICE-100% to bring proceeds to par.

  Improved Effective Hedge Swap
    (a) There is no upfront payment;
    (b) The investor pays the coupon and receives the floating rate amounts of $\alpha_t \times \text{LIBOR} + \lambda_t$;
    (c) The 2T parameters α_t and λ_t are determined in the simulation to fulfill two goals:
        1. Effective hedge versus interest rate movements on any kind,
        2. No (ex ante) loan embedded in the swap.

Table 10: The α and λ coefficients for T = 10Y and s = 2.275%

t	α_t	λ_t
1	1.19	5.40%
2	1.13	3.91%
3	1.09	2.58%
4	1.06	1.73%
5	1.05	1.18%
6	1.04	0.90%
7	1.03	0.80%
8	1.02	0.76%
9	1.01	0.73%
10	1.01	0.66%

4.0   Conclusion

We consider the instantaneous interest-rate risk of a bond portfolio. Our framework allows for general changes in interest rates, and does not require the specification of the yield curve dynamics or the estimation of such a model. We make two contributions. The paper analyzes the size of hedging imperfections arising from wide floating rate spreads in a traditional swap contract and subsequently proposes two new practical, effective and analytically tractable swap structures; *Structure 1*: An Improved Parallel Hedge Swap, which hedges against parallel shifts of the yield curve and *Structure 2*: An Improved Non-Parallel Hedge Swap, which hedges against any movement of the swap curve.

In analyzing the perceived weakness in the traditional IRS structure and highlighting the contributions of this paper we considered a traditional swap using Bloomberg market data of February 22nd 2010 where we structured a USD 500M 10-year deal swapped into 1-year LIBOR plus 2.275%. Left completely unhedged a drop of the swap curve by 1% would result in a loss of USD 49.56M. If swapped with a traditional interest rate swap at LIBOR plus 2.275%, interest rate risk is only hedged partially and a

drop of the swap curve by 1% would result in a loss of USD 5.14M, which is still more than 10% of the loss in case of no hedging at all. Appendix E depicting the simulation of the improved hedge swap structure for the earlier interest rate risk problem showed no cash flow losses occurring as a result of a shift in the benchmark rate, as expected due to the analytical structure of the hedge.

As a by-product to the derivation of the improved IRS models, we provide a simple methodology on how to effectively simulate and solve a given immunization problem (Appendices C - E). To reach the desired solution the swap structures introduce 2 new transaction parameters α and λ for parallel yield curve shifts or $2T$ new transaction parameters α_t and λ_t in the case of non-linear yield curve shifts.

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Appendix A: Bootstrapping the yield curve and its infinitesimal shifts

A.1: Infinitesimal shifts of the yield curve:

An infinitesimal shift of $s_t \longrightarrow s_t{'} = s_t + \delta s_t$

$$DF_t \longrightarrow DF_t{'} = DF_t + \delta DF_t$$

where

$$\delta DF_t = -\frac{1}{1+s_t}\left[\delta s_t \sum_{i=1}^{t} DF_i + s_t \sum_{i=1}^{t-1} \delta DF_i\right]$$

$f_t \longrightarrow f_t{'} = f_t + \delta f_t$ with

$$\delta f_t = \frac{1}{DF_t}\left[\delta DF_{t-1} - (1+f_t)\delta DF_t\right]$$

A.2: The classic bootstrapping:

$$DF_t = \frac{1-s_t \cdot \sum_{i=1}^{t-1} DF_i}{1+s_t}$$

$$f_t = DF_{t-1} / DF_t - 1$$

In classic bootstrapping (A.2) all discount factors DF_t and forward rates f_t can be found iteratively starting with DF_1 followed by DF_2, DF_3, ..., DF_T . All the forward rates f_t can then be derived. In order to find all the variations δDF_t and δf_t (e.g. for an infinitesimal parallel shift of the yield curve, as required in this paper), the equations above must be used iteratively in the following order: $\delta DF_1, \delta DF_2, ..., \delta DF_T$, and then immediately all the δf_t .

APPENDIX B: Deriving the 2*T* Parameters from the 2*T* Linear Equations.

$$[PV_t] \qquad -DF_t b_t + DF_t \left(\alpha_t f_t + \lambda_t \right) = 0 \qquad \text{(for } t = 1, ..., T)$$

Simplifying;

$$[PV_t] \qquad \lambda_t = b_t - \alpha_t f_t \qquad \text{(for } t = 1, ..., T)$$

where the associated IRS structure is given as

$$[HEDGE_t] \quad DF_T + \sum_{i=1}^{T} DF_i \left(\alpha_i f_i + \lambda_i \right) = DF_T^{(t)} + \sum_{i=1}^{T} DF_i^{(t)} \left(\alpha_i f_i^{(t)} + \lambda_i \right) \quad \text{(for } t = 1, ..., T)$$

Equation $[PV_t]$ has no loans embedded in the swap; hence all expected net cash flows

are zero. The requirement of an effective hedge against any movement of the swap curve

results in equation $[HEDGE_t]$. f_i (for $i = 1, ..., T$) represent the forward rates derived

from the original yield curve and $f_i^{(t)}$ represent the forward rates derived from a

modified yield curve with label (*t*):

$$f_i^{(t)} := f_i + \varepsilon \delta_{ti}, \text{ with } \varepsilon > 0 \text{ and } \delta_{ti} := \begin{cases} 1; \textit{if } t = i \\ 0; \textit{if } t \neq i \end{cases}.$$

We get $DF_i^{(t)} = \begin{cases} DF_i & ; \quad \textit{if } i < t \\ DF_i \cdot \dfrac{1 + f_t}{1 + f_t + \varepsilon} & ; \quad \textit{if } i \geq t \end{cases}.$

We substitute $[PV_t]$ in $[HEDGE_t]$ and get:

$$DF_T + \sum_{i=1}^{T} DF_i b_i = DF_T^{(t)} + \sum_{i=1}^{T} DF_i^{(t)} \left(\alpha_i f_i^{(t)} + b_i - \alpha_i f_i \right)$$

$$DF_T + \sum_{i=1}^{T} DF_i b_i = DF_T^{(t)} + \sum_{i=1}^{T} DF_i^{(t)} \left(b_i + \alpha_i (f_i^{(t)} - f_i) \right)$$

With $f_i^{(t)} = f_i + \varepsilon \delta_{ti}$ we get:

$$DF_T + \sum_{i=1}^{T} DF_i b_i = DF_T^{(t)} + \sum_{i=1}^{T} DF_i^{(t)} \left(b_i + \alpha_i \varepsilon \delta_{ti} \right)$$

$$0 = DF_T^{(t)} - DF_T + \sum_{i=1}^{T} (DF_i^{(t)} - DF_i) b_i + \alpha_t \varepsilon DF_t^{(t)}$$

With $DF_i^{(t)} - DF_i = DF_i(\dfrac{1+f_t}{1+f_t+\varepsilon} - 1) = DF_i \dfrac{1+f_t - 1 - f_t - \varepsilon}{1+f_t+\varepsilon} = -DF_i \dfrac{\varepsilon}{1+f_t+\varepsilon}$ we get:

$$0 = -DF_T \frac{\varepsilon}{1+f_t+\varepsilon} - \sum_{i=1}^{T} DF_i \frac{\varepsilon}{1+f_t+\varepsilon} b_i + \alpha_t \varepsilon DF_t \frac{1+f_t}{1+f_t+\varepsilon}$$

Simplifying

$$0 = -DF_T - \sum_{i=1}^{T} DF_i b_i + \alpha_t DF_t (1+f_t)$$

With $DF_t (1+f_t) = DF_{t-1}$:

$$DF_T + \sum_{i=1}^{T} DF_i b_i = \alpha_t DF_{t-1}$$

$$\alpha_t = \frac{DF_T + \sum_{i=t}^{T} b_i DF_i}{DF_{t-1}} = \frac{PV(bond\ cash\ flows)}{DF_{t-1}}.$$

We summarize the unique solution of the system of linear equations as:

$$\alpha_t = \frac{DF_T + \sum_{i=t}^{T} b_i DF_i}{DF_{t-1}}$$

$$\lambda_t = b_t - \alpha_t f_t = \frac{b_t DF_{t-1} - f_t \left((1+b_T) DF_T + \sum_{i=t}^{T-1} b_i DF_i \right)}{DF_{t-1}}$$

APPENDICES C-E: Simulated IRS model showing the effects of a shift in the benchmark yield curve.

APPENDIX C: Bootstrapping the USD swap curve to generate forward and discount rates

Time	BOOTSTRAPPING ORIGINAL CURVE					BOOTSTRAPPING SHIFTED CURVE				
t	s_t	DF_t	Q_t	z_t	f	s_t	DF_t	Q_t	z_t	f
0		1.000	0.00				1.000	0.00		
1	0.505%	0.995	0.99	0.504%	0.504%	-0.496%	1.005	1.00	-0.496%	-0.496%
2	1.171%	0.977	1.97	1.175%	1.850%	0.171%	0.997	2.00	0.172%	0.843%
3	1.806%	0.947	2.92	1.822%	3.128%	0.806%	0.976	2.98	0.813%	2.108%
4	2.329%	0.911	3.83	2.363%	4.003%	1.329%	0.948	3.93	1.348%	2.970%
5	2.747%	0.871	4.70	2.803%	4.585%	1.747%	0.915	4.84	1.782%	3.539%
6	3.071%	0.830	5.53	3.151%	4.909%	2.071%	0.881	5.72	2.125%	3.853%
7	3.319%	0.790	6.32	3.421%	5.051%	2.319%	0.848	6.57	2.389%	3.989%
8	3.511%	0.752	7.07	3.632%	5.125%	2.511%	0.815	7.38	2.596%	4.059%
9	3.665%	0.715	7.79	3.804%	5.194%	2.665%	0.782	8.17	2.765%	4.123%
10	3.797%	0.679	8.47	3.954%	5.306%	2.797%	0.751	8.92	2.910%	4.227%
11	3.909%	0.644	9.11	4.083%	5.384%	2.909%	0.720	9.64	3.036%	4.299%
12	4.016%	0.610	9.72	4.210%	5.619%	3.016%	0.689	10.33	3.158%	4.518%
13	4.103%	0.578	10.30	4.314%	5.567%	3.103%	0.659	10.98	3.258%	4.466%
14	4.172%	0.548	10.84	4.396%	5.470%	3.172%	0.632	11.62	3.337%	4.372%
15	4.232%	0.519	11.36	4.468%	5.486%	3.232%	0.605	12.22	3.407%	4.384%
16	4.260%	0.495	11.86	4.495%	4.901%	3.260%	0.583	12.80	3.434%	3.845%
17	4.288%	0.471	12.33	4.524%	4.990%	3.288%	0.561	13.36	3.463%	3.925%
18	4.316%	0.449	12.78	4.555%	5.083%	3.316%	0.539	13.90	3.493%	4.007%
19	4.344%	0.426	13.20	4.588%	5.180%	3.344%	0.518	14.42	3.525%	4.093%
20	4.372%	0.405	13.61	4.623%	5.281%	3.372%	0.497	14.92	3.557%	4.181%
21	4.383%	0.387	14.00	4.631%	4.792%	3.383%	0.479	15.40	3.566%	3.744%
22	4.395%	0.369	14.37	4.640%	4.835%	3.395%	0.462	15.86	3.576%	3.782%
23	4.406%	0.352	14.72	4.650%	4.880%	3.406%	0.445	16.30	3.587%	3.820%
24	4.418%	0.335	15.05	4.662%	4.927%	3.418%	0.428	16.73	3.598%	3.860%
25	4.430%	0.319	15.37	4.674%	4.977%	3.430%	0.412	17.14	3.610%	3.901%
26	4.437%	0.305	15.68	4.679%	4.795%	3.437%	0.397	17.54	3.615%	3.743%
27	4.444%	0.291	15.97	4.684%	4.827%	3.444%	0.383	17.92	3.621%	3.769%
28	4.451%	0.277	16.24	4.691%	4.860%	3.451%	0.369	18.29	3.627%	3.796%
29	4.458%	0.264	16.51	4.698%	4.895%	3.458%	0.355	18.65	3.634%	3.824%
30	4.465%	0.252	16.76	4.706%	4.931%	3.465%	0.342	18.99	3.641%	3.852%
31	4.464%	0.241	17.00	4.696%	4.418%	3.464%	0.331	19.32	3.634%	3.426%
32	4.464%	0.231	17.23	4.687%	4.415%	3.464%	0.320	19.64	3.628%	3.424%
33	4.463%	0.221	17.45	4.679%	4.412%	3.463%	0.309	19.95	3.621%	3.421%
34	4.462%	0.212	17.66	4.671%	4.408%	3.462%	0.299	20.25	3.615%	3.418%
35	4.462%	0.203	17.87	4.663%	4.404%	3.462%	0.289	20.54	3.610%	3.415%
36	4.461%	0.194	18.06	4.656%	4.400%	3.461%	0.280	20.82	3.604%	3.413%
37	4.460%	0.186	18.25	4.649%	4.396%	3.460%	0.270	21.09	3.599%	3.410%
38	4.460%	0.178	18.43	4.642%	4.392%	3.460%	0.261	21.35	3.594%	3.406%
39	4.459%	0.171	18.60	4.636%	4.388%	3.459%	0.253	21.60	3.589%	3.403%
40	4.458%	0.164	18.76	4.629%	4.383%	3.458%	0.244	21.85	3.584%	3.400%
41	4.455%	0.157	18.92	4.615%	4.037%	3.455%	0.237	22.08	3.573%	3.132%
42	4.451%	0.151	19.07	4.601%	4.014%	3.451%	0.230	22.31	3.562%	3.115%
43	4.448%	0.145	19.21	4.586%	3.989%	3.448%	0.223	22.54	3.551%	3.098%
44	4.444%	0.140	19.35	4.572%	3.964%	3.444%	0.216	22.75	3.541%	3.080%
45	4.441%	0.135	19.49	4.558%	3.938%	3.441%	0.210	22.96	3.530%	3.062%
46	4.437%	0.129	19.62	4.544%	3.911%	3.437%	0.204	23.17	3.519%	3.043%
47	4.434%	0.125	19.74	4.530%	3.883%	3.434%	0.198	23.36	3.509%	3.024%
48	4.430%	0.120	19.86	4.516%	3.855%	3.430%	0.192	23.56	3.498%	3.004%
49	4.427%	0.116	19.98	4.502%	3.826%	3.427%	0.186	23.74	3.488%	2.985%
50	4.423%	0.111	20.09	4.487%	3.796%	3.423%	0.181	23.92	3.477%	2.964%

APPENDIX D: Valuation and risk exposure under Traditional Hedge Swap

PV Traditional Hedge (original curve)				PV Traditional Hedge (shifted curve)			
32.142%	19.260%	68.464%	119.866%	24.938%	20.288%	75.667%	120.893%
CF float	CF fixed	CF cap	CF total	CF float	CF fixed	CF cap	PAY_t
		0.605%	0.605%			0.605%	0.605%
0.504%	2.275%	-	2.780%	- 0.496%	2.275%	-	1.780%
1.850%	2.275%	-	4.125%	0.843%	2.275%	-	3.118%
3.128%	2.275%	-	5.403%	2.108%	2.275%	-	4.383%
4.003%	2.275%	-	6.278%	2.970%	2.275%	-	5.245%
4.585%	2.275%	-	6.860%	3.539%	2.275%	-	5.814%
4.909%	2.275%	-	7.184%	3.853%	2.275%	-	6.128%
5.051%	2.275%	-	7.326%	3.989%	2.275%	-	6.264%
5.125%	2.275%	-	7.400%	4.059%	2.275%	-	6.334%
5.194%	2.275%	-	7.469%	4.123%	2.275%	-	6.398%
5.306%	2.275%	100.000%	107.581%	4.227%	2.275%	100.000%	106.502%

Profit/Loss	1.027%	(5.140)	Million USD for traditional swap

Footnotes

Profit/Loss = PV of traditional hedge (114.260%) - PV of shifted curve (112.932%)

CF Float = Libor cash flow

CF fixed = The spread cash flow

CF cap = Non-interest, that is capital cash flows for upfront payment and final redemption amount

APPENDIX E: Valuation and risk exposure under the Improved Hedge Swap Strategy

Calculating parameters of Improved Hedge Swap					PV Improved Hedge (original curve)				PV Improved Hedge (shifted curve)			
BOND			SWAP		33.634%	17.163%	67.858%	118.655%	25.871%	17.722%	75.062%	118.655%
Capital_t	Coupon_t	RECEIVE_t	alpha_t	lamda_t	CF float	CF fixed	CF cap	PAY_t	CF float	CF fixed	CF cap	PAY_t
-99.39%		- 99.395%										
-	6.000%	6.000%	**1.187**	**5.401%**	0.599%	5.401%	-	6.000%	- 0.588%	5.401%	-	4.813%
-	6.000%	6.000%	**1.133**	**3.905%**	2.095%	3.905%	-	6.000%	0.955%	3.905%	-	4.860%
-	6.000%	6.000%	**1.093**	**2.580%**	3.420%	2.580%	-	6.000%	2.305%	2.580%	-	4.885%
-	6.000%	6.000%	**1.068**	**1.726%**	4.274%	1.726%	-	6.000%	3.171%	1.726%	-	4.897%
-	6.000%	6.000%	**1.050**	**1.184%**	4.816%	1.184%	-	6.000%	3.717%	1.184%	-	4.901%
-	6.000%	6.000%	**1.039**	**0.902%**	5.098%	0.902%	-	6.000%	4.002%	0.902%	-	4.904%
-	6.000%	6.000%	**1.030**	**0.800%**	5.200%	0.800%	-	6.000%	4.107%	0.800%	-	4.907%
-	6.000%	6.000%	**1.022**	**0.764%**	5.236%	0.764%	-	6.000%	4.147%	0.764%	-	4.911%
-	6.000%	6.000%	**1.014**	**0.733%**	5.267%	0.733%	-	6.000%	4.181%	0.733%	-	4.914%
100.000%	6.000%	106.000%	**1.007**	**0.659%**	5.341%	0.659%	100.000%	106.000%	4.255%	0.659%	100.00%	104.915%

Profit/Loss	**0.000%**	**0.000**	**Million USD** for Improved swap

Footnotes

Profit/Loss = PV of original improved hedge (114.612%) - PV of shifted curve (114.612%)

Capital_t = Non-interest cash flows, that is the initial bond price and the final redemption amount

Receive_t = Sum of capital and interest cash flows

CF Float = Libor cash flow

CF fixed = The spread cash flow

CF cap = Non-interest, that is capital cash flows for upfront payment and final redemption amount