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Sequential Search with Incompletely Informed Consumers: Theory and Evidence from Retail Gasoline Markets

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Abstract

A large variety of markets, such as retail markets for gasoline or mortgage markets, are characterized by a small number of firms offering a fairly homogenous product at virtually the same cost, while consumers, being uninformed about this cost, sequentially search for low prices. The present paper provides a theoretical examination of this type of market, and confronts the theory with data on retail gasoline prices. We develop a sequential search model *with incomplete information* and characterize a perfect Bayesian equilibrium in which consumers follow simple reservation price strategies. Firms strategically exploit consumers being uninformed about their production cost, and set on average higher prices compared to the standard complete information model. Thus, consumer welfare is lower. Using data on the gasoline retail market in Vienna (Austria), we further argue that incomplete information is a necessary feature to explain observed gasoline prices within a sequential search framework.

JEL Classification: D40; D83; L13

Keywords: Sequential Search, Price Dispersion, Asymmetric Information

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1 Introduction

Consider a consumer who observes the price of gasoline at a gas station. Knowing that prices between different stations may vary considerably, the consumer must decide whether to buy at the observed price or search for a better deal elsewhere. When making this decision, she must estimate how much of the observed price is due to common factors affecting all gasoline stations in a similar way, e.g. the price of crude oil, and how much is due to idiosyncratic factors affecting the particular seller being visited. If the consumer believes that common factors are more relevant in determining the price, she might consider searching for a cheaper gas station not worthwhile and hence buy at the observed price. Conversely, if she believes that the station charges a particularly high price compared to other stations, she will probably consider it optimal to look for a better deal. A key feature of this problem is that the consumer must take her decision under *incomplete information*: she is uncertain about the gas station's input (production) cost. Moreover, information is *asymmetric*, since gasoline retailers are obviously aware of this cost; they will take this asymmetry into account when setting their prices.[1]

In this paper we study how information incompleteness and asymmetry affect equilibrium in a market like the one described above. To this end, we introduce these features into the sequential search model developed by Stahl (1989). In our model, finitely many firms sell a homogenous product on an oligopolistic market, with all firms facing the same stochastic production cost and being aware of its realization. Consumers have inelastic demand and engage in sequential search for low prices. Unlike Stahl's sequential search model, consumers do not observe the firms' production cost realization. Instead, they hold prior beliefs about the distribution of production costs and update these beliefs as they observe prices.

In this environment, we examine the properties of a *perfect Bayesian equilibrium satisfying a reservation prices property* (PBERP). In such an equilibrium, firms use mixed strategies and sample prices from an optimal distribution, while consumers employ a (possibly non-stationary) reservation price rule: observing a certain price, the consumer buys if the price is below her current reservation price, and searches for a lower price otherwise. At the relevant reservation price the consumer is indifferent between buying and searching for a better deal.

We show that, unlike in the standard search model where consumers know the firms' production cost, the equilibrium consumer search rule is history dependent. In particular, the reservation price in each search round depends on the prices already observed in previous rounds as consumers update beliefs about the production cost realization on the basis of their price ob-

[1]This feature is not only found in gasoline markets, but characterizes many environments such as insurance or mortgage markets.

servations. Consequently, if there exists an equilibrium where optimal search behavior in each round is characterized by a reservation price, then this reservation price must depend on the history of price observations. Fortunately, these reservation prices satisfy an important property: if a consumer observes her reservation price in the first search round and, being indifferent, continues to search, then her next rounds' reservation prices are higher than her first round reservation price. This property is key to our analysis, as it implies that in a PBERP no firm will set a price above this first round reservation price. Importantly, it allows us to characterize the first round reservation price.

Using this property allows us to specify a sufficient condition for the existence of a PBERP that guarantees existence in markets with (i) a small support of the production cost distribution, (ii) relatively large search costs, (iii) relatively many firms, or (iv) relatively few shoppers. We argue that this condition is necessary in the sense that, if it fails to hold, one can always find distributions of the production cost such that a PBERP does not exist. Moreover, we provide an example showing that even when the production cost is uniformly distributed, existence is not guaranteed if our condition is not satisfied. Thus, incomplete information introduces significant changes to the sequential search model with respect to the existence of reservation price equilibria.

At a more substantial level, we arrive at the following comparative statics results. Examining equilibrium price strategies used by firms, a first result shows that the lower bound of the price distribution is increasing in the cost level while its upper bound is independent of the cost level.[2] Thus, the extent of equilibrium price dispersion under incomplete information decreases as the cost level rises, which constitutes an important difference to the complete information setting where the extent of price dispersion is independent of the cost. Next, we highlight that prior beliefs of consumers play an important role in shaping equilibrium price distributions. Specifically, if consumers are more optimistic (pessimistic) that cost is low, then average prices in the market are lower (higher).

We assess the empirical importance of incomplete information within a consumer search framework. To this end, we consider data on gasoline prices charged in Vienna (Austria) in the period January 2007 until June 2009. We first argue that a search environment is indeed appropriate to analyze the Viennese gasoline retail market as a rank reversal test on the data suggests firms use mixed strategies, a key feature of consumer search. We then show that the two central properties of the incomplete information model presented above are supported by

[2] In a PBERP, the upper bound must be equal to the first round reservation price of consumers. Since the latter cannot depend of the cost realization which is unknown to consumers, this property carries over to the upper bound.

the data: (i) the extent of price dispersion is indeed lower for high cost levels than for low ones, (ii) prior beliefs of consumers are important in explaining price distributions as the production cost level alone cannot explain variations in price distributions.

Studying the welfare effects of incomplete information, we show that, from an ex-ante perspective, consumer welfare is unambiguously lower under incomplete information and profits are unambiguously higher as compared to the environment with complete information.

This paper contributes to a large and growing literature on equilibrium consumer search models starting from seminal contributions by Reinganum (1979), Varian (1980), Burdett and Judd (1983), and Stahl (1989). In terms of the research question being addressed, the papers most closely related to our paper are Benabou and Gertner (1993) and Dana (1994).[3] Both papers use, however, environments that substantially differ from the one studied in the present paper. Importantly, they do not adopt a sequential search protocol. Benabou and Gertner analyze a duopoly market where half the consumers observe one price and the other half observes the other price at no cost. The only decision consumers have to make is whether to also observe the price of the firm they have not yet observed at a search cost. Dana considers a model with two types of consumers (informed and uninformed) where the uninformed consumers are engaged in what he calls *newspaper search*. These consumers get a first price quote for free and, on the basis of this price, they decide whether or not to become fully informed about all prices by paying a search cost. Papers by Fershtman and Fishman (1992) and Fishman (1996) and the recent contributions by Yang and Ye (2008) and Tappata (2008) use frameworks similar to Dana (1994), but extend them to a dynamic setting. In such environments, these papers study asymmetric price adjustment to cost shocks, the so-called *rockets-and-feathers* pattern. To the best of our knowledge, our paper is the first to introduce incomplete information into a *sequential* consumer search model.

In a broader sense the current paper is related to recent work which elaborates on the role of information gathering and information processing in consumer search.[4] This literature focuses on obfuscation (Ellison and Wolitzky (2009), Ellison and Ellison (2009)), boundedly rational agents (see, e.g., Spiegler (2006)), or information gatekeepers on the internet (see, e.g., Baye and Morgan (2001)). Another strand of the literature makes progress on the policy implications of the consumer search literature on consumer protection policies (see, e.g., Armstrong, Vickers, and Zhou (2009)) or on the empirical implementation of consumer search models (see, e.g.,

[3]Earlier work by Diamond (1971) and Rothschild (1974) has analyzed optimal search behavior in a world where the price distribution is unknown, but exogenously given. Recently, Gershkov and Moldovanu (2009) uncover some formal relations between optimal stopping rules in the consumer search literature and the problem of ensuring monotone allocation rules in dynamic allocation problems.

[4]Baye, Morgan, and Scholten (2006) surveys a wide range of consumer search models .

Lach (2007), Hortaçsu and Syverson (2004) and Moraga-González and Wildenbeest (2008)).

The remainder of this paper is organized in the following way. In Section 2 we briefly discuss a standard sequential search model with *completely* informed consumers, establishing a theoretical benchmark for comparison of our incomplete information model. In Section 3 we develop our model with *incompletely* informed consumers, define a perfect Bayesian equilibrium and characterize its properties. In Section 4 we examine the effects of incomplete information on consumer and producer welfare. In Section 5, we assess its empirical relevance by confronting the model with data on retail gasoline prices. Finally, in Section 6 we conclude and discuss directions for future research. Proofs are provided in the Appendix.

2 Sequential search with completely informed consumers

We start our analysis by describing a sequential search model with *completely* informed consumers along the lines of Stahl (1989). This model will, at a later stage, serve as our benchmark to assess the implications of incomplete (asymmetric) information within the sequential search framework. Essentially, we modify Stahl's model along only two dimensions. First, to simplify the analysis we consider a model of inelastic demand as in Janssen, Moraga-Gonzalez, and Wildenbeest (2005). Second, we do not normalize marginal costs to zero; solving the model for positive marginal costs is inevitable for our purposes, because later on we want to analyze and compare situations under different marginal cost levels.

2.1 Model

We consider an oligopolistic market where N firms sell a homogenous good and compete in prices. Each firm $n \in \{1,...,N\}$ faces the same production technology and the same marginal production cost, denoted by c. Without loss of generality, we normalize fixed costs to zero. Each firms' objective is to maximize profits, taking the prices charged by other firms and the consumers' behavior as given.

On the demand side of the market we have a continuum of consumers with identical preferences. Each consumer $j \in [0,1]$ has inelastic demand normalized to one unit, and holds the same constant evaluation $v > 0$ for the good. Observing a price below v, consumers will thus either buy one unit of the good or search for a lower price. In the latter case, they have to pay a search cost s to obtain one additional price quote, i.e. search is *sequential*. A fraction $\lambda \in [0,1]$ of consumers, the *shoppers*, have zero search cost. These consumers sample all prices and buy at the lowest price. The remaining fraction of $1-\lambda$ consumers – the *non-shoppers* – have positive search costs $s > 0$. These consumers face a non-trivial problem when searching for low prices, as they have to trade off the search cost with the (expected) benefit from search. Consumers can always come back to previously visited firms incurring no additional cost, i.e. we are considering a model of *costless recall*.[5] We assume that v is large relative to c and s so that v is not binding. In this section consumers are informed about the cost realization c.

2.2 Equilibrium

In this model, there exists a unique symmetric Nash equilibrium where consumer behavior satisfies a *reservation price property*. Moreover, Kohn and Shavell (1974) and Stahl (1989) argue that the reservation prices are stationary. That is, the consumers' reservation prices are independent from the history of price observations and the number of firms left to be sampled (provided there is still at least one firm left) and can therefore simply be denoted by $\rho^k(c)$.[6]

To characterize this equilibrium it is useful to introduce some more notation: we denote, for a given production cost c, the distribution of prices charged by firms by $F^k(p|c)$, its density by $f^k(p|c)$, and the lower- and upper- bound of its support by $\underline{p}^k(c)$ and by $\bar{p}^k(c)$, respectively.

It is well-known that the presence of both shoppers and non–shoppers, $\lambda \in (0,1)$, implies that there does not exist an equilibrium in pure strategies and that there are no mass points in the equilibrium price distribution. The main reason behind this observation is that firms face a tradeoff between setting low prices to cater to the shoppers and setting high prices to extract profits from the non–shoppers. Also, the upper bound of the equilibrium price distribution must satisfy $\bar{p}^k(c) = \rho^k(c)$, i.e. in a symmetric equilibrium no firm will set a price higher than the reservation price $\rho^k(c)$. Given these two observations, the equilibrium price distribution can be characterized by

Proposition 2.1 *For $\lambda \in (0,1)$, the equilibrium price distribution for the cost realization c is given by*

$$F^k(p|c) = 1 - \left(\frac{1-\lambda}{\lambda N}\frac{\bar{p}^k(c)-p}{p-c}\right)^{\frac{1}{N-1}} \tag{1}$$

respectively

$$f^k(p|c) = \frac{1}{N-1}\frac{\bar{p}^k(c)-c}{(p-c)^2}\left(\frac{1-\lambda}{\lambda N}\right)^{\frac{1}{N-1}}\left(\frac{\bar{p}^k(c)-p}{p-c}\right)^{\frac{2-N}{N-1}} \tag{2}$$

with support on $[\underline{p}^k(c), \bar{p}^k(c)]$ with $\underline{p}^k(c) = \frac{\lambda N}{\lambda N+1-\lambda}c + \frac{1-\lambda}{\lambda N+1-\lambda}\bar{p}^k(c)$ and $\bar{p}^k(c) = \rho^k(c)$.

[5] Janssen and Parakhonyak (2007) analyze the case where this assumption is replaced by costly recall.

[6] We use the superscript k to indicate variables and parameters of the model with completely informed consumers who *k*now the production cost realization c.

The proof follows essentially Stahl (1989). For the reader's convenience we nevertheless report it in the appendix.

Having characterized the Nash equilibrium price distribution conditional on the reservation price $\rho^k(c)$, we turn to optimal consumer behavior. Given a distribution of prices $F^k(p|c)$ and an observed price p', it is straightforward to argue that the reservation price $\rho^k(c)$ is implicitly determined by

$$v - \rho^k(c) = v - s - \int_{\underline{p}^k(c)}^{\rho^k(c)} p f^k(p|c) dp.$$

Using the result that the equilibrium price distribution satisfies $\bar{p}^k(c) = \rho^k(c)$, this condition boils down to

$$\rho^k(c) = s + E^k(p|c). \tag{3}$$

(Janssen, Pichler, and Weidenholzer, 2009) show that the expected price conditional on the cost realization c, $E^k(p|c)$, can be computed as described in the following lemma:

Lemma 2.1 *The expected price conditional on the cost realization c, $E^k(p|c)$, is given by*

$$E^k(p|c) = c + \frac{\alpha}{1-\alpha} s, \tag{4}$$

where $\alpha = \int_0^1 \frac{1}{1+\frac{\lambda N}{1-\lambda} z^{N-1}} dz \in [0,1)$.

Note that (3) and (4) imply the following simple expression for the reservation price,

$$\rho^k(c) = \bar{p}^k(c) = c + \frac{s}{1-\alpha}. \tag{5}$$

The reservation price is thus a constant markup over the cost, with the size of the markup being determined by the model's parameters. Note further that, by Proposition 2.1, $\underline{p}^k(c)$ is a weighted average of c and $\rho^k(c)$. Consequently, it immediately follows that, provided $s > 0$, the lower bound satisfies $\underline{p}^k(c) > c$. Thus firms make positive profits when charging prices according to $F^k(p|c)$. Furthermore, the following result obtains:

Corollary 2.1 *The equilibrium price spread, i.e. the difference between the upper bound and the lower bound of the price distribution, is independent of the realized cost level c and given by*

$$\bar{p}^k(c) - \underline{p}^k(c) = \frac{\lambda N}{\lambda N + 1 - \lambda} \frac{s}{1-\alpha}. \tag{6}$$

The proof follows from (5) and proposition 2.1. What is interesting about Proposition 2.1 is that a change in c leads to a one to one shift in the price distribution, leaving the extent of price dispersion unaffected. This testable implication we will confront with actual data later on.

Note at this stage that, conditional on the cost c, the average price paid by a fraction $1-\lambda$ of consumers, i.e. the non-shoppers, is equal to $E^k(p|c)$ as given in (4). This is, however, not the (average) price paid by the λ shoppers who observe all prices in the market and buy at the cheapest firm. This latter price is given by $E^k(p_\ell|c)$, with $p_\ell = \min\{p_1, p_2, ..., p_N\}$. As firms choose prices randomly and independent from each other, it follows that the distribution of p_ℓ is given by

$$F_l^k(p_\ell|c) = 1 - [1 - F^k(p|c)]^N. \tag{7}$$

The techniques used to prove Lemma 2.1 provided in Janssen, Pichler, and Weidenholzer (2009) can then be applied to establish that:

Lemma 2.2 *The expected minimum-price conditional on the cost realization c, $E^k(p_\ell|c)$, is given by*

$$E^k(p_\ell|c) = c + \frac{\tilde{\alpha}}{1-\alpha}s, \tag{8}$$

where $\tilde{\alpha} = \int_0^1 \frac{1}{1+\frac{\lambda N}{1-\lambda}z^{\frac{N-1}{N}}}dz < \alpha$, $\tilde{\alpha} \in [0,1)$.

Finally, we compute the unconditionally expected prices $E^k(p) = \int E^k(p|c)dc$ and $E^k(p_\ell) = \int E^k(p_\ell|c)dc$. These expected prices will later on be important to assess consumer welfare in the economy, as $v - E^k(p_\ell)$ is the expected equilibrium consumer surplus attained by shoppers whereas $v - E^k(p)$ is expected equilibrium surplus of the $1-\lambda$ non-shoppers.[7] Formally, we obtain:

Corollary 2.2 *The unconditionally expected prices $E^k(p)$ and $E^k(p_\ell)$ are given by*

$$E^k(p) = E(c) + \frac{\alpha}{1-\alpha}s, \text{ and} \tag{9}$$

$$E^k(p_\ell) = E(c) + \frac{\tilde{\alpha}}{1-\alpha}s, \tag{10}$$

where $E(c) = \int_{\underline{c}}^{\bar{c}} cg(c)dc$.

The proof follows trivially from Lemmas 2.1 and 2.2. Note that both ex-ante expected prices take the form of a markup over the ex-ante expected cost, with the size of the respective markup being determined by the parameters λ, N, and s, respectively.

3 Sequential search with incompletely informed consumers

We now turn to the analysis of the incomplete information model and modify the model presented in Section 2 by postulating that consumers are uninformed about the firms' production

[7] We will compute expected consumer surplus in the very beginning of the game, i.e. before the cost level c is drawn from $g(c)$.

cost. Let nature randomly draw c from a continuous distribution $g(c)$ with compact support on $[\underline{c}, \bar{c}]$. Consumers do not know the cost realization and they all hold the same prior beliefs $\hat{g}$ about the production cost distribution and update their beliefs according to Bayes' rule as they observe prices. We first analyze the case where these beliefs are correct in the sense that $\hat{g}$ equals the actual cost distribution g.

3.1 On out-of-equilibrium beliefs

In the model with incompletely informed consumers, the exact specification of out-of-equilibrium beliefs plays an important role in determining reservation prices. To see this point, assume that consumers hold out-of-equilibrium beliefs that are such that, if a price above their reservation price is observed, they think that the lowest cost level has been realized with probability one and therefore continue to search. In such a case, in equilibrium no firm would set a price above the reservation price (more details will be given shortly) and therefore such a price observation is clearly an out-of-equilibrium event. Note that under these particular beliefs, one can support a (first round) "reservation price" with the property that a consumer who observes it will strictly prefer to buy instead of actually being indifferent between buying and searching for a lower price. In a complete information setting, this could never be a reservation price as consumers would then also be willing to buy at a slightly higher price. However, in the incomplete information case under these particular out-of-equilibrium beliefs, a consumer would prefer to search for lower prices thinking that the lowest cost level has been realized and thus that prices should be low. Consequently, there would be a discontinuity in the willingness of consumers to buy around this "reservation price".

However, we think that such a discontinuity is difficult to defend in a consumer search model and we certainly do not want the comparison between the complete and incomplete information settings to depend on the arbitrary choice of out-of-equilibrium beliefs. We therefore insist that, if at a reservation price consumers strictly prefer to buy, out-of-equilibrium beliefs should be such that consumers also should buy at a slightly higher price. This effectively defines the first round reservation price as the price at which the consumer is indifferent between buying and continuing to search, in a way similar to the familiar complete information search model. In the following, we limit attention to equilibria satisfying such a reservation prices property.

3.2 Equilibria with reservation prices property

We start by providing a formal definition of what we mean by *equilibria satisfying a reservation prices property.* This requires first to introduce some more notation. In particular, we denote by

$\rho_t(p_1, \ldots, p_{t-1})$ the reservation price of a consumer in search round t who has observed prices $p_1, \ldots, p_{t-1}$ in the $t-1$ previous search rounds. Note that unlike the complete information model, any reservation price $\rho_t(p_1, \ldots, p_{t-1})$ held by consumers has to be independent of the production cost and that the reservation price ρ_1 in the first round is not conditional on any price observation, and we write $\rho_1 = \rho$. We have: [8]

Definition 3.1 *A perfect Bayesian equilibrium satisfying a reservation prices property (PBERP) is characterized by:*

1) *each firm $n \in \{1,...,N\}$ uses a price strategy that maximizes its (expected) profit, given the competing firms' price strategies and the search behavior of consumers;*

2) *given the (possibly degenerate) distribution of prices, consumers search optimally; moreover, optimal consumer search is of the following form:*

    i) *after observing $p_t = \rho_t(p_1, \ldots, p_{t-1})$ in round t and $p_1, \ldots, p_{t-1}$ in previous rounds, the consumer is indifferent between buying and continuing to search;*

    ii) *after observing any $p_t < \rho_t(p_1, \ldots, p_{t-1})$ in round t and $p_1, \ldots, p_{t-1}$ in previous rounds, the consumer buys.*

In what follows, we concentrate on the characterization of this type of equilibrium and determine conditions for existence.

3.3 Properties of PBERP

We first examine the properties of a PBERP, assuming that such an equilibrium exists. In the next subsection, we consider the existence question. A first observation is that in a PBERP, the upper bound of the price distribution has to be equal to the reservation price of consumers in the very first search round, i.e. $\bar{p}(c) = \bar{p} = \rho$ for all $c \in [\underline{c}, \bar{c}]$. Suppose this was not the case and that for some c, $\bar{p}(c) > \rho$. If a firm charges $\bar{p}(c)$, it will not sell to shoppers in any PBERP, as $\bar{p}(c)$ does not have positive probability and therefore shoppers observe lower prices with probability one. Furthermore, a firm setting $\bar{p}(c)$ will not sell to non-shoppers either, as these consumers will continue to search after observing $\overline{p}$ in the first search round, and will then find a lower price in a subsequent search round with probability one. On the other hand, it can also not be the case that for some c, $\bar{p}(c) < \rho$ since firms could profitably deviate to a price equal to ρ because non-shoppers would continue to buy.

[8] This definition is an adaptation of the reservation price equilibrium defined by Dana (1994) to the case of sequential search.

Under asymmetric information firms face virtually the same maximization problem as in the complete information benchmark. The only major difference is that upper bound of the price distribution is now constant at $\bar{p} = \rho$ for all realizations of the cost c. Formally:

Proposition 3.1 *In any PBERP the equilibrium price distribution for the cost realization c is given by*

$$F(p|c) = 1 - \left(\frac{1-\lambda}{\lambda N}\frac{\bar{p}-p}{p-c}\right)^{\frac{1}{N-1}} \tag{11}$$

respectively

$$f(p|c) = \frac{1}{N-1}\frac{\bar{p}-c}{(p-c)^2}\left(\frac{1-\lambda}{\lambda N}\right)^{\frac{1}{N-1}}\left(\frac{\bar{p}-p}{p-c}\right)^{\frac{2-N}{N-1}} \tag{12}$$

with support on $[\underline{p}(c), \bar{p}]$ *with* $\underline{p}(c) = \frac{\lambda N}{\lambda N+1-\lambda}c + \frac{1-\lambda}{\lambda N+1-\lambda}\bar{p}$ *and* $\bar{p} = \rho$.

The proof is omitted, as it is a straightforward extension of the proof of Proposition 2.1. Inspection of (11) reveals that $F(p|c)$ first order stochastically dominates (FOSD) $F(p|c')$ whenever $c > c'$. Furthermore, we have that $\underline{p}(c)$ is increasing in c, implying that consumers who observe prices below $\underline{p}(\bar{c})$ can rule out certain (high) cost realizations. Finally, we can characterize $E(p|c)$ as in Janssen, Pichler, and Weidenholzer (2009)

$$E(p|c) = (1-\alpha)c + \alpha\bar{p}. \tag{13}$$

Using a numerical example, Figure 1 visualizes all these observations by plotting the price distributions $F(p|c)$ for different realizations of the production cost c. This figure points to the fact that the price spread is decreasing in c as stated in the following corollary:

Corollary 3.1 *If consumers are uninformed about the firms' cost realization, the price spread in a PBERP is equal to*

$$\bar{p} - \underline{p}(c) = \frac{\lambda N}{\lambda N + 1 - \lambda}(\bar{p} - c)$$

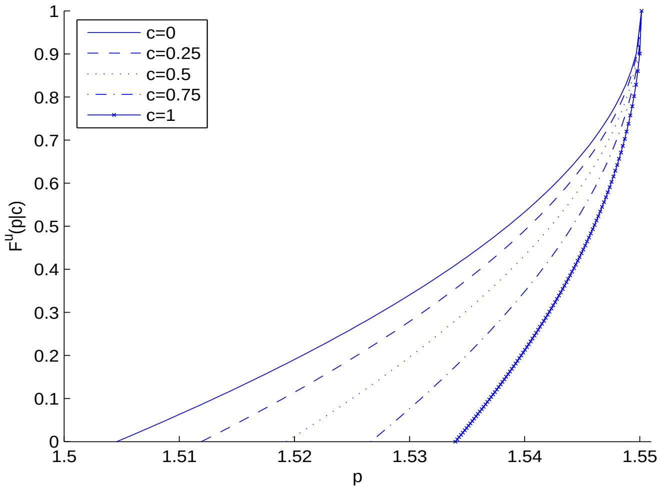
and therefore is decreasing in the cost level c.

Corollary 3.1 establishes a sharp contrast to the complete information model in which the price spread is independent of the realized cost level.

Let us now turn the focus on consumers' search behavior. Assuming consumers have correct prior beliefs about the production cost, $\hat{g}(c) = g(c)$, and use Bayesian updating after observing a price p, let $\delta(c|p)$ be the (posterior) probability density function of the production cost c conditional on a price observation p . By Bayes rule, we have that:

$$\delta(c|p) = \frac{g(c)f(p|c)}{\int_{\underline{c}}^{\bar{c}} g(c\prime)f(p|c')dc'}. \tag{14}$$

Figure 1: Price distributions



Parameters: $N = 3$, $s = 0.01$, $\lambda = 0.01$, $c \sim U(0,1)$

It remains to specify consumers' out-of-equilibrium beliefs, i.e. beliefs on the cost level for price observations not in the support of the equilibrium price distribution. As argued above, we want to avoid out-of-equilibrium beliefs that create a discontinuity in the willingness of consumers to buy around the reservation price. To this end, we assume that for a price observation above the upper bound of the price distribution consumers hold the same beliefs on the cost level as if they had observed the upper bound, i.e. $\delta(c|p) = \delta(c|\bar{p})$ for $p > \bar{p}$.[9]

In the following, we derive several lemmas that will prove useful to examine the properties of PBERP. First, we identify an important feature of Bayesian updating in our framework that plays a key role in our main results: a consumer who has observed a price $p \in [\underline{p}(\bar{c}), \bar{p}]$ will put more probability mass on higher realization of the production cost and less mass on lower realizations of the production cost than under the prior distribution $g(c)$.

Lemma 3.1 *For any $p \in [\underline{p}(\bar{c}), \bar{p}]$, the posterior distribution of cost levels $\delta(c|p)$ first order stochastically dominates the prior distribution $g(c)$.*

Lemma 3.1 implies that non-shoppers who have observed any $p \in [\underline{p}(\bar{c}), \bar{p}]$ expect a higher cost level $E(c|p)$ than if they hadn't observed any price, i.e.

$$E(c|p) = \int_{\underline{c}}^{\bar{c}} \delta(c|p) c dc > \int_{\underline{c}}^{\bar{c}} g(c) c dc = E(c).$$

[9] As at prices below the lower bound of the price distribution in the lowest cost scenario consumers buy regardless of their beliefs on the realized cost level, beliefs in these states are irrelevant.

Moreover, we find that the higher the price observed in the interval $p \in [\underline{p}(\overline{c}), \bar{p}]$, the more optimistic the consumer is about the possibility of finding low prices if she continues searching. Formally,

Lemma 3.2 *For all $p \in [\underline{p}(\overline{c}), \bar{p}]$, there is a unique cost level $\hat{c}$ such that*

$$\frac{\partial \delta(c|p)}{\partial p} \begin{cases} >0 & \text{if} \quad c < \hat{c} \\ =0 & \text{if} \quad c = \hat{c} \\ <0 & \text{if} \quad c > \hat{c}. \end{cases}$$

Consequently, for all $p, p' \in [\underline{p}(\overline{c}), \bar{p}]$ with $p' > p$, the posterior distribution of cost levels $\delta(c|p)$ FOSD $\delta(c|p')$.

Lemma 3.2 appears puzzling at first sight. However, the intuition behind it is readily seen: in the interval $[\underline{p}(\overline{c}), \bar{p}]$, the ratio of densities $f(p|c')/f(p|c)$ is increasing in p for any pair c, c' with $c' < c$. This implies that higher prices in $[\underline{p}(\overline{c}), \bar{p}]$ are *relatively* more likely under low costs than under high costs, which in turn explains why higher price observations in $p \in [\underline{p}(\overline{c}), \bar{p}]$ lead the consumer to become more optimistic about the cost realization.

We now move on to the characterization of reservation prices under *incomplete* information. Recall that in the *complete* information model, the reservation price is defined as the price at which the consumer is indifferent between buying now and continuing to search. In the present context, this would translate into defining the (first round) reservation price ρ by the indifference condition

$$v - \rho = v - s - E(p|\rho),$$

with $E(p|\rho) = \int_{\underline{c}}^{\bar{c}} E(p|c)\delta(c|\rho)dc$. The reservation price ρ would thus be implicitly given by

$$\rho = s + \int_{\underline{c}}^{\bar{c}} E(p|c)\delta(c|\rho)dc. \tag{15}$$

However, under *incomplete* information because of Bayesian updating reservation prices are not stationary and do dependent on the search history. The arguments used above may hence not be valid and it is not obvious that the first round reservation price should satisfy (15). In what follows, we however prove that (15) still provides a proper characterization of the reservation price. The intuition is the following: if a consumer in search round one observes the round one reservation price and decides to continue searching, she will find a price strictly below the round *two* reservation price with probability one in the next round and thus buys in round two. Lemma 3.3 establishes this result.

Lemma 3.3 *In any PBERP, after observing the upper bound of the price distribution in the first search round, a consumer's reservation price in the second search round satisfies $\rho_2(\rho) > \rho$.*

It follows that there does *not* exist a PBERP where consumers follow a stationary reservation price search rule. This is an important difference with the complete information model. Using equation (13) and $\rho = \bar{p}$ gives us

$$\begin{aligned} \bar{p} &= s + \int_{\underline{c}}^{\bar{c}} \Big((1-\alpha)c + \alpha\bar{p}\Big)\delta(c|\bar{p})dc \\ &= s + (1-\alpha)\int_{\underline{c}}^{\bar{c}} c\delta(c|\bar{p})dc + \alpha\bar{p}\int_{\underline{c}}^{\bar{c}} \delta(c|\bar{p})dc. \end{aligned}$$

Since $\int_{\underline{c}}^{\bar{c}} \delta(c|\bar{p})dc = 1$ and $\int_{\underline{c}}^{\bar{c}} c\delta(c|\bar{p})dc = E(c|\bar{p})$, we further have that the reservation price is implicitly defined by

$$\rho = \bar{p} = E(c|\bar{p}) + \frac{s}{1-\alpha}. \tag{16}$$

Substituting (16) into (13), we arrive at the following result:

Proposition 3.2 *In any PBERP the conditionally expected prices $E(p|c)$ and $E(p_\ell|c)$ in a PBERP are given by*

$$E(p|c) = c + \frac{\alpha}{1-\alpha}s + \alpha\left[E(c|\bar{p}) - c\right], \tag{17}$$

$$E(p_\ell|c) = c + \frac{\tilde{\alpha}}{1-\alpha}s + \alpha\left[E(c|\bar{p}) - c\right]. \tag{18}$$

From Proposition 3.2 the following result immediately follows:

Corollary 3.2 *In any PBERP the unconditionally expected prices $E(p)$ and $E(p_\ell)$ are given by*

$$E(p) = E(c) + \frac{\alpha}{1-\alpha}s + \alpha\left[E(c|\bar{p}) - E(c)\right], \tag{19}$$

$$E(p_\ell) = E(c) + \frac{\tilde{\alpha}}{1-\alpha}s + \alpha\left[E(c|\bar{p}) - E(c)\right]. \tag{20}$$

3.4 Existence of PBERP

Having established some properties any PBERP should satisfy, we now move to the existence question. Note that we have so far implicitly assumed that non-shoppers would like to buy at all prices below ρ. While this is straightforward to establish in the framework with complete information, it is not obvious under incomplete information since consumers update their beliefs about the true cost as they observe prices. In particular, after observing a price $p < \underline{p}(\bar{c})$ a consumer may suddenly think that the cost is very low and thus may decide to continue searching. Moreover, we need to verify that for all cost realizations firms find it optimal to set the prices implicitly specified above. In particular, we need that $\underline{p}(c) > c$ for all values of c. Again, under asymmetric information this condition is not automatically satisfied as the reservation price (and thereby the upper bound of the price distribution) is independent of the cost realization.

The next Proposition establishes a sufficient condition for the existence of PBERP with non-stationary reservation prices.

Proposition 3.3 *If*

$$\bar{c} - \underline{c} \leq \frac{\lambda N}{\lambda N + 1 - \lambda}\left(\frac{s}{1-\alpha}\right), \tag{21}$$

then a unique PBERP exists.

The proof is based on the following considerations. We first show that observing a price p with $\underline{p}(\bar{c}) < p < \rho$, an uninformed consumer prefers to buy instead of continuing to search and buy in a later round. Note that at these prices, consumers assign positive density to any cost realization c and by Lemma 3.1 become more pessimistic about the possibility of finding lower prices when continuing to search. We then examine lower price observations $p' < \underline{p}(\bar{c})$, where consumers can rule out certain high cost realizations. For a PBERP to exist, consumers must still find it optimal to buy at such prices. This, in turn, requires that consumers do not infer from observing a price $p' < \underline{p}(\bar{c})$ that the cost is low enough so that continued search pays off. To rule out this case, we exploit the idea that a consumer who finds it optimal to buy at a price p' if he *knows* the cost realization is $\underline{c}$, i.e. $p' \leq \rho^k(\underline{c})$, certainly has to find it optimal to buy in the unknown cost case at the same price. Consequently, by imposing $\underline{p}(\bar{c}) \leq \rho^k(\underline{c})$ we can guarantee that a consumer will find it optimal to buy at all price observations smaller than the reservation price ρ. This condition translates into inequality (21) characterizing the existence of a PBERP. Furthermore, inequality (21) also is sufficient to ensure that firms will set prices as specified above, i.e. $\underline{p}(c) > c$ holds for all $c \in [\underline{c}, \bar{c}]$.

It is interesting to see when the condition in Proposition 3.3 holds. Clearly, this is the case when the support of the cost distribution $\bar{c} - \underline{c}$ is small or s is large. More interestingly, it is also the case when N is large enough (for any given values of the other parameters). To see this, note that both $\frac{\lambda N}{\lambda N+1-\lambda}$ and α approach one as N approaches infinity; the RHS of inequality (21) therefore approaches infinity as well. Finally, note that when $\bar{c} \leq \underline{c} + Ns$, a PBERP exists also for small values of λ. To arrive at this observation, we evaluate

$$\frac{\lambda N}{\lambda N + 1 - \lambda} \frac{1}{1 - \int_0^1 \frac{1}{1+\frac{\lambda N}{1-\lambda} z^{N-1}} dz}$$

when λ is close to zero. Applying l'Hopital's Rule, we get that in a neighborhood of $\lambda = 0$

$$\frac{\frac{N}{(\lambda N+1-\lambda)^2}}{\int_0^1 \frac{N z^{N-1}/(1-\lambda)^2}{\left(1+\frac{\lambda N}{1-\lambda} z^{N-1}\right)^2} dz} = \frac{N}{\int_0^1 N z^{N-1} dz} = \frac{1}{\int_0^1 z^{N-1} dz} = N.$$

For λ close to 0, the right hand side of our inequality is thus approximately equal to $\underline{c} + Ns$.

We summarize our findings regarding the existence of PBERP in the following corollary:

Corollary 3.3 *A PBERP exists in environments with*

(i) a sufficiently small support of the cost distribution, $\bar{c} - \underline{c}$, and/or

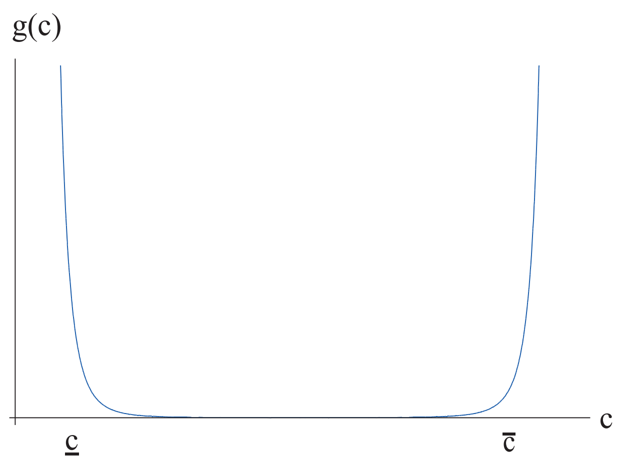
(ii) sufficiently large search costs s, and/or

(iii) sufficiently many firms N, and/or

(iv) a sufficiently small fraction of shoppers λ, provided that $\overline{c} \leq \underline{c} + Ns$ holds.

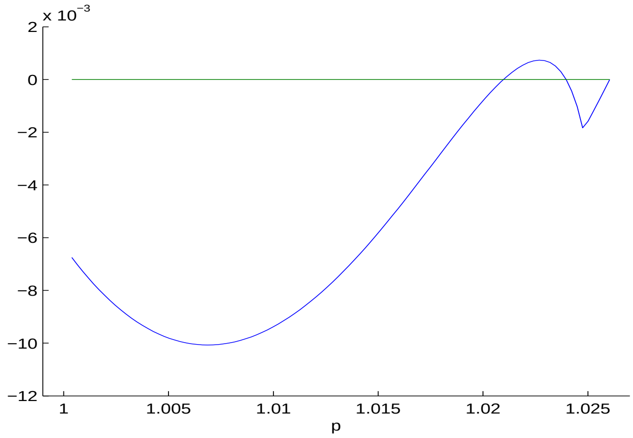
For general functions $g(c)$ the condition established in Proposition 3.3 is "almost necessary" in the following sense. If $\underline{p}(\bar{c}) > \rho^k(\underline{c})$, then one can construct a density function of the cost parameter, $g(c)$, that is concentrated on values close to the two extremes $\underline{c}$ and $\bar{c}$ (see Figure 2) such that, after observing a price smaller than $\underline{p}(\bar{c})$, consumers suddenly consider it extremely likely that the cost is close to $\underline{c}$. In particular, if a price observation p is in the interval $(\rho^k(\underline{c}), \underline{p}(\bar{c}))$ consumers will then prefer to search.

Figure 2: A cost distribution concentrated around the two extremes.



One may then wonder whether, if we restrict the prior cost distribution, existence of a PBERP may always be guaranteed. Considering a uniform distribution of production costs, the next example demonstrates that this is not the case. Figure 3 displays the net benefits of search in a duopoly market with search costs equal to $s = 0.00675$, a shopper-share equal to $\lambda = 0.025$, and production costs drawn from the uniform distribution $U(0,1)$. As can easily be seen, for this parameter constellation no PBERP exists: the consumer does not prefer to buy

Figure 3: Net benefits of search



Parameters: $N = 2$, $s = 0.00675$, $\lambda = 0.025$, $c \sim U(0,1)$

for all prices below the potential reservation price defined by equation (16).[10] While for prices between $\rho = 1.0260$ and $\underline{p}(\bar{c}) = 1.0248$ the consumer strictly prefers to buy, when observing prices slightly below $\underline{p}(\bar{c})$, the net benefits of search are increasing rapidly. Indeed, the net search benefits become positive for an interval of prices a bit below $\underline{p}(\bar{c})$. The reason is that when the consumer observes prices just below $\underline{p}(\bar{c})$, she infers that the expected production cost is relatively low and so is the expected price. When she would observe even lower prices, the search benefits increase rapidly and it becomes profitable not to buy at the observed price but to search for a lower price, even though the consumer would have bought had she observed a slightly higher price.

In case a PBERP does not exist, it is important to know what type of equilibrium does exist. Unfortunately, it turns out this is a very difficult issue to solve. One thing we can show is that allowing for more general reservation prices does not overcome the non-existence problem. Generally, a reservation price strategy is a strategy according to which consumers buy if, and only if, they observe a price at or below a certain cut-off price. The next results says that there are parameter values for which equilibria where consumers follow such strategies do not exist.

Proposition 3.4 *If s is relatively small or $\overline{c} - \underline{c}$ is relatively large and $g(c)$ has a relatively high probability mass close to $\underline{c}$, then an equilibrium where consumers follow a reservation price*

[10]Rothschild (1974) already observed that if consumers sample from an unknown distribution, it may happen that they prefer to buy at high prices, whereas they continue to search (and do not buy) at lower prices. Rothschild focuses, however, on the consumer search problem for a given (but unknown) price distribution. We show that these considerations actually are relevant and do arise in consumer search models where firms are strategically choosing prices.

strategy does not exist.

Together with the obvious fact that there cannot be a hole in the prices at which consumers decide to buy, Proposition 3.4 implies that for some parameter values consumers have to follow a mixed strategy in equilibrium. We leave it for further research to fully characterize these equilibria.

3.5 Distorted Priors

So far we have postulated that consumers hold optimal priors about the cost distribution $g(c)$, i.e. consumers effectively know the distribution from which the production cost is drawn. We have shown that these prior beliefs are an important determinant of prices in equilibrium, as they affect the reservation price of consumers (and thus equilibrium price distributions).

In this section, we relax the assumption that prior beliefs are necessarily correct and analyze the effects of distorted priors on the prices charged by firms. In particular, we examine an equilibrium where consumers' prior beliefs are characterized by

$$\hat{g}(c) = h(c)g(c),$$

where $h(c)$ is a function defined on $[\underline{c}, \bar{c}]$ which is either monotonically increasing in c or monotonically decreasing; the first case implies that consumers *overestimate* the cost level, while the latter case implies that consumers *underestimate* it.[11] With distorted priors we arrive at the following results.

Proposition 3.5 *When h is monotonically increasing, i.e. consumers have prior beliefs which are distorted towards higher cost levels, the average price and the average profit are higher as in the scenario where consumers hold optimal prior beliefs g. Conversely, when h is monotonically decreasing, the average price and the average profit are lower.*

This proposition shows a further important aspect of incomplete information within a search framework: consumers' reservation prices are affected by their priors. Consequently, the price setting behavior of firms is affected in the direction in which beliefs are distorted.

[11]Note that if $h(c)$ is increasing in c then $\hat{g}(c)$ FOSD $g(c)$ and if $h(c)$ is decreasing $g(c)$ FOSD $\hat{g}(c)$. In addition, note that first order stochastic dominance per se does not imply monotonicity of $h(c)$. In this sense our concept is stronger than FOSD.

4 The welfare implications of incomplete information

The examination of the welfare effects of incomplete information effectively boils down to a comparison of (i) the ex–ante expected price, (ii) the ex–ante expected lowest price, and (iii) the ex–ante expected firm profit in the two scenarios. This allows us, in turn, to assess the welfare implications for all three types of agents in the economy, i.e. non-shoppers, shoppers, and firms.

Assuming that a PBERP exists, and restricting attention to the case where consumer priors are again optimal, we find that:

Proposition 4.1 *In the PBERP of the sequential search model with incomplete information,*

- *the ex-ante expected price paid by non-shoppers, $E(p)$,*
- *the ex-ante expected price paid by shoppers, $E(p_\ell)$, and*
- *the ex-ante expected profit made by firms,*

are higher than in the complete information model. Consequently, consumer surplus is lower and producer surplus is higher.

Proposition 4.1 illustrates that consumer welfare is higher when the consumers are informed about the firms' production cost suggesting that policy interventions inducing observability of production cost benefit consumers.

Finally, we have:

Proposition 4.2 *The conditionally expected profits of firms are decreasing in the cost level c when consumers are uninformed about the cost realization, whereas these profits are independent of c when consumers are perfectly informed. In particular, conditionally expected profits under incomplete information are higher (lower) for low (high) cost realizations compared to when consumers are perfectly informed.*

This proposition highlights a further interesting and empirically testable difference between the complete and incomplete sequential consumer search frameworks.

5 The empirical relevance of asymmetric information: an example

Having examined the theoretical implications of incomplete information within a sequential consumer search framework, we next assess its empirical relevance. To this end, we confront the model with data on the retail gasoline market in Vienna, Austria. Specifically, we examine the properties of prices for Euro-super 95 gasoline (aka regular unleaded) together with a measure of its production cost based on the Amsterdam-Rotterdam-Antwerp spot market price. Note that it is not our goal to provide a full econometric analysis of the retail market for gasoline products.[12] Rather we want to provide some evidence that incomplete information and consumer search play an important role in shaping price distributions in the retail gasoline market.

Before proceeding to the empirical results, we give a detailed description of our data set and argue that the gasoline retail market in Vienna can be accurately described with a search framework.

5.1 Data description

Our first data set includes prices for Euro-super 95 gasoline at 231 stations in Vienna over the time period January 2007 until June 2009. In total, the sample contains 88.176 price observations. [13]

Our second data set includes a proxy for retailers' production costs. The measure is based on the *Daily Amsterdam-Rotterdam-Antwerp 10ppm Conventional Gasoline Regular Spot Price* series, which is available from the U.S. Energy Information Administration.[14] We convert the original data into Euro per litre using a gallon-to-litre ratio of 3.78541178 together with the USD-EUR exchange rates obtained from the European Central Bank.[15] Then we add Austrian indirect taxes and the value added tax following the guidelines provided in the *Oil Bulletin of the European Commission*.[16] To have cost data available on a daily basis, we use closing prices on Fridays to construct data for the weekends. The resulting series serves as our proxy for retailers' production cost.

Most of our empirical analysis will be based on the minimum, the average, and the maximum price observed in the market on a particular day. Note that our price data set contains many

[12]There are several extensive empirical analyses of gasoline markets available in the economics literature. See, among others, Hastings (2004), Lewis (2004, 2008), Hosken, McMillan, and Taylor (2008), Chandra and Tappata (2008), and Lach and Moraga-González (2009) for recent examples.

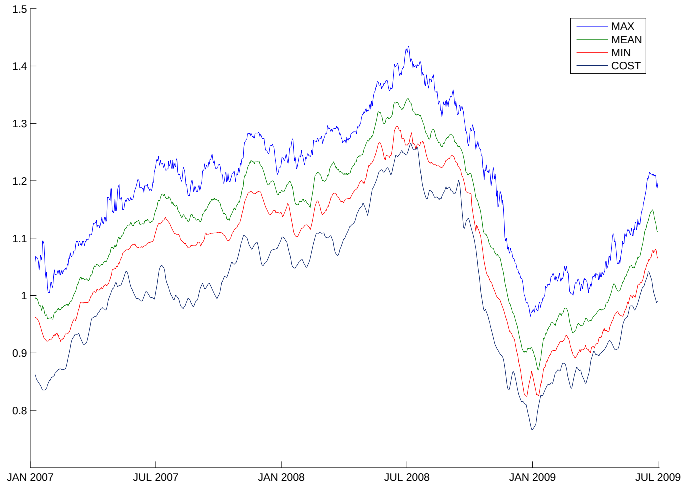
[13]The data were originally collected by the Austrian Automobile, Motorcycle- and Touring Club (ÖAMTC), who uses the data primarily to provide daily information about cheap gasoline stations on its website http://www.oeamtc.at. The website does not contain the time series data. Access to the full price data set is restricted, but the summary time series are available for download on the authors' websites.

[14]Available at http://tonto.eia.doe.gov/dnav/pet/hist/ru-10pp-ara5d.htm

[15]Available at http://www.ecb.int/stats/exchange/eurofxref/html/index.en.html

[16]Available at http://ec.europa.eu/energy/observatory/oil/bulletin_en.htm

Figure 4: Euro-super 95 prices and production costs



missing values, which creates some difficulties as it introduces high frequency variability in the minimum and maximum series. To remove this variability, we smooth the minimum, mean, and maximum series using a weekly moving average filter. For consistency, we employ the same filter on the production cost. Figure 4 illustrates the smoothed time series.

5.2 Is a consumer search model appropriate?

Before examining the role of incomplete information in shaping price distributions, we provide evidence that the gasoline retail market in Vienna can indeed be well characterized by a consumer search framework. To this end, we examine the properties of retail price dispersion over time.[17]

Visual inspection of Figure 4 reveals that there is indeed a significant extent of price dispersion: the maximum price and the minimum price differ by close to 12 Euro Cent on average. The size of dispersion becomes even more apparent when expressed in terms of the margin charged by retailers: while the minimum margin is on average 6 Cent, the maximum margin is on average three times as high at 18 Euro Cent per liter.

Price dispersion itself, however, is not a piece of hard evidence in favor of a costly consumer search framework, as it arises likewise in models of product differentiation. Although

[17]Note that throughout our analysis we will treat the whole city of Vienna as a single market for gasoline. This may be viewed as somewhat unrealistic, since not all stations compete directly with each other, but is assumed for the sake of simplicity.

Euro-super 95 gasoline is a fairly homogenous product, differences in retail prices may be due to differences in the retailers' attributes, such as brand type or location, among other things. Importantly, however, the dynamic properties of price dispersion are different for product differentiation models than for consumer search models. Under product differentiation, firms will set prices according to pure strategies. As a consequence, the relative price rankings of any pair of retailers should be roughly constant over time, provided that the characteristics of the products remain constant over time (which we think is plausible to assume in the present application). In search models, firms will however use mixed strategies, i.e. sample prices from an optimal distribution. Consequently, relative price rankings will not be constant over time: any retailer is expected to offer relatively low prices in some periods of time while setting relatively high prices in other periods. Using the terminology of Chandra and Tappata (2008), consumer search models display *temporal price dispersion*.

To test whether temporal price dispersion is present in our data, we follow the approach suggested by Chandra and Tappata and examine the price rankings of different gasoline stations over time. In particular, we analyze the *rank reversal* statistics for each pair (i, j) of stations in our sample. Labelling stations such that $p_{it} > p_{jt}$ is observed most of the time, where p_{it} and p_{jt} denote the prices of stations i and j at time t, respectively, the rank reversal statistic gives the proportion of observations for which $p_{jt} > p_{it}$:

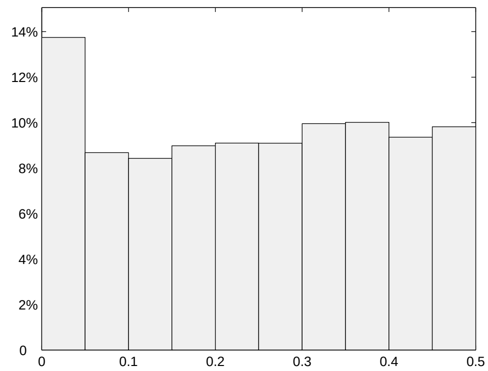
$$r_{ij} = \frac{1}{T_{ij}} \sum_{t=1}^{T_{ij}} I_{\{p_{jt} > p_{it}\}}.$$

I is an indicator function and T_{ij} gives the number of days in the sample on which price observations for both stations are available. Note that by construction $r_{ij} \in [0, 0.5]$. Rank reversals strictly greater than zero can be interpreted as evidence of temporal price dispersion, and thus that dispersion is due to consumer search rather than product differentiation.

Figure 5 presents a histogram of rank reversals for all pairs of stations in Vienna. Note further that 93% of station pairs feature a rank reversal statistic larger than zero, and that rank reversals are far from zero on average ($\bar{r} = 0.2445$).[18] From this evidence we conclude that a consumer search framework is indeed well suited to describe the retail gasoline market in Vienna.

[18]This general result is robust to modifications to the computation of rank reversals, such as including only pairs for which T_{ij} is large, or counting as a rank reversal only cases where prices differ noticeably.

Figure 5: Rank reversals



5.3 The role of incomplete information

Having argued that a search model is appropriate to analyze the retail gasoline market in Vienna, we proceed to discuss the role of incomplete information. In particular, we provide two pieces of empirical evidence that suggest the relevance of incomplete information in shaping price distributions.

Equilibrium price dispersion

We first focus again on the properties of equilibrium price dispersion. In particular, we examine whether the extent of dispersion as measured by the difference between the upper bound and the lower bound of the price distribution, i.e. the price spread, varies with the production cost level. Recall that in the complete information model, the price spread is independent of the cost level, while it is decreasing in c in the incomplete information framework (see Proposition 3.1). We exploit this difference to examine which of the two models is more in line with the gasoline price data.

To this end, we estimate by OLS the linear model

$$SPREAD_t = \theta_0 + \theta_c MC_t. \tag{22}$$

The variable $SPREAD_t$ serves as our proxy for the extent of price dispersion. It gives the difference between the observed maximum and the minimum price at date t, i.e. $SPREAD_t = MAXP_t - MINP_t$ with $MAXP_t$ and $MINP_t$ denoting the maximum and minimum price, respectively; by MC_t we denote the proxy for (marginal) production costs.[19]

[19] To keep the analysis as simple as possible, we do not control for product heterogeneity across retailers. The

The complete information framework suggests the parameter θ_c in (22) be zero, whereas it should be negative according to the incomplete information model. Table 1 summarizes the results of our regression and shows that θ_c is indeed significantly below zero, supporting the framework with incomplete information rather than complete information.

Table 1: Regression results

Coefficient	Point Est.	95% CI
θ_0	0.1563	[0.1440, 0.1687]
θ_c	-0.0377	[-0.0497, -0.0257]
R^2	0.0415	

Note that in both the incomplete and the complete information model, changes in the price spread can result from changes in the model parameters N, s, and λ. Variations in the spread over time might thus as well be due to variations in these parameters. Whereas it is easy to show that the number of firms N has been virtually constant over our sample period, this is more difficult to argue for the search cost, s, and the fraction of informed consumers, λ, since these are not directly observable in the data. However, we believe it is reasonable to assume that s and λ are not too volatile and, in particular, that both are not correlated with the evolution of the production cost level. Hence, it is safe to conclude from the results in Table 1 that the data suggest an important role for incomplete information in shaping price distributions.

We do not claim that the model from Section 3 is a proper device to study pricing behavior in the retail gasoline market. Importantly, our model is *static* while gasoline pricing is clearly a *dynamic* problem. Consequently, to have a full theoretical characterization of pricing behavior in gasoline markets one would need to work with a dynamic extension of the incomplete information model that would have to incorporate, among other things, features such as forward-looking agents and a role for reputation building. Constructing such a model is clearly not an easy exercise and goes beyond the scope of the present analysis.

Price distributions and the role of prior beliefs

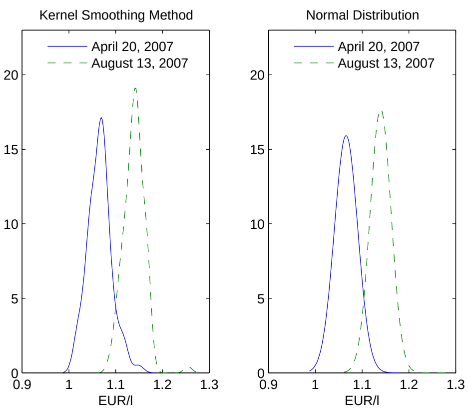
One issue where dynamics are important is in determining consumers' prior beliefs about the underlying cost for the gasoline retailers. In the static theory model of Section 3, we have that for a constant prior the upper bound of the equilibrium price distribution is a constant.

rank reversal statistics presented earlier make us confident, however, that this will not affect the results qualitatively.

Given the serial correlation present in the times series of the cost parameter, it is however more natural to assume that prior beliefs change over time. Here, we would like to discuss part of the observations we have on the gasoline retail market in Vienna from this perspective of adapting prior beliefs.

Let us start with a motivating observation. Inspection of Figure 4 reveals that in mid-April and mid-August 2007, the production cost of gasoline was at virtually the same level of slightly below 1 Euro per litre but, at the same time, price distributions differed noticeably. The price density estimates for April 20 and August 13 provided in Figure 6 further emphasize this observation: while the cost level on both dates was virtually identical at 97.7 Euro Cent per litre, prices in August have been by approximately 10 Cent lower than in April. Can a consumer search model be consistent with such a pricing behavior even if one takes the reasonable assumption that the parameters s, N and λ are roughly constant over time?

Figure 6: Price Density Estimates



If the underlying parameters were really constant, the complete information model would fail to explain the price distributions in Figure 6 as that model implies that price distributions depend on the cost level alone. In the *incomplete* information model, however, the prior beliefs of consumers about the production cost level are a further important determinant of prices. Consequently, the incomplete information model may be consistent with Figure 6 also under the assumption that s and λ are constant parameters. In particular, it seems plausible that priors are affected by the past evolution of production costs and that adjustment of prior beliefs happens in a sluggish way. This implies that consumers tend to underestimate the cost level in periods of increasing costs, whereas they overestimate it in periods of decreasing costs. Recall further that by Proposition 3.5 firms will find it optimal to set lower prices if consumers underestimate the cost level as compared to a scenario where consumers have optimal prior beliefs. Taken together, these observations suggest that, even for the same level of current marginal costs, prices should be lower in periods where costs have recently been increasing compared to periods where costs have been stable (or even decreasing).

While the arguments in the previous two paragraphs have been rather casual, the remainder of this section attempts to provide more formal evidence on the role of priors in shaping price distributions. We estimate by OLS several linear models of the minimum, average and maximum price, respectively, and use in addition to the current marginal cost a further explanatory variable reflecting whether costs have recently been increasing or decreasing. In particular, we construct a variable B_t^J that aims to capture the effect of prior beliefs. It formalizes the idea that prior beliefs in period t depend on the evolution of costs during the J days preceding date t. Formally, we add the variable

$$B_t^J = \frac{1}{J}\sum_{s=0}^{J-1} I_{\{MC_{t-s}/MC_{t-s-1}>1\}}$$

to each regression equation. B_t^J gives the fraction of days during the period $t-J, t-J+1, ..., t$ where costs have increased from one day to the other. For example, a value of $I_t^J = 0$ implies that costs have always been decreasing on a day-to-day basis for all J days before date t, whereas $I_t^J = 1$ implies that costs have always been increasing.[20] The estimated equations are given by

$$MINP_t = \beta_0^J + \beta_c^J MC_t + \beta_B^J B_t^J, \quad (23)$$

$$MEANP_t = \gamma_0^J + \gamma_c^J MC_t + \gamma_B^J B_t^J, \quad (24)$$

$$MAXP_t = \delta_0^J + \delta_c^J MC_t + \delta_B^J B_t^J. \quad (25)$$

The regression results for $J = 14$ are summarized in Table 2. Note that this particular value for J has been selected because it delivers the highest explanatory power according to the R^2 measure for the regression of the average price series, but we found our results to be qualitatively robust to different choices of J.

Our results show that the coefficient on the variable B_t^J is significant for all three equations. Its estimated value is negative, which confirms that prices are lower in periods where costs had been increasing in the recent past.[21] Interpreting B_t^J as a measure for the distortion in prior

[20] We have experimented with many other indicator variables, including past levels of the production cost, and found our results to be very robust to different specifications.

[21] For example, the estimated coefficient $\gamma_B^{14} = -0.0476$ can be interpreted as follows: for the same level of marginal cost at time t, the average price for Euro-super 95 in a (hypothetical) scenario where costs have always been *increasing* throughout the two weeks preceding date t would be 4.76 Euro Cent lower compared to a (hypothetical) scenario where costs had always been *decreasing* throughout the two weeks preceding date t.

Table 2: Regression results including B^{14}

Coefficient	Point Est.	95% CI
β_0^{14}	0.0932	[0.0800, 0.1063]
β_c^{14}	0.9911	[0.9784 , 1.0039]
β_B^{14}	-0.0448	[-0.0499,-0.0397]
R^2	0.9637	
γ_0^{14}	0.1458	[0.1333, 0.1583]
γ_c^{14}	0.9883	[0.9761 , 1.0005]
γ_B^{14}	-0.0476	[-0.0525,-0.0427]
R^2	0.9667	
δ_0^{14}	0.2569	[0.2437, 0.2702]
δ_c^{14}	0.9585	[0.9457, 0.9714]
δ_B^{14}	-0.0672	[-0.0723,-0.0620]
R^2	0.9609	

beliefs, this result provides further empirical support for the relevance of distorted priors within the incomplete information search framework.

6 Conclusion

In this paper we have analyzed a sequential consumer search model with incomplete (asymmetric) information about the common underlying production cost of firms. We have characterized a perfect Bayesian equilibrium of this model satisfying a reservation prices property. In this equilibrium, firms sample prices from an optimal distribution and, in each search round, a consumer buys if she observes a price below her current reservation price and searches for a better deal otherwise. Unlike in the standard consumer search model, the reservation prices under incomplete information are not stationary but differ across search rounds. This is due to consumers updating their beliefs about the production cost level when observing prices. We have further shown that an equilibrium with the properties just outlined exists for a relevant range of parameter values (such as low search costs or markets with relatively many firms), but there are cases where a reservation price equilibrium does not exist.

Comparing our environment to the complete information search model, we have shown that both the average price and the expected lowest price in the market are higher, and consumer welfare is thus lower, under incomplete information. We have furthermore demonstrated that the average profit margin charged by firms and the extent of equilibrium price dispersion are decreasing in the cost level, which is not the case in the complete information model. Moreover, we have highlighted that the consumers' prior beliefs about the production cost distribution play an important role in shaping equilibrium price distributions.

Confronting our model with data from the retail gasoline market in Vienna (Austria), we have finally investigated the empirical relevance of incomplete information. The evidence suggests that gasoline retailers do follow a mixed strategy when setting prices. The evidence also tells that (i) the extent of equilibrium price dispersion is decreasing in the level of production cost, and (ii) the production cost level alone cannot determine the price distribution. Both features point at the importance of incorporating incomplete information in consumer search models.

There are several directions for future research. From a theoretical perspective, the present paper does not answer the question which type of equilibria do exist in case a PBERP does not exist. From a more applied perspective, the current paper could be extended to build a reasonable dynamic model of the retail gasoline market, where consumers beliefs about retailers' cost are endogenously determined.

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Proofs

Proof of Proposition 2.1. If firms choose their prices according to a price distribution $F^k(p|c)$, firm i will, by setting a price p_i, expect to make a profit equal to

$$\pi_i(p_i, F^k(p|c), c) = \frac{1-\lambda}{N}(p_i - c) + \lambda\left(1 - F^k(p_i|c)\right)^{N-1}(p_i - c). \tag{26}$$

Since there are N firms, each firm can only expect to attract $\frac{1-\lambda}{N}$ non–shoppers, yielding the first part of equation (26). The second part of equation (26) expresses the idea that, if the other $N-1$ firms choose their strategies according to $F^k(p|c)$, the probability that firm $i's$ price is the lowest price offered in the market – thereby attracting all shoppers – is given by $\left(1 - F^k(p_i|c)\right)^{N-1}$.

Note that if a firm sets a price of $\bar{p}^k(c)$ it does not attract any shoppers and will make a profit of $\frac{1-\lambda}{N}(\bar{p}^k(c) - c)$. Further, note that in a mixed strategy equilibrium, each firm must be indifferent between charging any price in the support of $F^k(p|c)$. Hence, for all $p \in \left[\underline{p}^k(c), \bar{p}^k(c)\right]$, it has to be the case that

$$\frac{1-\lambda}{N}(p - c) + \lambda\left(1 - F^k(p|c)\right)^{N-1}(p - c) = \frac{1-\lambda}{N}(\bar{p}^k(c) - c). \tag{27}$$

Solving for $F^k(p|c)$ yields equation (11) and taking $\frac{\partial F^k(p|c)}{\partial p} = f^k(p|c)$ yields equation (12). Setting $F^k(p|c) = 0$ gives the lower bound $\underline{p}^k(c)$ of the price distribution. As argued in the main body of the paper, the upper bound $\bar{p}^k(c)$ is equal to the reservation price ρ^c. ■

Proof of Lemma 3.1 Recall that the probability density function of the production cost c conditional on a price observation p is given by

$$\delta(c|p) = \frac{g(c)f(p|c)}{\int_{\underline{c}}^{\bar{c}} g(c\prime)f(p|c')dc'} = \frac{g(c)}{\int_{\underline{c}}^{\bar{c}} g(c')\frac{f(p|c')}{f(p|c)}dc'} \equiv \frac{g(c)}{y(c;p)}.$$

Note that if $y(c;p)$ is monotonically decreasing in c, then $\delta(c|p)$ first order stochastically dominates $g(c)$. In the following, we show that $y(c;p)$ is indeed monotonically decreasing in c for price observations in the interval $[\underline{p}(\bar{c}), \bar{p}]$. To this end, note that for $p \in [\underline{p}(\bar{c}), \bar{p}]$ the function $y(c;p)$ is given by

$$\begin{aligned} y(c;p) &= \int_{\underline{c}}^{\bar{c}} g(c')\frac{\bar{p} - c'}{\bar{p} - c}\left(\frac{p - c}{p - c'}\right)^{\frac{N}{N-1}} dc' \\ &= \int_{\underline{c}}^{\bar{c}} g(c')\frac{\bar{p} - c'}{(p - c')^{\frac{N}{N-1}}}\frac{(p - c)^{\frac{N}{N-1}}}{\bar{p} - c}dc'. \end{aligned}$$

Its derivative with respect to c is hence given by

$$\frac{\partial y(c;p)}{\partial c} = \int_{\underline{c}}^{\bar{c}} g(c')\frac{\bar{p}-c'}{(p-c')^{\frac{N}{N-1}}}\left[\frac{-\frac{N}{N-1}(p-c)^{\frac{N}{N-1}-1}(\bar{p}-c)+(p-c)^{\frac{N}{N-1}}}{(\bar{p}-c)^2}\right]dc'$$
$$= \int_{\underline{c}}^{\bar{c}} g(c')\frac{\bar{p}-c'}{(p-c')^{\frac{N}{N-1}}}\frac{(p-c)^{\frac{N}{N-1}}}{(\bar{p}-c)^2}\left[1-\frac{N}{N-1}\frac{(\bar{p}-c)}{(p-c)}\right]dc' < 0$$

as $\left[1-\frac{N}{N-1}\frac{(\bar{p}-c)}{(p-c)}\right] < 0$. ■

Proof of Lemma 3.2. Taking the derivative of $\delta(c|p)$ with respect to p yields

$$\frac{\partial \delta(c|p)}{\partial p} = -\frac{[\delta(c|p)]^2}{g(c)}\int_{\underline{c}}^{\bar{c}} g(c')\frac{\partial \frac{f(p|c')}{f(p|c)}}{\partial p}dc'.$$

Restricting attention to prices in the interval $[\underline{p}(\bar{c}),\bar{p}]$, we have that

$$\frac{\partial \frac{f(p|c')}{f(p|c)}}{\partial p} = \frac{\bar{p}-c'}{\bar{p}-c}\frac{N}{N-1}\frac{c-c'}{(p-c')^2}\left(\frac{p-c}{p-c'}\right)^{\frac{1}{N-1}},$$

such that we obtain

$$\frac{\partial \delta(c|p)}{\partial p} = -\frac{[\delta(c|p)]^2}{g(c)}\int_{\underline{c}}^{\bar{c}} g(c')\frac{\bar{p}-c'}{\bar{p}-c}\frac{N}{N-1}\frac{c-c'}{(p-c')^2}\left(\frac{p-c}{p-c'}\right)^{\frac{1}{N-1}}dc'$$
$$= -\frac{[\delta(c|p)]^2}{g(c)}\frac{N}{N-1}\frac{(p-c)^{\frac{1}{N-1}}}{\bar{p}-c}\int_{\underline{c}}^{\bar{c}} g(c')\frac{(c-c')(\bar{p}-c')}{(p-c')^{\frac{2N-1}{N-1}}}dc'.$$

Since $\delta(c|p)$ is a density function for every p we have that $\int_{\underline{c}}^{\bar{c}}\delta(c|p)dc = 1$ for all p and consequently,

$$\frac{\partial[\int_{\underline{c}}^{\bar{c}}\delta(c|p)dc]}{\partial p} = \int_{\underline{c}}^{\bar{c}}\frac{\partial \delta(c|p)}{\partial p}dc = 0. \tag{28}$$

This in turn implies that $\frac{\partial \delta(c|p)}{\partial p}$ can neither be positive nor negative for all values of c.

In particular, since $\delta(c|p)$ is continuously differentiable, it follows that for all prices $p \in [\underline{p}(\bar{c}),\bar{p}]$ there exists (at least) one cost level $\hat{c}$ such that

$$\frac{\partial \delta(\hat{c}|p)}{\partial p} = 0.$$

Consequently, at this cost level

$$\int_{\underline{c}}^{\bar{c}} g(c')\frac{(\hat{c}-c')(\bar{p}-c')}{(p-c')^{\frac{2N-1}{N-1}}}dc' = 0.$$

For notational simplicity, let us introduce the function

$$\phi(p,c) = \int_{\underline{c}}^{\bar{c}} g(c') \frac{(c-c')(\bar{p}-c')}{(p-c')^{\frac{2N-1}{N-1}}} dc',$$

such that the above statement boils down to

$$\phi(p,\hat{c}) = 0.$$

To prove the lemma, it basically remains to show that (i) there exists only one unique $\hat{c}$ that satisfies $\phi(p,\hat{c}) = 0$; and (ii) $\phi(p,c) < 0$ for $c < \hat{c}$ and $\phi(p,c) > 0$ for $c > \hat{c}$. The last part is due to the fact that $\partial\delta(c|p)/\partial p$ and $\phi(p,c)$ have opposing signs.

Assume that there exist more than one values of $\hat{c}$ that satisfy $\phi(p,\hat{c}) = 0$. In such a case, at least one of these cost levels would have to satisfy

$$\frac{\partial\phi(p,c)}{\partial c}|_{c=\hat{c}} \leq 0.$$

This, however, cannot be true since

$$\frac{\partial\phi(p,c)}{\partial c} = \int_{\underline{c}}^{\bar{c}} g(c') \frac{(\bar{p}-c')}{(p-c')^{\frac{2N-1}{N-1}}} dc' > 0.$$

Consequently, there can only be a unique cost level $\hat{c}$ that satisfies $\phi(p,\hat{c}) = 0$, and $\phi(p,c) < 0$ for $c < \hat{c}$ and $\phi(p,c) > 0$ for $c > \hat{c}$ obtain trivially. Thus,

$$\frac{\partial\delta(c|p)}{\partial p} \begin{cases} > 0 & \text{if} \quad c < \hat{c} \\ = 0 & \text{if} \quad c = \hat{c} \\ < 0 & \text{if} \quad c > \hat{c}. \end{cases}$$

For $p' > p$ the posterior $\delta(c|p')$ puts more weight on low values of c and less weight on high values of c as compared to $\delta(c|p)$. Put differently, $\delta(c|p)$ first order stochastically dominates $\delta(c|p')$. ■

Proof of Lemma 3.3. Define $\rho_2(\overline{p})$ as the (hypothetical) price at which a consumer in round two would be indifferent between buying at that price and continuing to search after having observed $\overline{p}$ in the first search round. There are two cases to consider: (i) $\rho_2(\overline{p}) < \rho = \overline{p}$, and (ii) $\rho_2(\overline{p}) \geq \rho_1 = \rho = \overline{p}$. In the remainder of this proof, we argue that case (i) leads to an inconsistency, while case (ii) leads to a consistent procedure with ρ being indeed defined by

$$v - \rho = v - E(p|\rho) - s.$$

CASE (i): Assume that $\rho_2(\overline{p}) < \rho$, and let us introduce for notational simplicity $\widehat{p} = \rho_2(\overline{p})$. Note that if a consumer observes $\widehat{p}$ in the first search round, she would prefer to buy rather than continue to search. Formally, we have that $v-\widehat{p} \geq \pi^s(\widehat{p};N-1)$, where we denote by $\pi^s(\widehat{p};N-1)$ the payoff of a consumer who has observed $\widehat{p}$ in the first search round and continues to search optimally given that there are potentially still $N-1$ other firms to sample.

Next consider the hypothetical situation that there are in total $N+1$ firms in the market, but all of these set their prices according to the equilibrium price distribution in the market with N firms. Assume that a consumer has already observed two prices, $\widehat{p}$ and $\overline{p}$. If this consumer continues to search optimally after this hypothetical situation, her payoff is given by $\pi^s(\widehat{p},\overline{p};N-1)$. Furthermore, denote by $\pi^s(\widehat{p},\overline{p};N-2)$ the payoff if the consumer continues to search optimally after having observed $\widehat{p}$ and $\overline{p}$ and there are potentially only $N-2$ other firms to sample, as is true in our original market with N firms. Note further that, since $\rho_2(\overline{p}) = \widehat{p} < \rho$, we have that $v-\widehat{p} = \pi^s(\widehat{p},\overline{p};N-2)$.

At this stage, note that the benefits of search as defined above must satisfy

$$\pi^s(\widehat{p};N-1) > \pi^s(\widehat{p},\overline{p};N-1) \geq \pi^s(\widehat{p},\overline{p};N-2).$$

The second inequality is obvious: a consumer can never get a higher payoff of searching if she has the same price observations in her pocket and she has fewer search alternatives left. Regarding the first inequality, note that $\delta(c|\widehat{p},\overline{p})$ is the posterior distribution which is obtained by updating the belief $\delta(c|\widehat{p})$ using the price observation $\bar{p}$. As $\overline{p} > \underline{p}(\overline{c})$ we can apply (a modified version of) Lemma 3.1, taking $\delta(c|\widehat{p})$ instead of $g(c)$ as prior belief distribution, to obtain that $\delta(c|\widehat{p},\overline{p})$ FOSD $\delta(c|\widehat{p})$. Hence, consumers become more pessimistic about the underlying cost level having observed the price $\bar{p}$. Further, recall that we have that $F(p|c)$ FOSD $F(p|c')$ whenever $c > c'$, such that consumers expect higher prices when they expect higher costs. Consequently, it is strictly less attractive to continue searching after having observed both $\widehat{p}$ and $\overline{p}$ compared to a situation where only $\widehat{p}$ had been observed. Thus, we arrive at an inconsistency, because

$$v-\widehat{p} \geq \pi^s(\widehat{p};N-1) > \pi^s(\widehat{p},\overline{p};N-2) = v-\widehat{p}.$$

CASE (ii): now assume that $\rho_2(\overline{p}) \geq \rho = \overline{p}$. A consumer who has observed the upper bound ρ and continues to search will now buy in the next period at a price below ρ with probability one. Thus, a consumer who has observed the upper bound $\bar{p}$ and continues to search optimally will get a payoff of $\pi^s(\bar{p};N-1) = v-E(p|\overline{p})-s$. It follows that $v-\rho = v-E(p|\rho)-s$. ■

Proof of Proposition 3.3. For a PBERP to exist, a consumer must necessarily find it optimal to buy if she observes a price lower than the reservation price ρ, and firms must not make negative profits when choosing prices from the PBERP price distribution. In the following, we provide restrictions on the model's parameters such that these two conditions are satisfied. To this end, we proceed in two steps. In Part 1, we first show that at prices p with $\underline{p}(\bar{c}) < p < \rho$ an uninformed consumer prefers to buy instead of continuing to search and buy necessarily in the next round. Later on, we allow for more general search behaviors. In Part 2, we provide conditions such that a consumer also finds it optimal to buy when she observes a price in $[\underline{p}(\underline{c}), \underline{p}(\bar{c})]$. Finally, we show that these conditions already guarantee that firms make positive profits in equilibrium.

PART 1: Consider for the time being the following hypothetical scenario. A consumer observes the price p' and must decide between buying at p' immediately and visiting one more firm, provided that she must *necessarily* buy after having obtained this additional price quote. In such a situation, the consumer is indifferent between buying and searching if she observes the reservation price, i.e. if $p' = \rho$, as this price equates her net benefits of search to zero. Now assume that the consumer has observed a lower price p' in $(\underline{p}(\bar{c}), \rho)$. By Lemma 3.2, she is now more pessimistic about the cost and thus the possibility of finding lower prices compared to if she had observed ρ. Consequently, she must find it optimal to buy rather than search one more firm.

So far, we have argued that for prices p such that $\underline{p}(\bar{c}) < p < \rho$, the uninformed consumer prefers to buy instead of continuing to search and buy necessarily in the next round. We now consider more general search behaviors. In particular, it may easily be the case that the consumer, after continuing to search, may not want to buy after observing the next price, but instead prefers to continue searching at least one more time. We will now show that this cannot be optimal either if consumers observe prices p with $\underline{p}(\bar{c}) < p \leq \rho$.

If a consumer has observed t prices with $p' = \min(p_1, .., p_t) \geq \underline{p}(\bar{c})$, then her payoff from searching is given by

$$v - s - F(p'|p_1, \ldots, p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p|p < p', c)\delta(c|p_1, \ldots, p_t)dpdc - (1 - F(p'|p_1, \ldots, p_t))p'$$

where $F(p'|p_1, \ldots, p_t) = \int_{\underline{c}}^{\bar{c}} F(p'|c)\delta(c|p_1, .., p_t)dc$ is the subjective probability of finding a price lower than p' of a consumer who has observed prices $p_1, \ldots, p_t$. By Bayes rule, we have that:

$$\delta(c|p_1, .., p_t) = \frac{\delta(c|p_1, .., p_{t-1})f(p|c)}{\int_{\underline{c}}^{\bar{c}} \delta(c'|p_1, .., p_{t-1})f(p|c')dc'}. \tag{29}$$

In this sense, $\delta(c|p_1, .., p_t)$ is the distribution obtained from updating $\delta(c|p_1, .., p_{t-1})$ after the price observation p_t. We can again apply (a modified version of) Lemma 3.1, taking $\delta(c|p_1, .., p_{t-1})$ instead of $g(c)$ as prior distribution, to obtain that if $p_t \geq \underline{p}(\bar{c})$, then $\delta(c|p_1, .., p_t)$ first order stochastically dominates $\delta(c|p_1, .., p_{t-1})$. By induction, $\delta(c|p_1, .., p_t)$ FOSD $\delta(c|p')$.

Thus, as $E(p|c)$ is increasing in c and $F(p'|c)$ is decreasing in c, we have that

$$\begin{aligned} & v - s - F(p'|p_1,..,p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p|p < p', c)\delta(c|p_1,..,p_t)dc - (1 - F(p'|p_1,..,p_t))p' \\ & < v - s - F(p'|p_1,..,p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p|p < p', c)\delta(c|p')dc - (1 - F(p'|p_1,..,p_t))p' \\ & < v - s - F(p') \int_{\underline{c}}^{\bar{c}} E(p|p < p', c)\delta(c|p')dc - (1 - F(p'))p', \end{aligned}$$

where the last inequality follows from the fact that $E(p|p < p', c) < p'$ and that $F(p'|p_1,..,p_t) < F(p')$. Thus, as

$$v - p' > v - s - \int_{\underline{c}}^{\bar{c}} E(p|p < p', c)\delta(c|p')dc - (1 - F(p'))p',$$

it follows that if $p' = \min(p_1,..,p_t) \geq \underline{p}(\bar{c})$

$$v - p' > v - s - F(p'|p_1,..,p_t) \cdot \int_{\underline{c}}^{\bar{c}} E(p|p < p', c)\delta(c|p_1,..,p_t)dc - (1 - F(p'|p_1,..,p_t))p'.$$

Consequently, the consumer does not want to continue searching and then buy immediately in the next round after having observed t prices with $\underline{p}(\bar{c}) \leq p' = \min(p_1,..,p_t) \leq \rho$ for any t.

Let us finally consider the following, alternative search strategy: the consumer decides to continue searching in round t and, after having observed one more price, does not buy at any of the prices observed up to that moment if the newly observed price is larger than $\underline{p}(\bar{c})$. It is easy to see that, if the consumer searches in this way and then buys at a later moment, her payoff evaluated from period t onwards is smaller than

$$v - s - F(\widetilde{p}|p_1,..,p_{t+1}) \int_{\underline{c}}^{\bar{c}} E(p|p < \widetilde{p}, c)\delta(c|p_1,..,p_{t+1})dc - (1 - F(\widetilde{p}|p_1,..,p_{t+1}))\widetilde{p}$$

for some $\widetilde{p} \geq \underline{p}(\bar{c})$. Using the argument given above, it follows that this is not optimal as well. By induction, it follows that it is also not optimal to wait more than one period. Taken together, the arguments we have used so far show that a consumer will indeed buy if she observes a price in the interval $(\underline{p}(\bar{c}), \rho)$.

PART 2: Let us now consider consumer behavior if a price below $\underline{p}(\bar{c})$ is observed. By assumption, we have $\underline{p}(\bar{c}) \leq \rho^k(\underline{c})$, so that all these prices are below the reservation price in a model where (i) the consumers are informed about the cost realization and (ii) know this realization is equal to $\underline{c}$. We will argue that consumers should always buy at such prices.

As we consider prices $p' \leq \rho^k(\underline{c})$, it easily follows that

$$\begin{aligned} v - p' &\geq v - \rho^k(\underline{c}) = v - s - E^k(p|\underline{c}) \\ &> v - s - F^k(p'|\underline{c})E^k(p|p < p', \underline{c}) - (1 - F^k(p'|\underline{c}))p' \\ &\geq v - s - F(p') \int_{\underline{c}}^{\bar{c}} E(p|p < p', c)\delta(c|p')dc - (1 - F(p'))p'. \end{aligned}$$

Thus, the consumer prefers to buy at prices $p' \leq \rho^k(\underline{c})$ instead of continuing to search and buy then immediately. Furthermore, arguments similar to the ones used in Part 1 of this proof can be applied to establish that it does neither pay off to continue searching and then not to buy immediately after observing some price. Thus, under our assumption $\underline{p}(\overline{c}) \leq \rho^k(\underline{c})$, a reservation price strategy is optimal for the consumer.

To complete the existence part of the proof, we need to rewrite the condition $\underline{p}(\overline{c}) \leq \rho^k(\underline{c})$ in terms of the model's exogenous parameters and examine the profits made by firms given the consumers' behavior. Note first that we have

$$\begin{aligned}\underline{p}(\overline{c}) &= \frac{\lambda N}{\lambda N+1-\lambda}\overline{c} + \frac{1-\lambda}{\lambda N+1-\lambda}\left(\frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c|\bar{p})dc\right) \\ &< \overline{c} + \frac{1-\lambda}{\lambda N+1-\lambda}\left(\frac{s}{1-\alpha}\right), \\ \rho^k(\underline{c}) &= \underline{c} + \frac{s}{1-\alpha}\end{aligned}$$

such that certainly $\underline{p}(\overline{c}) \leq \rho^k(\underline{c})$ if

$$\overline{c} \leq \underline{c} + \frac{\lambda N}{\lambda N+1-\lambda}\left(\frac{s}{1-\alpha}\right).$$

To check that firms' profits are positive, it is sufficient to check that for all c, $\underline{p}(c) > c$. As $\underline{p}(c) = \frac{\lambda N}{\lambda N+1-\lambda}c + \frac{1-\lambda}{\lambda N+1-\lambda}\bar{p}$, this is the case if $\bar{p} > \overline{c}$. As $\bar{p} = \frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c|\bar{p})dc \geq \frac{s}{1-\alpha} + \underline{c}$. This inequality is automatically satisfied if $\overline{c} \leq \underline{c} + \frac{\lambda N}{\lambda N+1-\lambda}\left(\frac{s}{1-\alpha}\right)$. Uniqueness of the equilibrium is proved by showing that the reservation price is uniquely defined by

$$\overline{p} = \frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c|\bar{p})dc.$$

To show that this equation has a unique solution, we show that the RHS is decreasing in $\overline{p}$, which together with the fact that the LHS is increasing in $\overline{p}$, suffices. For this purpose, we have to evaluate the sign of

$$\frac{\partial\delta(c|\overline{p})}{\partial\overline{p}} = -\frac{[\delta(c|\overline{p})]^2}{g(c)}\int_{\underline{c}}^{\bar{c}} g(c')\frac{\partial\frac{f(\overline{p}|c')}{f(\overline{p}|c)}}{\partial p}dc'.$$

We have that

$$\frac{f(\overline{p}|c')}{f(\overline{p}|c)} = \left(\frac{\overline{p}-c}{\overline{p}-c'}\right)^{\frac{1}{N-1}}$$

and therefore

$$\frac{\partial\frac{f(\overline{p}|c')}{f(\overline{p}|c)}}{\partial\overline{p}} = \frac{1}{N-1}\frac{c-c'}{(\overline{p}-c')^2}\left(\frac{\overline{p}-c}{\overline{p}-c'}\right)^{\frac{2-N}{N-1}}.$$

It follows that if $\frac{\partial \frac{f(\overline{p}|c')}{f(\overline{p}|c)}}{\partial \overline{p}} > 0$ for some $\widetilde{c}$ it is positive for all $c > \widetilde{c}$. Moreover, since $\delta(c|\overline{p})$ is a density function we have $\int_{\underline{c}}^{\bar{c}} \delta(c|\overline{p})dc = 1$ and consequently we have

$$\frac{\partial[\int_{\underline{c}}^{\bar{c}} \delta(c|\overline{p})dc]}{\partial \overline{p}} = \int_{\underline{c}}^{\bar{c}} \frac{\partial \delta(c|\overline{p})}{\partial \overline{p}} dc = 0. \tag{30}$$

This in turn implies that

$$\frac{\partial \delta(c|\overline{p})}{\partial \overline{p}} \begin{cases} > 0 & \text{if} \quad c < \hat{c} \\ = 0 & \text{if} \quad c = \hat{c} \\ < 0 & \text{if} \quad c > \hat{c}. \end{cases}$$

Thus, the posterior $\delta(c|\overline{p})$ puts relatively more weight on low values of c the larger the values $\overline{p}$. Thus, the RHS is decreasing in $\overline{p}$. ■

Proof of Proposition 3.4. In a reservation price equilibrium, there is some ρ' such that consumers buy in the first round of search if, and only if, $p \leq \rho'$. It is clear that $\rho' \geq \rho^k(\underline{c})$ as otherwise consumers will buy even if they observe a price (slightly) above ρ'. Also, $\rho' \leq \rho$. This latter claim follows from the following observations. First, for all $\overline{p} \leq \rho^k(\underline{c})$, $v - \overline{p} < v - s - E(p|\overline{p})$, i.e., if the upper bound of the price distribution is relatively low consumers would prefer to buy at the upper bound rather than continuing to search. Second, for all $\overline{p} \geq \rho^k(\underline{c})$, $v - \overline{p} > v - s - E(p|\overline{p})$, i.e., if the upper bound of the price distribution is relatively high consumers would prefer to continuing to search if they observe a price equal to the upper bound rather than buy. Third, ρ is uniquely defined by $v - \rho = v - s - E(p|\rho)$. Thus, for all $\rho' > \rho$, $v - \rho > v - s - E(p|\rho)$, i.e., consumer prefer to buy immediately instead of continuing to search. Thus, it follows that the upper bound of the price distribution $\bar{p} = \rho'$ and $\rho^k(\underline{c}) \leq \bar{p} \leq \rho$.

Let us then consider the profits of firms. In an equilibrium it has to be the case that these profits are positive for all c. This is the case if, and only if, for all c, $\underline{p}(c) > c$. As $\underline{p}(c) = \frac{\lambda N}{\lambda N + 1 - \lambda} c + \frac{1-\lambda}{\lambda N + 1 - \lambda} \bar{p}$, this is the case if, and only if, $\bar{p} > \bar{c}$. A reservation price equilibrium therefore does not exist if $\bar{p} = \frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c|\bar{p})dc < \bar{c}$. This is the case if (i) s is relatively small enough or (ii) $\bar{c}$ and $\underline{c}$ are relatively far apart and $g(c)$ has a relatively high probability mass close to $\underline{c}$. ■

Proof of Proposition 3.5. Let the reservation price of consumers with $\hat{g}(c)$ be denoted by $\hat{\rho}$. We denote by $E(p;\hat{g})$ the ex-ante expected price of a consumer with prior $\hat{g}$. As above, $E(p)$ refers to the ex-ante expected price of a consumer who holds the optimal prior $g(c)$. Note that the firms' pricing behavior is again characterized by Proposition 2.1 and Lemma 2.1, with the upper bound of the price distribution being $\hat{\rho}$. Let $\hat{\delta}(c|\hat{\rho})$ denote the posterior distribution of the cost conditional upon having observed $\hat{\rho}$,

$$\hat{\delta}(c|\hat{\rho}) = \frac{\hat{g}(c)}{\sqrt[N-1]{\hat{\rho}-c}\int_{\underline{c}}^{\bar{c}} \hat{g}(c') \sqrt[N-1]{\frac{1}{\hat{\rho}-c'}}dc'}.$$

We need to show that $E(p;\hat{g}) > E(p)$. Note that, by equations (16) and (19), this is equivalent to $\hat{\rho} > \rho$. In the following, we prove that this latter inequality indeed must always hold if h is an increasing function. In particular, we argue that the assumption $\hat{\rho} \leq \rho$ is internally inconsistent.

To arrive at this result, consider first the ratio between the posterior densities

$$\frac{\hat{\delta}(c|\hat{\rho})}{\delta(c|\rho)} = \frac{\frac{\hat{g}(c)}{\sqrt[N-1]{\hat{\rho}-c}\int_{\underline{c}}^{\bar{c}} \hat{g}(c') \sqrt[N-1]{\frac{1}{\hat{\rho}-c'}}dc'}}{\frac{g(c)}{\sqrt[N-1]{\rho-c}\int_{\underline{c}}^{\bar{c}} g(c') \sqrt[N-1]{\frac{1}{\rho-c'}}dc'}} = \frac{\hat{g}(c)}{g(c)} \sqrt[N-1]{\frac{\rho-c}{\hat{\rho}-c}} \frac{\int_{\underline{c}}^{\bar{c}} g(c') \sqrt[N-1]{\frac{1}{\rho-c'}}dc'}{\int_{\underline{c}}^{\bar{c}} \hat{g}(c') \sqrt[N-1]{\frac{1}{\hat{\rho}-c'}}dc'}. \tag{31}$$

Note that this ratio can be written as

$$\frac{\hat{\delta}(c|\hat{\rho})}{\delta(c|\rho)} = h(c) \sqrt[N-1]{\frac{\rho-c}{\hat{\rho}-c}}\omega \tag{32}$$

where $\omega = \frac{\int_{\underline{c}}^{\bar{c}} g(c') \sqrt[N-1]{\frac{1}{\rho-c'}}dc'}{\int_{\underline{c}}^{\bar{c}} \hat{g}(c') \sqrt[N-1]{\frac{1}{\hat{\rho}-c'}}dc'}$ is positive and independent of c. For $\hat{\rho} \leq \rho$, we would have that $\sqrt[N-1]{\frac{\rho-c}{\hat{\rho}-c}}$ is (weakly) increasing in c, and therefore $\frac{\hat{\delta}(c|\hat{\rho})}{\delta(c|\rho)}$ would be strictly increasing in c since h is by assumption strictly increasing in c. Consequently, if $\hat{\rho} \leq \rho$, then $\hat{\delta}(c|\hat{\rho})$ would first order stochastically dominate $\delta(c|\rho)$. This, in turn, would imply that $\int_{\underline{c}}^{\bar{c}} c\hat{\delta}(c|\hat{\rho})dc > \int_{\underline{c}}^{\bar{c}} c\delta(c|\rho)dc$ and by equation (16), that $\hat{\rho} > \rho$, which obviously is inconsistent with the initial assumption that $\hat{\rho} \leq \rho$. Consequently, when h is an increasing c, it must be true that $\hat{\rho} > \rho$ and therefore that $E(p;\hat{g}) > E(p)$. The same arguments can be used to show that $E(p;\hat{g}) < E(p)$ when h is strictly decreasing in c.

Note that in a mixed strategy equilibrium all prices in the support of the equilibrium price distribution have to yield the same profits as the profits made when charging the upper bound. It follows that expected profits are higher/lower if consumers overestimate/underestimate the cost distribution compared to the scenario when the prior distribution is optimal. ■

Proof of Proposition 4.1. By equations (9), (10), (19), and (20) it is obvious that both $E(p) > E^k(p)$ and $E(p_\ell) > E^k(p_\ell)$ hold if $E(c|\bar{p}) = \int \delta(c|\bar{p})dc > E(c) = \int g(c)dc$. By Lemma 3.1, $\delta(c|\bar{p})$ FOSD $g(c)$, such that $E(c|\bar{p}) > E(c)$ obtains trivially. As the expected price of each consumer type is higher under incomplete information, expected profits are higher and thus producer welfare is higher. Furthermore, as consumer welfare is inversely related to the expected price, we have that consumer welfare of both shoppers and non-shoppers is lower under incomplete information. ■

Proof of Proposition 4.2. Under incomplete information, each firm's conditionally expected profit $E(\Pi|c)$ amounts to

$$E(\Pi|c) = \frac{1-\lambda}{N}(\bar{p}-c) = \frac{1-\lambda}{N}\left(\frac{s}{1-\alpha} + \int_{\underline{c}}^{\bar{c}} c\delta(c|\bar{p})dc - c\right).$$

To see this, note that in a mixed strategy equilibrium each firm's expected profit must be equal to the profit resulting from charging the upper bound of the price distribution. Obviously, $E(\Pi|c)$ is a decreasing function of the cost realization c. Under complete information, we have that

$$E^k(\Pi|c) = \frac{1-\lambda}{N}\frac{s}{1-\alpha}$$

which clearly is independent of c. Finally, note that the difference in profits between is given by

$$E(\Pi|c) - E^k(\Pi|c) = \frac{1-\lambda}{N}\left(c - \int_{\underline{c}}^{\bar{c}} c'\delta(c'|\bar{p})dc'\right),$$

establishing that conditional expected profits under incomplete information are higher for low cost realizations, and lower for high cost realizations, compared to conditional expected profits under complete information. ■