ISSN: 2038-6087
Working Paper

Dipartimento di
Scienze
Economiche
Università di Cassino

7/2010

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The Underground Economy in a Matching Model of Endogenous Growth

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Abstract

A matching model will explain both unemployment and economic growth by considering the underground sector. Three problems can thus be simultaneously accounted for: (i) the persistence of underground economy, (ii) the ambiguous relationships between underground employment and unemployment, and (iii) between growth and unemployment. The key assumptions adopted are that entrepreneurial ability is heterogeneous across individuals; skill accumulation determines productivity growth in the regular sector and a positive externality on the underground sector; job-seekers choose whether or not to invest in education and skill depending on the expected wages in the two sectors. The conclusions are that the least able entrepreneurs set up underground firms, employ unskilled labour, and do not contribute to growth. Underground employment alleviates unemployment only if the monitoring rate is sufficiently low. Policies for entrepreneurship and monitoring would help both economic growth and employment.

JEL classification: E26, J23, J24, J63, J64, L26, O40

Keywords: entrepreneurship, underground economy, shadow economy, unemployment, human capital, endogenous growth, search and matching models

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Non-technical summary

This theoretical paper contributes to explaining three stylised facts at the same time, viz.:

(i) the underground economy appears to be persistent and widespread in most countries. This fact has also been called the 'shadow puzzle';
(ii) underground employment and unemployment exhibit an ambiguous relationship across countries;
(iii) economic growth and unemployment also exhibit an ambiguous relationship across countries and over time.

As far as we are aware, no study has attempted to deal with these three issues at the same time. In particular, no study has attempted to link the human capital-economic growth nexus to unemployment through the economy's composition in the regular and underground sectors.

The paper develops a search and matching model of equilibrium unemployment *à la* Mortensen and Pissarides in two sectors where entrepreneurial ability and human capital play a key role. The model is based on the following assumptions, which are supported by a variety of empirical studies:

- labour productivity is lower in the underground sector with respect to the regular sector;
- individuals are heterogeneous in their entrepreneurial abilities;
- irregular firms have lower entry costs and taxes than regular firms, but bear the risk of being discovered as unregistered and destroyed, according to the monitoring rate implemented;
- irregular firms employ unskilled labour, while regular firms employ skilled labour;
- education is costly, and individuals can choose whether or not to invest in education and become skilled;
- the education level determines productivity growth by producing externalities also in favour of the underground sector.

These assumptions make it possible to find an interior equilibrium where both sectors survive, thus providing an original explanation for the 'shadow puzzle'. In this equilibrium, individuals with an unprofitable level of entrepreneurial ability seek jobs as employees; individuals with just sufficient ability open vacancies in the underground sector, and the ablest individuals open vacancies in the regular sector. Expected profits and wages are higher in the regular sector. On this basis, individuals who search for jobs as employees choose whether or not to invest in education and to become skilled before entering the labour market. Therefore, the education level is higher in the regular sector, and the size of this sector can thus contribute to explain economic growth.

If education influences labour productivity with increasing returns when it is at low levels, and with decreasing returns at high levels, two relevant equilibria may emerge. The economy represented by the more efficient equilibrium displays a smaller underground sector, higher levels of entrepreneurial ability used, extra-profits, relative wages, skill, education, and greater productivity growth.

The model contributes to explaining the other two stylised facts by adopting a novel perspective in which the monitoring rate plays a key role. In fact, the model predicts that the relationship between the underground employment and unemployment (issue (*ii*)) is negative (positive), and the relationships between productivity growth and unemployment (issue (*iii*)) is positive (negative) if the monitoring rate is sufficiently low (high). These results may account for the difference between Latin American and EU transition countries vs. EU non-transition countries.

Policies for entrepreneurship, education, and monitoring would help both employment and economic growth.

1. Introduction

The study of the underground economy that adopts matching-type models is not new in the economic literature. Two aims are usually pursued: solving the 'shadow puzzle', i.e. the persistence of the underground economy in a variety of contexts and times (Boeri and Garibaldi, 2002, 2006); highlighting the ambiguous relationship between underground employment and unemployment (Bouev, 2002, 2005; Boeri and Garibaldi, 2002, 2006; Kolm and Larsen, 2003, 2010; Fugazza and Jacques, 2004; Bosch and Esteban-Pretel, 2009; Albrecht et al., 2009).

The study of endogenous economic growth that also adopts matching-type models was initiated by Pissarides' (1990) book, and by Aghion and Howitt (1994), so that the issue of the relationship between growth and unemployment has been both raised and addressed with new analytical tools (Laing et al., 1995; Aghion and Howitt, 1998; Mortensen and Pissarides, 1998; Pissarides, 2000; Mortensen, 2005). In fact, different authors obtain different results concerning the sign of the correlation between growth and unemployment, both across countries and across long periods of time in the same country (Aghion and Howitt, 1994; Bean and Pissarides, 1993; Caballero, 1993; Hoon and Phelps, 1997; Muscatelli and Tirelli, 2001). This ambiguity has been explained on the basis of theoretical assumptions about technological progress and the interest rate (see the next section).

However, as far as we are aware, no study has attempted to deal with the three issues at the same time, i.e. (*i*) the persistence of underground economy, also called the 'shadow puzzle', (*ii*) the ambiguous relationship between the underground employment and unemployment, (*iii*) the ambiguous relationship between growth and unemployment. This paper makes such an attempt by developing a new matching model with the following key assumptions. First, individuals are heterogeneous in their entrepreneurial ability, and they can use it to run either a regular firm or an underground firm, which has smaller entry costs and taxes, but also lower productivity. These assumptions, which are empirically well-founded (La Porta and Shleifer 2008), make it possible to find an interior equilibrium where both sectors survive, thereby adopting Baumol's (1990), Lucas's (1978) and Rauch's (1991) approach of heterogeneous talent allocation. In this equilibrium, individuals with an unprofitable level of entrepreneurial ability seek jobs as employees; individuals with just sufficient ability open vacancies in the underground sector, and the ablest individuals open vacancies in the regular sector. This solution of the 'shadow puzzle' is new and general, as evidenced by Lisi and Pugno (2010).

Another key assumption of our model states that regular firms employ skilled labour, while underground firms employ unskilled labour. This assumption is supported by a variety of evidence (Agénor and Aizenman, 1999; Boeri and Garibaldi, 2002, 2006; Bosch and Esteban-Pretel, 2009; Cimoli, Primi and Pugno, 2006; Kolm and Larsen, 2010). In the individual's choice setting, this assumption leads to the further analytical postulate that individuals who search for jobs as employees have already chosen whether or not to invest in education and to become skilled before entering the labour market. Empirical support is provided by the fact that employment in the underground sector and the education level within countries appear to be negatively correlated (Albrecht et. al., 2009; Cappariello and Zizza, 2009).

A further key assumption of our model receives rather usual support in the literature about the role of human capital in endogenous growth (Romer, 1986, 1988, 1989; Lucas, 1988; Rebelo, 1991; Stokey, 1991), as recently surveyed by Savvides and Stengos (2009). Specifically, the assumption states that the education level determines productivity growth (Laing et al., 1995) by producing externalities also in favour of the underground sector. Since the education level is higher in the regular sector, the size of this sector contributes to explaining economic growth. Therefore, the ultimate engine of economic growth is "good matching" between the ablest entrepreneurs and the most educated workers.

This conclusion is interesting for the debate on the role of the underground economy in economic development, and on the policy implications (de Soto, 1989; Johnson et al., 2000; Friedman et al., 2000; Farrell, 2004; Carillo and Pugno, 2004; Banerjee and Duflo, 2005; Cimoli, Primi and Pugno, 2006). In particular, our theoretical conclusion accounts for La Porta and Shleifer's (2008) empirical finding that growth needs those firms which are most productive, and which hence cannot be informal.

On the basis of these assumptions, our model aids understanding of not only the shadow puzzle (issue (*i*)), but also the ambiguous relationships between underground employment and unemployment (issue (*ii*)), and between growth and unemployment (issues (*iii*)). Issue (*ii*) has arisen in the literature because of an ambiguity in the results. According to Bouev's (2002, 2005) matching model, scaling down the underground sector may lead to a decrease in unemployment, whereas, according to Boeri and Garibaldi's (2002, 2006) matching model, attempts to reduce shadow employment will result in higher open unemployment. Issue (*iii*) has been effectively synthesised by Mortensen (2005), who shows that the correlation between average growth and average unemployment over the past ten years across 29 European countries is essentially zero.

By considering that the economy includes underground firms, which benefit from evading taxes and from lower wages, but are burdened by backward techniques and by the risk of being discovered as unregistered and destroyed according to a monitoring rate, our model yields the following conclusion about issue (*ii*). The proportion of underground employment is positively related with the unemployment rate if the monitoring rate is sufficiently high, whereas, conversely, the proportion of underground employment is negatively related with the unemployment rate if the monitoring rate is sufficiently low. Since the proportion of underground employment negatively contributes to economic growth, the conclusion about issue (*iii*) follows. Economic growth is negatively related with unemployment if the monitoring rate is sufficiently high, whereas economic growth is positively related with unemployment if the monitoring rate is sufficiently low.

The empirical plausibility of these conclusions can be shown by scatter diagrams on the growth/unemployment axes *vis-à-vis* Mortensen's (2005) synthesis, which eventually brings us to issue (*iii*). The groups of countries with the highest monitoring rate (captured by the 'rule of law' index), such as the EU non-transition countries, exhibit a negative correlation (Fig. 1). The groups of countries with the lowest monitoring rate, such as the EU transition countries and the Latin American countries, exhibit a positive, though less close, correlation (see Figs 1-2).[1]

========== *Figs. 1-2 about here (now at the end with related data)* =========

The rest of the paper is organised as follows: section 2 briefly reviews the literature on growth and unemployment in the matching framework; section 3 presents the model with underground sector and finds the steady-state solutions; section 4 extends the model to endogenous investment in education and finds the steady-growth solutions; while section 5 concludes with some remarks on policy implications. The appendices set out the relevant proofs and mathematical details.

2. A brief literature review

Before the recent papers of search and matching theory, economic growth was usually analysed in a framework without unemployment. This was an important shortcoming in the neoclassical literature, as acknowledged by Solow himself (1988), but it was justified by the

[1] The correlation coefficient between the growth rate and the unemployment rate for the group of EU non-transition countries is –0.30 if they report a high 'rule of law' (above 88), and –0.17 for the same group irrespective of the 'rule of law'. The correlation coefficient for the group of EU transition countries is –0.13 if the outlier Poland is included but 0.30 if it is excluded. The correlation coefficient for the group of Latin American countries is 0.43 if Chile, which records a high index of 'rule of law' (88), is excluded, and 0.39 if Chile is included.

mere cyclical nature of unemployment. The influential papers of Aghion and Howitt (1994, 1998), Mortensen and Pissarides (1998) and Pissarides (2000), enable us to study growth and unemployment in the same framework, linking the neoclassical growth theory (Solow, 1956) with the theory of the natural rate of unemployment (Friedman, 1968; Phelps, 1968). It has thus been recognised that unemployment has also a structural nature which persists over the business cycle.

The analysis of both growth and unemployment has concentrated on technological progress. As shown in Pissarides (2000), innovation can be introduced into search and matching models in two ways. First, this can be done by assuming that technological progress is disembodied, meaning that labour productivity in both old and new jobs grows at the exogenous rate of technological progress. Second, on assuming Schumpeter's notion of "creative destruction", technological progress is embodied in new jobs, meaning that labour productivity in old jobs does not grow.

As in the standard neoclassical model (Solow model), technological progress is disembodied in the sense that both old and new jobs benefit from higher labour productivity without it being necessary to replace their capital stock.[2] In the disembodied technological progress, the higher the technological progress, the lower is the discount rate. Hence, the present-discounted profits are higher and firms open more vacancies. This is the so-called "capitalization effect", which implies both higher growth and a lower steady-state unemployment rate (Pissarides, 2000).

When technological progress is embodied in new jobs, growth can come about through job destruction and the creation of new and more productive jobs, owing to the need to replace the capital stock. In the case of embodied technological progress, the rate of job destruction is endogenous, and it is higher at faster rates of growth. Hence, faster technological progress is associated with a higher steady-state unemployment rate (Aghion and Howitt, 1994, 1998).

According to Mortensen and Pissarides (1998), these opposite results found in the literature on growth and unemployment can be interpreted within a more general model in which the direction of the effect of productivity growth on unemployment depends only on the size of the updating cost. Formally, Mortensen and Pissarides (1998) find a critical renovation cost such that faster growth decreases unemployment if the updating cost is below

[2] This is the only form of technological progress that is consistent with a balanced-growth path.

this critical value, and it increases unemployment if the updating cost is above the critical cost.

Finally, according to Mortensen (2005), there is no clear prediction about how the unemployment rate and the aggregate growth rate should be correlated across countries or across time, and the net effect of growth on unemployment is unclear. Indeed, in Mortensen's model two opposite effects are at work: the negative effect of creative destruction on market tightness, since a more rapid rate of job destruction reduces the value of firm and entry, and the positive relationship between the creative destruction and labour market tightness implied by the steady-state equilibrium condition and the unemployment identity.

The present paper takes another look at the structural link between growth and unemployment by recognising that the economy usually includes an underground sector, which is backward and less attractive for educated people with respect to the regular sector.

The fact that education plays a key role in human capital formation and economic growth has been widely studied in the endogenous growth literature (Savvides and Stengos, 2009) since the pioneering works by Romer (1986) and Lucas (1988). In particular, Laing et al. (1995) use a matching framework to analyze the 'long-run' endogenous growth rate in an economy in which 'short-run' labour market frictions and investment in education are important for the economic growth process. In particular, the economic growth rate depends crucially on the human capital growth rate. They find that a higher contact rate of workers with vacancies leads to a higher rate of growth of human capital and a lower level of unemployment.

However, no study has attempted to link the human capital-economic growth nexus to unemployment through the economy's sectoral composition.

3. Model with underground sector and unemployment

3.1 The matching framework

The paper proposes a general model of equilibrium unemployment where individual wage bargaining prevails in the labour market (Mortensen and Pissarides, 1994; Pissarides, 2000). Numerous firms competitively produce a homogeneous product, but adopt different institutional and technological set-ups. They may be registered, and therefore pay a production tax and adopt a relatively advanced technology; or they may not be registered, and therefore evade taxes and adopt a less efficient technology. Hence non-registered firms form the underground or shadow sector of the economy, which is illegal because of the process employed, not because of the good being produced.

As is usual in matching-type models (Pissarides, 2000; Petrongolo and Pissarides, 2001), the meeting of vacant jobs and unemployed workers is regulated by an aggregate matching function $m_i = m(v_i, u)$, where $i \in \{r, s\}$ denotes the sector (*r = regular*, *s = shadow*), v_i measures the vacancies in the sector, and u measures the unemployed (who are the only job-seekers). By assumption, the matching function is non-negative, increasing and concave in both arguments and performs constant returns to scale, so that the job-finding rate, $g(\theta_i) = m(v_i, u)/u = m(\theta_i, 1)$, is positive, increasing and concave in the ratio of vacancies to unemployment, $\theta_i = v_i / u$. Analogously, the rate at which vacancies are filled, $f(\theta_i) = m(v_i, u)/v_i = m\left(1, \theta_i^{-1}\right)$, is a positive, decreasing and convex function of market tightness, θ_i. Further, the *Inada*-type conditions hold: $\lim_{\theta_i \to 0} f(\theta_i) = \lim_{\theta_i \to \infty} g(\theta_i) = \infty$; $\lim_{\theta_i \to \infty} f(\theta_i) = \lim_{\theta_i \to 0} g(\theta_i) = 0$.[3]

The Bellman equations specified to find infinite horizon steady-state solutions are:[4]

Value of …	***Underground sector***	***Regular sector***
a vacancy	$r \cdot V_s = -c_s + f(\theta_s) \cdot [J_s - V_s]$	$r \cdot V_r = -c_r + f(\theta_r) \cdot [J_r - V_r]$
a filled job	$rJ_s = x_s y_s - w_s + (\delta + \rho) \cdot [V_s - J_s]$	$rJ_r = x_r y_r - w_r - \tau + \delta [V_r - J_r]$
searching for a job	$r \cdot U_s = z + g(\theta_s) \cdot [W_s - U_s]$	$r \cdot U_r = z + g(\theta_r) \cdot [W_r - U_r]$
being employed	$r \cdot W_s = w_s + (\delta + \rho) \cdot [U_s - W_s]$	$r \cdot W_r = w_r + \delta \cdot [U_r - W_r]$

where V_i is the value of a vacancy; J_i is the value of a filled job; U_i is the value for seeking a job; W_i is the value for being employed; r is the instantaneous discount rate; c_i is the start-up cost; z is the opportunity cost of employment; x_i is entrepreneurial ability; y_i is labour productivity; w_i is the wage rate; τ is an exogenous production tax; ρ is the monitoring rate, i.e. the exogenous instantaneous probability of a firm being discovered (and destroyed) as unregistered; δ is the exogenous destruction rate.[5] The parameters r, c_i, z, τ, ρ and δ are always considered as positive and exogenous.

Empirical evidence suggests that underground employment is one of low productivity jobs (Agénor and Aizenman, 1999; Boeri and Garibaldi, 2002, 2006; Cimoli, Primi and

[3] The matching functions of the two sectors may be different, but evidence is lacking in this regard.
[4] Time is continuous, and individuals are risk neutral, live infinitely, and discount the future.
[5] The unemployed cannot search for jobs in both sectors at the same time (i.e. there is a directed search). However, irrespective of the sector, if an unemployed person fails to find a job, s/he falls back into the same pool of unemployment.

Pugno, 2006; Bosch and Esteban-Pretel, 2009). Therefore, our first key assumption is the following.

Assumption 1. *Labour productivity is lower in the underground sector with respect to the regular sector:* $y_s < y_r$.[6]

As usual, wages are assumed to be the outcome of a Nash bargaining problem:

$$w_i = \arg\max\left\{(W_i - U_i)^{\beta} \cdot (J_i - V_i)^{1-\beta}\right\} \Rightarrow (W_i - U_i) = \frac{\beta}{(1-\beta)} \cdot (J_i - V_i) \qquad \text{with } i \in \{r, s\}$$

where the parameter $\beta \in (0,1)$ is the surplus share for labour. Simple manipulations thus yield:

$$w_r = (1-\beta) \cdot rU_r(\theta_r) + \beta \cdot (x_r y_r - \tau - rV_r(\theta_r))$$

$$w_s = (1-\beta) \cdot rU_s(\theta_s) + \beta \cdot (x_s y_s - rV_s(\theta_s))$$

with $w_i'(\theta_i) > 0 \ \ \forall i$, since $V_i'(\theta_i) < 0$, and $U_i'(\theta_i) > 0 \ \ \forall i$.

The surplus of a job in each sector (divided between one entrepreneur and one worker by the wage) is defined as the sum of the worker's and firm's value of being on the job, net of the respective outside options, so that $S_i = J_i - V_i + W_i - U_i$. Using the Bellman equations, we get:

$$S_s = \frac{x_s \cdot y_s - z + c_s}{r + \delta + \rho + (1-\beta) \cdot f(\theta_s) + \beta \cdot g(\theta_s)}; \qquad S_r = \frac{x_r \cdot y_r - \tau - z + c_r}{r + \delta + (1-\beta) \cdot f(\theta_r) + \beta \cdot g(\theta_r)}.$$

Note that both the surplus and wages are heterogeneous within the two sectors, besides being different between them. This is due to the overall heterogeneity of entrepreneurial ability.

The expected present values of vacancies for firms can be also obtained, since $(J_s - V_s) = (1-\beta) \cdot S_s$ and $(J_r - V_r) = (1-\beta) \cdot S_r$, i.e.:

$$rV_s(x) = \frac{f(\theta_s) \cdot (1-\beta) \cdot (x_s \cdot y_s - z) - c_s \cdot (r + \delta + \rho + \beta \cdot g(\theta_s))}{r + \delta + \rho + (1-\beta) \cdot f(\theta_s) + \beta \cdot g(\theta_s)} \qquad [1]$$

$$rV_r(x) = \frac{f(\theta_r) \cdot (1-\beta) \cdot (x_r \cdot y_r - \tau - z) - c_r \cdot (r + \delta + \beta \cdot g(\theta_r))}{r + \delta + (1-\beta) \cdot f(\theta_r) + \beta \cdot g(\theta_r)} \qquad [2]$$

As in Fonseca et al. (2001), we ignore the range beyond which θ_i is large enough to turn rV_i negative. Hence, it must be that $\theta_i \in [0, \tilde{\theta}_i) \ \ \forall i$, where $\tilde{\theta}_i < \infty$ is the value such that $V_i(\tilde{\theta}_i) = 0$. Furthermore, since for $\theta_i = 0$ the vacancy would be always filled, the relevant interval for θ_i becomes $\theta_i \in (0, \tilde{\theta}_i) \ \ \forall i$, which implies $u \neq 0$, $v_i \neq 0 \ \ \forall i$.

[6] We neglect possibilities of moonlighting, so that workers can perform only one activity at a time.

3.2 Entrepreneurial ability and the underground sector

A key feature of the model is that the comparison between the expected profitability of posting vacancies in the two sectors depends on the entrepreneurial ability of individuals (x) (see Lisi and Pugno, 2010). More precisely, let us assume the following.

Assumption 2*. Entrepreneurial ability x is distributed over a unitary set of a continuum of infinitely-living individuals who expect to participate in production activity either as entrepreneurs or as workers. This ability can be measured in continuous manner, $x \in [0, x_{max}]$, following the known c.d.f. $F:[0,\ x_{\max}] \to [0,1]$.*

The individual must be endowed with a minimum level of entrepreneurial ability in order to open a vacancy, thus becoming an entrepreneur. As will shortly be made clear, this minimum level is required to enter the underground sector only, because the level of ability required to enter the regular sector is even higher. The minimum ability required to become an entrepreneur, labelled with $x_{\min}$, can thus be obtained from the zero-profit condition in the underground sector, i.e. from $V_s = 0$ in equation [1]:[7]

$$\lim_{v_S \to 0}\left[\frac{c_s}{f(\theta_s)} = \frac{(1-\beta_s)\cdot(x_s \cdot y_s - z)}{(r+\delta+\rho+\beta_s \cdot g(\theta_s))}\right] \Rightarrow x_{\min} = \frac{z}{y_s} > 0$$

Therefore, the zero-profit condition can be used to distinguish entrepreneurs from workers.

Lemma 1. *All the individuals endowed with $x > x_{\min}$, i.e. within the interval $F(x_{\max}) - F(x_{\min})$, expect to profitably open a vacancy, thus becoming entrepreneurs, while the individuals, labelled with $l \equiv F(x_{\min})$ and endowed with $x < x_{\min}$, will not post any vacancy, thus becoming workers.*

Note that entrepreneurs will earn extra-profit as a rent in posting vacancies, because ability is not tradeable.

Let us now define a threshold level of entrepreneurial ability $T \in]x_{min}, x_{max}]$ such that two entrepreneurs drawn from the two sectors yield equal expected profitability, i.e.:

$$V_r(x = T) = V_s(x = T) \qquad [3]$$

T can therefore be derived from equations [1], [2], and [3]:

$$T = \frac{\dfrac{\tau + z + c_r \cdot A}{A+1} - \dfrac{z + c_s \cdot B}{B+1}}{\dfrac{y_r}{A+1} - \dfrac{y_s}{B+1}} \qquad [4]$$

[7] In a framework in which the number of firms is fixed, the zero-profit condition is no longer used to determine the labour-market tightness (see Fonseca et al., 2001, and Pissarides, 2002).

with $A \equiv \frac{r+\delta+\beta\cdot g(\theta_r)}{(1-\beta)\cdot f(\theta_r)}$ and $B \equiv \frac{r+\delta+\rho+\beta\cdot g(\theta_s)}{(1-\beta)\cdot f(\theta_s)}$.

In order to have a positive expression on the *r.h.s.* of equation [4], the following restrictions are sufficient: $(\tau+z)>c_s$, $c_r>z$, while y_r must be sufficiently greater than y_s (see *Appendix A* for details). The first two restrictions are realistic;[8] the fourth restriction is necessary for the regular sector to be able to survive, and it qualifies our Assumption 1.

A further result can be obtained from these restrictions: the intercept of $V_r(x)$ is lower than the intercept of $V_s(x)$, and the slope of $V_r(x)$ is steeper than the slope of $V_s(x)$ (see Fig. 3).

========== *Fig. 3 about here (now at the end)* ==========

From the macroeconomic point of view, the entrepreneurs' indifference condition [3] implies that, given the set of entrepreneurs $1-l$, the share of entrepreneurs who open a vacancy in the regular sector is:

$$1-F(T)=v_r \qquad [5]$$

while the share

$$F(T)-l=v_s \qquad [6]$$

opens a vacancy in the underground sector. Entrepreneurs may thus post a vacancy and then fill the job, or fail to fill it, in one of the two sectors, so that it can be simply stated that $v_r=1-(v_s+l)$.[9] Hence, equation [4] can be re-written in a more general form as follows:

$$T=T(v_s) \qquad [7]$$

In this subsection *u* is taken as exogenous, because it is taken by entrepreneurs, so that equation [7], henceforth called *T*-curve, makes evident the relationship between the two variables v_s and *T*. It can thus be proved that $\partial T/\partial v_s<0$ under restrictions very similar to those for $T=T(v_s)>0$ (see again *Appendix A*). The negative relationship in equation [7] captures the wage cost effect, and the effect due to *search* or *congestion* externalities (see Pissarides, 2000). If the irregular vacancies increase, wages increase, and the probability of filling them is lower. Hence, it is more difficult to fill an irregular vacancy and fewer entrepreneurs enter the irregular sector.

[8] The value of the start-up cost in the underground sector c_s should be very low, since ease of entry is often one of the criteria used to define the informal sector (Gërxhani, 2004). By contrast, the start-up cost c_r is often very heavy because of excessive regulations, administrative burdens, licence fees, bribery (Bouev, 2005).

[9] In this model, the number of incumbent entrepreneurs, who run n_r + n_s firms, is exogenous, and adds to those who enter the market. Matters thus become simpler without loss of generality.

Equation [7] can be coupled with equation [6], which represents the distribution of ability across entrepreneurs. In this equation v_s is monotonically rising in *T* from $x_{\min}$ up to $x_{\max}$. Both equations [6] and [7] can thus be depicted in the diagram with axes $[v_s, T]$, as in Fig. 4. Equation [7] has been built for $T \in]x_{\min}, x_{\max}[$, so that its vertical start-point is higher than the intercept of equation [6].

Lemma 2. *A unique intersection between the two curves exists, thus determining the partial equilibrium of the model, since u is taken as given.*

========== *Fig. 4 about here (now at the end)* ==========

From this result, and from the previous one represented in Fig. 3, a further result follows, thus substantiating the statement that the minimum level of entrepreneurial ability to profitably open a new vacancy, i.e. $x_{\min}$, strictly regards the underground sector.

Lemma 3. *The less able entrepreneurs open irregular vacancies; the abler entrepreneurs open regular vacancies.*

3.3 Unemployment and the steady state general equilibrium

Although the economy has two sectors, we empirically observe a single rate of unemployment, which is defined thus:

$$u = l - n_r - n_s \qquad [8]$$

where n_r and n_s represent steady-state employment in the regular and underground sector, respectively. Since jobs arrive to unemployed workers at the rate $g(\theta_i)$, with $i \in \{r, s\}$, and regular and irregular filled jobs are destroyed at the rate δ and $(\delta + \rho)$, respectively, then in the steady-state equilibrium it must be that:

$$\delta \cdot n_r = u \cdot g(\theta_r) \qquad [9]$$

$$(\delta + \rho) \cdot n_s = u \cdot g(\theta_s) \qquad [10]$$

Given the assumptions in subsection 3.2, we can view $u \cdot g(\theta_r)$ and $u \cdot g(\theta_s)$ as the share of skilled and unskilled workers who find jobs, respectively. Steady-state unemployment is thus given by equations [8], [9] and [10]:

$$u = \frac{l}{\frac{g(\theta_r)}{\delta} + \frac{g(\theta_s)}{\delta + \rho} + 1} \qquad [11]$$

Equation [11] closes the model, since *u*, which was exogenous in the previous subsection, can now be determined, so that the results previously obtained in partial

equilibrium, also hold in general equilibrium. A restriction on the relative sizes of the two sectors is a sufficient condition that is common in most countries.

Lemma 4. *A steady-state general equilibrium with positive u exists, is unique and stable if* $v_s \leq v_r$ *(see Appendix B for proof).*

Therefore, this concluding proposition can be obtained.

Proposition 1. *The solutions for the four key variables* v_s, v_r, T *and* u *are obtained by considering: 1) the present discounted values of the vacancies, i.e. equations [1] and [2]; 2) the entrepreneurs' indifference condition between open vacancies in the two sectors, given their entrepreneurial ability distribution, and the threshold level of entrepreneurial ability, i.e. equations [3] and [4]; 3) the unemployment identity [8] and the equilibrium condition of the transition flows on the supply side of the labour market, i.e. equations [9] and [10].*

3.4 Discussion

The main result of the model of this section is that not only is there an interior solution whereby both the underground sector and the regular sector survive in equilibrium (Boeri and Garibaldi, 2006; Albrecht et. al., 2009), but this equilibrium is determined by allocating heterogeneous entrepreneurial ability between the two sectors (Rauch, 1991; Carillo and Pugno, 2004; Pugno, 2000). This may explain the so-called "shadow puzzle", i.e. the persistence of the underground sector despite advances in detection technologies and greater organisation by public authorities to reduce irregularities (issue (*i*) in section 1). This kind of explanation runs counter to the argument that the underground sector is an incubator of infant industries (see also La Porta and Shleifer, 2008; Rauch, 1991; Levenson and Maloney, 1998).

A number of other important results can be drawn from comparative statics exercises, although described in dynamic terms for shortness. A general exercise concerns the effects of the shift of the *T*-curve due to changes in some parameters. Its downward shift decreases both the (partial) equilibrium of v_s in Fig. 4, and the model's (general) equilibrium of v_s, and hence also θ_s. Therefore, this downward shift squeezes the proportion of the underground sector and expands the proportion of the regular sector, as clearly emerges from equations [5] and [6], and as can be easily derived from equations [8], [9] and [10] jointly.

The downward shift of the *T*-curve can thus increase overall output, because it increases the proportion of the most productive sector. The regular sector is in fact more productive than the underground sector for two reasons: the regular sector exhibits a greater labour productivity, and the most able entrepreneurs prefer this sector. In fact, for a greater

number of regular vacancies made possible by the shift of the abler entrepreneurs from the underground sector, both the number of regular matches, $m_r = m(v_r, u)$, and skilled employment, n_r, are greater because of the greater probability to find a regular job.

The downward shift of *T*-curve also increases the shadow wage gap, i.e. the wage differentials between the two sectors. This effect is due to the rise of the equilibrium level of v_r, since the wages are increasing functions with respect to the vacancies level.

The main policy implications can be drawn from the effects of the changes in the policy parameters on *T*, and hence on the proportion of the underground sector, i.e.:

$$\frac{\partial T}{\partial \rho} < 0;\ \frac{\partial T}{\partial \tau} > 0;\ \frac{\partial T}{\partial c_r} > 0.$$

In words, closer monitoring, lower taxation and lower start-up costs reduce the underground sector. This is in line with the conclusions of other models (see e.g. Friedman et al., 2000; Johnson et al., 2000; Sarte, 2000; Bouev, 2005).

An important new contribution of this model regards a much more controversial question, i.e. the ambiguous relationship between the underground economy and unemployment (issue (*ii*) in section 1).

Proposition 2. *The relationship between v_s and u is negative if ρ is sufficiently low (and $v_s \leq v_r$). The relationship between v_s and u is positive if ρ is sufficiently high (and $v_s \leq v_r$) (see Appendix C for proof).*[10]

This is an interesting result from the policy implications point of view. In fact, the role of the monitoring parameter is strengthened, since any policy intended to reduce the irregular sector may also reduce the unemployment rate if ρ is sufficiently high.[11]

4. Extensions to investment in education and productivity growth

4.1 A steady-growth solution of the model

This paper assumes that human capital accumulation is the primary engine of economic growth. In the growth literature, workers' human capital usually refers to "*the average level of educational attainment*" (Nelson and Phelps, 1966; Benhabib and Spiegel,

[10] A very small calibration value of monitoring is usual in the literature. Precisely, it ranges between 0.03 (Busato and Chiarini, 2004) and 0.06 (Boeri and Garibaldi, 2006).

[11] Bosch and Esteban-Pretel (2009) focus on the role of the job destruction rate. According to their matching model, policies that reduce the cost of formality (or those that increase the cost of informality) produce an increase in the share of formal employment while also reducing unemployment because the reallocation between formal and informal jobs has non-neutral effects on the unemployment rate, since informal jobs record much higher separation rates.

1994) or similarly to "*the average total years of schooling*" (Savvides and Stengos, 2009).[12] Specifically, education and schooling enable workers to absorb knowledge and acquire additional human capital once employed (Rosen, 1976; Stokey, 1991; Laing et al., 1995). Therefore, it can be stated that the higher the level of schooling or knowledge (k) and the larger the human capital accumulation (h), the higher is the rate of economic growth.

To simplify matters, and without loss of generality, we assume $h = k$, so that education and human capital will be used interchangeably. Then, let us specify a simple equation for the rate of productivity growth (γ):

$$\gamma = \gamma(h) \qquad \text{with } \gamma'(h) > 0,\ \gamma''(h) < 0 \qquad [12]$$

with the further property that $r > \gamma(h)\ \forall h$, in order to keep present values finite.

Since the education level and skill in the workers employed in the regular sector are higher than those in the underground sector (Albrecht et. al., 2009; Cappariello and Zizza, 2009), growth is expected to be faster in the regular sector. This link is assumed in the form of labour-augmenting technological progress *à la* Pissarides (2000),[13] where, specifically, workers' human capital plays two roles, as suggested by Laing et al. (1995). In fact, since human capital is firstly acquired through formal education, workers can be employed with an initial productivity (y_0) that depends on the level of schooling (h). Secondly, workers' productivity increases according to equation [12]. Let us then state the following assumption.

Assumption 3*. The total discounted value of productivity in the regular sector is given by:*

$$y_r(h) = \int_0^{\infty} e^{-r\cdot t} \cdot y_0(h) \cdot e^{\gamma(h)\cdot t}\, dt \Rightarrow \frac{y_0(h)}{r - \gamma(h)} \qquad [13]$$

where:

$$y_0 = y_0(h) \qquad \text{with } y_0'(h) > 0,\ \lim_{h\to 0} y_0 = 0,\ \lim_{h\to\infty} y_0 < \infty \qquad [14]$$

Productivity in the underground sector is given by:

$$y_s = \varphi \cdot y_r(h) \qquad \text{with } 0 < \varphi < 1 \qquad [15]$$

According to this assumption, the underground sector partially benefits from this process because of spill-over effects in the diffusion of knowledge. Therefore, both sectors can grow at the same rate $\gamma(h)$, while the level of productivity in the regular sector remains higher than that of productivity in the underground sector.

[12] Indeed, the latter is often used as a quantitative proxy in empirical estimations (Savvides and Stengos, 2009).

[13] In our terms, Pissarides's (2000) simple specification is: $y_r(h,t) = y_0 \cdot e^{\gamma(h)\cdot t}$.

In order to endogenise the rate of productivity growth, let us consider the optimal choice of education for individuals, given that schooling investment is costly (cf. Laing et al., 1995; Decreuse and Granier, 2007), and that only regular firms profitably employ educated workers. Formally:

Assumption 4. *Let the cost function of education be c(k), with* $c'(k) > 0$, $c''(k) > 0$ *and* $\partial c(0)/\partial k = 0$, *because of either a direct pecuniary cost or the disutility from scholastic effort. Each job-seeker in the regular sector solves the following programme, before entering the labour market:* [14]

$$max_{k \geq 0} \{U_r - c(k)\}$$

$$\Rightarrow \max_{k \geq 0} \left\{ \frac{z}{r + g(\theta_r)} + \frac{g(\theta_r)}{r + g(\theta_r)} \cdot W_r(w_r(k)) - c(k) \right\}$$

since $rU_r = z + g(\theta_r) \cdot [W_r - U_r] \Rightarrow U_r = \frac{z}{r + g(\theta_r)} + \frac{g(\theta_r)}{r + g(\theta_r)} \cdot W_r(w_r(k))$, and wage depends on both labour market tightness and productivity.

The job-seeker's investment in education that maximises the value of his/her future search (k^*) can be obtained by the usual condition:

$$\frac{g(\theta_r)}{r + g(\theta_r)} \cdot \frac{\partial w_r(k^*)}{\partial k} - \frac{\partial c(k^*)}{\partial k} = 0 \qquad [16]$$

This condition shows a positive relationship between θ_r and *k*, besides the implication that $k^* > 0$. In fact, a rise in θ_r increases the probability of finding a regular job, i.e. $g(\theta_r)$, and consequently both the regular matches and regular wages increase. Hence, in order to search for a job (work) in the regular sector, more workers choose to invest in education. In turn, the higher the optimal investment in education, the greater is human capital and the greater is the productivity level of the economy. Therefore, regular wages are higher also for the increase in the productivity level, while the increase in the size of the regular sector, i.e. θ_r, spurs economic growth by a higher investment in education.

It follows that, from a macroeconomic point of view, the investment in education is on the one hand negatively linked to the size of the underground sector, and on the other, positively linked to productivity growth of the economy through *Assumption 3* and the equation $h = k$. The following Proposition can thus be stated.

[14] Workers invest in education when young, and having completed their schooling, they search for employment (Laing et al., 1995).

Proposition 3. *The solution of the steady state model can be extended to include the optimal investment in education* (k^*)*, and the rate of productivity growth of the economy* (γ)*, thus finding a steady-growth solution.*

These results, together with *Proposition 2* of the previous section regarding the relationship between the underground economy and unemployment, help understand the relationship between economic growth and unemployment (issue (*iii*) in section 1). Indeed, the relationship between $\gamma(h)$ and *u* is positive if ρ is low, this relationship is negative if ρ is high, under the condition that $v_s \leq v_r$.

Our analysis is thus able to reconcile the conflicting results found in the literature on growth and unemployment. This suggestion is alternative to Aghion and Howitt's approach, nevertheless it refers to the structure of the economy. Since the condition $v_s \leq v_r$ is the usual condition throughout the world, the monitoring rate becomes a very important parameter. Not only does it affect the size of the underground sector, but it may positively affect both unemployment and economic growth.

4.2 The case of multiple equilibria

The extended model may also be adapted in order to account for a relevant case: that of regional dualism, i.e. the failure of the more backward region to catch up with the more developed region.

Let us assume that $y_0(h)$ is a logistic function, i.e. it performs increasing returns to human capital before the usual and eventual decreasing returns. This form may be due to thresholds in human capital, i.e. once human capital attains a certain threshold level (*critical mass*) productivity may reach a higher steady-state level (Azariadis and Drazen, 1990). This pattern has also received some empirical evidence (Savvides and Stengos, 2009).[15]

Under this assumption, the relationship between T and v_s may change significantly. Indeed, if the functions [13] and [15] are plugged into [4], then multiple equilibria become possible since the *T*–curve may display an increasing part in the middle, thus cutting the other curve twice, as depicted in Fig. 4 (dotted line).[16]

[15] The models which describe general nonlinearities in the relationship between growth and human capital do not provide specific functional forms (Savvides and Stengos, 2009). Azariadis and Drazen (1990) even study a step functional form, where thresholds are more than one.

[16] As shown by Savvides and Stengos (2009) – adapted from Azariadis and Drazen (1990) – a step functional form may generate the possibility of multiple equilibria, with different balanced growth paths. This growth process comes to an end when *"labour productivity attains the highest possible value and the system settles down on the ultimate stage of growth"* (Azariadis and Drazen, 1990, p. 517).

The two extreme equilibria may be labelled as “good” and “bad” because they define two different conditions where the proportion of the underground sector is small and, respectively, large, with the consequent desirable and undesirable characterisations. Specifically, in the “good” equilibrium one region exhibits higher productivity, a more efficient use of entrepreneurial ability, higher investment in education, greater employment of skilled workers, and, finally, a higher rate of economic growth with respect to the region in the “bad” equilibrium.

This result is interesting because it can represent an economy characterised by a uniform institutional set-up, as captured by the same parameters of the model, but with two regions that differ in their histories, as captured by the initial economic structure. The region that has inherited a greater proportion of the underground sector may converge towards the “bad” equilibrium. The region that has inherited a smaller proportion of the underground sector may converge towards the “good” equilibrium. However, the region in the “bad” equilibrium does not catch up with the other region, because it exhibits a lower steady-growth. This case seems to be the best fit with the Italian North-South divide, which is special but not unique in the world. This case is also interesting theoretically, because it shows the crucial importance of the allocation of entrepreneurship for economic development.

5. Conclusions

Several empirical studies clearly document that the underground sector persists with a different size in many and various countries around the world, thus raising the ‘shadow puzzle’. Related studies also show that a less clear pattern emerges in the relationship between the size of the underground sector and unemployment. Another unclear pattern has been observed in the literature on economic growth, i.e. the pattern regarding the relationship between growth and unemployment. However, microeconomic studies have found that underground firms employ relatively backward technology, less skilled and less educated workers, as well as less able entrepreneurs, i.e. lower quality inputs for growth. This microeconomic evidence has suggested useful links to build up a matching type of model that is able to account for both the ‘shadow puzzle’, and the two evidenced unclear patterns.

The assumption that entrepreneurial ability is a heterogeneous input for production is rather new in matching models. However, it can increase their explanatory power, because heterogeneous entrepreneurs can well-match to workers with different skills, thus forming firms with rather different productivity. In this way, less productive firms can persistently

survive by evading taxes, and can discourage human capital accumulation and hence productivity growth.

Monitoring firms' regularity appears to be the key parameter for determining whether or not unemployment is complementary with underground employment, and, consequently, whether unemployment is positively or negatively correlated with economic growth. As shown in Figures 1 and 2, low levels of monitoring appear to make unemployment positively correlated with economic growth, and high levels of monitoring appear to make unemployment negatively correlated with economic growth.

The paper has also been able to account for the special case of regional dualism, as in the Italian case, where the more backward South diverges from the North, although both regions share the same institutional set-up. This case may arise if non-linearities in the human capital accumulation function produce multiple equilibria in the size of the underground sector.

Finally, a number of policy implications follow from this analysis. Reducing the tax burden becomes especially effective if monitoring is at a high level, because underground firms are discouraged without raising unemployment. In the long run, this may also enhance growth. These same results follow if monitoring is itself increased. In the case of regional dualism, a one-shot change in the policy parameters may trigger an endogenous dynamic of convergence between the two regions. More generally, an effective policy should seek to increase entrepreneurial ability, typically through education, so that overall economic performance improves, both because of the sectoral composition effect, and because of the positive level effect of each firm.

Appendices

Appendix A: Properties of equation [4]

The threshold *T* is a special *x*, so that it must be positive since $x > x_{\min} \geq 0$. Hence, also the *r.h.s.* of [4] must be positive. Sufficient conditions for the positivity of the *r.h.s.* of [4] are:

$$\frac{y_r}{A+1} > \frac{y_s}{B+1} \qquad [A.1]$$

$$\frac{(\tau + z_r) + c_r \cdot A}{A+1} > \frac{z_s + c_s \cdot B}{B+1} \qquad [A.2]$$

Let us examine the limit of the previous key conditions for v_r (and v_s) which goes to zero.

- If $v_r \to 0$, then $A \to 0$ and $B \to \{0 < \overline{B} < \infty\}$, so that:

  $y_r > \frac{y_s}{\overline{B}+1}$, which is always true if $y_r > y_s$, and

  $(\tau + z) > \frac{z + c_s \cdot \overline{B}}{\overline{B}+1} \Rightarrow \overline{B} > \frac{z - (\tau + z)}{(\tau + z) - c_s}$, which requires as sufficient conditions that: $\tau > 0$, and $(\tau + z) > c_s$.

- If $v_s \to 0$, then $B \to 0$ and $A \to \{0 < \overline{A} < \infty\}$, so that:

  $\frac{y_r}{\overline{A}+1} > y_s \Rightarrow y_r > y_s \cdot (\overline{A}+1)$ which requires that y_r is sufficiently greater than y_s,

  $\frac{(\tau + z) + c_r \cdot \overline{A}}{\overline{A}+1} > z_s \Rightarrow \overline{A} > \frac{z - (\tau + z)}{c_r - z}$, with $c_r > z$ as a sufficient condition to hold.

The proof that $\partial T / \partial v_s < 0$ in [4] thus becomes straightforward, bearing in mind that $1 - l = v_s + v_r$, and that $\theta_i = v_i / u$. Since $\frac{\partial A}{\partial v_s} < 0$ and $\frac{\partial B}{\partial v_s} > 0$, the denominator of [4] is rising in v_s, i.e. $\frac{\partial}{\partial v_s}\left(\frac{y_r}{A+1} - \frac{y_s}{B+1}\right) > 0$, while, the numerator of [4] is decreasing in v_s:

$$\frac{\partial}{\partial A}\left(\frac{\tau + z + c_r \cdot A}{A+1}\right) = \frac{c_r - (\tau + z)}{(A+1)^2} > 0 \qquad \text{if } c_r > (\tau + z)$$

$$\frac{\partial}{\partial B}\left(\frac{z + c_s \cdot B}{B+1}\right) = \frac{c_s - z}{(B+1)^2} > 0 \qquad \text{if } c_s > z.$$

The complete restriction set of the parameters is thus: $c_r > (\tau + z) > c_s > z$. Note that these are sufficient but not necessary conditions to obtain $\partial T / \partial v_s < 0$.

Appendix B: Proof of Lemma 4

In order to prove the existence, uniqueness and stability of the solution for *u*, let us rewrite equation [11] as follows:

$$u = \frac{l}{\frac{g((1 - l - v_s)/u)}{\delta} + \frac{g(v_s/u)}{\delta + \rho} + 1} \qquad [B.1]$$

which, together with equations [4] and [6], form a system in the three unknowns v_s, T, and u. The existence and uniqueness of the solution of the subsystem [4] and [6] in v_s and T is given in the text and in the *Appendix A*. It is thus sufficient to prove that:

$\left(\frac{\partial T(v_s, u(v_s))}{\partial v_s}\right)<0$, where $u(v_s)$ is the explicit general form of [B.1].

This inequality can be studied in three steps. First: $\left(\frac{\partial T}{\partial A}\right)>0$, $\left(\frac{\partial T}{\partial B}\right)<0$, which has been proved in the *Appendix A* under the stated restrictions on the parameters. Second: $\left(\frac{\partial A}{\partial \theta_r}\right)>0$, $\left(\frac{\partial B}{\partial \theta_s}\right)>0$, which follows from the simple inspection of the definitions of A and B, given that $v_r = (1-l-v_s)$ and the definitions $\theta_s \equiv \left(\frac{v_s}{u(v_s)}\right)$ and $\theta_r \equiv \left(\frac{v_r}{u(v_r)}\right)$. Third: $\frac{\partial \theta_s}{\partial v_s}>0$, and $\frac{\partial \theta_r}{\partial v_s}<0$. Proof of the third step is thus in order.

Let us start by showing that the two latter inequalities require that: $-\frac{1}{\theta_r} < \frac{\partial u(v_s)}{\partial v_s} < \frac{1}{\theta_s}$. An explicit form of the middle term can be obtained by using the Cobb-Douglas specification of the matching function, which is usual in the literature (Petrongolo and Pissarides 2001). Hence, given that $m(\theta^{-1},1) \equiv f(\theta) = \theta^{-a}$, and $m(1,\theta) \equiv g(\theta) = \theta \cdot f(\theta) = \theta^{1-a}$:

$$\frac{\partial u(v_s)}{\partial v_s} = \frac{(1-a)\cdot\left[\delta^{-1}\theta_r^{-a} - (\delta+\rho)^{-1}\theta_s^{-a}\right]}{a\cdot\left(\delta^{-1}\theta_r^{1-a} + (\delta+\rho)^{-1}\theta_s^{1-a}\right)+1}$$

With some manipulations, it can be shown that this derivative lies between the range $\left(-\frac{1}{\theta_r}, \frac{1}{\theta_s}\right)$ if $0<v_s \le v_r$ and if a is not unrealistically low, i.e. $a \ge \frac{1}{2\cdot(\delta/\rho+1)}$. Indeed, if the estimate found in the literature is applied, a = 0.5, then the only restriction $0<v_s \le v_r$ is sufficient.

More detailed proofs are available on request from the authors.

Appendix C: Proof of Proposition 2

From the previous Appendix, it emerges that $\frac{\partial u}{\partial v_s}$ can be negative or positive, although within the range $\left(-\frac{1}{\theta_r}, \frac{1}{\theta_s}\right)$. A level of ρ can be obtained such that $\frac{\partial u}{\partial v_s} = 0$. This level is the following, by using the Cobb-Douglas specification:

$$\rho_0 = \delta\cdot\left[(\theta_r/\theta_s)^a - 1\right] \qquad [C.1]$$

A similar condition can be also obtained by the Beveridge Curve of both sectors. From equation [11], it is straightforward to get:

$$\frac{\partial u}{\partial v_r} = -\left\{\frac{l\cdot\delta\cdot(\delta+\rho)^2\cdot g'(\theta_r)}{\left[(\delta+\rho)\cdot g(\theta_r)+\delta\cdot g(\theta_s)+\delta\cdot(\delta+\rho)\right]^2}\right\} \overset{>0}{} < 0$$

$$\frac{\partial u}{\partial v_s} = -\left\{ \frac{l \cdot \delta^2 \cdot (\delta + \rho) \cdot \overset{>0}{g'(\theta_s)}}{[(\delta + \rho) \cdot g(\theta_r) + \delta \cdot g(\theta_s) + \delta \cdot (\delta + \rho)]^2} \right\} < 0$$

with $v_r > v_s$, i.e. $\theta_r > \theta_s$, and knowing that $g'(\theta_i) > 0$, $g''(\theta_i) < 0 \quad \forall i$, we obtain $g'(\theta_s) > g'(\theta_r)$. Hence, if there is no monitoring ($\rho = 0$), the unemployment rate increases when the irregular vacancies decreases, because the *Beveridge Curve* of the underground sector is steeper than the *Beveridge Curve* of the regular sector, i.e. $\partial u / \partial v_s > \partial u / \partial v_r$.[17] However, a positive level of monitoring is a necessary condition to preserve legal jobs. Indeed, there is a threshold level of monitoring which reverses the previous result, thus making the *Beveridge Curve* of the regular sector steeper:

$$\rho > \{\delta \cdot [g'(\theta_s) / g'(\theta_r) - 1]\} = \rho_0 \qquad \text{[C.1b]}$$

which is a positive value since $[g'(\theta_s) / g'(\theta_r)] > 1$.[18]

Therefore, $\frac{\partial u}{\partial v_s} > 0$ if $\rho > \rho_0$; whereas, $\frac{\partial u}{\partial v_s} < 0$ if $\rho < \rho_0$. In particular, if $v_s = v_r$, then $\frac{\partial u}{\partial v_s} > 0$ for every ρ (since $\rho_0 = 0$); while if v_s is especially small, then $\frac{\partial u}{\partial v_s} < 0$ for every ρ.

[17] Indeed, equation [11], like the standard *Beveridge Curve*, is a decreasing and convex function with respect to both v_r and v_s:

$$\frac{\partial^2 u}{\partial v_r^{\,2}} = \frac{-l \cdot \delta \cdot (\delta + \rho)^2 \cdot \overset{<0}{g''(\theta_r)} \cdot H^2 - \left\{ -l \cdot \delta \cdot (\delta + \rho)^2 \cdot g'(\theta_r) \cdot 2 \cdot H \cdot (\delta + \rho) \cdot g'(\theta_r) \right\}}{H^4} > 0$$

$$\frac{\partial^2 u}{\partial v_s^{\,2}} = \frac{-l \cdot \delta^2 \cdot (\delta + \rho) \cdot \overset{<0}{g''(\theta_s)} \cdot H^2 - \left\{ -l \cdot \delta^2 \cdot (\delta + \rho) \cdot g'(\theta_s) \cdot 2 \cdot H \cdot \delta \cdot g'(\theta_s) \right\}}{H^4} > 0$$

where $H \equiv [(\delta + \rho) \cdot g(\theta_r) + \delta \cdot g(\theta_s) + \delta \cdot (\delta + \rho)]$.

[18] Note that in the inverse case (i.e. $\rho < \rho_0$) we cannot ensure that the monitoring rate is positive, since ρ_0 may be a very small value.

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Figures

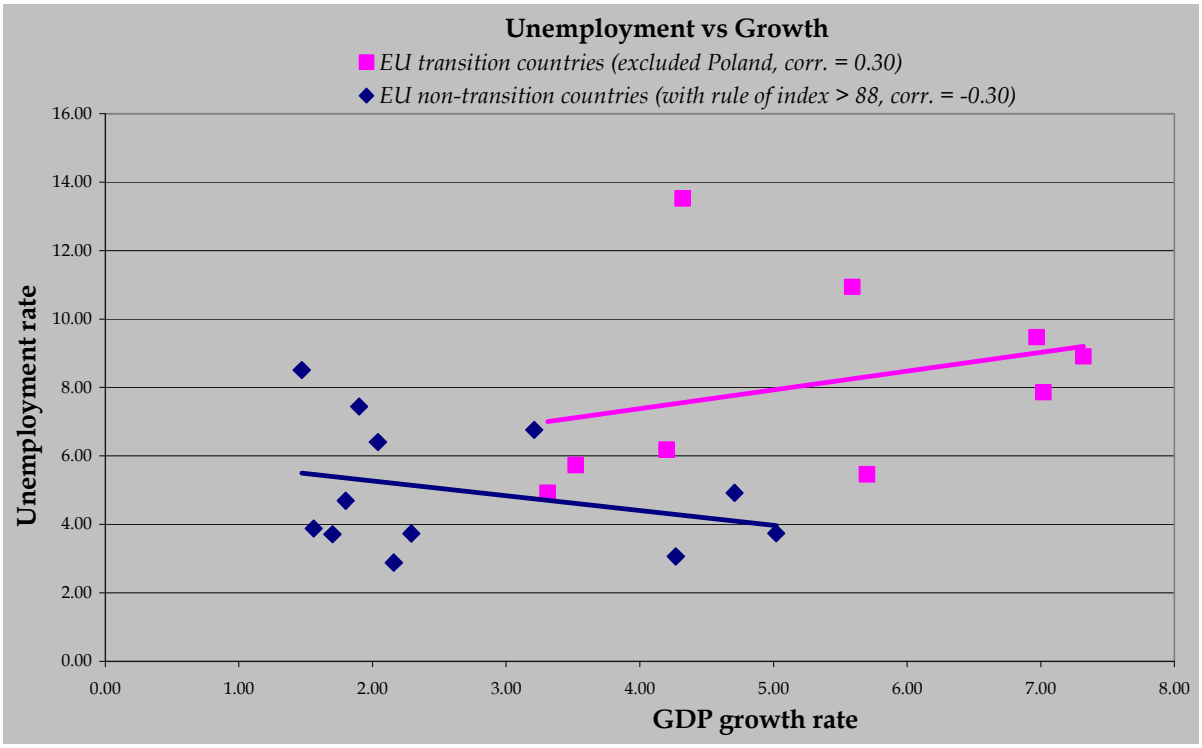

Figure 1. Unemployment vs Growth in EU countries (see Table 2 for the data details)

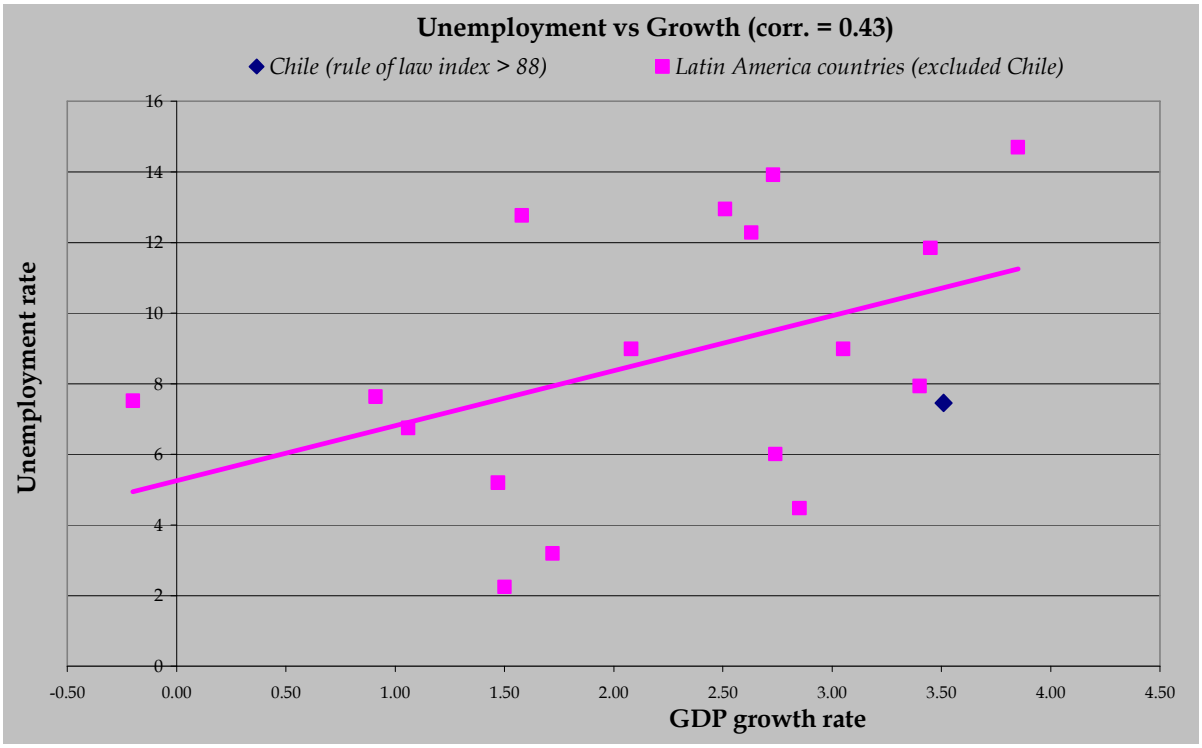

Figure 2. Unemployment vs Growth in Latin America countries (see Table 3 for the data details)

Table 1. Data for Figure 1

EU non-transition countries	**unemployment rate (%) ***	**GDP growth rate (%) ***	**Rule of Law **** (Percentile Rank ***)
Austria	3.73	2.29	99.0
Belgium	6.40	2.04	89.0
Cyprus	3.63	3.77	84.2
Denmark	3.88	1.56	99.5
Finland	6.76	3.21	97.6
France	7.44	1.90	90.0
Germany	8.51	1.47	93.3
Greece	7.98	3.98	73.2
Ireland	3.74	5.02	94.3
Italy	6.43	1.16	62.2
Luxembourg	3.06	4.27	96.2
Malta	4.69	1.80	91.4
Netherlands	2.88	2.16	94.7
Portugal	5.48	5.84	83.7
Spain	8.43	2.80	85.2
Sweden	4.92	4.71	98.1
United Kingdom	3.71	1.70	92.3

EU transition countries	**unemployment rate (%) ***	**GDP growth rate (%) ***	**Rule of Law **** (Percentile Rank ***)
Bulgaria	10.94	5.59	51.2
Czech Republic	6.18	4.20	77.0
Estonia	7.86	7.02	84.7
Hungary	5.73	3.52	76.1
Latvia	8.91	7.32	71.3
Lithuania	9.47	6.97	67.5
Poland	13.11	1.32	65.1
Romania	5.46	5.70	53.6
Slovakia	13.52	4.32	67.0
Slovenia	4.92	3.31	82.3

** (2000 - 2008) average.*

*Source: (*http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/themes*)*

*** Source: (*http://info.worldbank.org/governance/wgi/mc_countries.asp*)*

**** Percentile rank, from 0 (worst) to 100 (best). Precisely, according to the World Bank, the 'Rule of Law' index measures the quality of contract enforcement, the police, and the courts, as well as the likelihood of crime and violence.*

Tabel 2. Data for Figure 2

Latin America countries	Unemployment rate *	GDP growth rate **	Rule of Law index ***
Argentina	12.95	2.51	32.10
Bolivia	5.2	1.47	12.00
Brazil	8.99	2.08	46.40
Chile	7.46	3.51	88.00
Colombia	13.92	2.73	37.80
Costa Rica	6.01	2.74	62.70
Dominican Republic	14.7	3.85	33.00
Ecuador	8.99	3.05	9.10
El Salvador	6.75	1.06	30.60
Guatemala	2.25	1.50	12.90
Honduras	4.48	2.85	20.60
Mexico	3.2	1.72	29.70
Nicaragua	7.64	0.91	21.10
Panama	11.85	3.45	49.80
Paraguay	7.52	-0.20	15.30
Peru	7.94	3.40	25.80
Uruguay	12.77	1.58	65.60
Venezuela, R. B. de	12.28	2.63	2.90

* *(%) of labour force (2000-2008) average. Source:* http://data.worldbank.org/indicator/
** *(2000 - 2007) average.*
Source: Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.3, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, August 2009.
http://pwt.econ.upenn.edu/php_site/pwt_index.php
*** *Source: (*http://info.worldbank.org/governance/wgi/mc_countries.asp*). Percentile rank, from 0 (worst) to 100 (best).*

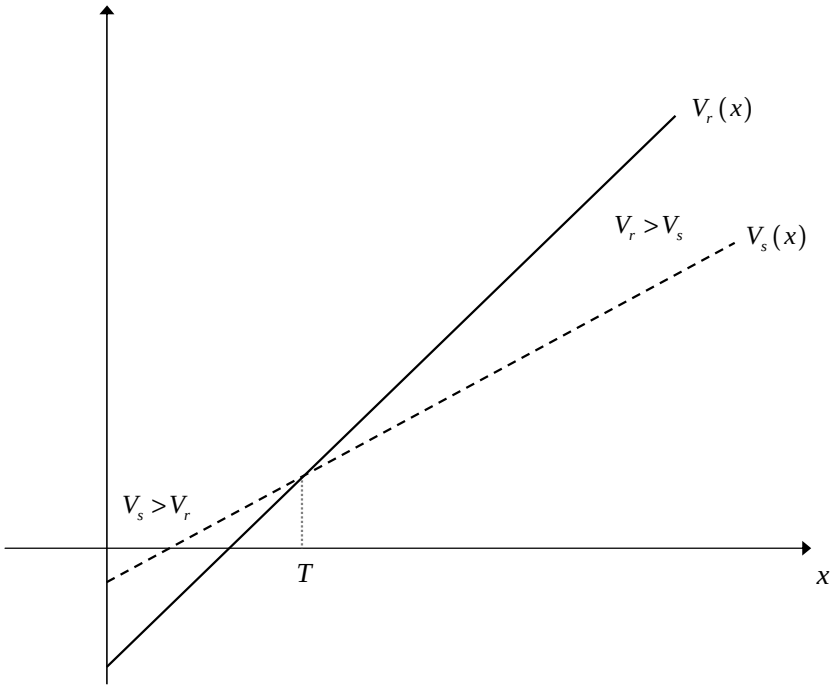

Figure 3. Entrepreneurs' indifference condition

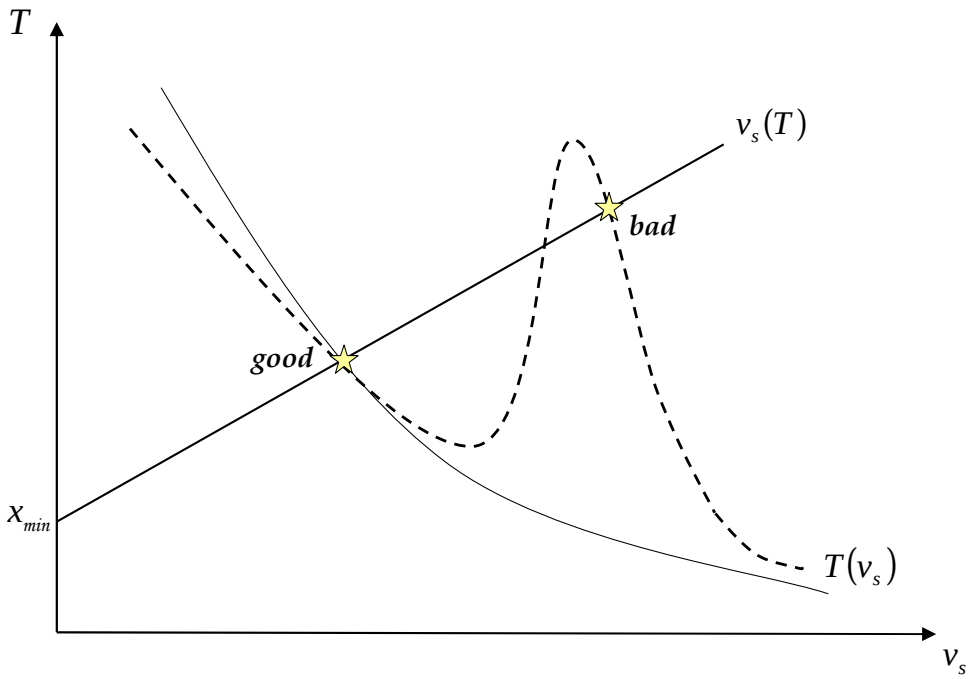

Figure 4. Interior equilibrium and multiple equilibria