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A Simple Model of Tax-Favored Retirement Accounts

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A Simple Model of Tax-Favored Retirement Accounts

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Abstract

To defend myopic workers against themselves, the government introduces a mandatory system but to help savers, it adds tax-favored retirement accounts. In a very simple model, where benefits are proportional to contributions, we compare three extreme systems: (i) the pure mandatory system, (ii) the asymmetric system, where only the savers participate in the voluntary system, (iii) the symmetric system, where both types participate proportionally to their wages. The symmetric voluntary system is welfare-superior to the asymmetric one as well as to the pure mandatory system, which in turn are equivalent to each other.

Keywords: mandatory pensions, tax-favored retirement accounts, voluntary contributions, subsidies.

JEL: H55, D91

Az önkéntes nyugdíjrendszer egy egyszerű modellje

András Simonovits

Összefoglaló

A rövidlátó dolgozókat a kormányzat úgy tudja saját maguk ellen megvédeni, hogy kötelező nyugdíjrendszert vezet be. De a takarékos dolgozók érdekében egy önkéntes nyugdíjrendszerrel egészíti ki a kötelező rendszert. Egy nagyon egyszerű modellben, amelyben mindkét rendszer nyugdíjai arányosak a befizetésekkel, három szélsőséges rendszert hasonlítunk össze: 1. a tisztán kötelező rendszert; 2. az aszimmetrikus rendszert, ahol csak a takarékosak vesznek részt az önkéntes rendszerben; 3. a szimmetrikus rendszert, amelyben mindkét típus tagjai a keresetükkel arányosan vesz részt a rendszerben. A társadalmi jóléti függvényt tekintve a szimmetrikus önkéntes rendszer egyaránt felülmúlja a tiszta kötelező és az aszimmetrikus önkéntes rendszert, amelyek viszont nagyjából ekvivalensek.

Tárgyszavak: kötelező nyugdíj, önkéntes nyugdíj, tagdíj, támogatás, adókedvezmény

JEL: H55, D91

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A Simple Model of Tax-Favored Retirement Accounts

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A Simple Model of Tax-Favored Retirement Accounts

András Simonovits

Abstract

To defend myopic workers against themselves, the government introduces a proportional mandatory system. But to help savers, it adds tax-favored retirement accounts, with proportional matching. In a very simple model, we compare three extreme systems: (i) the pure mandatory system, (ii) the asymmetric system, where only the savers participate in the voluntary system, (iii) the symmetric system, where both types participate proportionally to their wages. The symmetric voluntary system is welfare-superior to the asymmetric one as well as to the pure mandatory system, which in turn are close to each other.

JEL code: H55, D91

Key words: mandatory pensions, tax-favored retirement accounts, voluntary contributions, subsidies.

1. Introduction

In most developed countries, in addition to the mandatory (funded and/or unfunded, public or private) pension system, a voluntary pension system exists, providing tax and contribution subsidies. The voluntary pension system is formed by *tax-favored retirement accounts*. In the default case, these subsidized savings cannot be withdrawn until the owner retires. The proponents of such systems justify these subsidies like this: a mandatory system does not and need not ensure high enough pensions, and the mostly partially myopic (for short, myopic) workers must be made interested in raising their old-age incomes through a voluntary system. The opponents are afraid that these subsidies are poorly targeted, mostly subsidize the well-paid savers, while worsening the burden of the others by increasing the *tax expenditures*. Up to now these tax expenditures have generally been quite low, thus they may be neglected, but under a possible contraction of the mandatory system they may become much higher. In this paper, we will discuss the issue in a very simple model. Since there are no other taxes in the model, we will write earmarked taxes rather than tax expenditures, pretending that a special tax finances the subsidies. Following Feldstein (1987, Part I), we consider only two types: the *myope* (L) with a low discount factor and the *saver* (H) with a high discount factor. (In fact, with Feldstein, the myopes are fully myope and the savers are fully savers with discount factors 0 and 1, respectively.) To enhance the realism of the model, we introduce wage heterogeneity as well. To avoid having four types, we assume that the myopes' earning is less than or equal to the savers'.

We consider three simple systems consisting of different combinations of *pillars*: (i) the pure mandatory system, without voluntary pillar but with forced savings for the myopes; (ii) the sum of a mandatory pillar and the asymmetric voluntary pillar, with only the savers's participations; (iii) the sum of a mandatory pillar and the symmetric voluntary pillar, with both types participating proportionally to their wages. Without confusion, in case (ii) and (iii), we shall speak of asymmetric or symmetric voluntary system, respectively. Assuming that in both pension pillars, the benefits are proportional to contributions, the individually optimal decisions are easy to calculate, opening the door for further investigation. We posited a utilitarian social welfare, without discounting future utility (cf. Feldstein, 1987). Our main numerical results are as follows: The symmetric voluntary system is superior to the asymmetric one as well as to the pure mandatory system, which in turn are close to each other.

Starting with the Hungarian experiences, it should be emphasized that the newly granted benefits in the Hungarian mandatory pillar are almost proportional to the contributions, it is quite generous, replacing about 60–70% of the lifetime net wages, up to the triple of the average wage. In addition, the voluntary pillar is also generous: the current ceiling (on the sum of employee's and employer's contributions) is about 30% of the average gross wage and the matching rate varies between 30–50%. Nevertheless, the participation is quite modest, about 1/3 of the work force, while the average voluntary contribution is about 3.6% of the average wage. This is especially low if dormant accounts are taken into account (Matits, 2008). Our tentative results support those who criticize the Hungarian voluntary pillar for having too high ceilings and concentrated subsidies.

Turning to the international experience, let us underline that most pension systems deviate from the Hungarian system in a very important dimension: the mandatory or the voluntary pillar is progressive. For example, the US and the Czech mandatory pillars as well as the German and the Czech voluntary pillars are progressive. A proper evaluation of such systems needs modified models.

Among the large number of US studies, we single out the following ones: Poterba et al. (1996) estimate that the introduction of tax-favored retirement accounts significantly increased total savings, while Engen et al. (1996) find the opposite. Trying at a synthesis, Hubbard and Skinner (1996) guess that both trends are present but the positive trend outweighs the negative. Note that all the three studies identify savings and social welfare; further, concentrate on the former rather than on the latter. Bernheim (1999) gives an excellent survey on the topic. Love (2007) analyzes the impact of the age, the matching rate, the vesting policies and the withdrawal penalties on the participation rate. Baily and Kirkegaard (2009, p. 10) emphasize that "[t]the value of the tax breaks given to pensioners is very high in the US ... 1% of the GDP." Börsch-Supan et al. (2008) study the reform of the German system. OECD (2005) provides a useful overview.

Modeling the much more complex British system, Sefton et al. (2008) ask the following question: what is the impact of the introduction of pension credit on other pension savings? According to their model, there was only a small increase, because the increase in the pension savings of the lower-paid induced by the pension credit was almost counterbalanced by the decrease in the pension savings of the higher-paid.

Even more complex models are used by Imrohoroğlu et al. (1998) and Fehr et al. (2008). The latter emphasize the uncertainty of earning paths and longevity, and quantify the reduced quality of insurance following the setting up voluntary pension pillar. Admitting the virtues of these complex models, we still hope that our toy model has its own advantage of being simple.

A theoretical paper of Homburg (2006) considers the problem of rational prodigals, and argues for wage taxes and saving subsidies as a second-best solution.

The papers mentioned above follow an orthodox approach, because they heavily rely on time-consistency: as there is no new information, the workers do not change their saving behavior with the passage of time. The present paper also belongs to this group, it only deviates from orthodoxy by eliminating subjective discounting in the social welfare function.

Less orthodox models (e.g. Laibson, 1998; Diamond and Kőszegi, 2003) employ the hyperbolic discounting when explaining and evaluating the voluntary pension pillar. To give a simple example: some workers plans to pay monthly voluntary contributions of 10 units during 480 months to get additional pension benefit of 20 units during 240 months. But he immediately realizes that if he skips the first month voluntary contribution, then his monthly benefit is only reduced by 0.046 units, therefore he may safely skip the first month. But what happens if he goes on in the second, third etc. month?

Using behavioral economics, Choi et al. (2004) also find a quite unorthodox behavior: if the default option is changed, and the new employees are automatically enrolled into a pension fund, from which they can opt-out, then a much higher share will stay in the voluntary pillar than in the original default. Saez (2009, pp. 204–205) proves experimentally that "details matter ... [in] the take-up of financial incentives for retirement saving." For example, "increasing the effective match rate in the federal saver's credit from 25 percent to 100 percent raised take up by at most 1.3 percentage points."

Even the form of package matters. "[A] randomly selected subset of the treatment group members was presented with a 33 percent credit rebate (cash back) rather than a 50 percent match. While these two subsidies are economically equivalent, previous experiments ... have shown that a match presentation generates a higher take-up than a credit presentation." Being partial equilibrium models, the latter models neglect the tax burden of such schemes.

The structure of the remainder of the present paper is as follows: 2. The model framework. 3. Analytical results. 4. Numerical illustrations. 5. Conclusions.

2. The model framework

In this Section, we outline the model framework. First we determine the optimal voluntary contributions and savings chosen by the individual workers, then we define the welfare provided by various mandatory and voluntary systems.

Maximizing individual utility

We shall make the following extreme, nevertheless meaningful assumptions. The population and the economy are stationary, traditional saving does not yield interest. Every young-aged individual works and every old-aged individual is retired. Every worker is employed for a unit time period and every pensioner enjoys his retirement for a period of length μ, $0 < \mu < 1$. (In practice, the more one earns, the longer he lives on average; and the retirement age depends on the pension system, but here we neglect these relations.) Most existing systems superfluously differentiate between employer's and employee's mandatory contributions, but we assume a unified mandatory contribution. Contrary to practice, we prefer the total wage cost w to gross wages (their difference is the employer's contribution) and we calculate on its basis. Thus we assume that a worker with wage w pays a positive mandatory contribution τw, at least up to a ceiling $w_x > 0$. (The ceiling on the mandatory contributions τw_x will not play any role in this paper, but we display it, to stress its importance in reality, namely the higher the ceiling on mandatory contributions, the lower is the socially optimal ceiling on voluntary contributions.) In addition, the worker with wage w pays an *earmarked tax* θw into the budget, financing the voluntary pensions.

In addition to his wage, the worker has another parameter called *discount factor*: δ, $0 < \delta \leq 1$. We assume that some type (w, δ) prefers additional benefits over the mandatory ones, therefore he pays a *voluntary contribution* r over the mandatory contribution, where $r \in [0, r_x]$, and $r_x \geq 0$ is the *ceiling* on voluntary contribution. The government matches the voluntary contribution r according to a *matching–voluntary contribution* function $a(r)$. As was cited from Saez in the Introduction, this form is economically equivalent to the credit rebate. (Indeed, if the government immediately returns a from the extended voluntary contribution r, then this is equivalent to another system, where the voluntary contribution is only $r - a$ but the government adds *matching* a to the account.)

The pension paid as a life annuity consists of two terms: the earnings-related mandatory benefit $b(w)$ and the voluntary pension $[r + a(r)]/\mu$. (As a matter of fact, voluntary pensions are seldom paid as life annuity, but this is irrelevant here, because we do not discuss the distribution of consumption within the retirement period.)

Finally, there may exist types for whom even the maximal voluntary contribution r_x and the corresponding maximal subsidy a_x are insufficient. These types can *traditionally save* an additional sum, denoted by $s \geq 0$. We assume that the efficiency of this traditional saving is the same as that of the mandatory pillar, i.e. the corresponding life annuity is s/μ. Note that for an optimizing individual, $s > 0$ implies $r = r_x$!

The (intensity of) consumption of a worker and of a pensioner are, respectively

$$c = w - \tau w - \theta w - r - s \quad \text{and} \quad d = b(w) + [r + a(r) + s]/\mu.$$

(Both c and d are positive. Of course, the old-age consumption d means a lifetime pensioner consumption μd.)

We turn to the individual optimization. The *subjective* lifetime utility function of type (w, δ) consists of two terms: (i) the utility $u(\cdot)$ of worker consumption c and (ii) the utility $\mu\delta u(d)$ of the pensioner's consumption d. Here δ is the discount factor. In sum:

$$\hat{Z}(w, \delta, c, d) = u(c) + \mu\delta u(d).$$

The individual determines the pair (voluntary contribution, saving) $[r(w, \delta), s(w, \delta)]$ by maximizing his lifetime utility $\hat{Z}(w, \delta, c, d)$ under the lifetime budget constraint. Partly for the sake of simplicity, partly for bounded rationality, we assume that each worker takes the earmarked tax rate as given, i.e. does not consider the indirect impact of his or others' choices. Substituting the consumption equations into $\hat{Z}$, provides the subjective utility in another form:

$$Z(w, \delta, r, s) = u(w - \tau w - \theta w - r - s) + \mu\delta u(b(w) + [r + a(r) + s]/\mu).$$

The worker determines his *optimal* voluntary contribution $\tilde{r}$ and saving $\tilde{s}$ by taking the partial derivatives with respect to decisions r and s. (To avoid lengthy notations, we shall rarely use tilde for the optimum.) We must take into account the possibility of corner solutions. We assume that $b(w)$ and $a(r)$ are increasing concave functions, at least in the intervals $w_m \leq w \leq w_x$ and $0 \leq r \leq r_x$, respectively, where w_m is the minimal wage. Moreover, $b(0) \geq 0$ and $a(0) = 0$. To minimize the number of cases, for the time being, we assume that $b(w)$ and $a(r)$ are smooth functions. Here are the cases to be distinguished:

Zero voluntary contribution, zero saving, $r = 0$, $s = 0$:

$$Z'_r(w, \delta, 0, 0) = -u'(c) + \delta u'(d)[1 + a'(0)] \leq 0.$$

Positive voluntary contribution below ceiling, zero saving, $0 < r < r_x$, $s = 0$:

$$Z'_r(w, \delta, r, 0) = -u'(c) + \delta u'(d)[1 + a'(r)] = 0.$$

Maximal voluntary contribution, zero saving, $r = r_x$, $s = 0$:

$$Z'_s(w, \delta, r_x, 0) = -u'(c) + \delta u'(d) \leq 0.$$

Maximal voluntary contribution, positive saving, $r = r_x$, $s > 0$:

$$Z'_s(w, \delta, r_x, s) = -u'(c) + \delta u'(d) = 0.$$

Macro framework

In our model, workers have two characteristics: w and δ. We assume that their joint probability distribution is given by $(f_i)_{i=1}^I$ (possibly $i = (j,k)$) on the grid-points of the rectangle $w_m \le w \le w_x$ and $\delta_m \le \delta \le \delta_x$.

We assume that the government operates uniform contribution and tax rates τ and θ, where total mandatory contributions cover the total mandatory pension expenditures, while the earmarked taxes finance the subsidies. In formula:
Balance of the mandatory pensions

$$\sum_{i=1}^{I} f_i[\tau w_i - \mu b(w_i)] = 0.$$

Balance of the voluntary transfers

$$\sum_{i=1}^{I} f_i[\theta w_i - a(r(w_i, \delta_i))] = 0,$$

where $T_i = a(r(w_i, \delta_i)) - \theta w_i$ is the voluntary transfer *received* by type i. We also need the total savings, i.e. the aggregate traditional savings plus the aggregate voluntary savings, including the matching:

$$S = \sum_{i=1}^{I} f_i[s(w_i, \delta_i) + r(w_i, \delta_i) + a(r(w_i, \delta_i))].$$

Social welfare function

We also assume that the country is managed by a benevolent government which selects among various systems as to maximize an appropriately defined social welfare function. First of all, it removes discounting, and replaces subjective with *objective* utility functions:

$$U(w_i, \delta_i, c_i, d_i) = u(c_i) + \mu u(d_i).$$

(Note that U is independent of δ_i but to signal the second characteristic of the individual in aggregation, we still keep δ_i.)

The utilitarian social welfare function is the average of the individual objective utility functions, taken at the optima:

$$V = \sum_{i=1}^{I} f_i U(w_i, \delta_i, \tilde{c}_i, \tilde{d}_i).$$

If the government has a more egalitarian preference, it can choose a strictly concave scalar–scalar function ψ, and rely on a generalized utilitarian social welfare function:

$$V = \sum_{i=1}^{I} f_i \psi(U(w_i, \delta_i, \tilde{c}_i, \tilde{d}_i)).$$

The government looks for a mandatory contribution rate τ, an earmarked tax rate θ, and a pair of benefit and matching functions $b(\cdot), a(\cdot)$, which maximize the social welfare function under the budget constraints or more modestly, it selects among various systems on the basis of social welfare.

3. Analytical results

In this Section we shall outline some preliminaries, and then compare three systems mentioned in the Introduction: (i) the pure mandatory system, (ii) the asymmetric system, where only the savers participate in the voluntary pillar, (iii) the symmetric system, where both types' contribution rates are equal.

Preliminaries

We shall work with homogeneous linear benefit and matching functions with ceilings. Bounded homogeneous linear benefit–wage-function

$$b(w) = \beta \min(w, w_x),$$

where $\beta > 0$ is the gross *replacement ratio*.
Bounded homogeneous linear matching–voluntary contribution function

$$a(r) = \alpha \min(r,\ r_x),$$

where r_x is the *voluntary contribution's ceiling*, α is the *matching rate*, $a_x = \alpha r_x$ is the subsidy's ceiling. Then $a(r) = \min(\alpha r,\ a_x)$.

In the continuation, it is useful to apply a simple utility function, namely CRRA: $u(c) = \sigma^{-1} c^{\sigma}$, where $\sigma < 0$. As a special limiting case ($\sigma = 0$), Cobb–Douglas: $u(c) = \log c$ can also be very useful.

Since $u'(c) = c^{\sigma-1}$, therefore for the interior optimal consumption pair with matching, we have

$$c^{\sigma-1} = \delta(1+\alpha)d^{\sigma-1}, \quad \text{i.e.} \quad d = [\delta(1+\alpha)]^{1/(1-\sigma)}c.$$

We shall need the ratio of the optimal old- and young-age consumption:

$$\gamma(\delta, \alpha) = [\delta(1+\alpha)]^{1/(1-\sigma)}.$$

With this notation, the optimum condition reduces to

$$d = \gamma(\delta, \alpha)c \quad \text{with} \quad s \geq 0$$

and

$$d \geq \gamma(\delta, \alpha)c \quad \text{with} \quad s = 0.$$

For the homogeneous linear case (without ceilings), the balance equations are also simple: for example, $\mu\beta = \tau$.

As a start, we shall first analyze the pure mandatory pension system (unaccompanied by a voluntary pension pillar). We shall use notation x_+ for the positive part of the real number x: $x_+ = x$ if $x \geq 0$, 0 otherwise.

Theorem 1. *A mean discount factor δ^{o} in a pure mandatory pension system implies a mandatory contribution rate*

$$\tau = \frac{\gamma(\delta^{\mathrm{o}}, 0)}{\gamma(\delta^{\mathrm{o}}, 0) + \mu^{-1}}$$

with optimal consumption pairs and traditional saving

$$c = \frac{w}{1 + \mu\gamma(\delta^{\mathrm{o}}, 0)}, \quad d = \frac{\gamma(\delta^{\mathrm{o}}, 0)w}{1 + \mu\gamma(\delta^{\mathrm{o}}, 0)}, \quad s = \frac{\mu[\gamma(\delta, 0) - \gamma(\delta^{\mathrm{o}}, 0)]_+ w}{(1 + \mu\gamma(\delta^{\mathrm{o}}, 0))(1 + \mu\gamma(\delta, 0))}.$$

Proof. It is obvious that mandatory saving is d/μ, i.e. denoting the mandatory saving ratio by τ, the government can be represented by a discount factor $\delta^{\mathrm{o}} < 1$. If needed, we shall write $\tau = \tau(\delta^{\mathrm{o}})$. We could also write $\tau^{\mathrm{o}} = \tau(\delta^{\mathrm{o}})$, but it would be cumbersome.

The type (w, δ) will then choose the subjectively optimal consumption pair and traditional saving given above. (For discount factors lower than the mean, there would be no traditional saving at all.) ∎

Example 1. If the government sets its discount factor to 1, (first-best solution), then

$$c^* = \frac{w}{1 + \mu}, \quad d^* = \frac{w}{1 + \mu}, \quad \tau^* = \frac{1}{1 + \mu^{-1}}.$$

If the mandatory contribution rate is too high, implying little or no traditional saving, then the workers may restrain their labor supply or underreport their actual earnings. If the mandatory contribution rate is too low, then workers with low discount factor will have unacceptably low old-age consumption. As a compromise, the government sets a medium mandatory contribution rate and introduces a voluntary pension pillar, the subsidy of which is financed by an earmarked tax rate θ, which covers the resulting subsidies: $\theta\bar{w} = \alpha\bar{r}$, where average wage is $\bar{w}$. The government's hope is that at least some type will increase its total saving.

From now on we give up the assumption of pure mandatory system.

Inserting the consumption functions into the optimality conditions, after rearrangement, for any given θ, we obtain an optimum for each case. Four cases are to be distinguished.

Theorem 2. *For any given earmarked tax rate θ, the optimal solutions are classified as follows:*

Zero voluntary contribution, zero saving if

$$\beta > \gamma(\delta, \alpha)(1 - \tau - \theta).$$

Positive voluntary contribution, zero saving:

$$r = \frac{\gamma(\delta, \alpha)(1 - \tau - \theta) - \beta}{\gamma(\delta, \alpha) + \mu^{-1}(1 + \alpha)} w.$$

Maximal voluntary contribution, zero saving

$$\frac{\gamma(\delta,0)(1-\tau-\theta)-\beta}{\gamma(\delta,0)+\mu^{-1}}w \le r_x < \frac{\gamma(\delta,\alpha)(1-\tau-\theta)-\beta}{\gamma(\delta,\alpha)+\mu^{-1}(1+\alpha)}w,$$

Maximal voluntary contribution, positive saving

$$r = r_x \quad \text{and} \quad s = \frac{\gamma(\delta,0)(1-\tau-\theta)w - \beta w - [\gamma(\delta,0)+\mu^{-1}(1+\alpha)]r_x}{\gamma(\delta,0)+\mu^{-1}}.$$

Proof. We discuss the four cases one after the other.

(i) Inserting equations $d = \beta w$ and $c = (1-\tau-\theta)w$ into inequality $d > \gamma(\delta,\alpha)c$, yields

$$d = \beta w > \gamma(\delta,\alpha)(1-\tau-\theta)w.$$

determining domain 1 in the (w,δ)-plane, regardless of the wage.

(ii) Inserting equations $d = \beta w + (1+\alpha)r/\mu$ and $c = (1-\tau-\theta)w - r$ into $d = \gamma(\delta,\alpha)c$, yields the optimal voluntary contribution, assuming $0 \le r \le r_x$, defining domain 2, depending on the wage.

(iii) Inserting the equations into the inequality yields $\gamma(\delta,0)c \le d < \gamma(\delta,\alpha)c$, defining domain 3.

(iv) Inserting equations $d = \beta w + [(1+\alpha)r_x + s]/\mu$ and $c = (1-\tau-\theta)w - r_x - s$ into equation $d = \gamma(\delta,0)c$, yields the optimal saving. We must require $s \ge 0$, otherwise the worker would pay his voluntary contribution from credit. We have obtained domain 4.∎

Three systems

To compare the three pension systems (i)–(iii), we confine the in-depth analysis to the two-type case. Notation of types: L and H, relative frequencies f_L and f_H, wages w_L and w_H, and pensions $b_L = \beta w_L$ and $b_H = \beta w_H$ and with increasing discount factors: $0 < \delta_L < \delta_H < 1$. We shall call the types *myope* (L) and *saver* (H). Since in real life, typically the myopes' earning is less than or equal to the savers', we assume $w_L \le w_H$. As a normalization, we also assume that the average wage is unity: $f_L w_L + f_H w_H = 1$. We assume that the government chooses its discount factor between the two types': $\delta_L < \delta^{\text{o}} < \delta_H$.

Pure mandatory system: $\alpha = 0$

We reformulate Theorem 1 for the two-type case.

Theorem 1.* *The optimal consumption pair and the traditional saving in the proportional pure mandatory system are as follows:*

$$c_L = \frac{w_L}{1+\mu\gamma(\delta^{\text{o}},0)}, \quad d_L = \frac{\gamma(\delta^{\text{o}},0)w_L}{1+\mu\gamma(\delta^{\text{o}},0)}, \quad s_L = 0$$

and

$$c_H = \frac{w_H}{1+\mu\gamma(\delta^{\text{o}},0)}, \quad d_H = \frac{\gamma(\delta^{\text{o}},0)w_H}{1+\mu\gamma(\delta^{\text{o}},0)}, \quad s_H = \frac{\mu[\gamma(\delta_H,0)-\gamma(\delta^{\text{o}},0)]w_H}{(1+\mu\gamma(\delta^{\text{o}},0))(1+\mu\gamma(\delta_H,0))}.$$

Since d_L is too low, the government sets up tax-favored pension funds with a matching rate $\alpha > 0$, and ceiling $r_x > 0$ on the voluntary contributions. Rather than considering all the possibilities, we shall only discuss two special cases, to be called asymmetric and symmetric voluntary systems.

Asymmetric system

To simplify the calculations, first we assume that the matching rate α is so low or the mean discount factor δ^{o} is so high that the myopes do not participate at the voluntary pensions: $\delta_L(1+\alpha) \le \delta^{\mathrm{o}}$: *asymmetric system.*

There is another practical constraint: the savers do *not* pay too high voluntary contribution, i.e. their young-age consumption is higher than their old-age consumption: $c_H \ge d_H$, i.e. $\delta_H(1+\alpha) \le 1$. On the other hand, since $\delta^{\mathrm{o}} < \delta_H$, the savers always contribute to the voluntary pillar. Let us assume that the ceiling is so high that the savers' voluntary contribution is lower than the ceiling: $0 < r_H < r_x$, i.e. $s_H = 0$. We formulate

Theorem 3. *If the mean discount factor is high enough: $\delta_L(1+\alpha) \le \delta^{\mathrm{o}}$ and the ceiling r_x is high enough:*

$$r_x > r_H(\delta^{\mathrm{o}}) = \frac{[\gamma(\delta_H,\alpha)(1-\tau) - \mu^{-1}\tau]w_H}{\gamma(\delta_H,\alpha)(1+f_H\alpha w_H) + (1+\alpha)\mu^{-1}},$$

then H's optimal voluntary contribution r_H is equal to $r_H(\delta^{\mathrm{o}})$, while $s_H = 0$.

Proof. The interior optimality condition holds for H: $d_H = \gamma(\delta_H,\alpha)c_H$.

Then the earmarked tax balance is very simple: $\theta = f_H\alpha r_H$. Therefore $c_H = (1-\tau)w_H - (1+\alpha f_H w_H)r_H$ and $d_H = \beta w_H + (1+\alpha)r_H/\mu$. Substituting c_H and d_H into H's optimum condition:

$$\beta w_H + (1+\alpha)r_H/\mu = \gamma(\delta_H,\alpha)[(1-\tau)w_H - (1+f_H\alpha w_H)r_H].$$

After rearrangement, we have the voluntary contribution. ∎

Remark. It is obvious that the bill of savers' 'perfection' is partly paid by the myopes, their young-age consumption is reduced, while their old-age consumption remains invariant:

$$c_L = \frac{w_L}{1+\mu\gamma(\delta^{\mathrm{o}},0)} - \alpha f_H r_H w_L \quad \text{and} \quad d_L = \frac{\gamma(\delta^{\mathrm{o}},0)w_L}{1+\mu\gamma(\delta^{\mathrm{o}},0)}.$$

Typically the welfare provided by the asymmetric voluntary system is close to that of the pure mandatory one.

Symmetric system

Before discussing the third system, let us introduce type i's *voluntary contribution rate* ρ_i: $r_i = \rho_i w_i$, $i = L, H$.

In comparison to the asymmetric system, it seems to be more appropriate if the government sets such a low ceiling and such a high matching rate that both voluntary contribution rates are equal: $\rho_L = \rho_H = \rho$ and H's voluntary contribution reaches the ceiling: $\rho w_H = r_x$. We shall call this system *symmetric.*

Theorem 4. *a) If the matching rate is moderate:* $(1+\alpha)\delta_H \le 1$ *and the mean discount factor* δ^{o} *is also moderate:*

$$\delta_L \le \delta^{\mathrm{o}} \le \delta_0 = (1+\alpha)\delta_L,$$

then the optimal voluntary contribution ratio is equal to

$$\rho(\tau) = \frac{\gamma(\delta_L,\alpha)}{\gamma(\delta_L,\alpha)+\mu^{-1}} - \frac{\tau}{1+\alpha},$$

and the voluntary contribution's ceiling is reached: $r_x = \rho(\tau)w_H = r_H$*, while H's traditional saving ratio is equal to*

$$\frac{s_H}{w_H} = \frac{\gamma(\delta_H,0)}{\gamma(\delta_H,0)+\mu^{-1}} - \tau - (1+\alpha)\rho(\tau)$$

b) For a moderate fixed contribution rate (see in Corollary 1), the symmetric system is welfare-superior to the pure mandatory system.

Proof. a) In the symmetric system, $\theta = \alpha\rho$, hence L's optimum condition

$$\mu^{-1}[\tau + (1+\alpha)\rho] = \gamma(\delta_L,\alpha)[1-\tau-(1+\alpha)\rho]$$

yields the optimal voluntary contribution rate, which in turn yields the ceiling on voluntary contributions.

To determine H's traditional saving, substitution into $d_H = \gamma(\delta_H,0)c_H$ yields

$$\mu^{-1}\{[\tau+(1+\alpha)\rho]w_H + s_H\} = \gamma(\delta_H,0)\{[1-\tau-(1+\alpha)\rho]w_H - s_H\}.$$

Solving for s_H/w_H, gives the result.

b) The optimal solution of type H is the same in the symmetric voluntary system as in the pure mandatory system, because the earmarked tax and the voluntary contribution are financed from the reduction of traditional savings. The optimal solution of type L in the former consists of higher old-age consumption and lower young-age consumption than does the latter, but preserving their order and the their weighted sum. Thus the objective utility of the symmetric voluntary system is higher than that of the pure mandatory one. ∎

We continue with

Conjecture 1. *For a sufficiently low mean discount factor* δ^{o}*, the asymmetric voluntary system provides a social welfare close to the pure mandatory one's.*

Finally, we formulate an interesting corollary to Theorem 2, which outlines the equivalence between various combinations of mandatory and symmetric voluntary systems. Let τ_0 correspond to δ_0 (pure mandatory system) and let τ_L stand for the minimal mandatory contribution rate consistent with nonnegative traditional saving intensions, for given α and δ^{o}.

Corollary 1. *Under the condition of Theorem 2, adding a symmetric voluntary system with*

$$\rho[\tau]=\frac{\tau_0-\tau}{1+\alpha}\quad\text{and}\quad r_x[\tau]=\rho[\tau]w_H$$

to the mandatory pillar provides essentially the same solution, i.e. the combinations $(\tau,\rho[\tau])$ *are equivalent in the interval* $\tau_L\le\tau\le\tau_0$.

Proof. Note that $\rho(\tau)=\rho[\tau]$. ∎

Remark. Contrary to Theorem 4b, here the mandatory contribution rate τ varies together with the parameters of the symmetric voluntary system.

4. Numerical illustration

We continue our analysis with numerical illustrations. We assume that the time spent at retirement is half as long as that of working: $\mu=0.5$ and the CRRA parameter is $\sigma=-1$. Basically we follow the logic of the previous section.

For the time being, we assume that every worker has a unit total wage and we vary the discount factor and the ceiling on mandatory contributions to study their impacts.

As a baseline case, we calculate the optimal consumption pairs plus the mandatory contribution rate for four corresponding discount factors. Each case has a name, two have an abbreviations: myope (L) and saver (H), and two have a symbol: o (mean)and government (*). Table 1 presents the optimal young- and old-age consumption and the saving.

Table 1. *Discounting and optimal consumption pair: no matching*

Type i	Discounting factor δ_i	Worker consumption c_i	Pensioner consumption d_i	Pension-saving rate τ_i
Myopic (L)	0.15	0.838	0.324	0.162
Mean (°)	0.225	0.808	0.383	0.192
Saver (H)	0.5	0.739	0.522	0.261
Government (*)	1	0.667	0.667	0.333

Remark: $w=1$. We display $10U+100$ rather than U.

Table 1 displays that the lower the discount factor, the higher is the worker consumption and the lower is the pensioner consumption, and the corresponding saving or mandatory contribution rate. (The value of the consumption ratio depends on the exponent of the utility function, σ. The higher the absolute value of σ, the higher is the ratio of the pensioner's consumption to the worker's.)

From now on we move on to the two-type case, with relative frequencies $f_L=2/3$ and $f_H=1/3$, wage rates $w_L=1/2$ and $w_H=2$, yielding $\bar{w}=1$. We assume different discount factors $\delta_L = 0.15$, $\delta_H = 0.5$ (first and third rows in Table 1). The government chooses $\delta = 0.225$, i.e. the corresponding medium mandatory contribution rate is $\tau = 0.192$ (second row in Table 1).

Table 2 displays a desaggregated picture. The last column contains the efficiency of the system in terms of the pure mandatory one, where e defines the real number, by which multiplying the wages, the modified pure mandatory system becomes welfare equivalent to the voluntary system (either asymmetric or symmetric).

Table 2. *Comparison of mandatory and voluntary pensions*

Earning w_i	Voluntary contri- bution r_i	Traditi- onal saving s_i	Worker consumption c_i	Pensioner consumption d_i	Volun- tary transfer T_i	Life- time utility U_i	Efficiency e
Pure mandatory system ($\alpha = 0$)							1
0.5	0	0	0.409	0.183	0	48.167	
2.0	0	0.157	1.478	1.045	0	88.447	
Asymmetric voluntary system ($\alpha = 0.5$)							0.989
0.5	0	0	0.391	0.192	–0.013	48.372	
2.0	0.152	0	1.413	1.224	0.02	88.840	
Symmetric voluntary system ($\alpha = 1$)							1.038
0.5	0.006	0	0.393	0.215	0	51.265	
2.0	0.023	0.092	1.478	1.045	0	88.447	

The pure mandatory system is only displayed as a benchmark, with relative efficiency 1. Note the unacceptably low old-age consumption of the myope: $d_L = 0.183$.

The mandatory pillar with an asymmetric voluntary pillar only makes things a little bit worse because the matching rate is too low to help the myope ($\alpha = 0.5$) and the ceiling on voluntary contribution is high enough ($r_x = 0.152$) to allow the saver to appropriate the benefits. The earmarked tax rate creates a net transfer from the myopes to the savers. The young-age consumption of the former slightly diminishes, just to help raise the savers' old-age consumptions. The pure mandatory pillar can achieve the same social welfare as the asymmetric voluntary system with 1.1 percent lower wages.

The mandatory pillar with a symmetric voluntary pillar redresses the injustice: the matching rate is raised to 1, while the ceiling is lowered to 0.023. Now the myope's old-age consumption rises from $d_L = 0.183$ to 0.215, young-age consumption drops from $c_L = 0.409$ to 0.393, while c_H and d_H remain invariant. The pure mandatory pillar can achieve the welfare of the symmetric voluntary system by increasing wages uniformly by 3.8 percent.

In harmony with Conjecture 1 and Theorem 4b, the social welfare provided by the pure mandatory and asymmetric voluntary systems are close too each other, and are dominated by the symmetric voluntary system.

Finally, in the interval $0.15 \le \delta^o \le 0.3$, defining $\tau(\delta^o)$, $\rho(\tau)$ decreases, while the outcome remains the same.

5. Conclusions

We have constructed a simple overlapping generations model, where in addition to the proportional (contributive) mandatory system, there is a tax-favored proportional retirement pillar, financed from earmarked taxes. The voluntary contribution and the traditional saving are determined by the workers maximizing their subjective utility functions, while the corresponding earmarked tax rate and the ceiling on voluntary contributions are calculated by the government. In our "general equilibrium" model, we have done the first theoretical and numerical calculations. The proportional tax-favored pillar with high ceiling and low matching (resembling Hungary) is poorly targeted, when the mandatory pillar is also proportional and generous: it helps just those who do not need this help, asymmetry. It is socially more attractive to diminish radically the ceiling and enhance the matching: symmetry. The results seem to be acceptable but a lot of further analytical arguments and numerical trials are needed to confirm our tentative deductions. For progressive mandatory or voluntary pillars, the evaluation will be different. Further complications arise if behavioral anomalies are taken into account.

References

Baily, M.N. and Kirkegaard, J.F. (2009): *US Pension Reform: Lessons from other Countries,* Petersons Books.

Bernheim, B. D. (1999): Taxation and Saving, NBER WP 7061, *Auerbach, A.J. and Feldstein, M. (2002)*: Handbook of Public Economics, Amsterdam, Elsevier.

Börsch-Supan, A.; Reil-Held, A. and Schunk, D. (2008): "Saving Incentives, Old-age Provision and Displacement Effects: Evidence from the Recent German Pension Reform", *Journal of Pension Economics and Finance 7*, 295–319.

Choi, J., Laibson, D., Madrian, B. and Metrick, A. (2004): "For Better or Worse: Default Effects and 401(k) Saving Behavior", Wise, D. ed. (2004): *Perspectives in the Economics of Aging, Chicago, University of Chicago Press* 81–121.

Diamond, P. and Kőszegi, B. (2003): "Quasi-hyperbolic Discounting and Retirement", *Journal of Public Economics, 87*, 1839–1872.

Engen, E. M., Gale, W. G. and Scholz, J. (1996): "The Illusory Effects of Savings Incentives on Saving", *Journal of Economic Perspectives 10:4*, 111–138.

Fehr, H., Habermann, C. and Kindermann, F. (2008): "Tax-Favored Retirement Accounts: Are they Efficient in Increasing Savings and Growth?, *FinanzArchiv, Public Finance Analysis 64*, 171–198.

Feldstein, M. S. (1987): "Should Social Security Means Tested?", *Journal of Political Economy 95*, 468–484.

Homburg, S. (2006): "Coping with Rational Prodigal: A Theory Social Security and Saving Subsidies", *Economica 73,* 47–58.

Hubbard, R. G. and Skinner, J. S. (1996): "Assessing the Effectiveness of Saving Incentives", *Journal of Economic Perspectives 10:4*, 73–90.

Imrohoroğlu, A., Imrohoroğlu, S and Joines, D. H. (1998): "The Effect of Tax-favored Retirement Accounts on Capital Accumulation", *American Economic Review 88*, 749–768.

Laibson, D. (1998): "Life-Cycle Consumption and Hyperbolic Discount Functions", *European Economic Review 42*, 861–871.

Love, D. A. (2007): "What can the Life-cycle Model Tell us about 401(k) Contributions and Participation?", *Journal of Pension Economics and Finance 6*, 147–185.

Matits, Á. (2008): "Voluntary Pension Funds: The Third Pillar", Gál, R., Iwasaki, I. and Széman, Zs. eds. (2008): *Assessing Intergenerational Equity*, Budapest, Akadémiai Publisher, 111–135.

OECD (2005): *Tax-Favored Retirement Saving*, OECD Economic Studies, 39, Paris.

Poterba, J. M., Venti, S. F. and Wise, D. A. (1996): "How Retirement Savings Program Increase Savings", *Journal of Economic Perspectives 10:4*, 91–112.

Saez, E. (2009): "Details Matter: The Impact of Presentation and Information on the Take-up of Financial Incentives for Retirement Savings", *American Economic Journal: Economic Policy 1:1*, 204–228.

Sefton, J., van de Ven, J. and Weale, M. (2008): "Means Testing Retirement Benefits: Fostering Equity or Discourageing Saving?", *Economic Journal, 118*, 556–590.

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