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Andreas Haufler und Frank Stähler:

Tax competition in a simple model with heterogeneous firms: How larger markets reduce profit taxes

Tax competition in a simple model with heterogeneous firms: How larger markets reduce profit taxes*

Andreas Haufler[†]

University of Munich and CESifo

Frank Stähler[‡]

University of Würzburg and CESifo

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Abstract

An important puzzle in corporate taxation is that effective tax rates have fallen significantly while tax revenue has simultaneously risen in most countries. Moreover, the gross profitability of firms seems to be lower in high-tax countries, even though standard models of international investment would yield the opposite conclusion. We offer an explanation for these stylized facts by setting up a simple two-country model of tax competition with heterogenous firms. In this model a unique, asymmetric Nash equilibrium can be shown to exist, provided that countries are sufficiently different with respect to their exogenous market conditions. In equilibrium the larger country levies the higher tax rate and attracts the high-cost firms. A simultaneous expansion of both markets intensifies tax competition and causes both countries to reduce their tax rates, despite higher corporate tax bases.

Keywords: tax competition, heterogeneous firms, imperfect competition

JEL Classification: F21, F15, H25, H73

[†]Seminar for Economic Policy, Akademiestr. 1/II, D-80799 Munich, Germany. Phone: +49-89-2180-3858, fax: +49-89-2180-6296, e-mail: Andreas.Haufler@lrz.uni-muenchen.de

[‡]Department of Economics, University of Würzburg, Sanderring 2, D-97070 Würzburg, Germany. Phone: +49-931-31-82950, fax: +49-931-31-87025, e-mail: frank.staehler@uni-wuerzburg.de

1 Introduction

The development of corporate taxes under conditions of increasing capital and firm mobility has been a prominent field of study for some time.[1] Nevertheless, several important puzzles remain that have not been explained by the existing literature in a satisfactory way. We argue in this paper that an important shortcoming of the existing theory of corporate taxation and capital tax competition is that it typically assumes all firms to be identical. By incorporating firm heterogeneity into a simple model of corporate tax competition we arrive at conclusions that are consistent with some of the main developments of corporate taxation in the industrialized countries over the last decades.

A first empirical puzzle is that corporate income tax (CIT) rates have fallen significantly, yet at the same time corporate tax revenue, as a fraction of GDP, has increased in most countries. This is summarized in Table 1 for selected OECD countries. The table shows that statutory corporate tax rates have been strongly reduced in virtually all countries in the sample since the mid-1980s. This downward trend is still clearly visible when using the 'effective average tax rate', which accounts for the simultaneous broadening of tax bases that has occurred in many countries.[2] At the same time, corporate tax revenue has risen in all of the smaller OECD countries in the sample, whereas the picture for the larger countries is somewhat more mixed. In the (weighted) OECD average, however, there is a clear increase in corporate tax collections.

Several arguments have been put forth to explain the co-existence of falling tax rates and rising tax revenues. One is that a rising share of companies has chosen to incorporate, partly in order to benefit from lower corporate tax rates, as compared to personal income taxes. Empirical studies indicate, however, that this argument can explain only a fraction of the observed increase in corporate tax revenues.[3] Secondly, several studies have found empirical evidence that multinational firms have engaged in income shifting

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[1] Wilson (1999) and Fuest et al. (2005) provide thorough surveys of the theoretical literature.

[2] See Devereux et al. (2002) for an elaboration of this concept, and for a more detailed overview of the trends in capital income taxation since the 1980s. A recent summary of these trends is found in Auerbach et al. (2008).

[3] de Mooij and Nicodème (2007) estimate that increased incorporation has raised the corporate tax-to-GDP ratio by some 0.25 percentage points since the early 1990s, other things being equal. This increase is substantially lower than the average increase in CIT revenues shown in Table 1, even though the latter incorporate the negative effect of falling tax rates.

Table 1: Corporate taxation in selected OECD countries[*]

	statutory tax rate[a]		effective average tax rate[b]		CIT revenue (% of GDP)	
	1985	*2005*	*1985*	*2005*	*1985*	*2005*
large countries (> 20 million)						
Australia	50	30	37	26	2.6	6.0
Canada	45	36	28	28	2.7	3.5
France	50	34	34	25	1.9	2.4
Germany	63	38	45	32	2.2	1.7
Italy	46	37	31	26	3.1	2.8
Japan	56	40	45	32	5.7	4.2
Spain	35	35	27	26	1.4	3.9
United Kingdom	40	30	28	24	4.7	3.4
United States	50	39	32	29	1.9	3.0
∅ *large countries*[c]	50.3	37.4	34.6	28.6	2.8	3.2
small countries (< 20 million)						
Austria	61	25	37	22	1.4	2.2
Belgium	45	34	35	26	2.2	3.5
Finland	60	26	45	21	1.4	3.3
Greece	44	32	36	21	0.7	3.2
Ireland	10	13	5	11	1.1	3.4
Netherlands	43	32	34	25	3.0	3.8
Norway	51	28	36	24	7.3	11.8
Portugal	55	28	48	20	2.2[e]	2.8
Sweden	60	28	45	21	1.7	3.7
Switzerland	35	34	26	25	1.7	2.6
∅ *small countries*[c]	46.8	29.2	35.2	22.5	2.3	3.9
∅ *all countries*[c]	49.9	36.7	34.7	28.1	2.7	3.3
∅ *total OECD*[d]					2.6	3.9

[*] The table lists all countries for which effective average tax rates are available since 1985.

[a] including local taxes

[b] base case: real discount rate: 10%, inflation rate: 3.5%, depreciation rate: 12.25%, rate of economic rent: 10% (financial return: 20%)

[c] weighted average in sample, countries weighted by GDP in 2005

[d] weighted average of all OECD countries; see OECD (2008)

[e] 1990

Sources: Devereux *et al.* (2002); www.ifs.org.uk/publications.php?publication_id=3210

OECD (2008), Table 12 http://dx.doi.org./10.1787/443744327555

OECD (2009). http://statlinks.oecdcode.org/302009011P1T010.XLS

from high-tax to low-tax countries (e.g. Hines and Rice, 1994; Huizinga and Laeven, 2008). This can explain why small, low-tax countries have experienced a higher growth of corporate tax revenues as compared to larger countries (see Table 1), but it cannot explain an increase in *average* tax collections.[4] It is therefore at least suggestive to explain the divergent developments of corporate tax rates and corporate tax revenues in the past two decades by an increase in the overall profitability of the incorporated sector during this period. This proposition is supported by the evidence obtained in detailed country studies for the United Kingdom (Devereux et al., 2004) and Germany (Becker and Fuest, forthcoming).[5]

Explaining the growth of CIT revenue (at least partly) by rising overall profits in the corporate sector raises a further question, however. Standard optimal tax theory would predict that, other things being equal, tax rates should rise when the tax base is enlarged, because a marginal tax rate increase generates more additional revenue. Hence, the trends summarized in Table 1 can only be explained when tax competition has simultaneously intensified so that countries find it optimal to reduce tax rates, despite larger corporate tax bases.

A final puzzle in corporate taxation arises from the empirical evidence showing that the gross profitability of firms is higher, on average, in low-tax countries (e.g. Becker et al., 2009). Standard models of international investment would suggest the opposite conclusion, as capital owners should be compensated for higher tax payments by a higher gross return to capital in high-tax countries. One explanation for this apparent conflict is that multinational firms shift income from high-tax to low-tax countries in order to reduce their worldwide tax payments. Tax practitioners remind us, however, that income shifting is limited by existing rules to trade at arm's-length prices, which cannot be costlessly circumvented.[6] It is therefore not clear that profit shifting alone is able to

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[4]The profit shifting argument alone can even be expected to lead to *falling* corporate tax revenues in the OECD average, as profits are also shifted from (high-tax) OECD countries into tax havens.

[5]The aggregate increase in profitability may be closely related to sectoral shifts within the incorporated sector, in particular, the expansion of the (until recently) highly profitable financial sector. In both the United States and the United Kingdom the share of corporate tax revenues collected from this sector rose from around 10% in the early 1980s to more than 25% of total CIT revenue in 2003 (Auerbach et al., 2008, Figure 5).

[6]In the United States, one indicator for the limited possibilities of firms to minimize their tax payments is the rising importance of corporate losses that are not deductible from tax. As Auerbach (2006, p. 14) concludes, this development "casts some doubt on the importance of tax planning strategies as a vehicle for reducing corporate taxes".

resolve the discrepancy between theoretical and empirical results. An alternative explanation is that there is a real difference in the gross-of-tax profitability of investments in high-tax and in low-tax countries. Several recent studies from the international trade literature show that firms within an industry differ strongly by their productivity (see Tybout, 2003 and Bernard et al., 2007 for surveys). Hence, if high-productivity firms have a distinct incentive to locate in low-tax countries, then the observed relationship between taxes and profitability can be explained even in the absence of profit shifting. As a result, the empirical importance of profit shifting may have been overestimated, due to the failure to account for productivity differences between individual firms.

Our argument in the present paper is that the expansion of profitable markets in conjunction with heterogeneity between firms offers a possible explanation for these stylized facts in a coherent theoretical framework. To develop this argument we set up a simple model of tax competition where a fixed number of firms differs exogenously in their costs of producing a homogeneous good. In this model firms sort according to their cost structure and high-cost firms locate in the high-tax country, whereas low-cost firms settle in the low-tax country. This is because (labor) costs of production are deductible from the corporate tax base and the value of this deduction is higher for high-cost firms, and it matters more in the high-tax country. A core simplification in our benchmark model is that each firm produces only one unit of output, in the country of its choice. We also show, however, that the basic properties of our simple model carry over to an extended setting where firms' output choices are endogenous.

One important analytical problem arising from firm heterogeneity is that governments' best response functions are generally not continuous, as each country has an incentive to marginally underbid its neighbor's tax rate and attract the low-cost (and hence high-profit) firms. The simplicity of our benchmark model allows us to prove the existence of a unique Nash equilibrium in tax rates, provided that countries are sufficiently different with respect to exogenous market conditions. In this equilibrium the larger country levies a higher tax rate than its smaller neighbor, and it attracts the high-cost firms. Based on this asymmetric Nash equilibrium we then consider changes in market size that raise the overall profitability of firms. In particular, we show that a simultaneous expansion of both markets will cause the pivotal firm to react more sensitively to corporate taxation, rendering tax competition more aggressive. In the new equilibrium this leads to falling tax rates in both countries, even though corporate tax bases are simultaneously enlarged.

Our model is directly related to the recent theoretical literature on trade and firm heterogeneity, starting with Melitz's (2003) well-known model of a monopolistically competitive industry in which firms draw their productivity randomly. We incorporate some of the basic findings of this line of research into the literature on taxation and foreign direct investment. The latter has analyzed the interaction between taxes and firm location in models of industry agglomeration (see Ludema and Wooton, 2000; Kind et al., 2000; Baldwin and Krugman, 2004; Borck and Pflüger, 2006), or in models that explicitly allow for heterogeneous *countries*, in particular with respect to market size (e.g. Ottaviano and van Ypersele, 2005; Haufler and Wooton, forthcoming). However, with few recent exceptions, the heterogeneity of *firms* has been neglected so far in the international tax literature.

A first analysis of tax competition in the presence of heterogeneous firms is Burbidge et al. (2006). In this paper each firm's productivity also differs across regions, however. This feature eliminates the sorting of firms on the basis of tax rates only, leading to a smooth trade-off for tax policy that is very different from the one studied here. Closer to our setting is Davies and Eckel (forthcoming). Their framework differs from ours in that they use a model of monopolistic competition and allow for endogenous firm entry. Accordingly the focus of their analysis is on the normative question of whether tax competition distorts the equilibrium number of firms in the industry, whereas our analysis aims to explain existing trends in corporate tax policy. Another difference is that the model of Davies and Eckel is considerably more complex than ours. As a result, the authors are not able to establish sufficient conditions for the existence of a pure-strategy Nash equilibrium in tax rates. Krautheim and Schmidt-Eisenlohr (2009) also present a model of tax competition with firm heterogeneity, but they focus on the competition for book profits between a large country and a tax haven, rather than on the location decision of mobile firms. Finally, some recent studies address tax policy issues in settings that involve the sorting of heterogeneous firms in the presence of international tax differentials (Becker and Fuest, 2007; Baldwin and Okubo, forthcoming). These studies are not cast in a tax competition framework, however.

The paper is organized as follows. In section 2 we describe our basic tax competition model. Section 3 asks under which conditions a Nash equilibrium in pure strategies exists in this model. Section 4 analyzes the effects of a unilateral and a general increase in market size on tax rates and tax revenue in each country. Section 5 extends the basic model to allow for variable outputs of firms. Section 6 concludes.

2 The model

2.1 Firms

We consider a region of two countries $i \in \{1, 2\}$ in which two goods are produced. Our focus lies on the market for a homogeneous good x, which is served by a total of N firms. Entry to the x-industry is restricted because one unit of a specific factor ('capital') is needed to produce at all, and the supply of this factor is fixed at N.[7] Importantly, the N firms differ in their costs of production. Specifically we assume that costs are drawn randomly and independently from a uniform distribution with $c \in [\underline{c}, \overline{c}]$. These costs reflect the firm-specific employment of labor that is needed to produce one unit of output, irrespective of where the output is produced. Each firm, identified by its unit cost c, decides in which country to settle and produce output. Locations differ with respect to their market potential, in a way described in more detail below. Firms decide on their location, knowing both countries' markets and their own costs, and forming rational expectations about the location of their rivals. Due to restricted entry to sector x, all firms in this sector will make positive profits in equilibrium.[8]

The model is closed by the presence of a second, numeraire sector which produces output under perfectly competitive conditions using labor only. In each country one unit of labor is required per unit of the numeraire good; hence free trade in this good equalizes wages across countries at unity. As no profits are generated in the numeraire sector, it remains in the background throughout our analysis. Aggregate labor supplies in each country are exogenously given and labor is immobile across countries.

The aim of our analysis is to establish conditions under which a Nash equilibrium in taxes exists in a model of heterogeneous firms, and to determine the effects that market enlargement has on tax rates and tax revenues in each country. These effects are derived in a two-stage game where governments first determine their tax policy and firms then decide on their location. To keep our analysis as simple as possible, we focus

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[7]As we will see below it is immaterial for the present analysis how this factor endowment is distributed between residents of countries 1 and 2. The factor owners could also be located in a third country (the rest of the world).

[8]Alternatively, the model could also endogenize the number of firms such that each firm makes a start-up investment after which it learns its cost. Ex ante, firms would make expected zero profits, and not all firms would be profitable. Our results do not change if countries set taxes after potential entrants have made the investment, but before profitable firms decide on their location.

only on the location decision of firms in our benchmark model. Hence we assume that, irrespective of its costs, each firm produces exactly one unit of output in equilibrium. Even though output choices are fixed, it is still true in our benchmark model that the lowest-cost firms are the most valuable from the perspective of host countries, in the sense that attracting them yields the largest gain in tax base. Hence the model retains the essential qualitative characteristics of firm heterogeneity for tax policy decisions. In Section 5 we analyze an extended version of our benchmark model where firms' output choices are endogenized and show that the basic properties of our benchmark model carry over to this more general setting.

We parameterize the attractiveness of a location by A_i and assume, with no loss of generality, that country 1 is the more attractive region so that $A_1 \geq A_2$. The simplest possible interpretation of this model is that each firm chooses the country where to locate and then produces only for the local market. In this case A_i acts as an indicator of market size. We assume that the (inverse) demand function for good x is linear in each country and given by $p_i = A_i - n_i$, where p_i is the price of good x and n_i is the number of firms, and hence total production, in country i. The gross profits of a firm with costs c locating in country i are then $A_i - n_i - c$. Alternatively, the model is also able to accommodate an integrated market in which firms make an export platform investment in one of the two countries and sell to the other market from this location (cf. Ekholm et al., 2007). In that case, output can be freely sold in the common market, but (exogenous) business conditions differ in the two countries and are more favorable in country 1. Assuming that costs are composed of unit production costs and agglomeration costs that are linearly increasing in the number of local firms, this setup is formally identical to the case of local markets. In the following we mostly adopt the interpretation of differences in local market size and refer to country 1 and country 2 as the 'large' and the 'small' country, respectively. In several places of our analysis we emphasize, however, the equivalent interpretation of the model based on an integrated economy with platform investment.

Each country levies a proportional profit tax at rate t_i on the gross profits earned by the firms in the x-industry that locate within its jurisdiction. All (labor) costs of production are deductible from the profit tax base. Let $\pi_i(c) \equiv (A_i - n_i - c)(1 - t_i)$ denote the net-of-tax profit of a firm with cost c in country i. Firms locate in the country in which the expected net-of tax profit is larger. We denote by $\hat{c}$ the costs of the firm that is just indifferent between locating in country 1 and in country 2. If firms

7

do not all locate in the same country (that is, if $\underline{c} < \hat{c} < \bar{c}$) the following arbitrage condition must hold for this firm:

$$(A_1 - n_1 - \hat{c})(1 - t_1) = (A_2 - n_2 - \hat{c})(1 - t_2), \qquad n_1 + n_2 = N. \tag{1}$$

Through the arbitrage condition (1) the pivotal firm with costs $\hat{c}$ is just compensated for the higher tax in, say, country 1, either by a larger market in country 1 or by a smaller number of competitors.

To see how changes in production costs affect firms' location choice, we differentiate $\pi_i(c)$ with respect to c and obtain

$$\frac{\partial \pi_i}{\partial c} = -(1 - t_i) \quad \forall\, i \in \{1, 2\}. \tag{2}$$

Hence a given increase in costs will lead to a smaller reduction in net profits in the high-tax country. The reason is that (labor) costs are deductible from the corporate tax base and this deduction is more valuable, the higher is the tax rate. Together with the arbitrage condition of the pivotal firm [eq. (1)], this implies that all firms with costs $c > \hat{c}$ will locate in the high-tax country, whereas all firms with costs $c < \hat{c}$ will prefer to locate in the low-tax region.

With the high-tax country attracting the high-cost firms, and given our assumption that costs are uniformly distributed in the interval $[\underline{c}, \bar{c}]$, the critical cost level $\hat{c}$ determines the (expected) number of firms that locate in each country by

$$n_i = \frac{\bar{c} - \hat{c}}{\bar{c} - \underline{c}}\, N, \qquad n_j = \frac{\hat{c} - \underline{c}}{\bar{c} - \underline{c}}\, N, \qquad i \neq j,\; t_i > t_j. \tag{3}$$

From (3) and the arbitrage condition (1), we can derive the critical cost level $\hat{c}$ as a function of the exogenous parameters and the endogenous tax rates t_i. This depends on which country chooses the lower tax rate and thus attracts the low-cost firms. Denoting by a subscript I (II) the regime where $t_1 > t_2$ ($t_1 < t_2$) and assuming interior solutions we get

$$\hat{c}_I = \frac{(A_2 + \underline{c}\phi)(1 - t_2) - (A_1 - N - \underline{c}\phi)(1 - t_1)}{\Theta_I} \qquad \text{if } t_1 > t_2, \tag{4a}$$

$$\hat{c}_{II} = \frac{(A_1 + \underline{c}\phi)(1 - t_1) - (A_2 - N - \underline{c}\phi)(1 - t_2)}{\Theta_{II}} \qquad \text{if } t_1 < t_2, \tag{4b}$$

where

$$\Theta_I \equiv (\phi + 1)(1 - t_2) + (\phi - 1)(1 - t_1) > 0, \tag{5a}$$

$$\Theta_{II} \equiv (\phi + 1)(1 - t_1) + (\phi - 1)(1 - t_2) > 0, \tag{5b}$$

8

and

$$\phi \equiv \frac{N}{(\overline{c} - \underline{c})} \geq 1. \tag{6}$$

In expression (6) we thus assume that there is a minimum density of firms, relative to the cost spread between the firms with the highest and those with the lowest costs of production. This condition is sufficient to ensure that Θ_I and Θ_{II} are always positive.

Our further analysis is based on the following definition.

Definition: *Let $\hat{c}$ be the critical value of costs for the firm that is indifferent between locating in country 1 or country 2. In Regime I, country 1 chooses the higher tax rate ($t_1 > t_2$) and $\hat{c}$ is given by (4a). In Regime II, country 2 chooses the higher tax rate ($t_2 > t_1$) and $\hat{c}$ is given by (4b).*

Equations (3) and (4a)–(4b) determine the number of firms that locate in each country, as a function of both tax rates. Differentiating these equations with respect to t_1 and t_2 we can unambiguously sign

$$\frac{\partial \hat{c}_I}{\partial t_1} = \frac{(1 - t_2)\left[(A_1 - N - \underline{c}\phi)(\phi + 1) + (A_2 + \underline{c}\phi)(\phi - 1)\right]}{\Theta_I^2} > 0,$$

$$\frac{\partial \hat{c}_I}{\partial t_2} = -\frac{(1 - t_1)}{(1 - t_2)} \frac{\partial \hat{c}_I}{\partial t_1} < 0; \tag{7a}$$

$$\frac{\partial \hat{c}_{II}}{\partial t_1} = \frac{-(1 - t_2)\left[(A_1 + \underline{c}\phi)(\phi - 1) + (A_2 - N - \underline{c}\phi)(\phi + 1)\right]}{\Theta_{II}^2} < 0,$$

$$\frac{\partial \hat{c}_{II}}{\partial t_2} = -\frac{(1 - t_1)}{(1 - t_2)} \frac{\partial \hat{c}_{II}}{\partial t_1} > 0. \tag{7b}$$

In Regime I, where country 1 is the high-tax region, a rise in this country's tax rate will raise $\hat{c}_I$. Since country 2 hosts all firms with cost levels between $\underline{c}$ and $\hat{c}$ in this regime, this implies a rising number of firms in country 2 and accordingly a fall in n_1. An increase in t_2 will instead reduce $\hat{c}_I$ and thus increase the number of firms in the high-tax country 1. In Regime II the signs of both derivatives are reversed.

2.2 Governments

Our analysis is based on the assumption that countries set taxes non-cooperatively before firms decide on their location.[9] Moreover, we assume that the objective of each

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[9]For evidence that OECD countries compete over corporate taxes, see Devereux et al. (2008).

government is to maximize (expected) profit tax revenue T_i in the x-industry. This assumption implies that governments value tax revenue highly in comparison to consumer and producer surplus. One common explanation is that governments are of a Leviathan type and are therefore mostly interested in tax revenue. Alternatively, governments could be politically forced by the working population to maximize revenue from the corporate income tax, for example because capital is perceived to be gaining from globalization, whereas labor is losing. Whatever its underpinnings, revenue maximization is a frequent assumption in the tax competition literature. Revenue maximization is a particularly plausible assumption when the firms in the x-industry are owned by foreigners so that the host country can increase domestic tax revenue at the expense of the profit income of foreign shareholders.[10]

In Regime I the expected tax revenues of the two countries are given by

$$T_1^I = t_1 \int_{\hat{c}}^{\bar{c}} \left[A_1 - \phi(\bar{c} - \hat{c}_I) - c\right] \phi \, dc, \qquad T_2^I = t_2 \int_{\underline{c}}^{\hat{c}} \left[A_2 - \phi(\hat{c}_I - \underline{c}) - c\right] \phi \, dc, \qquad (8)$$

where the relevant expression for $\hat{c}$ is given in (4a) and ϕ is in (6). Differentiating country 1's objective function with respect to t_1, using the Leibniz integration rule and canceling ϕ yields the following regime-specific first-order condition:

$$\frac{\partial T_1^I}{\partial t_1} = \int_{\hat{c}}^{\bar{c}} \left[A_1 - \phi(\bar{c} - \hat{c}_I) - c\right] dc - t_1 \frac{\partial \hat{c}_I}{\partial t_1} \left[A_1 - 2\phi(\bar{c} - \hat{c}_I) - \hat{c}_I\right] = 0.$$

Integrating the first term further gives

$$\frac{\partial T_1^I}{\partial t_1} = (\bar{c} - \hat{c}_I) \left[A_1 - \phi(\bar{c} - \hat{c}_I) - \frac{(\bar{c} + \hat{c}_I)}{2}\right] - t_1 \frac{\partial \hat{c}_I}{\partial t_1} \left[A_1 - 2\phi(\bar{c} - \hat{c}_I) - \hat{c}_I\right] = 0. \quad (9a)$$

Similarly, the first-order condition for country 2's government reads

$$\frac{\partial T_2^I}{\partial t_2} = (\hat{c}_I - \underline{c}) \left[A_2 - \phi(\hat{c}_I - \underline{c}) - \frac{(\hat{c}_I + \underline{c})}{2}\right] + t_2 \frac{\partial \hat{c}_I}{\partial t_2} \left[A_2 - 2\phi(\hat{c}_I - \underline{c}) - \hat{c}_I\right] = 0. \quad (9b)$$

The interpretation of these first-order conditions is straightforward. The first term in (9a) and (9b) gives the increase in revenues induced by a higher tax rate at an unchanged tax base. This effect is unambiguously positive. The second terms give the change in the tax base resulting from a small tax increase. Note first from (7a) that in both (9a) and (9b) the second terms have the opposite sign as the squared brackets in

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[10]In fact, in the interpretation of our model as one with export platform investment and costless trade, revenue maximization is equal to welfare maximization when profits accrue to foreigners, because consumer surplus in each country is fixed by the exogenous number of firms N.

these terms. The squared brackets in turn combine two distinct effects. A tax increase induces some firms to relocate to the other country, but the profits of the remaining firms rise due to lower market output and accordingly higher prices. For an interior equilibrium to exist, the first of these effects must dominate the second so that the tax base falls in the country that marginally raises its tax rate. A sufficient condition ensuring this is:

$$A_i - 2N - \bar{c} > 0 \quad \forall\, i. \tag{10}$$

Throughout the following analysis we assume that this condition is met. In verbal terms it states that if all but one firms locate in the same country, attracting the last firm (with the highest cost level $\bar{c}$) will still raise aggregate profits in that country.

In Regime II the first-order conditions for the two countries' optimal tax rates are derived and interpreted analogously. These conditions are given in the appendix.

3   Existence of equilibrium

In this section we ask under which conditions a Nash equilibrium in taxes exists in the present model. A fundamental existence problem arises in the presence of heterogeneous firms because country i's payoff is not continuous at the other country's tax rate t_j. If country i overbids country j's tax rate, then it will attract the high-cost (and thus low-profit) firms. If instead country i underbids t_j, then it attracts the low-cost (high-profit) firms. Hence, each country's tax revenue experiences an upward jump when the tax rate is set marginally lower than in the competing jurisdiction. Accordingly, best response functions can also be discontinuous in our model. These properties require a thorough analysis of equilibrium existence and uniqueness.

As a starting point for our analysis, a natural benchmark is the case where market conditions in the two countries are identical (i.e., $A_1 = A_2$). In this case it is also natural to focus on a symmetric situation with $t_1 = t_2$ and ask whether this situation can represent a Nash equilibrium. With identical market conditions and taxes, all firms are indifferent as to their location and each firm will thus locate in each jurisdiction with probability $q = 0.5$. In a situation where both countries choose the same tax rate the expected per-firm tax revenue for each country is thus

$$T_i|_{t_1 = t_2} = tq \int_{\underline{c}}^{\overline{c}} (A_i - Nq - c) \frac{1}{\overline{c} - \underline{c}} \, dc = \frac{t}{2} \left[A_i - \frac{N}{2} - \frac{(\overline{c} + \underline{c})}{2} \right] \quad \forall\, i. \tag{11a}$$

In contrast, if country i slightly underbids country j, it will still get half of all firms, but now the low-cost firms will self-select into country i. Hence country i's per firm tax revenue becomes

$$T_i|_{t_i=t_j-\varepsilon} = t \int_{\underline{c}}^{(\overline{c}+\underline{c})/2} \left(A_i - \frac{N}{2} - c \right) \frac{1}{\overline{c} - \underline{c}} \, dc = \frac{t}{2} \left[A_i - \frac{N}{2} - \frac{(\overline{c} + \underline{c})}{2} + \frac{(\overline{c} - \underline{c})}{4} \right] \quad \forall \, i.$$
(11b)

Comparing (11a) and (11b) shows that profits and tax revenue are unambiguously larger for a country that marginally underbids its neighbor, because sales are the same, but aggregate costs are lower in the low-tax country. Hence for any positive, common tax rate $t_1 = t_2$ there is an incentive for each country to marginally underbid the other, in order to attract the more profitable firms. Thus $t_1 = t_2 > 0$, with strict inequality, cannot be a Nash equilibrium pair of taxes. A situation with $t_1 = t_2 = 0$ and hence $T_1 = T_2 = 0$ can also not be an equilibrium, because each country can gain by setting a positive tax rate and still attract some firms, obtaining strictly positive tax revenue. Hence there cannot be a symmetric, pure-strategy Nash equilibrium with $t_1 = t_2$.

This result implies that, unlike in tax competition models with homogeneous firms, a situation of perfect symmetry is not a suitable starting point when firms differ in their productivity levels.[11] In the following we will therefore focus on asymmetric situations and consider in turn the cases where the larger country 1 either has the lower or the higher tax rate than its smaller neighbor.

We first ask whether an equilibrium can exist in Regime II. This yields:

Proposition 1 *There cannot be an interior tax competition equilibrium in Regime II, where the larger country (country 1) has the lower tax rate.*

Proof: See the appendix.

The technical proof for the proposition is relegated to the appendix. The intuition for Proposition 1 is, however, straightforward. In its tax optimum, each country equates the marginal revenue gains and the marginal revenue losses resulting from a small tax increase [see the discussion of (9a)–(9b)]. If a Regime II equilibrium with $t_1 < t_2$ existed, the larger country 1 would clearly have the larger tax base as it would host more firms

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[11]It is seen from (11a) and (11b) that there is no revenue gain from marginally underbidding the neighboring country when $\overline{c} = \underline{c}$ and thus the production costs of all firms are identical. This is the reason why symmetric tax equilibria generally exist in models with homogenous firms (e.g. Ottaviano and van Ypersele, 2005; Haufler and Wooton, forthcoming).

and these firms would also be more profitable. Hence the marginal gains from a tax increase would be unambiguously larger for country 1 than for country 2. Moreover, if an interior tax equilibrium existed in Regime II, a marginal tax increase of country 1 would cause those firms to leave the country which have the highest costs among the firms that locate in country 1 and hence are least attractive from the perspective of this country. In contrast, a tax increase by country 2 would create an outflow of the firms which have the lowest costs and hence are most valuable from the perspective of country 2. Hence the marginal costs of a small tax increase would be lower for country 1 as compared to country 2. As country 1 would face higher benefits but lower costs from a tax increase, as compared to country 2, it is impossible in such a situation that the marginal gains and the marginal losses from a small tax increase can be equal in both countries simultaneously.[12]

From Proposition 1 we know that a Nash equilibrium in pure strategies, if it exists at all, can only arise in Regime I, with the larger country having the higher tax rate. It can be shown that a Regime I equilibrium will indeed exist when $A_1 \geq A_2 + N$ and hence the size or productivity advantage of country 1 is sufficiently large. From (4a) this implies that country 1 would attract all firms, if tax rates were equal in the two countries (i.e., $\hat{c}_I = \underline{c}$). To establish this result we need to focus on situations in which the own effects of tax rates on marginal tax revenues dominate the cross effects. This is formalized by assuming that the determinant of the Jacobian matrix

$$|J| = \frac{\partial^2 T_1^I}{\partial t_1^2} \frac{\partial^2 T_2^I}{\partial t_2^2} - \frac{\partial^2 T_1^I}{\partial t_1 \partial t_2} \frac{\partial^2 T_2^I}{\partial t_1 \partial t_2} > 0$$

is strictly positive. Given this assumption, we are able to prove both existence and uniqueness of the equilibrium, as summarized in the following proposition:

Proposition 2 *There exists a unique Nash equilibrium in pure strategies in Regime I with $t_1 > t_2 \geq 0$ if country 1 is sufficiently large relative to country 2, i.e., $A_1 \geq A_2 + N$, and if $|J| > 0$.*

Proof: See the appendix.

To provide an intuition for this proof, we start from a situation where country 2's tax rate is zero and country 1's tax rate is at the highest possible level, denoted $\bar{t}_1$, that

---

[12]Note that this argument does not rely on the specific setup of our benchmark model. Hence it carries over to the case where firms' output choices are endogenous (Section 5).

is consistent with attracting all firms to this country. If, at $\bar{t}_1$, country 1's marginal tax revenues are negative [see eq. (9a)], then $\bar{t}_1 > t_2 = 0$ must be a Nash equilibrium because a deviation from $\bar{t}_1$ in either direction leads to tax revenue losses for country 1, whereas country 2 cannot improve upon the outcome of zero tax revenue.[13] If, in contrast, country 1's marginal tax revenues are positive at $\bar{t}_1$, then it will want to raise its tax rate above $\bar{t}_1$. Once t_1 has been increased such that $\hat{c}_I > \underline{c}$, country 2 is also able to attract some firms. Hence it has an incentive to raise its own tax rate above zero while still underbidding country 1's tax rate, in order to maintain a positive tax base. In this case a mutually optimal set of tax rates with $t_1^* > t_2^*$ will exist, which leads to an interior equilibrium with a positive number of firms in each country.[14]

Proposition 2 is our first central result, establishing sufficient conditions for the existence of a unique Nash equilibrium in our simple tax competition model with heterogeneous firms. In this equilibrium the larger country is able to levy the higher tax rate.[15] Proposition 3 in Davies and Eckel (forthcoming) has a similar flavor, but the authors are not able to place explicit conditions on the exogenous variables of their model that ensure the existence of a tax competition equilibrium. Clearly, this is the result of the simpler structure of our model. Given our assumptions, the sufficient condition $A_1 \geq A_2 + N$ has a straightforward interpretation. It implies that the home market advantage must be sufficiently large so that, in the absence of taxes, *all* firms would want to settle in the larger country 1. In the location equilibrium with taxes, each firm then trades off the net location advantage of country 1 (taking into account the larger number of competitors in this country) against the tax advantage of country 2.

Note, finally, that Proposition 2 is not exhaustive in the sense that an equilibrium may also exist if $A_1 \in [A_2, A_2 + N]$, that is, country 1 is larger than country 2 but it will not host all firms when tax rates are equal. The difficulty that arises in this case is that country 2 can secure a positive tax base with a strictly positive tax rate. This makes it potentially interesting for country 1 to underbid country 2's tax rate, in order to attract all firms. Hence, if $A_1 - A_2 < N$, any candidate equilibrium must be immune against such an underbidding strategy by country 1. We know that underbidding is

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[13]Strictly speaking, country 2 is indifferent between all tax rates $t_2 \geq 0$, as its tax base is zero.

[14]If $A_1 = A_2 + N$ then only an interior equilibrium can arise, because country 1 could only attract all firms by setting a tax rate of zero, which would yield zero revenue.

[15]The result that the large country levies the higher tax rate is familiar from the literature on capital tax competition (Bucovetsky, 1991; Wilson, 1991). The novel element in a model with heterogeneous firms is that the large country, by imposing the higher tax, also attracts the low-profitability firms.

always profitable when countries are identical ($A_1 = A_2$). However, the more asymmetric countries become, the lower is the taxing power of country 2 and thus the lower is both the tax rate and the tax base of country 2 in an asymmetric candidate equilibrium. Hence country 1 can only secure a small additional tax base by underbidding country 2, and doing so requires a large drop in country 1's tax rate. This implies that the incentive for country 1 to underbid its smaller neighbor will monotonically fall as the size asymmetry grows. From this discussion we expect that there is a critical size difference $(A_1 - A_2)^c < N$ such that an asymmetric pure-strategy equilibrium with the properties of Proposition 2 exists, once this critical threshold is surpassed.[16]

4 Market expansion and tax competition

As we have discussed in the introduction, there are several empirical indications that the expansion of highly profitable services, in particular in the banking and finance sector, has contributed to rising corporate profits in many OECD countries during the last decades. In the following we capture this development by an exogenous increase in the size of either one or both markets in our model, as given by the parameters A_i. This exogenous market expansion raises the profitability of all firms in equilibrium.

Our analysis is based on a situation where country 1 is sufficiently large, relative to country 2, so that Proposition 2 guarantees the existence of a unique Nash equilibrium. Furthermore, in what follows we focus on an interior tax equilibrium where $t_1^* > t_2^* > 0$ and $\hat{c}_I > \underline{c}$. We then consider small changes in the exogenous model parameters. By a continuity argument, we assume that an equilibrium still exists after the small perturbation of the initial equilibrium has taken place.

Our comparative static analysis starts from the optimal tax conditions in Regime I, as given in (9a)–(9b). Perturbing this equation system yields the following responses of optimal tax rates to an exogenous parameter change $d\xi$:[17]

$$\frac{dt_i}{d\xi} = \frac{1}{|J|} \left[-\frac{\partial^2 T_j}{\partial t_j^2} \frac{\partial^2 T_i}{\partial t_i \partial \xi} + \frac{\partial^2 T_i}{\partial t_i \partial t_j} \frac{\partial^2 T_j}{\partial t_j \partial \xi} \right]. \tag{12}$$

Our first concern is the slope of best response functions. This is ambiguous for coun-

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[16]These expectations are confirmed by simulations that we have carried out to identify $(A_1 - A_2)^c$ numerically. See also Table 2 in Section 5.

[17]From here on, we suppress the regime index, as all expressions refer to Regime I.

try 1:

$$\frac{\partial^2 T_1}{\partial t_1 \partial t_2} = \left(-\frac{\partial \hat{c}}{\partial t_2} - t_1 \frac{\partial^2 \hat{c}}{\partial t_1 \partial t_2}\right)[A_1 - 2\phi(\bar{c} - \hat{c}) - \hat{c}] - t_1 \frac{\partial \hat{c}}{\partial t_1} \frac{\partial \hat{c}}{\partial t_2} (2\phi - 1) \gtrless 0. \quad (13a)$$

The ambiguity arises from the effects collected in the round bracket in the first term of (13a). The first term in this bracket is positive as $\partial \hat{c}/\partial t_2 < 0$ from (7a). Intuitively, an increase in t_2 increases the number of firms in the large country 1 by lowering $\hat{c}$ and this makes it more attractive for country 1 to raise its own tax rate. The second term in the round bracket is negative, however, as

$$\frac{\partial^2 \hat{c}}{\partial t_1 \partial t_2} = \frac{1}{\Theta^3} \{[2 - t_1 - t_2 + \phi(t_1 - t_2)][(A_1 - N - \underline{c}\phi)(\phi + 1) + (A_2 + \underline{c}\phi)(\phi - 1)]\} > 0.$$

This term captures the fact that a rise in t_2 lowers $\hat{c}$ and thus increases the profitability of the marginal firm in country 1. This causes country 1's marginal firm to respond more elastically to changes in t_1 and tends to decrease country 1's tax rate, other things being equal. The second term in (13a) is positive from (7a) and (6). Overall, country 1's best response function can thus be upward or downward sloping.

In contrast, country 2's best response is always upward sloping. This is seen from

$$\frac{\partial^2 T_2}{\partial t_2 \partial t_1} = \left(\frac{\partial \hat{c}}{\partial t_1} + t_2 \frac{\partial^2 \hat{c}}{\partial t_1 \partial t_2}\right)[A_2 - 2\phi(\hat{c} - \underline{c}) - \hat{c}] - t_2 \frac{\partial \hat{c}}{\partial t_1} \frac{\partial \hat{c}}{\partial t_2} (2\phi + 1) > 0, \quad (13b)$$

where (7a) is used to sign the effects on the marginal firm and $\partial^2 \hat{c}/(\partial t_1 \partial t_2) > 0$. An increase in t_1 raises the tax base of country 2 and thus raises the benefits for country 2 to also increase its tax rate. At the same time an increase in t_1 raises $\hat{c}$ and lowers the profitability of the marginal firm in country 2. This reduces country 2's marginal costs of taxation, as it causes its marginal firm to react less sensitively to an increase in t_2.

The difference in the slopes of the best responses are caused by the opposite effects that changes in t_1 and t_2 have on the profitability of the pivotal firm. The changing identity of this firm affects in turn the sensitivity with which it responds to a tax increase in the neighboring region. This (additional) interdependence between the tax policies in the two countries is thus caused only by the presence of firm heterogeneity.

4.1 Isolated market expansion in country 2

We first consider a unilateral increase in the market size of country 2, so that $d\xi = dA_2$. Hence we analyze a 'catching-up' process of the region which has the smaller market

16

in our analysis.[18] We analyze how this change affects tax rates in both countries and consider country 1's tax response first. The impact effect of an increase in country 2's market potential on the tax rate in country 1 is

$$\frac{\partial^2 T_1}{\partial t_1 \partial A_2} = -\left[A_1 - 2\phi(\bar{c} - \hat{c}) - \hat{c}\right]\left[\frac{\partial \hat{c}}{\partial A_2} + \frac{\partial^2 \hat{c}}{\partial t_1 \partial A_2}\right] - t_1 \frac{\partial \hat{c}}{\partial t_1}\frac{\partial \hat{c}}{\partial A_2}(2\phi - 1) < 0, \quad (14)$$

where

$$\frac{\partial \hat{c}}{\partial A_2} = \frac{(1 - t_2)}{\Theta} > 0, \quad (15)$$

from (4a) and $\partial \hat{c}^2/\partial t_1 \partial A_2 > 0$ from (7a) have been used to sign the effects. Hence country 1's tax rate unambiguously falls, upon impact, when A_2 is increased. On the one hand the larger market size of country 2 leads some firms to relocate to this country. This reduces country 1's tax base and hence the marginal benefit of a tax increase [the first part of the first term in (14)]. On the other hand, the expansion of country 2's market increases the profits of the pivotal firm, making this firm more sensitive to tax changes. This increases the marginal costs of a tax increase in country 1 [the second part of the first term in (14)]. Finally, the second term in (14) is also negative, as the marginal firm in country 1 is more profitable after the parameter change, and hence country 1 loses a larger tax base when losing the marginal firm.

The impact change in the tax rate of country 2 is given by

$$\begin{aligned}
\frac{\partial^2 T_2}{\partial t_2 \partial A_2} &= \frac{(\hat{c} - \underline{c})}{2}\left[2 - (2\phi + 1)\frac{\partial \hat{c}}{\partial A_2}\right] + \frac{\partial \hat{c}}{\partial A_2}\left[A_2 - \phi(\hat{c} - \underline{c}) - \frac{\hat{c} + \underline{c}}{2}\right] \\
&+ t_2 \frac{\partial \hat{c}}{\partial t_2}\left[1 - (2\phi + 1)\frac{\partial \hat{c}}{\partial A_2}\right] + t_2 \frac{\partial^2 \hat{c}}{\partial t_2 \partial A_2}[A_2 - 2\phi(\hat{c} - \underline{c}) - \hat{c}], \quad (16)
\end{aligned}$$

where $\partial \hat{c}/\partial A_2$ is given in (15) and $\partial \hat{c}/(\partial t_2 \partial A_2) = -(1 - t_1)(\phi - 1)/\Theta^2 \leq 0$ from (7a).

The first two terms in (16) give the increase in country 2's tax base, induced directly by A_2 and also indirectly by the increase in $\hat{c}$ following the change in A_2. These terms are both positive as country 2 can raise more additional revenue by increasing its tax rate. The third term gives the loss in tax base that country 2 experiences when losing the marginal firm. This effect is ambiguous because there are offsetting effects on the profitability of the pivotal firm in country 2 (as given in the squared bracket of this term): market size has risen in country 2, but the marginal firm in this country also faces higher costs as before. Finally, the fourth effect is negative, because the increase

---

in the size of market 2 will make the pivotal firm respond more sensitively to a tax increase in country 2. Consequently, we cannot sign (16) in general.

We can, however, sign the overall effect in (16), if we assume that the density of firms is sufficiently low, relative to the cost spread (that is, ϕ is sufficiently small). Assume, for example, that $\phi \simeq 1$. In this case, $\partial^2 \hat{c} / \partial t_2 \partial A_2 = 0$ and $\partial \hat{c} / \partial A_2 = 1/2$, leading to

$$\frac{\partial^2 T_2}{\partial t_2 \partial A_2} = \frac{1}{2} \left[A_2 - (\hat{c} - \underline{c}) - t_2 \frac{\partial \hat{c}}{\partial t_2} \right] > 0.$$

Intuitively in this case the profitability of the marginal firm falls from the perspective of country 2, and this firm does not respond more elastically to an increase in t_2. Hence the impact response of t_2 to the parameter change is unambiguously positive. We summarize these results in:

Proposition 3 *An isolated market expansion in country 2 reduces the tax rate of country 1 upon impact (for an unchanged level of t_2). If the density of firms is sufficiently small (ϕ is low) then an increase in A_2 raises the tax rate of country 2 upon impact.*

Note that Proposition 3 only makes a statement about the impact effect of the change in A_2 on optimal tax rates. To this must be added the indirect effects that result from the best response of each country to the initial change in the other country's tax rate. These indirect effects tend to reduce t_2, whereas their effect on t_1 is ambiguous. In general, the total effects on changes in both countries' tax rates can therefore not be signed without imposing further restrictions on the model. Nevertheless, in many cases it can be expected that the total effects of the change are of the same sign as the impact effects given in Proposition 3. In these cases our model thus predicts a convergence of tax rates when the size of the two markets becomes more similar. In particular, one implication of our model is that the growing size of small, peripheral markets may impose downward pressure on the tax rates in larger countries, even if tax rates in the small countries simultaneously rise.

4.2   Market expansion in both countries

Our above analysis has already shown that a growing size of one market may cause the government of the competing country to reduce its tax rate. In the following we consider a simultaneous expansion of both markets, as given by $d\xi = dA_1 + dA_2 \equiv dA$. This specification aims to capture the fact that common technological and economic factors,

such as the IT 'revolution' or global economic integration, have expanded the markets of different potential host countries simultaneously. This is most clearly expressed in the interpretation of our model as one with export platform investment (see Section 2.1), where a positive shock occurs to the overall size of the common, integrated market in which the investment is made.

Our analysis starts from an initial situation where $A_1 = A_2 + N$ ensuring, from Proposition 2, that an interior tax equilibrium in pure strategies exists. We perturb the initial equilibrium and show that the exogenous market expansion will reduce tax rates in both countries, despite the resulting higher profitability of firms. Our results for this case refer to the total effects on tax policy, including the strategic responses of each country to the tax policy change induced in the neighboring jurisdiction.

We first show that the impact effect of the common market expansion dA on the tax rate of the larger country 1 is unambiguously negative (see the appendix):

$$
\begin{aligned}
\frac{\partial^2 T_1}{\partial t_1 \partial A} &= (\bar{c} - \hat{c}) \left[1 + \frac{2\phi - 1}{2} \frac{\partial \hat{c}}{\partial A} \right] - \frac{\partial \hat{c}}{\partial A} \left[A_1 - \phi(\bar{c} - \hat{c}) - \frac{(\bar{c} + \hat{c})}{2} \right] \\
&\quad - t_1 \frac{\partial \hat{c}}{\partial t_1} \left[1 + (2\phi - 1) \frac{\partial \hat{c}}{\partial A} \right] - t_1 \frac{\partial^2 \hat{c}}{\partial t_1 \partial A} [A_1 - 2\phi(\bar{c} - \hat{c}) - \hat{c}] < 0, \quad (17)
\end{aligned}
$$

where $\partial^2 \hat{c}/(\partial t_1 \partial A) > 0$ follows from (7a) and

$$
\frac{\partial \hat{c}}{\partial A} = \frac{(t_1 - t_2)}{\Theta} > 0. \quad (18)
$$

These effects are similar in structure to those in equation (16). The first two terms in (17) describe the change in the profit tax base of country 1. The first effect is positive from the common increase in market size, but the second effect is negative as the simultaneous increase in A_1 and A_2 raises $\hat{c}$ at *unchanged* tax rates [see (18)]. The third effect is negative because the higher profits of the marginal firm imply a larger loss in country 1's tax base when this firm relocates. Finally, the fourth effect is also negative, indicating that the marginal firm will react more sensitively to an increase in t_1 after its profitability has risen. It is shown in the appendix that the positive first effect is dominated by the other three. Hence the impact effect of a simultaneous and equal increase in A_1 and A_2 on country 1's tax rate is unambiguously negative.

Proceeding analogously for country 2, it turns out that the impact effect on this coun-

try's tax rate is exactly zero (see the appendix):

$$
\begin{aligned}
\frac{\partial^2 T_2}{\partial t_2 \partial A} &= (\hat{c} - \underline{c})\left[1 - \frac{2\phi + 1}{2}\frac{\partial \hat{c}}{\partial A}\right] + \frac{\partial \hat{c}}{\partial A}\left[A_2 - \phi(\hat{c} - \underline{c}) - \frac{(\hat{c} + \underline{c})}{2}\right] \\
&+ t_2\frac{\partial \hat{c}}{\partial t_2}\left[1 - (2\phi + 1)\frac{\partial \hat{c}}{\partial A}\right] + t_2\frac{\partial^2 \hat{c}}{\partial t_2 \partial A}[A_2 - 2\phi(\hat{c} - \underline{c}) - \hat{c}] = 0, \quad (19)
\end{aligned}
$$

where $\partial \hat{c}/\partial A > 0$ is given in (18) and $\partial^2 \hat{c}/(\partial t_2 \partial A) < 0$ follows from (7a).

The first two effects are now both positive, because the increase in $\hat{c}$ also works to increase the tax base of country 2. The third effect is ambiguous for country 2, because there are counteracting effects on the profitability of its marginal firm. Finally, the fourth term in (19) is negative. In sum, these effects just offset each other and the induced impact effect on t_2 is zero. The intuition for this result is that the initial condition $A_1 = A_2 + N$ is maintained by the simultaneous and equal increase in market size. In this case we know from our discussion of Proposition 2 that the taxing power of country 2 depends only on the tax rate set by country 1.

The critical difference between (17) and (19) is that a simultaneous expansion of both markets increases the profitability of firms and thus raises the importance of the initial difference in profit tax rates. Hence, if tax rates were (hypothetically) held fixed at their initial levels, some of the moderate-cost firms that initially located in country 1 would find it in their interest to relocate to country 2, once the positive shock has occurred in both countries. It is this relocation of firms as a result of higher gross profits which forces country 1, but not country 2, to lower its tax rate upon impact.

We have now determined all the terms that are needed to evaluate the total effect of the simultaneous increase in A_1 and A_2 on the optimal tax rates. Substituting (13a), (13b), (17) and (19) in (12) yields

$$
\frac{dt_1}{dA} = \frac{1}{|J|}\left(-\frac{\partial^2 T_j}{\partial t_j^2}\frac{\partial^2 T_i}{\partial t_i \partial A}\right) < 0, \quad \frac{dt_2}{dA} = \frac{1}{|J|}\left(\frac{\partial^2 T_2}{\partial t_1 \partial t_2}\frac{\partial^2 T_1}{\partial t_1 \partial A}\right) < 0. \quad (20)
$$

Hence we get:

Proposition 4 *Starting from an initial equilibrium where country 1 is substantially larger than country 2 ($A_1 = A_2 + N$), a simultaneous and equal expansion of both markets, $dA_1 = dA_2 > 0$, reduces equilibrium tax rates in both countries.*

Proposition 4 is illustrated with the help of Figure 1, where panels (a) and (b) correspond to the cases where country 1's best response function is downward or upward

Figure 1: Market expansion in both countries
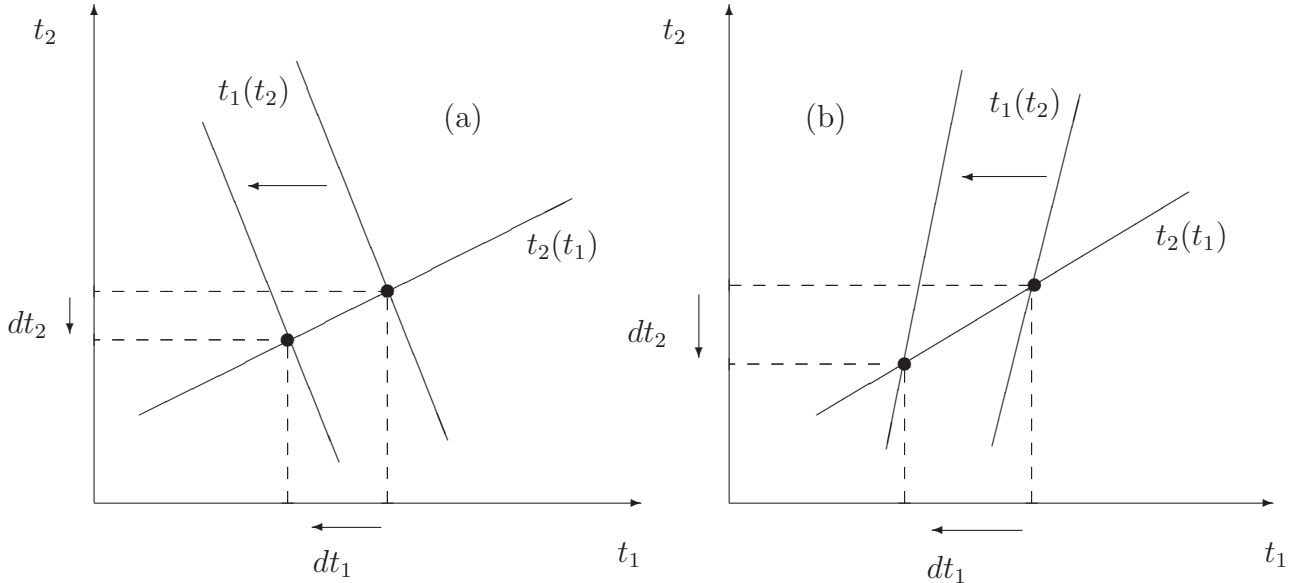
sloping, respectively.[19] Upon impact, the expansion of both countries' markets shifts the best response function of country 1 to the left, but it does not cause a shift in the best response function of country 2. In the new equilibrium, country 1's tax rate is thus unambiguously reduced, no matter whether this country's best response is downward or upward sloping. The fall in t_1 will in turn lead to a downward adjustment of country 2's tax rate, as the equilibrium moves along this country's upward sloping best reply.

Finally we determine the effects of the simultaneous change in market size on (expected) tax revenues in both countries. From the envelope theorem these are given by

$$
\frac{dT_i}{dA} = \underbrace{\frac{\partial T_i}{\partial A}}_{direct\ effect} + \underbrace{\frac{\partial T_i}{\partial t_j}\frac{dt_j}{dA}}_{indirect\ effect} \quad \forall\, i, j. \tag{21}
$$

We distinguish again between the direct (impact) effect for given tax rates, and the indirect effect due to an adjusted tax rate of the neighboring country. From eq. (8) the direct effects in (21) are simply the first two terms in (17) and (19), respectively, multiplied by the corresponding tax rate. This yields an ambiguous impact effect of the common increase in market size on tax revenues in country 1. While the profit tax base of the firms remaining in country 1 is increased, some firms will relocate to country 2 as they react more sensitively to the pre-existing tax differential. Hence the

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[19]The figure is confined to the tax ranges where best responses are continuous in both countries.

net effect on the profit tax base in country 1 is uncertain. In contrast, the profit tax base of country 2 rises from both the higher profitability of the firms that are located in country 2 initially, and from the relocation of firms to this country following the external shock. Hence the direct effect of the simultaneous and equal increases in A_1 and A_2 on country 2's tax revenues is unambiguously positive. We sum up these results in our final proposition:

Proposition 5 *Starting from an initial equilibrium with $A_1 = A_2 + N$, a simultaneous and equal increase in market size $dA_1 = dA_2 > 0$ unambiguously raises the tax base in country 2 upon impact, whereas the effect on country 1's tax base is ambiguous.*

The indirect effects in (21) are negative for both countries, however. It is straightforward to establish that $\partial T_i / \partial t_j > 0 \; \forall \, i$, that is, each country benefits from a tax increase in the other country. Since the expansion of markets lowers tax rates in both countries by Proposition 4, each country will accordingly lose from the induced tax reduction in the neighboring jurisdiction. Therefore, even though both markets expand and the aggregate profit tax base in the two countries taken together will unambiguously rise, the effect on tax revenues is ambiguous in both countries.

Clearly, the results from our simple model cannot be more than suggestive when they are compared with the actual developments in corporate tax policy among the OECD countries (see Table 1). Nevertheless, there are some notable consistencies. Firstly, we have shown that an overall increase in the profitability of firms can be accompanied by falling tax rates, as the rise in firms' profits triggers more severe tax competition (Proposition 4). Moreover, and contrary to what is often argued, our model implies that more aggressive tax competition following the expansion of markets is initiated by the large, high-tax country, whereas the small country merely responds to this tax cut by adjusting its own tax downward. Against this background it is interesting to recall that the first major corporate tax reforms were initiated by the United Kingdom and the United States in the mid-1980s, following a period of deregulation and GDP growth in both of these economies.[20] Only then did smaller countries in Europe and elsewhere start to enact similar reforms. Finally, we have shown that an overall increase in firm profitability raises the tax base of the small country upon impact, whereas the effect on the large country's tax base is ambiguous (Proposition 5). These differences are

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[20]The United Kingdom lowered its statutory corporate tax rate in several stages from 52% in 1982 to 35% in 1986. The United States reduced its corporate tax rate from 50% to 39% in 1987.

consistent with the empirical finding that corporate tax revenue in the small, low-tax OECD countries has unambiguously risen, whereas the effect on tax revenue in the large, high-tax countries has been mixed.

With respect to the existing literature, the distinguishing feature of the present analysis has been the focus on expanding markets as the underlying shock to tax policies, in conjunction with firm heterogeneity. The previous literature on the subject has mostly relied on exogenous reductions in (broadly defined) 'mobility costs' of firms, in order to motivate stiffer tax competition and falling levels of capital taxation (see, e.g., Kind et al., 2005). In the present model we have shown that declining tax rates can alternatively, and perhaps surprisingly, be explained by a general increase in market potential. In particular, this argument also applies to an increase in the size of a common market, which raises the profitability of an export platform investment made by multinational firms.[21] We show that, as a result of its rising profitability, the pivotal firm becomes more sensitive to profit taxes and optimal tax rates in all countries may fall. Thus tax competition for heterogeneous multinationals does not ease when the target region as a whole becomes more attractive.

5 Extension: The model with variable outputs

In this section we briefly consider an extended version of our model where each firm, when deciding to locate in a particular country, simultaneously decides to set up a plant of a certain capacity. Hence the location and output choices are made simultaneously, before each firm learns the cost structure of its competitors.[22]

Let $q_i(c)$ denote the output of a firm of type c locating in country i. This output is determined from maximizing expected after-tax profits $\pi_i = (1 - t_i)q_i(p_i - c)$. The first-order condition for a firm in country i is $p_i(q_i + \hat{Q}_{-i}) + q_i p'(q_i + \hat{Q}_{-i}) - c = 0$, where $\hat{Q}_{-i}$ denotes the aggregate expected output of all rival firms in country i and p' is the derivative of the demand function. With linear demand, the optimal output choice of a firm with cost c in country i is

$$q_i(c) = (A_i - \hat{Q}_{-i} - c)/2 \ . \tag{22}$$

---

[21]Firm heterogeneity is also observed among multinationals, which are found to be more productive, on average, than firms which export or serve their local market only. See Helpman et al. (2004).

[22]Formally, this is a Bayesian Cournot game similar to Long et al. (2009), which is extended to the simultaneous location choice of firms.

Using (22) leads to optimized after-tax profits equal to $\pi_i = (1 - t_i)[q_i(c)]^2 \ \forall \ i \in \{1, 2\}$. If an interior equilibrium exists with firms locating in both countries, then there must be a firm with a critical cost level $\hat{c}$ that is indifferent between the two locations:

$$(1 - t_1)[q_1(\hat{c})]^2 = (1 - t_2)[q_2(\hat{c})]^2 \iff q_1(\hat{c}) = \sqrt{\frac{(1 - t_2)}{(1 - t_1)}} \ q_2(\hat{c}). \qquad (23)$$

It follows from (23) that the pivotal firm produces a higher level of output in the high-tax country. In Regime I, where country 1 chooses the higher tax rate, the pivotal firm's output will thus be higher in country 1.

Where do firms with costs slightly above $\hat{c}$ locate? To answer this question, we consider the effects of a small increase in costs on after-tax profits in each of the two countries, starting from the critical cost level $\hat{c}$. Using (22) and (23) yields

$$\frac{d\pi_1(\hat{c})}{dc} = -(1 - t_1)q_1(\hat{c}) = -(1 - t_2)\sqrt{\frac{(1 - t_1)}{(1 - t_2)}} \ q_2(\hat{c}),$$

$$\frac{d\pi_2(\hat{c})}{dc} = -(1 - t_2)q_2(\hat{c}). \qquad (24)$$

Equation (24) shows that the negative effects on after-tax profits of an increase in production costs are smaller in absolute value in the high-tax country 1. As maximized profits are monotonic in c this implies that all firms with $c > \hat{c}$ will have higher after-tax profits in country 1, whereas all firms with $c < \hat{c}$ have higher after-tax profits in country 2. Hence, as in our benchmark model [cf. eq. (2)], high-cost firms will locate in the high-tax country in the location equilibrium.

It is then straightforward to derive the rival firms' aggregate output in each country, $\hat{Q}_{-1}$ and $\hat{Q}_{-2}$, and the expressions for expected tax revenue in each country. This is done in the Appendix. The extended model is too complex, however, to be solved analytically. We therefore carried out simulation analyses, whose primary aim is to establish conditions under which a Nash equilibrium in taxes exists in the extended model.

Following our argument in Section 3 above, a non-cooperative tax equilibrium should exist, if the difference in the market size parameters A_i is sufficiently large. More precisely, a sufficient condition for existence should be that, if tax rates were (hypothetically) set at the same level in both countries, all production would take place in the larger market. In contrast to our benchmark model this condition cannot solely be expressed in terms of exogenous variables, because aggregate production is now endoge-

Table 2: Nash equilibria in the extended model with variable output

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A_1	t_1	t_2	T_1	T_2	$\hat{c}$	Q_1	Q_2	$\hat{q}_1$	$\hat{q}_2$
CASE 1: $A_2 = 60$, $N = 3$, $\bar{c} = 13$, $\underline{c} = 10$									
70	–	–	–	–	–	–	–	–	–
75	0.835	0.759	190.63	97.33	11.29	22.77	14.88	13.31	11.53
80	0.845	0.746	226.36	94.49	11.25	24.98	14.55	14.27	11.64
90	0.861	0.722	307.25	89.18	11.18	29.45	13.92	16.18	11.80
CASE 2: $A_2 = 30$, $N = 2$, $\bar{c} = 2$, $\underline{c} = 0$									
33	–	–	–	–	–	–	–	–	–
35	0.614	0.524	60.13	33.02	0.85	7.52	5.17	6.54	6.08
40	0.649	0.478	88.14	27.74	0.74	9.24	4.64	7.33	6.27
45	0.678	0.435	121.85	23.11	0.65	11.00	4.16	8.15	6.40

<u>Notes :</u> Q_i: total production in country i; $\hat{q}_i$: production per firm in country i $(= Q_i/n_i)$

nous. Nevertheless, our simulations show that the basic argument made in Section 3 carries over to the extended model. The simulation results are presented in Table 2.

The two cases shown in Table 2 yield qualitatively similar results; hence it is sufficient to discuss Case 1 in more detail. Here we fix the value of $A_2 = 60$ and consider different values of A_1. For $A_1 = 70$ the size differential between the two markets is too small to permit the existence of a pure-strategy Nash equilibrium. For values of $A_1 \geq 75$, however, a Nash equilibrium exists with country 1 as the high-tax country.[23] Corresponding to our benchmark model, column (6) shows that the expected number of firms in country 1 exceeds that in country 2. Moreover, aggregate output is also higher in country 1 [columns (7)–(8)]. Finally, columns (9) and (10) show that the average output per firm is higher in country 1, even though this country hosts the high-cost firms. This is due to the discontinuous jump in output at the cost level $\hat{c}$ [cf. eq. (23)].

These results show that the basic properties of our benchmark model carry over to an extended setting with variable outputs of firms.

---

[23]To determine the existence of a pure-strategy Nash equilibrium, we compute tax rates and tax revenues in a *constrained* equilibrium where country 1 is exogenously taken to be the high-tax country. We then test whether country 1 can increase its tax revenue by marginally undercutting the tax rate that country 2 chooses in this constrained equilibrium. A (unconstrained) Nash equilibrium in taxes exists iff country 1 can *not* increase its revenues by switching to Regime II.

6 Conclusions

In this paper we have employed a simple model of tax competition for heterogeneous, internationally mobile firms. This model leads to clear predictions regarding the interaction between market conditions, the cost structure of firms locating in a particular country, and tax policies. A first result is that firms sort according to their productivity, with low-cost firms settling in the low-tax country. This result is consistent with the empirical observation that the average gross profitability of investments is higher in low-tax countries. It thus offers a rivaling explanation for this stylized fact, which has so far been exclusively ascribed to income shifting by multinational firms. This is a potentially important finding from a policy perspective because the perceived empirical importance of income shifting has been a major motivation for several recent corporate tax reforms (such as the German tax reform of 2008). To the extent that low corporate tax bases in high-tax countries are indeed caused by low-profitability investments, rather than by profit shifting, the focus of these reforms may have been partly misplaced.

A second result of our analysis is that a common increase in the market potential of host countries can lead to intensified tax competition and reduced tax rates in both countries while corporate tax bases are simultaneously increased. The reason is that the pivotal firm in each country will experience higher profits, causing it to react more sensitively to changes in tax rates. This finding offers one possible explanation for the puzzling fact that falling tax rates and increasing corporate tax receipts have occurred simultaneously in many OECD countries. At the same time, our results are also consistent with the observation that the growth in corporate tax revenue seems to have been more robust among small, low-tax countries than among their larger, high-tax neighbors.

In order to bring these results forward, an important assumption in the present paper has been that the output of each firm is fixed. It is known from the recent international trade literature that when firm size is endogenized, then expanding the size of a common market will benefit the productive firms more than proportionately, shifting market shares from high-cost to low-cost firms. As our extended model has shown, the small country still levies the lower profit tax rate in this case, and it attracts the low-cost firms. However, tax competition may become more severe when firm size is endogenous because gross profits - and hence corporate tax revenue - increase more

than proportionately with a decrease in production costs. A further extension of our analysis would be to give governments a second instrument in the competition for the most profitable firms, by letting them choose to which extent production costs are tax-deductible. This would allow to study the tax-rate-cut-cum-base-broadening reforms, which have occurred in many countries, in a setting with firm heterogeneity. We leave these extensions to future research.

Appendix

Optimal tax rates in Regime II

In Regime II, tax revenues in the two countries are given by

$$T_1^{II} = t_1 \int_{\underline{c}}^{\hat{c}} \left[A_1 - \phi(\hat{c} - \underline{c}) - c \right] \phi dc, \qquad T_2^{II} = t_2 \int_{\hat{c}}^{\overline{c}} \left[A_2 - \phi(\overline{c} - \hat{c}) - c \right] \phi dc, \quad \text{(A.1)}$$

where the relevant expression for $\hat{c}$ is now given in (4b). Differentiating with respect to t_i yields the regime-specific first-order conditions

$$\frac{\partial T_1^{II}}{\partial t_1} = (\hat{c}_{II} - \underline{c}) \left[A_1 - \phi(\hat{c}_{II} - \underline{c}) - \frac{(\hat{c}_{II} + \underline{c})}{2} \right] + t_1 \frac{\partial \hat{c}_{II}}{\partial t_1} \left[A_1 - 2\phi(\hat{c}_{II} - \underline{c}) - \hat{c}_{II} \right] = 0,$$

$$\frac{\partial T_2^{II}}{\partial t_2} = (\overline{c} - \hat{c}_{II}) \left[A_2 - \phi(\overline{c} - \hat{c}_{II}) - \frac{(\overline{c} + \hat{c}_{II})}{2} \right] - t_2 \frac{\partial \hat{c}_{II}}{\partial t_2} \left[A_2 - 2\phi(\overline{c} - \hat{c}_{II}) - \hat{c}_{II} \right] = 0.$$

$$\text{(A.2)}$$

Proof of Proposition 1

Rewriting the two expressions in (A.2) yields

$$t_1^{II} = (\hat{c}_{II} - \underline{c}) \left(\frac{-\partial \hat{c}_{II}}{\partial t_1} \right)^{-1} \frac{[A_1 - \phi(\hat{c}_{II} - \underline{c}) - 0.5(\hat{c}_{II} + \underline{c})]}{[A_1 - 2\phi(\hat{c}_{II} - \underline{c}) - \hat{c}_{II}]}, \qquad \text{(A.3)}$$

$$t_2^{II} = (\overline{c} - \hat{c}_{II}) \left(\frac{\partial \hat{c}_{II}}{\partial t_2} \right)^{-1} \frac{[A_2 - \phi(\overline{c} - \hat{c}_{II}) - 0.5(\overline{c} + \hat{c}_{II})]}{[A_2 - 2\phi(\overline{c} - \hat{c}_{II}) - \hat{c}_{II}]}. \qquad \text{(A.4)}$$

A Regime II equilibrium is defined by $t_1 < t_2$. Hence for such an equilibrium to exist, *at least one* of the three positive terms in (A.3) must be smaller than the corresponding term in (A.4). In the following we compare the three terms in turn, always starting from the assumption that $t_1 < t_2$.

(i) $(\hat{c}_{II} - \underline{c}) < (\overline{c} - \hat{c}_{II}) \Longrightarrow 2\hat{c}_{II} - \overline{c} - \underline{c} < 0$

Using (4b) and performing straightforward manipulations we obtain

$$2\hat{c}_{II} - \overline{c} - \underline{c} = \frac{1}{\Theta_{II}}\left\{2(A_1 - A_2)(1 - t_2) + (t_2 - t_1)[2A_1 - N - (\overline{c} + \underline{c})]\right\} > 0. \quad \text{(A.5)}$$

This is unambiguously positive in Regime II because (i) $A_1 > A_2$ by our convention that country 1 is the larger one, (ii) $t_2 > t_1$ by the definition of Regime II, and (iii) the term in the squared bracket must be positive from the condition for an interior tax optimum (10). Hence the first condition for $t_1^{II} < t_2^{II}$ is *not* fulfilled.

(ii) $(-\partial\hat{c}_{II}/\partial t_1)^{-1} < (\partial\hat{c}_{II}/\partial t_2)^{-1} \Longrightarrow (-\partial\hat{c}_{II}/\partial t_1) > (\partial\hat{c}_{II}/\partial t_2)$

It is immediately seen from (7b) that this condition is also *not* fulfilled in Regime II, where $t_1 < t_2$.

(iii) $\Gamma_I \equiv [A_1 - \phi(\hat{c}_{II} - \underline{c}) - 0.5(\hat{c}_{II} + \underline{c})][A_2 - 2\phi(\overline{c} - \hat{c}_{II}) - \hat{c}_{II}]$
$\qquad -[A_1 - 2\phi(\hat{c}_{II} - \underline{c}) - \hat{c}_{II}][A_2 - \phi(\overline{c} - \hat{c}_{II}) - 0.5(\overline{c} + \hat{c}_{II})] < 0$

Multiplying out the terms on the RHS, rearranging terms and performing several straightforward manipulations yields, in a first step

$$\begin{aligned}
\Gamma_I &= 0.5(A_1 - A_2)(\overline{c} - \hat{c}_{II}) + A_2\phi(2\hat{c}_{II} - \overline{c} - \underline{c}) - (A_1 - A_2)\phi(\overline{c} - \hat{c}_{II}), \\
&+ 0.5(A_2 - \hat{c}_{II})(\overline{c} - \underline{c}) + \hat{c}_{II}\phi(2\hat{c}_{II} - \overline{c} - \underline{c}) - 2\phi(\hat{c}_{II}^2 - \overline{c}\underline{c}).
\end{aligned}$$

Combining the last two terms and expanding this can be rewritten as

$$\Gamma_I = 0.5(A_1 - A_2 + 2n)(\overline{c} - \hat{c}_{II}) + 0.5(A_2 - 2N - \hat{c}_{II})(\overline{c} - \underline{c}) + \Delta, \quad \text{(A.6)}$$

where the first two terms are positive from $A_1 > A_2$ and (10) and

$$\Delta = (A_1 - A_2)\phi(\overline{c} - \hat{c}_{II}) + (A_2 - \underline{c})\phi(2\hat{c}_{II} - \overline{c} - \underline{c}).$$

To sign Δ we substitute (A.5), rearrange terms, expand with $2(A_2 - \underline{c})(A_1 - A_2)(t_2 - t_2)$ and use the fact that $(\overline{c} - \hat{c}_{II})/(\overline{c} - \underline{c}) < 1/2$ from (A.5). This yields

$$\begin{aligned}
\Delta &> \frac{\phi}{\Theta_{II}}\left\{(t_2 - t_1)\left[(A_1 - A_2)(A_2 - \overline{c} + (\hat{c}_{II} - \underline{c})) + 2(A_2 - \underline{c})(A_2 - 0.5N - 0.5(\overline{c} + \underline{c}))\right]\right. \\
&\quad + \left.(A_1 - A_2)\left[(A_2 - 0.5N - \underline{c})(2 - t_1 - t_2)\right]\right\} > 0,
\end{aligned}$$

which is unambiguously positive from $A_1 > A_2$, eq. (10) and $t_2 - t_1 > 0$ in Regime II. Hence we unambiguously obtain $\Gamma_I > 0$ so that the third condition for $t_1^{II} < t_2^{II}$ is also violated. Thus there cannot be an interior Nash equilibrium in Regime II where the tax rates are given by (A.3) and (A.4) and $t_1 < t_2$ holds. $\square$

Proof of Proposition 2

The proof proceeds in three steps: (i) we show that reaction functions exist in the relevant range; (ii) we identify two tax functions, one which makes country 1 attract all firms and one which makes country 2 attract all firms, and show when an interior solution exists; (iii) we demonstrate that the reaction functions will intersect exactly once in the case of an interior equilibrium.

(i) Reaction functions exist if there is one and only one optimal t_i as a response to each t_j. This is true if the second-order conditions for an interior maximum are fulfilled. From (9a) and (9b) the second-order conditions of the two countries' optimal tax problems are given by

$$\frac{\partial^2 T_1^I}{\partial t_1^2} = \left(-2\frac{\partial \hat{c}_I}{\partial t_1} - t_1\frac{\partial^2 \hat{c}_I}{\partial t_1^2}\right)[A_1 - 2\phi(\bar{c} - \hat{c}_I) - \hat{c}_I] - t_1\left(\frac{\partial \hat{c}_I}{\partial t_1}\right)^2(2\phi - 1) < 0, \quad \text{(A.7)}$$

$$\frac{\partial^2 T_2^I}{\partial t_2^2} = \left(2\frac{\partial \hat{c}_I}{\partial t_2} + t_2\frac{\partial^2 \hat{c}_I}{\partial t_2^2}\right)[A_2 - 2\phi(\hat{c}_I - \underline{c}) - \hat{c}_I] - t_2\left(\frac{\partial \hat{c}_I}{\partial t_2}\right)^2(2\phi + 1) < 0. \quad \text{(A.8)}$$

In (A.7) and (A.8) both terms are then unambiguously negative from (7a) and (6) as $\partial \hat{c}_I/\partial t_1^2 > 0$ and $\partial \hat{c}_I/\partial t_2^2 < 0$. Hence both countries' tax revenue functions are strictly concave in Regime I and reaction functions are well-defined.

(ii) Next, we identify two tax schemes for country 1, as functions of the tax rate of country 2. First, consider (4a) and determine country 1's largest possible tax rate which still induces all firms to locate in country 1, i.e., $\hat{c}_I = \underline{c}$. We denote this tax rate by τ:

$$\tau(t_2) \equiv \frac{A_1 - A_2 - N + t_2(A_2 - \underline{c})}{A_1 - N - \underline{c}}, \quad \tau(0) = \frac{A_1 - A_2 - N}{A_1 - N - \underline{c}} \geq 0, \quad \tau(1) = 1. \quad \text{(A.9)}$$

Expression (A.9) shows that τ is linearly increasing with t_2. If $A_1 = A_2 + N$, then $\tau(0) = 0$. Second, consider (4a) again and determine country 1's smallest tax rate which makes all firms locate in country 2, i.e., $\hat{c}_I = \bar{c}$. This tax is denoted by σ:

$$\sigma(t_2) \equiv \frac{A_1 - A_2 + N + t_2(A_2 - N - \bar{c})}{A_1 - \bar{c}}, \quad \sigma(0) = \frac{A_1 - A_2 + N}{A_1 - \bar{c}} > 0, \quad \sigma(1) = 1. \quad \text{(A.10)}$$

Expression (A.10) shows that σ also increases linearly with t_2. Of course, $\sigma(t_2) > \tau(t_2)$ for all $t_2 \in [0, 1[$. Both tax schemes are shown in Figure 2.

Any equilibrium must be found strictly in between the $\tau-$ and the $\sigma-$lines in Figure 2: there is no equilibrium on the $\sigma-$line because country 1 would attract no firm but could do so with a positive tax rate. Furthermore, there is no equilibrium on the $\tau-$line for

Figure 2: $\tau(t_2)$ and $\sigma(t_2)$
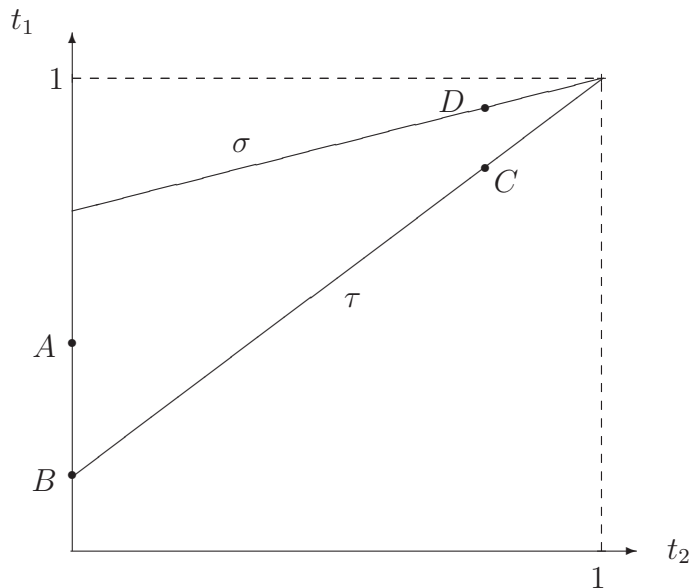
$t_2 > 0$ because country 2 has no tax base but could generate tax revenues by reducing t_2. The only equilibrium candidate on the τ–line is $\tau(0)$. At this point, given by point B in Figure 2, country 2 cannot do better by lowering its tax rate, as this would imply negative tax revenue if it attracted any firms. Whether $\tau(0)$ is an equilibrium depends on the marginal tax revenues of country 1 at $\tau(0)$:

$$
\begin{aligned}
\frac{\partial T_1}{\partial t_1}(\tau(0),0) &= (\bar{c} - \underline{c})\left(A_1 - N - \frac{\underline{c} + \bar{c}}{2}\right) \\
&\quad - \frac{(A_1 - N - \underline{c})(A_1 - 2N - \underline{c})(A_1 - A_2 - N)}{(1 + \phi)(A_1 - A_2 - N) + 2\phi(A_2 - \underline{c})}.
\end{aligned}
\tag{A.11}
$$

If (A.11) is negative, then $t_1 = \tau(0)$, $t_2 = 0$ is the unique equilibrium, featuring full agglomeration of firms in country 1. If (A.11) is positive, country 1's best response to $t_2 = 0$ is a tax rate $t_1 > \tau(0)$. This is indicated in Figure 2 by a point such as A.[1]

(*iii*) We now turn to existence and uniqueness of the equilibrium if (A.11) is positive. We have already determined two points on the reaction curves: if (A.11) is positive, country 1's reaction curve starts at A while $t_2 = 0$ is still country 2's best response to $t_1 = \tau(0)$; see point B. We evaluate country 1's marginal tax revenues (9a) along the τ–line and country 2's marginal tax revenues (9b) along the σ–line. The derivatives are

---

[1]It is easily shown that $\partial T_1/\partial t_1(\tau(0),0)$ increases with A_2. Thus an interior candidate equilibrium (as opposed to an equilibrium with full agglomeration) is likely, if country 2 is not too small.

denoted by the superscripts τ and σ and are given by

$$
\frac{\partial T_1^\tau}{\partial t_1} = (\bar{c} - \underline{c})\left(A_1 - N - \frac{\underline{c} + \bar{c}}{2}\right) \tag{A.12}
$$
$$
- \tau(t_2)\frac{(1 - t_2)((A_1 - N - \phi\underline{c})(\phi + 1) + (A_2 + \phi\underline{c})(A_1 - A_2 - N))}{(\phi + 1)(1 - t_2) + (\phi - 1)(1 - \tau(t_2))},
$$

$$
\frac{\partial T_2^\sigma}{\partial t_2} = (\bar{c} - \underline{c})\left(A_2 - N - \frac{\underline{c} + \bar{c}}{2}\right) \tag{A.13}
$$
$$
+ t_2\frac{(1 - \sigma(t_2))((A_1 - N - \phi\underline{c})(\phi + 1) + (A_2 + \phi\underline{c})(A_1 - A_2 - N))}{(\phi + 1)(1 - t_2) + (\phi - 1)(1 - \sigma(t_2))}.
$$

Note that $\partial T_1/\partial t_1(\tau(0), 0) = \partial T_1^\tau/\partial t_1(t_2 = 0)$. Thus, if (A.11) is positive, we find that

$$
\frac{\partial T_1^\tau}{\partial t_1}(t_2 = 0) > 0, \qquad \frac{\partial T_2^\sigma}{\partial t_2}(t_2 = 0) > 0, \tag{A.14}
$$
$$
\lim_{t_2 \to 1}\frac{\partial T_1^\tau}{\partial t_1} = -\infty, \qquad \lim_{t_2 \to 1}\frac{\partial T_2^\sigma}{\partial t_2} = -\infty.
$$

Since tax revenues are a continuous and twice differentiable function of tax rates, (A.14) proves that at least one tax rate t_2, with $0 < t_2 < 1$, must exist such that $\partial T_1^\tau/\partial t_1 = 0$; this is shown by a point such as C in Figure 2. Similarly, at least one tax rate t_1 with $0 < t_1 < 1$ must exist such that $\partial T_2^\sigma/\partial t_2 = 0$; see point D in Figure 2. Point C is thus also on the reaction curve of country 1 and D is on the reaction curve of country 2. Since reaction functions are continuous, they must run from A to C for country 1 and from B to D for country 2. Consequently, they must intersect at least once. Due to $|J| > 0$, they can intersect only once because any second intersection would imply $|J| < 0$ from the continuity of best responses. This proves existence and uniqueness for the interior candidate equilibrium. $\square$

Proof of Proposition 4

To sign (17), it suffices to compare the first and the third term on the RHS of the equation. The squared bracket in the first term is unambiguously smaller than the corresponding bracket in the third term. Hence a sufficient condition for (17) to be negative is that $(\bar{c} - \hat{c}) < t_1(\partial\hat{c}/\partial t_1)$. But this last inequality is implied by the first-order condition for country 1's initial tax rate (9a). In (9a), the squared bracket in the first term is unambiguously larger than the squared bracket in the second term when $\phi > 1$ [cf. (6)]. Hence $(\bar{c} - \hat{c}) < t_1(\partial\hat{c}/\partial t_1)$ must hold for the two terms to sum to zero.

To sign (19), we simplify the value for $\hat{c}_I$ in (4a) when $A_1 = A_2 + N$ holds in the initial equilibrium. This yields

$$\hat{c}_I|_{A_1=A_2+N} = \frac{A_2(t_1 - t_2) + \underline{c}\,\phi\,(2 - t_1 - t_2)}{t_1 - t_2 + \phi(2 - t_1 - t_2)} \tag{A.15}$$

This also yields simplified derivative properties

$$\frac{\partial \hat{c}}{\partial t_1} = \frac{2(1 - t_2)\phi(A_2 - \underline{c})}{\Theta_I^2}, \quad \frac{\partial \hat{c}}{\partial t_2} = \frac{-2(1 - t_1)\phi(A_2 - \underline{c})}{\Theta_I^2}, \quad \frac{\partial^2 \hat{c}}{\partial t_2 \partial A} = \frac{-2(1 - t_1)\phi}{\Theta_I^2}. \tag{A.16}$$

We decompose $\partial^2 T_2/(\partial t_2 \partial A) = \psi_1 + \psi_2$, where ψ_1 is the sum of the first and the third term in (19), whereas ψ_2 stands for the second and the fourth term. Using (A.15)–(A.16) and performing straightforward manipulations gives

$$\Psi_1 = \left[(\hat{c} - \underline{c}) + t_2 \frac{\partial \hat{c}}{\partial t_2}\right]\left[\frac{t_1 - t_2}{2} + 2\phi(1 - t_1)\right]\frac{1}{\Theta_I} - t_2\left(\phi + \frac{1}{2}\right)\frac{\partial \hat{c}}{\partial t_2}\frac{\partial \hat{c}}{\partial A} \tag{A.17}$$

From (9b) we obtain through simple manipulations

$$\left[t_2 \frac{\partial \hat{c}}{\partial t_2} + (\hat{c} - \underline{c})\right]\left[A_2 - \phi(\hat{c} - \underline{c}) - \frac{(\hat{c} + \underline{c})}{2}\right] = t_2\frac{\partial \hat{c}}{\partial t_2}\left(\phi + \frac{1}{2}\right)(\hat{c} - \underline{c})$$

Substituting this in (A.17) and using (A.15)–(A.16) and (18) gives

$$\psi_1 = t_2\frac{\partial \hat{c}}{\partial t_2}\left(\phi + \frac{1}{2}\right)(t_1 - t_2)\frac{1}{\Theta_I}\left[\frac{(A_2 - \underline{c})(t_1 - t_2)/2 + 2\phi(1 - t_1)}{\{A_2 - \phi(\hat{c} - \underline{c}) - (\hat{c} + \underline{c})/2\}\Theta_I} - 1\right].$$

Again using (A.15)–(A.16) and (5a) shows that the term in squared brackets is zero and hence $\psi_1 = 0$.

Next we sum over the second and the fourth term in (19). This gives

$$\psi_2 = \frac{1}{\Theta_I^2}\left\{(t_1 - t_2)\Theta_I\left[A_2 - \phi(\hat{c} - \underline{c}) - \frac{(\hat{c} + \underline{c})}{2}\right] - 2\phi(1 - t_2)t_2[A_2 - 2\phi(\hat{c} - \underline{c}) - \hat{c}]\right\}.$$

Substituting t_2 from (9b) into the second term and using (A.15)–(A.16) shows that $\psi_2 = 0$. Hence $\partial^2 T_2/(\partial t_2 \partial A) = 0$. Using these results in (20) yields Proposition 4. $\square$

Appendix to Section 5

When all high-cost firms locate in country 1, a firm that wants to locate in this country expects an aggregate output of all rival firms equal to

$$\hat{Q}_{-1} = (N - 1)\int_{\hat{c}}^{\overline{c}}\frac{(A_1 - \hat{Q}_{-1} - c)}{2}\frac{dc}{\overline{c} - \underline{c}} = \frac{(N - 1)[2(A_1 - \hat{Q}_{-1}) - \hat{c} - \overline{c}](\overline{c} - \hat{c})}{\overline{c} - \underline{c}}.$$

Solving this for $\hat{Q}_{-1}$ gives

$$\hat{Q}_{-1} = \frac{(N-1)(2A_1 - \hat{c} - \overline{c})(\overline{c} - \hat{c})}{2[(N+1)\overline{c} - (N-1)\hat{c} - 2\underline{c}]}. \tag{A.18}$$

Similarly, for a firm contemplating to locate in country 2 the expected output of all rival firms in this country is

$$\hat{Q}_{-2} = (N-1) \int_{\underline{c}}^{\hat{c}} \frac{(A_2 - \hat{Q}_{-2} - c)}{2} \frac{dc}{\overline{c} - \underline{c}} \quad \Rightarrow \quad \hat{Q}_{-2} = \frac{(N-1)(2A_2 - \hat{c} - \underline{c})(\hat{c} - \underline{c})}{2[(N-1)\hat{c} - (N+1)\underline{c} + 2\overline{c}]}. \tag{A.19}$$

The expected tax revenues in the two countries are given by

$$T_1 = t_1 \int_{\hat{c}}^{\overline{c}} [q_1(c)]^2 \frac{dc}{\overline{c} - \underline{c}} = t_1 \frac{(A_1 - \hat{c} - \hat{Q}_1)^3 - (A_1 - \overline{c} - \hat{Q}_1)^3}{12(\overline{c} - \underline{c})} ,$$

$$T_2 = t_2 \int_{\underline{c}}^{\hat{c}} [q_2(c)]^2 \frac{dc}{\overline{c} - \underline{c}} = t_2 \frac{(A_2 - \underline{c} - \hat{Q}_2)^3 - (A_2 - \hat{c} - \hat{Q}_2)^3}{12(\overline{c} - \underline{c})} , \tag{A.20}$$

where $\hat{Q}_{-1}$ and $\hat{Q}_{-2}$ are given in (A.18) and (A.19), respectively.

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